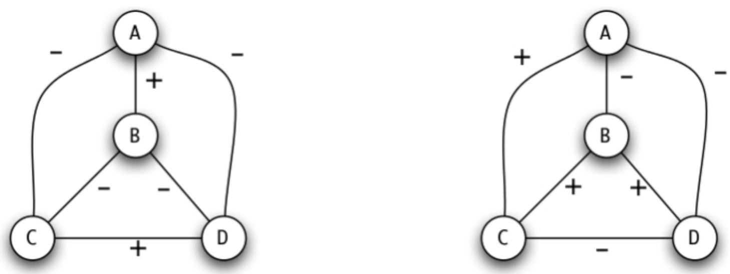
Homophily (Continued) and Positive and Negative relationships

Social Networks - July 2020

MCQ Assignment - Week 5

1. Assuming $-$ indicating *positive* relationship and $+$ indicating *negative* relationship, which of the following graphs is/are balanced?



(a) (b)

A. Only (b)
B. Neither (a) nor (b)
C. Both (a) and (b)
D. Only (a)

ANSWER: B
A graph is said to be balanced if all its triangles are balanced. In graph (a): the three triangles have parity (-, +, -), (-, +, -) and (-, -, +) respectively. This indicates that all the triangles have one negative edge (interchanged), which means they all are imbalanced. Hence graph (a) is not balanced. In graph (b): the three triangles have parity (+, -, +), (-, -, +) and (+, +, -) respectively. This indicates that two out of three triangles have 1 positive edges, and the other one is having one negative relationship which means it is not balanced. Since all the triangles of graph (b) are not balanced, this graph will be imbalanced.

2. Count the number of unstable triangles in the graphs shown in Figure 1, where solid edges indicate positive ties and dotted edges indicate negative ties.

A. 8,5,6
B. 8,4,6
C. 7,3,7
D. 7,3,6

ANSWER: D
For figure 1(a) we count the number of unstable triangles and observe that out of the total possible triangles (10), there are 7 triangles which are unstable. Figure 1(b) is the complement of figure 1(a), in

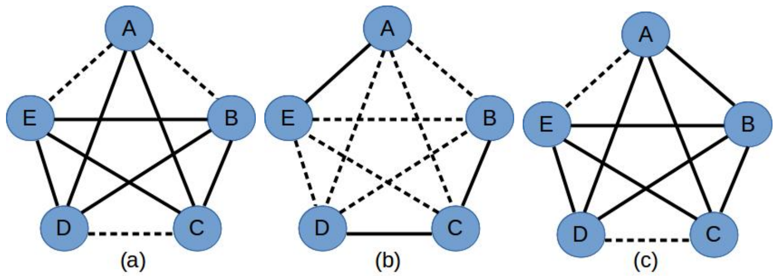


Figure 1: The graphs. Solid edges indicate positive ties and dotted edges indicate negative ties.

the sense that the positive and negative edges are inverted. Thus the number of unstable triangles can be directly concluded to be equal to 3 (=10-7). Figure 1(c) has 6 unstable triangles, obtained through counting.

3. Can we have a complete signed graph on 4 nodes (K_4) and 6 nodes (K_6) respectively, each having exactly one unstable triangle?

A. K_4 - Yes K_6 - Yes
B. K_4 - No K_6 - No
C. K_4 - No K_6 - Yes
D. K_4 - Yes K_6 - No

ANSWER: B
The proof structure is as follows.
We begin by first showing that it is not possible to have a single unstable triangle in K_4. This would imply that the same holds good for K_6 as well, since K_4 is a proper subgraph of a K_6 graph. Therefore, a possibility of such a construction for K_5 would imply a possibility for K_4, which we would have proved to be impossible.
Part A: Showing the impossibility in K_4 Let us assume that it is possible to have a labeled K_4 graph with only one unstable triangle. Without loss of generality, let us assume that the triangle ABC is the only instable one. The triangle could either be labeled as shown in Figure 2(a) or as in Figure 2(b).

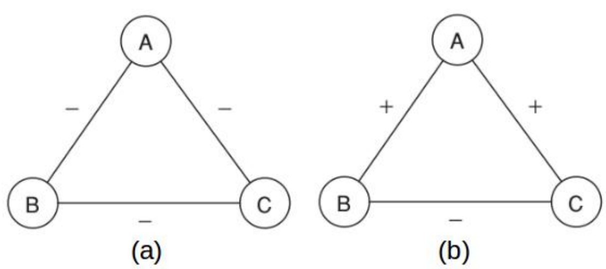


Figure 2: Signed Network

Now the aim would be to introduce the last node D and label the remaining edges DA, DB, DC such that no more unstable triangles are created in both the cases.

Case 1: triangle ABC is labeled as in Figure 2(a)

Let us consider the triangle DAB. Since one of the edges is negative (AB), for the triangle to be stable, one of the remaining edges must be labeled positive while the other negative. Let us say DA was negative and DB positive. Then a similar argument in triangle DAC would ensure that the edge DC is labeled positive. This would make the triangle DBC also unstable, increasing the number of unstable triangles. It can be seen that the same problem persists in case we had begun by labeling DA positive instead. Both these sub-cases are illustrated in Figure 3.

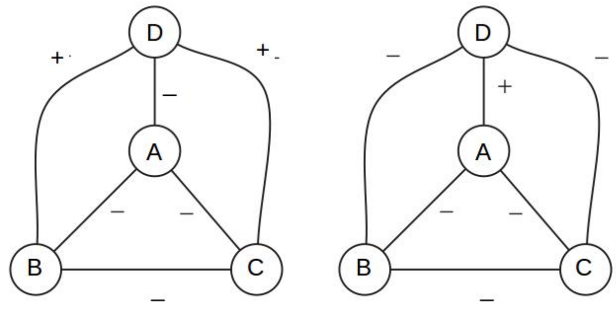


Figure 3

Case 1: triangle ABC is labeled as in figure 2(b)
An argument similar to Case 1 can be made. Let us consider the triangle DAB. Since one of the edges is positive (AB), for the triangle to be stable, both of the remaining edges must be labeled positive or both negative. Let us say DA, DB were negative. Then a similar argument in triangle DAC would ensure that the edge DC is labeled negative. This would make the triangle DBC also unstable (with all edges negative), increasing the number of unstable triangles. It can be seen that the same problem persists in case we had begun by labeling DA, DB positive instead. Both these sub-cases are illustrated in Figure 4

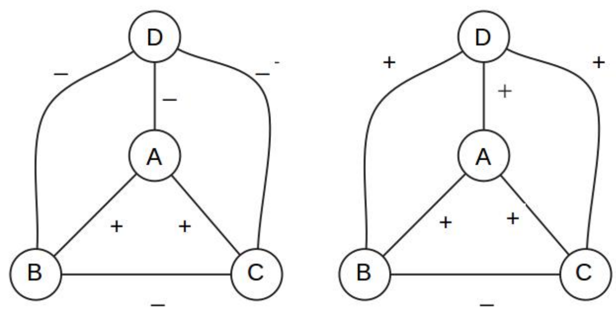


Figure 4

4. Does the network given in Figure 5 exhibit homophily? (Nodes are divided into two types represented by different colors)

   A. Yes
   B. No

ANSWER: B
5 of the 18 edges in the graph are cross-gender. Since $p = 2/3$ and $q = 1/3$ in the above network,

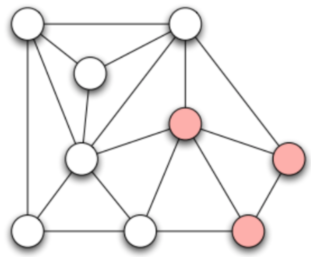


Figure 5: The network

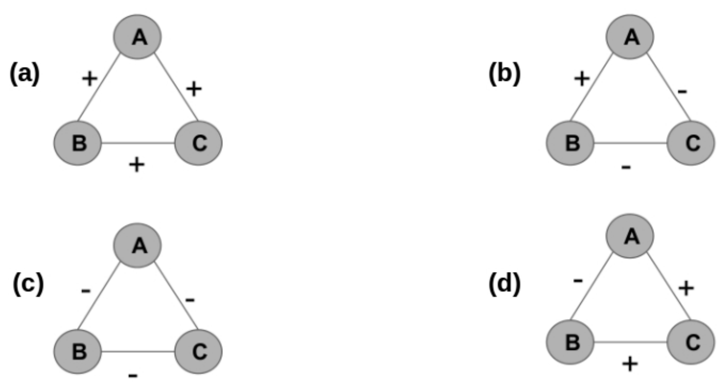
compare the fraction of cross-gender edges to the quantity $2pq = 4/9 = 8/18$. As $8/18 > 5/18$, hence, the network does not exhibit homophily.

5. A signed triangular network is stable if it has .... +ve relationships.

A. only 2
B. no
C. odd number of
D. even number of

ANSWER: C
A signed triangular network is stable if it has odd number of positive relationships.

6. A complete graph is 'weakly balanced' if there is not even a single triangle of which of the following type?:



A. (a)
B. (b)
C. (c)
D. (d)

ANSWER: D
Weak Structural Balance Property: There is no set of three nodes such that the edges among them

consist of exactly two positive edges and one negative edge. Hence (d) is not allowed to be present in the graph.

7. Given a threshold of 4 in Schelling's model configuration shown in Figure 6, how many agents are satisfied?

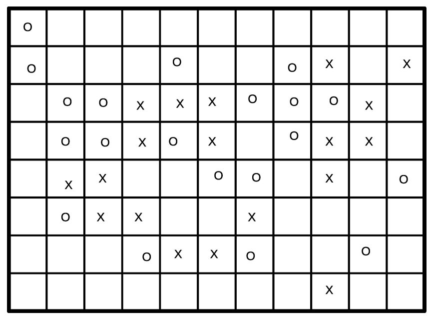
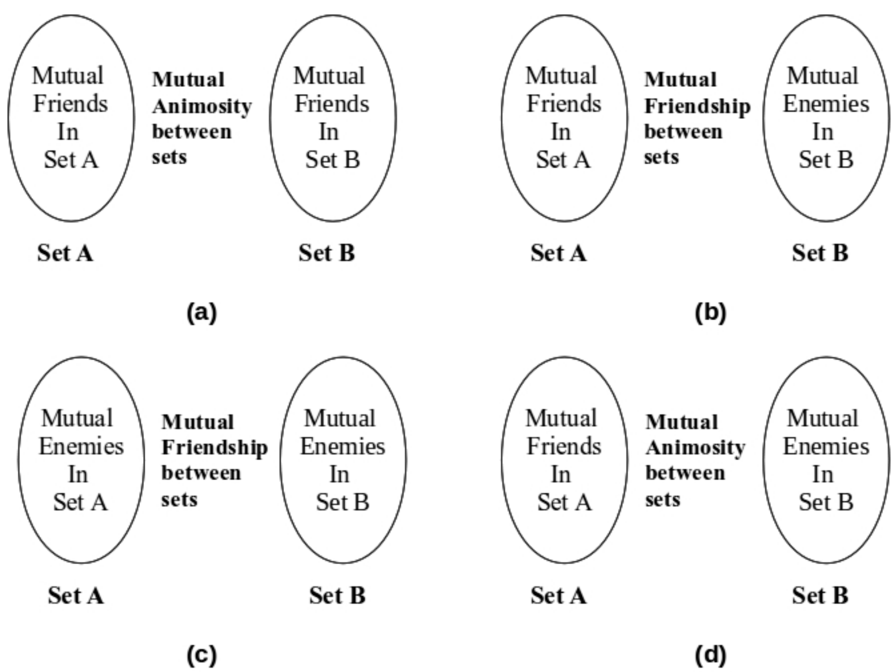


Figure 6: Schelling's Model

A. 3
B. 4
C. 5
D. 6

ANSWER: C
5 agents satisfied.

8. Which of the following structures of graphs is/are balanced?



A. Both (a) and (c)
B. Only (a)
C. Only (b)
D. Both (b) and (d)

ANSWER: B
A graph that is balanced, can be divided into two components such that all the nodes inside one component are friends to each other, all the nodes in the second component are also friends to each other, however, the nodes in the first component are enemies to the nodes in the second component. Hence, only the first graph is balanced.