

# 12.385

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## Question

Given the following matrix equation

$$A_{m \times n} X_{n \times 1} = b_{m \times 1}$$

the nature of this system of equations is

- |                                |                                     |
|--------------------------------|-------------------------------------|
| 1) over determined if $m > n$  | 3) even determined if $m = n$       |
| 2) under determined if $m < n$ | 4) determined by rank of the matrix |

## Solution

Given the system of equations

$$A_{m \times n} X_{n \times 1} = b_{m \times 1},$$

As  $m$  determine number of equations and  $n$  number of unknowns

- If  $m > n$ , there are more equations than unknowns *over-determined*.
- If  $m < n$ , there are fewer equations than unknowns *under-determined*.
- If  $m = n$ , the system is *even-determined* (square system).

However, just knowing  $m$  and  $n$  does **not** guarantee a solution, because some equations may be **linearly dependent**.

- The rank of  $A$  gives the number of **independent equations**.
- For a square system ( $m = n$ ), a unique solution exists only if  $\text{rank}(A) = n$ .
- If  $\text{rank}(A) < n$ , the system may have **no solution or infinitely many solutions**.
- For non-square systems ( $m \neq n$ ), the rank still determines if a solution exists and how many solutions are possible.

## Conclusion:

The actual nature of the system depends on the **rank of the matrix  $A$** , not just on the number of equations and unknowns.