AMATH 351 Summer 19 HW01

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Q1

Q1-a

a first order, 1-D, autonomous, linear, not homogeneous, constant-coefficient ODE

Q1-b

a first order, 3-D, autonomous, non-linear, NA, NA ODE

Q1-c

an n-th order, 1-D, non-autonomous, linear, homogeneous, not constant coefficient ODE

Q1-d

a first order, 1-D, autonomous, non-linear, NA, NA ODE

Q2

Q2-a

$$\frac{d^{10}y}{dt^{10}} = y^2$$

Q2-b

$$\frac{dx}{dt} + \frac{dy}{dt} = x + y$$

Q3

Q3-a

$$\text{Want to proof } \frac{d}{dt}(f+g) = \frac{df}{dt} + \frac{dg}{dt}$$

On LHS, by definition of differentiation,

$$\begin{aligned}\frac{d}{dt}(f+g) &= \lim_{h\to 0} \frac{f(t+h)+g(t+h)-(f(t)+g(t))}{h} \\ &= \lim_{h\to 0}(\frac{f(t+h)-f(t)}{h} + \frac{g(t+h)-g(t)}{h})\end{aligned}$$

By definition of differentiation,

$$\begin{aligned}\lim_{h\to 0} \frac{f(t+h)-f(t)}{h} &= \frac{df}{dt} \\ \lim_{h\to 0} \frac{g(t+h)-g(t)}{h} &= \frac{dg}{dt}\end{aligned}$$

This proved that

$$\frac{d}{dt}(f+g) = \frac{df}{dt} + \frac{dg}{dt}$$

Q3-b

$$\text{Want to proof } \frac{d^n}{dt^n}(f+g) = \frac{d^n f}{dt^n} + \frac{d^n g}{dt^n}$$

□