# clDice - a Novel Topology-Preserving Loss Function for Tubular Structure Segmentation

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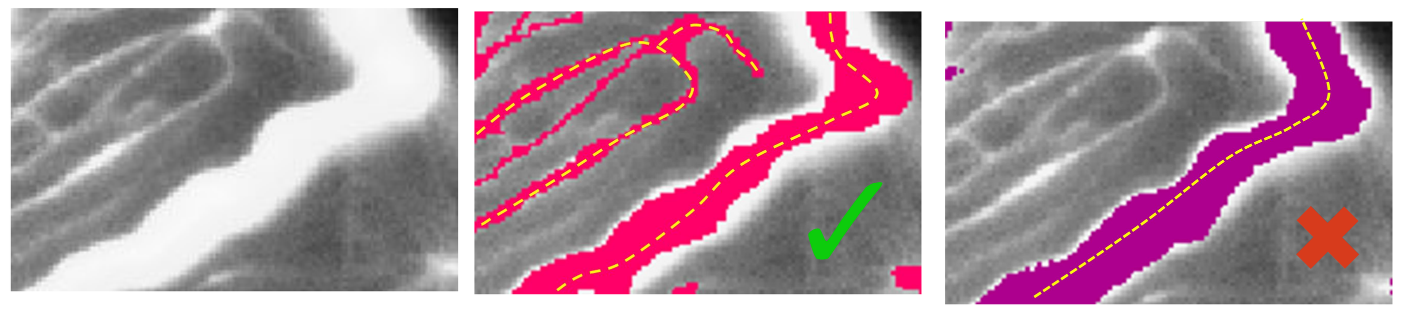
## Motivation



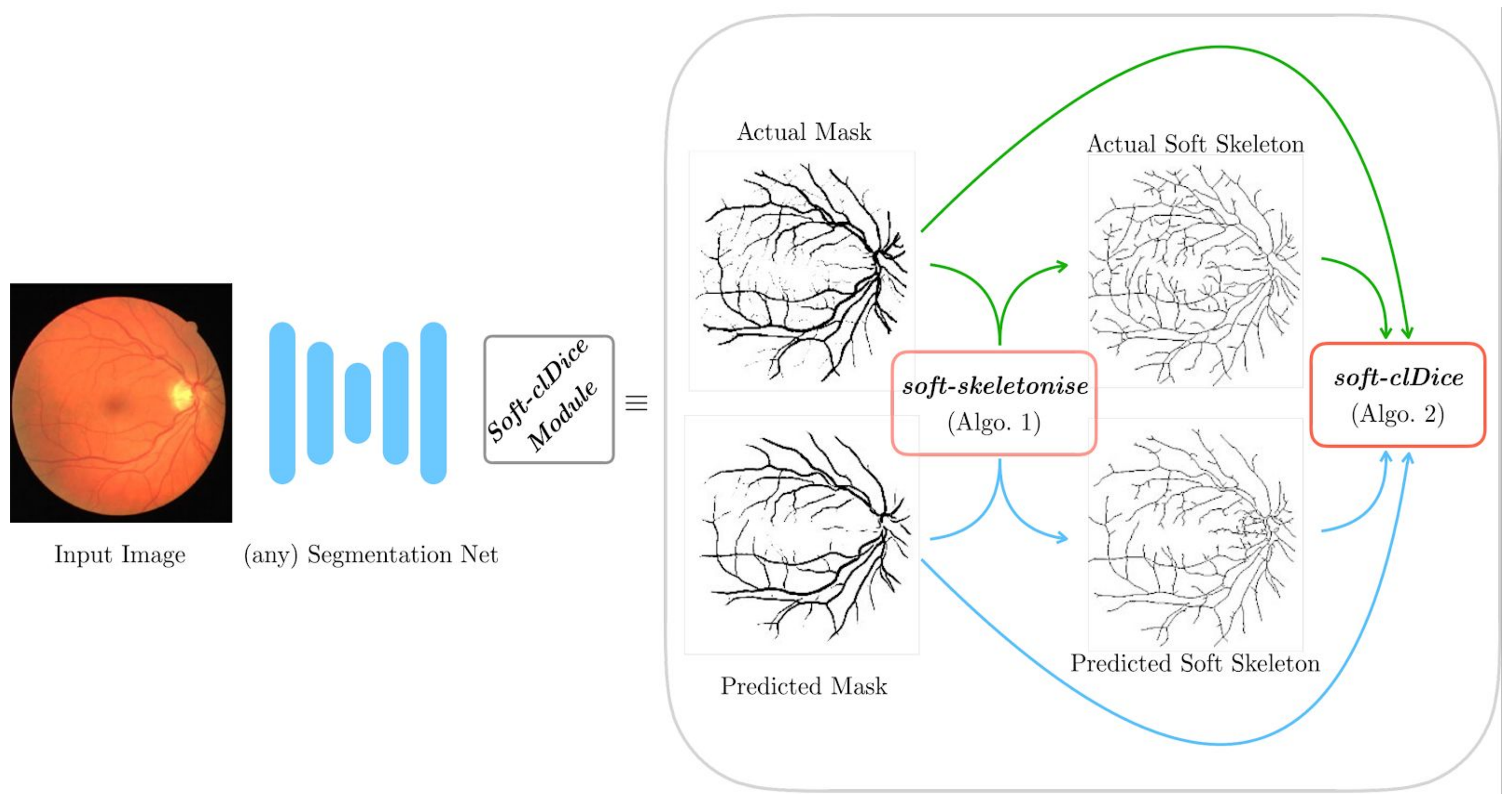
Out of two segmentations of tubular data (similar dice score) we prefer the one with better connectivity - **red is superior to purple**.

## Problem & Research Question

Q1. What is a good pixelwise measure to benchmark segmentation algorithms for tubular, linear and curvilinear structure segmentation while guaranteeing the preservation of the network-topology?

Q2. Can we use this improved measure as a loss function for neural networks?

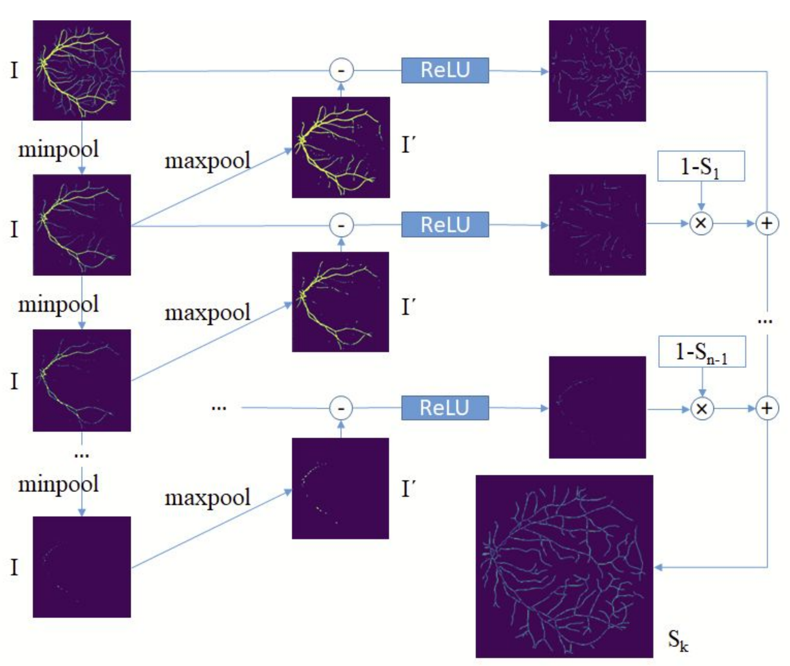
## clDice Formulation




## Soft Skeletonization

**Algorithm 1:** *soft-skeleton*

**Input:** $I, k$
 $I' \leftarrow maxpool(minpool(I))$
 $S \leftarrow ReLU(I - I')$
**for** $i \leftarrow 0$ **to** $k$ **do**
 $I \leftarrow minpool(I)$
 $I' \leftarrow maxpool(minpool(I))$
 $S \leftarrow S + (1 - S) \circ ReLU(I - I')$
**end**
**Output:** $S$




## A computationally efficient overlap-based loss for curvilinear structure segmentation with theoretical topology guarantee.

### Theory

1. **Optimal clDice for foreground and background guarantees a predicted segmentation to be homotopy equivalent to the ground truth label.** □
2. **clDice as a loss achieves homotopy equivalence via a minimum error correction between prediction and ground truth label.**

### Loss Function

**Algorithm 2:** *soft-clDice*

**Input:** $V_P, V_L$
 $S_P \leftarrow soft\text{-}skeleton(V_P)$
 $S_L \leftarrow soft\text{-}skeleton(V_L)$
 $Tprec(S_P, V_L) \leftarrow \frac{|S_P \circ V_L| + \epsilon}{|S_P| + \epsilon}$
 $Tsens(S_L, V_P) \leftarrow \frac{|S_L \circ V_P| + \epsilon}{|S_L| + \epsilon}$
 $clDice \leftarrow$
 $2 \times \frac{Tprec(S_P, V_L) \times Tsens(S_L, V_P)}{Tprec(S_P, V_L) + Tsens(S_L, V_P)}$
**Output:** $clDice$

In practice, volumetric segmentation is desired, over pure skeletons, therefore we combine soft-clDice with soft-Dice.

$$\mathcal{L}_c = (1 - \alpha)(1 - soft\mathbf{Dice}) + \alpha(1 - soft\mathbf{clDice})$$

$\alpha$ is a tunable parameter.

## Quantitative Results

| Dataset | Network | Loss | Dice | Accuracy | *clDice* | $\beta_0$ Error | $\beta_1$ Error | SMD [4] | $\chi_{error}$ | Opt-J F1 [7] |
|---|---|---|---|---|---|---|---|---|---|---|
| Roads | FCN | *soft-dice* | 64.84 | 95.16 | 70.79 | 1.474 | 1.408 | 0.1216 | 2.634 | 0.766 |
| | | $\mathcal{L}_c, \alpha = 0.1$ | 66.52 | 95.70 | 74.80 | 0.987 | 1.227 | 0.1002 | 2.625 | 0.768 |
| | | $\mathcal{L}_c, \alpha = 0.2$ | **67.42** | **95.80** | 76.25 | **0.920** | 1.280 | **0.0954** | 2.526 | 0.770 |
| | | $\mathcal{L}_c, \alpha = 0.3$ | 65.90 | 95.35 | 74.86 | 0.974 | 1.197 | 0.1003 | 2.448 | 0.775 |
| | | $\mathcal{L}_c, \alpha = 0.4$ | 67.18 | 95.46 | **76.92** | 0.934 | **1.092** | 0.0991 | **2.183** | **0.803** |
| | | $\mathcal{L}_c, \alpha = 0.5$ | 65.77 | 95.09 | 75.22 | 0.947 | 1.184 | 0.0991 | 2.361 | 0.782 |
| | U-NET | *soft-dice* | 76.23 | 96.75 | 86.83 | 0.491 | 1.256 | 0.0589 | 1.120 | 0.881 |
| | | $\mathcal{L}_c, \alpha = 0.1$ | **76.66** | **96.77** | 87.35 | 0.359 | **0.938** | 0.0457 | 0.980 | 0.878 |
| | | $\mathcal{L}_c, \alpha = 0.2$ | 76.25 | 96.76 | 87.29 | **0.312** | 1.031 | **0.0415** | 0.865 | 0.900 |
| | | $\mathcal{L}_c, \alpha = 0.3$ | 74.85 | 96.57 | 86.10 | 0.322 | 1.062 | 0.0504 | 0.827 | 0.913 |
| | | $\mathcal{L}_c, \alpha = 0.4$ | 75.38 | 96.60 | 86.16 | 0.344 | 1.016 | 0.0483 | **0.755** | **0.916** |
| | | $\mathcal{L}_c, \alpha = 0.5$ | 76.45 | 96.64 | **88.17** | 0.375 | 0.953 | 0.0527 | 1.080 | 0.894 |
| | Mosinska et al. | [29, 17] | - | 97.54 | - | - | 2.781 | - | - | - |
| | Hu et al. | [17] | - | 97.28 | - | - | 1.275 | - | - | - |
| CREMI | U-NET | *soft-dice* | 91.54 | 97.11 | 95.86 | 0.259 | 0.657 | 0.0461 | 1.087 | 0.904 |
| | | $\mathcal{L}_c, \alpha = 0.1$ | 91.76 | **97.21** | 96.05 | 0.222 | 0.556 | **0.0395** | 1.000 | 0.900 |
| | | $\mathcal{L}_c, \alpha = 0.2$ | 91.66 | 97.15 | 96.01 | 0.231 | 0.630 | 0.0419 | 0.991 | 0.902 |
| | | $\mathcal{L}_c, \alpha = 0.3$ | **91.78** | 97.18 | **96.21** | **0.204** | **0.537** | 0.0437 | **0.919** | **0.913** |
| | | $\mathcal{L}_c, \alpha = 0.4$ | 91.56 | 97.12 | 96.09 | 0.250 | 0.630 | 0.0444 | 0.995 | 0.902 |
| | | $\mathcal{L}_c, \alpha = 0.5$ | 91.66 | 97.16 | 96.16 | 0.231 | 0.620 | 0.0455 | 0.991 | 0.907 |
| | Mosinska et al. | [29, 17] | 82.30 | 94.67 | - | - | 1.973 | - | - | - |
| | Hu et al. | [17] | - | 94.56 | - | - | 1.113 | - | - | - |
| DRIVE retina | FCN | *soft-Dice* | 78.23 | 96.27 | 78.02 | 2.187 | 1.860 | 0.0429 | 3.275 | 0.773 |
| | | $\mathcal{L}_c, \alpha = 0.1$ | 78.36 | 96.25 | 79.02 | 2.100 | 1.610 | 0.0393 | 3.203 | 0.777 |
| | | $\mathcal{L}_c, \alpha = 0.2$ | **78.75** | 96.29 | 80.22 | 1.892 | 1.382 | 0.0383 | 2.895 | 0.793 |
| | | $\mathcal{L}_c, \alpha = 0.3$ | 78.29 | 96.20 | 80.28 | 1.888 | **1.332** | **0.0318** | 2.918 | **0.798** |
| | | $\mathcal{L}_c, \alpha = 0.4$ | 78.00 | 96.11 | 80.43 | 2.036 | 1.602 | 0.0423 | 3.141 | 0.764 |
| | | $\mathcal{L}_c, \alpha = 0.5$ | 77.76 | 96.04 | **80.95** | **1.836** | 1.408 | 0.0394 | **2.848** | 0.794 |
| | U-Net | *soft-Dice* | 74.25 | 95.63 | 75.71 | 1.745 | 1.455 | 0.0649 | 2.997 | 0.760 |
| | | $\mathcal{L}_c, \alpha = 0.5$ | **75.21** | **95.82** | **76.86** | **1.538** | **1.389** | **0.0586** | **2.737** | **0.767** |
| | Mosinska et al. | [29, 17] | - | 95.43 | - | - | 2.784 | - | - | - |
| | Hu et al. | [17] | - | 95.21 | - | - | 1.076 | - | - | - |
| Vessap data | FCN, 1 ch | *soft-dice* | 85.21 | **96.03** | 90.88 | 3.385 | 4.458 | 0.00459 | 5.850 | 0.862 |
| | | $\mathcal{L}_c, \alpha = 0.5$ | **85.44** | 95.91 | **91.32** | **2.292** | **3.677** | **0.00417** | **5.620** | **0.864** |
| | FCN, 2 ch | *soft-dice* | 85.31 | 95.82 | 90.10 | 2.833 | 4.771 | 0.00629 | 6.080 | 0.849 |
| | | $\mathcal{L}_c, \alpha = 0.1$ | 85.96 | 95.99 | 91.02 | 2.896 | **4.156** | 0.00447 | 5.980 | 0.860 |
| | | $\mathcal{L}_c, \alpha = 0.2$ | **86.45** | **96.11** | 91.22 | 2.656 | 4.385 | 0.00466 | 5.530 | 0.869 |
| | | $\mathcal{L}_c, \alpha = 0.3$ | 85.72 | 95.93 | 91.20 | 2.719 | 4.469 | **0.00423** | 5.470 | 0.866 |
| | | $\mathcal{L}_c, \alpha = 0.4$ | 85.65 | 95.95 | **91.65** | 2.719 | 4.469 | **0.00423** | 5.670 | 0.869 |
| | | $\mathcal{L}_c, \alpha = 0.5$ | 85.28 | 95.76 | 91.22 | **2.615** | 4.615 | 0.00433 | **5.320** | **0.870** |
| | U-Net, 1 ch | *soft-dice* | 87.46 | 96.35 | 91.18 | 3.094 | 5.042 | 0.00549 | 5.300 | 0.863 |
| | | $\mathcal{L}_c, \alpha = 0.5$ | **87.82** | **96.52** | **93.03** | **2.656** | **4.615** | **0.00533** | **4.910** | **0.872** |
| | U-Net, 2 ch | *soft-dice* | 87.98 | 96.56 | 90.16 | 2.344 | 4.323 | 0.00507 | 5.550 | 0.855 |
| | | $\mathcal{L}_c, \alpha = 0.1$ | 88.13 | 96.59 | 91.12 | 2.302 | 4.490 | 0.00465 | 5.180 | **0.872** |
| | | $\mathcal{L}_c, \alpha = 0.2$ | 87.96 | 96.74 | 92.52 | 2.208 | **3.979** | 0.00342 | **4.830** | 0.861 |
| | | $\mathcal{L}_c, \alpha = 0.3$ | 87.70 | 96.71 | 92.56 | **2.115** | 4.521 | **0.00309** | 5.260 | 0.858 |
| | | $\mathcal{L}_c, \alpha = 0.4$ | **88.57** | **96.87** | **93.25** | 2.281 | 4.302 | 0.00327 | 5.370 | 0.868 |
| | | $\mathcal{L}_c, \alpha = 0.5$ | 88.14 | 96.74 | 92.75 | 2.135 | 4.125 | 0.00328 | 5.390 | 0.864 |

➔ **clDice improves connectivity and volumetric metrics**
➔ **On par with state-of-the-art with less computation cost**

## Results

### Qualitative Results

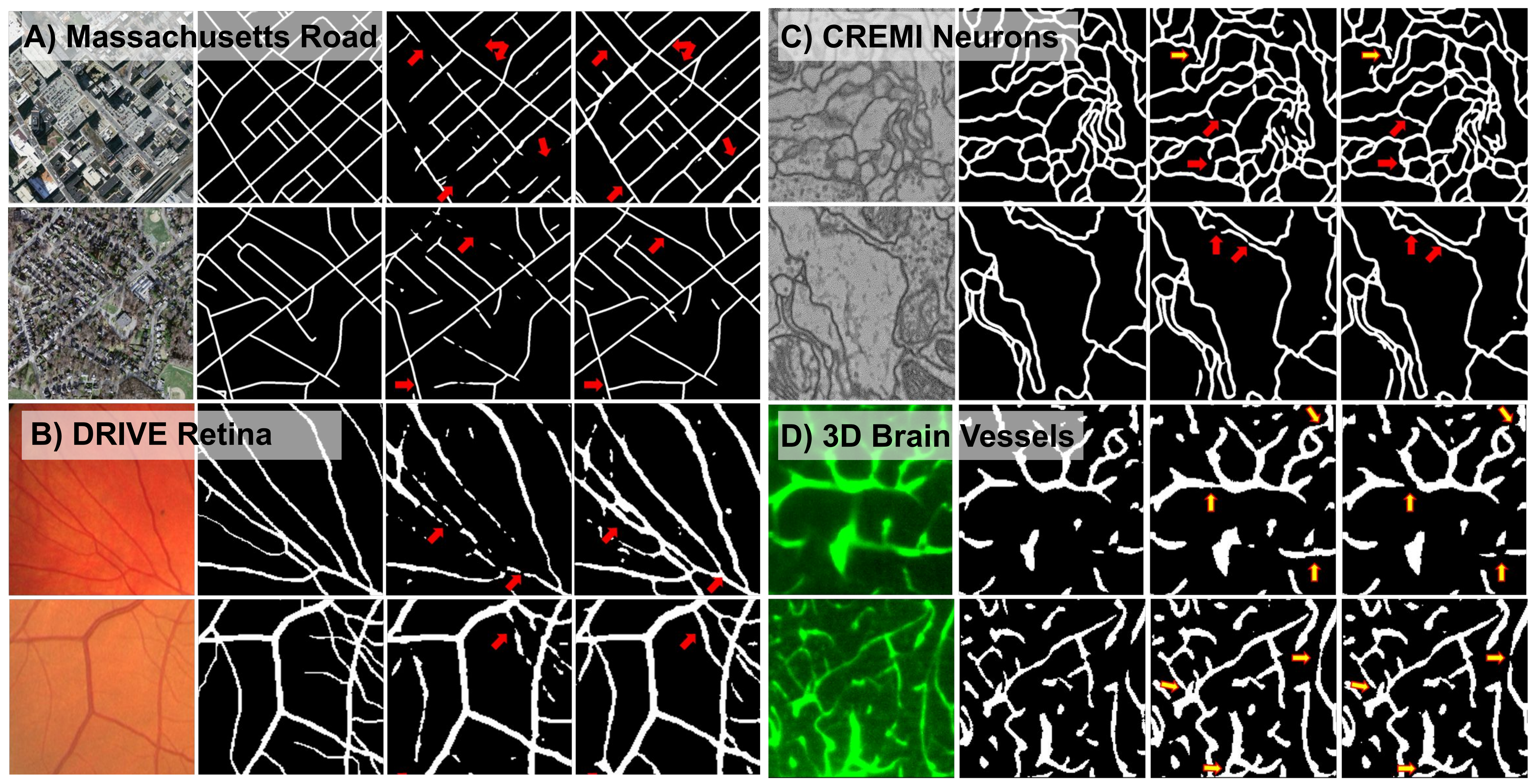
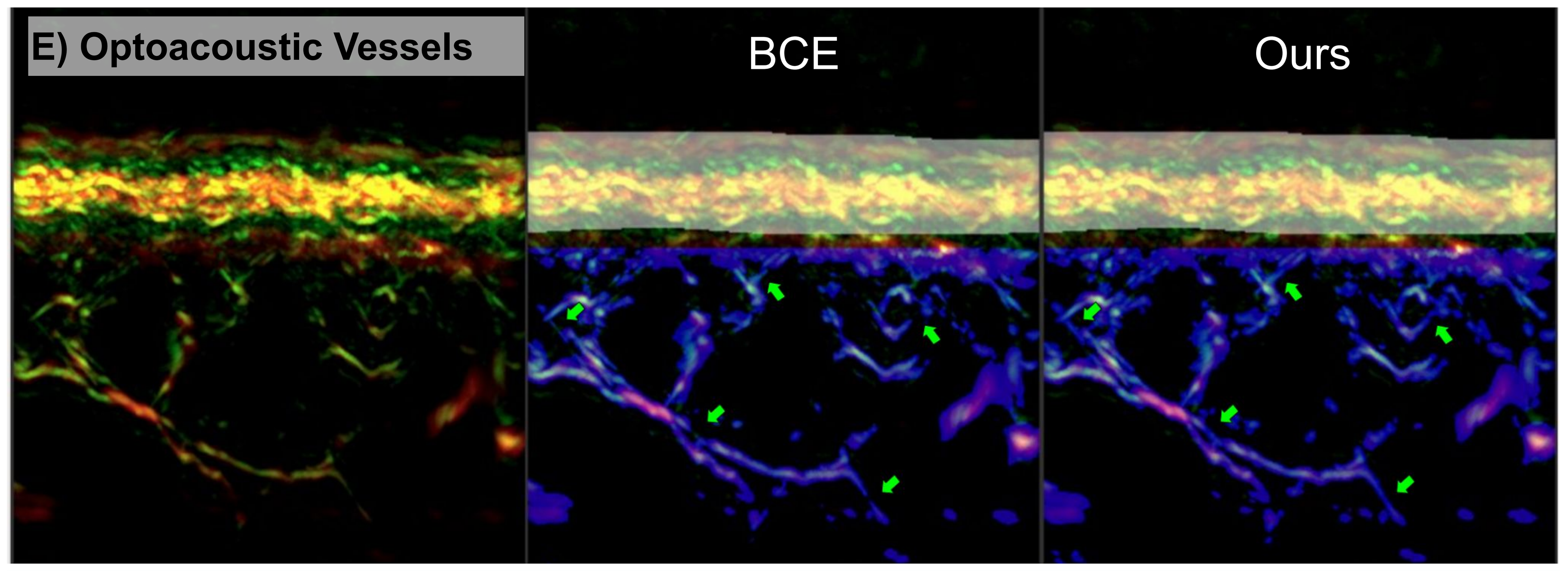



Image Label Soft-Dice Ours Image Label Soft-Dice Ours

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github.com/jocpae/clDice

See the full paper