Cascades

Social Networks - July 2020
MCQ Assignment - Week 7

1. Consider two actions A and B. The payoff associated with the action A is 20 while the payoff associated with action B is 5. In such a case, what is the threshold fraction of neighbors that should have adopted A, in order for a node to adopt the action A?

   A. 1/3
   B. 1/4
   C. 1/5
   D. 1/3

   ANSWER: C
   Let a be the payoff for product A and b be the payoff for product B. We have seem that in such a case, the threshold on the fraction of neighbors which should adopt A for it to be adopted by a node, q = b/(b+a) = 5/5+20 = 1/5.

2. Assume that the actions A and B yield every player a payoff of a and b. Further assume that there are two friends Ram and Rahim; Ram decides to adopt action A while Rahim decides to adopt action B. What are the payoffs that they get?

   A. Ram- 0, Rahim- 0
   B. Ram- a, Rahim- b
   C. Ram- a, Rahim- 0
   D. Ram- 0, Rahim- b

   ANSWER: A
   Both Ram and Shyam have a choice between two possible behaviors, labeled A and B. There is an incentive for them to have their behaviors match. The payoffs are defined as follows.

   1. if Ram and Shyam both adopt behavior A, they each get a payoff of $a > 0$;
   2. if they both adopt B, they each get a payoff of $b > 0$; and
   3. if they adopt opposite behaviors, they each get a payoff of 0.

3. Consider a set of initial adopters of behavior A, with a threshold of q for nodes in the remaining network to adopt behavior A. Given the following two statements,

   1. **Statement 1**: If the remaining network contains a cluster of density greater than $1 - q$, then the set of initial adopters will not cause a complete cascade.
   2. **Statement 2**: Whenever a set of initial adopters does not cause a complete cascade with threshold q, the remaining network must contain a cluster of density greater than $1 - q$.

   Choose the correct option from the following.

   A. Both Statement 1 and Statement 2 are true.
   B. Both Statement 1 and Statement 2 are false.
   C. Statement 1 is true but Statement 2 is false.
   D. Statement 2 is true but Statement 1 is false.

ANSWER: A
There can not exist a complete cascade on a network iff there exists a cluster of density $1-q$.

4. In the network shown in Figure 1, what is the density of the cluster comprised by the set of nodes $\{5, 6, 7, 8\}$?

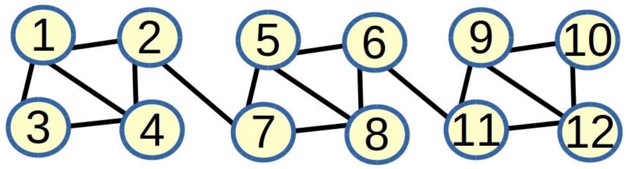

Figure 1: The network

A. 1/3
B. 1/2
C. 2/3
D. 3/2

ANSWER: C
We say that density of a cluster is p if each node in the cluster(given set) has at least a p fraction of its network neighbors in the cluster(the set). For the given set, nodes 5 and 8 have 1 fraction of their neighbors in the same set, nodes 6 and 7 has 2/3 fraction of its neighbors in the same set. Overall, every node in this set has at least 2/3 fraction of their neighbors in the same set. Hence, the density of the cluster is 2/3.

5. Given a network as shown in Figure 2. Assume that initially every node in this network has adopted behavior B. Next, a new behavior A is introduced in the network and the nodes v and w are the initial adopters of this behavior A, i.e., nodes v and w now have adopted behavior A and rest of the nodes have adopted behavior B. The payoff associated with A is $a = 3$ and the payoff associated with B is $b = 2$. After the introduction of this new behavior A in the network, all the nodes will start weighing their options and might change their behavior. This leads to a cascade in the network. After two iterations, which nodes would have adopted the behavior A?

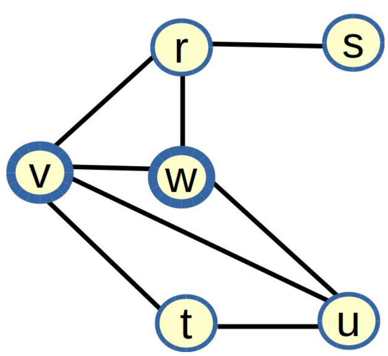

Figure 2: Network for diffusion

A. v, w, r
B. v, w, t, s
C. v, w, r, s, t
D. v, w, r, s, t, u

ANSWER: B
Refer to Figure 3.

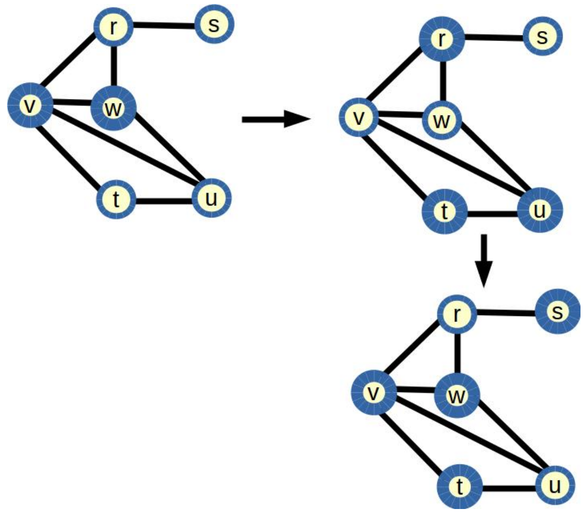

Figure 3: Nodes in thick boundaries are the ones who have adopted behavior A.

6. Given a network as shown in Figure 4. Assume that initially every node in this network has adopted behavior B. Next, a new behavior A is introduced in the network and the nodes 1 and 3 are the initial adopters of this behavior A, i.e., nodes 1 and 3 now has adopted behavior A and rest of the nodes have adopted behavior B. The payoff associated with A is $a = 3$ and the payoff associated with B is $b = 1$. After the introduction of this new behavior A in the network, all the nodes will start weighing their options and might change their behavior. This leads to a cascade in the network. When the cascade ends, which all are the nodes who have adopted the behavior A.

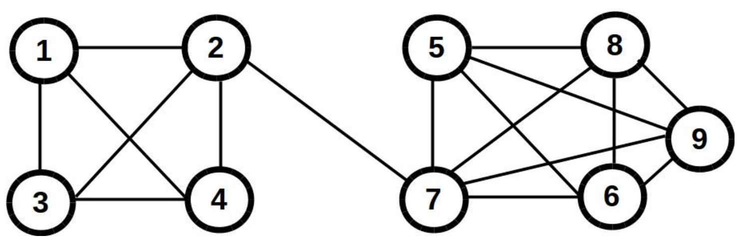

Figure 4: Network for diffusion

A. $1, 3, 2$
B. $1, 3, 2, 4$
C. $1, 3, 2, 4, 7$
D. $1, 3, 2, 4, 5, 6, 7, 8$

ANSWER:B

1. First iteration-
   - Node 1 calculates its payoff. It gets a payoff of $1 \times 3 = 3$ for action A since there is 1 of its neigbor, i.e. node 3 who has adopted A. It gets a payoff of $2 \times 1 = 2$ for action B since there are 2 of its neigbours, i.e. nodes 4 and 2, who has adopted B. Hence, it adopts A since that is yielding a higher payoff.

- Node 3 calculates its payoff. It gets a payoff of $1 \times 3 = 3$ for action A since there is 1 of its neigbor, i.e. node 1 who has adopted A. It gets a payoff of $2 \times 1 = 2$ for action B since there are 2 of its neigbours, i.e. nodes 4 and 2, who has adopted B. Hence, it adopts A since that is yielding a higher payoff.
- Node 2 calculates its payoff. It gets a payoff of $2 \times 3 = 6$ for action A since there are 2 of its neigbors 1 and 3 who have adopted A. It gets a payoff of $2 \times 1 = 2$ for action B since there are 2 of its neigbours, i.e. 4 and 7, who has adopted B. Hence, it adopts A since that is yielding a higher payoff.
- Node 4 calculates its payoff. It gets a payoff of $2 \times 3 = 6$ for action A since there are 2 of its neigbors 3 and 1 who has adopted A. It gets a payoff of $1 \times 1 = 1$ for action B since there is only one of its neigbour, i.e. 2, who has adopted B. Hence, it adopts A since that is yielding a higher payoff.

2. Second iteration-
   - Node 7 calculates its payoff. It gets a payoff of $1 \times 3 = 3$ for action A since there is 1 of its neigbors 2 who has adopted A. It gets a payoff of $1 \times 4 = 4$ for action B since there are 4 of its neigbours who has adopted B- 5, 6, 8 and 9. Hence, it does not adopt A since that is yielding a low payoff. Rest of the nodes 5, 6, 8 and 9 have none of their neighbors having adopted A, hence they keep adopting B only and the cascade stops.

7. Consider the following network with green nodes following the action A and red nodes following the action B at time t = 1 (Figure 5).

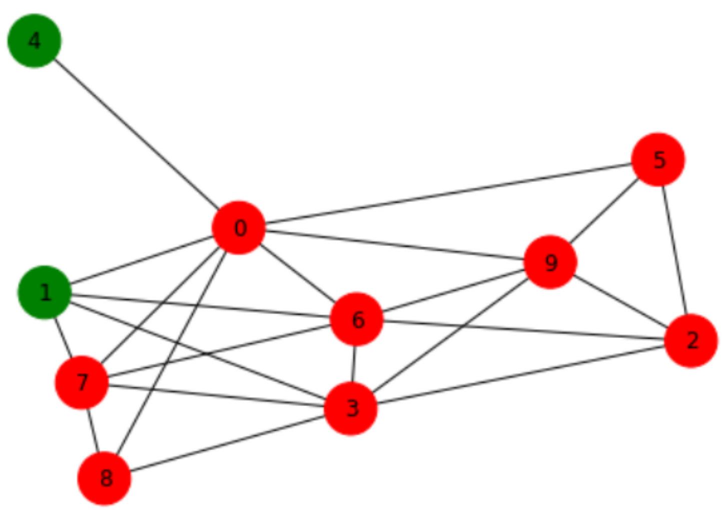

Figure 5: Network for diffusion

The payoff for action A is 6 and for B it is 5. Each one in this network changes his/her action based on his/her friends actions. What will happen after one complete iteration.

A. All nodes adopt action A
B. All nodes adopt action B
C. Some nodes adopt action A and other nodes adopt action B
D. Insufficient data

ANSWER: B
Node 0: Has 2 neighbors with payoff 6 but 5 neighbors with payoff 5. Hence it adopts B.

Nodes 3,6,7 : Have 1 neighbor with payoff 6 but ≥ 2 neighbors with payoff 5. Hence, they adopt B.
Nodes 1,2,4,5,8,9: All of their neighbors have adopted B, so they adopt B.

8. Let v be a node in a graph with $deg(v) = d$. Suppose that a p fraction of $v's$ neighbors have behavior A, and a $(1-p)$ fraction have behavior B. Behavior A has a payoff of a and behavior B has a payoff of b. Then A is a better choice for v if

   A. $p \geq \frac{a}{b}$
   B. $p \geq \frac{b}{a}$
   C. $p \geq \frac{b}{a+b}$
   D. $p \geq \frac{a}{a+b}$

   ANSWER: C
   Suppose that a p fraction of $v's$ neighbors have behavior A, and a $(1-p)$ fraction have behavior B; that is, if v has d neighbors, then pd adopt A and $(1-p)d$ adopt B. So if v chooses A, it gets a payoff of pda, and if it chooses B, it gets a payoff of $(1-p)db$. Thus, A is the better choice if $pda \geq (1-p)db$, or, rearranging terms, iff $p \geq \frac{b}{a+b}$.