Bisection Method

(repeated application of intermediate value property)

Let $f(x)$ be continuous between a & b

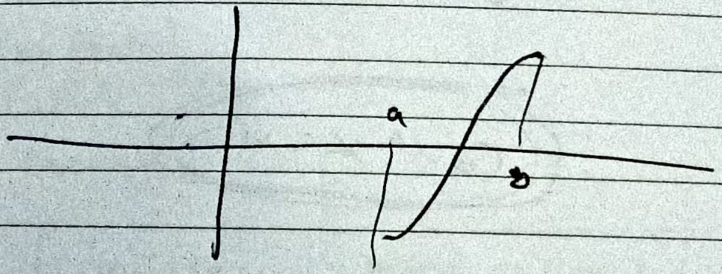
Let $f(a)$ be $-ve$
and $f(b)$ be $+ve$

Then first approximation of the root is
$$x_1 = \frac{a+b}{2}$$

if $f(x_1)=0$, then x is root of $f(x)=0$

Otherwise the root lies between a and x_1 or x_1 and b accordingly as $f(x_1)$ is positive or negative then we bisect the interval as before and continue the process until the root is found

$f(x) = x^3 - x - 1 = 0$. find real root.



$f(-1) = -1+1-1 = -1$ $\quad$ $f(1) = 1-1-1 = -1$

$f(0) = 1-1-1 = -1$

$f(2) = 8-2-1 = 5$

one real root lies between $x \in (1,2)$

now taking average $= \frac{1+2}{2} = \frac{3}{2}$

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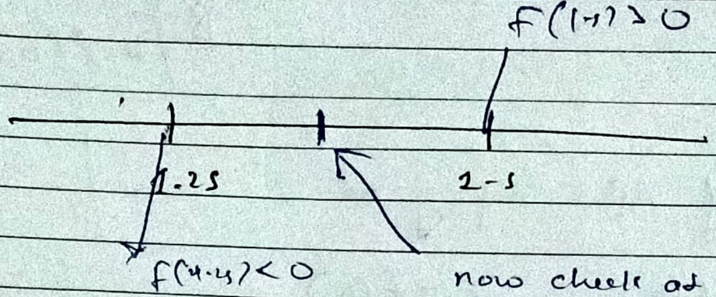
$f\left(\frac{3}{2}\right) = f(1.5) = (1.5)^3 - (1.5) - 1 = \frac{7}{8} > 0$

average $= \dfrac{\left(1+\frac{3}{2}\right)}{2} = 1.25$

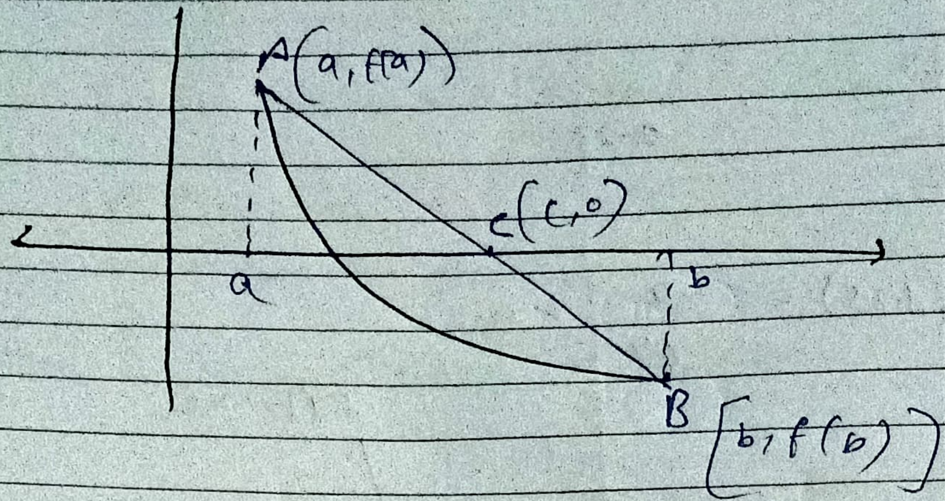
$f(1.25) = \dfrac{-9}{64} < 0$



now check at $\left(\dfrac{1.5+1.25}{2}\right) = \dfrac{2.75}{2} = \underline{\underline{1.375}}$

and so on until we don't find x for which $f(x) = 0$.

False Position Method (Regula- falsi method)



Slope of AB = Slope of AC.

$$\frac{f(b)-f(a)}{b-a} = \frac{0-f(a)}{c-a}$$

$$[f(b)-f(a)][c-a] = af(a) - bf(a)$$

$$cf(b) - cf(a) - af(b) + \cancel{af(a)} = \cancel{af(a)} - bf(a)$$

$$c[f(b)-f(a)] = af(b) - bf(a)$$

$$\boxed{c = \frac{af(b) - bf(a)}{f(b) - f(a)}}$$

Now check $f(c)$

here $f(c) < 0$

$\therefore$ root should between [a and c]

in the similar way it goes on

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Newton Raphson Method (tangent method)

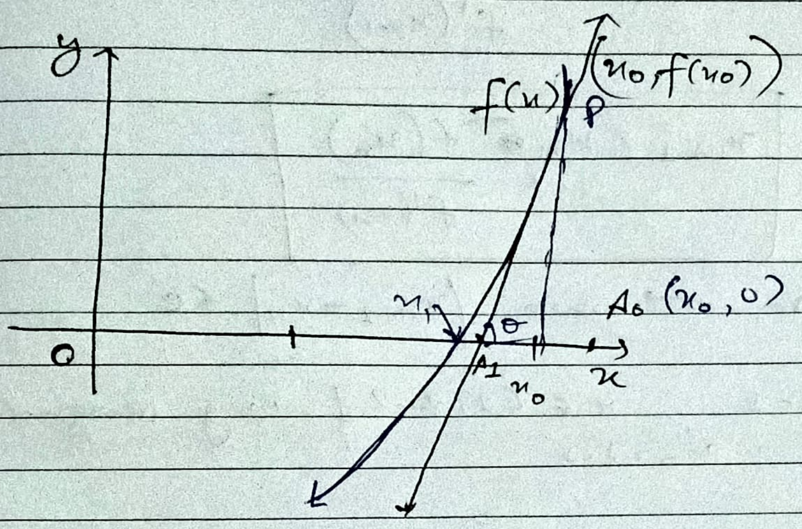
$f(x) = \cos x - x$ $\quad [a_0, b_0]$

$f(a_0) \cdot f(b_0) < 0$

f is continuous.

$f'(x)$ exist.

$[0, 1]$



In $\Delta PA_1A_0 \longrightarrow \tan\theta = f'(x_0) = \dfrac{f(x)}{A_1A_0}$

$$A_1A_0 = \frac{f(x)}{f'(x)}$$

$$OA_0 - OA_1 = \frac{f(x_0)}{f'(x_0)}$$

$$x_0 - x_1 = \frac{f(x_0)}{f'(x_0)}$$

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

$x_1 \longrightarrow$ root of the function

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Similarly $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $\quad x_2$ is more close than x_1

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$ $\quad x_3$ is more close than x_2

$\vdots$

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

when to terminate $\longrightarrow$ when $|x_{n+1} - x_n| < \epsilon$

Q. $f(x) = \tan x - x - 1$ $\quad x \in [1, 1.2]$ using Newton-Raphson method. $\quad x_0 = 1.1$

$f'(x) = \frac{1}{1+x^2} - 1 = \frac{-x^2}{x^2+1}$

$f'(x_0) = -0.54751$

~~$x_1 = 1.1 +$~~

$f(x_0) = -0.13524$

$x_1 = x_0 + \frac{f(x_0)}{f'(x_0)} = \cancel{1.1} + \left(\cancel{\frac{-0.54751}{-0.13524}}\right)$

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n	x_n	$f(x_n)$	$f'(x_n)$	$h_n = x_n + \frac{f(x_n)}{f'(x_n)}$
0	1.1	−0.13524	−0.54751	1.3470
1	1.3470	−0.9948	−0.0575	17.54787875

$f(x) = 3x - \cos x - x$ $[0.5 \quad 0.7]$ $f'(x) = 3 + \sin x$

n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
0	0.6	~~0.37466~~ −0.02533	3.5646	0.59289
1	0.59289	−0.05064	3.5587	0.57866
2	0.57866	−0.10120	3.5469	0.55012
3	0.55012	−0.20206		

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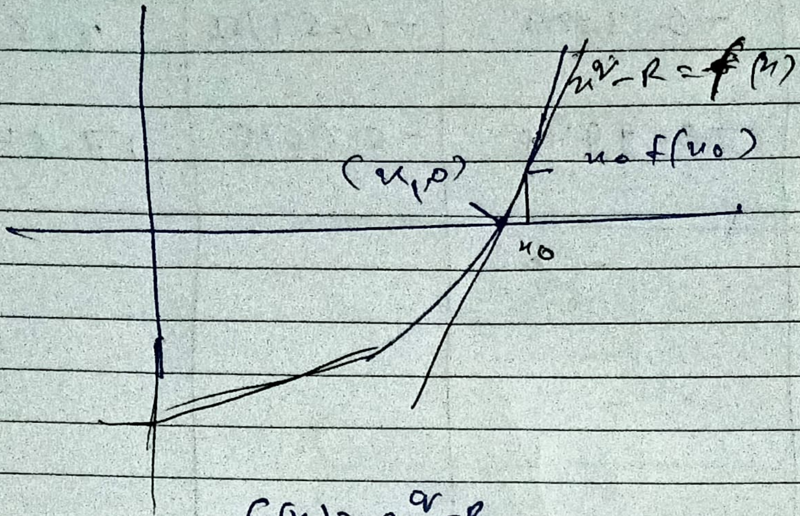
Que if $x^q = R$. find iteration formula.



$f(x) = x^q - R$

$f'(x) = q x^{q-1} = \tan\theta = \dfrac{f(x_0)}{x_0 - x_1}$

$$x_0 - x_1 = \frac{f(x_0)}{q x_0^{q-1}} = \frac{x_0^q - R}{q x_0^{q-1}}$$

$$x_1 = x_0 + \frac{R - x_0^q}{q x_0^{q-1}}$$

$$\therefore \boxed{x_{n+1} = x_n + \frac{R - x_n^q}{q x_n^{q-1}}}$$

taking $R=2$ and $q=2$

$x^2 = 2$.

(finding $\sqrt{2}$)

x_0

~~n x_n $f(x_n)$ $f'(x_n)$~~

$$x_{n+1} = x_n + \frac{2 - x_n^2}{2x_n}$$

$$\downarrow$$

$$x_{n+1} = \frac{x_n^2 + 2}{2x_n}$$

n	x_n	x_{n+1}
0	1	1.5
1	1.5	1.41666
2	1.41666	1.41421
3	1.41421	1.414213

Q. find $3^{1/5}$

Using same formula

$$3^{1/5} = x$$

$$x^5 = 3$$

$$q = 5 \qquad k = 3$$

$$x_{n+1} = x_n + \frac{3 - x_n^5}{5x_n^4} = \frac{4x_n^5 + 3}{5x_n^4}$$

n	x_n	x_{n+1}
0	1	1.66667
1	1.6667	1.240711

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