AMATH 352 Fall 2019 Practice Final Exam

Friday, December 12

This exam is scheduled for 110 minutes. Show all your work and provide full reasoning. Credit will not be given if
your work is not shown. No notes, external scratch paper or calculators are allowed. Use answer boxes when they are
provided. If you wish to use the restroom, you must leave your phone at the front of the classroom first.

Name & ID #: _____

Problem	Points	Score
1	?	
2	?	
3	?	
4	?	
5	?	
6	?	

Total

1.	For the following statements decide whether they are true or false. If false, provide a counterexample. If true, explain your reasoning.
	(a) Suppose A is an $m \times n$ matrix with $m > n$ and $\vec{b} \in \mathbb{R}^m$. Then solving the normal equations will always give the least-squares solution of $A\vec{x} = \vec{b}$ for any such matrix A and vector \vec{b} . True \Box False \Box
	(b) Every basis of \mathbb{R}^6 consists of 6 vectors. True \square False \square

(c) If $A\vec{x} = \vec{b}$ has a solution for every \vec{b} then A has an LU factorization.

True
False

(d) The following is a subspace of \mathbb{R}^3

$$S = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} : x_1 + x_2 + x_3 = 1 \right\}.$$

True False

2. Find the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}.$$

$$\lambda_1 = \square \quad ec{v}_1 = \left(egin{matrix} \square \ \square \end{matrix}
ight) \quad \lambda_2 = \square \quad ec{v}_2 = \left(egin{matrix} \square \ \square \end{matrix}
ight)$$

3. Find the determinant of A:

$$A = \begin{pmatrix} 2 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

4. Let

$$A = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 3 & 1 & 2 \\ 2 & 0 & 2 & 3 \end{array} \right].$$

Find a basis \mathcal{B}_{K} for the kernel of A and find a basis \mathcal{B}_{I} for the image of A.

$$\mathcal{B}_{ ext{K}} = \left\{ egin{array}{c} \mathcal{B}_{ ext{I}} = \left\{ egin{array}{c} = \left\{ egin{array}{c} \mathcal{B}_{ ext{I}} = \left\{ egin{array}{c} = \left\{ e$$

5. Solve for the least-squares solution of $A\vec{x} = \vec{b}$ where

$$A = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ 2 & 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

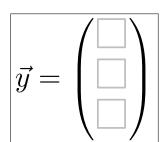
$$\vec{x} = \begin{pmatrix} \Box \\ \Box \end{pmatrix}$$

6. Find the value of \vec{y} that minimizes

$$\|A\vec{y} - \vec{b}\|_2^2 + \|\vec{y}\|_2^2,$$

when

$$A = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \end{pmatrix}.$$



7. For $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ find a matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

$$P = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}, D = \begin{bmatrix} \square & 0 & 0 \\ 0 & \square & 0 \\ 0 & 0 & \square \end{bmatrix}$$

8. Find the singular values of

$$A = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ 2 & 1 \end{pmatrix}.$$