AMATH 301 Winter 2019 **Name** _____

Instructor: Benjamin Liu **NetID** _____

Practice Exam: February 13, 2019 **Student ID** _____

Time Limit: 30 Minutes

Please write your name, NetID (first part of your UW email address) and student ID number on the front of the exam

This exam contains 4 problems. You will have 25 minutes to complete it.

This is a closed book exam. You are not allowed to use notes, a calculator, your phone, or to access the internet.

You are required to show your work on each problem on this exam (except multiple choice). Points can be lost for correct answers if your work is not complete. If you need more space, please raise your hand and I will bring you extra paper. Staple any extra paper to your exam when you turn it in.

1. (a) (2 points) You need to generate a vector `v` consisting of 900 points between π and $\sqrt{71}$. Which is a better choice to generate these values, colon or linspace? Write a single line of code to produce the vector `v`.

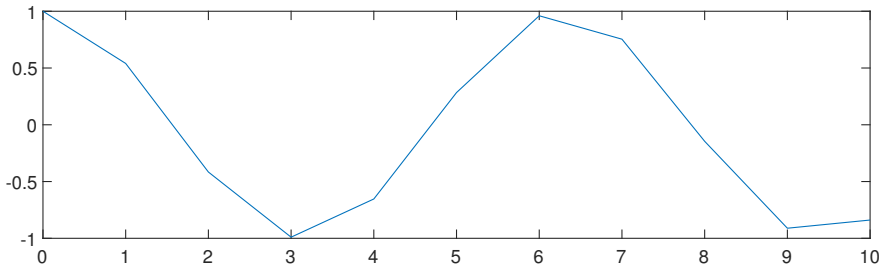
   (b) (2 points) You wish to test parameter values of α between 0 and 1. You choose to sweep from 0 up to 1, advancing by 0.005 each time. Write a single line of code to produce a vector `alphas` that contains all of the values of α that you will test.

   (c) (3 points) You need to generate a plot of the function $\cos(x)$ over the domain $[a, b]$ where a and b are integers. You reason that you will need at least ten points per unit length so that the plot of the cosine function looks smooth. Which is a better choice: colon or linspace? Write a few lines of MATLAB code to make this plot, assuming that `a` and `b` are already defined.

   (d) (3 points) A student generated a plot of cosine between $0 \leq x \leq 10$ by creating a vector `xs` in MATLAB and then executing the following

   ```
   plot(xs,cos(xs));
   ```

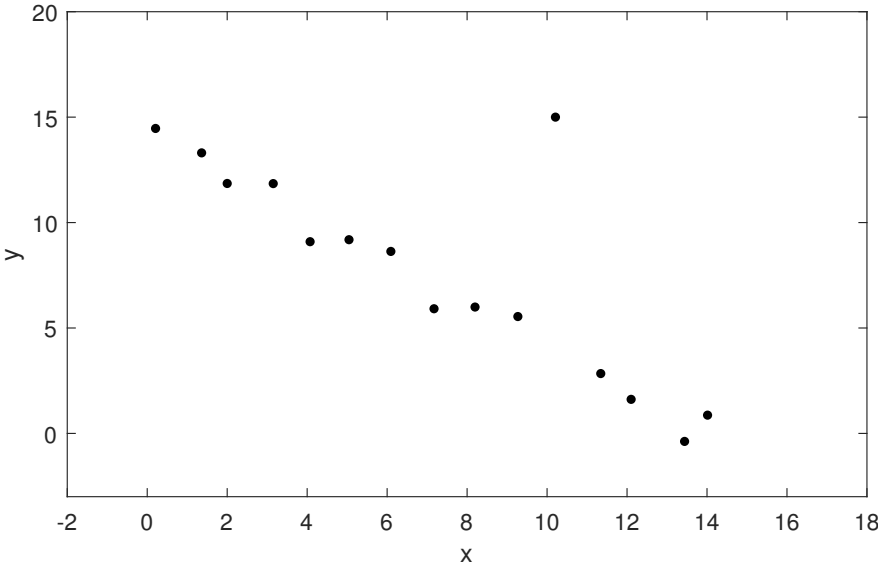
   They observe the following result,



   This figure does not look very good. Suggest a fix that will produce a smoother result.

2. (9 points) A researcher is using Successive Over-relaxation (SOR) to solve a linear system $Ax = b$. They know that the magnitude e_{50} of the error of the 50–th iterate is 10^{-2}, and that the iterates have begun to converge at a rate corresponding to the dominant eigenvalue $|\lambda_{\max}|$ of the SOR iteration matrix M. Find an expression for the iterate number k for when the magnitude of the error e_k is less than 10^{-6}. Your expression should be in terms of $|\lambda_{\max}|$.

3. A set of n data points $\{(x_k, y_k\}_{k=1}^n$ is shown plotted in the figure below.



On top of the figure, draw lines of best fit according to each of the following types of error, with the plot style given in parentheses.

(a) (3 points) Maximum or L_∞ error (dotted line)

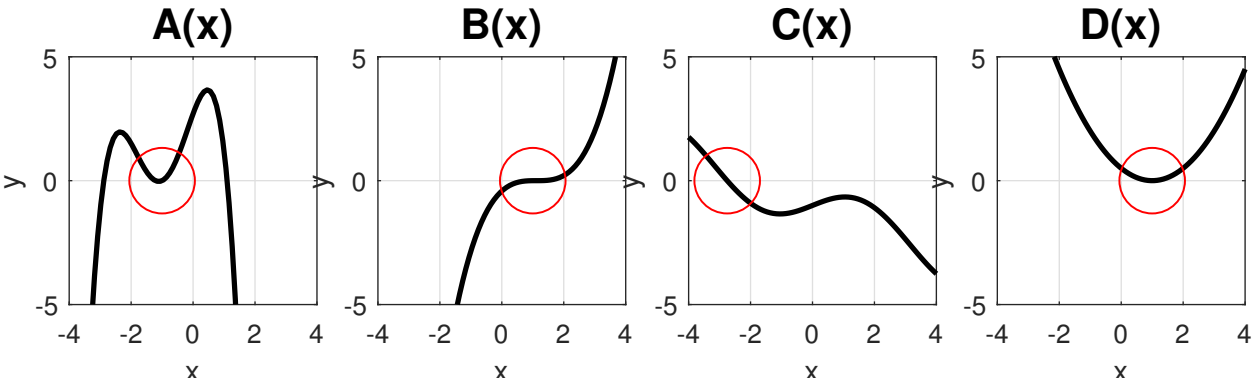
(b) (2 points) Average or L_1 error (dashed line)

(c) (2 points) Root-mean-square or L_2 error (solid line)

(d) (4 points) The data seems to have an outlier around $x = 10$. Which of lines of best fit produces the best fit to the true underlying trend that generated the data in this case?

(e) (4 points) Would linear interpolation on the data do a good job of approximating the true value near $t = 10$, by avoiding the outlier? Why or why not? Write one or two sentences and give specific reasons.

4. Consider the following functions, $A(x), B(x), C(x), D(x)$,



Each of these functions has a single root, shown circled, that we wish to find numerically using the Bisection method. State reasonable starting values that could be used to begin the Bisection method in each case. If no appropriate guess exists, or the method would not work for the function, state this clearly and write a sentence justifying this conclusion.

(a) (4 points) Function $A(x)$.

(b) (4 points) Function $B(x)$.

(c) (4 points) Function $C(x)$.

(d) (4 points) Function $D(x)$.

Problem	Points	Score
1	10	
2	9	
3	15	
4	16	
Total:	50	