

12.281

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Question

\mathbf{A} is a 2×2 matrix with $\det \mathbf{A} = 2$. Then $\det 2\mathbf{A}$ is

Solution

Given

$$\det(\mathbf{A}) = 2 \quad (1)$$

We want to find $\det(2\mathbf{A})$.

As For any $n \times n$ matrix A and scalar k , we have

$$\det(k\mathbf{A}) = k^n \det(\mathbf{A}) \quad (2)$$

Since \mathbf{A} is 2×2 , $n = 2$. Therefore,

$$\det(2\mathbf{A}) = 2^2 \det(\mathbf{A}) \quad (3)$$

$$= 4 \times 2 \quad (4)$$

$$= 8 \quad (5)$$

Verification using eigenvalues:

Let the eigenvalues of \mathbf{A} be λ_1 and λ_2 .

We know:

$$\det(\mathbf{A}) = \lambda_1 \lambda_2 = 2 \quad (6)$$

The eigenvalues of $2\mathbf{A}$ are:

$$2\lambda_1 \quad \text{and} \quad 2\lambda_2 \quad (7)$$

Thus,

$$\det(2\mathbf{A}) = (2\lambda_1)(2\lambda_2) \quad (8)$$

$$= 4\lambda_1 \lambda_2 \quad (9)$$

$$= 4 \times 2 \quad (10)$$

$$= 8 \quad (11)$$