12.385

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Question

Given the following matrix equation

$$A_{m \times n} X_{x \times 1} = b_{m \times 1}$$

the nature of this system of equations is

1) over determined if $m > n$
2) under determined if $m < n$
3) even determined if $m = n$
4) determined by rank of the matrix

Solution

Given the system of equations

$$\mathbf{A}_{m \times n} \mathbf{X}_{n \times 1} = \mathbf{b}_{m \times 1},$$

As m determine number of equations and n number of unknowns

- If $m > n$, there are more equations than unknowns *over-determined*.
- If $m < n$, there are fewer equations than unknowns *under-determined*.
- If $m = n$, the system is *even-determined* (square system).

However, just knowing m and n does **not** guarantee a solution, because some equations may be **linearly dependent**.

- The rank of A gives the number of **independent equations**.
- For a square system ($m = n$), a unique solution exists only if $\text{rank}(A) = n$.
- If $\text{rank}(A) < n$, the system may have **no solution or infinitely many solutions**.
- For non-square systems ($m \neq n$), the rank still determines if a solution exists and how many solutions are possible.

Conclusion:

The actual nature of the system depends on the **rank of the matrix** A, not just on the number of equations and unknowns.