

## Some Useful Formulas

**Conditional Probability:**  $\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$ , provided that  $\mathbb{P}(B) \neq 0$ .

**Rule of Total Probability:** If  $B_1, B_2, \dots, B_k$  constitute a partition of the sample space, then  $\mathbb{P}(A) = \mathbb{P}(A|B_1)\mathbb{P}(B_1) + \mathbb{P}(A|B_2)\mathbb{P}(B_2) + \dots + \mathbb{P}(A|B_k)\mathbb{P}(B_k)$

**Bayes' Rule:** If  $B_1, B_2, \dots, B_k$  constitute a partition of the sample space, and  $\mathbb{P}(A) \neq 0$ , then  $\mathbb{P}(B_r|A) = \frac{\mathbb{P}(A|B_r)\mathbb{P}(B_r)}{\mathbb{P}(A|B_1)\mathbb{P}(B_1) + \mathbb{P}(A|B_2)\mathbb{P}(B_2) + \dots + \mathbb{P}(A|B_k)\mathbb{P}(B_k)}$

### Expectation, Mean and Variance:

If  $X$  is a discrete r.v., and  $f(x)$  is its probability distribution function, then

$$\mu = \mathbb{E}[X] = \sum_x x f(x) \text{ and } \mathbb{V}ar(X) = \mathbb{E}[(X - \mu)^2] = \sum_x (x - \mu)^2 f(x) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

If  $X$  is a continuous r.v., and  $f(x)$  is its probability density function, then

$$\mu = \mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x) dx \text{ and } \mathbb{V}ar(X) = \mathbb{E}[(X - \mu)^2] = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

### Moment Generating Function:

If  $X$  is a discrete r.v., and  $f(x)$  is its probability distribution function, then  $M_X(t) = \mathbb{E}[e^{tX}] = \sum_x e^{tx} f(x)$ .

If  $X$  is a continuous r.v., and  $f(x)$  is its probability density function, then  $M_X(t) = \mathbb{E}[e^{tX}] = \int_{-\infty}^{\infty} e^{tx} f(x) dx$ .

#### Discrete Uniform Distribution:

$$f(x; k) = \frac{1}{k} \text{ for } x = 1, 2, \dots, k$$

Parameter:  $k$  is a positive integer

$$\text{Mean and Variance: } \mu = \frac{k+1}{2} \text{ and } \sigma^2 = \frac{k^2-1}{12}$$

#### Bernoulli Distribution:

$$f(x; \theta) = \theta^x (1 - \theta)^{1-x} \text{ for } x = 0, 1$$

Parameter:  $0 < \theta < 1$

$$\text{Mean and Variance: } \mu = \theta \text{ and } \sigma^2 = \theta(1 - \theta)$$

#### Binomial Distribution:

$$b(x; n, \theta) = \binom{n}{x} \theta^x (1 - \theta)^{n-x} \text{ for } x = 0, 1, 2, \dots, n$$

Parameters:  $n$  is a positive integer and  $0 < \theta < 1$

$$\text{Mean and Variance: } \mu = n\theta \text{ and } \sigma^2 = n\theta(1 - \theta)$$

#### Geometric Distribution:

$$g(x; \theta) = \theta(1 - \theta)^{x-1} \text{ for } x = 1, 2, 3, \dots$$

Parameter:  $0 < \theta < 1$

$$\text{Mean and Variance: } \mu = \frac{1}{\theta} \text{ and } \sigma^2 = \frac{1-\theta}{\theta^2}$$

#### Negative Binomial Distribution:

$$b^*(x; k, \theta) = \binom{x-1}{k-1} \theta^k (1 - \theta)^{x-k} \text{ for } x = k, k+1, k+2, \dots$$

Parameter:  $0 < \theta < 1$

$$\text{Mean and Variance: } \mu = \frac{k}{\theta} \text{ and } \sigma^2 = \frac{k(1-\theta)}{\theta^2}$$

#### Poisson Distribution:

$$p(x; \lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ for } x = 0, 1, 2, \dots$$

Parameter:  $\lambda > 0$

$$\text{Mean and Variance: } \mu = \lambda \text{ and } \sigma^2 = \lambda$$

#### Hypergeometric Distribution:

$$h(x; n, N, M) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}} \text{ for } x = 0, 1, \dots, n$$

Parameters:  $n, N, M$ .

$$\text{Mean and Variance: } \mu = \frac{nM}{N} \text{ and } \sigma^2 = \frac{nM(N-M)(N-n)}{N^2(N-1)}$$

#### Continuous Uniform Distribution:

$$f(x; \alpha, \beta) = \frac{1}{\beta - \alpha} \text{ for } \alpha < x < \beta, 0 \text{ elsewhere.}$$

$$\text{Mean and Variance: } \mu = \frac{\alpha + \beta}{2} \text{ and } \sigma^2 = \frac{(\beta - \alpha)^2}{12}$$

#### Exponential Distribution:

$$f(x; \lambda) = \lambda e^{-\lambda x} \text{ for } x > 0, 0 \text{ elsewhere.}$$

$$\text{Mean and Variance: } \mu = \frac{1}{\lambda} \text{ and } \sigma^2 = \frac{1}{\lambda^2}$$