Vis Comput
DOI 10.1007/s00371-016-1324-y

SURVEY

New advances for haptic rendering: state of the art

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Abstract During the last decade, haptics has been a new emerged and interesting subject for many researchers, which can be classified into three topics such as human haptics, machine haptics and computer haptics. Haptic rendering is the most important technology for computer haptics, which means the process of calculating the force or tactile feedback to give the user a sense of touch or interaction with the virtual object. This paper provides a detailed and comprehensive survey of the methods and technologies for haptic rendering in the past 5 years, mainly from 2010–2015, including haptic rendering for rigid–rigid interaction, haptic rendering for rigid–deformable interaction, haptic rendering for rigid–fluid interaction, haptic rendering for image- and video-based interaction, and texture and tactile rendering. The main research efforts and the typical algorithms are discussed, and the new ideas and research progresses are investigated, then the conclusions and future directions are summarized.

Keywords Haptics · Haptic rendering · Survey

1 Introduction

As the new technology emerged in recent years, haptics has been an interesting topic for many researchers, and also has shown a good application potential in many areas, for example, from industry to medicine, education to service, and entertainment to military [1]. Haptic rendering is the most important technology for computer haptics, which means the process of calculating the force or tactile feedback to give the user a sense of touch with the virtual object [2]. Different haptic rendering algorithms have been proposed in the past 20 years. According to the way of the probing object modeled, they can be categorized as point-based methods, line-based methods, triangle mesh-based methods, and volume mesh-based methods [3]. According to the number of degrees of freedom (DoFs), they can be classified as 3-DoF haptic rendering methods and 6-DoF haptic rendering methods. According to the control type, they can be classified as admittance-based rendering methods and impedance-based rendering methods. According to the feedback type, they can be classified as force rendering methods and tactile rendering methods.

Early haptic rendering methods are mainly focused on point-based 3-DoF rendering, which has the benefit of providing a pen-like tool-based interaction for efficient computation with a stable force feedback. As the wide applications of haptic simulation in engineering and medicine, haptic rendering has the trend of moving from 3-DoF rendering to 6-DoF rendering, from simple point-based rendering to complex object-based rendering, and from rigid object rendering to deformable object rendering. A diverse set of new haptic rendering methods has been proposed in recent years, this paper provides a detailed and comprehensive survey of haptic rendering emerged in the past 5 years from 2010–2015. Normally haptic rendering includes force rendering and tactile rendering. Currently because force feedback devices are much more popular than tactile feedback devices, in this paper we mainly focus on force rendering. However, we also give a simple classification and discussion for tactile rendering. According to the interaction type, the different haptic rendering methods for force feedback can be classified as: haptic rendering for rigid–rigid interaction, haptic rendering for rigid–deformable interaction, haptic rendering

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Published online: 18 October 2016

for rigid–fluid interaction, and haptic rendering for image- and video-based interaction.

The rest of the paper is organized as follows. The main challenges for haptic rendering are summarized in Sect. 2. Haptic rendering for rigid–rigid interaction is discussed and compared in Sect. 3, which includes penalty-based method, constraint-based method, impulse-based method, and implicit surface-based method. Haptic rendering for rigid–deformable interaction is discussed and compared in Sect. 4, several new rendering methods of deformable objects are introduced and analyzed in this section. Haptic rendering for rigid–fluid interaction is discussed in Sect. 5, haptic rendering for image- and video-based interaction is discussed in Sect. 6, and texture and tactile rendering is discussed in Sect. 7. Then, conclusions and future directions are summarized in Sect. 8.

2 Challenges for haptic rendering

Haptic rendering is an interesting topic which has attracted many researchers in recent years; however, the successful applications of haptic rendering algorithms are handicapped by several challenges. Here, we mainly discuss the challenges for force rendering, which can be organized from the following two aspects.

Three challenges can be summarized from the property of haptic rendering. (1) Force modeling technology. Most of the current haptic rendering algorithms only use the simplified model based on penetration depth to calculate the force or torque feedback according to Hooke law, which don't think about the material characteristics of real touch such as hardness, visco-elasticity, no homogeneity, and anisotropy. Physics-based modeling is a promising approach for accurate force model of haptic rendering. However, because of the complexity of physical phenomena, it is still not clear to understand the physical law behind many physical phenomena. Therefore, it is not easy to develop an accurate analytic force model for credible haptic rendering. For example, for haptics-based cutting and suturing of medical simulation, according to the cutting theory, there are many parameters such as cutting speed, cutting depth, tool geometry, the angles between the cutting blades and the bone surface, and so on, which will affect the resultant cutting force. Also for haptics-based dental drilling, because the theory of bone drilling is still not clearly explored, some researchers use the metal machining theory for drilling force modeling. However, because the complex fracture phenomenon of the bone drilling differs from the continuous metal drilling, the force model is still different from real surgery. (2) Number of degrees of freedom. More degrees of freedom of haptic rendering algorithm mean that it can support more complicate haptic interaction. Normally 3-DoF haptic rendering method can only support single-point-based interaction without torque feedback, which is much different from the real interaction with human hand. 6-DoF haptic rendering algorithm can support object–object interaction with force and torque feedback; however, it will bring high computation cost for real-time collision detection and haptic rendering. (3) Physics simulation technology. All the haptic rendering algorithms share a common idea: they use a virtual replica of the haptic device, which is called as haptic probe or virtual tool, to simulate the physics-based interaction between this probe and virtual object, and then to compute and render force/torque based on the deviation of the haptic probe and the true device configuration. Therefore, physics simulation technology is very important to realize physics-based interaction. For rigid–rigid haptic interaction, the dynamic behavior of rigid object should satisfy physics law such as Newton's laws of motion. For rigid–deformable haptic interaction, the deformation of soft object should be as realistic as the real world according to accuracy and real-time. For rigid–fluid-based haptic interaction, the simulation process should satisfy the fluid mechanics. However, currently, there are still challenges of physics simulation technology for complex environment with a large number of rigid objects, or deformable objects with hundreds of thousands of polygons, or large-scale fluids.

Three challenges can be also summarized from the performance of haptic rendering. (1) Stability of haptic rendering. Stability refers to the stable running of the haptic device with high-update rate [4]. Several factors will affect the stability of haptic rendering. For example, for haptic rendering of high-hardness objects, if the stiffness value in the force model exceeds the maximum stiffness of haptic device, it will generate unstable force feedback. During haptic cutting of volume-based models, when the material is directly removed by haptic device, the discontinuity in the geometric model will also produce discontinuous force feedback and lead to unexpected vibrations. For distributed haptic rendering over internet, the network delay, jitter, and packet loss occurred during the transmission of HIP will also cause unstable force feedback. The multi-thread simulation will also affect the stability of haptics rendering. Normally the haptic rendering thread runs at high-fresh rate, while the graphic rendering thread runs at low fresh rate. The incompatibility of computation time among different threads will lead to the discrepancy of data for haptic rendering. For example, the haptic rendering thread needs to obtain the penetration depth from physics simulation thread for force computation. However, the abrupt change of penetration depth between two continuous frames will lead to the abrupt change of force feedback, which resulting in the instability of haptic rendering. (2) Fidelity of haptic rendering. Fidelity means that the force feeling during haptic simulation is similar to that in real situation [4]. Although most of haptic rendering methods can provide force or torque feedback to the user, there is still a large difference for the fidelity of haptic rendering between the simulation environment and the real environment. One reason is that the range of force feedback provided by haptic rendering algorithm is different from that of real force feedback. For example, for MIS colonoscopy surgery, the estimated forces and torques for a procedure performed with an adult size colonoscopy are in the range of about ± 40 N and ± 1 Nm, respectively [5]. However, most of the commercial haptic devices cannot meet this requirement because they can only provide limited continuous force/torque feedback (the maximum exertable force for PHANTOM desktop is only 7.9 N). The simplified force model will also affect the fidelity of haptic rendering. For example, to relieve computation cost and achieve adequate high update rate, some simplifications of the force model will be adopted, however this will weaken the fidelity of haptic rendering in return. (3) High-update rate for complex models. Because the human sensory-motor system requires a fast-update rate (at least 1 kHz) to feel the stable stimuli applied to the user, haptic rendering algorithms should also run at high-update rate to provide realistic and stable feedback. However, this high-update rate is often difficult to reach when working in complex environments. Complex models with millions of vertice and meshes will bring heavy computation load for real-time collision detection and high-update rate haptic rendering. Most of today's haptic rendering methods are limited by the complexity of virtual models.

It is difficult to quantitatively compare the different haptic rendering methods. Until now there is still no standard method to evaluate haptic rendering. Therefore, in this paper we review the different haptic rendering methods from the following perspectives: (1) force modeling technology; (2) degrees of freedom; (3) physics simulation technology for haptic rendering; (4) stability of haptic rendering; (5) fidelity of haptic rendering; (6) supporting complex models for haptic rendering;

3 Haptic rendering for rigid–rigid interaction

The typical haptic rendering methods for rigid–rigid interaction can be summarized as shown in Table 1. According to the implementation approach, they can be classified as four categories: penalty-based method, constraint-based method, impulse-based method, and implicit surface-based method.

Penalty-based method calculates the force feedback based on the tool's penetration depth into virtual objects. Penalty-based method has been widely used for early haptic rendering, because of its simplicity and less computational cost. However, it may allow interpenetration between the graphic tool and virtual environments, and also it may result in strong instability problem because of the difficulty of tuning the parameters properly. Li et al. [6,7] extended the traditional penalty-based method based on the idea of generalized penetration depth. Normally for penalty-based haptic rendering, the penalty forces are calculated based on the amount of translational penetration depth only. However, for generalized penetration depth-based method, the penetration depth is defined to include both the translational and rotational motions. The validation experiments show that the proposed method can produce more accurate and stable haptic feedback than traditional method. Virtual coupling [13] uses a spring-damper model to connect the haptic device with the virtual tool. The motion of haptic device can be converted as the force/torque acting on the virtual tool, and the same force/torque is used for haptic rendering. The dynamics of virtual tool can be simulated by the coupling force/torque combined with the collision force/torque. Although virtual coupling method can overcome the limitation of penalty-based method, it still suffers from the degraded transparency problem, and it cannot ensure stable haptic rendering for high stiffness. Hou [8] proposed an adaptive haptic rendering algorithm for virtual coupling. The method can adjust the parameters of virtual coupling automatically according to mass values of the simulated virtual tools. Therefore it can overcome the virtual tool displacement problem caused by the large mass values, and keep a stable force/torque feedback in a dynamic virtual environment. However, the proposed method needs to be further validated for its generality for diffierent haptic devices and its performance for complex virtual models.

The main idea of constraint-based method is to constrain the pose of graphic tool on the surface of virtual object to be free of penetration, while the haptic tool can penetrate into objects. Constraint-based method has many advantages than penalty-based method, such as stable force feedback and realistic haptic interaction. However, the main drawback is the high-computation cost for collision detection and constraint solution. The typical constraint-based haptic rendering methods for generic polygonal models are the early 3-DoF god-object method [14], and the extended 6-DoF god-object method [15]. However, for 6-DoF constraint-based haptic rendering, the continuous collision detection is time-consuming for complex scenarios involving multiple contacts, and explicit Euler integration for target configuration computation of the god-object can cause enough inaccuracy which will make it penetrate the surface of a cavity or even vibrate between two surfaces in a cluttered environment. Wang et al. [9] presented an improved constraint-based 6-DoF haptic rendering method for fine manipulation in narrow space during dental surgery. For example, in dental probing, the probe is constrained by both the tooth and the gingival in a narrow cavity between them. Frequent constraint changes or contact switches occur during the tool's movement. How to satisfy the narrow space constraints on the manipulated tool and, the accurate and stable force sensa-

Table 1 Summary of typical haptic rendering methods for rigid–rigid interaction

Method	Property			Performance		
	Dynamics simulation	DoF	Force model	Stability	Fidelity	Supporting complex model
Generalized penetration depth-based method [6,7]	Not discussed	6	Springdamper model	An iterative local optimization method to decrease the subtle rotation motion which can cause instability	Projecting the force direction onto velocity direction of the moving object to get same haptic feedback as real world	Supporting general polygonal models with tens of thousands of triangles with update rate of 200–500Hz
Adaptive haptic rendering algorithm [8]	Open Dynamic Engine (ODE) for physics simulation	6	Mass–spring–damper model	Automatically adjust the parameters of virtual coupling for stable haptic rendering; The force/torque magnitude can be automatically saturated to the maximum values of device	Overcoming the virtual tool displacement problem caused by the large mass values	Need further validation for complex models
Configuration-based optimization method [9]	A configuration-based optimization algorithm to simulate the quasi-static motion of virtual tool	6	Spring force model	The stability of force feedback can be maintained without using virtual coupling to avoid the loss of realistic feeling	Accuracy of haptic rendering affected by two factors: geometric error of using spheres to approximate object's shape and Taylor expansion	Both collision detection and haptic rendering can be maintained at 1 kHz for the experimental scenarios with multiple contacts because of a sphere tree representation of virtual objects
Method proposed by Wang et al. [10]	Physics simulation based on impulse theory	6	Force model based on impulse theory	Stable haptic feedback for the rotational speed increased from 15,000 to 30,000 rpm	Translational velocity, rotational velocity, and material properties have been considered for haptic rendering, also vibration perception during burring increases fidelity	The refreshing rate can be maintained around 500–1000 Hz for bone models involved a tooth (589 triangles) and a jaw (882 triangles)
Method proposed by Moustakas [11]	No physics simulation for implicit surface-based method	6	Spring force model	A local smoothing operation is applied to the distance map for stable force feedback in the areas around the edges	Friction and texture is added for haptic rendering to avoid unrealistic touching such as slippery surface	The computational complexity is greatly reduced compared to mesh-based haptic rendering for complex models because of using implicit function
God-Finger method [12]	Havok physics for collision detection and physics simulation	3	Spring-damper model	By simulating a contact area from a single contact point, it can provide stable haptic interaction without rotation oscillation	More realistic interaction in a similar manner as actual finger pads for better visual and haptic fidelity	The method can be performed on single PC with 60 Hz for visual rendering and 1000 Hz for haptic rendering because of less computation cost compared with soft hand method, however more evaluations are needed for complex scenarios

tion of fine and sharp features in complex contact scenarios at 1 kHz update rate is a big challenge. The method uses sphere tree representation of objects for real-time collision detection, and the spring force model without virtual coupling for stable and realistic haptic rendering. A configuration-based optimization method is developed to compute contact configurations of the graphic tool to avoid visual artifacts such as perceptible penetration or separation between the graphic tool and the virtual object. Talvas et al. [12] proposed a god-finger method for realistic haptic interaction with rigid objects. Normally, single-point-based haptic rendering method uses only one contact point to touch the virtual object, which is different from the actual finger pad. God-finger method calculates the contact area between a god-object and virtual objects for haptic interaction, which can simulate the slight deformation of human finger pad to match the shape of a touched object. Comparing with god-object method, god-finger method has more realistic behavior for visual and haptic rendering; however, currently it can only support 3-DoF haptic rendering with rigid objects, 6-DoF haptic rendering for deformable objects and multiple god-fingers rendering should be taken into account in the future.

Impulse-based method was first proposed for rigid body dynamics simulation [16], where a virtual object is moved by a series of impulses upon contact/collision rather than forces based on penetration depth. For impulse-based method, the contact states between a moving object and a static object can be classified as separation and impulse contacts, and a continuous contact state is regarded as a series of micro-impulses. Therefore, it can simultaneously meet the requirements of physical accuracy and computational efficiency for haptic rendering. Wang et al. [10] proposed an impulse-based method for haptic rendering of bone-burring simulation. Because of the extremely high rotational speeds of bone-burring, the traditional methods such as constraint-based methods are not suitable for haptic rendering of the interaction. The proposed force model is based on impulse theory, which includes translational velocity, rotational velocity, and material properties. Also, the vibration force during burring can increase the reality of haptic simulation. However, the real forces produced by the actual bone-burring operations should be measured and the parameters of force model can be refined.

Implicit surface-based method can be regarded as a direct haptic rendering method. Implicit surface has the benefit of quick collision detection and simple calculation. Therefore, it provides an efficient approach for haptic rendering. By transforming the polygon-based model into implicit surface representation, the collision detection and penetration depth computation can be simplified as the calculation of implicit function value. Kim et al. [17] proposed an early 3-DoF haptic rendering algorithm based on implicit surface. The volumetric model is represented by the implicit surface with potential value in a 3D regular grid, and force feedback can be calculated based on the function value of implicit surface. Moustakas [11] presented an improved haptic rendering method based on support plane mapping and implicit surface. The proposed method can support both 3-DoF and 6-DoF haptic rendering. Comparing with the state-of-the-art mesh-based haptic rendering method, implicit surface-based method can significantly reduce the computation cost, and also can provide smooth and realistic haptic interaction with collision, friction, and texture feedback.

4 Haptic rendering for rigid-deformable interaction

Early haptic rendering methods mainly focus on rigid–rigid interaction. However, haptic rendering for deformable objects is more challenging and attracts more attention in recent years. Two difficulties hamper the progress of haptic rendering for deformable objects: (1) physics simulation for deformable object is more expensive than dynamic simulation of rigid object; (2) high-haptic update rate cannot be easily achieved for complex deformable models. The typical haptic rendering methods for rigid–deformable interaction emerged in recent 5 years are summarized as shown in Table 2.

Several approaches have been proposed to simulate the soft object deformation during haptic interaction, such as mass-spring method, shape matching method, and mesh-free method [29]. Early deformation simulation is mainly based on mass-spring model, because of its simple structure and high-computation efficiency. However, mass-spring methods will produce physically inaccurate results, and also lack a clear parameter selection mechanism to achieve a desired response. Physics-based modeling has got wide applications for haptics-based deformation simulation. Several approaches have been proposed in recent years. (1) Finite element method has become the mainstream method for haptics-based deformation simulation, such as linear FEM [30], Non-linear FEM [31], co-rotational linear FEM [27], extended FEM [29], and so on. For example, Peterlı'k et al. [27] uses the co-rotational FEM combined with an implicit integration to simulate haptic interaction of both light models (e.g., flexible needles) and soft-tissues. This approach can handle both volumetric objects and hollow structures, and can capture accurately the geometrical nonlinearities of the deformations. However, it does not account for material nonlinearities such as hyperelastic, viscoelastic, or viscoplastic behaviors. Barbic and James [25] proposed the reduced nonlinear FEM model for haptic rendering of multisite contact with large deformation of geometrical highly complex objects. One deformable object is represented by

Table 2 Summary of typical haptic rendering methods for rigid–deformable interaction

Method	Property			Performance		
	Deformation simulation	DoF	Force model	Stability	Fidelity	Supporting complex model
Method proposed by Zhang et al. [18]	Layered rhombus-chain-connected model	6	The virtual contact force is given by the force sensor installed on the admittance device	Both haptic simulation and qualitative analyses showed that the proposed method did not reflect any vibration and unstable force in haptic interaction	Compared with finite element method (FEM), the proposed method provides a more realistic force feeling based on the threshold for the force that the human can reliably discriminate	Haptic update rate can be realized at 1000 Hz for the virtual liver composed by 3116 nodes and 6228 triangular meshes and the virtual lung composed by 15,298 nodes and 29,460 triangular meshes
Method proposed by Tian et al. [19]	Shape matching method	3	Force is defined as the gradient of potential energy of soft tissue based on particle constraint	Stable haptic rendering can be guaranteed by computation efficiency and unconditionally stable time integration scheme of shape matching	The recorded curve of force feedback is similar with the measurement results	Haptic thread can maintain a high update rate of 1 KHz while keeping 27.6 Hz for 3D rendering during the needle insertion into virtual liver
Method proposed by Hem and Choi [20]	Local geometry deformation based on benchmarking data from offline FEM	3	Analytical force model based on tool size, shape, and material properties of deformable object	Comparing with FEM and mass spring model, the proposed method can be directly run in the 1 kHz haptic servo loop for stable haptic rendering	The force errors between the proposed method and FEM are less than 7 %, which is around the just-noticeable difference (JND) of human force sensation	Supporting real-time haptic interaction for high-resolution surface model which contains 10,000 vertices, 29,601 edges, and 19,602 triangles at 1 kHz refresh rate, however as the mesh resolution increases, the computation time per step grows proportionally
Method proposed by Neubauer et al. [21]	A precomputed reduced order model with FEM	6	Sum of penalty force (spring force model) and reaction force (FEM)	Force feedback is much stable than the other similar methods without collision volume	More realistic tissue deformation and more intuitive force feeling because the surface will start deforming as soon as pressure is applied	For complex model with about 190,000 surface vertices, haptic rendering was maintained at 1 kHz while only a small portion of endoscope was inserted, however it dropped to the minimum of about 500 Hz when endoscope was fully inserted
Method proposed by Gonzaleza et al. [22,23]	Every possible deformation are precomputed offline	3	Spring force model	Stable haptic rendering can be guaranteed by avoiding updating the distance field of deformable object	The realism of force feedback needs to be further validated	Supporting haptic interaction with two Stanford bunnies containing 286 vademecum modes at 1 KHz

Table 2 continued

Method	Property			Performance		
	Deformation simulation	DoF	Force model	Stability	Fidelity	Supporting complex model
Method proposed by Wang et al. [24]	Sphere tree model with springs	6	Spring damper model	Stable force feedback under either large or small collision velocities with a large coupling stiffness in the stable impedance range of haptic device, however, instabilities will occur for a detailed sphere tree representation with explicit methods	Realistic haptic feedback by simulating responsive changes of force/torque caused by switches of contact states and a resistance force threshold, however, geometric error of sphere tree representation will lead to perceptible force/ torque artifact while sliding along surface	Supporting complex haptic interaction of dental surgery with hybrid contacts and frequent contact switches in one single thread with 1 kHz update rate
Method proposed by Barbic and James [25]	Reduced nonlinear FEM model	6	Spring force model with quasi-static damping	Multi-resolution point-contact model and nested pointshell structure enable stable haptic rendering of large models at fast update rate	The fidelity of haptic feedback needs to be compared with other methods	Supporting geometrically complex deformable objects with multisite contacts and large deformation and rotation at real-time kilohertz rates such as deformable dinosaur, hose and dragon with millions of points
Method proposed by Mafi et al. [26]	FEM	3	Spring force model	Hardware-based approach can solve a large system of equations at an update rate of 100–1000 Hz for stable haptic rendering	A survey of graduate students showed that the rendered soft objects using the proposed platform are felt realistic	Real-time simulation of 3D linear elastic deformation models with 1500 nodes can be maintained at an update rate of up to 2500 Hz
Method proposed by Peterlı'k et al. [27]	Co-rotational FEM	6	Constraint-based force model	Stable haptic rendering can be guaranteed by a multirate compliant mechanism and by setting the time limitation of timed Gauss-Seidel method to a sufficiently short period	The recorded force profiles were proved to be accurate and similar to profiles reported elsewhere	Supporting interactive simulation of flexible needle insertion through soft anatomical structures where 120 constraints can be handled on single PC
Method proposed by Knott and Kuhlen [28]	The contact model with an implicit temporal integration scheme creates an intermediate mechanical contact representation for dynamic behavior simulation of objects	6	Force feedback based on contact model	Stable haptic rendering can be achieved by overcoming the artifacts of common multi-rate scheme	The accuracy and quality of haptic rendering can be deomonstraed by several scenarios with one or more haptic devices	The size of the region modeled by the contact representation can be automatically adapted to the complexity of the geometry model in contact for real-time simulation

the pointshell organized as a nested tree structure and the other one by a signed-distance field. Then, collision detection and haptic computation can be performed in a single loop running at 1000 frames per second. However, this contact force and torque cannot simply be sent to the haptic device because each point in contact adds to the overall stiffness of the system and if enough points are added, maximum renderable stiffness of the haptic device is easily exceeded. Therefore the quasi-static damping is introduced to maintain the stability of haptic rendering. The proposed method is the first realization of haptic rendering of geometrically complex deformable objects with several simultaneous contact sites on a 6-DoF haptic device. However, currently some limitations such as the modification or cutting of the models, the self-collision processing, and the simulation of friction are not supported. (2) The precomputed approach or the offline computation method can be also used for haptic simulation of deformable object. Gonzaleza et al. [23] provides an offline precomputed solution for real-time collision contact and haptic interaction between two non-linear deformable solids. One solid is modeled as a pointshell and the other is in turn equipped with a signed distance field. Then at every haptic cycle, collision with the first solid is detected by checking the position of its boundary nodes against this distance field, and the penalty force can be determined by querying the points of first solid against the distance field associated to second one. Because the high-dimensional distance field can be computed offline to contain any deformed configuration, stable haptic rendering can be guaranteed with 1 kHz update rate. However, currently, the cost of offline computation for these vademecums is expensive such as more than several hours. (3) The layered rhombus-chain-connected model [18] is also a physics-based method for haptic interaction of deformable objects. By an efficient representation of virtual object with the layered rhombus-chain-connected model, the surface deformation can be calculated according to the sum of relative displacements of each chain structure unit, and the resultant force of springs can be calculated as the same value of the external force. Compared with FEM, the proposed method can simulate the haptic behavior of soft objects in real time because of its simple structure and fast computation; therefore, it can provide better haptic behavior than that of the FEM with improved stability and perception ability. However, the main drawback is that it is only suitable for calculation between force and deformation in a local region of a single-point contact, or a several-point contact where the several contact points are set a little far from each other. (4) Shape matching [19] is a particle-based method for haptic interaction of deformable objects. The virtual object can be modeled by a group of particle clusters, and each cluster is a voxel or cube with eight particles, and each particle is associated with a mass, an initial position, a current position, as well as a goal position. When the constrained particles are inserted into shape matching clusters, the corresponding movement serves as the target position to trigger the shape matching-based deformation. Then, force feedback can be formulated as the gradient of the potential energy of the soft tissue based on particle constraint. Four interaction forces including stiffness force, friction force, cutting force, and clamping force are taken into account for haptic rendering. (5) The sphere tree-based method [24] aims to provide a unified representation of rigid haptic tool and deformable objects for real-time collision detection and haptic rendering. Sphere tree-based collision detection can handle any type of complex polygon models without assumption about the underlying geometry and topology. Deformation of soft tissues can be simulated by updating the hierarchical sphere tree with springs based on contact force, and constraint-based optimization method is used to avoid penetration between rigid haptic tool and soft objects. All the computation including collision detection, deformation simulation, and haptic rendering can be implemented in a single thread with 1 kHz update rate. Because of the simplicity and efficiency of sphere tree representation, the proposed rendering method can simulate large deformation and force response of a tongue under multi-region contacts, and also can simulate the force response when a dental probe inserts into a narrow periodontal pocket with hybrid contacts. (6) Hardware acceleration and parallel computation provides a new approach for real-time haptic interaction with deformable objects. For example, Altomonte et al. [32] studied the deformable objects with haptic feedback on graphics processing unit (GPU). Taking advantage of the computational power of the graphics hardware, the whole computational process can be moved from CPU to GPU. Haptic forces were also computed on GPU and the results were asynchronously transferred to CPU. Then the deformation of complex models based on mass-spring model and haptic interaction with soft objects can be rendered at a frame rate higher than 1 KHz. A hardware-based parallel computation approach is also proposed by Mafi et al. [26]. A linear elastic model based on FEM is formalized for soft object deformation, a new hardware-based parallel implementation of a preconditioned conjugate gradient (PCG) algorithm is implemented to solve the linear equations, and concurrent utilization of a large number of fixed-point computing units on a field-programmable gate array (FPGA) device is employed to yield a very fast solution. Compared to programmable processors such as multicore CPUs and GPUs, FPGAs provide the flexibility of customizing the computing hardware architecture to the problem and accelerate the calculations by efficiently employing a large number of simple parallel computing units. However, generalization of the proposed design to multiple-FPGA configurations to simulate the nonlinear deformation and modeling complex interactions such as cutting and suturing should be exploited in the future.

Multirate simulation scheme is an efficient method for haptic rendering of deformable objects. Although some researchers tried to perform deformation simulation, collision detection, and contact force/torque computation in one single loop, such as the sphere tree-based method [24] and reduced deformable model-based method [25], the main challenge is that these methods require fast enough collision detection and physics simulation algorithms to perform expensive computation in a very short time, which is difficult for nowadays haptic rendering. Multi-thread simulation has been widely used for haptic interaction. For example, the collision detection or the simulation of a complex deformable object are often performed asynchronously at a lower frequency, while force/torque computation and haptic rendering run at the haptic rate. However, data inconsistency between different threads will bring the instability for haptic rendering. A multirate compliant mechanism [27] is proposed for stable haptic rendering of multi-thread surgical simulation. The deformation of virtual objects is computed based on FEM at low-rate in the simulation loop and the haptic feedback is modeled and solved using constraints in the haptic loop with high-update rate. The interaction forces are computed based on an intermediate representation named as multirate compliant mechanism shared between the two loops. The main advantage is that it can overcome the limitation of refresh rate for simulating deformable objects by integrating the constraint solver into the control loop of haptic device with fully implicit scheme for complex haptic interactions. However, the stability of haptic rendering will be affected by the duration of the time step, which is highly constrained by the complexity of the model given by the size of the deformable meshes and the number of constraints. Also, the model topological changes such as cutting are not supported by the current method. Knott and Kuhlen [28] proposed an accurate and adaptive contact model for multi-rate multi-point haptic rendering of deformable objects. The method can overcome the artifact of common multi-rate simulation for haptic rendering when the contact situation quickly changes and the intermediate representation is not able to reflect the changes due to the lower update rate. The contact model can accurately handle convex and concave contact regions, and the size of the region can be automatically adapted to the complexity of the geometry model for real-time simulation. The proposed method can support bimanual haptic interaction with deformable objects.

5 Haptic rendering for rigid-fluid interaction

Haptic rendering of rigid-fluid interaction has promising application in medical, industrial, and entertainment environment; however, it has been ignored for a long time. Several early haptic fluid interaction methods have been proposed [33–35], however, these methods mainly focus on 3-DoF haptic rendering, nonviscous fluid, or simply "swiping" the surface of small amounts of fluid with simple objects. Therefore, there is still a lack of efficient models and algorithms to handle complex 6-DoF haptic rendering with viscous fluids.

Cirio et al. [36] proposed a 6-DoF haptic rendering algorithm for rigid-fluid interaction-based on Smoothed-Particle Hydrodynamics (SPH). A unified particle model is proposed to represent the rigid haptic tool and the fluid to avoid the expensive collision detection computation. Force feedback can be computed by the definition of the smoothing volume. When an external particle coupled with a haptic device enters the smoothing volume, haptic forces can be seamlessly computed between the external particle and the fluid particles based on the sum of pressure and viscosity forces. GPU-based parallel computation is introduced to improve the haptic performance. The proposed method is the first 6-DoF haptic rendering for viscous fluids, which can support many rich and complex manipulations such as stirring, pouring, shaking, and scooping as shown in Fig. 1. Comparing with early haptic fluid rendering methods, the proposed method has better real-time performance such as rendering at 70Hz with up to 32,000 particles. However, the stability of haptic rendering should be investigated in the next step and the update rate should be improved further for more complex fluids. Based on SPH for fluid simulation, they also proposed a vibrotactile rendering method for solid-fluid interaction [37]. The main idea comes from the fact that both acoustic and tactile vibrations share a common physical source. Therefore, the proposed vibrotactile model is based on bubble-based vibrations, which can be divided into three components: the initial high frequency impact with the fluid surface, a cavity oscillation created when the body enters the fluid, and a set of small bubble harmonics. During solid-fluid interaction, the impact produces a "slap" and projects droplets, while the object penetration creates a cavity that is filled with air. Then the cavity is sealed at the surface, creating an air bubble that vibrates due to pressure changes. The

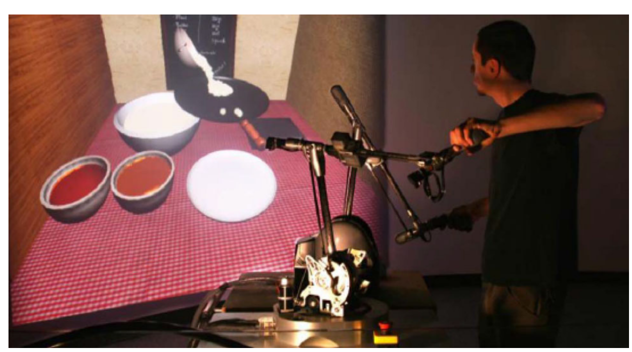

Fig. 1 Haptic rendering for fluid based on SPH [36]

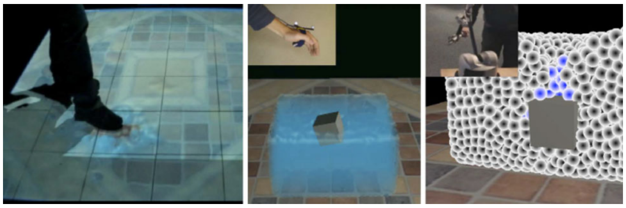

Fig. 2 Vibrotactile rendering of fluid with different haptic devices [37]

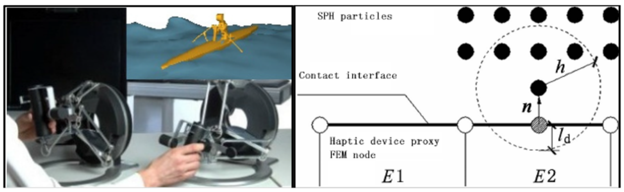

Fig. 3 Haptic rendering of fluid based on SPH and FEM [38]

vibrotactile signal can be output by a specific haptic device such as shoes, gloves, or a 6-DoF haptic device as shown in Fig. 2.

Wang and Wang [38] proposed a hybrid method for haptic rendering of rigid–fluid interaction, where the solid haptic proxy is modeled by FEM mesh and the fluid is modeled by SPH. As shown in Fig. 3, the interface between haptic device proxy and fluid is modeled as the node-surface contact. For each particle, a neighbor search of FEM elements in a predefined radius is executed to obtain the surface by traversing all the nodes. If penetration between a particle and the surface occurs with a depth, a parameterized penalty force will be generated and applied to particle to impede the movement. GPU-based acceleration has been implemented to improve system performance. The proposed method can validly deal with fluid–solid interaction and the solid deformation phenomenon, and at the same time can obtain the smoother force feedback than a rigid proxy.

Hover et al. [39] proposed a data-driven approach for haptic rendering of viscous fluid. Interaction data can be acquired during a recording session by manipulating a sample fluid with a probing tool, which is equipped with a force sensor to obtain the associated force signals. Then, the raw captured force data can be used to synthesize appropriate forces for haptic rendering. During haptic simulation, the user is not restricted to move the tool in the same way as during the recording. Therefore, an offline interpolation of the feedback force for such new interactions from the observed data is necessary. During real-time simulation, forces can be linearly interpolated from this precalculated multidimensional data set. However, the accuracy of force model and the fidelity of haptic rendering are difficult to be guaranteed as real solid-fluid interaction.

6 Haptic rendering for image and video based interaction

Haptic rendering for 2D image-based interaction can be implemented by two approaches: calculating force directly from image, or reconstructing 3D models from 2D image followed by collision detection and haptic computation. The former is mainly based on image processing technology such as contour segmentation, gray value calculation and template matching, which can allow fast computation of force from image pixel with high haptic update rate. However, only the visible part of the shape in the image can be felt by this approach. The other approach is mainly based on 3D geometry reconstruction technology, such as building the depth information from depth image, building the stereoscopic scene from 2D image, or converting 2D sketches into 3D solid models.

Sourin et al. [40] have carried out an intensive research on image-based haptic rendering in recent years. They proposed the idea of tangible image for haptic rendering of medical simulation [41–43]. For example, for haptics-based knee surgery simulation, the arthroscopic panoramic images are stitched from real arthroscopic videos, and then augmented with function-defined 3D models. Two depth maps, one from the 2D image and the other from the 3D models, are extracted and mapped into a uniform depth mask according to the same coordinate system. During the simulation loop, collisions between HIP and the depth value of the corresponding pixel can be checked in real-time to calculate force feedback. The force magnitude can be calculated based on the center value of the depth mask, which determines the amount of penetration into the height field. The force direction is the surface normal at the center of the depth mask, which can be calculated by taking cross-product of two vectors that share the center of the depth mask as a common vertex. The main advantages of this method are the high-quality graphic rendering as realistic as actual surgery, and the high-update-rate haptic rendering with 1 kHz. They also proposed the 3D haptic model for image-inspired haptic rendering [44]. Unlike the common geometry reconstruction process by accurately calculating the central projection parameters from 2D image, the 3D haptic models are created in a 3D haptic modeling space distorted by the central (perspective) projection so that if the haptic models were projected onto the image with parallel orthographic projection, their boundaries would match the contours of the respective parts of the image. Then these 3D haptic models can be placed in front of the image plane, and at the same time the 3D virtual representation of the HIP is also projected onto the same image plane. Since the 3D haptic models are invisible for the user, the analytic FRep functions (variants of implicit functions) are used to define them without graphic rendering. As shown in Fig. 4, after the haptic models are sketched and built, the haptic forces can be

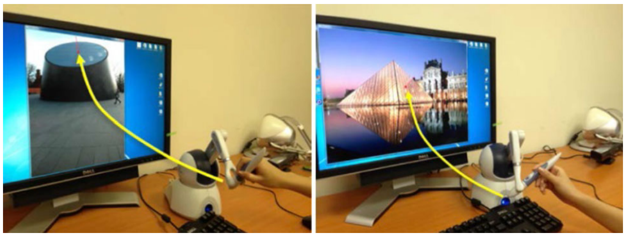

Fig. 4 Stroke-based sketching of haptic models for haptic rendering of 2D image [44]

computed both from the reconstructed 3D haptic models and from the image itself. The force for haptic rendering consists of three components: contact force, friction force, and texture force, which can realistically simulate the image as the actual 3D scenes.

Ryden and Chizeck [45,46] proposed a 6-DoF haptic rendering method for streaming point clouds from depth camera. The depth image is captured in real-time at 30 Hz using a Kinect depth camera, and then is transformed into a point cloud where every point in Cartesian space corresponds to a pixel. A real-time processing algorithm is carried out to filter the streaming point clouds and then the surface normals can be calculated for every point. A quick collision detection algorithm is also developed to find out if a specific point in the point cloud is colliding with the virtual haptic tool. When the virtual tool is in contact, the points of contact can be used to form motion constraints for the virtual tool, which can be used to compute a constrained motion to move the virtual tool toward the configuration of the haptic device without violating any contact constraints. Force/torque feedback can be calculated as a function of the difference between the virtual haptic tool and haptic device configuration. This is the first real-time 6-DoF haptic rendering algorithm for discontinuous streaming point clouds derived from depth sensors.

7 Texture and tactile rendering

Comparing with force rednering, texture and tactile rendering is a new research topic and the progress is still falling behind. There are several tactile rendering methods proposed in recent years.

The first approach for texture rendering is the synthesis method based on real surface [47]. This method needs to design a mechanism to control the real surface to appropriate the shape and curve of the virtual surface, and then the texture of virtual surface can be simulated by the exploration on the real surface. A typical example of this method is the sandpaper system [48]. Bordegoni et al. [49–51] also developed several haptic systems with tactile feedback for product esthetic design. For example, the T'nD sanding tool system is equipped with a surface patch tool to simulate the interaction of a piece of sandpaper. When the user leans his hand over the tool and moves or rotates freely in space to feel the surface of the object, the system can automatically compute the information about the surface curvature according to contact position, and then use this information to shape the tool to follow the curvature of the explored virtual surface. The T'nD scraping tool system can simulate the modeling of a piece of clay by removing material using a sheet of metal to generate high quality surface. The haptic strip system can deform (bending and twisting) a physical continuous strip which is actuated by a mechanical modular mechanism and sensors, to exactly reproduce the shape of a curve on the virtual surface. When the user explores on the virtual surface, the tactile feedback can be reproduced by the physical surface.

The second approach for texture rendering is force feedback-based method based on single-point haptic device [52]. This method is designed to simulate the process of manipulating a tool to explore on the object surface, and the corresponding texture can be rendered by the single-point haptic device through force feedback. Because of the wide applications of commercial single-point haptic devices, several implementations have been realized for this approach. For example, Sierra and Pai [53] proposed a stochastic texture model to simulate object surface roughness haptically supported by a variety of force-feedback mechanisms, Otaduy and Lin proposed [54] a Sinusoidal force model for haptic texture rendering of surface roughness, Song et al. [55] proposed a method to generate force feedback from 2D image captured from object surface for texture rendering using single-point haptic device. Culbertson et al. [56] presented a data-driven approach for modeling and rendering virtual texture from data recorded during unconstrained tool-surface interactions. When a user drags the tool across a texture, the recorded high-frequency signals such as speed, force, and acceleration can be segmented and modeled as a piecewise autoregressive (AR) model. Then each AR model is labeled with the source segment's median force and speed values and stored in a Delaunay triangulation to create a model set, and these model sets can be used to create realistic virtual textures in real time by synthetic vibration signals. The results showed that the proposed method can accurately capture and recreate the roughness of real texture; however, surface hardness and slipperiness are not supported by this approach. Based on this method, they developed one hundred haptic texture models enclosed as an open-source haptic toolkit HaTT (Penn Haptic Texture Toolkit) for texture rendering of 3D objects [57]. However, most of current force feedback devices are heavy and nonportable, which will restrict the possible contexts of application for texture rendering. Also, the mechanical structure of force feedback devices such as the single point stylus held by a set of solid shafts will also limit the range of possible exploration movements.

The third approach for tactile rendering is vibration-based method with multi-point tactile array [58]. This approach needs to design a special needle shaped array device, which each unit can generate vibration based on the characteristics of object surface. Therefore, when the user's hand contacts the tactile array, he can feel the different vibration and force just as touching the real surface. Several actuators (electromagnetic actuator, pneumatical actuators, piezoactuators, and smart materials actuators) can be used to represent and render a sort of virtual textures; however, they lack the active role of the user. Ahmaniemi et al. [59] proposed a dynamic vibrotactile texture rendering method based on wavetable synthesis driven by the user's hand movements. Hand gestures can be easily measured by commercial inertial motion sensors such as accelerometers and gyroscopes, and a vibrotactile actuator can provide realistic tactile feedback. Although this method provides a novel approach for creating virtual textures using simple motion sensors and a single vibrotactile actuator, the main shortcoming is that the bandwidth of the tactile actuator is limited, which can only provide limited frequency response and power. Sarakoglou et al. [60] developed a novel tactile rendering method to display the shape of surface to an area of the fingertip through a 4×4 array of tactors moving perpendicularly to the skin surface. The tactors are spring loaded and are actuated remotely by dc motors through a flexible tendon transmission. Hoshi et al. [61] proposed a noncontact tactile rendering method based on airborne ultrasound. It can render tactile feelings (e.g., textures, furs, particles, and so on) to users to feel virtual objects without mechanical touches on any display devices. Comparing with the conventional tactile rendering methods such as attaching tactile devices on user's fingers or palms, the proposed noncontact method has the advantages such as avoiding the interference that the tactile display hides the visual images, allowing users to move their arms, hands, and fingers freely to provide tactile feedback anywhere within 3D free space, and removing undesired touch feelings originated from the contact between the device and the skin. However, several issues need to be solved in the future, for example, intensifying the total force by enlarging the device area so that more transducers can be driven, controlling the pressure distribution and the force direction for a user touching the virtual objects from arbitrary angles, preventing audible sound for ear protection, and limiting ultrasound intensity for the safety of human bodies.

The fourth approach for tactile rendering is electrotactile-based method. The main idea is that electrotactile display can deliver very weak and controlled current pulses to embedded electrodes, which can produce touch sensations by passing a small electric current through the skin. Different from electrovibration-based method where there is no direct contact between the finger and the electrode, the electrotactile-based method is based on the excitation of the cutaneous nerve fibers with electric charge. Therefore, it has the advantages such as containing no moving components, maintaining good contact with the skin, and silent. Altinsoy and Merchel [62] proposed an electrotactile-based tactile rendering method for handheld device with touch screen, and two experiments were conducted to determine the stimulation parameters for the texture reproduction. The results showed that a high current magnitude and a low pulse frequency are more suitable to represent rough surfaces. However, this method requires dual-hand contact with the device, because there is no tactile feedback with only single-hand contact.

8 Conclusion and future direction

From the review, the following conclusions can be summarized.

1. *For haptic rendering of rigid–rigid interaction* Penalty-based method and constraint-based method are the most popular methods for haptic rendering of rigid objects. Penalty-based methods define the contact constraints as springs whose elastic energy increases when the haptic tool penetrates into a virtual object, then the penalty force can be generated as the negative gradient of the elastic energy to push the graphic tool toward a nonpenetrating configuration. The main advantage of penalty-based method is the low computation cost; however, it may lead to interpenetration and pop-through effect. Virtual coupling is used to maintain the stability of penalty-based haptic rendering. However, it can lead to incorrect force feeling by modifying the force orientation applied to the user, and also it may lead to the loss of detailed perception of force change which has been filtered by virtual coupling. Comparing to penalty-based method, constraint-based method is a global analytical approach which uses a linear complementary model for the normal and friction contact constraints solved by differential algebraic equations or mesh-based continuous collision detection. Although this is a more expensive computation process, it can avoid the pop-through problem and guarantee the visual and haptic realism. Impulse-based method is a high-computational efficient method to support a high-refreshing rate of haptic rendering. It can directly process the change of velocity arisen from colliding bodies immediately after transient contact. Therefore, it is more suitable for haptic rendering where high speed is the main factor of force feedback. Compared to constraint-based method, it is much easier to be implemented. Also, the feedback force is more credible because it is based on the physical law rather than the simplified spring force model. Implicit surface-based method is the most efficient method for high-update-rate haptic rendering, because it uses mesh-free collision detection without expensive computation. However, a preprocessing stage is

needed to approximate the model with appropriate implicit surface, which will affect the accuracy of haptic rendering. Also it cannot support dynamic change of virtual models, such as haptics-based cutting and suturing.

2. *For haptic rendering of rigid-deformable interaction* Most of today's deformation simulation algorithms have shortcomings for haptic rendering of rigid-deformable interaction. For example, geometry deformation method lacks an efficient mechanical basis and its accuracy is only decided by geometrical function. The offline precomputation method is unable to model nonlinear behavior or accommodate for topological changes during haptic interaction. The layered rhombus-chain-connected model needs to be investigated the physical meaning of the model parameters and also to be validated the behavior of the model for complex object with thousands of nodes. FEM is the most popular method for deformation simulation and has been used as the benchmark for other methods. However, it is very limited for real-time haptic interaction due to the overwhelming computational cost, unless the number of mesh nodes is greatly reduced. Therefore, the reduced model combined with FEM is a promising approach for haptic rendering of rigid-deformable interaction. The suitable reduced models include classic linear modal vibration models, reduced nonlinear models, low-resolution deformable models with embedded geometry, multiresolution models, and articulated rigid or flexible multibodies or skinned deformable models [25]. Hardware acceleration also provides an efficient approach for deformation simulation and haptic interaction. Combining efficient deformation simulation algorithm with GPU or parallel computation platform can significantly improve the performance of haptic rendering. However, for complex deformable models which contain hundreds of thousands of polygons or nodes, it is still challenging to achieve stable and credible haptic rendering with current technologies. The trade-off between the deformation accuracy and haptic update rate is difficult to be determined. Several physics engines have been developed for haptics-based deformation simulation in recent years, such as Bullet [63], PhysX [64], and SOFA [65]. However, all of these physics engines have their limitations for complex deformable objects. For example, Bullet and PhysX can only support deformation simulation which is modeled by mass-spring model. SOFA supports FEM-based deformation simulation; however, its performance is determined by the number of nodes in the FEM model.

3. *For haptic rendering of rigid-fluid interaction* Fluid has the particularity such as highly deformable shape and dynamics. Comparing with haptic rendering of rigid–deformable interaction, haptic rendering of fluid involves more expensive computation. Real-time fluid simulation and credible force model are the main challenges for haptic rendering. However, only few researchers focus on this topic in recent years. SPH is the main approach for fluid simulation and haptic interaction, because it has the advantages such as the transparent handling of multiple contact points, the parallelizability of computations and their scalability, the seamless use of arbitrary-shaped objects, and the computation of a fully dynamic world. Some researchers try to provide a unified method for haptic rendering of deformable solid and fluid. For example, Kim et al. [66,67] proposed the micro-lump model combined with the bond graph model to merge deformable solid and fluid into one haptic rendering model. However, currently this method is still in an initial stage of concept framework, which needs more experiments to demonstrate its validity. Some haptic rendering methods only use the pressure as the feedback force on a freeform surface of the boundary between the proxy and fluid medium. However, the stability of these methods is heavily depended on the density of the discrete fluid. For instance, if the proxy is immersed into a low-density fluid, an oscillatory vibration force will be generated. Therefore, the force model for haptic rendering of rigid-fluid interaction should consist by two main components: pressure and viscosity forces.

4. *For haptic rendering of image and video based interaction* Direct haptic rendering of 2D static image can calculate the force feedback based on the color temperature, luminance, or image gradients without generating a depth map. Because it does not need expensive collision detection and complex force model, it can be used for photograph perceiving or contour tracing for visually impaired or blind people. However, because of the incomplete perception of object in the 2D image, converting image information directly into haptic force has limited performance and can only get limited applications. Reconstructing 3D geometry from 2D image or augmenting 2D image by invisible 3D object are promising approaches for image driven haptic simulation. Many technologies have been proposed for automatic 3D object reconstructing form 2D image, such as shape from shading, computer vision or stereo projection. However, the expensive computation of 3D reconstruction process and the accuracy of the reconstructed 3D model are main challenges for haptic rendering. Augmenting 2D image by invisible 3D object can be implemented by function-based modeling. However, the registration between the 3D model and 2D image is a great challenge, and only rigid 3D object can be supported by this approach. Haptic rendering of video-based interaction is a new research topic and has potential applications in many areas such as social communication, entertainment, education, and training. For example, during video call through skype, if haptic rendering can be provided, the "two-way remote touch" would be possible, and the principle "What You See Is What You Touch" can be realized. However, the challenges such as timing latency, haptic resolution, and interaction with moving objects are

still unresolved. Until now only few researchers focus on this topic.

5. *For texture and tactile rendering* Comparing with force rendering, the progress of tactile rendering is much slower. Two challenges hamper the development of tactile rendering. First, the mechanism of human haptics with tactile feedback is much more complex than force feedback, which has not been clearly exploited. Second, the available texture rendering device is still absent in the market. Therefore, until now only few tactile rendering methods can get applications in medical and industrial simulation. The synthesis method based on real surface is the early proposed method for tactile rendering. However, because of the difference of real surface and virtual surface, it is difficult to use the limited types of real surface such as sandpaper to represent the characteristics of virtual texture. Force feedback-based method can be widely supported by today's commercial haptic devices. However, the development of an accurate force model to represent virtual texture is also a challenge. Vibration-based method can simulate the mechanism of touching the real surface with human hand. However, because of the availability of commercial actuators, the quality of vibration-based tactile rendering is strongly influenced by the important physical properties of actuators such as bandwidth, frequency response, maximum feedback amplitude, resolution, and latency. Noncontact rendering method and electrotactile-based method are new promising approaches for tactile rendering. However, they are still in the lab-developing stage. Therefore, their stability and fidelity need to be further investigated for practical use.

Although haptic rendering has been received intensive research over the last decade, and significant progresses have been achieved. However, there are still many challenges remained affecting the performance of haptic rendering. The future research should focus on the following directions.

1. *Efficient collision detection for haptic rendering* Collision detection and contact queries are the computational bottleneck of haptic rendering. The haptic force between the virtual objects must be computed from contact information. Therefore, determining whether two virtual objects collide each other is not enough for haptic rendering, the additional information, such as penetration depth, contact points, and contact normals, also need to be calculated. Collision detection becomes more difficult and computationally more expensive as the complexity of the models increases. For the conventional collision detection algorithms, such as the space subdivision-based method and hierarchical bounding volume-based method, their performances are limited by the complexity of virtual models. For example, the existing exact collision detection methods can barely execute contact queries for 6-DoF haptic rendering between pairs of objects with 1000 triangles in complex contact scenarios at force update rates of 1 kHz [68]. Several new collision detection algorithms have also been proposed in recent years, such as distance filed-based method, multiresolution-based method, sphere tree-based method, particle base method, and so on. However, all these algorithms have their limitations for haptic rendering. For example, some algorithms can be only used for specific geometry structure of models such as convex objects, or can only support a specific application of haptic rendering such as rigid objects. Until now no efficient collision detection algorithm can be used for all haptic rendering.

2. *Efficient force modeling technology* The spring-damping force model is widely used for haptic rendering. In spite of its easy implementation and high computation efficiency, it is still a simplified method which cannot reflect the essential characteristics of haptic interaction. Physics-based force model is an accurate method for haptic rendering. However, because of the complication and uncertainty of physical phenomena, it is still difficult to design an analytic force model for haptic rendering, or the force model is too complex for real-time haptic rendering. The determination of proper values for the parameters in the force model is also a great challenge. For example, for haptics-based medical simulation, the biomechanical properties of human soft tissue such as hardness, visco-elasticity, no homogeneity, and anisotropy, are difficult to assign appropriate values. Normally, they are set by human experience. Measurement-based approach has been proposed to determine the proper parameter values for accurate force model. However, because of the unavailability of human organs, normally the experiments are performed based on similar animal organs, which lead to a big difference for haptic rendering.

3. *Wearable haptic device for tactile rendering* Tactile rendering devices are facing serious difficulties for their implementation. The spatio-temporal acuity of the finger skin, its large force range, and the ergonomic constraints in the area of the hand propose challenging performance and design requirements which are difficult to meet. For example, the mechanism design, driving, and control of haptic devices with tactile feedback are much more difficult than force feedback devices. Wearable device is the future direction of haptic device with tactile feedback. For the next generation of wearable haptic device, it should be inexpensive so as to be affordable and accepted by the user. It should be light and portable so as to avoid wearer fatigue. It should be compact and simple, especially around the user's hand to not to occlude virtual objects visually. It should be carefully designed so that the user can put on or take off easily.

4. *Human perception characteristics for haptic rendering* Understanding the perception characteristics of human people has the importance for haptic rendering. Several issues of haptic rendering propose the need of exploiting

human perception characteristics. For example, the design of multi-resolution representation of complex models and the selection of the appropriate contact resolution can take advantage of perceptual observations made by psychophysics researchers. Therefore, if the multi-resolution haptic rendering algorithms can be developed based on the psychophysics of touch, it is possible to achieve high-fidelity haptic rendering of complex models consisting of hundreds of thousands of geometric primitives. For haptics-based deformable object simulation, it is very difficult to achieve high update rate of 1 KHz for haptic rendering and at the same time obtain the real-time physics-based deformation simulation with high accuracy. Understanding the psychophysics of touch and perceptual factors can help the user to design the appropriate trade-off between haptic update rate and deformation accuracy. Also, for tactile and texture rendering, understanding the influence of various parameters on the accuracy and magnitude of sensory outputs can help the user to design advanced rendering algorithm.

5. *Evaluation of haptic rendering* The effectiveness of different haptic rendering methods should be analyzed and evaluated by an efficient quantitative or qualitative method. Although in this paper the haptic rendering is reviewed from the perspectives such as physics simulation technology, force modeling technology, degrees of freedom, stability of haptic rendering, fidelity of haptic rendering, and supporting high update rate for complex models, it is still not a systematic or standard evaluation method. However, no research paper can be found in recent years which are focusing on this topic, and until now there is still no appropriate evaluation model or efficient evaluation indices for haptic rendering.

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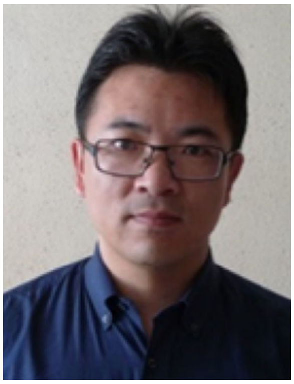

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