12.489

EE25BTECH11060 - V.Namaswi

Question

Matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 2 \\ 3 & 7 & 2 \\ 5 & 1 & 7 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix}$$

are given. If vector $\mathbf{x}$ is the solution to the system of equations $\mathbf{Ax} = \mathbf{b}$, which of the following is true for $\mathbf{x}$

(a) Solution does not exist
(b) Infinite solutions exist
(c) Unique solution exists
(d) Five possible solutions exist

Solution

Given,

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 2 \\ 3 & 7 & 2 \\ 5 & 1 & 7 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix} \tag{1}$$

Form the augmented matrix:

$$\left(\begin{array}{ccc|c} 2 & 0 & 2 & 4 \\ 3 & 7 & 2 & 4 \\ 5 & 1 & 7 & 5 \end{array}\right) \tag{2}$$

According to Gaussian elimination: Replace

$$R_2 \to R_2 - \frac{3}{2}R_1, \quad R_3 \to R_3 - \frac{5}{2}R_1$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 2 & 4 \\ 0 & 7 & -1 & -2 \\ 0 & 1 & 2 & -5 \end{array}\right) \tag{3}$$

Replace

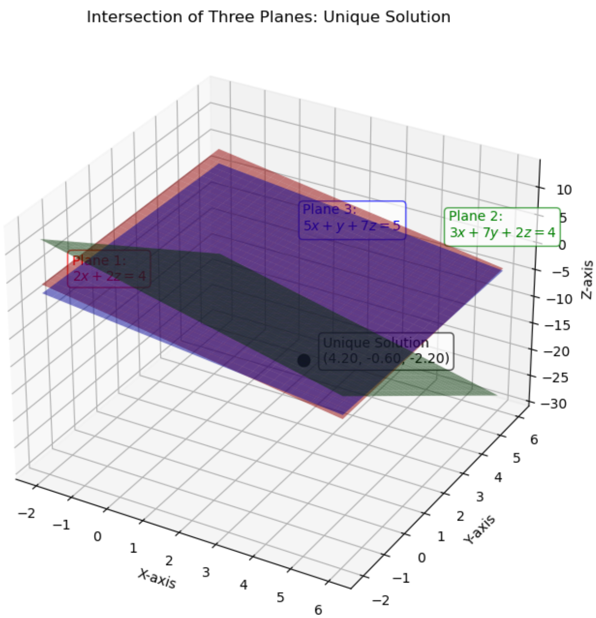
$$R_3 \to R_3 - \frac{1}{7}R_2$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 2 & 4 \\ 0 & 7 & -1 & -2 \\ 0 & 0 & \frac{15}{7} & \frac{-33}{7} \end{array}\right) \tag{4}$$

Now on back-substitute:

$$\mathbf{x} = \begin{pmatrix} \frac{21}{5} \\ -\frac{3}{5} \\ -\frac{11}{5} \end{pmatrix} \tag{5}$$

Hence, a unique solution exists.



$$\tag{6}$$