

Substitution
Approach

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ 2T(n/2) + n & \text{if } n>1 \end{cases}$$

$$T(n) = 2T(n/2) + n.$$

$$= 2[2T(n/4) + n/2] + n.$$

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$$= 2[2T(n/4) + n/2] + n.$$

$$= 4T(n/4) + n + n$$

$$= 4T(n/4) + 2n.$$

$$= 2^2 T(n/2^2) + 2n.$$

\vdots

$$= 2^k T(n/2^k) + kn.$$

$$= n + T(1) + \log_2 n \cdot n$$

$$= n + 1 + \log_2 n \cdot n.$$

$$= n + n \log n.$$

$$\Rightarrow \underline{\underline{O(n \log n)}}$$

$$2^k = n.$$

$$\therefore k = \log_2 n.$$

