AMATH 301
Homework 3
Due: Friday, January 25, 2019

Truss Bridge

Learning goal: See an example of a problem that involves a system of linear equations

Consider the truss bridge diagrammed in Figure 1. The bridge is composed of steel beams that are connected at joints or *nodes*. Each beam exerts a force, F_j, on the nodes that it touches. When the bridge is at rest, these forces balance. A positive force means that the beam is in tension: it is being pulled apart. A negative force means that the beam is in compression: being pushed inwards.

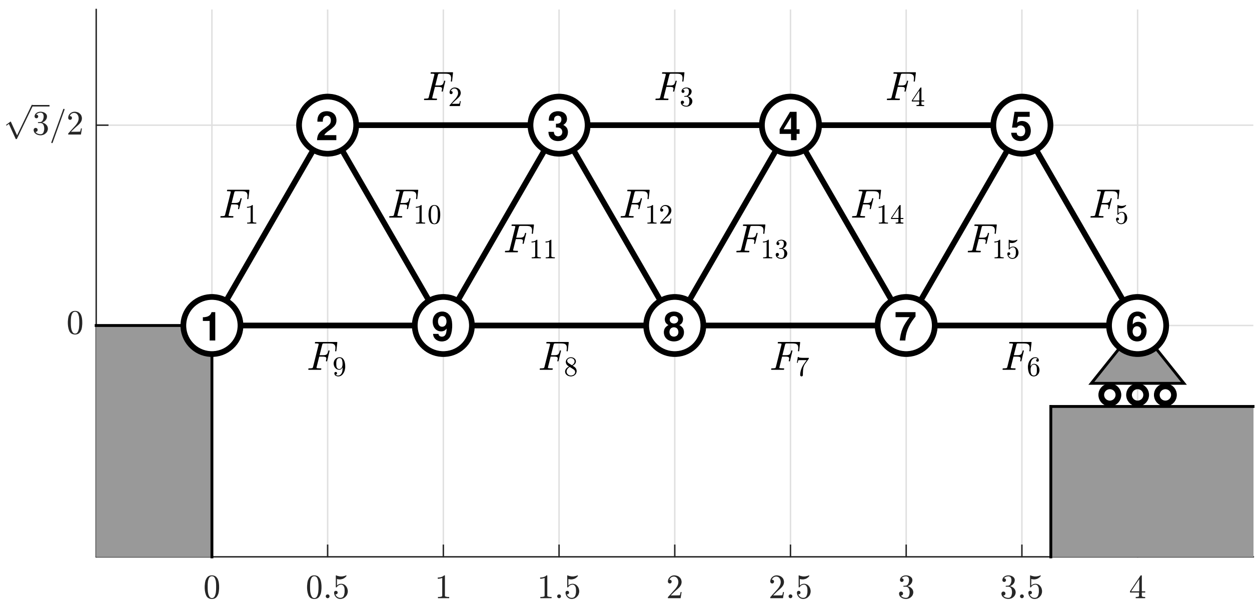


Figure 1: A truss bridge. [1]

Cars and trucks will pass over this bridge, and we would like to figure out what forces will act on the bridge as its load changes. Let W_7, W_8, W_9 be the weights, measured in Newtons, of vehicles on the bridge located at nodes $7, 8, 9$, respectively.

We can find equations for the unknown forces with Newton's second and third laws of motion. These laws yield the following fifteen equations that balance the forces on the nine nodes in the bridge that are free to move.

[1] The cart-like object supporting the right end of the bridge at $x = 4$ is a roller bearing. `https://theconstructor.org/structures/bridge-bearings-types-details/18062/`

- x- and y-forces sum to zero on node (2).

$$-\frac{1}{2}F_1 + F_2 + \frac{1}{2}F_{10} = 0 \tag{1}$$

$$-\frac{\sqrt{3}}{2}F_1 - \frac{\sqrt{3}}{2}F_{10} = 0 \tag{2}$$

- x and y-forces sum to zero on node (3)

$$-F_2 + F_3 - 0.5F_{11} + 0.5F_{12} = 0 \tag{3}$$

$$-\frac{\sqrt{3}}{2}F_{11} - \frac{\sqrt{3}}{2}F_{12} = 0 \tag{4}$$

- x and y-forces sum to zero on node (4)

$$-F_3 + F_4 - \frac{1}{2}F_{13} + \frac{1}{2}F_{14} = 0 \tag{5}$$

$$-\frac{\sqrt{3}}{2}F_{13} - \frac{\sqrt{3}}{2}F_{14} = 0 \tag{6}$$

- x and y-forces sum to zero on node (5)

$$-F_4 + \frac{1}{2}F_5 - \frac{1}{2}F_{15} = 0 \tag{7}$$

$$-\frac{\sqrt{3}}{2}F_5 - \frac{\sqrt{3}}{2}F_{15} = 0 \tag{8}$$

- x and y-forces sum to zero on node (6)

$$-\frac{1}{2}F_5 - F_6 = 0 \tag{9}$$

- x and y-forces sum to zero on node (7)

$$F_6 - F_7 - \frac{1}{2}F_{14} + \frac{1}{2}F_{15} = 0 \tag{10}$$

$$\frac{\sqrt{3}}{2}F_{14} + \frac{\sqrt{3}}{2}F_{15} - W_7 = 0 \tag{11}$$

- x and y-forces sum to zero on node (8)

$$F_7 - F_8 - \frac{1}{2}F_{12} + \frac{1}{2}F_{13} = 0 \tag{12}$$

$$\frac{\sqrt{3}}{2}F_{12} + \frac{\sqrt{3}}{2}F_{13} - W_8 = 0 \tag{13}$$

- x and y-forces sum to zero on node (9)

$$F_8 - F_9 - \frac{1}{2}F_{10} + \frac{1}{2}F_{11} = 0 \tag{14}$$

$$\frac{\sqrt{3}}{2}F_{10} + \frac{\sqrt{3}}{2}F_{11} - W_9 = 0 \tag{15}$$

Problem 1 (Scorelator).

Learning goal: Write a system of linear equations in matrix-vector form $Ax = b$.

Equations (1–15) taken together constitute a system of linear equations. Convert them (paper and pencil might be useful here) into a single matrix-vector equation of the form

$$Ax = b.$$

Preserve the order of the equations in the rows of the matrix A, and let the j–th element of the vector x correspond to F_j. Save the matrix A to file as **A1.dat**.

Problem 2 (Scorelator).

Learning goal: Solve linear systems with the backslash operator.

Solve the system for force vector x when the weights are $W_7 = 800, W_8 = 900, W_9 = 13000$ using the backslash operator "\". Save the vector x to the file **A2.dat**.

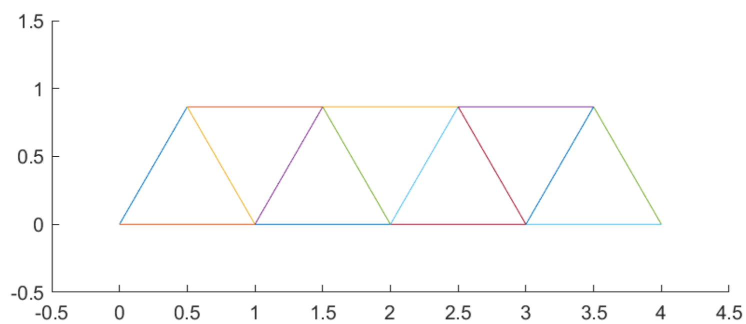
Save the largest-magnitude force experienced among all of the steel beams to file as **A3.dat**.

Hint: Look up how to find the largest element in a vector. Remember that a negative value can have larger magnitude than a positive one, so really we care about the force with largest absolute value.

Problem 3 (Writeup).

Learning goal: Use matrices to store and retrieve information, not in a linear-algebra sense.

In this problem you will write MATLAB code to create a diagram of the truss bridge like the one below. In the next problem we will use this to display the forces acting within the bridge.



Create a 9×2 matrix called `nodes` to store a list of x- and y-coordinates of the nodes in the bridge. In each row, the first column should hold the x-coordinate and the second column the y-coordinate.

Next create a 15×2 matrix `beams`. Each row will represent a beam by listing the 'starting' and 'ending' node. For example, the tenth row, representing the tenth beam, should be `[2, 9]` since the tenth beam connects the second and ninth nodes.

Begin your plotting code with the following commands. These commands will change the position of the figure window and axis scaling so that the size is consistent.

```
clf; % clear the figure window
set(gcf,'position',[20 50 600 250],'paperpositionmode','auto')
hold on
% Code to plot goes here!
axis equal; % make aspect ratio 1:1
axis([-.5 4.5 -.5 1.5]);
print(gcf,'-dpng','truss_bridge_beams.png');
```

Use a loop to iterate and draw the fifteen beams by drawing straight-line connections between each beam's endpoints. For example, to plot the twelfth beam by connecting the third and ninth nodes, we can use the code

```
mybeam = [3 9];
plot( nodes(mybeam,1), nodes(mybeam,2) );
```

In the above, the `[3 9]` indicates which nodes are at the endpoints of the beam and the `1` and `2` retrieve the first column (x-coordinate) and second column (y-coordinate) of `nodes`.

Save your plot to file as `truss_bridge_beams.png`. Note that depending on your version of MATLAB, the lines you plot may be different colors than those shown above.

Problem 4 (Writeup).

Learning goal: Construct a diagram to aid in the visual interpretation of the solution of a problem.

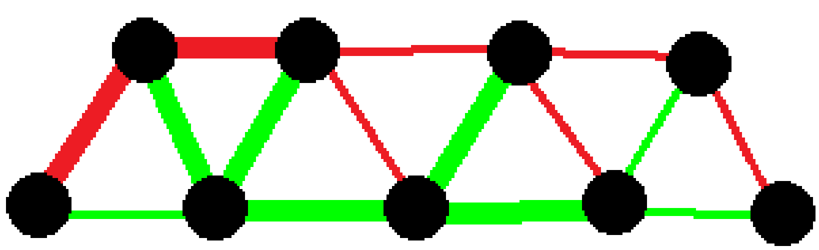
Copy your code from the previous problem. Add code to set the style of the lines (beams) to indicate the force acting on each beam from Problem 2.

1. Color the beams red for compression (when the force is negative) and green for tension (when the force is positive). You will want to use an `if` statement to decide which color to use based on the `sign` of the force F_j on the j–th beam.

2. Set the line width equal to the magnitude of the force divided by 1000. Use the `abs` command to calculate magnitude.

Finally, plot large dots at the positions of the nodes for stylistic effect. To achieve this effect, plot black dots with the marker type `'.'`, with the size of the markers set to be `80`.

A crude version of what your plot should look like is shown below. Save your plot to file as `truss_bridge_forces.png`



Problem 5 (Scorelator).

Learning goal: Use the `lu()` command to compute LU factorizations and use them to solve linear systems.

Since we are going to repeatedly solve the system $Ax = b$ in the next problem, it makes sense to use LU decomposition.

Use the `lu` command to compute the LU factorization $LU = A$ of the matrix A that you found in Problem 1 as follows,

```
[L,U,P] = lu(A);
```

Save the matrices L, U, and P into the files **A4.dat**, **A5.dat**, and **A6.dat**, respectively.

Solve the system $Ax = b$ for the same values of W_7, W_8, W_9 you used in Problem 2 using LU factorization. Save the intermediate result y (obtained through forward substitution, preceding backward substitution for x) to the file **A7.dat** and the final answer to the file **A8.dat**.

Problem 6 (Scorelator).

Learning goal: Interpret the solution of a linear system in terms of an application problem.

Now suppose we add weight to the truck parked at node 9 in increments of 0.1, starting with the weight $W_9 = 13\,000$ and increasing to some large weight until the bridge collapses. Each beam can withstand 20 000 Newtons of tensile or compressive force (positive or negative). Therefore the bridge will collapse when any one force F_j is larger than 20 000 in magnitude, or equivalently when the maximum of the absolute values of all forces is greater than or equal to 20 000.

Use a loop to iterate through the weights $W_9 = 13000.0, 13000.1, 13000.2, \ldots$ and so on. Solve the system $Ax = b$ repeatedly for the forces within the bridge, until the force on at least one of the beams exceeds the breaking point. Use the LU decomposition that you calculated in the previous problem to solve the system $Ax = b$ efficiently; if you forget to do this your code may take longer to run.

Save the first value for the weight W_9 that causes a collapse to the file **A9.dat**. Save the vector of all forces on the bridge at this time to the file **A10.dat**.

Problem 7 (Writeup).

Copy your code from Problem 4 to visualize the forces on the bridge when it is at the breaking point you found in Problem 6.

Save your plot to file as `truss_bridge_breakingpoint_forces.png`