

## **Assignment- 6**

The Ford-Fulkerson algorithm finds augmenting paths from the source  $s$  to the sink  $t$  in the residual network and augments flow along those paths until no such path exists.

The proposed redefinition removes all reverse edges, that is, no edges lead into  $s$  in the residual graph. Even after this redefinition, FORD-FULKERSON still correctly computes a maximum flow because:

### 1) No Augmenting Path Uses Edges Into $s$ :

- An augmenting path in Ford-Fulkerson is a path from  $s$  to  $t$  in the residual network along which every edge has positive residual capacity.
- An augmenting path goes from  $s$  to  $t$ . Such a path never goes back to  $s$ .
- Hence, edges into  $s$  are never used by any valid augmenting path..

### 2) No Loss of Feasible Paths:

- The algorithm terminates when there are no more augmenting paths from  $s$  to  $t$ . At that point, the flow is maximum.
- Removing edges into  $s$  from the residual network does not remove any possible path from  $s$  to  $t$ . Therefore, it does not affect the discovery of augmenting paths.

### 3) Flow Undoing is possible:

- If flow needs rerouting, the algorithm uses reverse edges in cycles away from  $s$ , so a direct reverse edge into  $s$  isn't needed.

In conclusion, since edges into  $s$  are never used in any augmenting  $s$ - $t$  path, removing them does not affect the algorithm's ability to find such paths. Thus, Ford-Fulkerson will still correctly determine when no augmenting path remains and will compute a maximum flow.