

12.697

Namaswi-EE25BTECH11060

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# Question

Given the matrix

$$\mathbf{A} = \begin{pmatrix} 5 & 3 \\ 2 & 0 \end{pmatrix},$$

find the value of

$$\det(\mathbf{A}^4 - 5\mathbf{A}^3 + 6\mathbf{A}^2 + 21\mathbf{I})$$

The characteristic equation of **A** is

$$\det(A - \lambda I) = 0 \quad (1)$$

$$\det \begin{pmatrix} 5 - \lambda & 3 \\ 2 & -\lambda \end{pmatrix} = 0 \quad (2)$$

$$(5 - \lambda)(-\lambda) - (3 \cdot 2) = -5\lambda + \lambda^2 - 6 = 0 \quad (3)$$

$$\lambda^2 - 5\lambda - 6 = 0 \quad (4)$$

Factorizing:

$$(\lambda - 6)(\lambda + 1) = 0 \quad (5)$$

Hence, the eigenvalues are:

$$\lambda_1 = 6, \quad \lambda_2 = -1 \quad (6)$$

According to eigenvalue property

If  $f(\mathbf{A}) = \mathbf{A}^4 - 5\mathbf{A}^3 + 6\mathbf{A}^2 + 21\mathbf{I}$ , then the eigenvalues of  $f(\mathbf{A})$  are:

$$f(\lambda_i) = \lambda_i^4 - 5\lambda_i^3 + 6\lambda_i^2 + 21$$

So Find,

$$f(6) = 6^4 - 5 \cdot 6^3 + 6 \cdot 6^2 + 21 \quad (7)$$

$$= 1296 - 1080 + 216 + 21 \quad (8)$$

$$= 453 \quad (9)$$

$$f(-1) = (-1)^4 - 5(-1)^3 + 6(-1)^2 + 21 \quad (10)$$

$$= 1 + 5 + 6 + 21 \quad (11)$$

$$= 33 \quad (12)$$

Determinant of  $f(A)$

$$\det(f(A)) = f(\lambda_1) \cdot f(\lambda_2) \quad (13)$$

$$= 453 \cdot 33 \quad (14)$$

$$= 14949 \quad (15)$$

```
#include <stdio.h>

int main() {
    // Eigenvalues of the matrix A
    double lambda1 = 6;
    double lambda2 = -1;

    // Function  $f(\lambda) = \lambda^4 - 5\lambda^3 + 6\lambda^2 + 21$ 
    double f_lambda1 = lambda1*lambda1*lambda1*lambda1
                      - 5*lambda1*lambda1*lambda1
                      + 6*lambda1*lambda1
                      + 21;
```

```
#include <stdio.h>

int main() {
    // Eigenvalues of the matrix A
    double lambda1 = 6;
    double lambda2 = -1;

    // Function  $f(\lambda) = \lambda^4 - 5\lambda^3 + 6\lambda^2 + 21$ 
    double f_lambda1 = lambda1*lambda1*lambda1*lambda1
                      - 5*lambda1*lambda1*lambda1
                      + 6*lambda1*lambda1
                      + 21;
```

```
double f_lambda2 = lambda2*lambda2*lambda2*lambda2
                - 5*lambda2*lambda2*lambda2
                + 6*lambda2*lambda2
                + 21;
// Determinant of f(A) = f(lambda1) * f(lambda2)
double det = f_lambda1 * f_lambda2;
printf("The determinant of A^4 - 5A^3 + 6A^2 + 21I is: %.01f\\n", det);
return 0;
}
```