Basic R: Matrices

Nanfang Hong (U92430309)

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Matrix problems

1. Suppose

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

(a) Check that $A^3 = \mathbf{0}$

```
A <- matrix(c(1,1,3,5,2,6,-2,-1,-3), nrow = 3, byrow = TRUE)
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    3
## [2,]    5    2    6
## [3,]   -2   -1   -3
```

```
A %*% A %*% A
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    0    0    0
```

(b) Replace the third column of A by the sum of the second and third columns

```
A[,3] <- A[,2] + A[,3]
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    4
## [2,]    5    2    8
## [3,]   -2   -1   -4
```

2. Create the following matrix B with 15 rows

$$B = \begin{bmatrix} 10 & -10 & 10 \\ 10 & -10 & 10 \\ \cdots & \cdots & \cdots \\ 10 & -10 & 10 \end{bmatrix}$$

Calculate the 3x3 matrix $B^T B$. You can make this calculation with the function crossprod(). See the documentaion.

```
B <- matrix(c(10, -10, 10), 15, 3, byrow = TRUE)
B
```

```
##       [,1] [,2] [,3]
##  [1,]   10  -10   10
##  [2,]   10  -10   10
```

```
##  [3,]   10  -10   10
##  [4,]   10  -10   10
##  [5,]   10  -10   10
##  [6,]   10  -10   10
##  [7,]   10  -10   10
##  [8,]   10  -10   10
##  [9,]   10  -10   10
## [10,]   10  -10   10
## [11,]   10  -10   10
## [12,]   10  -10   10
## [13,]   10  -10   10
## [14,]   10  -10   10
## [15,]   10  -10   10
```

```
crossprod(B)
```

```
##       [,1]  [,2]  [,3]
## [1,]  1500 -1500  1500
## [2,] -1500  1500 -1500
## [3,]  1500 -1500  1500
```

3. Create a 6 x 6 matrix matE with every element equal to 0. check what the functions row() and col() return when applied to matE.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

```
matE <- matrix(rep(0,36), nrow = 6, byrow = TRUE)
```

```
# With a little experimentation you would see
# that the specified pattern is in the |1|'s
row(matE)-col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0   -1   -2   -3   -4   -5
## [2,]    1    0   -1   -2   -3   -4
## [3,]    2    1    0   -1   -2   -3
## [4,]    3    2    1    0   -1   -2
## [5,]    4    3    2    1    0   -1
## [6,]    5    4    3    2    1    0
```

```
# so you use the locations of the 1's to modify matE
matE[abs(row(matE)-col(matE))==1] <- 1
matE
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    1    0    0    0    0
## [2,]    1    0    1    0    0    0
## [3,]    0    1    0    1    0    0
## [4,]    0    0    1    0    1    0
## [5,]    0    0    0    1    0    1
## [6,]    0    0    0    0    1    0
```

4. Look at the help for the function outer(). Now, create the following patterned matrix:

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```
a <- 0:4
A <- outer(a,a,"+")
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    5
## [3,]    2    3    4    5    6
## [4,]    3    4    5    6    7
## [5,]    4    5    6    7    8
```

5. Create the following patterned matrices. Your solutions should be generalizable to enable creating larger matrices with the same structure.

(a)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 4 & 0 & 1 \\ 3 & 4 & 0 & 1 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

```
a <- 0:4
A <- outer(a,a,"+")
A <- A %% 5
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    0
## [3,]    2    3    4    0    1
## [4,]    3    4    0    1    2
## [5,]    4    0    1    2    3
```

(b)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 8 & 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```
a <- 0:9
A <- outer(a,a,"+")
A <- A %% 10
A
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]    0    1    2    3    4    5    6    7    8     9
##  [2,]    1    2    3    4    5    6    7    8    9     0
##  [3,]    2    3    4    5    6    7    8    9    0     1
##  [4,]    3    4    5    6    7    8    9    0    1     2
##  [5,]    4    5    6    7    8    9    0    1    2     3
##  [6,]    5    6    7    8    9    0    1    2    3     4
##  [7,]    6    7    8    9    0    1    2    3    4     5
##  [8,]    7    8    9    0    1    2    3    4    5     6
##  [9,]    8    9    0    1    2    3    4    5    6     7
## [10,]    9    0    1    2    3    4    5    6    7     8
```

(c)

$$\begin{bmatrix} 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 \\ 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 \\ 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 \\ 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

```
a <- 0:8
b <- 9:1
A <- outer(a,b,"+")
A <- A %% 9
A
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    0    8    7    6    5    4    3    2    1
##  [2,]    1    0    8    7    6    5    4    3    2
##  [3,]    2    1    0    8    7    6    5    4    3
##  [4,]    3    2    1    0    8    7    6    5    4
##  [5,]    4    3    2    1    0    8    7    6    5
##  [6,]    5    4    3    2    1    0    8    7    6
##  [7,]    6    5    4    3    2    1    0    8    7
##  [8,]    7    6    5    4    3    2    1    0    8
##  [9,]    8    7    6    5    4    3    2    1    0
```

6. Solve the following system of linear equations by setting up and solving the matrix equation $\mathbf{Ax} = \mathbf{y}$.

$$x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 7$$
$$2x_1 + x_2 + 2x_3 + 3x_4 + 4x_5 = -1$$
$$3x_1 + 2x_2 + x_3 + 2x_4 + 3x_5 = -3$$
$$4x_1 + 3x_2 + 2x_3 + x_4 + 2x_5 = 5$$
$$5x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 = 17$$

```
A <- matrix(rep(0, 25), nrow = 5, byrow = TRUE)
A <- abs(row(A)-col(A)) + 1
y <- c(7, -1, -3, 5, 17)
solve(A, y)
```

```
## [1] -2  3  5  2 -4
```

7. Create a 6 x 10 matrix of random integers chosen from 1,2,. . . ,10 by executing the following two lines of code:

```
set.seed(75)
aMat <- matrix(sample(10, size=60, replace=TRUE), nr=6)
aMat
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,]    3    6    7    7    2    4    3    7    1     4
## [2,]    1    9    8    7    2    6   10    9    5     2
## [3,]    7   10    8    4   10    5    4    8    4     4
## [4,]    4    3    1    1    3    3    9    7    4     2
## [5,]    1    8    1    9    9    8    1    3    7     7
## [6,]    2    6    7    5    6   10    4    6   10     1
```

Use the matrix you have created to answer these questions:

(a) Find the number of entries in each row which are greater than 4.

```
rowSums(ifelse(aMat > 4, TRUE, FALSE))
```

```
## [1] 4 7 6 2 6 7
```

(b) Which rows contain exactly two occurrences of the number seven?

```
rowsum1 <- rowSums(ifelse(aMat == 7, TRUE, FALSE))
names(rowsum1) <- paste("Row", 1:length(rowsum1))
print(names(rowsum1[rowsum1 == 2]))
```

```
## [1] "Row 5"
```

(c) Find those pairs of columns whose total (over both columns) is greater tnan 75. The answer should be a matrix with two columns; so, for example, the row (1,2) in the output matrix means that the sum of columns 1 and 2 in the original matrix is greater than 75. Repeating a column is permitted; so, for exalmple, the final output matrix could contain the rows (1,2), (2,1), and (2,2).

```
# Ask Dr. Brian what the output is desired?
colsum1 <- colSums(aMat)
outPutMat <- outer(colsum1, colsum1, "+")
outPutMat
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]   36   60   50   51   50   54   49   58   49    38
##  [2,]   60   84   74   75   74   78   73   82   73    62
##  [3,]   50   74   64   65   64   68   63   72   63    52
##  [4,]   51   75   65   66   65   69   64   73   64    53
##  [5,]   50   74   64   65   64   68   63   72   63    52
##  [6,]   54   78   68   69   68   72   67   76   67    56
##  [7,]   49   73   63   64   63   67   62   71   62    51
##  [8,]   58   82   72   73   72   76   71   80   71    60
##  [9,]   49   73   63   64   63   67   62   71   62    51
## [10,]   38   62   52   53   52   56   51   60   51    40
```

```
index <- which(outPutMat > 75)
colMat <- matrix(c(ceiling(index / 10), index %% 10), ncol = 2)
colMat
```

```
##      [,1] [,2]
## [1,]    2    2
## [2,]    2    6
## [3,]    2    8
## [4,]    6    2
## [5,]    6    8
## [6,]    8    2
## [7,]    8    6
## [8,]    8    8
```

What if repetitions are not permitted? Then only (1,2) from (1,2),(2,1) and (2,2) would be permitted.

```
del <- which((colMat[ ,1] < colMat[ ,2]) == FALSE)
colMat[-del, ]
```

```
##      [,1] [,2]
## [1,]    2    6
## [2,]    2    8
## [3,]    6    8
```

8. Calculate

(a) $\sum_{i=1}^{20} \sum_{j=1}^{5} \frac{i^4}{(3+j)}$

```
sum(sapply(1:20, function(i) (i ^ 4) / (3 + (1:5))))
```

```
## [1] 639215.3
```

(b) $\sum_{i=1}^{20} \sum_{j=1}^{5} \frac{i^4}{(3+ij)}$

```
sum(sapply(1:20, function(i) (i ^ 4) / (3 + i * (1:5))))
```

```
## [1] 89912.02
```

(c) $\sum_{i=1}^{10} \sum_{j=1}^{i} \frac{i^4}{(3+ij)}$

```
a <- sapply(1:20, function(i) (i ^ 4) / (3 + i * (1:i)))
sum(sapply(1:20, function(x) sum(a[[x]])))
```

```
## [1] 137295.9
```