HW3

2022-10-17

Contents

Question 1

a.1

```
dis_1 <- rbinom(1000000, 12, 0.2)
mean(dis_1)
```

```
## [1] 2.398445
```

```
var(dis_1)
```

```
## [1] 1.91506
```

```
sd(dis_1)
```

```
## [1] 1.383857
```

The mean is 2.4, varaiance is 1.92, standard deviation is 1.387.

a.2

```
12*0.2
```

```
## [1] 2.4
```

```
12*0.2*(1-0.2)
```

```
## [1] 1.92
```

```
sqrt(12*0.2*(1-0.2))
```

```
## [1] 1.385641
```

The mean is 2.4, varaiance is 1.92, standard deviation is 1.38.

a.3

```
seq_a <- seq(0, 12)
exv_a <- sum(seq_a * dbinom(seq_a, 12, 0.2) )
var_a <- sum(seq_a**2 * dbinom(seq_a, 12, 0.2)) - exv_a**2
std_a <- sqrt(var_a)
exv_a
```

```
## [1] 2.4
```

```
var_a
```

```
## [1] 1.92
```

```
std_a
```

```
## [1] 1.385641
```

The mean is 2.4, varaiance is 1.92, standard deviation is 1.386.

b.1

```
exp_b <- rexp(1000000, 0.03)
exv_b <- mean(exp_b)
var_b <- var(exp_b)
std_b <- sd(exp_b)
exv_b
```

```
## [1] 33.30641
```

```
var_b
```

```
## [1] 1109.795
```

```
std_b
```

```
## [1] 33.31358
```

The mean is 33.3, varaiance is 1109.34, standard deviation is 33.3.

b.2

```r
exv_b2 <- 1/0.03
var_b2 <- 1/(0.03**2)
std_b2 <- sqrt(var_b2)
exv_b2
```

```
## [1] 33.33333
```

```r
var_b2
```

```
## [1] 1111.111
```

```r
std_b2
```

```
## [1] 33.33333
```

The mean is 33.33, varaiance is 1111.11, standard deviation is 33.33.

c.1

```r
poi_c <- rpois(1000000, 8)
exv_c <- mean(poi_c)
var_c <- var(poi_c)
std_c <- sd(poi_c)
exv_c
```

```
## [1] 7.999677
```

```r
var_c
```

```
## [1] 8.009963
```

```r
std_c
```

```
## [1] 2.830188
```

The mean is 7.99, varaiance is 8.00, standard deviation is 2.83.

c.2

```r
exv_c2 <- 0.4 * 20
var_c2 <- 0.4 * 20
std_c2 <- sqrt(var_c2)
exv_c2
```

```
## [1] 8
```

```r
var_c2
```

```
## [1] 8
```

```r
std_c2
```

```
## [1] 2.828427
```

The mean is 8, varaiance is 8, standard deviation is 2.828.

d.1

```r
dis_d <- runif(1000000, 0, 6)
exv_d <- mean(dis_d)
```

```r
var_d <- var(dis_d)
std_d <- sd(dis_d)
exv_d
```

```
## [1] 3.001929
```

```r
var_d
```

```
## [1] 2.999141
```

```r
std_d
```

```
## [1] 1.731803
```

The mean is 2.99, varaiance is 2.99, standard deviation is 1.73.

d.2

```r
exv_d2 <- (0 + 6)/2
var_d2 <- 6**2 /12
std_d2 <- sqrt(var_d2)
exv_d2
```

```
## [1] 3
```

```r
var_d2
```

```
## [1] 3
```

```r
std_d2
```

```
## [1] 1.732051
```

The mean is 3, varaiance is 3, standard deviation is 1.732.

Question 2

a

```r
ex_1 <- 0.3 * 0.8* 9000000
ex_2 <- 0.6 * 0.9 * 0.7 * (2000000 + 8000000)/2 + 0.6 * 0.9 * 0.3 *((2000000 + 8000000)/2 - (1 + 300000
ex_1
```

```
## [1] 2160000
```

```r
ex_2
```

```
## [1] 2457000
```

Expected value for product 1 is 2160000.

Expected value for product 2 is 2457000.

b

```r
set.seed(123)
sim <- sample(c(0,1), 1000000, prob = c(0.7, 0.3), replace = TRUE) * sample(c(0,1), 1000000, prob = c(0
ave <- mean(sim)
prob <- mean(sim >= 500000)
ave
```

product 1

```
## [1] 2164100
```

```
prob
```

```
## [1] 0.22662
```

For product 1, the mean is 2164100, the probability to get money back is 0.227.

```
set.seed(234)
sim_2 <- sample(c(0,1), 1000000, replace = TRUE, prob = c(0.4, 0.6)) * sample(c(0,1), 1000000, replace =
for(i in 1:length(sim_2)){
  if(sim_2[i] == 1)
    sim_2[i] <- sim_2[i] * runif(1, 2000000, 8000000)
}

for(j in 1:length(sim_2)){
  if (sim_2[j] > 0){
    infri <- runif(1, 1, 3000000)
    sim_2[j] <- sim_2[j] - sample(c(0, infri), 1, prob = c(0.7, 0.3))
  }
}

ave_2 <- mean(sim_2)
prob_2 <- mean(sim_2 >= 500000)
ave_2
```

product 2

```
## [1] 2451333
```

```
prob_2
```

```
## [1] 0.529351
```

For product 2, the mean is 2451333, the probability to get money back is 0.529.

c

```
mean(sim > sim_2)
```

```
## [1] 0.193661
```

About 19% simulations in product 1 make more money than product 2.

d

```
mean(sim >= 7000000)
```

```
## [1] 0.110368
```

```
mean(sim_2 >= 7000000)
```

```
## [1] 0.067209
```

Product 1 has 11% change to earn more than 7000000 dollars.

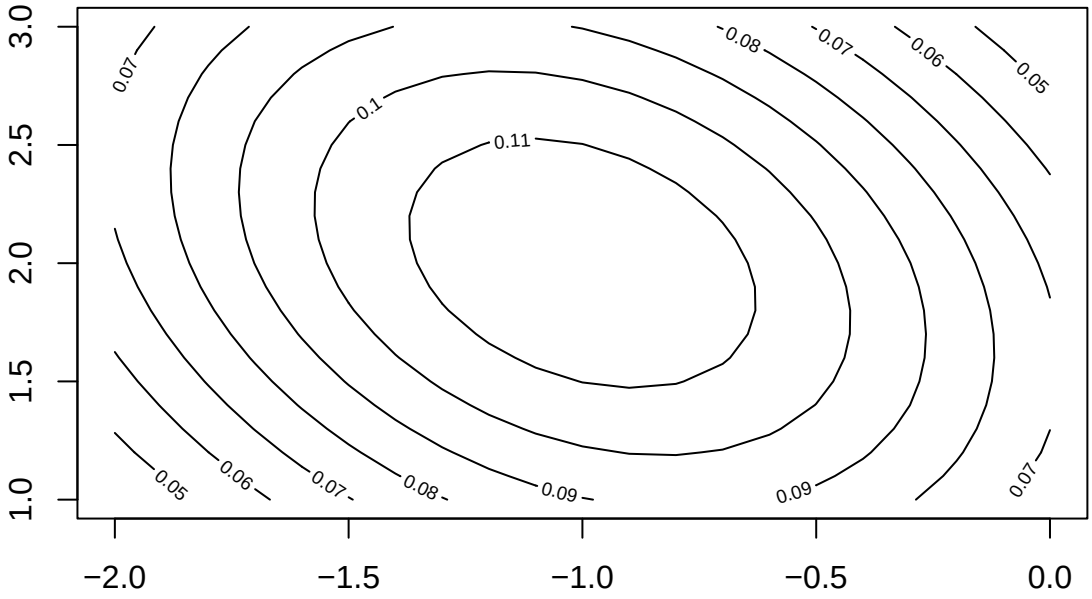
Product 2 has 6.7% change to earn more than 7000000 dollars.

Thus product 1 is more likely to earn more than 7000000 in revenue.

Question 3

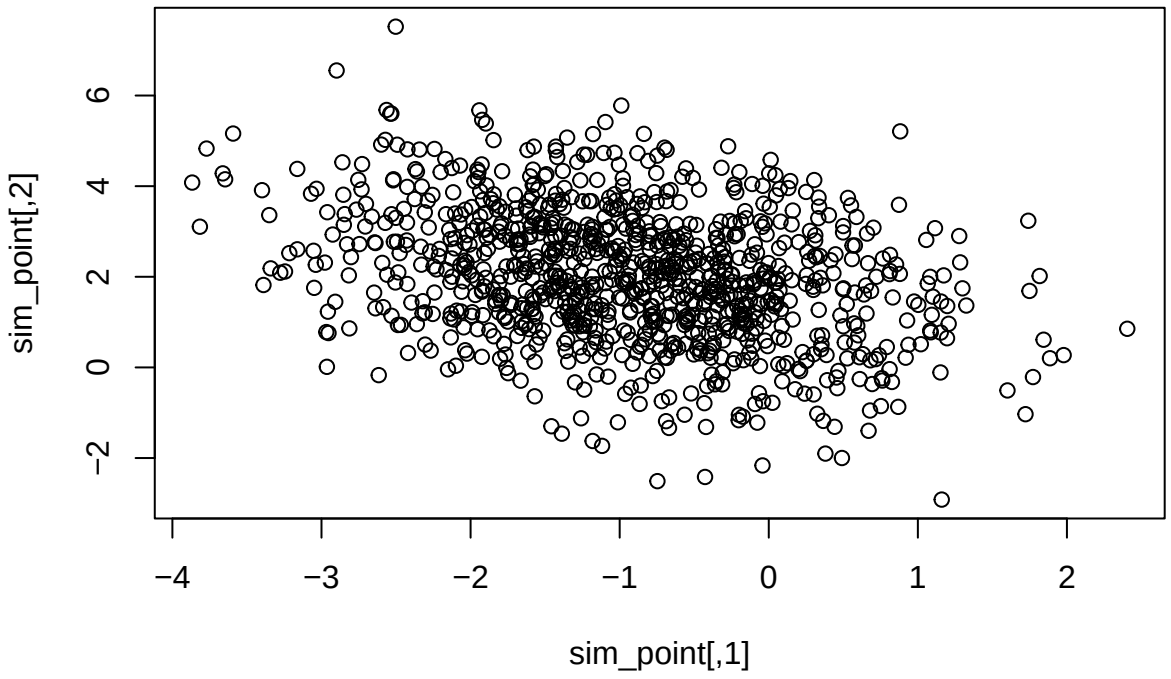
a

```
library(mvtnorm)
cor_xy <- -0.3
var_x <- 1
var_y <- 2
mean_x <- -1
mean_y <- 2
cov_xy <- cor_xy * sqrt(var_y) * sqrt(var_x)
sigma <- cbind(c(var_x, cov_xy), c(cov_xy, var_y))
x <- seq(-2, 0, by = 0.1)
y <- seq(1, 3, by = 0.1)
z <- matrix(nrow = length(x), ncol = length(y))
for (i in 1:length(x)){
  for (j in 1:length(y)){
    z[i, j] <- dmvnorm(c(x[i], y[j]), c(-1, 2), sigma)
  }
}
contour(x, y, z)
```



b

```
library(MASS)
sim_point <- mvrnorm(1000, mu=c(-1, 2), Sigma = sigma)
plot(sim_point)
```



c

```
set.seed(12)
sim_1m <- mvrnorm(1000000, mu=c(-1, 2), Sigma = sigma)

ex_1m <- mean(sim_1m[,1])
ey_1m <- mean(sim_1m[,2])
var_x <- var(sim_1m[,1])
var_y <- var(sim_1m[,2])
ex_xy <- mean(sim_1m[,1] + sim_1m[,2])
var_xy <- var(sim_1m[,1] + sim_1m[,2])
e_xpy <- ex_xy - ex_1m - ey_1m
var_xpy <- var_xy - var_x - var_y - 2*cov_xy

ex_5 <- c()
for(i in 1:1000000){
  if(sim_1m[i,2] > 2.5 && sim_1m[i,2] < 3.5){
    ex_5[i] = sim_1m[i,1]
  }
}
```

```
ex_1m
```

c1

```
## [1] -0.99962
```

E(x) is -0.999.

```
ey_1m
```

c2

```
## [1] 1.999719
```

E(y) is 1.999.

```
var_x
```

c3

```
## [1] 1.000693
```

```
var_y
```

```
## [1] 2.003877
```

Var(x) is 1, Var(y) is 2.

```
e_xpy
```

c4

```
## [1] 2.220446e-16
```

```
var_xpy
```

```
## [1] -0.00123035
```

E(X+Y) is 2.220446e-16, Var(X+Y) is -0.00123035.

```
mean(ex_5,na.rm = T)
```

c5

```
## [1] -1.205132
```

E(X|Y = 3) is -1.205132.

Question 4

a

```
set.seed(114)
ave_4 <- c()
for( i in 1:2000){
  sim_4 <- rexp(100, 1/3)
  ave_4[i] <- mean(sim_4)
}
ave_4
```

```
##    [1] 3.093626 2.586359 2.884363 4.087608 3.000990 2.470159 2.973753 3.418532
##    [9] 2.761419 3.334130 3.047855 2.646745 2.802615 3.221310 2.979259 2.947192
##   [17] 2.978004 3.395453 2.948985 3.169757 2.760048 3.413342 2.999129 2.692455
##   [25] 3.049262 2.475747 3.155847 2.794990 2.778041 3.154182 2.763178 2.931079
##   [33] 3.670131 3.118613 3.364622 2.654921 3.130400 2.519387 3.122455 2.644901
##   [41] 2.957149 2.884617 3.287428 2.905765 3.127545 2.440917 3.122568 2.941158
##   [49] 3.537767 2.830374 3.107352 3.297627 2.805648 2.693841 2.692076 2.653746
##   [57] 2.653670 2.798801 3.043826 2.754434 2.771565 3.152841 2.724917 2.890484
```

```
##   [65] 3.360175 3.172211 3.816509 2.726427 3.642599 3.351090 2.947819 3.184969
##   [73] 2.910365 3.001693 3.152571 2.927271 3.221052 2.631326 2.947208 2.508259
##   [81] 3.321639 2.925810 3.295370 2.580373 3.219436 3.128549 2.767641 3.390953
##   [89] 3.109129 3.055033 2.458195 3.041064 3.613613 2.871225 2.773599 2.645094
##   [97] 2.501304 3.318565 2.574131 3.025980 3.206634 3.047356 3.037310 3.339971
##  [105] 2.656728 3.332077 3.055547 3.132032 2.695883 2.980965 2.609486 2.706855
##  [113] 3.409296 3.291280 2.736015 3.105590 2.523939 2.867000 3.139520 2.836665
##  [121] 3.056760 2.620952 2.687404 3.134513 2.993453 3.062541 3.033435 2.841124
##  [129] 3.294219 3.250208 2.807577 2.779637 2.923878 3.623482 2.980295 3.421883
##  [137] 2.505507 3.578987 3.268564 2.770271 3.182076 2.380155 3.401728 3.254219
##  [145] 3.148525 2.673796 3.306727 2.660853 2.735735 2.866975 2.750732 3.058424
##  [153] 3.019258 3.177205 2.675036 2.882969 3.158505 2.596660 3.005568 2.833441
##  [161] 2.894040 3.211673 3.020862 2.762276 2.789927 3.365277 2.803019 3.052797
##  [169] 3.178407 2.883474 2.835425 2.983454 2.807785 2.981743 2.526292 2.309193
##  [177] 2.994717 3.180346 3.297204 3.113356 3.151251 3.252172 2.925023 2.666945
##  [185] 2.667955 3.129185 2.800912 3.188947 2.988190 2.637530 2.978318 2.896417
##  [193] 2.977746 2.788570 3.012147 2.819926 2.745704 3.050810 3.257433 3.151613
##  [201] 2.906481 2.941963 2.961100 3.205613 3.386622 3.029408 2.540361 3.081201
##  [209] 2.950269 3.310553 3.035122 2.839876 2.687552 2.930164 2.967454 2.843488
##  [217] 2.896192 3.566483 2.919544 2.754850 2.722419 2.823640 2.843009 2.435125
##  [225] 2.800060 3.071082 3.111172 3.364498 3.344496 2.620480 2.809189 2.811831
##  [233] 3.259435 2.959631 3.618820 2.942236 2.737233 3.244311 2.858952 2.836227
##  [241] 2.710571 2.863885 2.634540 3.109493 3.141719 2.630747 2.919395 3.335031
##  [249] 3.221072 2.850425 3.450452 2.917221 3.669742 2.697606 2.995561 3.289731
##  [257] 3.069337 2.584866 2.978947 3.039614 3.543119 4.075672 3.800667 3.281714
##  [265] 2.896139 3.301841 3.363760 3.151526 3.144966 2.611689 2.529772 2.704811
##  [273] 2.880028 2.839385 3.236325 3.097923 3.054084 2.825082 3.185048 2.566205
##  [281] 2.739876 3.067594 2.833174 3.079014 2.875457 3.514463 2.802802 2.808551
##  [289] 2.414378 2.967431 2.816552 3.072380 3.090041 2.850680 2.748524 2.963721
##  [297] 3.198664 3.140459 3.240217 3.106565 2.821448 3.050227 3.227286 3.152006
##  [305] 2.811819 3.098409 2.988927 3.065219 3.211382 2.916407 3.152471 3.446018
##  [313] 3.666768 2.801709 2.908290 2.892779 2.651434 3.212559 2.931391 2.719190
##  [321] 3.220690 3.010341 3.098355 3.278133 2.908224 2.605215 3.132996 2.637674
##  [329] 3.445718 2.868117 2.911343 3.078714 2.926461 3.727683 2.942328 2.955440
##  [337] 3.125126 3.253386 2.753280 2.645414 3.099688 3.259707 2.934261 2.497883
##  [345] 2.797399 2.590717 3.368107 2.662301 3.352730 2.627482 2.713987 2.976799
##  [353] 3.241188 3.210824 3.466885 3.108000 3.036876 3.167320 3.309056 3.039609
##  [361] 2.636187 3.042671 3.355395 3.070340 3.240400 3.763594 3.018104 2.758851
##  [369] 2.961995 3.252570 3.462803 2.827610 3.156190 2.954314 3.014373 2.714254
##  [377] 2.855634 2.920350 3.100849 2.747932 3.144980 3.011073 2.871427 3.373309
##  [385] 3.447425 2.766484 3.367623 2.596430 2.680496 3.205965 2.964045 3.013099
##  [393] 2.999324 3.259777 2.926665 2.662144 2.949957 2.710269 3.172904 3.445811
##  [401] 3.114521 2.573556 2.719724 2.770979 3.343417 2.664095 3.391741 2.577778
##  [409] 2.934725 2.983497 2.682766 3.551847 2.862882 2.548737 2.691283 3.028391
##  [417] 3.059606 2.835101 2.952034 2.644493 3.076388 2.820022 3.158205 3.106801
##  [425] 3.396573 3.293231 2.939128 3.071766 2.648155 3.250088 3.394843 3.031925
##  [433] 3.066679 2.353939 3.051561 3.060740 3.192073 2.962739 2.782051 3.187721
##  [441] 2.597329 3.472122 3.251001 3.038847 2.958864 2.669885 2.844230 3.332068
##  [449] 2.951323 2.977819 3.391797 3.175769 2.844494 2.549092 2.628374 2.954142
##  [457] 3.390903 3.566284 3.151516 3.358636 2.589077 3.486579 2.838971 3.187341
##  [465] 3.034773 3.070875 2.757692 2.811869 2.677154 2.967723 3.420890 3.114306
##  [473] 2.843441 3.341895 2.843837 2.716942 3.532425 2.503821 3.384904 3.038978
##  [481] 2.986602 3.038596 3.290383 2.794774 2.722083 2.973568 3.563257 2.591123
##  [489] 2.712588 3.046826 2.579400 3.180642 3.021742 2.817192 3.449094 2.935842
```

```
##   [497] 2.859578 2.738485 3.025923 2.432993 3.074309 2.511338 2.822425 2.834305
##   [505] 2.705821 2.914273 3.465025 3.382958 2.917557 2.661661 2.910698 2.765475
##   [513] 2.693236 2.855419 3.315423 3.000056 3.233959 3.779521 3.398984 2.909983
##   [521] 2.950194 2.814512 3.244470 3.341852 3.251074 3.305637 2.802467 3.203083
##   [529] 2.722302 3.119115 3.344510 2.653070 3.699055 3.283756 3.402960 3.417682
##   [537] 3.415970 3.190547 2.350918 2.702164 2.518829 3.186106 3.153959 3.083600
##   [545] 2.863225 2.922863 2.796046 3.235897 3.101579 2.962332 3.062802 2.996972
##   [553] 3.017910 3.260246 3.130968 2.895220 2.921418 3.214124 2.760952 3.238452
##   [561] 3.193170 2.478774 2.771043 2.813082 2.511076 2.732625 3.284458 2.820210
##   [569] 2.785353 3.014677 2.997442 3.133269 3.346759 3.669710 3.242116 2.688765
##   [577] 3.290929 3.065309 2.741334 3.023930 3.280370 2.863578 2.953170 2.911108
##   [585] 2.892845 3.201048 2.893560 2.412940 3.127283 3.162366 2.926896 3.167988
##   [593] 3.330025 2.939753 3.177403 2.832254 3.378794 2.740579 2.879879 2.904063
##   [601] 3.130451 3.623750 2.713941 3.385625 3.122389 2.776102 2.502414 2.700568
##   [609] 3.490491 2.914758 3.098735 3.301688 3.225168 3.035262 3.404610 3.085624
##   [617] 2.658727 3.647966 2.691533 2.846129 2.535402 3.259072 3.229227 2.561407
##   [625] 2.749339 2.991554 2.910002 2.924368 3.295161 3.072297 3.597917 2.682341
##   [633] 2.844647 2.760827 3.161957 3.259965 2.493589 3.474357 3.030465 2.774187
##   [641] 2.406940 3.316268 3.238519 2.661759 2.748150 3.088067 3.373006 3.533670
##   [649] 2.831558 2.967216 3.002424 2.696401 2.674027 2.646542 3.036609 2.506244
##   [657] 2.978667 3.127248 2.764614 3.174333 3.110815 3.413547 3.425545 2.944186
##   [665] 2.926525 2.759647 2.958294 3.237256 2.760674 2.929479 2.947372 2.467460
##   [673] 2.511828 3.057422 2.514805 3.104469 2.940213 3.395860 2.851003 2.892565
##   [681] 3.127362 3.285082 3.162246 2.631689 2.448342 3.330420 2.913034 3.173601
##   [689] 2.932589 3.140153 3.140864 2.757111 3.041972 2.394384 3.102357 3.503867
##   [697] 3.096104 3.215064 3.175041 2.927632 2.986838 2.889216 2.914763 2.909611
##   [705] 2.841350 3.370050 3.321087 2.974625 2.953959 2.706336 2.922565 2.597292
##   [713] 3.040051 3.159354 2.767106 3.253536 3.161559 2.947347 3.709457 3.131929
##   [721] 3.161446 2.909422 2.733234 3.136116 3.040169 2.927104 3.400987 3.248967
##   [729] 2.914688 2.999051 3.400730 2.834064 3.013561 2.897093 3.331234 2.944927
##   [737] 3.327021 2.478586 2.883000 3.291465 2.821372 3.148536 2.700899 2.960283
##   [745] 3.084041 2.704040 2.762923 2.987880 3.274867 3.064355 3.606036 3.155269
##   [753] 3.219814 2.502596 3.246382 3.176501 3.434672 3.256838 3.116734 3.270388
##   [761] 2.716277 3.022278 2.963173 2.795658 2.970498 3.254602 3.364908 3.368932
##   [769] 3.372198 3.021515 2.481554 2.970156 3.164856 2.615206 2.697317 3.068136
##   [777] 3.059169 3.530193 3.089128 2.610834 2.913007 3.222896 3.113625 2.794850
##   [785] 2.632652 2.856160 2.670598 2.727312 2.518344 2.955899 3.300240 3.081785
##   [793] 3.073166 3.552271 2.341600 3.244676 3.006214 2.806509 3.004895 3.262451
##   [801] 2.886066 3.218673 2.928970 2.646462 2.956673 3.195892 3.235874 2.977132
##   [809] 3.969141 3.205508 3.058203 3.385300 2.835386 3.162640 3.465251 2.457016
##   [817] 2.746670 3.195144 3.572208 3.021669 2.667769 2.931066 3.039985 3.033077
##   [825] 3.254636 3.199665 2.798588 2.925691 3.260654 2.449551 2.389411 3.069218
##   [833] 2.643857 3.650443 2.928620 2.814720 2.650962 3.533163 2.519963 3.290536
##   [841] 3.371781 3.045690 2.728448 3.297100 3.042008 3.152767 2.182110 2.895746
##   [849] 3.501960 2.556044 3.389886 3.211194 2.647319 2.985695 2.780332 3.404347
##   [857] 2.555225 2.945157 3.550115 3.552691 2.904015 2.840192 3.103592 3.673683
##   [865] 2.983744 2.753595 3.115834 3.041133 3.028200 2.954587 3.181590 3.298688
##   [873] 2.883313 2.752229 3.427575 2.545483 2.990839 2.934391 3.013132 3.006288
##   [881] 3.193845 2.357412 2.827789 2.848737 2.683265 3.329484 2.633353 3.294133
##   [889] 3.109293 3.191054 3.158520 3.135031 2.489237 2.573614 3.071797 2.694612
##   [897] 3.223376 3.281239 2.624041 3.223815 3.304595 2.730496 2.900802 3.592894
##   [905] 3.023613 3.487513 3.085383 3.064676 2.730754 2.710559 3.067266 3.533792
##   [913] 3.729037 2.566322 3.256358 3.321208 2.802945 2.989463 2.583246 2.940296
##   [921] 3.609320 2.892455 3.078475 3.456843 3.334473 2.535777 3.148154 3.107584
```

```
##  [929] 3.497064 2.825275 2.693741 3.142112 2.645769 3.327896 3.419246 2.719880
##  [937] 2.941113 2.586202 3.001099 2.725823 2.954853 3.084188 3.354692 3.140997
##  [945] 3.083583 2.635787 3.111431 3.161416 2.945366 3.294343 2.918275 3.180142
##  [953] 2.488760 2.558490 2.743379 2.983122 3.270245 3.630399 2.689380 3.421792
##  [961] 3.268839 2.879749 2.964412 2.511057 3.520754 2.598828 2.762631 3.108754
##  [969] 2.810244 3.139329 2.707839 2.653454 2.930041 3.459760 2.674882 3.087205
##  [977] 2.787139 2.958564 3.302695 3.063606 2.548887 2.857184 3.176436 3.223833
##  [985] 2.768719 3.140592 3.135785 3.676984 2.815577 3.103142 2.526518 2.914026
##  [993] 3.100892 3.311916 2.976528 3.068211 2.808394 3.427713 2.956238 3.555983
## [1001] 3.089482 2.441351 3.525406 2.704080 2.793740 2.756718 3.358825 3.090434
## [1009] 2.822778 2.868482 3.106698 3.153731 2.072678 2.889645 3.072300 3.550772
## [1017] 2.829019 3.551586 2.986960 3.268882 3.266853 2.722351 2.817580 3.042733
## [1025] 2.980918 2.681903 2.843000 3.621909 3.196627 2.922987 2.940328 3.020152
## [1033] 3.422287 3.181791 2.980620 3.535263 2.510054 3.460340 3.206929 2.793191
## [1041] 2.982665 3.085976 3.116933 3.210722 3.175434 2.645180 2.749566 3.049881
## [1049] 3.332344 2.671623 2.817269 2.741354 3.257963 2.522080 2.645697 3.323479
## [1057] 3.141202 3.052238 3.356892 2.938051 3.282909 3.040515 2.663980 3.698087
## [1065] 3.011809 2.780599 2.642443 3.138355 2.328548 3.117774 3.165768 2.499931
## [1073] 3.064381 2.704222 2.892894 3.152979 2.891133 2.909055 3.374805 4.044929
## [1081] 2.896325 3.104055 3.455143 3.171943 3.127068 2.855945 2.654664 2.858519
## [1089] 2.958178 3.035960 2.784742 2.657341 2.866409 2.670676 2.681778 3.205893
## [1097] 2.983343 3.137098 2.549007 3.083095 2.594350 3.044188 3.400171 3.287558
## [1105] 3.027528 3.400100 2.949467 3.185745 3.195502 2.493981 3.022360 3.257244
## [1113] 3.254231 2.862096 3.008189 3.023245 2.896948 3.177379 3.272097 3.214289
## [1121] 2.653934 3.024764 2.866562 3.368255 3.339271 2.627982 2.843956 3.313297
## [1129] 3.007871 2.830393 3.024095 3.161922 2.917053 2.756966 2.790385 3.078952
## [1137] 3.422430 3.478919 2.416086 2.686286 3.775631 3.118723 2.345220 2.785981
## [1145] 2.759307 2.598684 2.926058 3.302393 3.329676 3.029992 2.918885 2.871991
## [1153] 2.879318 2.610899 2.844947 2.723101 3.236908 3.006039 3.372023 3.185387
## [1161] 2.996453 2.911422 2.847212 2.956400 2.874859 3.315412 3.544890 3.008387
## [1169] 3.209758 2.773615 3.299254 3.242875 3.196915 2.908692 3.107473 2.784905
## [1177] 3.204867 3.081788 2.934031 2.903065 3.426466 2.590263 3.336416 2.973083
## [1185] 2.803207 3.003607 3.177019 3.276266 3.287485 3.226564 2.953017 2.384061
## [1193] 3.386095 3.268455 2.578372 2.605189 2.864755 2.976017 3.588221 3.065676
## [1201] 3.016958 3.318288 2.769178 2.954251 2.632298 2.748477 3.079164 3.537600
## [1209] 2.590298 3.104499 2.651648 2.532945 2.692872 2.863720 2.557120 2.823054
## [1217] 3.066373 2.761570 3.073010 3.199338 3.265878 2.674954 2.991245 3.027810
## [1225] 3.404846 2.483970 2.915388 3.107081 3.323750 3.076150 2.990881 3.292798
## [1233] 3.092273 2.874685 3.410728 3.422028 2.758534 2.637739 2.888727 2.744360
## [1241] 3.783711 3.083906 3.198631 2.614654 3.060729 3.287718 3.069624 3.596855
## [1249] 2.552666 3.022756 3.362396 3.215261 3.011257 3.066901 2.914730 3.685211
## [1257] 3.009634 2.790634 3.487029 2.922821 3.018767 3.227418 3.183492 2.947060
## [1265] 2.621834 2.891077 2.940553 2.442497 2.672398 3.201645 3.397086 2.426746
## [1273] 3.050375 3.214346 2.281196 2.634872 3.528368 3.164224 3.367281 2.963000
## [1281] 2.782435 3.340052 2.542116 2.580296 3.190596 3.283808 2.943201 3.260840
## [1289] 2.927163 2.996992 2.443614 3.457547 2.731266 2.910898 3.090071 2.969387
## [1297] 3.035343 3.180575 2.775428 2.937922 3.228510 2.935404 3.076205 3.007881
## [1305] 2.477767 3.698956 2.835722 3.300115 3.148525 3.063152 2.865365 2.813674
## [1313] 2.861842 2.868983 2.631800 2.924172 3.181415 3.187090 3.553391 3.582601
## [1321] 3.048507 2.799455 2.867693 3.553184 3.467375 2.562572 2.916298 3.264081
## [1329] 3.222078 2.631093 2.792496 3.038688 2.775528 2.778890 2.969034 3.335216
## [1337] 3.160620 2.946054 3.242157 2.875321 3.173393 2.964874 2.536413 2.585431
## [1345] 3.291840 3.573871 3.530999 3.710526 3.364626 3.467346 3.225064 3.160250
## [1353] 3.492364 2.931556 3.531904 2.736308 2.363863 2.939398 2.961357 2.806763
```

```
## [1361] 2.696499 3.047986 3.199830 3.121637 2.726889 2.954057 3.304805 3.265812
## [1369] 3.221663 2.694492 3.220091 3.117431 2.783345 3.023932 2.827989 2.717370
## [1377] 2.614444 2.377177 3.506683 2.675463 2.953987 3.187059 3.000230 3.136916
## [1385] 2.975414 3.285646 2.748727 3.375705 3.174088 3.095469 2.646547 3.248003
## [1393] 2.781857 2.830004 3.322691 3.013797 2.376910 2.790199 2.858920 2.832659
## [1401] 2.788601 2.454382 2.666304 2.805256 2.800986 3.405841 3.181286 3.466362
## [1409] 3.367173 3.175268 3.186558 2.806869 3.159622 3.273713 3.363832 2.905272
## [1417] 2.853564 2.332824 3.262683 2.419861 3.250124 2.528049 2.649702 3.080835
## [1425] 2.183882 3.292910 3.036703 2.785930 3.055605 3.270526 2.885932 3.259044
## [1433] 2.382449 2.815667 2.714745 3.140877 2.790928 3.429010 2.996210 2.951643
## [1441] 2.640718 2.897043 2.607176 3.104131 2.778762 2.439686 2.661370 2.623101
## [1449] 2.780096 2.770507 2.782778 2.985080 3.410919 3.125787 2.718480 2.923794
## [1457] 3.454452 2.736980 3.284399 2.756138 2.911958 2.653742 2.734988 2.836379
## [1465] 2.924702 2.985390 3.221494 3.855788 3.582629 2.699671 3.179590 2.980027
## [1473] 3.225339 2.943099 2.422884 2.807185 2.537793 2.805116 2.667193 2.819429
## [1481] 3.169753 3.344268 2.789745 3.022822 2.929308 2.833719 3.376606 3.139652
## [1489] 2.972331 3.335913 2.849381 3.638671 3.275604 3.068000 2.823236 3.131424
## [1497] 2.878551 3.020341 2.976752 3.279973 2.523576 3.143649 3.013996 3.431691
## [1505] 2.748995 3.089066 3.334228 3.398662 3.580084 3.119923 2.866568 3.350019
## [1513] 3.173087 2.900443 3.102286 3.198549 2.859197 2.684859 2.897822 2.763763
## [1521] 2.729814 3.383242 3.100442 2.980907 2.529709 2.851046 3.202113 3.026235
## [1529] 3.193945 2.624956 3.142017 2.851767 2.530164 3.023715 2.609981 3.241365
## [1537] 3.540620 3.247694 3.646124 2.989640 2.622686 3.070992 2.813638 3.238375
## [1545] 2.613727 2.400325 2.640773 2.553422 3.002370 2.921942 3.009949 2.855490
## [1553] 4.082575 3.064978 2.916157 2.412298 3.182717 3.188165 2.169622 3.096352
## [1561] 2.891905 3.090286 3.445374 3.534851 3.045112 2.872197 2.884842 3.022418
## [1569] 3.054472 2.743138 3.185484 3.336624 2.751212 3.367661 3.168653 3.201298
## [1577] 3.528657 3.150690 2.625198 3.060623 3.044912 3.122199 2.986277 3.574817
## [1585] 3.457574 2.773064 2.723337 2.607880 3.417636 3.405315 2.583733 3.010774
## [1593] 3.177669 3.179952 2.538076 2.784969 2.628921 2.385769 2.757979 2.649290
## [1601] 2.734322 3.043179 3.138082 3.232413 2.779460 2.705189 2.810339 3.075749
## [1609] 3.371222 3.107842 3.013091 3.125184 2.578723 3.034718 2.648031 3.220720
## [1617] 3.015818 3.371319 2.943667 2.436298 2.591098 3.314713 2.899528 3.623459
## [1625] 3.255503 2.962287 3.148377 3.010396 2.875043 2.654208 3.266995 3.058789
## [1633] 2.996529 3.301524 3.172491 3.249479 3.249486 3.361268 3.412779 2.916372
## [1641] 3.438070 2.882406 3.273329 3.266854 2.806755 2.845610 2.865090 2.727473
## [1649] 2.527742 2.661674 2.601034 3.521853 2.657068 2.697712 3.451547 2.981476
## [1657] 3.263000 2.475482 2.788332 2.809314 2.944819 3.355284 2.654813 3.048339
## [1665] 3.231306 2.921005 2.922969 3.498251 2.690434 3.439124 2.982369 3.325679
## [1673] 3.191240 2.864418 3.058191 3.064225 3.340931 2.857836 3.084367 2.963510
## [1681] 2.376124 2.680188 3.060663 2.903653 3.060191 3.177176 2.984147 2.656781
## [1689] 2.921813 2.703503 3.631199 2.483766 2.801718 2.632091 3.147769 3.105341
## [1697] 3.706614 3.058531 3.802291 2.520127 2.911844 2.691013 2.988056 3.060277
## [1705] 3.446972 2.839929 3.228125 2.942418 2.855273 3.031013 3.118961 3.190109
## [1713] 2.669610 2.539875 3.182818 2.991270 2.889833 2.764086 3.150402 2.837276
## [1721] 3.286728 2.618254 2.126773 3.459429 3.001213 2.612387 2.752064 2.799640
## [1729] 3.314565 3.619715 2.292891 3.706399 3.222847 3.396531 2.938558 2.728429
## [1737] 3.632298 3.205576 3.159848 2.805452 2.948504 2.779298 2.968812 2.855549
## [1745] 2.739321 3.097379 2.699615 2.978778 3.228592 3.367795 2.836172 2.950124
## [1753] 2.670182 2.981838 3.040976 3.418154 3.755251 2.540390 3.483980 3.342372
## [1761] 3.216653 2.952486 2.576899 2.976634 3.181917 3.034523 2.831262 3.248999
## [1769] 3.149769 3.289251 3.342876 3.397760 3.248310 2.944140 3.136659 2.753281
## [1777] 3.211440 3.390333 3.010593 3.441073 2.273591 2.610411 2.938987 3.151017
## [1785] 3.031571 3.258672 3.057169 3.173754 3.810477 3.287704 2.966662 2.680038
```

```
## [1793] 2.800898 3.241142 3.304812 3.188094 3.409905 3.214882 2.910847 3.229666
## [1801] 3.313191 2.774314 2.692792 3.141982 3.270868 3.060254 2.722887 2.931640
## [1809] 2.918607 3.216806 2.710552 2.934784 3.242795 2.754084 2.759757 2.869798
## [1817] 3.240315 2.703967 2.586482 3.113879 3.046209 3.487245 2.540912 3.487460
## [1825] 3.653968 3.119570 2.611854 3.023411 2.768923 3.360732 2.989780 3.014093
## [1833] 3.065648 2.978160 3.320861 2.612846 2.375672 2.610347 2.429628 2.770585
## [1841] 2.797769 2.784822 2.580528 2.760007 3.335530 2.782476 2.964629 2.776760
## [1849] 3.062560 3.051235 2.930192 3.219237 2.671938 2.533911 3.153475 2.694873
## [1857] 2.857115 2.979717 2.562559 3.041594 3.145247 3.404202 2.923615 2.467990
## [1865] 3.407977 2.849991 3.186539 2.813156 3.161952 2.653058 3.080221 3.367412
## [1873] 3.507579 2.615914 3.086773 2.818747 3.241817 2.923062 3.720797 2.679878
## [1881] 2.865124 3.234914 2.979522 2.546858 3.289193 2.993363 3.453397 3.499160
## [1889] 3.268282 2.872631 2.976820 3.362112 2.747760 2.885159 3.003310 2.931755
## [1897] 2.729001 3.280518 3.176134 2.926432 2.704751 2.914732 3.313143 3.058508
## [1905] 2.796456 2.931797 3.204972 2.904681 2.961896 2.787037 2.996644 2.650727
## [1913] 2.909337 2.615529 2.818391 2.854872 3.155222 3.371023 3.302537 2.860712
## [1921] 3.441446 3.124401 3.289250 2.682473 3.212596 3.230966 3.408239 3.026580
## [1929] 2.382192 2.500161 2.947508 2.637981 3.342336 3.123541 2.604419 2.973420
## [1937] 2.929313 2.959497 3.022735 3.043814 3.281992 2.817794 2.161843 2.680799
## [1945] 3.090690 2.913607 2.999476 2.907590 2.624101 2.915688 3.220097 3.145646
## [1953] 2.873402 3.043844 2.611528 2.992218 3.373314 3.431875 2.397730 2.907421
## [1961] 2.732508 3.302185 3.188328 3.053807 2.551144 3.239128 2.423047 3.198095
## [1969] 3.054253 3.357560 3.101060 3.059800 2.535369 3.099247 2.982813 2.750625
## [1977] 2.713131 2.233228 3.039319 3.133972 3.248789 3.573901 3.228806 2.536136
## [1985] 3.184412 2.889690 3.187453 3.443181 2.660498 2.808989 2.963313 3.021908
## [1993] 2.368119 2.539801 3.498098 2.852362 2.765808 2.498683 3.629073 3.122352
```

The mean value of each simulation is showed above.

b

```
mean(ave_4)
```

```
## [1] 3.004053
```

```
var(ave_4)
```

```
## [1] 0.0890649
```

The mean value is 3.00, variance is 0.89.

c

```
set.seed(514)
ave_1k <- c()
for( i in 1:2000){
  sim_1k <- rexp(1000, 1/3)
  ave_1k[i] <- mean(sim_1k)
}

ave_10k <- c()
for( i in 1:2000){
  sim_10k <- rexp(10000, 1/3)
  ave_10k[i] <- mean(sim_10k)
}
```

```
ave_100k <- c()
for( i in 1:2000){
  sim_100k <- rexp(100000, 1/3)
  ave_100k[i] <- mean(sim_100k)
}

ave_1m <- c()
for( i in 1:2000){
  sim_1m <- rexp(1000000, 1/3)
  ave_1m[i] <- mean(sim_1m)
}

mean(ave_1k)
```

```
## [1] 3.001768
```

```
var(ave_1k)
```

```
## [1] 0.009134064
```

```
mean(ave_10k)
```

```
## [1] 3.000299
```

```
var(ave_10k)
```

```
## [1] 0.000888794
```

```
mean(ave_100k)
```

```
## [1] 3.000131
```

```
var(ave_100k)
```

```
## [1] 8.838508e-05
```

```
mean(ave_1m)
```

```
## [1] 3.000082
```

```
var(ave_1m)
```

```
## [1] 9.006417e-06
```

Thus: For n = 100, mean is 3, variance is 0.89.

For n = 1000, mean is 3, variance is 0.009.

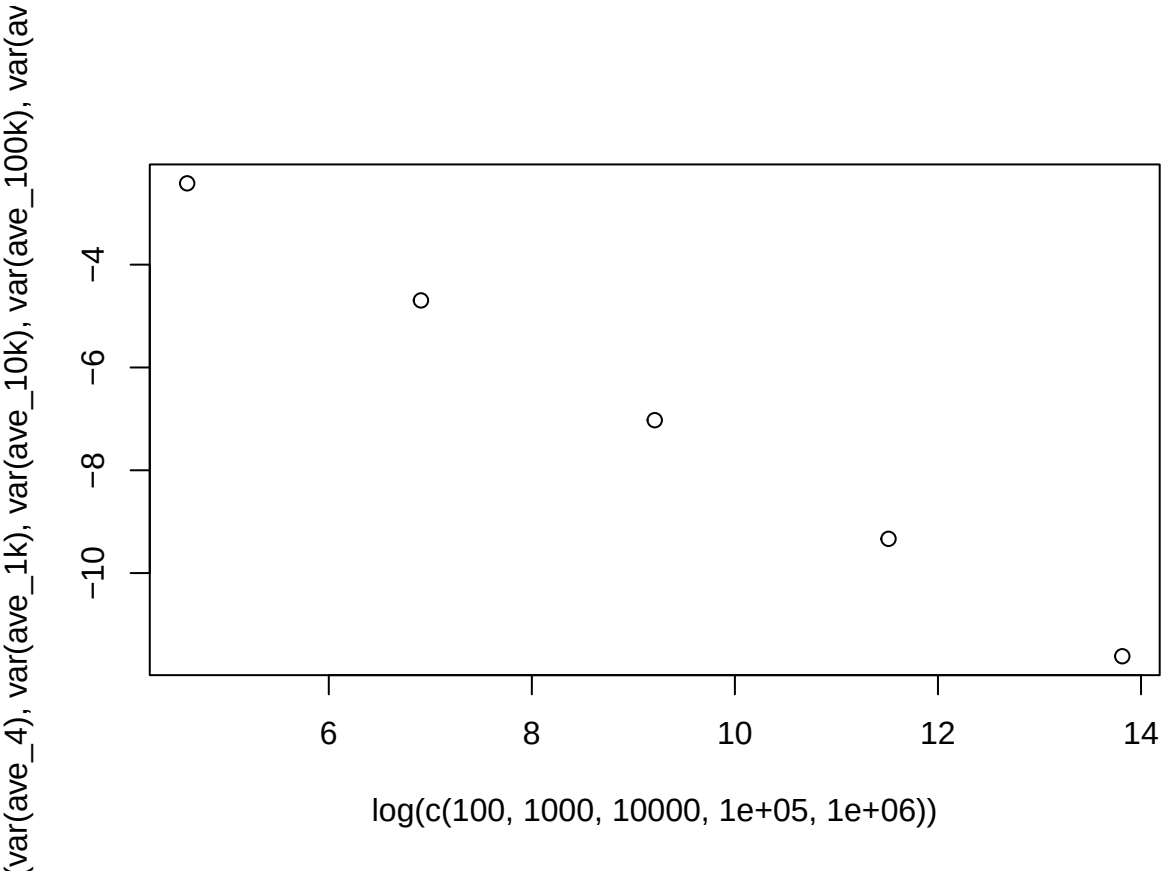
For n = 10000, mean is 3, variance is 0.00089.

For n = 100000, mean is 3, variance is 0.00009.

For n = 1000000, mean is 3, variance is 0.000009.

e

```
plot(x = log(c(100,1000,10000,100000,1000000)), y = log(c(var(ave_4),var(ave_1k),var(ave_10k),var(ave_1
```




```
slope = (log(var(ave_4))-log(var(ave_1m)))/(log(100) - log(1000000))
slope
```

```
## [1] -0.9987886
```

We can see that, the shape is close to a straight line with slope -0.99., and it is expected with central limit theorem.

Question 5

a

$$p(x) = \frac{1}{x^2}$$

$$cdf(x) = \int_1^x \frac{1}{x^2} dx$$

$$cdf(x) = (-1)x^{-1} - (-1)1^{-1} = 1 - \frac{1}{x}$$

Verify:

$$cdf(1) = 1 - 1 = 0$$

$$cdf(x_{lim \to \infty}) = 1 - 0 = 1$$

$$cdf(x_{lim \to \infty}) - cdf(1) = 1 - 0 = 1$$

Thus we verified the minimum is 0 and the maximum is 1.

b

$$y = 1 - \frac{1}{x}$$

The quantile function is:

$$q(y) = \frac{1}{1-y}$$

c

```
set.seed(1919)
ave_100 <- c()
for( i in 1:2000){
    seq_5<- runif(100)
    sim_100 <- 1/(1 - seq_5)
  ave_100[i] <- mean(sim_100)
}

ave_1k <- c()
for( i in 1:2000){
    seq_5<- runif(1000)
    sim_1k <- 1/(1 - seq_5)
  ave_1k[i] <- mean(sim_1k)
}


ave_10k <- c()
for( i in 1:2000){
    seq_5<- runif(10000)
    sim_10k <- 1/(1 - seq_5)
  ave_10k[i] <- mean(sim_10k)
}

ave_100k <- c()
for( i in 1:2000){
    seq_5<- runif(100000)
    sim_100k <- 1/(1 - seq_5)
  ave_100k[i] <- mean(sim_100k)
}

ave_1m <- c()
for( i in 1:2000){
    seq_5<- runif(1000000)
    sim_1m <- 1/(1 - seq_5)
  ave_1m[i] <- mean(sim_1m)
}

mean(ave_100)
```

```
## [1] 13.41435
```

```
var(ave_100)
```

```
## [1] 7335.809
```

```
mean(ave_1k)
```

```
## [1] 24.35754
```

```
var(ave_1k)
```

```
## [1] 168245.6
```

```
mean(ave_10k)
```

```
## [1] 16.35707
```

```
var(ave_10k)
```

```
## [1] 1374.035
```

```
mean(ave_100k)
```

```
## [1] 21.05908
```

```
var(ave_100k)
```

```
## [1] 10273.89
```

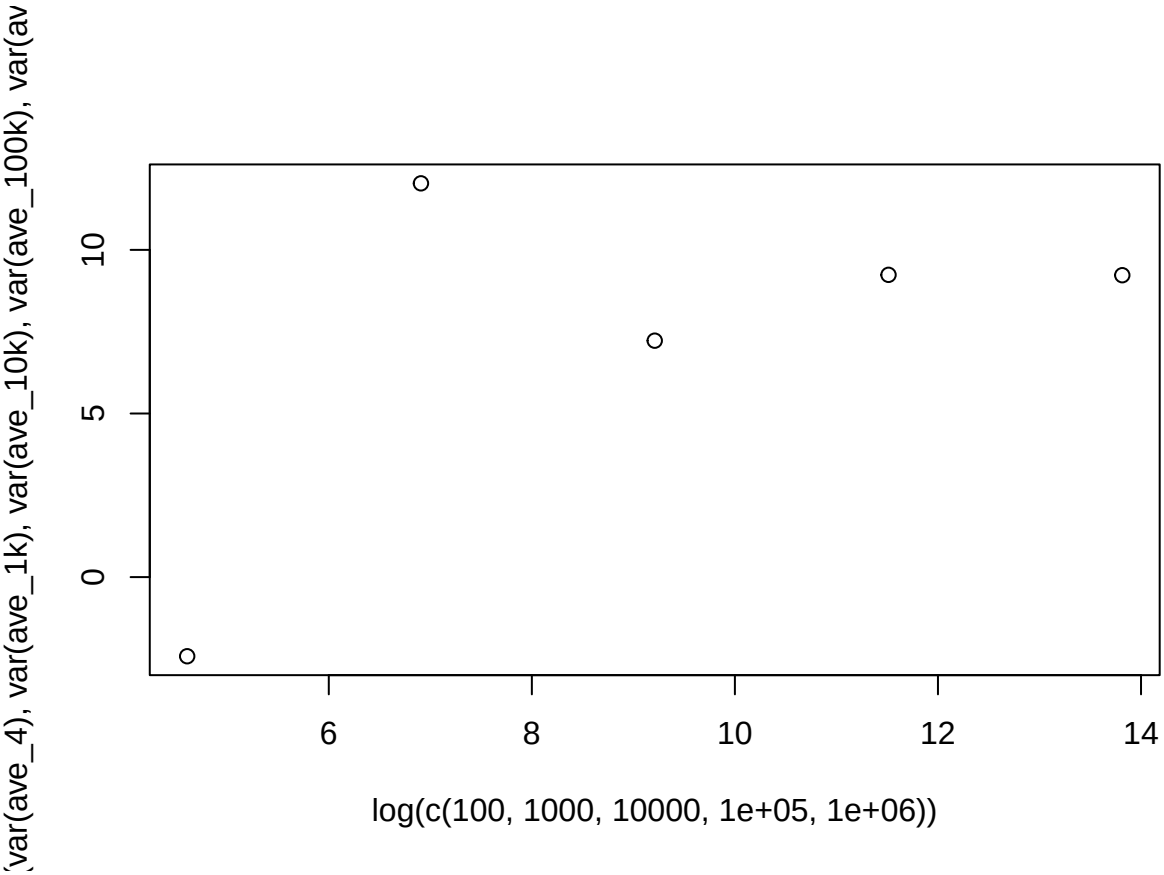
```
mean(ave_1m)
```

```
## [1] 22.57929
```

```
var(ave_1m)
```

```
## [1] 10154.04
```

```
plot(x = log(c(100,1000,10000,100000,1000000)), y = log(c(var(ave_4),var(ave_1k),var(ave_10k),var(ave_1
```




```
slope = (log(var(ave_4))-log(var(ave_1m)))/(log(100) - log(1000000))
slope
```

```
## [1] 1.264233
```

We can see that, there is no convergence behavior in this experiment, as the number of simulation increases the variance does not decrease, but rather get close to a value around 10000. Now with limited tries I am not confident in this conclusion, but I think this is still a random result, the result has no relation to the simulartion number.