PX902 Pharmacometrics Research	**授课日期: 2020年10月20日**

Bayesian Inference for Beginners

讲师: 洪楠方 | 撰写: 洪楠方

助教: | 最后更新: *October 20, 2020*

1 First Example

$P_{X,Y}(\cdot,\cdot)$	$P_{X,Y}(0,\cdot)$	$P_{X,Y}(1,\cdot)$
$P_{X,Y}(\cdot,0)$	0.1	0.2
$P_{X,Y}(\cdot,1)$	0.3	0.4

Table 1: a joint distribution

We start by a motivating example, a joint distribution as shown by table 1. We use marginal distribution $P_X(\cdot)$ as the prior, and conditional distribution $P_{Y|X}(\cdot|\cdot)$ as the likelihood, as derived in table 2.

$P_X(\cdot)$	
$P_X(0)$	0.4
$P_X(1)$	0.6

$P_{Y\|X}(\cdot\|0)$	
$P_{Y\|X}(0\|0)$	0.25
$P_{Y\|X}(1\|0)$	0.75

$P_{Y\|X}(\cdot\|1)$	
$P_{Y\|X}(0\|1)$	1/3
$P_{Y\|X}(1\|1)$	2/3

Table 2: prior and likelihood

The point of Bayesian inference is that the posterior is sampled and normalized in entire parameter space. Here, the parameter space is $\mathcal{X} = \{0,1\}$. Suppose we see a datum $Y = 0$, we ask the conditional distribution, the posterior, $P_{X|Y}(\cdot|0)$.

Since "Posterior $\propto$ Likelihood $\times$ Prior", we sample entire parameter space, which is $\mathcal{X} = \{0,1\}$. The empirical counts of X would be the posterior, see table 3.

$P_{Y\|X}(0\|\cdot) \times P_X(\cdot)$	Normalized
0.1	1/3
0.2	2/3

$P_{X\|Y}(\cdot\|0)$	
$P_{X\|Y}(0\|0)$	1/3
$P_{X\|Y}(1\|0)$	2/3

Table 3: prior and likelihood

2 Second Example

A coin has an unknown probability Θ of facing head, and our prior knowledge of such probability is $\texttt{Beta}_{(2,2)}(\cdot)$. Suppose we observed an i.i.d. sequence of tosses HHHTH, what is the posterior $p_{\Theta|Y_1^5}(\cdot|\texttt{HHHTH})$?

$$p_{\Theta|Y_1^5}(\theta|\texttt{HHHTH}) \propto p_{Y_1^5|\Theta}(\texttt{HHHTH}|\theta) \times \texttt{Beta}_{(2,2)}(\theta) = \underbrace{\theta^4(1-\theta) \times \texttt{Beta}_{(2,2)}(\theta)}_{\int_0^1 = 1/28}$$

normalized to:

$$p_{\Theta|Y_1^5}(\theta|\mathtt{HHHTH}) = 28 \times \theta^4(1-\theta) \times \mathtt{Beta}_{(2,2)}(\theta)$$

The graphics of prior and posterior were shown by figure 1 and 2.

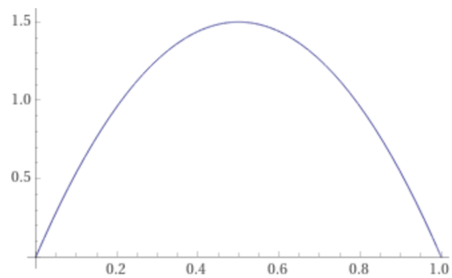

Figure 1: the prior $\mathtt{Beta}_{(2,2)}(\cdot)$

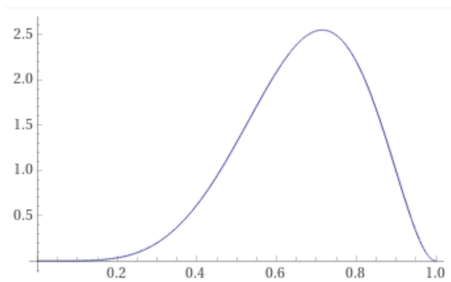

Figure 2: the posterior after observing `HHHTH`

Similarly, if observed 14 `H` in tosses, the posterior shown in figure 3.

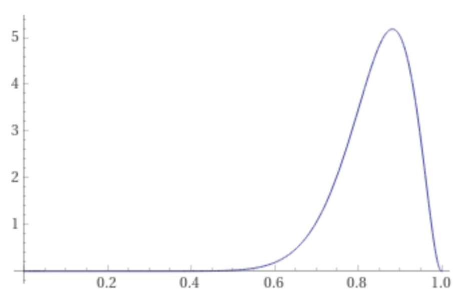

Figure 3: the posterior after observing 14`H` in 15 tosses, more spiky and skewed