Simple R Functions

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1.

(a) Write functions `tmpFn1` and `tmpFn2` such that if `xVec` is the vector $(x_1, x_2, ..., x_n)$, then `tmpFn1(xVec)` returns vector $(x_1, x_2^2, ..., x_n^n)$ and `tmpFn2(xVec)` returns the vector $(x_1, \frac{x_2^2}{2}, ..., \frac{x_n^n}{n})$.

Here is `tmpFn1`

```
tmpFn1 <- function(xVec){
  return(xVec^(1:length(xVec)))
}

## simple example
a <- c(2, 5, 3, 8, 2, 4)

b <- tmpFn1(a)
b
```

```
## [1]    2   25   27 4096   32 4096
```

and now `tmpFn2`

```
tmpFn2 <- function(xVec2){

  n = length(xVec2)

  return(xVec2^(1:n)/(1:n))
}




c <- tmpFn2(a)
c
```

```
## [1]    2.0000   12.5000    9.0000 1024.0000    6.4000  682.6667
```

(b) Now write a function `tmpFn3` which takes 2 arguments x and n where x is a single number and n is a strictly positive integer. The function should return the value of

$$1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^n}{n}$$

```
tmpFn3 <- function(x, n){
  return(1 + sum(sapply(1:n, function(i) x ^ i / i)))
}
```

2. Write a funciton `tmpFn(xVec)` such that if `xVec` is the vector $x = (x_1, ..., x_n)$ then `tmpFn(xVec)` returns the vector of moving averages:

$$\frac{x_1 + x_2 + x_3}{3}, \frac{x_2 + x_3 + x_4}{3}, ..., \frac{x_{n-2} + x_{n-1} + x_n}{3}$$

```
tmpFn <- function(xVec){
  n2 <- 1:(length(xVec) -2)
  return((xVec[n2] + xVec[n2 + 1] + xVec[n2 + 2]) / 3)
}
```

Try out your function. `tmpFn`(c(1:5,6:1))

```
tmpFn(c(1:5, 6:1))
```

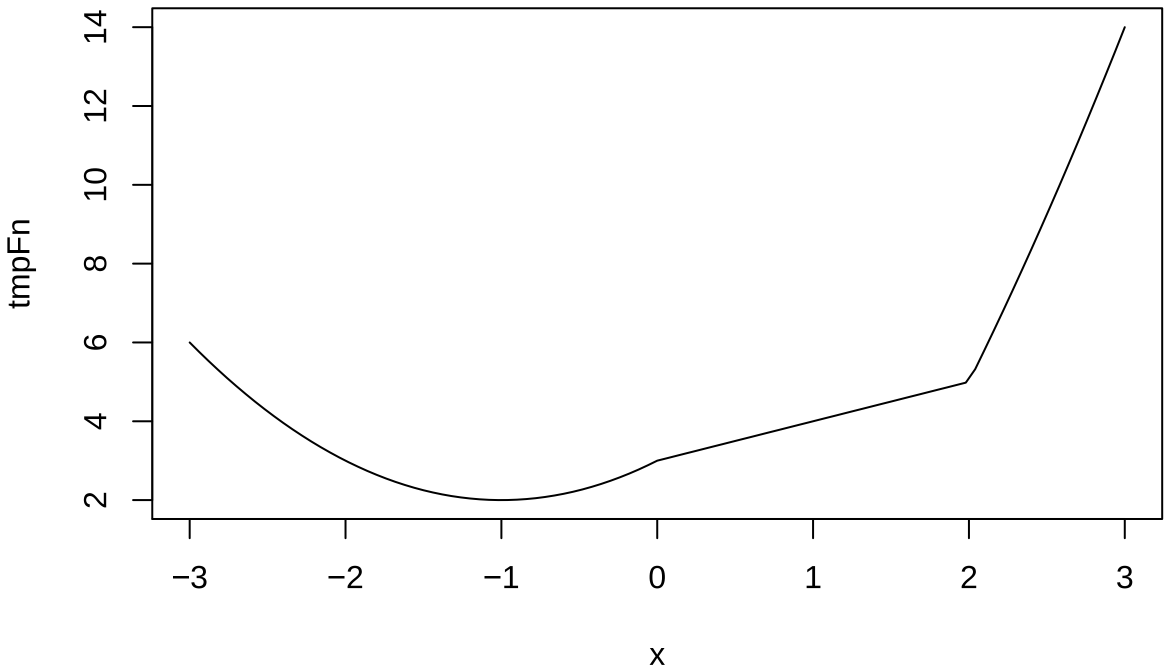
```
## [1] 2.000000 3.000000 4.000000 5.000000 5.333333 5.000000 4.000000 3.000000
## [9] 2.000000
```

3. Consider the continuous function

$$f(x) = \begin{cases} x^2 + 2x + 3 & if \quad x < 0 \\ x + 3 & if \quad 0 \leq x < 2 \\ x^2 + 4x - 7 & if \quad 2 \leq x \end{cases}$$

Write a function `tmpFn` which takes a single argument `xVec`. the function should return the vector the values of the function $f(x)$ evaluated at the values in `xVec`.
Hence plot the function $f(x)$ for $-3 < x < 3$.

```
tmpFn <- function(xVec){
  return(
    (xVec < 0) * (xVec ^ 2 + 2 * xVec + 3)
  + (xVec >= 0 & xVec < 2) * (xVec + 3)
  + (xVec >= 2) * (xVec ^ 2 + 4 * xVec - 7)
  )
}
x <- seq(-3, 3, 0.01)
plot(tmpFn, -3, 3)
```



4. Write a function which takes a single argument which is a matrix. The function should return a matrix which is the same as the function argument but every odd number is doubled.

Hence the result of using the function on the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

should be:

$$\begin{bmatrix} 2 & 2 & 6 \\ 10 & 2 & 6 \\ -2 & -2 & -6 \end{bmatrix}$$

```r
tmpFn <- function(xMat){
  return(
    (xMat %% 2 == 0) * xMat
  + (xMat %% 2 != 0) * (xMat * 2)
  )
}
xMat <- matrix(c(1, 1, 3, 5, 2, 6, -2, -1, -3), nrow = 3, byrow = TRUE)
tmpFn(xMat)
```

```
##      [,1] [,2] [,3]
## [1,]    2    2    6
## [2,]   10    2    6
## [3,]   -2   -2   -6
```

5. Write a function which takes 2 arguements n and k which are positive integers. It should return the nxn matrix:

$$\begin{bmatrix} k & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & k & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & k & 1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & k & \cdots & 0 & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \cdots & k & 1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & k \end{bmatrix}$$

```
tmpFn <- function(k, n){
  xMat <- matrix(rep(0, n ^ 2), nrow = n)
  xMat <- row(xMat) - col(xMat)
  xMat[abs(xMat) == 1] <- 1
  xMat[abs(xMat) != 1] <- 0
  xMat <- xMat + diag(k, nrow = n)
  return(xMat)
}
tmpFn(2, 5)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    2    1    0    0    0
## [2,]    1    2    1    0    0
## [3,]    0    1    2    1    0
## [4,]    0    0    1    2    1
## [5,]    0    0    0    1    2
```

6. Suppose an angle α is given as a positive real number of degrees.

If $0 \leq \alpha < 90$ then it is quadrant 1. If $90 \leq \alpha < 180$ then it is quadrant 2.
if $180 \leq \alpha < 270$ then it is quadrant3. if $270 \leq \alpha < 360$ then it is quadrant 4.
if $360 \leq \alpha < 450$ then it is quadrant 1.
And so on . . .

Write a function `quadrant(alpha)` which returns the quadrant of the angle α.

```
quadrant <- function(alpha){
  return(1 + floor((alpha %% 360) / 90))
}
```

7.

(a) Zeller's congruence is the formula:

$$f = ([2.6m - 0.2] + k + y + [y/4] + [c/4] - 2c)mod7$$

where $[x]$ denotes the integer part of x; for example $[7.5] = 7$.

Zeller's congruence returns the day of the week f given:

k = the day of the month
y = the year in the century
c = the first 2 digits of the year (the century number)
m = the month number (where January is month 11 of the preceding year, February is month 12 of the

preceding year, March is month 1, etc.)
For example, the date 21/07/1‘963 has $m = 5, k = 21, c = 19, y = 63$;
the date 21/2/63 has $m = 12, k = 21, c = 19, andy = 62$.
Write a function `weekday(day,month,year)` which returns the day of the week when given the numerical inputs of the day, month and year.
Note that the value of 1 for f denotes Sunday, 2 denotes Monday, etc.

(b) Does your function work if the input parameters day, month, and year are vectors with the same length and valid entries?

```
weekday <- function(day, month, year){
  kVec <- day
  yVec <- ifelse(
    month > 2,
    year - floor(year / 100) * 100,
    year - floor(year / 100) * 100 - 1
  )
  cVec <- floor(year / 100)
  mVec <- ifelse(month > 2, month - 2, month + 10)
  return(
    (
     floor(2.6 * mVec - 0.2)
   + kVec
   + yVec
   + floor(yVec/4)
   + floor(cVec/4)
   - 2 * cVec
    ) %% 7 + 1
  )
}
```

28/1/2018 is Sunday, output should be 1

```
weekday(28, 1, 2018)
```

```
## [1] 1
```

8.

(a) Suppose $x_0 = 1$ and $x_1 = 2$ and

$$x_j = x_{j-1} + \frac{2}{x_{j-1}} \text{ for } j = 1, 2, \dots.$$

Write a function `testLoop` which takes the single argument n and returns the first $n - 1$ values of the sequence $\{x_j\}_{j \geq 0}$: that means the values of $x_0, x_1, x_2, ..., x_{n-2}$

(b) Now write a function `testLoop2` which takes a single argument `yVec` which is a vector. The function should return

$$\sum_{j=1}^{n} e^j$$

where n is the length of `yVec`.

```
testLoop2 <- function(yVec){
  return(sum(exp(1:length(yVec))))
}
```

9.

Solution of the difference equation $x_n = rx_{n-1}(1 - x_{n-1})$, with starting value x_1

(a) Write a function `quadmap( start, rho, niter )` which returns the vector $(x_1, ..., x_n)$ where $x_k = rx_{k-1}(1 - x_{k-1})$ and
`niter` denotes n,
`start` denotes x_1, and
`rho` denotes r.
Try out the function you have written:
• for $r = 2$ and $0 < x_1 < 1$ you should get $x_n \to 0.5$ as $n \to \infty$.
• try `tmp <- quadmap(start=0.95, rho=2.99, niter=500)`
Now switch back to the Commands window and type:
`plot(tmp, type="l")`
Also try the plot `plot(tmp[300:500], type="l")`

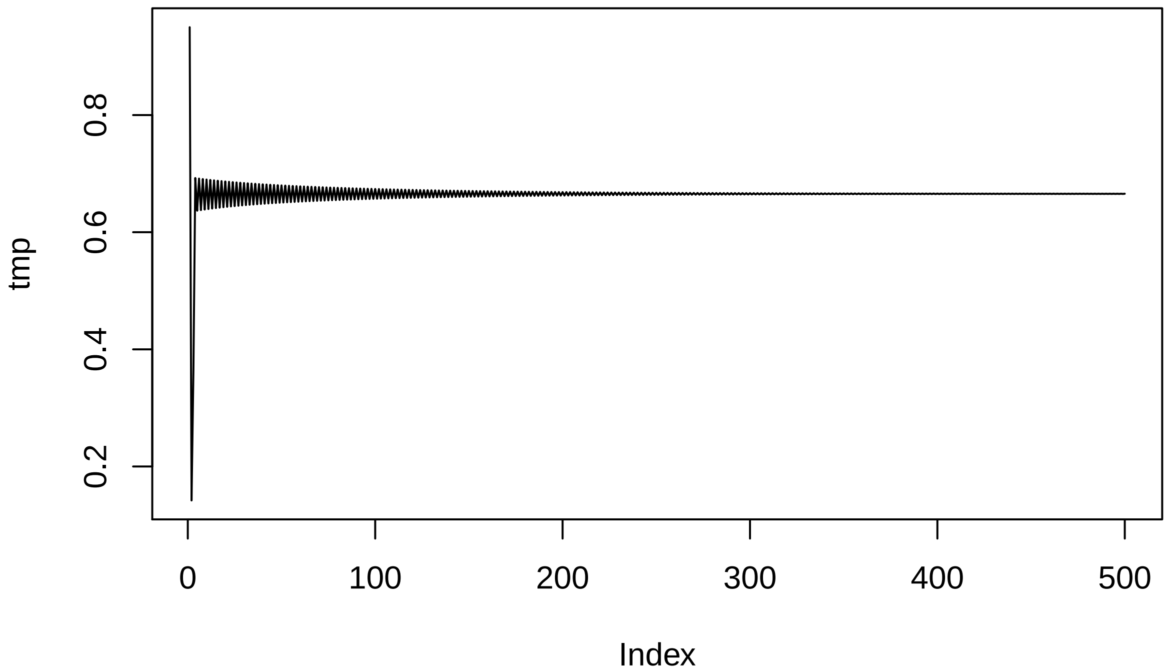
```
quadmap <- function(start, rho, niter){
  yVec <- c(start)
  loop <- 1
  while (loop != niter){
    yVec <- rbind(yVec, rho * yVec[loop] * (1 - yVec[loop]))
    loop <- loop + 1

  }
  return(yVec)
}
```

Try out

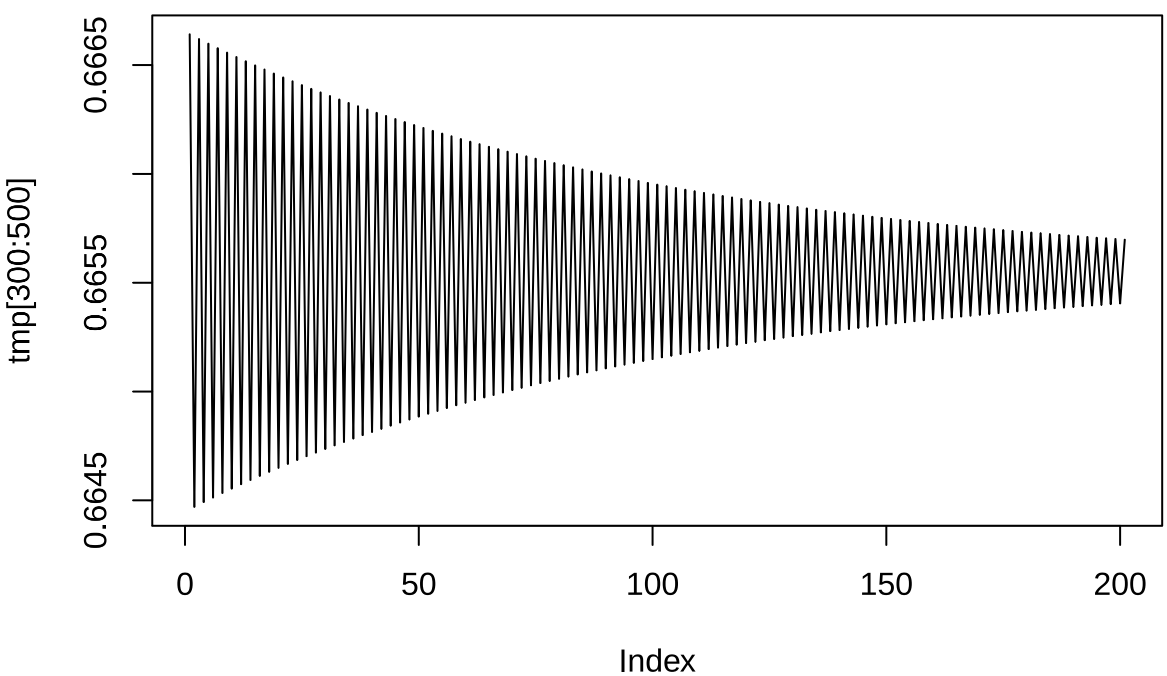
```
quadmap(0.1, 2, 9)
```

```
##           [,1]
## yVec 0.1000000
##      0.1800000
##      0.2952000
##      0.4161139
##      0.4859263
##      0.4996039
##      0.4999997
##      0.5000000
##      0.5000000
```

```
tmp <- quadmap(start=0.95, rho=2.99, niter=500)
plot(tmp, type="l")
```




```
plot(tmp[300:500], type="l")
```



(b) Now write a function which determines the number of iterations needed to get $|x_n - x_{n-1}| < 0.02$. So this function has only 2 arguments: `start` and `rho`. (For `start=0.95` and `rho=2.99`, the answer is 84.)

```
determineNumber <- function(start, rho){
  yVec <- quadmap(start, rho, 2)

  loop <- 2
  while (abs(yVec[length(yVec)] - yVec[length(yVec) - 1]) >= 0.02){
    yVec <- rbind(yVec, rho * yVec[loop] * (1 - yVec[loop]))
    loop <- loop + 1
```

```
  }
  return(length(yVec) - 1)
}
determineNumber(0.95, 2.99)
```

```
## [1] 84
```

10.

(a) Given a vector $(x_1, ..., x_n)$, the sample autocorrelation of lag k is defined to be

$$r_k = \frac{\sum_{i=k+1}^{n}(x_i - \bar{x})(x_{i-k} - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Thus

$$r_1 = \frac{\sum_{i=2}^{n}(x_i - \bar{x})(x_{i-1} - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{(x_2 - \bar{x})(x_1 - \bar{x}) + \cdots + (x_n - \bar{x})(x_{n-1} - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Write a function `tmpFn(cVec)` which takes a single argument `xVec` which is a vector and returns a `list` of two values: r_1 and r_2.

In particular, find r_1 and r_2 for the vector $(2, 5, 8, ..., 53, 56)$.

```
tmpFn <- function(xVec, k){
  xVecMean <- mean(xVec)
  lowerSum <- sum((xVec - xVecMean) ^ 2)
  upperSum <- sum(sapply((k+1):length(xVec), function(i) (xVec[i] - xVecMean) * (xVec[i-k] - xVecMean))
  return(upperSum / lowerSum)
}

tmpFn(seq(2,56,3), 1)
```

```
## [1] 0.8421053
```

```
tmpFn(seq(2,56,3), 2)
```

```
## [1] 0.6859649
```

(b) (Harder.) Generalise the function so that it takes two arguments: the vector `xVec` and an integer `k` which lies between 1 and $n - 1$ where n is the length of `xVec`.
The function should return a vector of the values $(r_0 = 1, r_1, ..., r_k)$
If you used a loop to answer part (b), then you need to be aware that much, much better solutions are possible–see exercises 4 (Hint: `sapply`.)

```
tmpFn4 <- function(xVec, k){
  outPut <- rep(0, k)
  for (i in 1:k) outPut[i] <- tmpFn(xVec, i)
  return(outPut)
}

tmpFn4(seq(2,56,3), 2)
```

```
## [1] 0.8421053 0.6859649
```