Growth and Development Economics
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Homework 2, due Friday Feb 8 at 1pm

Question 1. Praying for Rain: The Welfare Cost of Seasons.

1. Assume an economy with $n = 1,000$ households that live for 40 years (from age 16 to age 55) and that face repeteadly the same seasonal risk across years.

   Household i of age t in season m obtains a level consumption defined as the following random variable:

   $$c_{m,t} = z\left[e^{g(m)}e^{-\sigma_\varepsilon^2/2}\varepsilon_t\right]$$

   where $e^{g(m)}$ keeps track of the deterministic seasonal component of consumption common to all households. There is also an idiosyncratic nonseasonal stochastic component with $E\left[e^{-\sigma_\varepsilon^2/2}\varepsilon_t\right] = 1$ where $\ln \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. That is, at each period t (i.e., during 12 months) agents differ in ε_t. Households also differ in their permanent level of consumption, $z = e^{-\sigma_u^2/2}u$, which we draw at the begining of their lifetimes from $\ln u \sim N(0, \sigma_u^2)$. Assume that $\sigma_u^2 = \sigma_\varepsilon^2 = 0.2$. Also, assume that the seasonal component common to all households follows the deterministic process in Table 1.

   Household i lifetime utility is:

   $$W(z) = \sum_{t=1}^{40} \beta^{12t}\left[\sum_{m=1}^{12} \beta^{m-1}\left[u(c_{m,t})\right]\right]$$

   where $u(c_{m,t}) = \frac{c_{m,t}^{1-\eta}}{1-\eta}$ with $\eta = 1$. The discount factor β is such its annual value is 0.99.

   (a) Compute the welfare gains of removing the seasonal component from the stream of consumption separately for each degree of seasonality in Table 1.

   (b) Compute the welfare gains of removing the nonseasonal consumption risk.

   (c) Compare and discuss your results in (a) and (b).

   (d) Redo for $\eta = \{2, 4\}$.

2. We now add an stochastic seasonal component to consumption. We assumie that this seasonal risk is "pro-seasonal." That is, seasonal risk (i.e., the variance of logged consumption generated by seasons) increases in seasons when consumption is above its annual mean, and decreases otherwise.

   Precisely, household i in season m obtains a level consumption defined as the following random variable:

   $$c_{m,t} = z\left[e^{g(m)}e^{-\sigma_m^2/2}\varepsilon_m e^{-\sigma_\varepsilon^2/2}\varepsilon_t\right] \quad (1)$$

Table 1: Deterministic Seasonal Component, $e^{g(m)}$

Month	Degree of seasonality		
	Middle	High	Low
Jan	0.863	0.727	0.932
Feb	0.691	0.381	0.845
Mar	1.151	1.303	1.076
Apr	1.140	1.280	1.070
May	1.094	1.188	1.047
Jun	1.060	1.119	1.030
Jul	1.037	1.073	1.018
Aug	1.037	1.073	1.018
Sep	1.037	1.073	1.018
Oct	1.002	1.004	1.001
Nov	0.968	0.935	0.984
Dec	0.921	0.843	0.961
Average	1.000	1.000	1.000

Table 2: Stochastic Seasonal Component, σ_m^2

Month	Degree of seasonality		
	Middle	High	Low
Jan	0.085	0.171	0.043
Feb	0.068	0.137	0.034
Mar	0.290	0.580	0.145
Apr	0.283	0.567	0.142
May	0.273	0.546	0.137
Jun	0.273	0.546	0.137
Jul	0.239	0.478	0.119
Aug	0.205	0.410	0.102
Sep	0.188	0.376	0.094
Oct	0.188	0.376	0.094
Nov	0.171	0.341	0.085
Dec	0.137	0.273	0.068
Average	0.200	0.400	0.100

where now seasonal risk consists of a deterministic component, $e^{g(m)}$, and an stochastic component with $E\left[e^{-\sigma_m^2/2}\varepsilon_m\right] = 1$ and $\ln \varepsilon_m \sim N(0, \sigma_m^2)$.

Using the same lifetime utility as in the previous item, do the following:

(a) Compute the welfare gains of removing the seasonal component (all combinations of deterministic and stochastic) from the stream of consumption separately for each degree of seasonality in Table 1 and 2.

(b) Compute the welfare gains of removing the nonseasonal consumption risk.

(c) Compare and discuss your results in (a) and (b).

(d) Redo for $\eta = \{2, 4\}$.

Question 2. Adding Seasonal Labor Supply.

1. Assume labor supply follows an stochastic process analogous to (1). Household i lifetime utility is:

$$W(z) = \sum_{t=1}^{40} \beta^{12t} \left[\sum_{m=1}^{12} \beta^{m-1} \left[u(c_{m,t}, h_{m,t}) \right] \right]$$

where $u(c_{m,t}, h_{m,t}) = \ln c_{m,t} - \kappa \frac{h_{m,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$, with $\nu = 1.0$. Then calibrate κ to get the monthly hours that match the average weekly hours worked per adult in poor countries, as reported by Bick et al (2018), i.e, $28.5 * 30/7$.

(a) Assume a deterministic seasonal component and a stochastic seasonal component for labor supply both of which are highly positively correlated with their consumption counterparts. Then, compute the welfare gains of removing seasons isolating the effects of consumption and leisure.

(b) Assume a deterministic seasonal component and a stochastic seasonal component for labor supply both of which are highly negatively correlated with their consumption counterparts. Then, compute the welfare gains of removing seasons isolating the effects of consumption and leisure.

(c) How do your answers to (a) and (b) change if the nonseasonal stochastic component of consumption and leisure are correlated?

Question 3. Empirical Evidence [Optional]

1. Use the data on consumption and labor supply for Uganda LSMS-ISA 2009/10, 2010/11, 2011/12 and 2013/14 to identify the mean and variance of (logged) consumption and labor supply generated by seasons and by aging. To do this, specify the following model:

$$m(\ln x_{i,m,t,a}) = Const. + \beta_t \mathbf{1}_t + \beta_a \mathbf{1}_a + \beta_m \mathbf{1}_m$$

where $m(.)$ is a moment of interest, in our case mean and variance, and $x = \{c, h\}$ captures the variables of interest (either consumption or labor supply). We are also interested in the covariance of consumption and hours. Plot the estimated values for β_a and β_m for those three moments. Compute the welfare gains of removing seasons using your estimated seasonal effects for meand and variance of logged consumption and labor supply and discuss your results. Redo using information on the covariance of consumption and labor supply.