

# 12.489

EE25BTECH11060 - V.Namaswi

## Question

Matrix

$$A = \begin{pmatrix} 2 & 0 & 2 \\ 3 & 7 & 2 \\ 5 & 1 & 7 \end{pmatrix}, b = \begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix}$$

are given. If vector  $\mathbf{x}$  is the solution to the system of equations  $\mathbf{Ax} = \mathbf{b}$ , which of the following is true for  $\mathbf{x}$

- |                              |                                   |
|------------------------------|-----------------------------------|
| (a) Solution does not exist  | (c) Unique solution exists        |
| (b) Infinite solutions exist | (d) Five possible solutions exist |

## Solution

Given,

$$A = \begin{pmatrix} 2 & 0 & 2 \\ 3 & 7 & 2 \\ 5 & 1 & 7 \end{pmatrix} \quad b = \begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix} \quad (1)$$

Form the augmented matrix:

$$\left( \begin{array}{ccc|c} 2 & 0 & 2 & 4 \\ 3 & 7 & 2 & 4 \\ 5 & 1 & 7 & 5 \end{array} \right) \quad (2)$$

According to Gaussian elimination: Replace

$$R_2 \rightarrow R_2 - \frac{3}{2}R_1, \quad R_3 \rightarrow R_3 - \frac{5}{2}R_1$$

$$\left( \begin{array}{ccc|c} 2 & 0 & 2 & 4 \\ 0 & 7 & -1 & -2 \\ 0 & 1 & 2 & -5 \end{array} \right) \quad (3)$$

Replace

$$R_3 \rightarrow R_3 - \frac{1}{7}R_2$$

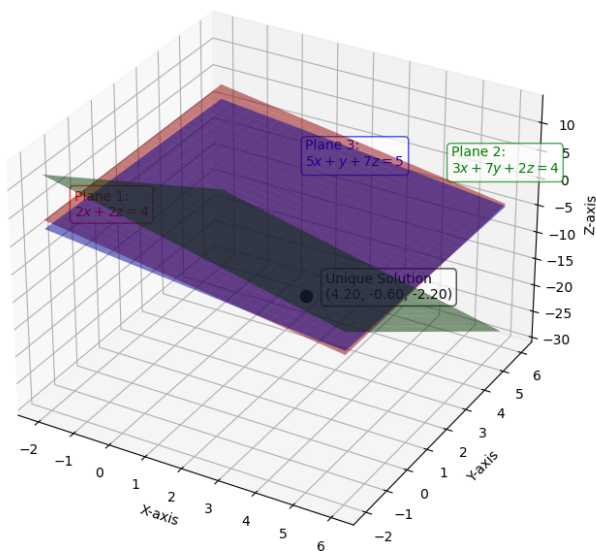
$$\left( \begin{array}{ccc|c} 2 & 0 & 2 & 4 \\ 0 & 7 & -1 & -2 \\ 0 & 0 & \frac{15}{7} & \frac{-33}{7} \end{array} \right) \quad (4)$$

Now on back-substitute:

$$\mathbf{x} = \begin{pmatrix} \frac{21}{5} \\ -\frac{3}{5} \\ -\frac{11}{5} \end{pmatrix} \quad (5)$$

Hence, a unique solution exists.

Intersection of Three Planes: Unique Solution



(6)