

## Homework 1 (Due: 9/19)

For problem 1, scan your proof and submit it on i-campus. For problem 2, submit your R script file on i-campus (please only include R function).

1. Suppose that  $X|\theta \sim N(0, 1/\theta)$  and  $\theta \sim \text{Gamma}(r/2, 2/r)$ . Show that  $X \sim t_r$ .
2. Make **R** function to compute the MLE of  $(\mu_1, \mu_2, p)$  for the mixture model  $X \sim pN(\mu_1, 1) + (1 - p)N(\mu_2, 1)$  using the Newton-Raphson method. The function name should be your last name attached the last digit of your student id number. For example, if your name is Younghee Kim and student id number is 2022345678, your function name should be Kim8. For your function, the input argument should contain one-dimensional data vector and the vector of the initial value of  $(\mu_1, \mu_2, p)$ . For example,

```
Kim8=function(x,ini_vector){  
  .  
  .  
  .  
  est_mu1=....  
  est_mu2=....  
  est_p=...  
  return(c(est_mu1,est_mu2,est_p))  
}
```

For the stopping criterion for your algorithm, if the difference of the consecutive log-likelihood values is less than  $10^{-10}$ , stop your algorithm. I expect your function works as follows:

```
> # Data generation  
> n=500;p=0.3;mu1=-2;mu2=2  
> x1=rnorm(n,mu1,1)  
> x2=rnorm(n,mu2,1)  
> z=rbinom(n,1,p)  
> x=z*x1+(1-z)*x2  
>  
>  
> Kim8(x,c(-1,1,0.5))  
Newton-Raphson is converged in 8 steps  
[1] -2.0343353  2.0371738  0.3516917
```