Urban Traffic Control Structure Based on Hybrid Petri Nets

Angela Di Febbraro, *Member, IEEE*, Davide Giglio, *Member, IEEE*, and Nicola Sacco

***Abstract*—An urban network of signalized intersections can be suitably modeled as a hybrid system, in which the vehicle flow behavior is described by means of a time-driven model and the traffic light dynamics are represented by a discrete event model. In this paper, a model of such a network via hybrid Petri nets is used to state and solve the problem of coordinating several traffic lights with the aim of improving the performance of some classes of special vehicles, i.e., public and emergency vehicles. The proposed model has been validated using real traffic data relevant to the city of Torino, Italy. Some relevant experimental results are reported and discussed.**

***Index Terms*—discrete event systems, hybrid systems, modeling, Petri nets (PNs), signaling control, traffic control (transportation), traffic responsive strategies.**

I. Introduction

THE REQUEST of efficient traffic services continues to grow, making the need for an effective management of mobility stronger, especially inside and around cities. In the course of the last decade, mobility-management problems have started to be addressed through the implementation and operation of *intelligent transportation systems* (ITS). In this framework, the main objectives are to improve both the flow distribution of private traffic and the services offered by public transportation and parking facilities, while reducing the environmental impact of road traffic.

Before the introduction of ITS, *traffic-signaling control* has often been the only measure applied to regulate vehicle flow inside cities. In traffic-signaling control, the control strategies usually consist of changing the intersection's stage specification, the relative green duration of each stage, the intersection's cycle time, and/or the offset between cycles for successive intersections, thus influencing traffic conditions via traffic signals' operation, according to the time-varying behavior of the incoming traffic [1].

The synchronization issues related to resource sharing and conflict solving, proper to traffic systems, make the synthesis of an adequate model of the traffic behavior of paramount importance; for instance, to suitably coordinate traffic lights. The problem dates back to the 1960s and since then a lot of work has been done on optimal signal timings, either neglecting the time variance of route choices and traffic volumes, as in [12], or taking it into account, as in [13]. A variety of models, methods, and architectures have been proposed and applied for controlling urban traffic via signalized intersections, with the aim of optimizing transportation network performances.

Control strategies employed for traffic-signaling control are primarily divided into *fixed-time* and *traffic-responsive* strategies. Fixed-time strategies consider a given time of a specific day (for example, the morning rush hour of a working day) and determine the optimal splits (that is, the optimal green durations); the optimal cycle time; and, sometimes, also the optimal staging, based on historical values about the traffic demand over the considered signalized urban area. Such strategies have been accomplished in the past through the solution of specific nonlinear [2], linear [3], or binary-mixed-integer-linear [4], [5] mathematical programming problems, with the aim of minimizing the total time spent by vehicles in the area or by maximizing of the intersection capacity. Moreover, the most known and most frequently applied signal-control strategy is the traffic network study tool (TRANSYT), which optimizes the overall travel time, delays, and number of stops [6]. On the other hand, traffic-responsive strategies make use of real-time measurements (typically acquired by means of inductive loops or pattern-recognition digital cameras) to calculate in real time the suitable signal settings. In the last two decades, some tools have been developed in order to apply traffic-responsive strategies to signalized urban areas. The most known and widely applied are split cycle offset optimization technique (SCOOT), the traffic-responsive version of TRANSYT [7], and optimization policies for adaptive control (OPAC), which makes use of the solution of a set of dynamic programming optimization problems in a rolling horizon environment [8]. Finally, a further class of traffic-responsive strategies are based on the store-and-forward model [9], [10]. Such strategies minimize traffic network occupancy by solving a quadratic programming optimization problem. They can then be actually solved by using standard commercial codes, even for a large-scale network.

It is worth noting that an effective responsiveness cannot be achieved by implementing offline methods at an increasing frequency [11]. In this connection, new models and methods that allow a better representation of dynamics within transportation systems are welcome.

Manuscript received November 30, 2003; revised July 15, 2004 and July 31, 2004. The Associate Editor for this paper was F.-Y. Wang.

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Digital Object Identifier 10.1109/TITS.2004.838180

In this work, an urban transportation network is considered to be a *hybrid system*, including both continuous-time and discrete event components. The growing attention of researchers that was recently paid to hybrid systems has also provoked a further increase in research interest on modeling and controlling *discrete event systems* (DES) [15], considered to be standalone systems or components of more complex systems, e.g., hybrid systems. It is well known that Petri nets (PNs) [16] play a key role among the modeling methodologies for DES, since they are able to capture the precedence relations and interactions among the concurrent and asynchronous events that are typical of DES. It was in [17] that PN capabilities were first applied to traffic control, proposing to represent via colored PNs a network of signalized intersections. Afterward, some other works have been published in the related literature (see, for instance, [18]–[21]).

In this paper, hybrid Petri nets (HPNs), as described in [22], are proposed to model an urban transportation network [23]–[26]. Modeling the urban transportation networks as hybrid systems by means of HPNs allows us to take advantage of modeling the traffic flows as fluids, taking into account at once the event-driven dynamics of the traffic lights and its influence on the flow dynamics.

Several kinds of control problems can be addressed and solved by means of the proposed model, with special reference to the issues of the following:

- traffic light plan optimization;
- dynamic routing;
- special vehicle control.

The first class of problems gathers all the control strategies that act with the aim of reducing traffic congestion, minimizing queues at the intersections, and so on. These problems usually need information about traffic flows and/or information about queues at the intersections, which are provided by the model (see, for instance, [27]). The second class of problems concerns the routing of single vehicles or platoons of vehicles in an urban traffic network with the aim of reducing congestion or travel times. To get an idea, the reader can refer, for instance, to [28]. Finally, the third class gathers the problems stated with the aim of improving the performances of privileged vehicles, such as public or emergency vehicles, as the problem faced by the authors in [29]–[31] for an isolated intersection and extended here to consider a network of intersections.

This paper is organized as follows. In Section II, the adopted urban traffic model is briefly recalled to be used for control purposes. Then, the control structure applied to the urban traffic network is introduced in Section III, where the basics of the adopted HPNs are recalled (Section III-A) and the HPN model (Section III-B) is fully described. In the second part of this paper (Section IV), the control problem aimed at the determination of the optimal traffic light plan based on the requests coming from emergency and/or public vehicles is presented. In the last part of this paper (Section V), some case studies relevant to the city of Torino (in the northwest region of Italy) are described, which validate the proposed approach making use of real traffic data coming from some test sites of such a city. Finally, in Section VI, some concluding remarks and indications of current and future work are reported.

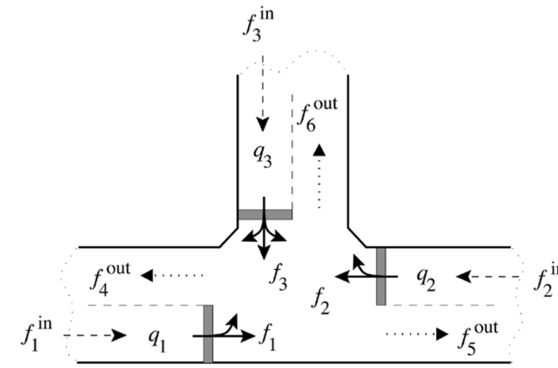

Fig. 1. Two-phase signalized intersection I_h.

II. Urban Traffic Model

An *urban transportation network* (UTN) is the couple

$$\text{UTN} = \{I, R\} \tag{1}$$

where $I = \{I_h, h = 1, \dots, H\}$ is the set that gathers the H signalized intersections in the network and $R = \{R_k, k = 1, \dots, K\}$ is the set that gathers K different roads in the urban area.

Then, in an UTN, signalized intersections are those elements of the net that connect two or more roads and consist of the physical area occupied by vehicles crossing the intersections, as well as a part of the adjacent roads from which vehicle flows come and to which vehicle flows go. In the proposed model, the abovementioned vehicle flows are represented by means of a time-driven macroscopic model that has been proved to be suitable for control purposes, for instance in [27] and [32]. On the other hand, road stretches are those network elements which connect two adjacent signalized intersections.

Note that the definition given previously easily leads to a modular modeling approach, as those described by the authors in [33], which is very useful in building the model of a large urban network.

A. Intersection Model

Consider the traffic area reported in Fig. 1, which consists of a particular intersection I_h with three incoming directions (roads), namely, R_1, R_2, and R_3, and three outgoing directions (roads), namely, R_4, R_5, and R_6, and the adjacent roads. Flows at the intersection are assumed to be ruled by a three-phase traffic light. Let IN_h and OUT_h be the sets of indices of incoming and outgoing directions, respectively; that is, $\text{IN}_h = \{1, 2, 3\}$ and $\text{OUT}_h = \{4, 5, 6\}$. In Fig. 1, the flow $f_j^{\text{in}}, j \in \text{IN}_h$ represents the (input) vehicle flow approaching the intersection from the adjacent road R_j; the flow $f_j, j \in \text{IN}_h$ represents the flow crossing the intersection, entering from the incoming direction j, when enabled by the traffic light; and the flow $f_i^{\text{out}}, i \in \text{OUT}_h$ represents the flow leaving the intersection toward direction i. Moreover, $q_j, j \in \text{IN}_h$ denotes the queue of vehicles waiting for the green light at an incoming direction

TABLE I
PHASES OF THE INTERSECTION OF FIG. 1

Phase	Enabled flows	Length
1st	f_1	$\vartheta_1 = \vartheta_1^g + \vartheta_1^a$
2nd	f_2	$\vartheta_2 = \vartheta_2^g + \vartheta_2^a$
3rd	f_3	$\vartheta_3 = \vartheta_3^g + \vartheta_3^a$

j. It is worth noting that the traffic flows of any road are sampled by means of inductive loops suitably placed under the road.

In general, at each signalized intersection, a multiphase traffic light rules the vehicle flows by means of the light signal, which can show the three usual values: green; amber; and red. Traffic lights are suitable to be modeled as discrete event systems, thus making the whole system a hybrid system, as introduced by the authors in [24]. For what concerns the traffic light dynamics, each phase is characterized by the flows that are enabled within the phase. As a consequence, only the green and amber lights are taken into account, since all the incoming flows that do not find a green or amber light are supposed to find the red one. Coming back to the example of Fig. 1, the flows enabled during each phase of the traffic light are reported in Table I, where ϑ_φ^g and $\vartheta_\varphi^a, \varphi = 1, 2, 3$ represent the lengths of the green and amber times of phase φ.

Then, as regards the traffic flow dynamics within I_h, each input flow $f_j, j \in \mathrm{IN}_h$—enabled by the traffic lights—is given, at each time τ, by

$$f_j(\tau) = \begin{cases} f_{s_j}, & \text{if } q_j(\tau) > 0 \\ \min\left\{f_j^{\text{in}}(\tau), f_{s_j}\right\}, & \text{if } q_j(\tau) = 0 \end{cases} \tag{2}$$

where f_{s_j} is the (experimentally determined) saturation flow. It is obvious that $f_j(\tau) = 0$ when the input flow f_j is not enabled by the traffic lights.

The queue dynamics is ruled by

$$\dot{q}_j(\tau) = f_j^{\text{in}}(\tau) - f_j(\tau) \tag{3}$$

for any $j \in \mathrm{IN}_h$ and where $f_j(\tau)$ is given by (2) for any $\tau \in \mathbb{R}$ such that f_j is enabled by the traffic lights and is equal to zero otherwise.

Finally, the output flows can be calculated by associating with the intersection I_h an *origin–destination* matrix $A(\tau)$, whose generic element $\alpha_{i,j}(\tau)$ indicates the percentage of vehicles coming from direction $j, j \in \mathrm{IN}_h$ and going toward direction $i, i \in \mathrm{OUT}_h$, at time instant τ. Thus, at τ, the output flows are given by

$$f_i^{\text{out}}(\tau) = \sum_{j \in \mathrm{IN}_h} \alpha_{i,j}(\tau) \cdot f_j(\tau) \tag{4}$$

for any $i \in \mathrm{OUT}_h$.

B. Road Model

For what strictly concerns the traffic behavior in road $R_k, k = 1, \ldots, K$, it is worth saying that the model presented here allows representing only unidirectional flows. For this reason, any bidirectional road stretch is split into two separate road stretches, as exemplified in Fig. 2.

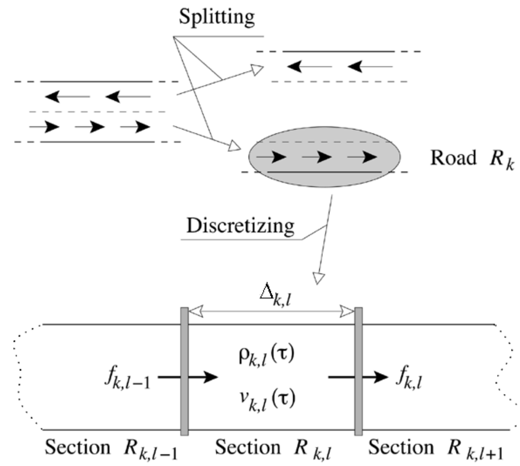

Fig. 2. Road section $R_{k,l}$.

Each unidirectional road stretch $R_k, k = 1, \ldots, K$ is spatially discretized into L_k sections $R_{k,l}, l = 1, \ldots, L_k$, of different lengths $\Delta_{k,l}$. Then, the traffic behavior can be described using a well-known second-order macroscopic model, first introduced in [34]. In such a model, the traffic behavior is described by means of the following "aggregate" variables (Fig. 2):

- $\rho_{k,l}(\tau)$, which is the vehicle density, i.e., the number of vehicles per unit length, in section $R_{k,l}$ at time τ;
- $f_{k,l}(\tau)$, which indicates the traffic flows, i.e., the volume of vehicles; it represents the instantaneous number of vehicles exiting section $R_{k,l}$ toward section $R_{k,l+1}$, at time τ;
- $v_{k,l}(\tau)$; that is, the vehicle speed that depends on the former variables by means of

$$v_{k,l}(\tau) = \frac{f_{k,l}(\tau)}{\rho_{k,l}(\tau)}. \tag{5}$$

The dynamics of the traffic density in a section $R_{k,l}$ depends on its input and output flows, and its section length, through

$$\dot{\rho}_{k,l}(\tau) = \frac{1}{\Delta_{k,l}}\left[f_{k,l-1}(\tau) - f_{k,l}(\tau) + f_{k,l}^r(\tau) - f_{k,l}^w(\tau)\right] \tag{6}$$

where $f_{k,l}^r(\tau)$ [respectively, $f_{k,l}^w(\tau)$] represents the secondary flow that enters (respectively, exits) section $R_{k,l}$.

On the other hand, the vehicle volume exiting section $R_{k,l}$ is given by

$$f_{k,l}(\tau) = \mu \cdot F[\rho_{k,l}(\tau)] + (1-\mu) \cdot F[\rho_{k,l+1}(\tau)] \tag{7}$$

where

$$F[\rho_{k,l}(\tau)] = v_f \cdot \rho_{k,l}(\tau) \cdot \left\{1 - \left[\frac{\rho_{k,l}(\tau)}{\rho_{\max}}\right]^l\right\}^m \tag{8}$$

is the *fundamental traffic diagram* (see [35] for details) and where μ, the free speed v_f, the maximum admissible density $\rho_{\max}, l$, and m are parameters dependent on the road stretch topology.

It is worth stressing that, in real cases, most of the roads linking intersections in urban areas are quite short, so that they usually consist of a single road section. For this reason, and for the sake of simplicity, in the following the index l of a section will be dropped. In this case, the output flow of road R_k coincides with the flow that enters the queue at the downstream intersection at its incoming direction k, namely, $f_k^{\rm in}(\tau)$, and depends on the traffic light dynamics of the downstream intersection, since the dynamics of $\rho_{k+1}(\tau)$, that is, the vehicle density in the queue at the end of road R_k, depends on the traffic lights. On the other hand, the flow entering road R_k corresponds to the flow exiting the upstream intersection toward its output direction k, namely, $f_k^{\rm out}(\tau)$.

C. Event-Driven Dynamics of the Model

In the above sections, urban traffic networks have been said to be hybrid systems and their time-driven dynamics has been described. In this section, some discrete event peculiarities of the proposed model are taken into account.

The events characterizing an urban transportation network can be divided into three classes.

1) *Phase changes:* This class of events affects continuous dynamics, since each phase change corresponds to both enabling and disabling different incoming flows. The phase changes are determined by traffic lights, which have their own event-driven dynamics.
2) *Control requiring events:* When an incoming flow $f_j^{\rm in}$, which enters the queue q_j, changes owing to traffic conditions, the queue dynamics changes and the traffic light optimal plan can have to be recalculated.
3) *Special events:* This class of events gathers all extraordinary events such as, for instance, car accidents, which can block an incoming/outgoing direction (at least partially), leading to changes in the incoming/outgoing flow rates or turning percentages.

It is easy to note that, in the model expressed by (2) and (4), the event-driven behavior can be represented by simply changing the parameters f_{s_j} and $\alpha_{i,j}$. For instance, a car may (partially) block an incoming direction, thus reducing the saturation flow and/or changing the turning percentages. This means that, after each event occurrence, only such parameters have to be recomputed, whereas the net structure remains unchanged.

III. Control Structure of the Urban Traffic Network

In this section, the proposed control structure is presented. Such a structure is thought to be able to perform traffic optimization by changing the length of traffic light phases under the hypotheses of both regular and special conditions; that is, when a public or emergency vehicle asks for a privilege.

To this aim, consider the control architecture represented in Fig. 3. Each generic intersection I_h is equipped with the following.

1) A *local controller* regulates the intersection under the hypothesis of regular conditions, by applying an optimal responsive traffic light plan (see Section IV-A for details).

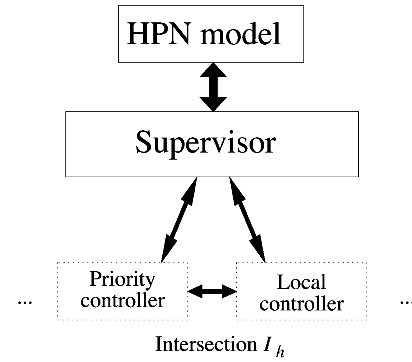

Fig. 3. Proposed control structure.

2) A *priority controller* can force a modified traffic light plan when a particular event occurs (see Section IV-B for details).

In addition, a supervisor coordinates all the local and priority controllers and solves all the problems involving several intersections. The urban traffic network model is implemented into the supervisor by means of an HPN, which can be built by means of a modular procedure, making the representation of the UTN a simple task, as described in [33]. Moreover, it will be shown later in this section that the HPN closely follows the behavior of the adopted urban traffic network model.

A. Background on HPNs

The hybrid formalism on which the proposed model is based is here introduced. The reader can find a more detailed presentation of HPNs in [22], whereas [16] gives a detailed description of the (classical) discrete PNs.

Definition 1: An *HPN structure* is the 5-tuple

$$\mathrm{HPN} = \{P, T, \mathrm{Pre}, \mathrm{Post}, h\} \tag{9}$$

where P is the finite set of $n = |P|$ places[1] and T is the finite set of $m = |T|$ transitions. $P \cap T = \emptyset$, i.e., the sets of places and transitions are disjoint. The set of places P (respectively, transitions T) is split into two subsets P^D and P^C (respectively, T^D and T^C) gathering, respectively, the discrete and continuous places (respectively, transitions). Pre is the *preset* function[2]

$$\mathrm{Pre}_{i,j}\colon (P \times T) \rightarrow \begin{cases} \mathbb{R}_0^+, & \forall p_i \in P^C \\ \mathbb{N}_0^+, & \forall p_i \in P^D \end{cases} \tag{10}$$

that assigns a weight to any arc between a transition t_j and its input place p_i. Post is the *postset* function

$$\mathrm{Post}_{i,j}\colon (P \times T) \rightarrow \begin{cases} \mathbb{R}_0^+, & \forall p_i \in P^C \\ \mathbb{N}_0^+, & \forall p_i \in P^D \end{cases} \tag{11}$$

that assigns a weight to any arc between a transition t_j and its output place p_i. $h : P \cup T \rightarrow \{D, C\}$ is a hybrid function that indicates whether a node is discrete (D) or continuous (C). △

[1] $|X|$ indicates the number of elements in the set X.

[2] $\mathbb{R}_0^+$ and $\mathbb{N}_0^+$ indicate the set of nonnegative real and integer number, respectively.

From a graphical point of view, discrete places are represented by circles and discrete transitions are represented by bars, whereas continuous places are represented by double circles and continuous transitions are represented by boxes. Finally, arcs are represented by arrows.

Places and transitions can have both discrete and continuous inputs according to the following definitions. For each transition t_j, ${}^D t_j$ and ${}^C t_j$ denote the sets of its input discrete and continuous places, respectively, whereas t_j^D and t_j^C are the sets of its output discrete and continuous places. The whole input and output place sets of transition t_j are defined as ${}^\bullet t_j = {}^D t_j \cup {}^C t_j$ and $t_j^\bullet = t_j^D \cup t_j^C$, respectively. On the other hand, for each place p_i, ${}^D p_i$ and ${}^C p_i$ denote the sets of its input discrete and continuous transitions, whereas p_i^D and p_i^C are the sets of its output discrete and continuous transitions. In this case, the whole input and output transition sets of p_i are ${}^\bullet p_i = {}^D p_i \cup {}^C p_i$ and $p_i^\bullet = p_i^D \cup p_i^C$, respectively.

Definition 1 only gives the structure of HPNs, whereas their dynamic evolution is described through the *marking* and two *firing rules*.

Definition 2: The marking of an HPN is the function

$$\mathcal{M} : P \rightarrow \begin{cases} \mathbb{N}_0^+, & \forall p_i \in P^D \\ \mathbb{R}_0^+, & \forall p_i \in P^C \end{cases} \tag{12}$$

which assigns a nonnegative integer number (the number of "tokens") to each discrete place and a nonnegative real number to each continuous place. △

The marking is an instantaneous representation of the state of the system described by means of an HPN structure. It evolves following the rules given here.

Rule 1: Firing of discrete transitions:

1) A discrete transition t_j is enabled, i.e., it can fire, if and only if

$$\mathcal{M}(p_i) \geq \mathrm{Pre}_{i,j}, \qquad \forall p_i \in {}^\bullet t_j. \tag{13}$$

2) Discrete transition t_j starts firing as soon as it is enabled.
3) Firing of t_j lasts ϑ_j time units, where $\vartheta_j \geq 0$ is a nonnegative deterministic or stochastic number. If $\vartheta_j > 0$, then t_j is named a *timed transition*, whereas if $\vartheta_j = 0$, then t_j is an *immediate transition*. From a graphical point of view, timed transitions are represented by thick bars, whereas immediate transitions are represented by thin bars.
4) Upon completion of the firing of t_j, the marking of input places and of output places changes as

$$\begin{cases} \mathcal{M}'(p_i) = \mathcal{M}(p_i) - \mathrm{Pre}_{i,j}, & \forall p_i \in {}^\bullet t_j \\ \mathcal{M}'(p_l) = \mathcal{M}(p_l) + \mathrm{Post}_{l,j}, & \forall p_l \in t_j^\bullet \end{cases} \tag{14}$$

where $\mathcal{M}'(p_i)$ and $\mathcal{M}'(p_l)$ indicate the markings of input place p_i and of output place p_l, respectively, after the firing of t_j. □

Rule 2: Firing of continuous transitions:

1) Continuous transition t_j is *strongly enabled* if and only if every place in ${}^\bullet t_j$ meets the following conditions:
   - $\mathcal{M}(p_i) \geq \mathrm{Pre}_{i,j}, \forall p_i \in {}^D t_j$;
   - $\mathcal{M}(p_i) \geq 0, \forall p_i \in {}^C t_j$.
2) Firing of a continuous transition t_j is not instantaneous, but happens with a constant speed v_j, which is a nonnegative real number.
3) If only the first bulleted point is satisfied, then t_j is *weakly enabled* if and only if all $p_i \in {}^C t_j$ such that $\mathcal{M}(p_i) = 0$ are fed, i.e., there exists at least an enabled continuous transition $t_k \in {}^C p_i$, which fires with a constant speed $v_k > 0$. In this case, t_j can fire with a speed $\tilde{v}_j = \min\{v_j, v_k\}$. □

The marking evolution of the continuous places, with respect to the time, can be described easily by means of the equations given in the following definition. In this connection, let $\dot{\mathcal{M}}(p_i, \tau)$ denote the marking of place p_i at time instant τ.

Definition 3: (balance equation):

The evolution of the marking of a continuous place p_i is given by the differential relation

$$\dot{\mathcal{M}}(p_i, \tau) = \sum_{k \in \mathrm{IN}_i} \mathrm{Post}_{i,k} \cdot v_k - \sum_{j \in \mathrm{OUT}_i} \mathrm{Pre}_{i,j} \cdot v_j \tag{15}$$

where the sets IN_i and OUT_i gather the indices of the enabled transitions belonging to ${}^C p_i$ and p_i^C respectively; v_k and v_j are the firing speeds of transitions t_k and t_j, respectively; and $\dot{\mathcal{M}}(p_i, \tau)$ is the first derivative of $\mathcal{M}(p_i, \tau)$.

Whenever $\mathcal{M}(p_i, \tau)$ becomes zero, all $t_j \in p_i^C$ can become disabled or weakly enabled. In the latter case, the firing speed of any transition $t_j, j \in \mathrm{OUT}_i$ must fulfill the constraint

$$\sum_{k \in \mathrm{IN}_i} \mathrm{Post}_{i,k} \cdot v_k \leq \sum_{j \in \mathrm{OUT}_i} \mathrm{Pre}_{i,j} \cdot v_j. \tag{16}$$

△

Summing up, an HPN is composed of a "classical" (discrete) PN and a continuous PN, the "fluid" version of usual timed PNs (see, for instance, [36]). Note that the states (i.e., markings) of the two PNs making up an HPN, i.e., the continuous and discrete ones, usually affect each other. However, it is possible to prove that, to keep an integer number of tokens in a discrete place, p_i connected to a continuous transition t_j, place p_i must be both an input and an output place for t_j, i.e., $p_i \in {}^D t_j$ and $p_i \in t_j^D$, and the arcs connecting p_i and t_j must have the same weights. In this case, the firing of a continuous transition cannot modify the marking of the discrete places of the HPN.

B. HPN Model

The HPN model of the intersection in Fig. 1 is depicted in Fig. 4. Such a net is composed by three continuous PNs modeling the vehicle flows entering the intersection and the queue and three continuous PNs modeling the vehicle flows leaving the intersection and the downstream roads. In addition, a discrete PN represents the traffic light. The marking of the continuous part of the net represents the number of vehicles that are in each place. Although a detailed description of places and transitions is reported in Tables II and III, respectively, it is worth noting that the net in Fig. 4 is built by joining the elementary modules

A) that represents a generic phase of the traffic light;
B) that represents a generic incoming direction;
C) that represents a generic road section.

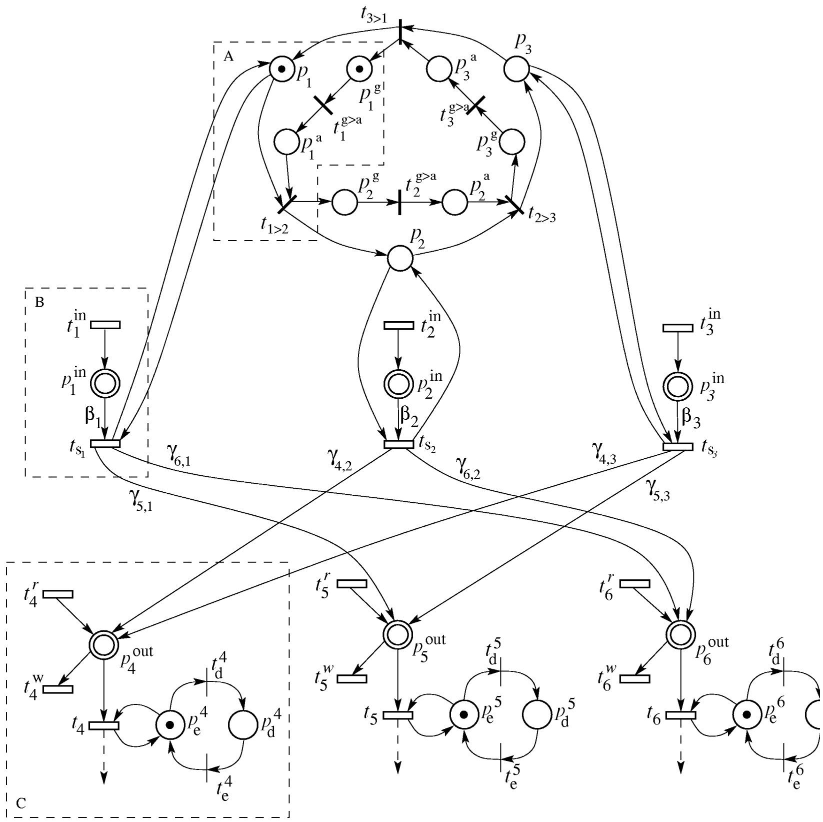

Fig. 4. HPN model of the intersection of Fig. 1.

TABLE II
PLACES OF THE HPN IN FIG. 4

Place	Meaning
$p_j^{\rm in}$, $j \in {\rm IN}_h$	vehicles are in the queue at incoming direction j
$p_i^{\rm out}$, $i \in {\rm OUT}_h$	vehicles are leaving the intersection towards direction i
p_φ	the φ-th phase is enabled
p_φ^g	green period of φ-th phase
p_φ^a	amber period of φ-th phase
$p_{\rm d}^i$	vehicles cannot flow through road R_i
$p_{\rm e}^i$	vehicles can flow through road R_i

It is this modularity that makes the HPN model easy to use in urban traffic network modeling.

For what concerns the state (i.e., the marking) evolution, first consider the discrete PN, which models the traffic light. As soon as the first phase begins, the marking of the net is the one depicted in Fig. 4, which represents the beginning of the green time of the first phase of the cycle. In this state, the transition $t_1^{g>a}$ is enabled and fires for ϑ_1^g s, where ϑ_1^g is the duration of the green time of the first phase. The end of firing means that the green time ends and the amber one begins. After the firing, the token in p_1^g moves to p_1^a, enabling the transition $t_{1>2}$ that fires for ϑ_1^a s, being ϑ_1^a the duration of the amber time of the first phase. During the second and third phases, the system evolves analogously to what is said for the first phase, until a first phase of a new cycle begins.

For what concerns the flow model part of the net in Fig. 4, it will be shown here as it evolves following the time-driven relation (4). The fact is, (4) can be rewritten as

$$\begin{aligned} f_i^{\rm out}(\tau) &= \left[\sum_{j \in {\rm IN}_h} \alpha_{i,j}(\tau) \cdot \beta_j(\tau)\right] \cdot f \\ &= \left[\sum_{j \in {\rm IN}_h} \gamma_{i,j}(\tau)\right] \cdot f \end{aligned} \tag{17}$$

where $f = 1$ veh/s is the unitary flow rate, $\beta_j(\tau) = f_j(\tau)/f$ represents the ratio between the jth incoming flow rate and the unitary flow rate $\gamma_{i,j}(\tau) = \alpha_{i,j}(\tau) \cdot \beta_j(\tau), j \in {\rm IN}_h, i \in {\rm OUT}_h$ represents the ratio between the flow entering from direction j and exiting toward direction i and the rate f. It is clear that, since all the flows $f_j, j \in {\rm IN}_h$ depend on the traffic light phases, the coefficients β_j and $\gamma_{i,j}$ do also. Such coefficients, which have been "artificially" introduced in (17), are used as arc weights in the HPN-based representation of a signalized intersection described here, thus guaranteeing the equivalence between the analytical and HPN-based models.

To this aim, consider the balance equation of the marking of places $p_j^{\rm in}, j \in {\rm IN}_h$, which is given by

$$\dot{\mathcal{M}}\left(p_j^{\rm in}, \tau\right) = v_j^{\rm in}(\tau) - \beta_j(\tau) \cdot v_{s_j} \tag{18}$$

where $v_j^{\rm in}(\tau)$ and v_{s_j} are initially assumed to be the firing speed of transitions $t_j^{\rm in}$ and t_{s_j}, respectively. It is easy to note that, due

TABLE III
TRANSITIONS OF THE HPN IN FIG. 4

Transition	Meaning	Speed/Delay
t_{s_j}, $j \in \mathrm{IN}_h$	vehicles leave the queue at incoming direction j that is, approach and cross the intersection	f_{s_j}
t_j^{in}, $j \in \mathrm{IN}_h$	vehicles enter the queue at incoming direction j	f_j
t_i, $i \in \mathrm{OUT}_h$	vehicles leave the intersection towards direction i	f_i
$t_\varphi^{g>a}$	φ-th green phase ends and φ-th amber phase begins	ϑ_φ^g
$t_{\varphi>\varepsilon}$	φ-th phase ends and ε-th one begins	ϑ_φ^a
t_{d}^i, $i \in \mathrm{OUT}_h$	the road stretch R_i becomes blocked	-
t_{e}^i, $i \in \mathrm{OUT}_h$	the road stretch R_i becomes unblocked	-
t_i^r, $i \in \mathrm{OUT}_h$	vehicles enter the road stretch R_i from a secondary road	f_i^r
t_i^w, $i \in \mathrm{OUT}_h$	vehicles exit from road stretch R_i towards a secondary road	f_i^w

to the definition of β_j, (18) coincides with the queue dynamics (3) if $\mathcal{M}(p_j^{\mathrm{in}}, \tau) = q_j(\tau), \forall j \in \mathrm{IN}_h$ and if the firing speed of transitions t_j^{in} and t_{s_j}, that is, $v_j^{\mathrm{in}}(\tau)$ and v_{s_j}, are set equal to $f_j^{\mathrm{in}}(\tau)$ and f, respectively.

In addition, the equation of the marking evolution of a place $p_i^{\mathrm{out}}, i \in \mathrm{OUT}_h$, which represents the road stretch toward which vehicles exit, is given by

$$\dot{\mathcal{M}}\left(p_i^{\mathrm{out}}, \tau\right) = \sum_{j=1}^{3} \gamma_{i,j}(\tau) \cdot v_{s_j} - v_i(\tau) + v_i^r(\tau) - v_i^w(\tau) \tag{19}$$

where $v_i(\tau), v_i^r(\tau)$, and $v_i^w(\tau)$ are initially assumed to be the firing speed of transitions t_i, t_i^r, and t_i^w, respectively. Such an equation gives the traffic density in road R_i if $\mathcal{M}(p_i, \tau) = \rho_i(\tau) \cdot \Delta_i$, if the firing speed of transitions t_{s_j}, t_i, t_i^r and t_i^w are set to be equal to $f, f_i(\tau), f_i^r(\tau)$, and $f_i^w(\tau)$, respectively, and if the weights $\gamma_{i,j}$ are the parameters defined in (17).

In addition, the HPN model of the road stretches, which essentially consists of a continuous PN, has a discrete part that can enable or disable the vehicle flows when some parameters have to be changed, as done in [24] and [25].

Then, one can conclude that the proposed HPN model evolves, reproducing the hybrid dynamics of the considered urban transportation network.

IV. Control Problem

The problem of controlling traffic in an urban transportation network so as to increase the performances of "special" vehicles is now addressed. The proposed control algorithms are based on the previously described HPN model. In this framework, the categories of vehicles taken into account are those performing public services (buses) or those performing emergency services (fire brigades, ambulances, police, etc.). In both cases, the main objective is to minimize the travel time from a specific origin to a specific destination, in order to maximize the efficiency of the considered classes of vehicles, hereafter called *privileged* vehicles.

Although the basic traffic regulation problem is the same for both considered classes of vehicles, a higher priority might be guaranteed to emergency vehicles, so as to make their travel times as short as possible. This means that such vehicles are allowed to provoke perturbations in the traffic light nominal plan more significantly than the others (buses). In addition, public vehicles follow *a priori* determined routes, whereas emergency vehicles can choose the best route among all possible routes. This consideration implies that the problem of controlling public vehicles can be considered to be a subproblem of the more general problem of setting an "optimal route" for emergency vehicles. On the other hand, such a subproblem is very easy to solve; for this reason, it will be faced before the more general one and then generalized to the problem of controlling emergency vehicles.

A. Optimal Responsive Traffic Light Plan

Before introducing the special vehicle problem, it is worth describing the environment in which special vehicles works. Such an environment consists of a traffic area in which the traffic lights perform a traffic-responsive plan, in the sense that they dynamically change the phase splits with the aim of optimizing the performance indexes of the overall traffic system by, for example, shortening and equalizing the queues at the intersections.

This problem has been faced by many authors in the past and two valuable example can be found in [27] and [37]. Obviously, all of the solutions provided in these works assume the knowledge of the traffic behavior at each time or, more often, at any fixed sampling time. For this reason, many traffic models have been developed, for example in [17], [19], and [21]. In particular, the control problem proposed by Lei and Ozguner in [27] needs, as inputs, the data relevant to the traffic flows approaching an intersection, which can be provided by the previously introduced model. For this reason, in this section, a control problem similar to the one addressed by [27] is briefly described.

Consider a generic intersection with m incoming directions and a Φ-phase traffic light. The controller finds the optimal solution to the problem characterized by the cost function

$$\min_{\vartheta_1,\ldots,\vartheta_\Phi} \left\{ \int_{\tau_0}^{\tau_0+\vartheta_1+\cdots+\vartheta_\Phi} \sum_{j\in\mathrm{IN}_h} q_j^2(\tau) + \sum_{j\in\mathrm{IN}_h} \sum_{i\in\mathrm{IN}_h, i\neq j} \left[(q_j(\tau) - q_i(\tau)\right]^2 d\tau \right\} \tag{20}$$

where the first term represents the total number of vehicles waiting in the queues to be minimized, whereas the second term forces an equalization among all the queue lengths. The cost function (20) is subject to constraints

$$\vartheta_C^{\min} \leq \sum_{\varphi=1}^{\Phi} \vartheta_\varphi \leq \vartheta_C^{\max} \tag{21}$$

$$\vartheta_\varphi^{\min} \leq \vartheta_\varphi \leq \vartheta_\varphi^{\max} \quad \forall \varphi = 1, \ldots, \Phi \tag{22}$$

which guarantee the minimum and maximum time that vehicles have to wait before getting the green signal.

In (20)–(22), τ_0 represents the time instant at which a traffic light cycle begins; $\vartheta_\varphi, \varphi = 1, \ldots, \Phi$ is the duration of the phase $\varphi, \vartheta_C^{\min}$ and $\vartheta_C^{\max}, \vartheta_C^{\max} > \vartheta_C^{\min} > 0$ are, respectively, the minimum and maximum lengths of a traffic light cycle, whereas $\vartheta_\varphi^{\min}$ and $\vartheta_\varphi^{\max}, \vartheta_\varphi^{\max} > \vartheta_\varphi^{\min} > 0$ are, respectively, the minimum and maximum admissible durations of the traffic light phase φ. In addition, the optimization problem (20) must fulfil the constraints relevant to the queue dynamics equations (2) and (3).

Such a problem can be also extended to the case of several synchronized adjacent intersections with the aim of improving the traffic system performances more and more. See [27] for more details.

B. Local Priority Control Problem

In this section, the problem of managing a priority request is stated and solved for an isolated intersection. This is accomplished through a simple control algorithm, which also is the basis of the more complex algorithm proposed in a following section for the management of emergency vehicles in a network of intersections.

When a privileged vehicle asks the priority controller of the downstream intersection for a phase modification, two different control actions can be implemented, according to the following algorithm.

Algorithm 1: Priority control at isolated intersections:

Step 1) Let $\bar{\tau}$ be the time instant at which a privileged vehicle asks for a phase modification and $\tilde{\tau}$ the time instant, computed by the priority controller (which has the knowledge of the local traffic conditions), at which the privileged vehicle is expected to arrive.

Step 2) If at $\tilde{\tau}$ the light for the direction the privileged vehicle comes from is expected to be green, then there will be no need for intervention (and then the algorithm ends).

Step 3) If the red light is expected, the local priority controller has to decide whether

- *Step 3.1* extending the current green time until the privileged vehicle crosses the intersection;
- *Step 3.2* anticipating the next green time, shortening the current red one accordingly; note that this modification is subject to (22), which implies that the privileged vehicles may have to wait for the green time.

In this connection, the priority controller has to solve the quite simple integer programming problem

$$\min_{y_a, y_b} [(1-y_a)(1-y_b)\, C + y_a\, C_a + y_b\, C_b] \tag{23}$$

subject to constraints

$$y_a + y_b \leq 1 \tag{24}$$

$$y_a, y_b \in \{0, 1\} \tag{25}$$

to determine whether it is more convenient to concede the privilege or not. The binary control variable y_a assumes value 1 if the privilege request is satisfied by extending the current green time, whereas y_b assumes value 1 if the privilege request is satisfied by shortening the current red time. Otherwise, they assume value 0. The costs C, C_a, and C_b are due to the fact that the modifications to the traffic light plan introduced by the privileged vehicle represent suboptimal solutions to the local control problem defined by (20)–(22). Note that, for this reason, during the subsequent cycles, the local regulator may have to find a new optimized plan by evaluating the perturbations introduced by the privileged vehicle. Due to this last consideration, it is reasonable to suppose that such perturbations become negligible after two or three cycles. On the other hand, since the traffic plan can be changed before the privileged vehicle arrives to the intersection, these costs have to be defined and calculated over a time period starting when the privilege is requested and finishing when the second traffic light cycle after the modification ends (let τ_{2c} be such a time instant). The considered costs are as follows.

C represents the cost of not conceding any privilege; it depends on the kind of privileged vehicle and on the traffic condition and is defined by

$$C = C_0 + \sum_{j \in \mathrm{IN}_h} \frac{\varsigma_j}{\tau_{2c} - \bar{\tau}} \int_{\bar{\tau}}^{\tau_{2c}} [\alpha \cdot q_j(\tau)]^2 d\tau \tag{26}$$

where C_0 is a constant cost term, due to the fact that the privileged vehicle has to wait, and α is the maximum admissible displacement for a privileged vehicle; in general, this quantity is greater for emergency vehicles than for buses.

C_a is defined as

$$C_a = \sum_{j \in \mathrm{IN}_h} \frac{\xi_j}{\tau_{2c} - \bar{\tau}} \int_{\bar{\tau}}^{\tau_{2c}} \left[q_j^a(\tau) - q_j(\tau)\right]^2 d\tau \tag{27}$$

which represents the mean square deviations of the queue lengths in the perturbed case from the queue lengths when no perturbation occurs, from the time instant at which the privilege is requested until the end of the second traffic light cycle after the current one.

C_b is defined as

$$C_b = \sum_{j \in \mathrm{IN}_h} \frac{\psi_j}{\tau_{2c} - \bar{\tau}} \int_{\bar{\tau}}^{\tau_{2c}} \left[q_j^b(\tau_c) - q_j(\tau_c)\right]^2 d\tau \tag{28}$$

which represents the mean square deviations of the queue lengths in the perturbed case from the queue lengths when no perturbation occurs, from the time instant at which the privilege is requested until the end of the second traffic light cycle after the current one.

Weights ς_j, ξ_j, and $\psi_j, j \in \mathrm{IN}_h$, in (26), (27), and (28), respectively, represent the importance of each incoming direction j. The weights corresponding to the directions than are enabled together with the direction from which the privileged vehicle comes are usually smaller that the weights of the other directions, since they are already advantaged by the privilege concession. Moreover, $q_j^a(\tau)$ and $q_j^b(\tau), j \in \mathrm{IN}_h$ represent the perturbed queue lengths at the incoming direction j, whereas

$q_j(\tau), j \in \mathrm{IN}_h$ is the queue length without perturbations at the incoming direction j. It is worth noting that all the costs in (26), (27), and (28) can be easily calculated since ς_j, ξ_j and $\psi_j, j \in \mathrm{IN}_h$ are known parameters, whereas all $q_j^a(\tau), q_j^b(\tau)$, and $q_j(\tau), j \in \mathrm{IN}_h$ can be determined by means of the traffic model proposed in Section II. In particular, $q_j(\tau)$ can be determined by knowing the nominal and the perturbed traffic light plan determined by the local controller, whereas $q_j^a(\tau)$ and $q_j^b(\tau)$ can be calculated by knowing the perturbed traffic light plans required to the priority controller.

C. Public Vehicle Performance Optimization

The problem of optimizing the performances of the public vehicles is characterized by two peculiarities.

1) At each bus stop, the public vehicle may wait for people getting on and off. This waiting time is not *a priori* known, although it can be characterized as a stochastic variable, often Gaussian; that is, $\approx N(\mu, \sigma^2)$.
2) Public vehicles have to follow a fixed route; then, their route cannot be optimized.

These two peculiarities suggest to face the problem of managing the priority of public vehicles as a sequence of local priority problems.

An example of the problem of optimizing the performances of public vehicles will be discussed in Section V-A.

D. Emergency Vehicle Performance Optimization

The problem of optimizing the performances of privileged vehicles at isolated intersections, introduced in Section IV-B, is used here to solve the problem of optimizing the performances of emergency vehicles in a network of intersections. The proposed control procedure consists of an algorithm that finds the optimal route by trying to change the traffic light phases conveniently. It is worth stressing that the problem addressed in this section differs from the problem of finding the shortest path between two points of a traffic network, since each intersection is asked to modify its traffic light timing with the aim of shortening the travel time. In other words, in this framework the weights of the arcs of the graph can be reduced by the control procedure.

The basic idea introduced here is that an urban traffic network, such as the one depicted in Fig. 5, can be seen as a weighted graph, such as the one in Fig. 6, where each node represents a signalized intersection and each arc represents a road linking two intersections. Weight $w_{i,j}$ represents the travel time that is necessary to reach I_j starting from I_i. In addition, the waiting times in the queues have to be taken into account. Note that whereas $w_{i,j}$ depends on the traffic behavior at the time of the privilege request, the waiting time at the intersection depends directly on the traffic light plan.

Suppose that a vehicle has to reach a particular destination, starting from a particular origin, as soon as possible. If the travel time in each link and the waiting times due to the expected red lights and to the presence of the queues are known, the resulting problem takes on the form of a *shortest path problem* (SPP). In this paper, this problem will be combined with the intersection priority controller problem defined in Section IV-B, with the aim of minimizing the travel times of emergency vehicles, making them encounter as few red lights as possible.

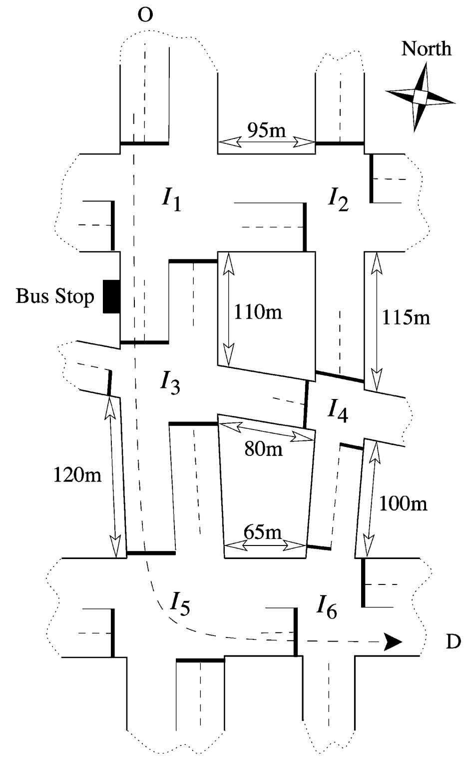

Fig. 5. Real case.

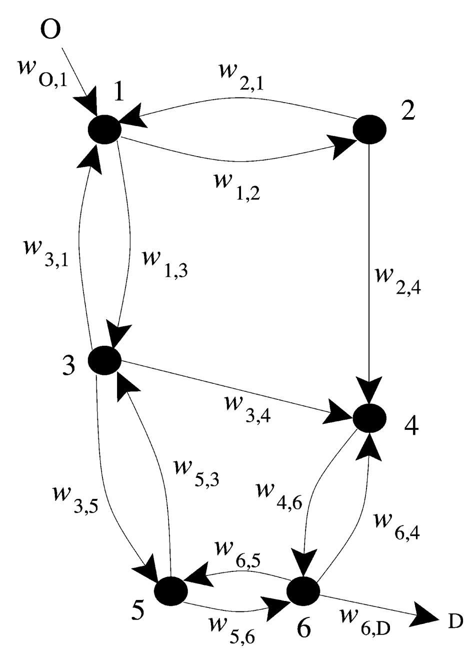

Fig. 6. Graph representing the real case of Fig. 5.

This problem can be solved by means of the following algorithm. It is based on the hypothesis that the traffic behavior has a slow dynamics; this implies that the traffic plan can be considered to keep constant during some traffic light cycles after the privilege request. Note that travel and waiting times are not constant, but depend on the traffic conditions when the privilege request is made.

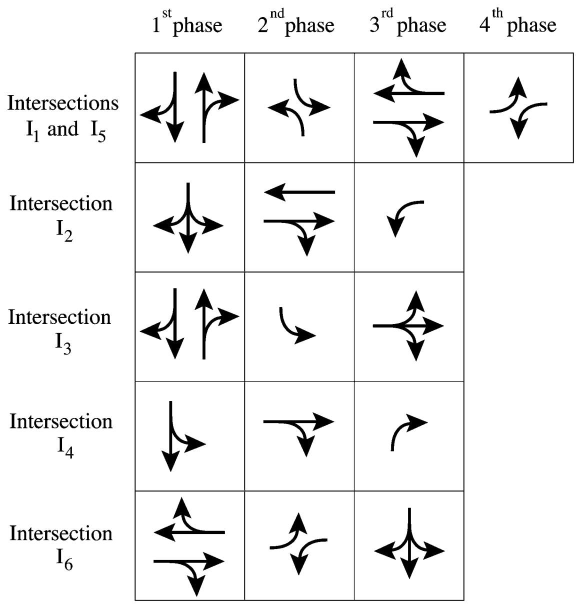

Fig. 7. Enabled flows at each intersections of the net in Fig. 5.

Algorithm 2: Control algorithm for emergency vehicles:

Step 1) When a privileged vehicle asks, at $\bar{\tau}$, for a traffic plan modification, it communicates its position and destination to the supervisor (again see Fig. 3); then, the supervisor determines, by knowing the travel times over each arc of the net at $\bar{\tau}$, the set

$$\mathcal{SP}(\bar{\tau}) = \{\mathcal{P}_1, \ldots, \mathcal{P}_N\} \tag{29}$$

which gathers the N shortest paths according to the traffic conditions at $\bar{\tau}$; set $i = 1$.

Step 2) For any intersection I_h included in route $\mathcal{P}_i$, the supervisor asks to the corresponding priority controller to solve the priority control problem described in Section IV-B, providing the time instant $\tilde{\tau}_{i,h}$ and the direction from which the vehicle is expected to come.

Step 3) Each local controller solves the local problem described in Section IV.B and determines its own optimal timing; note that, due to the structure of the problem (23), the emergency vehicle might encounter a red light at any intersection of the route $\mathcal{P}_i$.

Step 4) The supervisor computes the real travel time T_i on the route $\mathcal{P}_i$ by means of the traffic model of Section II.A, on the basis of the traffic conditions and the traffic light timings determined in Step 3); then, if $i \leq N$, set $i = i+1$ and go to Step 2). Otherwise, go to Step 5).

Step 5) Among all the possible routes $\mathcal{P}_i, i = 1, \ldots, N$, the supervisor chooses the route $\mathcal{P}_i$ such that $i = \arg\min\{T_i\}$; then, it communicates the route to the privileged vehicles and to the involved priority controllers.

TABLE IV
PHASE LENGTHS OF THE INTERSECTIONS IN FIG. 5 WITH THEIR CONSTRAINTS (22)

Intersection	Phases				
	Kind	1st	2nd	3rd	4th
I_1	minimum	28 s	12 s	20 s	10 s
I_1	nominal	30 s	15 s	25 s	13 s
I_1	maximum	36 s	18 s	27 s	16 s
I_2	minimum	22 s	11 s	18 s	–
I_2	nominal	25 s	15 s	20 s	–
I_2	maximum	29 s	17 s	24 s	–
I_3	minimum	28 s	12 s	18 s	–
I_3	nominal	31 s	15 s	20 s	–
I_3	maximum	36 s	18 s	24 s	–
I_4	minimum	15 s	17 s	10 s	–
I_4	nominal	18 s	20 s	13 s	–
I_4	maximum	21 s	25 s	16 s	–
I_5	minimum	30 s	12 s	20 s	10 s
I_5	nominal	33 s	15 s	24 s	14 s
I_5	maximum	38 s	18 s	28 s	18 s
I_6	minimum	24 s	12 s	15 s	–
I_6	nominal	27 s	15 s	18 s	–
I_6	maximum	30 s	18 s	21 s	–

TABLE V
OFFSET BETWEEN CYCLES OF THE INTERSECTIONS IN FIG. 5

Intersections	Offset
$I_1 \rightarrow I_3$	8 s
$I_3 \rightarrow I_5$	8.5 s

V. CASE STUDIES

In this section, two examples that are relevant to the public and emergency vehicles priority control will be discussed, with the aim of proving the efficacy of the proposed control procedures.

First of all, consider the system in Fig. 5, which represents a portion of the traffic network of the Italian city of Torino with six signalized intersections. The flows enabled during each traffic light phase and the relevant lengths are reported in Fig. 7 and Table IV, respectively. Moreover, the cycles of the intersections I_1, I_2, I_4, and I_6 begin at the same time τ_0. On the other hand, the cycles of the intersections I_3 and I_5 begin after a certain offset, which is reported in Table V.

A. Public Vehicles

As an example of public vehicle-priority control, consider a bus approaching I_1 with the aim of following the route indicated in Fig. 5 by means of a dashed arrow. It is easy to note, by analyzing Figs. 5 and 7, that, in the best case, the public vehicle should find the first phase at I_1, the first phase at I_3, the second phase at I_5, and, finally, the first phase at I_6.

TABLE VI
COSTS FOR THE PUBLIC VEHICLE CONTROL PROBLEMS

Intersection	C_0	C_1	C_2
I_3	8.51	**7.98**	9.87
I_6	11.17	10.05	**9.43**

In the considered real case, the privileged vehicle arrives at the first intersection at $\tau_1 = \tau_0 + 15$ s and finds the green signal. Then, it arrives at I_3 at time

$$\begin{aligned}\tau_2 &= \tau_1 + w_{1,3} + n_{1,3} \\ &= 15\ s + 9.2\ s + 18\ s = 42.2\ s\end{aligned} \tag{30}$$

where $w_{1,3}$ is the time the vehicle needs to reach I_3 from I_1 and $n_{1,3}$ is the time it spends at the bus station. At time τ_2, it finds that the second phase has just begun. For this reason, at I_3, a local priority problem is solved (see the computed costs in Table VI) and, as a result, the phases are changed by extending the first phase. Thus, the bus can continue its run without stopping. After crossing I_3, it arrives at I_5 at time

$$\tau_3 = \tau_2 + w_{3,5} = 42.2\ s + 10\ s = 52.2\ s \tag{31}$$

$w_{3,5}$ being the time that the vehicle needs to reach I_5 from I_3, then finding the suitable phase to cross the intersection. Finally, it arrives at I_6 at time

$$\tau_4 = \tau_3 + w_{5,6} = 52.2\ s + 5.1\ s = 57.3\ s \tag{32}$$

where $w_{5,6}$ is the time the vehicle needs to reach I_6 from I_5, then finding the red signal due to the third phase. The solution provided by the local controller consists of shortening the present third phase, thus beginning a new cycle, as is clear from the cost analysis in Table VI. Then, in this particular case, the privileged vehicle can reach the destination without stopping at any intersection.

Consider the time diagrams in Fig. 8, where the traffic light phases of I_1, I_3, I_5, and I_6 are reported in the nominal and perturbed cases. In such diagrams, the gray boxes represent the extensions (respectively, shortenings) of the phases of I_3 (respectively, I_6), whereas the dashed and thick lines represent the trajectory (with respect to the time) of the public vehicle. As is clear in Fig. 8, the application of the proposed control strategy reduces the travel time of the public vehicle by about 40 s.

It is worth finally stressing that such a timing perturbation introduces a nonoptimality in the queue dynamics, but such a performance worsening is just taken into account in the costs of Table VI and, in addition, the optimal behavior can be recovered by the local controller discussed in Section IV.A.

B. Emergency Vehicles

Consider an emergency vehicle asking the supervisor for a "privileged route" to reach the (destination) location D from the (origin) location O. As is clear in Fig. 5, the set of the shortest paths is

$$\begin{aligned}\mathcal{SP} &= \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3\} \\ &= \{(I_1, I_3, I_5, I_6), (I_1, I_2, I_4, I_6), (I_1, I_3, I_4, I_6)\}.\end{aligned} \tag{33}$$

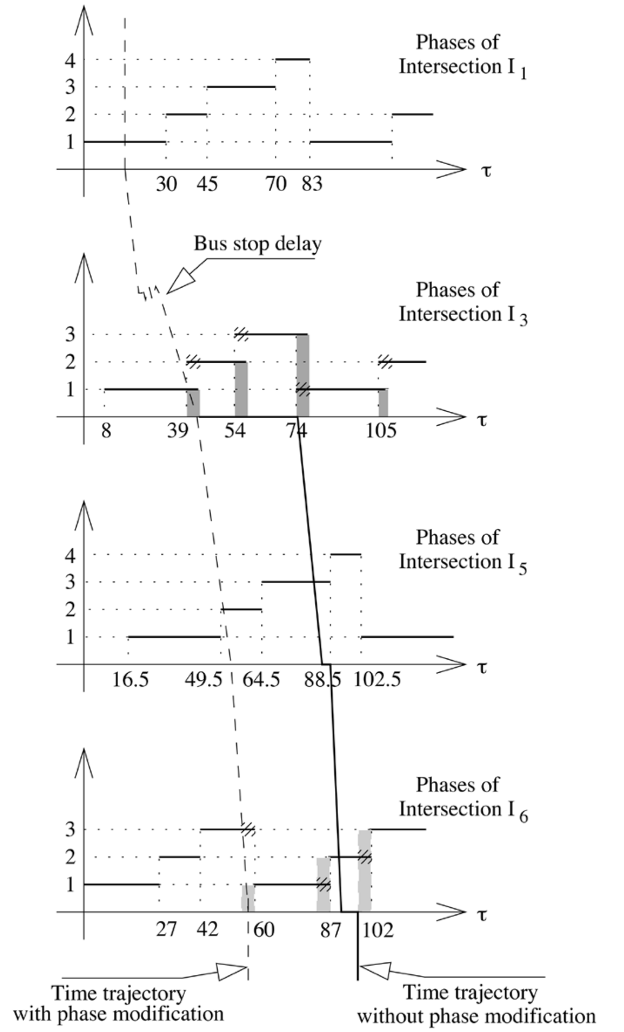

Fig. 8. Time diagrams of the phases of the net in Fig. 5 and trajectories of the public vehicle.

TABLE VII
WEIGHTS OF THE ARCS OF THE GRAPHS IN FIG. 5

Weight	Value
$w_{O,1}$	-
$w_{1,2}$ and $w_{2,1}$	7.5 s
$w_{1,3}$ and $w_{3,1}$	9.2 s
$w_{2,4}$	8.6 s
$w_{3,5}$ and $w_{5,3}$	10 s
$w_{3,4}$	6.3 s
$w_{4,6}$ and $w_{6,4}$	7.5 s
$w_{5,6}$ and $w_{6,5}$	5.1 s
$w_{6,D}$	-

Then, as regards the length of the routes, they are 295 m for $\mathcal{P}_1$, 310 m for $\mathcal{P}_2$, and 290 m for $\mathcal{P}_3$. Thus, from a topological point of view, the shortest route results to be the third one. However, the travel times on the three admissible routes depend on the traffic light signal the emergency vehicle finds at each

TABLE VIII
COSTS OF THE PROBLEM OF THE EXAMPLE IN SECTION V-B

Route	Intersections and costs		T_i	C_i^{tot}
$\mathcal{P}_1$	I_5 $C_0 = 14.57$, $C_1 = 14.92$, $C_2 = \mathbf{13.02}$	I_6 $C_0 = \mathbf{18.11}$, $C_1 = 20.37$, $C_2 = 19.09$	35 s	31.13
$\mathcal{P}_2$	I_1 $C_0 = 15.12$, $C_1 = 24$, $C_2 = \mathbf{14.54}$	I_4 $C_0 = 13.39$, $C_1 = 29.11$, $C_2 = \mathbf{12.08}$	30.5 s	**26.62**
$\mathcal{P}_3$	I_3 $C_0 = 15$, $C_1 = 25.07$, $C_2 = \mathbf{14.38}$	I_4 $C_0 = 15.24$, $C_1 = \mathbf{15.03}$, $C_3 = 17.12$	**24.8** s	29.4

intersection. The weights of the graph in Fig. 6 are reported in Table VII.

Suppose that the privileged vehicle arrives at the north incoming direction of I_1 at $\tau_1 = \tau_0 + 25$ s. The supervisor has to choose one among the following routes.

$\mathcal{P}_1$: If the vehicle follows the first route, it crosses I_1 toward I_3 without stopping. Then, due to the traffic behavior, it reaches I_3 at time

$$\tau_2^{\mathcal{P}_1} = \tau_1 + w_{1,3} = 25\ s + 9.2\ s = 34.2\ s \tag{34}$$

and crosses it without stopping toward I_5, where it arrives at

$$\tau_3^{\mathcal{P}_1} = \tau_2^{\mathcal{P}_1} + w_{3,5} = 34.2\ s + 10\ s = 44.2\ s \tag{35}$$

finding the first phase whereas it needs the second. Thus, a local priority control problem has to be solved. The resulting costs are those reported in Table VIII. As is clear, the best choice for the controller is to shorten the first phase. With this change, the privileged vehicle can cross the intersection toward I_6 where it arrives at

$$\tau_4^{\mathcal{P}_1} = \tau_3^{\mathcal{P}_1} + w_{5,6} = 44.2\ s + 5.1\ s = 49.3\ s \tag{36}$$

finding the third phase instead of the needed first. Thus, a second control problem has to be solved. From the cost analysis (again see Table VIII), it is clear that the best choice is to do nothing; that is, the vehicle has to wait until a new cycle begins in $\tau_4^{\mathcal{P}_1} = 60$ s. At this time, it can proceed. The travel time on this route is $T_1 = 35$ s and the total cost is $C_1^{\text{tot}} = 31.13$.

$\mathcal{P}_2$: If the vehicle follows the second route, it finds, at I_1, the first phase of the cycle instead of the second one, which it needs. Thus, a local priority control problem has to be solved at I_1. Because the results are those reported in Table VIII, the best choice is to shorten the first phase; however, due to the presence of a constraint on the minimum length of the first phase, the vehicle should have to wait for $r_1^{\mathcal{P}_2} = 3$ s. Then, it can proceed toward I_2, where it arrives at

$$\begin{aligned}\tau_2^{\mathcal{P}_2} &= \tau_1 + r_1^{\mathcal{P}_2} + w_{1,2} \\ &= 25\ s + 3\ s + 7.5\ s = 35.5\ s\end{aligned} \tag{37}$$

and it can proceed, without stopping, toward I_4, where it arrives at

$$\tau_3^{\mathcal{P}_2} = \tau_2^{\mathcal{P}_2} + w_{2,4} = 35.5\ s + 8.6\ s = 44.1\ s. \tag{38}$$

At this time, it finds the third phase, whereas it needs the first. Then, another local priority control problem has to be solved, which gives the costs reported in Table VIII. As is clear in such a table, the best choice for the controller is to shorten the current (third) phase, but as for the previous problem, it has to wait for $r_2^{\mathcal{P}_2} = 3.9$ s, due to the presence of a constraint on the minimum length of the third phase. After this waiting time, the privileged vehicle can cross the intersection toward I_6, where it arrives at

$$\begin{aligned}\tau_4^{\mathcal{P}_2} &= \tau_3^{\mathcal{P}_2} + r_2^{\mathcal{P}_2} + w_{4,6} \\ &= 44.1\ s + 3.9\ s + 7.5\ s = 55.5\ s\end{aligned} \tag{39}$$

finding the third phase that gives it the right to turn left toward its destination. The travel time on this route is $T_2 = 30.5$ s and the total cost is $C_2^{\text{tot}} = 26.62$.

$\mathcal{P}_3$: If the vehicle follows the third route, it can cross the intersection toward I_3 without stopping. Due to the traffic behavior, it arrives at I_3 at

$$\tau_2^{\mathcal{P}_3} = \tau_1 + w_{1,3} = 25\ s + 9.2\ s = 34.2\ s \tag{40}$$

during the first phase, as in $\mathcal{P}_1$. However, now it needs the second one. Then, a local priority control problem has to be solved. As is clear in Table VIII, the best choice is to shorten the first phase, but, in any case, the vehicle has to wait for $r_2^{\mathcal{P}_1} = 1.8$ s, because of the presence of a minimum length constraint. Then, with this change, the privileged vehicle can cross the intersection toward I_4, where it arrives at

$$\begin{aligned}\tau_3^{\mathcal{P}_3} &= \tau_2^{\mathcal{P}_3} + r_1^{\mathcal{P}_3} + w_{3,4} \\ &= 34.2\ s + 1.8\ s + 6.3\ s = 42.3\ s\end{aligned} \tag{41}$$

finding the third phase instead of the second one. In such a scenario, a second control problem has to be solved and, in this case, the best choice is to extend the current (second) phase (again see Table VIII). Finally, the vehicle can cross the intersection toward I_6, where it arrives at

$$\tau_4^{\mathcal{P}_3} = \tau_3^{\mathcal{P}_3} + w_{4,6} = 42.3\ s + 7.5\ s = 49.8\ s \tag{42}$$

finding the third phase, which is the one it needs to reach its destination D. The travel time on this route is $T_3 = 24.8$ s and the total cost is $C_3^{\text{tot}} = 29.4$.

Then, by analyzing the results summarized in Table VIII, one can conclude that the choice of $\mathcal{P}_3$ minimizes the travel time, whereas the choice of $\mathcal{P}_2$ introduces the minimum perturbation in the traffic network in terms of "abnormal" queue length with respect to the nominal case. Thus, the supervisor decides whether to choose $\mathcal{P}_2$ or $\mathcal{P}_3$, depending on the selected criterium, i.e., the minimum travel time or the minimum perturbation cost. Note that both these solutions provide a significant decrement of the travel time.

VI. Conclusion

In this paper, the hybrid modeling formalism based on HPNs has been adopted to represent an urban network of signalized intersections. Such a model, which has been validated through real traffic data about the Italian city of Torino, has been used to state and solve the problem of coordinating several traffic lights with the aim of improving the performances of some classes of special vehicles, i.e., public and emergency vehicles. Some experimental results relevant to two real case studies have been reported and discussed.

Work is in progress on the statement and solution of distributed control problems related with the occurrences of discrete events perturbing the nominal traffic behavior, which are not caused by special vehicles or with the presence of several interacting special vehicles.

References

[1] M. Papageorgiou, C. Diakaki, V. Dinopoulou, A. Kotsialos, and Y. Wang, "Review of road traffic control strategies," *Proc. IEEE*, vol. 91, pp. 2043–2067, Dec. 2003.

[2] R. Allsop, "SIGSET: A computer program for calculating traffic capacity of signal-controlled road junctions," *Traffic Eng. Control*, vol. 12, pp. 58–60, 1971.

[3] ——, "SIGCAP: A computer program for assessing the traffic capacity of signal-controlled road junctions," *Traffic Eng. Control*, vol. 17, pp. 338–341, 1976.

[4] G. Improta and G. Cantarella, "Control systems design for an individual signalised junction," *Transp. Res. B*, vol. 18, pp. 147–167, 1984.

[5] J. Little, "The synchronization of traffic signal by mixed-integer-linear programming," *Oper. Res.*, vol. 14, pp. 568–594, 1966.

[6] D. Robertson, "TRANSYT method for area traffic control," *Traffic Eng. Control*, vol. 10, pp. 276–281, 1969.

[7] P. B. Hunt, D. I. Robertson, and R. D. Bretherton, "The SCOOT on-line traffic signal optimization technique," in *Traffic Eng. Control*, vol. 23, 1982, pp. 190–192.

[8] N. Gartner, "OPAC: A Demand-responsive strategy for traffic signal control," U.S. Dept. Transp., Washington, DC, Transp. Res. Rec. 906, 1983.

[9] D. Gazis and R. Potts, "The oversaturated intersection," in *Proc. 2nd Int. Symp. Traffic Theory*, 1963, pp. 221–237.

[10] ——, *Traffic Theory*. Boston, MA: Kluwer, 2002.

[11] N. Gartner, "Prescription for Demand-responsive urban traffic control," U.S. Dept. Transp., Washington, DC, Transp. Res. Rec. 881, 1982.

[12] R. E. Allsop, "Delay-minimizing settings for fixed-time traffic signals at a single road junction," *J. Inst. Math. Appl.*, vol. 8, pp. 164–185, 1971.

[13] M. J. Smith, "A local traffic control policy which automatically maximizes the overall travel capacity of an urban network," *Traffic Eng. Control*, vol. 21, pp. 298–302, 1980.

[14] V. Mauro and C. D. Taranto, "UTOPIA," in *Proc. Int. IFAC/IFIP/IFORS Symp. Control, Computers, and Communications in Transportation*, 1989.

[15] C. G. Cassandras and S. Lafortune, *Introduction to Discrete Event Systems*. Norwell, MA: Kluwer, 1999.

[16] T. Murata, "Petri nets: Properties, analysis, and applications," *Proc. IEEE*, vol. 77, pp. 541–580, Apr. 1989.

[17] F. Di Cesare, P. Kulp, K. Gile, and G. List, *The Applications of Petri Nets to the Modeling Analysis and Control of Intelligent Urban Traffic Networks*, ser. Lecture Notes in Computer Science. Berlin, Germany: Springer-Verlag, 1994, vol. 815.

[18] J. L. Gallego, J. L. Farges, and J. J. Henry, "Design by Petri nets of an intersection signal controller," *Transport. Res. C*, vol. 4, pp. 231–248, 1996.

[19] A. Tzes, S. Kim, and W. R. McShane, "Applications of Petri networks to transportation network modeling," *IEEE Trans. Veh. Technol.*, vol. 45, pp. 391–400, May 1996.

[20] J. Wang, C. Jin, and Y. Deng, "Performance analysis of traffic networks based on stochastic timed Petri net models," in *Proc. IEEE Int. Conf. Engineering of Complex Computer Systems (ICECCS'99)*, 1999, pp. 77–85.

[21] C. Tolba, D. Lefebvre, P. Thomas, and A. El Moudni, "Continuous Petri nets models for the analysis of traffic urban networks," *Proc. Int. Conf. IEEE Trans. Systems, Man, Cybern.*, vol. 2, pp. 1323–1328, 2001.

[22] H. Alla and R. David, "On hybrid Petri nets," *J. Discrete Event Dynam. Syst.*, vol. 8, no. 1–2, pp. 9–40, 2001.

[23] J. Lee and S. Song, "Modeling urban transportation systems with hybrid systems: An overview," in *Proc. IEEE Int. Conf. Intelligent Trasportation Systems*, vol. 2, Oct. 2003, pp. 1269–1272.

[24] A. Di Febbraro and N. Sacco, "A control-oriented model of urban traffic behavior based on hybrid Petri nets," in *Proc. 9th IEEE Med. Conf. Control and Automation*, Dubrovnik, Croatia, June 2001.

[25] ——, "Experimental validation of a hybrid Petri-net based model of urban transportation networks," in *Proc. IFAC Int. Conf. Analysis and Design of Hybrid Systems (ADHS'03)*, June 2003, pp. 111–115.

[26] ——, "On modeling urban transportation networks via hybrid Petri nets," *Control Eng. Practice*, vol. 12, no. 10, pp. 1225–1239, 2004.

[27] J. Lei and U. Ozguner, "Decentralized hybrid intersection control," in *Proc. 40th IEEE Int. Conf. Decision and Control*, vol. 2, Dec. 2001, pp. 1237–1242.

[28] P. Gaetani and R. Minciardi, "Dynamic models and optimal control methods for route guidance in urban traffic networks," in *Proc. IEEE 5th Int. Conf. Intelligent Transportation Systems*, Sept. 2002, pp. 454–459.

[29] A. Di Febbraro and N. Sacco, "Traffic light regulation to privilege some classes of vehicles in an urban transportation system," in *Proc. IEEE Int. Conf. Computational Engineering in Systems Applications (CESA'03_*, July 2003.

[30] A. Di Febbraro, D. Giglio, and N. Sacco, "On controlling privileged vehicles by means of coordinated traffic lights," in *Proc. IEEE Int. Conf. Intelligent Transportation Systems*, Oct. 2003, pp. 1318–1323.

[31] A. Di Febbraro and N. Sacco, "A HPN-based control structure for privileged vehicle performance optimization," in *Proc. IFAC Int. Workshop on Discrete Event Systems (WODES'04)*, Sept. 2004, pp. 43–48.

[32] M. V. D. Berg, A. Hegyi, B. D. Schutter, and J. Hellendoorn, "A macroscopic traffic flow model for integrated control of freeway and urban traffic networks," in *Proc. 42th IEEE Int. Conf. Decision and Control*, vol. 3, Dec. 2003, pp. 2774–2779.

[33] A. Di Febbraro, D. Giglio, and N. Sacco, "Modular representation of urban traffic systems based on hybrid Petri nets," in *Proc. IEEE Int. Conf. Intelligent Transportation Sysiems*, Aug. 2001, pp. 866–871.

[34] H. J. Payne, "Models of freeway traffic and control," in *Sim. Council Proc.*, vol. 1, 1971, pp. 51–61.

[35] M. Papageorgiou, *Applications of Automatic Control Concepts to Traffic Flow Modeling and Control*. Berlin, Germany: Springer-Verlag, 1983.

[36] H. Alla and R. David, "Continuous and hybrid Petri nets," *J. Syst. Circuits Comput.*, vol. 8, no. 1, pp. 159–188, Feb. 1998.

[37] F. Balduzzi and A. Di Febbraro, "On modeling signalized intersections via first-order hybrid Petri nets," in *Proc. Eur. Control Conf.*, Porto, Portugal, Sept. 2000, pp. 1333–1338.

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