$$X = \begin{bmatrix} 2 & 100 \\ 4 & 200 \\ 6 & 300 \end{bmatrix}_{3\times 2} \rightarrow Z \begin{bmatrix} \\ \\ \end{bmatrix}_{3\times 1}$$

1. Standa

PCA PROBLEM [Example].

1. $$X = \begin{bmatrix} 2 & 100 \\ 4 & 200 \\ 6 & 300 \end{bmatrix}_{3\times 2} \longrightarrow Z = \begin{bmatrix} z_1 & ? \\ z_2 & ? \\ z_3 & ? \end{bmatrix}_{3\times 1}$$

$d = 1$

Sol^n Std each coloum. $\boxed{n = 3}$

$$z = \frac{x - \bar{x}}{\sigma} \qquad \frac{x - \mu}{\sigma}$$

μ_1 = mean 1st col^n = $2+4+6/3$ = $\boxed{4 = \mu_1}$

μ_2 = 100 2^{n} col = $100+200+30/3$ = $\boxed{200 = \mu_2}$

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2$$

col 1 $= \frac{1}{3-1}\left[(2-4)^2 + \cancel{(4-4)^2} + (6-4)^2\right]$

$$\sigma^2 = \frac{1}{2}[4+4]$$

$$\boxed{\sigma_1 = \sqrt{4} = 2}$$

Col 2 $= \frac{1}{3-1}\left[(100-200)^2 + (200-200)^2 + (300-200)^2\right]$

$$= \frac{1}{2}\left[(-100)^2 + 0 + (100)^2\right]$$

$$= \frac{1}{2}[10{,}000 + 10{,}000)$$

$$\boxed{\sigma_2 = 100}$$

Feat. column - 1 $= z_{11} = \frac{(2-4)}{2} = -1$ $\quad z = \frac{x - \text{mean}\ (\mu)}{\text{std.}}$

$$z_{12} = \frac{(4-4)^2}{2} = 0$$

$$z_{13} = \frac{(6-4)}{2} = 0$$

Feat col - 2 $= z_{21} = \frac{(100-200)}{100} = (-1)$

$$z_{22} = \frac{(200-200)}{100} = 0$$

$$z_{23} = \frac{(300-200)}{100} = 1$$

$$X_{std} = \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \Longrightarrow \text{step 1 Std.}$$

Step - 2

$X^T x$ Co-variance matrix

$$\frac{1}{n-1}\left(X^T_{std} \cdot X_{std}\right) \Longrightarrow \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}_{2\times 3} \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}_{3\times 2.}$$

$$= \begin{bmatrix} -1\times -1 + 0 + 1\times 1 & -1\times -1 + 0 + 1 \\ -1\times -1 + 0 * 1\times 1 & -1\times -1 + 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} +1 + 0 + 1 & +1 + 0 + 1 \\ +1 + 0 + 1 & +1 + 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\frac{1}{n-1}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}_{2\times 2}$$

$$= \frac{1}{3-1}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}_{2\times 2.} \longrightarrow X^T x \text{ [Co-variance matrix]}$$

Shift

Step - 3

Compute eigen Values & Vectors of Co-variance matrix

determinant $(A - \lambda I) = 0$

$$A - \lambda I \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix} \quad \text{— (1)}$$

$$\det \begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix}$$

$$= [(1-\lambda)(1-\lambda)] - [(1\times1)] = (1-\lambda)^2 - 1$$

$$= \cancel{1} - \lambda - \lambda + \lambda^2 - \cancel{1}$$

$$= 2\lambda + \lambda^2 = 0$$

$$= \lambda(\lambda - 2) = 0$$

$\lambda_1 = 0 \quad \lambda_2 = 2$ eigen Value.

Eign Vector.

$$(A - \lambda I)v = 0 \qquad v = \begin{bmatrix} x \\ y \end{bmatrix}$$

For $\lambda = 2$ — (1)

$$(A - 2I) \cdot \begin{bmatrix} 1-2 & 1 \\ 1 & 1-2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$(A - 2I)v \Rightarrow \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-1x + y = 0 \rightarrow x_1 = y \qquad x_1 = t$

$1x - y = 0 \qquad \boxed{x = y} \qquad y = t$

$\lambda = 2 \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ eigen Vector for $\lambda_1 = 0$

$$(A - 0I)v = \begin{bmatrix} 1-0 & 1 \\ 1 & 1-0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{matrix} x+y=0 \\ x+y=0. \end{matrix} \quad \begin{matrix} x=-y \\ x=-y \end{matrix}$$

$$\boxed{x = -y}$$

$$v_1 = \begin{bmatrix} -1 \\ +1 \end{bmatrix}$$

Sq and add $\sqrt{1^2+1^2}$ $= \sqrt{2}$ $= \frac{1}{\sqrt{2}}.$

$\sqrt{-1^2+1^2}$ $-1+1$ $= 1/\sqrt{2}.$

$$Z_{pca} = \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ +1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$Z_{pca} = \begin{bmatrix} (-1 \times 1/\sqrt{2}) + (-1 \times 1/\sqrt{2}) \\ (0 \times 1/\sqrt{2}) + (0 \times 1/\sqrt{2}) \\ (1 \times 1/\sqrt{2}) + (1 \times 1/\sqrt{2}) \end{bmatrix} \qquad \begin{bmatrix} (-1) \times (-1/\sqrt{2}) + (-1) \times (1/ \\ (0 \times 1/\sqrt{2}) + (0 \times 1/\sqrt{2}) \\ (1 \times (1/\sqrt{2})) + (0 \times 1/\sqrt{2}) \end{bmatrix}$$

$$= \begin{bmatrix} -2/\sqrt{2}. \\ 0 \\ 2/\sqrt{2}. \end{bmatrix}_{3 \times 1}$$

Tranformed data point

$$Z_{pca} = X_{std} \times V_2$$