Concept of Mathematics

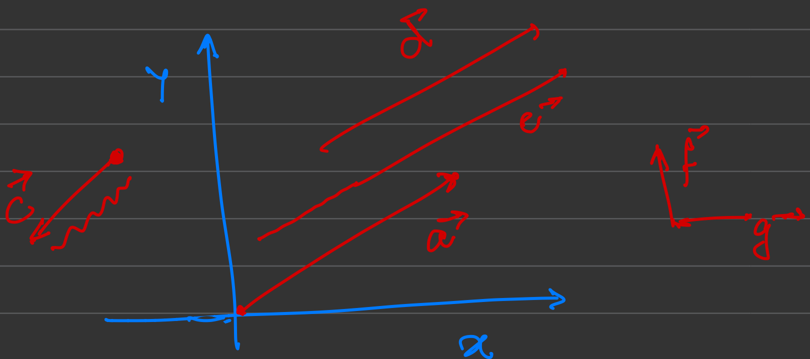
① Vectors
② Differentiation
③ Partial Differentiation
④ Gradient of a function
⑤ maxima & minima of a function

Vector: it is an object which has a magnitude and a direction

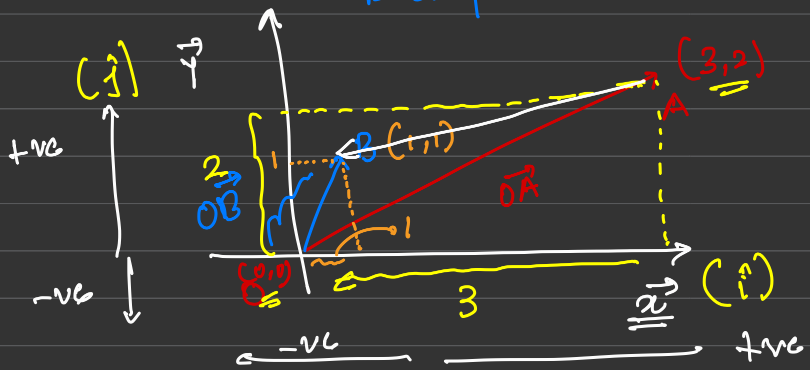
Vector

O → North → 5km/hr (velocity)

O 5km/hr (speed) → Scalar



Decomposed into its projections



$$\vec{OA} = 3\hat{i} + 2\hat{j}$$
$$\vec{OB} = 1\hat{i} + 1\hat{j}$$
$$\vec{AB} = \vec{OB} - \vec{OA}$$
$$\vec{AB} = 1i + 1j - (3i + 2j)$$
$$\vec{AB} = -2i - 1j$$

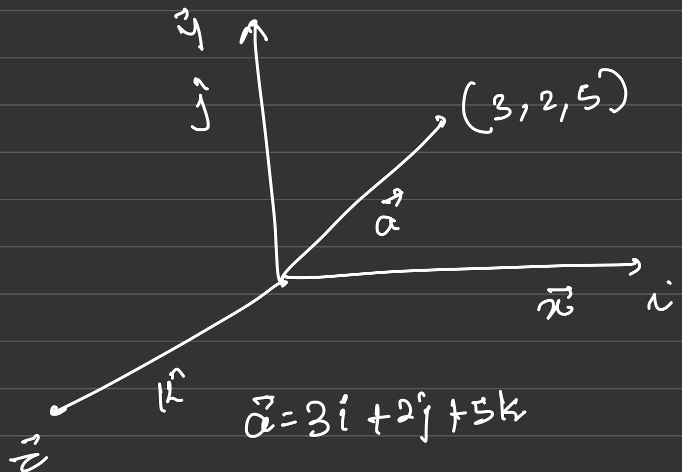
$$|\vec{AB}| = \sqrt{(2)^2 + (-1)^2}$$

$$|\vec{AB}| = \sqrt{5}$$
$$\vec{x} = a\hat{i} + b\hat{j}$$

$$|\vec{x}| = \sqrt{a^2 + b^2}$$

$$\vec{x} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$|\vec{x}| = \sqrt{a^2 + b^2 + c^2}$$

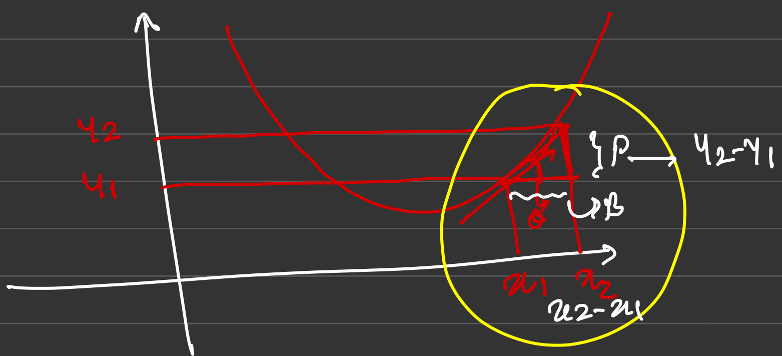


$\vec{a} = 3\hat{i} + 2\hat{j} + 5\hat{k}$

Differentiation

$F(x)$

$\dfrac{dF(x)}{dx} \rightarrow$ rate of change of the function w.r.t to the variable



$\tan\theta = P/B$

$\tan\theta = (y_2 - y_1)/(x_2 - x_1)$

$dx \to 0$ $\boxed{\dfrac{dF(x)}{dx}}$ $\quad$ $\Delta x \to 0$ $\boxed{\dfrac{\Delta y}{\Delta x} = dy/dx}$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\lim_{h \to 0} \boxed{\frac{f(x_1 + h) - f(x_1)}{x_1 + h - x_1}}$$

Rules of Differentiation

① Power Rule.

$$f(x) = x^n$$

$$f'(x) = dy/dx = n \times x^{n-1}$$

$$f(x) = 5 \times x^5$$

$$f'(x) = 5 \times \frac{d(x^5)}{dx}$$

$$= 5 \times 5 \times x^4$$

$$= 25x^4$$

② Product Rule

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d\cos(x)}{dx} = -\sin x$$

$$h(x) = f(x) \times g(x)$$

$$h'(x) = f(x) \times g'(x) + f'(x) \times g(x)$$

$$h(x) = (x^4 + 2)\cos x$$

$$h'(x) = -(x^4+2)\sin x + \cos x\,(4x^3)$$

$$h'(x) = -\sin x\, x^4 - 2\sin x + 4x^3\cos x$$

Partial Differentiation

$$f(x, y) = x^4 y$$

$\dfrac{\partial f(x,y)}{\partial x}$ / $\dfrac{\partial f(x,y)}{\partial y}$ → differentiate the function w.r.t y keeping x constant

↓

differentiate the function w.r.t x, keeping y constant

$$\frac{\partial (x^4 y)}{\partial y} = x^4$$

$$\boxed{\frac{\partial (x^4 y)}{\partial x} = y \times 4x^3}$$

Gradient

$$f(x,y) \rightarrow \frac{\partial f(x,y)}{\partial y}$$

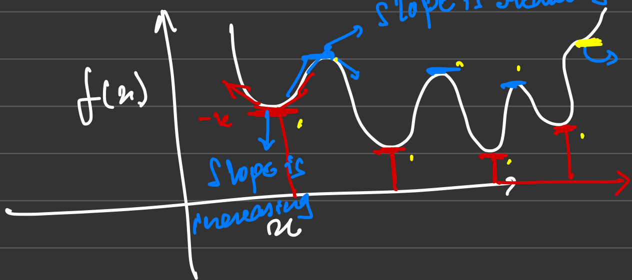
$$f(x,y) \rightarrow \frac{\partial f(x,y)}{\partial x}$$

$$f(x,y) = 2x^2 + 4y$$

$$\nabla f = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix} = \begin{matrix} 4x \\ 4 \end{matrix}$$

$$\nabla f = \begin{bmatrix} 4x \\ 4 \end{bmatrix}$$

Maxima & minima of a function



$$\boxed{f'(x) = 0}$$

Critical Points
- Points at which the slope of the function

→ i.e the slope of the tangent at that point is zero.

Red Points → minima

$$\boxed{f''(x) > 0} \quad \text{Red}$$

$$\frac{d f'(x)}{dx}$$

$f''(x) < 0$ → maxima
↳ Blue

$f''(x) = 0$ → inflection points

Calculation of the maxima & minima of a Bivariate Function.

$f(x, y)$ Local minima and local maxima points

$$p = \boxed{\frac{\partial f(x,y)}{\partial x}}$$ Partial differentiation

$$\boxed{q = \frac{\partial f(x,y)}{\partial y}}$$

Solve for $p=0$ and $q=0$
Critical points.

$$r = \frac{\partial^2 f(x,y)}{\partial x^2}$$

$$s = \frac{\partial^2 f(x,y)}{\partial (xy)}$$

$$t = \frac{\partial^2 f(x,y)}{\partial y^2}$$

for all the critical points (a,b)

if $\boxed{(rt - s^2 > 0)}$ and

$r > 0$ (a,b) is a local minima

$r < 0$ (a,b) is a local maxima

$rt - s^2 = 0$ test fails

$rt - s^2 < 0$ it's a saddle point