AMATH 352 Fall 2019
Practice Final Exam

Friday, December 12

This exam is scheduled for 110 minutes. Show all your work and provide full reasoning. Credit will not be given if your work is not shown. No notes, external scratch paper or calculators are allowed. Use answer boxes when they are provided. If you wish to use the restroom, you must leave your phone at the front of the classroom first.

Name & ID #: ______________________________

Problem	Points	Score
1	?	
2	?	
3	?	
4	?	
5	?	
6	?	

Total	

1. For the following statements decide whether they are true or false. If false, provide a counterexample. If true, explain your reasoning.

   (a) Suppose A is an $m \times n$ matrix with $m > n$ and $\vec{b} \in \mathbb{R}^m$. Then solving the normal equations will always give the least-squares solution of $A\vec{x} = \vec{b}$ for any such matrix A and vector $\vec{b}$.

   True ☐ False ☐

   (b) Every basis of $\mathbb{R}^6$ consists of 6 vectors.

   True ☐ False ☐

(c) If $A\vec{x} = \vec{b}$ has a solution for every $\vec{b}$ then A has an LU factorization.

True ☐ False ☐

(d) The following is a subspace of $\mathbb{R}^3$

$$S = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} : x_1 + x_2 + x_3 = 1 \right\}.$$

True ☐ False ☐

2. Find the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}.$$

$$\lambda_1 = \square \quad \vec{v}_1 = \begin{pmatrix} \square \\ \square \end{pmatrix} \quad \lambda_2 = \square \quad \vec{v}_2 = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

3. Find the determinant of A:

$$A = \begin{pmatrix} 2 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

$$\boxed{\det A = \square}$$

4. Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 3 & 1 & 2 \\ 2 & 0 & 2 & 3 \end{bmatrix}.$$

Find a basis $\mathcal{B}_{\mathrm{K}}$ for the kernel of A and find a basis $\mathcal{B}_{\mathrm{I}}$ for the image of A.

$$\mathcal{B}_{\mathrm{K}} = \left\{ \qquad \right\}, \mathcal{B}_{\mathrm{I}} = \left\{ \qquad \right\}$$

5. Solve for the least-squares solution of $A\vec{x} = \vec{b}$ where

$$A = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ 2 & 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

$$\boxed{\vec{x} = \begin{pmatrix} \square \\ \square \end{pmatrix}}$$

6. Find the value of $\vec{y}$ that minimizes

$$\|A\vec{y} - \vec{b}\|_2^2 + \|\vec{y}\|_2^2,$$

when

$$A = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \end{pmatrix}.$$

$$\boxed{\vec{y} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}}$$

7. For $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ find a matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

$$P = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}, D = \begin{bmatrix} \square & 0 & 0 \\ 0 & \square & 0 \\ 0 & 0 & \square \end{bmatrix}$$

8. Find the singular values of

$$A = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ 2 & 1 \end{pmatrix}.$$

$\sigma_1 = \square \qquad \sigma_2 = \square$