classmate

Date

Page

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1) $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{2\sigma}\right)^2}$

$x_1, x_2, \ldots, x_n \rightarrow$ sample size of n

$L(x_1, x_2, \ldots, x_n) = f(x_1) \cdot f(x_2) \ldots f(x_n)$

$\Rightarrow \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\right.$

taking ln

$\Rightarrow \ln L = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{x_i-\mu}{2\sigma^2}\right)^2$

Take partial der wrt μ

$\Rightarrow \frac{\partial \ln L}{\partial \mu} = 0 + \sum_{i=1}^{n} -\left(\frac{\cancel{2}(x_i-\mu)}{\cancel{2}\sigma^2}\right) = 0$

$= \sum_{i=1}^{n} (x_i - \mu) = 0$

$\Rightarrow n\bar{x} - n\mu = 0$

$\Rightarrow \boxed{\bar{x} = \mu} \rightarrow$ Hence $\theta = \bar{x}$ is sample mean

Taking der wrt to σ^2

$$\frac{\partial \ln L}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} \frac{-(x_i-\mu)^2}{2\sigma^2} = 0$$

$$= -n + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{\sigma^2} = 0$$

$$\Rightarrow n = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2}$$

$$\Rightarrow \sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n} (x_i-\mu)^2\right)$$

Hence

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n} (x_i-\mu)^2$$

2) Binomial dist

$\hookrightarrow {}^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

Taking log on both sides

$$\log L = \sum_{i=1}^{n} \left(\log({}^nC_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i} \right)$$

$$\Rightarrow \log L = \sum_{i=1}^{n} \log({}^nC_{x_i}) + \log\theta \sum_{i=1}^{n} x_i + \log(1-\theta) \sum (n-x_i)$$

diff wrt θ

$$\Rightarrow \frac{d \log L}{d\theta} = 0$$

$$\Rightarrow \frac{1}{\theta}\sum x_i - \frac{1}{1-\theta}\sum (n-x_i) = 0$$

$$\Rightarrow \frac{1}{\theta}\sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta}\sum x_i = 0$$

$$\Rightarrow \frac{1}{\theta(1-\theta)} \sum x_i = \frac{n^2}{1-\theta} \Rightarrow \boxed{\theta = \frac{\sum x_i}{n^2}}$$