Emergency Traffic-Light Control System Design for Intersections Subject to Accidents

Liang Qi, MengChu Zhou, *Fellow, IEEE*, and WenJing Luan

***Abstract*—Petri nets (PNs) are well utilized as a visual and mathematical formalism to model discrete-event systems. This paper uses deterministic and stochastic PNs to design an emergency traffic-light control system for intersections providing emergency response to deal with accidents. According to blocked crossing sections, as depicted by dynamic PN models, the corresponding emergency traffic-light strategies are designed to ensure the safety of an intersection. The cooperation among traffic lights/facilities at those affected intersections and roads is illustrated. For the upstream neighboring intersections, a traffic-signal-based emergency control policy is designed to help prevent accident-induced large-scale congestion. Deadlock recovery, livelock prevention, and conflict resolution strategies are developed. We adopt a reachability analysis method to verify the constructed model. To our knowledge, this is the first paper that employs PNs to model and design a real-time traffic emergency system for intersections facing accidents. It can be used to improve the state of the art in real-time traffic accident management and traffic safety at intersections.**

***Index Terms*—Discrete-event systems, Petri nets, intelligent transportation systems, traffic light control, accident at intersections, traffic congestion.**

I. Introduction

URBAN road intersections are one of the most accident-prone zones in road networks. For different countries and regions, between 30% and 60% of all injury accidents and up to one third of the fatalities occur at intersections [1]. This is due mainly to the fact that accident scenarios at intersections are among the most complex ones, since different categories of road users interact in limited areas with crossing trajectories. To some extent, the high incidence of traffic accidents turns intersections into the hotspots of the primary accident-induced accidents called secondary ones which can bring much more severe damage. Besides, the injured person may suffer secondary injury when moved incautiously. Consequently, it is imperative to effectively enforce the scientific management of intersections having an accident (called accidental intersections in the following discussions) to prevent secondary accidents and injuries.

Traffic incident management (TIM) makes a systematic effort to detect, respond to, and remove traffic accidents. It aims to offer the rapid recovery of traffic safety and capacity [2] and lead to many measurable benefits [3], such as decrease in fuel consumption, accident duration, secondary accidents, and traffic jams. In the past thirty decades, ITS technologies were recognized as valuable tools and being used worldwide in traffic accident detection [4]–[6], verification [6], response [7], and communication [8]. In order to alleviate the induced congestion and reduce the possibility of potential secondary accidents, the accident at an intersection should be cleared as quickly as possible. However, the injured person in the accident should not be moved without rendering first aid unless the casualty is in immediate danger. So far, there are not many studies about how to guarantee the safety of an accidental intersection.

Traffic signals have been effectively used to manage conflicting requirements for the use of road space by allocating the right of way to different sets of mutually compatible traffic movements during distinct time intervals. Much work has been done to develop various strategies in order to promote traffic efficiency, which can be categorized as fixed-timed, traffic responsive [9] and predictive control ones [10]. The first one is widely used in our present urban traffic systems due to its easy implementation and low cost. The second one controls intersections based on inputs detected by physical sensors such as loop detectors or cameras. Its examples are SCOOT [11] and SCATS [12]. Some other works are carried out to improve the safety of road traffic. For example, signal strategies are adopted by Huang *et al.* [13] to keep the traffic safety at intersections of railroads and roadways. Weng *et al.* [14] propose the use of TPNs to model parallel railroad level crossing traffic safety control systems. Huang *et al.* [15] design a traffic control system for emergency vehicles. It clearly describes the traffic light behaviors to support the smooth movement of emergency vehicles. Urgent spectacles appear when emergency vehicles are encountering red light phases at a crossing zone. They can be avoided and emergency vehicles are ensured with the highest priority to go through intersections through such system.

Decreasing those conflicting movements can greatly reduce the potential crashes [1]. Thus, at an accidental intersection, there is a strong demand that traffic flow from some directions should be immediately stopped to prevent the confliction such that any secondary accidents can be avoided from happening,

Manuscript received May 10, 2015; revised July 20, 2015; accepted July 25, 2015. This work was supported in part by the Hong Kong, Macao, and Taiwan Science and Technology Cooperation Program of China under Grant 2013DFM10100. The Associate Editor for this paper was F. Chu. (*Corresponding author: MengChu Zhou.*)

L. Qi and W. Luan are with the Department of Computer Science and Technology, School of Electronics and Information Engineering, Tongji University, Shanghai 201804, China (e-mail: qiliangsdkd@163.com; wenjingmengjing@163.com).

M. Zhou is with The Key Laboratory of Embedded System and Service Computing, Ministry of Education, Tongji University, Shanghai, 201804 China and also with the Helen and John C. Hartmann Department of Electrical and Computer Engineering, Newark College of Engineering, New Jersey Institute of Technology, Newark, NJ 07102 USA (e-mail: zhou@njit.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TITS.2015.2466073

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

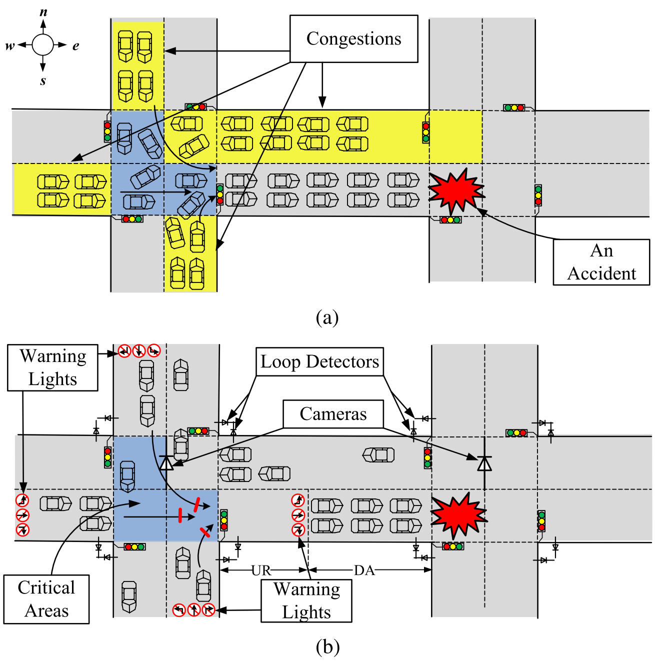

Fig. 1. An accidental intersection and its up-stream neighboring one. (a) Traffic under no emergency control. (b) Traffic under emergency control.

and the injured person can obtain enough time to be removed safely. It is the traffic signal operation strategy that can help us do the work. In fact, different emergency strategies should be adopted according to different positions of blockage induced by accidents at an intersection.

On the other hand, the above stop-flow strategies may induce large-scale congestion while the neighboring intersections are blocked. As an example, Fig. 1(a) illustrates an accidental intersection and its induced congestion. An accident is occurring at the right intersection and blocks it. When the road with two lanes between the two intersections, is full of vehicles, the blue heavy-shaded areas (called critical areas in the following discussions) in the left intersection may be blocked by the corresponding traffic flow. Their blockage induces heavier congestion as shown in the yellow light-shaded areas. An emergency traffic light strategy can be adopted at these neighboring intersections cooperating with the accidental one to stop the traffic flow driving towards its blocked area as shown in Fig. 1(b). Note that if the clearance of the accident costs little time, the former may not be blocked in some cases, e.g., off-peak traffic periods, and thus such strategy may not be needed.

Petri nets (PNs) [16], [17] are a well-founded model applied to the simulation and analysis of a complex discrete event dynamic system. They can well model its important characters such as synchronization, concurrency and cooperative control. Due to their graphical expression and mathematical rigor, they are an effective description and analysis tool for urban traffic [18] and especially for traffic light control systems [19]–[22]. List and Cetin use Petri nets to model the control of signalized intersections in [19], where the signal control logic is designed for eight-phase traffic and the traffic operation safety rules are enforced through this model. Huang *et al.* [20]–[22] model traffic signal alternation that consists of two, six and eight-phase transitions using timed Petri nets. They aim to model complex phase transitions of traffic lights and analyze the related properties via occurrence graphs. Huang *et al.* [13] pioneered in a parallel railroad level crossing (LC) control system by using Deterministic and Stochastic Petri nets (DSPNs). Its critical scenarios are extracted and avoided by controlling the phase of traffic light operations. This keeps the traffic safety related to intersections of railroads and roadways. PNs have also been used in assessment studies of transportation systems [23], [24], modeling and management of collective motion [25], and simulation of container terminals operations [26]. Reachability tree analysis methods [27] and many simulation tools are used to analyze the constructed models.

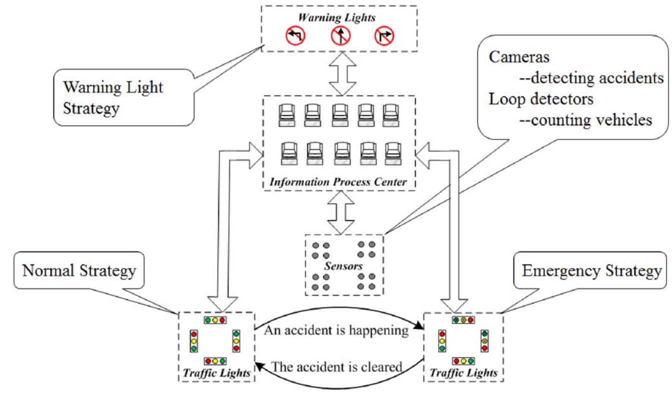

Fig. 2. Architecture of an emergency traffic-light control system for intersections.

This work adopts DSPNs to design a traffic-light control system for an accidental intersection and its up-stream neighboring intersections preventing secondary injuries and accident-induced traffic congestion. Sensing actuators can be installed at an intersection, e.g., inductive loops are adopted to detect vehicle motion via the vibration [28] while cameras are used to percept road traffic collisions at an intersection visually [29], [30]. Here, three kinds of facility are installed at each intersection as shown in Fig. 1: a set of traffic lights, several sensors and warning lights. The architecture of our proposed system is described in Fig. 2. Sensors are used to gather real-time traffic at intersections: cameras are used to detect accidents, and loop detectors are used to count the number of vehicles. Warning facility is used for sending the accidental warning signals to vehicles. An Information Processing Center (IPC) is deployed. By processing the real time traffic information obtained by sensors, normal traffic light strategies and emergency ones are designed. Also, the facility needs proper cooperation, which can be described by the PN model. Through these PN-based formal models, the correctness of the control system can be ensured. Then the system can be implemented directly with a hardware design.

The rest of this paper is organized as follows. Section II presents a signalized intersection, traffic network, and traffic-light control. Section III reviews DSPNs, and constructs a normal traffic light strategy and other facility's operation models with DSPNs. Section IV presents the emergency strategies including emergency traffic light strategies for an accidental intersection as well as for its up-stream neighboring ones according to the critical crossing sections blocked by an accident, and emergency warning light strategies. Some deadlock recovery, livelock prevention, and conflict resolution strategies are presented. Property analysis of the constructed Petri net models

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

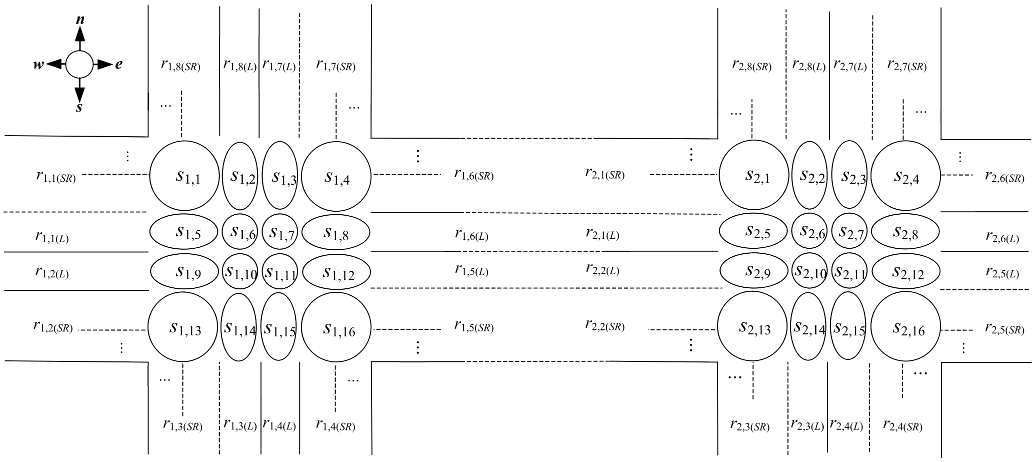

Fig. 3. Traffic network.

is conducted in Section V. Concluding remarks are made in Section VI.

II. Signalized Intersections

In this section, firstly, a traffic network is presented. Then a set of traffic light phase transitions used are designed. We adopt the idea from [31]–[34] to divide each intersection into a finite number of parts called crossing sections, and regulate their occupation by traffic flows at corresponding traffic light phases. We present some basic notations that will be used in the following content: $\mathbf{R}$ is a real number set, $\mathbf{R}^+$ is the set of positive real numbers, $\mathbf{N} = \{0, 1, 2, \ldots\}$ is a natural number set, $\mathbf{N}^+ = \mathbf{N}/\{0\}$ is the positive integer set, $\mathbf{N}_k = \{0, 1, 2, \ldots, k\}$ and $\mathbf{N}_k^+ = \{1, 2, \ldots, k\}$ where $k \in \mathbf{N}^+$.

A. Traffic Network

In this paper, we consider a two-way grid network as shown in Fig. 3. In the network, each intersection I_i, $i \in \mathbf{N}^+$, connects eight roads $r_{i,1} - r_{i,8}$ which model eastbound, westbound, southbound and northbound directions, respectively. A road is a physical area used by vehicles to pass from one intersection to another. As shown in Fig. 3, intersections I_i, where $i \in \mathbf{N}_2^+$ connect eight roads $r_{i,1} - r_{i,8}$, where in $r_{i,1}$, $r_{i,3}$, $r_{i,5}$, and $r_{i,7}$, vehicles coming out from I_i drive to west, south, east and north, respectively; in $r_{i,2}$, $r_{i,4}$, $r_{i,6}$, and $r_{i,8}$, vehicles coming from west, south, east and north, respectively, enter I_i. Each road $r_{i,j}$, $j \in \mathbf{N}_8^+$ is characterized by at least two lanes which are categorized into two groups: $r_{i,j(L)}$ denoting the left-most lane of $r_{i,j}$ that is used for vehicles turning left, and named as a left-turn lane (L-lane for short); and $r_{i,j(SR)}$ denoting another lane that is used for vehicles driving straight and turning right, and named as a straight-right lane (SR-lane for short). Vehicles entering the intersection and turning left are out from $r_{i,j(L)}$ while those driving straight or turning right are out from $r_{i,j(SR)}$. Note that, $r_{1,6}$ $(r_{1,5})$ and $r_{2,1}$ $(r_{2,2})$ are the same road. For convenience, we denote $r_{i'i}$ as the road transferring vehicles from I_i' to I_i. I_i' is called an up-stream neighboring intersection of I_i. Thus, $r_{1,6}$ $(r_{1,5})$ and $r_{2,1}$ $(r_{2,2})$ are denoted by r_{21} (r_{12}). Each intersection has four up-stream neighboring intersections.

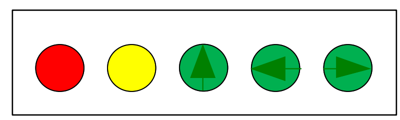

Fig. 4. Five signal lights in a light set.

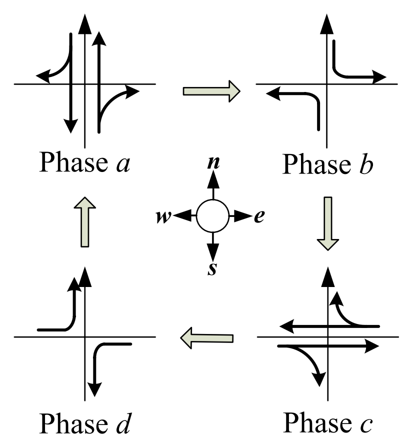

Fig. 5. Four phase transitions of the traffic control system.

B. Traffic-Light Control

In this paper, four traffic light sets are deployed at directions north ($\boldsymbol{n}$), east ($\boldsymbol{e}$), south ($\boldsymbol{s}$), and west ($\boldsymbol{w}$). Each set has five signal lights: a red one (R), a yellow one (Y), three green ones with a straight arrow (GS), a left turn arrow (GL), and a right turn arrow (GR), respectively, as shown in Fig. 4. The flows are ruled by four-phase traffic light rules shown in Fig. 5, where four phases form a cycle, i.e., from Phases a to d, and then back to a. For convenience, we denote X_n, Y_e, Z_s, and W_w as traffic lights with certain colors along directions $\boldsymbol{n}$, $\boldsymbol{e}$, $\boldsymbol{s}$, and $\boldsymbol{w}$, respectively, where $X, Y, Z, W \in \{R, Y, GS, GL, GR\}$. The detailed operations are described as follows.

1) Phase a: GR and GS signals turn on in the northbound and southbound directions, i.e., GR_n, GS_n, GR_s, and GS_s are on, and the red signals are displayed in the east-westward directions, i.e., R_e and R_w are on. The straight-driving traffic flow is from north (south) to south (north)

denoted by F_{ns} (F_{sn}), and right-turning one towards west (east) denoted by F_{nw} (F_{se}).

2) Phase b: GL signals turn on in the northbound and southbound directions, i.e., GL_n and GL_s are on, and the red signals are displayed in the east-westward directions, i.e., R_e and R_w are on. The left-turning traffic flow is from north (south) to east (west) denoted by F_{ne} (F_{sw}).
3) Phase c: GR and GS signals turn on in the eastbound and westbound directions, i.e., GR_e, GS_e, GR_w, and GS_w are on, and the red signals are displayed in the north-southward directions, i.e., R_n and R_s are on. The straight-driving traffic flow is from east (respectively, west) to west (respectively, east) denoted by F_{ew} (F_{we}), and the right-turning one towards north (south) denoted by F_{en} (F_{ws}).
4) Phase d: GL signals turn on in the eastbound and westbound directions, i.e., GL_e and GL_w are on, and the red signals are displayed in the north-southward directions, i.e., R_n and R_s are on. The left-turning traffic flow is from east (west) to south (north) denoted by F_{es} (F_{wn}).

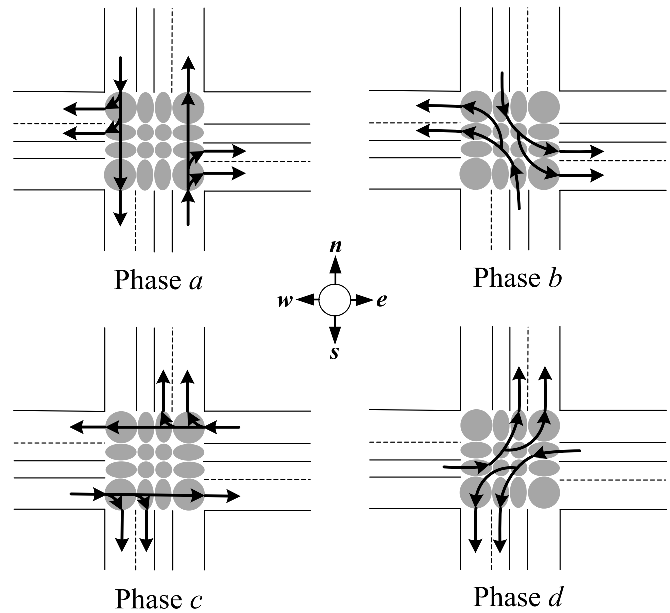

Fig. 6. Crossing section occupation during the four phases.

C. Crossing Sections

In order to model the behavior of vehicles when crossing an intersection, according to [31]–[34], we divide each intersection into a finite number of physical spaces, which can easily and efficiently depict vehicle driving paths within an intersection in a microscopic representation way. The parts in which an intersection is physically divided are named as crossing sections. According to the lanes we divided previously, three kinds of crossing sections are categorized: L-section that is the intersection of two L-lanes, LS-section that is the intersection of an L-lane and SR-lane, and SR-section that is the intersection of two SR-lanes. As shown in Fig. 3, each intersection I_i is divided into 16 crossing sections $s_{i,j}$, $j \in \mathbf{N}_{16}^{+}$, where there are 4 L-sections, i.e., $s_{i,6}$, $s_{i,7}$, $s_{i,10}$, and $s_{i,11}$, 8 LS-sections, i.e., $s_{i,2}$, $s_{i,3}$, $s_{i,5}$, $s_{i,8}$, $s_{i,9}$, $s_{i,12}$, $s_{i,14}$, and $s_{i,15}$, and 4 SR-sections, i.e., $s_{i,1}$, $s_{i,4}$, $s_{i,13}$, and $s_{i,16}$. Notice that, there are 12 vehicle movement paths during the four phases discussed in the former sub-section. We define a section-occupation path over a traffic flow F as a set of physical space sequences, formally as $l_F = \{r_{i,m} \rightarrow s_{i,j1} \rightarrow s_{i,j2}, \ldots \rightarrow s_{i,jk} \rightarrow r_{i,n} | i \in \mathbf{N}^{+}$, and m, $n \in \mathbf{N}_8^{+}$, $r_{i,m}$ and $r_{i,n}$ are two lanes connected by intersection $I_i, s_{i,j1}, s_{i,j2}, \ldots, s_{i,jk}$ are k crossing sections$\}$, i.e., a vehicle in F can move along a path from one lane $r_{i,m}$, through a sequence of crossing sections, and to another lane $r_{i,n}$. The section-occupation paths at intersection I_i for traffic flows are clearly depicted in Fig. 6 and formally presented next:

- Fig. 6 indicates the mentioned traffic flows of phase a coming from $r_{i,4(SR)}$ and $r_{i,8(SR)}$ and driving straight towards $r_{i,7(SR)}$ and $r_{i,3(SR)}$, respectively, and turning right towards $r_{i,5}$ and $r_{i,1}$, respectively. F_{ns} has one section-occupation path: $l_{Fns} = \{r_{i,8(SR)} \rightarrow s_{i,1} \rightarrow s_{i,5} \rightarrow s_{i,9} \rightarrow s_{i,13} \rightarrow r_{i,3(SR)}\}$; F_{sn} has one path: $l_{Fns} = \{r_{i,4(SR)} \rightarrow s_{i,16} \rightarrow s_{i,12} \rightarrow s_{i,8} \rightarrow s_{i,4} \rightarrow r_{i,7(SR)}\}$; F_{nw} has two paths: $l_{Fnw} = \{r_{i,8(SR)} \rightarrow s_{i,1} \rightarrow r_{i,1(SR)}, r_{i,8(SR)} \rightarrow s_{i,1} \rightarrow s_{i,5} \rightarrow r_{i,1(L)}\}$; and F_{se} has two paths: $l_{Fse} = \{r_{i,4(SR)} \rightarrow s_{i,16} \rightarrow r_{i,5(SR)}, r_{i,4(SR)} \rightarrow s_{i,16} \rightarrow s_{i,12} \rightarrow r_{i,5(L)}\}$. Note that lane-changing movements for the straight driving flow are forbidden in all intersections. In addition, during the right-turning flow, $s_{i,12}$ (respectively, $s_{i,5}$) may not be occupied until the proceeding area of $s_{i,16}$ (respectively, $s_{i,1}$) in $r_{i,5(SR)}$ (respectively, $r_{i,1(SR)}$) is blocked by vehicles or other obstacles.
- Fig. 6 indicates the mentioned traffic flows of phase b coming from $r_{i,4(L)}$ and $r_{i,8(L)}$ and turning left towards $r_{i,1}$ and $r_{i,5}$, respectively, where F_{sw} has two section-occupation paths: $l_{Fsw} = \{r_{i,4(L)} \rightarrow s_{i,15} \rightarrow s_{i,10} \rightarrow s_{i,5} \rightarrow r_{i,1(L)}, r_{i,4(L)} \rightarrow s_{i,15} \rightarrow s_{i,10} \rightarrow s_{i,6} \rightarrow s_{i,1} \rightarrow r_{i,1(SR)}\}$; and F_{ne} has two paths as well: $l_{Fne} = \{r_{i,8(L)} \rightarrow s_{i,2} \rightarrow s_{i,7} \rightarrow s_{i,12} \rightarrow r_{i,5(L)}, r_{i,8(L)} \rightarrow s_{i,2} \rightarrow s_{i,7} \rightarrow s_{i,11} \rightarrow s_{i,16} \rightarrow r_{i,5(SR)}\}$. Note that, the left-turn flow from south has to occupy $s_{i,15}$ and then $s_{i,10}$, while those from north must occupy $s_{i,2}$ and then $s_{i,7}$.

Traffic flow in phases c and d can be analyzed similarly like that in phases a and b, but in a reverse order. When we divide the intersection into several crossing sections, and the relevant section-occupation paths are defined exactly for each traffic flow, the impact of blockage of crossing sections towards traffic flow with regard to the traffic light phases becomes apparent.

III. Normal Traffic Light Strategy and Other Facility's Operation Models

In this section, the DSPN concept is reviewed. Then we present a DSPN construction process for the normal traffic light strategy, the involved sensors' operation as well as warning lights' operation.

A. DSPN Model

A PN is a kind of bipartite directed graphs populated by three types of objects. They are places, transitions, and directed

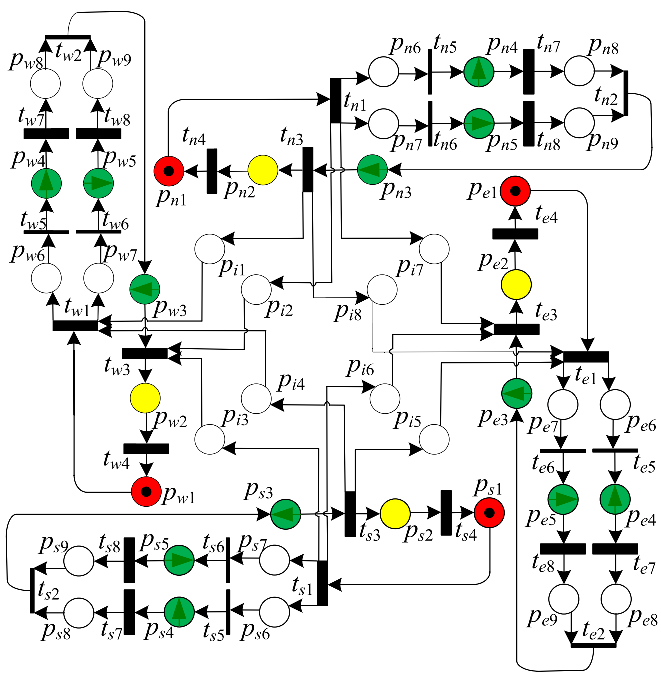

Fig. 7. DSPN model of a normal traffic light strategy at intersection I_i.

arcs connecting places to transitions and transitions to places [17]. Deterministic and stochastic delays can be associated with transitions in PNs, leading to DSPNs [35], [36].

Definition 1: A DSPN is a 8-tuple $\Sigma = (P, T, F, W, H, M_0, \tau, \lambda)$, where

1) P is a finite set of places;
2) $T = T_I \cup T_d \cup T_e$ is a finite set of transitions, partitioned into three disjoint sets, T_I, T_d, and T_e, representing immediate, deterministic and exponential ones, respectively, with $P \cup T \neq \varnothing$ and $P \cap T = \varnothing$;
3) $F \subseteq (P \times T) \cup (T \times P)$ is a set of directed arcs;
4) $H \subseteq P \times T$ is a set of inhibitor arcs from P to T with $H \cap F = \varnothing$;
5) $W : (P \times T) \cup (T \times P) \to \mathbf{N}$ is a weight function, where $W(f) > 0$ if $f \in F \cup H$ and otherwise $W(f) = 0$;
6) $M : P \to \mathbf{N}$ is a marking function, where $\forall\, p \in P$, $M(p)$ is the number of tokens in p, and M_0 denotes the initial marking;
7) $\tau : T_d \to \mathbf{R}^+$ is the firing time for deterministic transitions;
8) $\lambda : T_e \to \mathbf{R}^+$ is the firing rate vector whose element $\lambda(t)$ is the firing rate of transition t that is associated with exponentially distributed time delay.

In a DSPN, an immediate transition can fire immediately when it becomes enabled; a deterministic transition t is attached with a fixed enabling duration $\tau(t)$ that is known, and if t becomes enabled, it waits for a time duration $\tau(t)$, and after that, fires instantaneously; and a stochastic transition t is associated with an exponentially distributed random delay function with a firing rate $\lambda(t)$, and its enabling duration complies with the exponential distribution. Graphically, immediate, deterministic and stochastic transitions are represented by thin, thick black and empty bars, respectively. An inhibitor arc is used to decrease the model complexity. It links a place p to a transition t with a small circle attached to t. By default, as for usual arcs, an inhibitor arc has a weight of 1. Thereby, it prevents t from firing as long as p is marked. It can also have a weight n, in which case, it forbids the firing of t while there are at least n tokens in p. For convenience, G is denoted as the global time and G_0 the initial time.

When a DSPN model is built, according to its firing rules [35], [36], its reachability graph can be constructed. In order to reduce the size of the graph, each arc corresponds to a sequence of transitions containing only one timed transition (deterministic or stochastic one). Besides, we put them in parentheses and then attached to the corresponding arc if more than one transition fires at the same time.

TABLE I

MEANING OF TRANSITIONS AND PLACES AND TRANSITION DELAYS OF FIG. 7 WHERE $\chi \in \{\boldsymbol{n}, \boldsymbol{e}, \boldsymbol{s}, \boldsymbol{w}\}$

P	Meaning	T	Meaning	τ (Sec.)
$p_{\chi 1}$	R	$t_{\chi 1}$	Changing R to GS and GR	5
$p_{\chi 2}$	Y	$t_{\chi 7}$ and $t_{\chi 8}$	Changing GS and GR to GL	20
$p_{\chi 3}$	GL	$t_{\chi 3}$	Changing GL to Y	10
$p_{\chi 4}$	GS	$t_{\chi 4}$	Changing Y to R	3
$p_{\chi 5}$	GR	others	Immediate ones with no meaning but for a control purpose	
$p_{\chi 6}$-$p_{\chi 9}$, p_{i1}-p_{i8}	No meaning but for a control purpose			

B. Traffic-Light Control Model

We are interested in a control policy for an intersection where the following statements are true [13]:

- The order of the traffic lights forms a cycle;
- The system starts when the lights are all red;
- A short yellow duration is adopted among transitions from green to red; and
- To insure traffic safety, all traffic lights are all red when the phase transition is completed.

The DSPN model of normal strategy of traffic lights is depicted in Fig. 7, where we specify that the capacity of each place is 1 (i.e., the net is 1-bounded). The meanings of places and transitions with their enabling durations are given in Table I. For convenience, let $\chi \in \{\boldsymbol{n}, \boldsymbol{e}, \boldsymbol{s}, \boldsymbol{w}\}$, $\xi \in \{\boldsymbol{n}, \boldsymbol{s}\}$, and $\zeta \in \{\boldsymbol{e}, \boldsymbol{w}\}$. Suppose that the global time is $G_0 = 0$ at the initial state, that is, only $p_{\chi 1}$ contains a token and $t_{\xi 1}$ is enabled at G_0. They fire after 5 seconds when $G = 5$ where a token is moved from $p_{\xi 1}$ to $p_{\xi 6}$ and $p_{\xi 7}$, respectively, enabling and firing $t_{\xi 5}$ and $t_{\xi 6}$, immediately, while each of p_{i2}, p_{i3}, p_{i6}, and p_{i7} obtains a token. $p_{\xi 4}$ and $p_{\xi 5}$ obtain a token when $t_{\xi 5}$ and $t_{\xi 6}$ fire, thus resulting in that $t_{\xi 7}$ and $t_{\xi 8}$ are enabled. It denotes that the traffic lights GR_ξ and GS_ξ are turning on from the global time 5 to 25. This means that the lights change from R_ξ to GR_ξ and GS_ξ and the duration time of GR_ξ and GS_ξ are 20 seconds. $t_{\xi 7}$ and $t_{\xi 8}$ with $\tau(t_{\xi 7}) = \tau(t_{\xi 8}) = 20$ fire after

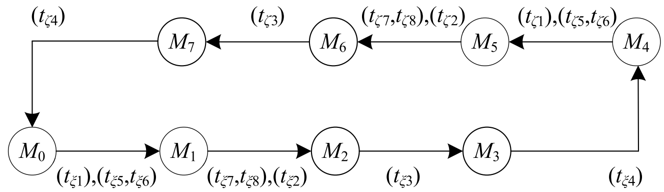

Fig. 8. The reachability graph of the model in Fig. 7, where $\xi \in \{\boldsymbol{n}, \boldsymbol{s}\}$ and $\zeta \in \{\boldsymbol{e}, \boldsymbol{w}\}$.

20 seconds at $G = 25$. Then a token is moved from $p_{\xi 4}$ and $p_{\xi 5}$ to $p_{\xi 8}$ and $p_{\xi 9}$, respectively, enabling and firing $t_{\xi 2}$ with a token deposited to $p_{\xi 3}$. It denotes that the traffic lights GL_ξ are turning on from the global time 25 to 35. That means the light changes from GL_ξ to Y_ξ and the duration time of GL_ξ is 10 seconds. Then $t_{\xi 3}$ is fired resulting in that $p_{\xi 2}$ are marked with a token. At the same time, p_{i1}, p_{i4}, p_{i5}, and p_{i8} obtain a token. Then $t_{\xi 4}$ with $\tau(t_{\xi 4}) = 3$ and $t_{\zeta 1}$ with $\tau(t_{\zeta 1}) = 5$ are enabled and fire at $G = 38$ and $G = 40$, respectively. The duration time of Y_ξ is 3 seconds. Traffic light turns to GS_ζ and GR_ζ after Y_ξ turns to R_ξ. Meanwhile, $t_{\xi 1}$ is disabled because the 1-capacity places p_{i2}, p_{i3}, p_{i6} and p_{i7} contain tokens. Then the traffic lights turn successively to GS_ζ and GR_ζ, GL_ζ, Y_ζ, and return to their initial state when $G = 73$. Transitions $t_{\xi 1}$ enabled at $G = 70$ fire when $G = 75$, and the same transition firing sequence is repeatedly executed. The reachability graph is presented in Fig. 8, where $M_0 - M_7$ are all markings, and for convenience, we denote M as the state of the lights. Thus $M_0 = (R_n, R_e, R_s, R_w)$, $M_1 = (GS_n, GR_n, R_e, GS_s, GR_s, R_w)$, $M_2 = (GL_n, R_e, GL_s, R_w)$, $M_3 = (Y_n, R_e, Y_s, R_w)$, $M_4 = (R_n, R_e, R_s, R_w)$, $M_5 = (R_n, GS_e, GR_e, R_s, GS_w, GR_w)$, $M_6 = (R_n, GL_e, R_s, GL_w)$, and $M_7 = (R_n, Y_e, R_s, Y_w)$. Since this graph is a finite circle containing all transitions, the DSPN model in Fig. 7 is live and reversible. Thus, the time necessary to complete a circle from marking M_i, $i \in \mathbf{N}_7$ to the same one is 70 except that the time needed for M_0 to M_0 for the first time is 73. Note that the order of the traffic lights forms a cycle directing the traffic light phases operating in a cycle "$a \to b \to c \to d \to a$".

C. Other Facility's Operation Models

We deploy sensors including cameras and loop detectors at intersections to help us collect the traffic data, i.e., accident information and the number of vehicles in roads. With these sensors we can build the emergency traffic light strategies. In detail, a camera is employed at each intersection I_i, for the purpose of gathering accident information including its occurrence, position and clearance [29], [30]. Eight detectors that are denoted by $C_{i,j}$, $j \in \mathbf{N}_8^+$ and installed at the connection of roads r_j and intersection I_i, respectively, are used to count the number of vehicles driving from $r_{i,j}$ (I_i) into I_i $(r_{i,j})$. Besides, the electronic display or warning lights are warning facility used for sending the accidental warning signals to vehicles. Here we adopt a warning light as the warning facility example. In fact, a warning light set is installed at the midpoint of a road $r_{i'i}$. Each set has three lights, as shown in Fig. 9, denoted by $w_{Li',i}$, $w_{Si',i}$, and $w_{Ri',i}$, warning vehicles that intend to turn left, drive straight and turn right at the downstream I_i, respectively. Thus, when the downstream traffic flow in $r_{i'i}$ is interrupted, the lights can warn traffic flow not entering the downstream conflict crossing sections. Now we construct a DSPN-based model for each kind of facility as follows.

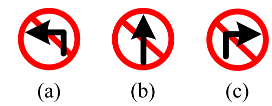

Fig. 9. Three warning lights installed at the midpoint of road $r_{i'i}$: (a) $w_{Li',i}$, (b) $w_{Si',i}$, and (c) $w_{Ri',i}$.

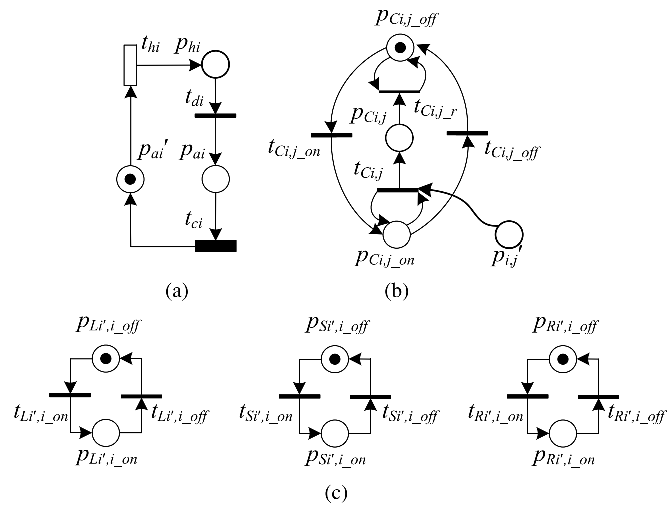

Fig. 10. Models of sensors and warning lights at intersection I_i: (a) A DSPN of the accident detection with camera S_i, (b) A DSPN of vehicle counting with loop detector $C_{i,j}$ where $j \in \mathbf{N}_8^+$, and (c) DSPNs of warning lights $w_{Li',i}$, $w_{Si',i}$, and $w_{Ri',i}$.

TABLE II
MEANING OF PLACES AND TRANSITIONS IN FIG. 10 WHERE $j \in \mathbf{N}_8^+$, AND $\delta \in \{L, S, R\}$

P	Meaning	T	Meaning
p_{hi}	An accident is happening	t_{hi}	Having an accident
p_{ai}	An accident is detected	t_{di}	Detecting that an accident is happening
p_{ai}'	No accident is detected	t_{ci}	Detecting that the accident is cleared
p_{Ci,j_on}	$C_{i,j}$ is on	t_{Ci,j_on}	Turning $C_{i,j}$ on
p_{Ci,j_off}	$C_{i,j}$ is off	t_{Ci,j_off}	Turning $C_{i,j}$ off
$p_{Ci,j\_}$	Vehicles that are counted by $C_{i,j}$	$t_{Ci,j}$	Counting vehicles by $C_{i,j}$
$p_{i,j}'$	Vehicles that have passed $C_{i,j}$	t_{Ci,j_r}	Resetting the count of vehicles by $C_{i,j}$ to 0
$p_{\delta i',i\_on}$	$w_{\delta i',i}$ is on	$t_{\delta i',i\_on}$	Turning $w_{\delta i',i}$ on
$p_{\delta i',i\_off}$	$w_{\delta i',i}$ is off	$t_{\delta i',i\_off}$	Turning $w_{\delta i',i}$ off

1) We build a DSPN model of the accident detection of a camera at intersection I_i as shown in Fig. 10(a) (see Table II for the meaning of the places and transitions in this figure). The firing of t_{hi} depicts the occurrence of an accident. It is an infrequent event and we adopt an exponential transition to depict it. t_{di} models that the accident is detected by the camera. A token is filled in p_{ai} if t_{di} fires. Then t_{ci} fires if the clearing process is finished.

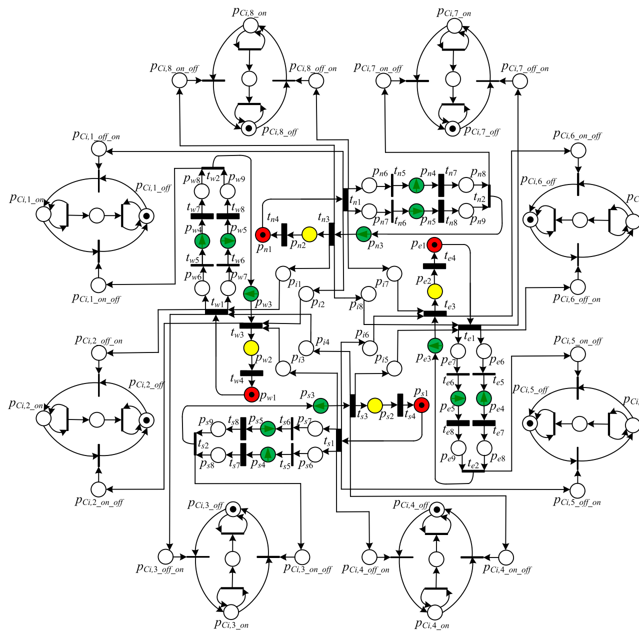

Fig. 11. Counter operations controlled by a traffic light strategy.

The duration of accident clearance is described by the enabling duration of a deterministic transition, i.e., t_{ci}. Note that when t_{ci} has a stochastic time delay, the DSPN analysis is more complex. Finally, we suppose that there is no overlap between consecutive accidents, as modeled by place $p_{ai}{}'$ containing only one token.

2) The DSPN model of vehicle counting of a loop detector $C_{i,j}$ is described in Fig. 10(b). Its initial marking is that there exists a token in p_{Ci,j_off} describing that $C_{i,j}$ is not working. When $C_{i,j}$ starts to work, t_{Ci,j_on} fires by moving a token from p_{Ci,j_off} to p_{Ci,j_on}. By firing $t_{Ci,j}$, $C_{i,j}$ counts vehicles that have passed $C_{i,j}$ and are denoted by tokens in $p_{i,j}{}'$. Note that $p_{i,j}{}'$ is used as the interface between the loop detector and traffic flows. The total number of vehicles sensed by $C_{i,j}$ during its working interval time in a phase cycle equals to the number of tokens in $p_{Ci,j}$. The firing of t_{Ci,j_off} depicts that $C_{i,j}$ is closed. Then a token is filled in p_{Ci,j_off} and t_{Ci,j_r} is enabled if $p_{Ci,j}$ contains any tokens. By firing t_{Ci,j_r} repeatedly, $C_{i,j}$ is reset resulting in the vehicle count being cleaned.

3) We build the warning light model as shown in Fig. 10(c) describing that warning lights $w_{\delta i',i}$, $\delta \in \{L, S, R\}$ switches between on and off depicted by $p_{\delta i',i\_on}$ and $p_{\delta i',i\_off}$, respectively. Transitions $t_{\delta i',i\_on}$ and $t_{\delta i',i\_off}$ are immediate.

Fig. 11 focuses on the loop detectors' switch work controlled by traffic light strategies at intersection I_i. The loop detectors are controlled by the traffic light strategy such that the sensors from the corresponding directions are on when vehicles are permitted to pass while others sleep for saving energy. For example, at the initial state in the model, all loop detectors do not work. Then $t_{\xi 1}$, $\xi \in \{\boldsymbol{n}, \boldsymbol{s}\}$ fires, and $t_{\xi 5}$ and $t_{\xi 6}$ fire immediately, such that GS_i and GR_i are on and Phase a begins. Loop detectors $C_{i,1}$, $C_{i,4}$, $C_{i,5}$, and $C_{i,8}$ supervising

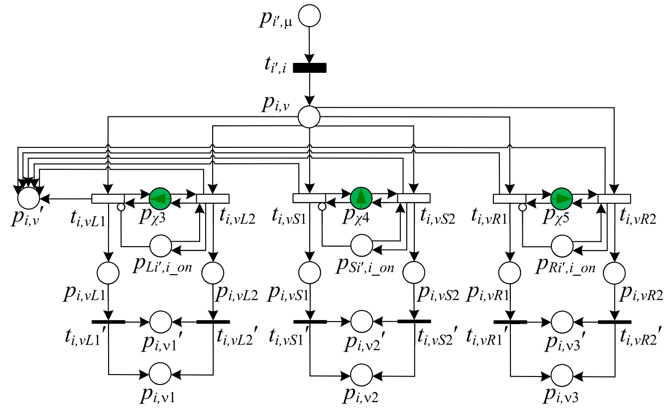

Fig. 12. DSPN model of traffic flow, where $\chi \in \{\boldsymbol{n}, \boldsymbol{e}, \boldsymbol{s}, \boldsymbol{w}\}$.

TABLE III

MEANING OF PLACES AND TRANSITIONS IN FIG. 12 WHERE $\delta \in \{L, S, R\}$, AND $\mu,\ v,\ v1 - v3 \in \mathbf{N}_8^+$

P	Meaning	T	Meaning
$p_{i',\mu}$	Vehicles coming from I_i' into $r_{i',\mu}$	$t_{i',i}$	Passing through $r_{i'i}$
$p_{i,v}$	Vehicles to drive from $r_{i,v}$ into I_i	$t_{i,v\delta 1}$ ($t_{i,v\delta 2}$)	To drive from $r_{i,v}$ into I_i and towards direction δ with a rate $\lambda_{i,v\delta 1}$ ($\lambda_{i,v\delta 2}$) when $w_{\delta i',i}$ is off (on) with $\lambda_{i,v\delta 1}$>$\lambda_{i,v\delta 2}$
$p_{i,v}{}'$	Vehicles that have driven from $r_{i,v}$ into I_i	$t_{i,v\delta 1}{}'$ ($t_{i,v\delta 2}{}'$)	Driving towards direction δ from I_i when $w_{\delta i',i}$ is off (on)
$p_{i,v\delta 1}$ ($p_{i,v\delta 2}$)	Vehicles that have driven from $r_{i,v}$ into I_i and that will drive towards direction δ when $w_{\delta i',i}$ is off (on)		
$p_{i,v1}$-$p_{i,v3}$ ($p_{i,v1}{}'$-$p_{i,v3}{}'$)	Vehicles that have driven from I_i into $r_{i,v1}$-$r_{i,v3}$, respectively		

the corresponding roads are on while others remain off. The number of vehicles driving out or entering into roads $r_{i,1} - r_{i,8}$ can be obtained, respectively. In order to reduce the model complexity and improve its clarity and readability, the switch work control model will not link the traffic flows to be modeled next. Thus $p_{i,j}{}'$ in Fig. 10(b) is omitted Fig. 11.

D. Traffic Flow Model

We build the traffic flow model in Fig. 12 (see Table III for the meaning of the places and transitions in this figure). We only consider the traffic flow at a single road $r_{i'i}$ and its downstream intersection I_i as an example. Then the whole traffic flow in the network can be composed through the model in Fig. 12. A token in $p_{i',\mu}$ represents a vehicle that has just driven from intersection I_i' into road $r_{i',\mu}$. Then after a certain time duration $\tau(t_{i',i})$, $t_{i',i}$ fires. It means that the vehicle passes through road $r_{i',i}$ and is ready to drive from $r_{i,v}$ into I_i. Note that $r_{i',\mu}$, $r_{i,v}$, and $r_{i'i}$ are the same road. The vehicle can turn left, or drive straight, or turn right by firing $t_{i,vL1}$, $t_{i,vS1}$, or $t_{i,vR1}$. A token is generated in $p_{i,v}{}'$ representing that the vehicle has driven from $r_{i,v}$ into I_i. Then by firing $t_{i,vL1}{}'$, $t_{i,vS1}{}'$, or $t_{i,vR1}{}'$, the vehicle drives from I_i into roads $r_{i,v1}$, $r_{i,v2}$ or $r_{i,v3}$, with a token deposited in $p_{i,v1}$ and $p_{i,v1}'$, or $p_{i,v2}$ and $p_{i,v2}'$, or $p_{i,v3}$ and $p_{i,v3}'$, respectively. The turning rate is described by exponential transitions $t_{i,vL1}$, $t_{i,vS1}$, and $t_{i,vR1}$ with firing rates $\lambda_{i,vL1}$, $\lambda_{i,vS1}$, and $\lambda_{i,vR1}$, respectively. When warning lights turn on, i.e., there is a token in $p_{\delta i',i\_\text{on}}$, $t_{i,v\delta1}$ is not enabled, $t_{i,v\delta2}$ is enabled and can fire with a firing rate $\lambda_{i,v\delta2} < \lambda_{i,v\delta1}$. It means that fewer vehicles choose a direction indicated by the warning light that is on. Note that if there is a token in $p_{i,v}'$, i.e., a vehicle has passed from $r_{i,v}$ into I_i, it will be sensed by $C_{i,v}$ as described in Fig. 10(b). We will not pay more attention to the traffic flow model but concentrate on the control logic of the emergency system in this paper due to page limitation.

IV. Emergency Strategies

In this section, we propose emergency traffic light strategies for an accidental intersection I_i as well as for its up-stream neighboring ones, and build emergency warning light strategies by using DSPNs.

A. Dynamic PN Models of Critical Crossing Sections

First we define critical crossing section sets over a traffic flow movement as follows.

$CS_F = \{S_n | n \in \mathbf{N}^+\}$ denotes n critical crossing section sets over traffic flow movement $F \in \{F_{ns}, F_{sn}, F_{nw}, F_{se}, F_{ne}, F_{sw}, F_{ew}, F_{we}, F_{en}, F_{ws}, F_{es}, F_{wn}\}$ where S_i, $i \in \mathbf{N}_n^+$ is a set of crossing sections called a critical crossing section set and

1) each section-occupation path of F is blocked if there exists an S_i such that $\forall\, s \in S_i$ is occupied; and
2) for each S_i, there is no $S_j \in CS_F$, $j \in \mathbf{N}_n^+$ and $j \neq i$, such that $S_i \subset S_j$.

We can obtain CS_F by the following algorithm.

Algorithm 1: Critical crossing section computation

Input: A traffic flow F with m section-occupation paths, i.e., $l_F = \{l_h | h \in \mathbf{N}_m^+\}$,
Output: CS_F
Step 1: Obtain the set of relevant crossing sections of each path in l_F, denoted by $\{Sl_h | h \in \mathbf{N}_m^+\}$, where if $l_h = r_h \to s_{h1} \to s_{h2}, \ldots \to s_{hk} \to r_h', l_h \in l_F, k \in \mathbf{N}^+$, then $Sl_h = \{s_{h1}, s_{h2}, \ldots, s_{hk}\}$;
Step 2: $CS_F = \{S_i = \cup_{h \in \mathbf{N}_m^+} \{s_h'\} | i \in \mathbf{N}_n^+, s_h' \in Sl_h\}$;
Step 3: if $S_i \subseteq S_j$, S_i, $S_j \in CS_F$, delete S_j from CS_F.

According to the algorithm, we obtain the critical crossing section set CS for all traffic flow movements.

$$CS = \{CS_{Fsn}, CS_{Fse}, CS_{Fns}, CS_{Fnw}, CS_{Fsw}, CS_{Fne}, CS_{Few}, CS_{Fen}, CS_{Fwe}, CS_{Fws}, CS_{Fwn}, CS_{Fes},\}$$

where

$$\begin{aligned}
CS_{Fsn} &= \{\{s_{i,4}\}, \{s_{i,8}\}, \{s_{i,12}\}, \{s_{i,16}\}\}\\
CS_{Fse} &= \{\{s_{i,16}\}\}\\
CS_{Fns} &= \{\{s_{i,1}\}, \{s_{i,5}\}, \{s_{i,9}\}, \{s_{i,13}\}\}\\
CS_{Fnw} &= \{\{s_{i,1}\}\}\\
CS_{Fsw} &= \{\{s_{i,15}\}, \{s_{i,10}\}, \{s_{i,5}, s_{i,6}\}, \{s_{i,1}, s_{i,5}\}\}\\
CS_{Fne} &= \{\{s_{i,2}\}, \{s_{i,7}\}, \{s_{i,11}, s_{i,12}\}, \{s_{i,12}, s_{i,16}\}\}\\
CS_{Few} &= \{\{s_{i,1}\}, \{s_{i,2}\}, \{s_{i,3}\}, \{s_{i,4}\}\}\\
CS_{Fen} &= \{\{s_{i,4}\}\}\\
CS_{Fwe} &= \{\{s_{i,13}\}, \{s_{i,14}\}, \{s_{i,15}\}, \{s_{i,16}\}\}\\
CS_{Fws} &= \{\{s_{i,13}\}\}\\
CS_{Fwn} &= \{\{s_{i,9}\}, \{s_{i,6}\}, \{s_{i,3}, s_{i,7}\}, \{s_{i,3}, s_{i,4}\}\}\\
CS_{Fes} &= \{\{s_{i,8}\}, \{s_{i,11}\}, \{s_{i,10}, s_{i,14}\}, \{s_{i,13}, s_{i,14}\}\}.
\end{aligned}$$

Note that, in this paper, each traffic flow movement corresponds to at most two section-occupation paths. The above critical crossing section computation method is still valid for complex traffic flow with more paths. An intersection can be divided into smaller crossing sections, for example, a road is categorized into three lanes and a right-turning lane is distinguished from an SR-lane. There will be more crossing sections and more section-occupation paths for each traffic flow as a result.

The detailed position of an accident can be determined and described by a camera. Given a set of crossing sections, denoted by BS, occupied by an accident, the following properties about the critical crossing section can be obtained.

Property 1: If $\exists S \in CS_F$ such that $S \subseteq BS$, then F is blocked.

In the network discussed in this paper, we can have the following property.

Property 2: If F_{se} (respectively, F_{en}, F_{nw}, F_{ws}) is blocked, then F_{sn} (respectively, F_{ew}, F_{ns}, F_{we}) is blocked.

According to Properties 1 and 2, we present an algorithm to describe the PN model indicating the corresponding traffic flow movements. Let F_{BS} denote a set of traffic flow movements blocked by BS. A token in places $p_{\chi,S}$, $p_{\chi,R}$, $p_{\chi,L}$, and $p_{\chi,LSR}$, $\chi \in \{\boldsymbol{n}, \boldsymbol{e}, \boldsymbol{s}, \boldsymbol{w}\}$ denotes that the straight, right, left, and left-straight-right movements of traffic flow from direction χ need to be stopped by emergency traffic light strategies, respectively.

Algorithm 2: Traffic flow blockage model

Input: $BS \subseteq \{s_{i,j}, j \in \mathbf{N}_{16}^+\}$
Output: tokens to be filled in $p_{\chi,S}$, $p_{\chi,R}$, $p_{\chi,L}$, and $p_{\chi,LSR}$, $\chi \in \{\boldsymbol{n}, \boldsymbol{e}, \boldsymbol{s}, \boldsymbol{w}\}$.
Step 1: Compute the blocked $F_{BS} \subseteq \{F_{ns}, F_{sn}, F_{nw}, F_{se}, F_{ne}, F_{sw}, F_{ew}, F_{we}, F_{en}, F_{ws}, F_{es}, F_{wn}\}$ according to Property 1;

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

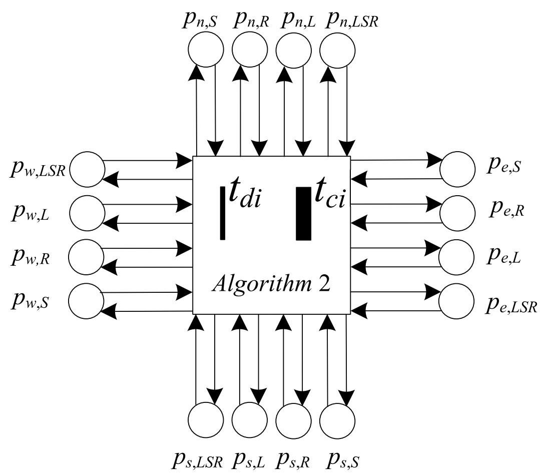

Fig. 13. Dynamic PN models of critical crossing sections.

Step 2: if F_{sn} (respectively, F_{ew}, F_{ns}, and F_{we}) $\in F_{BS}$, a token is filled in $p_{s,S}$ (respectively, $p_{e,S}$, $p_{n,S}$, and $p_{w,S}$);

Step 3: if F_{se} (respectively, F_{en}, F_{nw}, andF_{ws}) $\in F_{BS}$, a token is filled in $p_{s,R}$ and $p_{s,S}$ (respectively, $p_{e,R}$ and $p_{e,S}$, $p_{n,R}$, and $p_{n,S}$, and $p_{w,R}$ and $p_{w,S}$);

Step 4: if F_{sw} (respectively, F_{es}, F_{ne}, and F_{wn}) $\in F_{BS}$, a token is filled in $p_{s,L}$ (respectively, $p_{e,L}$, $p_{n,L}$, and $p_{w,L}$);

Step 5: if F_{se}, F_{sn}, and F_{sw} (respectively, F_{en}, F_{ew}, and F_{es}, F_{nw}, F_{ns}, and F_{ne}, and F_{ws}, F_{we}, and F_{wn}) $\in F_{BS}$, a token is filled in $p_{s,LSR}$ (respectively, $p_{e,LSR}$, $p_{n,LSR}$, and $p_{w,LSR}$).

Note that the tokens filled in $p_{\chi,S}$, $p_{\chi,R}$, $p_{\chi,L}$, and $p_{\chi,LSR}$ are generated by t_{di} and when the accident is cleared, these tokens are removed by t_{ci}. Also note that several paths may be blocked according to the location and range of the accident, and thus more than one place in Fig. 13 can obtain a token. Here we adopt Algorithm 2 to decide the arcs to be added to connect $p_{\chi,S}$, $p_{\chi,R}$, $p_{\chi,L}$, and $p_{\chi,LSR}$ with transitions t_{di} and t_{ci}. If a token needs to be deposited in $p \in \{p_{\chi,S}, p_{\chi,R}, p_{\chi,L}, p_{\chi,LSR}\}$, two arcs (t_{di}, p) and (t_{ci}, p) are drawn. Thus, dynamic PN models of critical crossing sections are constructed as shown in Fig. 13.

We design emergency traffic light strategies at the accidental intersection such that the traffic lights of the corresponding blocked directions should be kept in red till the accident is cleared while other lights are under normal operations.

B. Emergency Traffic Light Strategies for an Accidental Intersection

The emergency traffic light strategies are designed based on the PN model in Fig. 14, where $\chi \in \{\boldsymbol{n}, \boldsymbol{e}, \boldsymbol{s}, \boldsymbol{w}\}$, p_α, $p_\beta \in \{p_{i1} - p_{i8}\}$, and the dotted double sided arrow denoted by $f_1 = (p_a, t_{\chi 1})$ represent an arrow satisfying that if it is from $t_{\chi 1}$ to p_α (respectively, from p_α to $t_{\chi 1}$), then the arrow $f_2 = (p_a, t_{\chi,LSR})$ is from $t_{\chi,LSR}$ to p_α (respectively, from p_α to $t_{\chi,LSR}$). The arrow directions related to p_β have similar property. We denote $\bar{\chi}$ as the reverse direction of χ, i.e., if χ is the direction $\boldsymbol{n}$ (respectively, $\boldsymbol{e}$, $\boldsymbol{s}$, and $\boldsymbol{w}$), $\bar{\chi}$ represents its reverse direction

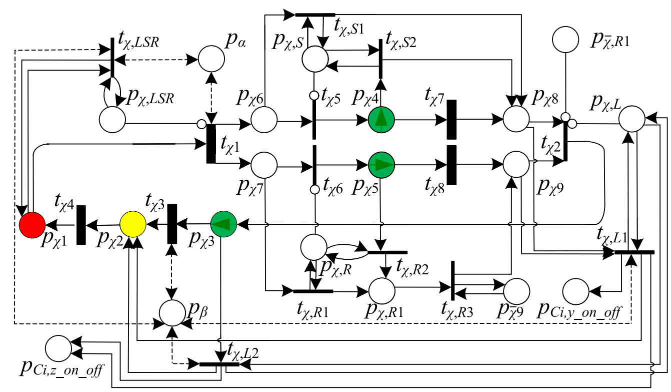

Fig. 14. Emergency traffic light strategy model at I_i.

$\boldsymbol{s}$ (respectively, $\boldsymbol{w}$, $\boldsymbol{n}$, and $\boldsymbol{e}$). Now we discuss the four different emergency traffic light strategies as follows:

1) If there exists a token in place $p_{\chi,S}$, the straight traffic flow from direction χ should be forbidden. It is controlled by the traffic light GS_χ that should be off. There are two conditions for this emergency strategy:
   - If $p_{\chi 4}$ contains a token, then $t_{\chi,S2}$ is enabled and fires immediately by removing the token from $p_{\chi 4}$ and generating a token into $p_{\chi 8}$. Hence, GS_χ is turned off.
   - If $p_{\chi 4}$ contains no token, because of the inhibitor arc from $p_{\chi,S}$ to $t_{\chi 5}$, $t_{\chi 5}$ is prevented from firing. If $p_{\chi 6}$ contains a token, then $t_{\chi,S1}$ is enabled and fires immediately by removing the token from $p_{\chi 6}$ and depositing a token into $p_{\chi 8}$. Hence, GS_χ remains off.

   Notice that, $t_{\chi 2}$ is used for the synchronization of lights GS_χ and GR_χ. In the emergency strategy, a token stays in $p_{\chi 8}$ until GR_χ finishes its time duration.
2) If there exists a token in $p_{\chi,R}$ and $p_{\chi,S}$, the right-turning and straight traffic flows from direction χ are stopped. They are controlled by GS_χ and GR_χ that should be off. There are two conditions for this emergency strategy:
   - If both $p_{\chi 4}$ and $p_{\chi 5}$ contain a token, then both $t_{\chi,S2}$ and $t_{\chi,R2}$ are enabled and fire immediately by removing tokens from them and filling a token into $p_{\chi 8}$ and $p_{\chi,R1}$, respectively. Hence, GS_χ and GR_χ are turned off.
   - If $p_{\chi 4}$ and $p_{\chi 5}$ contain no token, $t_{\chi 5}$ and $t_{\chi 6}$ are not enabled because of the inhibitor arcs from $p_{\chi,S}$ to $t_{\chi 5}$ and from $p_{\chi,R}$ to $t_{\chi 6}$. When both $p_{\chi 6}$ and $p_{\chi 7}$ get a token, $t_{\chi,S1}$ and $t_{\chi,R1}$ are enabled and fire, depositing a token into $p_{\chi 8}$ and $p_{\chi,R1}$, respectively. Hence, GS_χ and GR_χ remain off.

   If $p_{\chi,R1}$ contains a token, in order to continue the phase transitions of the traffic-light system, a token needs to be filled in $p_{\chi 9}$ by firing $t_{\chi,R3}$, which will immediately lead to the next state of the traffic light GL_χ, i.e., $p_{\chi 3}$ gets a token after the firing of $t_{\chi 2}$. But at this time, $GS_{\bar{\chi}}$ and $GR_{\bar{\chi}}$ are opened which lead to a conflict between the left-turning traffic from χ and straight and right-turning traffic from $\bar{\chi}$. In order to resolve it problem, we add two arcs $(p_{\bar{\chi}9}, t_{\chi,R3})$ and $(t_{\chi,R3}, p_{\bar{\chi}9})$. In order to make the firing of $t_{\chi,R3}$ before that of $t_{\bar{\chi}2}$, an inhibitor arc $(p_{\chi,R1}, t_{\bar{\chi}2})$ is added.

3) If there exists a token in place $p_{\chi,L}$, the left-turning flow from direction χ is stopped. Thus traffic light GL_χ should be off. There are two conditions for this emergency strategy:
   - If $p_{\chi 3}$ contains a token, then $t_{\chi,L2}$ is enabled and fires by removing a token from $p_{\chi 3}$, depositing a token into $p_{\chi 2}$. Thus GL_χ is turned off. In this situation, both $t_{\chi 2}$ and $t_{\chi 3}$ do not fire. In Fig. 11, p_{Ci,y_on_off} and p_{Ci,z_on_off} cannot obtain a token, where if $\chi \in \{\boldsymbol{n}, \boldsymbol{e}, \boldsymbol{s}, \boldsymbol{w}\}$, $y \in \{7, 5, 3, 1\}$, and $z \in \{8, 6, 4, 2\}$, respectively. The corresponding loop detectors should be turned off when $t_{\chi,L1}$ fires, and thus we add two arcs $f_1' = (t_{\chi,L1}, p_{Ci,z\_on\_off})$ and $f_2' = (t_{\chi,L1}, p_{Ci,z\_on\_off})$.
   - If $p_{\chi 3}$ contains no token, $t_{\chi 2}$ is not enabled because of the inhibitor arc from $p_{\chi,L}$ to it. When both $p_{\chi 8}$ and $p_{\chi 9}$ get a token, $t_{\chi,L1}$ is enabled and fires, depositing a token into $p_{\chi 2}$. Thus GL_χ remains off. In this situation, $t_{\chi 3}$ does not fire. When $t_{\chi,L2}$ fires, the corresponding loop detectors should be turned off by adding an arc $f_3' = (t_{\chi,L2}, p_{Ci,z\_on\_off})$.

   In the operation of the traffic-light control system in Fig. 7, a token is deposited into or removed from p_β at direction χ during a traffic light transition cycle. When GL_χ is changed to Y_χ and $t_{\chi 3}$ fires, a token is deposited into or removed from p_β. However, in the emergency one, $t_{\chi 3}$ does not fire. In order to preserve the token in p_β, we add dotted double sided arrows $f_3 = (p_\beta, t_{\chi,L1})$ and $f_4 = (p_\beta, t_{\chi,L2})$.

4) If there exists a token in place $p_{\chi,LSR}$, all traffic flows from direction χ are forbidden. Consequently, lights GS_χ, GR_χ, and GL_χ should be off. There are two conditions for this emergency strategy:
   - If $p_{\chi 1}$ contains a token, because of the inhibitor arc from $p_{\chi,LSR}$ to $t_{\chi 1}$, $t_{\chi 1}$ is not enabled. At this condition, $t_{\chi,LSR}$ fires and a token is deposited into or removed from each of p_α and p_β. Note that both p_α and p_β have a token or neither of them has one. Thus R_χ is kept on.
   - If $p_{\chi 1}$ contains no token, all green lights are turned off due to the tokens in $p_{\chi,S}$, $p_{\chi,R}$ and $p_{\chi,L}$. If $p_{\chi 2}$ has one, $t_{\chi 4}$ fires to deposit a token into $p_{\chi 1}$. Thus R_χ is turned on.

   Note that, in this emergency operation, both $t_{\chi 1}$ and $t_{\chi 3}$ do not fire. The dotted double sided arrows f_2 and $f_5 = (p_\beta, t_{\chi,LSR})$ preserve the token in p_α and p_β.

C. Deadlock Recovery and Livelock Prevention

Is the emergency traffic light strategy model in Fig. 14 deadlock-free? This subsection answers this question and presents a control strategy to address a deadlock issue.

There is a deadlock if GL_χ, GS_χ, $GL_{\bar{\chi}}$, and $GS_{\bar{\chi}}$ need to be off. At this situation, both $p_{\chi,R1}$ and $p_{\bar{\chi},R1}$ obtain a token, and a circular wait happens that $t_{\chi,R3}$ waits for a token in $p_{\bar{\chi}9}$ to be generated by firing $t_{\bar{\chi},R3}$ while $t_{\bar{\chi},R3}$ waits for a token in $p_{\chi 9}$ which is generated by firing $t_{\chi,R3}$. This circular wait generates a deadlock. To resolve it, we design a deadlock recovery model in Fig. 15. If both of $p_{\chi,R1}$ and $p_{\bar{\chi},R1}$ obtain a token, $t_{\chi\bar{\chi},R}$ fires, generating a token in to each of $p_{\chi 9}$ and $p_{\bar{\chi}9}$. Note that this decreases the whole traffic light phase cycle duration of the control system, and the saved time is used for other phases.

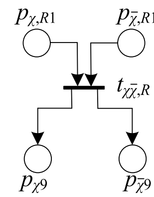

Fig. 15. A deadlock recovery model.

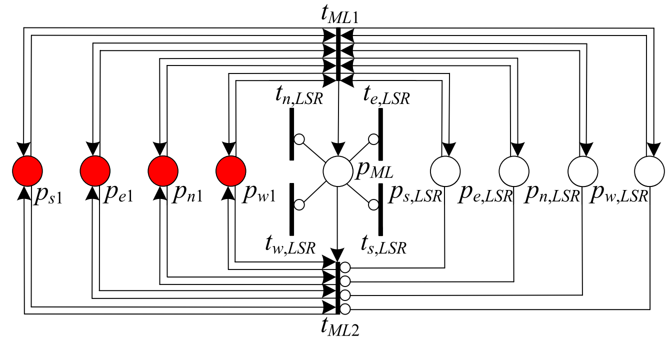

Fig. 16. A livelock prevention model.

In the special situation that all the traffic flow movements need to be forbidden, each of p_{s1}, p_{e1}, p_{n1}, and p_{w1} contains a token, that is the state of the signal is $M_0 = (R_n, R_e, R_s, R_w)$. In the traffic control model, there will be a firing transition circle $\sigma = (t_{n,LSR}, t_{s,LSR}), (t_{e,LSR}, t_{w,LSR})$ such that $M_0[\sigma\rangle M_0$. The sequence of these transitions fire infinitely, thereby forming a livelock which has no benefit to traffic flows. In order to prevent it, we design the model as shown in Fig. 16. If all $p_{\chi 1}$ and $p_{\chi,LSR}$ contain tokens, transition t_{ML1} fire, generating a token in p_{ML}. Because of the inhibitor arcs from p_{ML}, transitions $t_{n,LSR}$, $t_{s,LSR}$, $t_{e,LSR}$, and $t_{w,LSR}$ cannot fire such that σ is prevented. When the accident is cleared, tokens in $p_{\chi,LSR}$ are removed and transition t_{ML2} fires by consuming the token in p_{ML}, and the traffic light control system returns to its normal operations.

D. Warning Light Strategies

In this section, we propose warning light strategies by using DSPNs. Let I_i be an accidental intersection, I_i' an (supposed western) up-stream neighboring intersection of I_i, and $I_{i1}' - I_{i3}'$ the (northern, western, and southern) up-stream neighboring intersections of I_i' except I_i. Road $r_{i'i}$ is at the χ direction of I_i, and traffic flows from roads $r_{i1'i'} - r_{i3'i'}$ to enter road $r_{i'i}$ turn left, drive straight and turn right, respectively, at I_i'. Strategies of warning lights at $r_{i'i}$ are shown in Fig. 17. A warning light $w_{\delta i',i}$, $\delta \in \{L, S, R\}$ turns on by depositing a token in $p_{\delta i',i\_on}$ if there is a token in $p_{\chi,\delta}$. It turns off if there is no token in $p_{\chi,\delta}$ by transferring the token from $p_{\delta i',i\_on}$ to $p_{\delta i',i\_off}$.

At this situation, the emergency strategies of warning lights at $r_{i1'i'} - r_{i3'i'}$, as shown in Fig. 18, are adopted to decrease traffic flow to enter $r_{i'i}$. In the model, if there is a token in $p_{\chi,\delta}$, $t_{\chi,\delta}$ fires and deposits a token in $p_{i,\chi}$. It describes

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

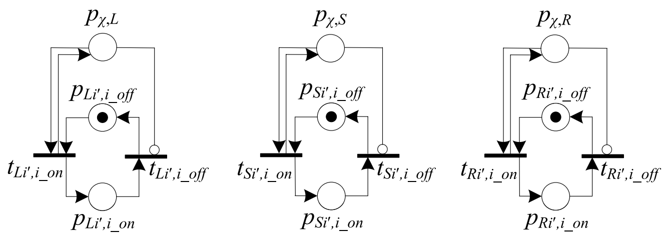

Fig. 17. Models of strategies of warning lights at $r_{i'i}$.

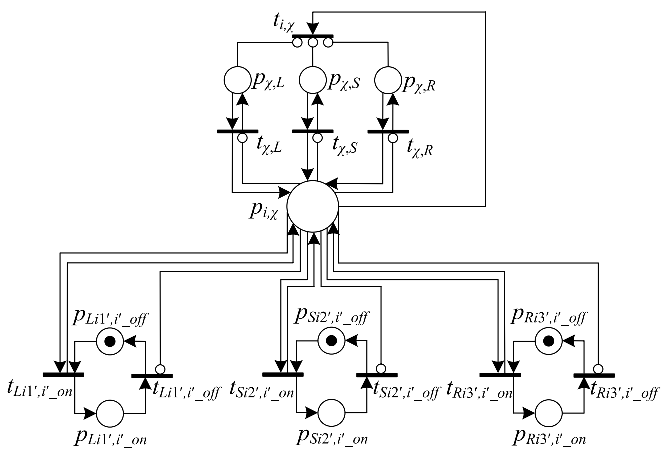

Fig. 18. Models of strategies of warning lights at $r_{i1'i'} - r_{i3'i'}$.

that traffic flow in the road at the χ direction of I_i is interrupted by an accident. Then $t_{Li1'i'\_on}$, $t_{Si2'i'\_on}$, and $t_{Ri3'i'\_on}$ are enabled and fire by depositing a token in $p_{Li1'i'\_on}$, $p_{Si2'i'\_on}$, and $p_{Ri3'i'\_on}$, respectively. Thus, warning lights $w_{Li1'i'}$, $w_{Si2'i'}$, and $w_{Ri3'i'}$ at roads $r_{i1'i'} - r_{i3'i'}$, respectively, are on. If there is a token in $p_{\chi,\delta}$, the accident is detected to be cleared. $t_{i,\chi}$ fires and removes the token in $p_{i,\chi}$. Then tokens in $p_{Li1'i'\_on}$, $p_{Si2'i'\_on}$, and $p_{Ri3'i'\_on}$ are transferred into $p_{Li1'i'\_off}$, $p_{Si2'i'\_off}$, and $p_{Ri3'i'\_off}$ by firing $t_{Li1'i'\_off}$, $t_{Si2'i'\_off}$ and $t_{Ri3'i'\_off}$, respectively. Thus, $w_{Li1'i'}$, $w_{Si2'i'}$, and $w_{Ri3'i'}$ are turned off.

E. Emergency Strategies for Up-Stream Neighboring Intersections

In this section, we adopt the assumptions and symbols in the former section. We design traffic light emergency strategies for the up-stream neighboring intersections of I_i, i.e., I_i'. There are two cases at which the emergency strategies should be adopted.

1) All traffic flows in $r_{i'i}$ are stopped at I_i by its emergency strategies, i.e., there is a token in $p_{\chi,LSR}$.
2) Some of traffic flows in $r_{i'i}$ are stopped at I_i and vehicle count in road $r_{i'i}$ sensed by loop detectors reaches a maximal number Ω.

Note that in Case 1, any vehicles to enter $r_{i'i}$ cannot pass through I_i because of the on-going red traffic light signal. Thus, emergency traffic light strategies are adopted immediately to stop the traffic flowing into $r_{i'i}$ at I_i'. In Case 2, if road $r_{i'i}$ is occupied with enough vehicles, the crossing sections of I_i', e.g., the blue heavy-shaded areas (called critical areas) in Fig. 1 may

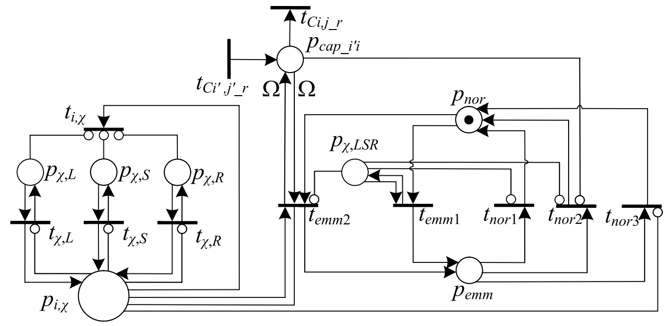

Fig. 19. Transformation between the normal strategy and emergency ones.

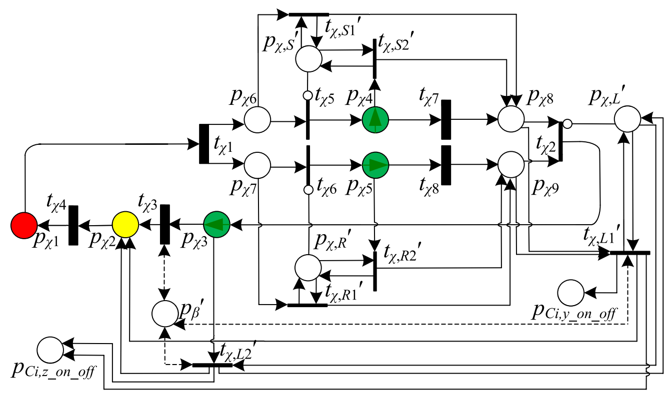

Fig. 20. Emergency traffic light strategy model at I_i'.

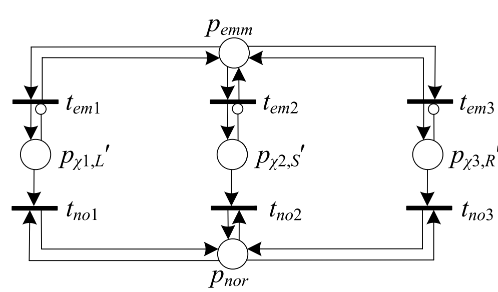

Fig. 21. Emergency traffic light strategy control model at I_i'.

TABLE IV
MEANING OF PLACES

P	Meaning
$p_{cap\_i'i}$	The number of vehicles detected in $r_{i'i}$
$p_{i,\chi}$	The neighboring eastbound intersection has an accident blocking the traffic flow from $r_{i'i}$
p_{nor}	Normal traffic light strategy is in use
p_{emm}	Emergency traffic light strategy is in use

be blocked. Thus, if vehicle count in $r_{i'i}$ reaches Ω, emergency strategies are also needed.

Now we propose emergency traffic light strategies by using DSPNs in Figs. 19–21. The meaning of their transitions and places is shown in Tables IV and V where all transitions are immediate ones.

As shown in Fig. 19, if there exists a token in $p_{\chi,LSR}$, then t_{emm1} is enabled and fires. As a result, a token is transferred

TABLE V
MEANING OF TRANSITIONS

T	Meaning
t_{emm1}	Changing traffic light control from a normal strategy to an emergency one when all flows in $r_{i'i}$ are stopped at I_i
t_{emm2}	Changing traffic light control from a normal strategy to an emergency one when some flows in $r_{i'i}$ are stopped at I_i and vehicle count in $r_{i'i}$ reaches Ω
t_{nor1} and t_{nor2}	Changing traffic light control back to a normal one when the accident is cleared
t_{nor3}	Changing traffic light control back to a normal one when there is no vehicle in $r_{i'i}$
Others	No meaning but for a control purpose

from p_{nor} into p_{emm} indicating that a normal strategy is changed to an emergency one. The strategy is suspended only if the accident is cleared, i.e., there is no token in $p_{\chi,LSR}$ such that t_{nor1} fires by transferring the token from p_{emm} into p_{nor}.

If some of traffic flows in $r_{i'i}$ are stopped and others are permitted to go through I_i, at least one token exists in $p_{\chi,L}$, $p_{\chi,S}$ or $p_{\chi,R}$, and there is no token in $p_{\chi,LSR}$. Then $p_{i,\chi}$ receives a token. $p_{cap\_i'i}$ describes the number of vehicles in $r_{i'i}$ that have been counted by loop detectors $C_{i,j}$ and $C_{i',j'}$ installed at the entrance and exit of $r_{i'i}$, respectively. A directed arc is drawn from $p_{cap\_i'i}$ to t_{emm2} with a weight Ω. If the number of vehicles in $r_{i'i}$ reaches Ω, i.e., Ω tokens are in $p_{cap\_i'i}$, t_{emm2} is enabled and fires such that a token is transferred from p_{nor} into p_{emm} indicating that the normal strategy is changed into an emergency one. The strategy is suspended if: 1) The accident is cleared, i.e., there is no token in $p_{\chi,L}$, $p_{\chi,S}$, and $p_{\chi,R}$ such that t_{nor1} fires transferring the token from p_{emm} into p_{nor}; and 2) there is no vehicles in road $r_{i'i}$, i.e., there is no token in $p_{cap\_i'i}$, and then t_{nor2} fires to transfer the token from p_{emm} into p_{nor}. At the second condition, the emergency strategies are executed if the number of vehicles in$r_{i'i}$ reaches Ω again.

The emergency traffic light strategy is presented in Fig. 20. Similar to the control strategy at the accidental intersection, traffic lights $GL_{\chi1}$, $GS_{\chi2}$, and $GR_{\chi3}$ need to be kept off while other lights are controlled under their normal operations, where $\chi1-\chi3$ are the directions of road $r_{i1'i'}-r_{i3'i'}$. Thus a token is deposited in $p_{\chi1,L}{}'$, $p_{\chi2,S}{}'$, and $p_{\chi3,R}{}'$ as shown in Fig. 21.

Now, let's give a method to compute Ω. First, $r_{i'i}$ is divided into two distinct zones: a downstream queue storage area (DA) and an up-stream "reservoir" (UR) as shown in Fig. 1. When UR is full of vehicles, emergency traffic light control strategies at I_i' are adopted. First, we define the following notation to describe the traffic conditions:

- L_{RD}, L_{UR}, L_{DA}: the length of road $r_{i'i}$, sections UR, and DA, respectively.
- L_h: the mean "headway" (distance between the fronts of successive vehicles in the same lane of traffic) of two queuing vehicles.
- L_h': the least mean "headway" of two successive queuing vehicles when the following one starts to move from the queuing status.
- a: the mean acceleration value of vehicles.
- v: the maximal speed of the vehicle.

We do our computation based on the following assumptions:

- The values of the above notations are known as constants except L_{UR} and L_{DA}.
- The two intersections are installed with same signal phases, as depicted in Table I.
- Normally, after phases c and d in each cycle, there is no vehicle remaining in DA.

Theorem 1: If the vehicle queue in DA is not full yet to force the stop of a vehicle in UR, we have the maximal number of vehicles that $r_{i'i}$ can hold as follows:

$$\Omega = \left\lfloor \frac{2L_{RD}}{v\sqrt{\frac{2(L_h'-L_h)}{a}}+L_h} \right\rfloor \tag{1}$$

where $\lfloor x \rfloor$ is a floor function, representing the integer equal to or less than $x \in \mathbf{R}$.

Proof: The maximal number of vehicles that $r_{i'i}$ can hold such that I_i' cannot be blocked is to be computed in such way that the last vehicle in DA starts to move while the first one in UR reaches DA. According to these assumptions, we can obtain the following formulae:

$$\frac{at^2}{2} = L_h' - L_h \tag{2}$$

$$\frac{L_{DA}}{L_h} = \frac{\Omega}{2} \tag{3}$$

$$\frac{\Omega t}{2} = \frac{L_{UR}}{v} \tag{4}$$

$$L_{RD} = L_{DA} + L_{UR} \tag{5}$$

where t denotes the mean time interval of the start-driving of a queuing vehicle and its successive one. Then we can obtain the maximal number of vehicles that $r_{i'i}$ can hold as (1).

V. Property Analysis

In this section we discuss the properties of the proposed DSPN models. Note that, all DSPNs except those in Figs. 10(a) and 12 contain no exponential transitions.

A. Conflict Resolution

Conflicts appear in the DSPNs in Figs. 18, 19, and 21. In Figs. 18 and 19, if at least two of places $p_{\chi,L}$, $p_{\chi,S}$, and $p_{\chi,R}$ contain a token, a conflict for transitions $t_{\chi,L}$, $t_{\chi,S}$, and $t_{\chi,R}$ arises, i.e., only one token can be deposited in $p_{i,\chi}$, and the firing of one transition prevent that of another. In order to resolve the conflict, we define that firing priorities of $t_{\chi,L}$, $t_{\chi,S}$, and $t_{\chi,R}$ decrease, for example, if $t_{\chi,L}$ and $t_{\chi,S}$ are enabled, $t_{\chi,L}$ fires because of its highest firing priority. In fact, the firing sequence will not affect the resultant state of the model, i.e., $p_{i,\chi}$ receives a token. Another conflict appears among $t_{Li1'i'\_\mathrm{on}}$, $t_{Si2'i\_\mathrm{on}}$, and $t_{Ri3'i'\_\mathrm{on}}$ in Fig. 18. We define their priorities in a decreasing order. Because the transitions are all immediate ones, $p_{Li1'i'\_\mathrm{on}}$, $p_{Si2'i'\_\mathrm{on}}$, and $p_{Ri3'i'\_\mathrm{on}}$ receive a token in no

TABLE VI
TRANSITIONS ASSOCIATED WITH TIME

Set	T	$Type$
A_1	$t_{\chi 1}$, $t_{\chi 3}$, $t_{\chi 4}$, $t_{\chi 7}$ and $t_{\chi 8}$ in Fig. 7	T_d
A_2	t_{hi} in Fig. 10(a)	T_e
A_3	t_{ci} in Fig. 10(a)	T_d
A_4	$t_{i,v\delta 1}$ and $t_{i,v\delta 2}$ in Fig. 12	T_e
A_5	$t_{i',i}$ in Fig. 12	T_d

time as a result. Similarly, we give priorities to $t_{em1} - t_{em3}$ and $t_{no1} - t_{no3}$ to resolve the corresponding conflicts.

B. Reachability Analysis

We verify the properties and correctness of the DSPNs through reachability analysis. The reachability graph is used for reachability analysis. It can be constructed in a manual way. In this paper we adopt a construction method that each arc corresponds to a sequence of transitions containing only one timed transition (deterministic or stochastic one). Besides, we put them in parentheses and then attached to the corresponding arc if more than one transition fires at the same time. Obviously, this construction method can greatly reduce the size of the graph. The reachability analysis can also be done through a tool such as the TimeNet [37]. Some recently proposed reachability analysis methods can be used to analyze large and unbounded Petri nets [27], [38]–[42].

In the proposed models in this paper, transitions associated with time are categorized into five sets $A_1 - A_5$ as shown in Table VI: deterministic transitions in A_1 model the time duration of traffic signals; the exponential transition in A_2 describes the occurrence of an accident; the deterministic transitions in A_3 represents the disposal of the accident; exponential transitions in A_4 depict the turning rate of vehicles at I_i; and the transition in A_5 shows the vehicle's driving time from I_i' to I_i. Our purpose is to verify the validity of the control logic of the designed strategies, and thus we have not paid more attention to the traffic flow model. The exponential transition increases the reachability analysis complexity. Fortunately, there exists only one exponential transition in our cases, and we let it fire at a finite number of time intervals during the construction of the reachability graph. Besides, the control logic validity can be verified without a simulation method.

As a result, we can construct the reachability graph of the DSPN model in Fig. 11, and the graph of a composition model of the ones in Fig. 10(a) and Figs. 13–21 by removing place $p_{cap\_i'i}$, transitions t_{Ci,j_r} and t_{Ci',j'_r} and their related arcs in Fig. 19 that are related to the no-safe traffic flow model in Fig. 12. We do not present them here due to space limitation. Their analyses suggest that the DSPN models are live and reversible.

VI. CONCLUSION

This work uses DSPNs to design a traffic-light control system at an intersection dealing with accidents. Normal and emergency traffic light strategies are designed for both the accidental intersection and its up-stream neighboring ones. Emergency warning light strategies are presented. The PN-based control logics are proved to be valid. Comparing with [13] and [14], apart from considering the safety of a different system, our contributions mainly are that 1) we adopt the concept of a crossing section to construct a PN-based dynamic model indicating the blockage of traffic flows; 2) we control the traffic lights from four directions, respectively; and 3) the traffic lights cannot only be controlled but also in turn control the other facilities including loop detectors and warning lights. Future work will construct a complete traffic flow model. Based on the model, some simulations can be done to show the advantages of the proposed system in traffic accident management and flow congestion prevention comparing with the traditional self-evolution case. To our knowledge, this is the first work to propose an emergency system for intersections facing accidents based on PNs. It can potentially be used to improve the state of the art in real-time traffic accident management at urban road intersections.

REFERENCES

[1] P. Pyykonen, M. Molinier, and G. A. Klunder, "Traffic monitoring and modeling for intersection safety," in *Proc. IEEE ICCP*, 2010, pp. 401–408.

[2] K. Ozbay and P. Kachroo, *Incident Management in intelligent Transportation Systems*. Norwood, MA, USA: Artech House, 1999.

[3] "Traffic incident management handbook," Office Travel Manage., Fed. Highway Admin., Washington, DC, USA, 2000.

[4] B. M. Williams and A. Guin, "Traffic management center use of incident detection algorithms: Findings of a nationwide survey," *IEEE Trans. Intell. Transp. Syst.*, vol. 8, no. 2, pp. 351–358, Jun. 2007.

[5] S. Y. Chen, W. Wang, and H. V. Zuylen, "Construct support vector machine ensemble to detect traffic incident," *Expert Syst. Appl.*, vol. 36, no. 8, pp. 10976–10986, Oct. 2009.

[6] B. A. Coifman and R. Mallika, "Distributed surveillance on freeways emphasizing incident detection and verification," *Transp. Res. A, Policy Practice*, vol. 41, no. 8, pp. 750–767, Oct. 2007.

[7] K. G. Zografos, K. N. Androutsopoulos, and G. M. Vasilakis, "A real-time decision support system for roadway network incident response logistics," *Transp. Res. C, Emerging Technol.*, vol. 10, no. 1, pp. 1–18, Feb. 2002.

[8] R. N. Mussa and J. E. Upchurch, "Monitoring urban freeway incidents by wireless communications," *Transp. Res. Rec.*, vol. 2001, no. 1748, pp. 153–160, Jan. 2007.

[9] M. Papageorgiou, C. Diakaki, V. Dinopoulou, A. Kotsialos, and Y. Wang, "Review of road traffic control strategies," *Proc. IEEE*, vol. 91, no. 12, pp. 2043–2067, Dec. 2003.

[10] S. F. Cheng, M. A. Epelman, and R. L. Smith, "CoSIGN: A parallel algorithm for coordinated traffic signal control," *IEEE Trans. Intell. Transp. Syst.*, vol. 7, no. 4, pp. 551–564, Dec. 2006.

[11] D. I. Robertson and R. D. Bretherton, "Optimizing networks of traffic signals in real-time—the SCOOT method," *IEEE Trans. Veh. Technol.*, vol. 40, no. 1, pp. 11–15, Feb. 1991.

[12] P. Lowrie, "SCATS: A traffic responsive method for controlling urban traffic," Roads Traffic Authority, Sydney, NSW, Australia, Tech. Rep., 1990.

[13] Y. S. Huang, Y. S. Weng, and M. C. Zhou, "Critical scenarios and their identification in parallel railroad level crossing traffic emergency systems," *IEEE Trans. Intell. Transp. Syst.*, vol. 11, no. 4, pp. 968–977, Dec. 2010.

[14] Y. S. Weng, Y. S. Huang, Y. L. Pan, and M. Jeng, "Design of traffic safety control systems for railroads and roadways using timed Petri nets," *Asian J. Control*, vol. 17, no. 2, pp. 626–635, Mar. 2015.

[15] Y. S. Huang, Y. S. Weng, and M. C. Zhou, "Design of traffic safety control systems for emergency vehicle preemption using timed Petri nets," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 4, pp. 2113–2120, Aug. 2015.

[16] Z. W. Li, N. Q. Wu, and M. C. Zhou, "Deadlock control of automated manufacturing systems based on Petri nets—a literature review," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 42, no. 4, pp. 437–462, Jul. 2012.

[17] M. C. Zhou and F. DiCesare, *Petri Net Synthesis for Discrete Event Control of Manufacturing Systems*. London, U.K.: Kluwer, 1993.

[18] K. M. Ng, M. B. I. Reaz, and M. A. M. Ali, "A review on the applications of Petri nets in modeling, analysis and control of urban traffics," *IEEE Trans. Intell. Transp. Syst.*, vol. 14, no. 2, pp. 858–870, Jun. 2013.

[19] G. F. List and M. Cetin, "Modeling traffic signal control using Petri nets," *IEEE Trans. Intell. Transp. Syst.*, vol. 5, no. 3, pp. 177–187, Sep. 2004.

[20] Y. S. Huang and T. H. Chung, "Modeling and analysis of urban traffic lights control systems using timed CP-nets," *J. Inf. Sci. Eng.*, vol. 24, no. 3, pp. 875–890, 2008.

[21] Y. S. Huang and T. H. Chung, "Modeling and analysis urban traffic light control systems," *J. Chin. Inst. Eng.*, vol. 32, no. 1, pp. 85–95, 2009.

[22] Y. S. Huang, Y. S. Weng, and M. C. Zhou, "Modular design of urban traffic-light control systems based on synchronized timed Petri nets," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 2, pp. 530–539, Apr. 2014.

[23] H. A. P. Blom, J. Krystul, G. J. Bakker, M. B. Klompstra, and B. Klein Obbink, "Free flight collision risk estimation by sequential Monte Carlo simulation," in *Stochastic Hybrid Systems; Recent Developments and Research Trends*, C. G. Cassandras and J. Lygeros, Eds. New York, NY, USA: Taylor & Francis, 2007.

[24] M. Ghazel, "Using stochastic Petri nets for level-crossing collision risk assessment," *IEEE Trans. Intell. Transp. Syst.*, vol. 10, no. 4, pp. 668–677, Dec. 2009.

[25] P. Lino and G. Maione, *Applying a Discrete Event System Approach to Problems of Collective Motion in Emergency Situations*ser. Pedestrian and Evacuation Dynamics 2008, Berlin, Germany: Springer-Verlag, 2010, pp. 465–477.

[26] G. Maione and M. Ottomanelli, "A Petri net model for simulation of container terminal operations," in *Advanced OR and AI Methods in Transportation*, A. Jaszkiewicz, Ed. *et al.*, Poznan, Poland: House Poznan Univ. Technol., 2005, pp. 373–378.

[27] S. Wang, M. C. Zhou, Z. Li, and C. Wang, "A new modified reachability tree approach and its applications to unbounded Petri nets," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 43, no. 4, pp. 932–940, Jul. 2013.

[28] W. T. Ma *et al.*, "A wireless accelerometer-based automatic vehicle classification prototype system," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 1, pp. 104–111, Feb. 2014.

[29] Y. K. Ki and D. Y. Lee, "A traffic accident recording and reporting model at intersections," *IEEE Trans. Intell. Transp. Syst.*, vol. 8, no. 2, pp. 188–194, Jun. 2007.

[30] W. Hu, X. Xiao, D. Xie, T. Tan, and S. Maybank, "Traffic accident prediction using 3-D model-based vehicle tracking," *IEEE Trans. Veh. Technol.*, vol. 53, no. 3, pp. 677–694, May 2004.

[31] A. Di Febbraro, D. Giglio, and N. Sacco, "On using Petri nets for representing and controlling signalized urban areas: New model and results," in *Proc. IEEE ITSC*, 2009, pp. 771–778.

[32] A. Di Febbraro and D. Giglio, "On representing signalized urban areas by means of deterministic-timed Petri nets," in *Proc. IEEE ITSC*, 2004, pp. 372–377.

[33] A. Di Febbraro, D. Giglio, and N. Sacco, "On applying Petri nets to determine optimal offsets for coordinated traffic light timings," in *Proc. IEEE ITSC*, 2002, pp. 773–778.

[34] A. Di Febbraro, D. Giglio, and N. Sacco, "Modular representation of urban traffic systems based on hybrid Petri nets," in *Proc. IEEE ITSC*, 2001, pp. 867–872.

[35] G. Ciardo and C. Lindemann, "Analysis of deterministic and stochastic Petri nets," in *Proc. IEEE 5th Int. Workshop*, Oct. 1993, pp. 160–169.

[36] M. A. Marsan and G. Chiola, "On Petri nets with deterministic and exponentially distributed firing times," in *Advances in Petri Nets*. Berlin, Germany: Springer-Verlag, 1987, pp. 146–161.

[37] A. Zimmermann, M. Knoke, A. Huck, and G. Hommel, "Towards version 4.0 of TimeNET," in *Proc. 13th GI/ITG Conf. Meas., Modeling Eval. Comput. Commun. Syst.*, 2006, pp. 477–480.

[38] Z. H. Ding, M. C. Zhou, and S. G. Wang, "Ordinary Differential Equation Based Deadlock Detection," *IEEE Trans. on Systems, Man, and Cybernetics: Systems*, vol. 44, no. 10, pp. 1435–1454, Oct. 2014.

[39] M.-D. Gan, S.-G. Wang, M.-C. Zhou, J. Li, and Y. Li, "A survey of reachability trees of un-bounded Petri nets," *Acta Automatica Sinica*, vol. 41, no. 4, pp. 686–693, Apr. 2015.

[40] L. Pan, Z. Ding, and M. C. Zhou, "A Configurable State Class Method for Temporal Analysis of Time Petri Nets," *IEEE Trans. on Systems, Man, and Cybernetics: Systems*, vol. 44, no. 4, pp. 482–493, Apr. 2014.

[41] S. G. Wang, M. D. Gan, and M. C. Zhou, "Macro liveness graph and liveness of ω-independent unbounded nets," *Science China Information Sciences*, vol. 58, no. 3, pp. 032201:1–032201:10, Mar. 2015.

[42] S. G. Wang, M. D. Gan, M. C. Zhou, and D. You, "A reduced reachability tree for a class of unbounded Petri nets," *IEEE/CAA J. Automatica Sinica*, vol. 2, no. 4, pp. 353–360, 2015.

[43] S. Wang, M. Zhou, M. Gan, D. You, and Y. Li, "New Reachability Trees for Unbounded Petri Nets," in *Proc. of 2015 IEEE Int. Conf. on Robotics Automation (ICRA)*. Seattle, WA, May 26-30, 2015, pp. 3862–3867.

Liang Qi received the B.S. and M.S. degrees from Shandong University of Science and Technology, Qingdao, China, in 2009 and 2012, respectively. He is currently working toward the Ph.D. degree with the Department of Computer Science and Technology, School of Electronics and Information Engineering, Tongji University, Shanghai, China.

He is the author or coauthor of eight technical papers in journals and conference proceedings. His current research interests include Petri nets and the control and optimization of large complex systems in transportation applications.

MengChu Zhou (S'88–M'90–SM'93–F'03) received the B.S. degree in control engineering from Nanjing University of Science and Technology, Nanjing, China, in 1983; the M.S. degree in automatic control from Beijing Institute of Technology, Beijing, China, in 1986; and the Ph.D. degree in computer and systems engineering from Rensselaer Polytechnic Institute, Troy, NY, USA, in 1990. In 1990, he joined the New Jersey Institute of Technology, Newark, NJ, USA, where he is now a Distinguished Professor with the Helen and John C. Hartmann Department of Electrical and Computer Engineering, Newark College of Engineering. He is the author or coauthor of over 600 publications, including 12 books, 300+ journal papers (the majority in IEEE Transactions), and 28 book chapters. His research interests include Petri nets, the Internet of Things, and transportation systems. Dr. Zhou is a fellow of IFAC and AAAS. He is the Founding Editor of the IEEE Press Book Series on Systems Science and Engineering.

WenJing Luan received the B.S. and M.S. degrees from Shandong University of Science and Technology, Qingdao, China, in 2009 and 2012, respectively. She is currently working toward the Ph.D. degree with the Department of Computer Science and Technology, School of Electronics and Information Engineering, Tongji University, Shanghai, China.

Her current research interests include Petri nets and smart cities.