AMATH 383 HW 3

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Table 3.3

Year	Balance (9 decimals)	Balance (10 decimals)
0	58000.000	58000.0000
1	221908.000	221908.0000
2	739902.518	739902.5186
3	1317242.863	1317242.8633
4	63585.171	63585.1704
5	242211.462	242211.4599
6	792846.671	792846.6656
7	1285569.152	1285569.1569
8	184212.474	184212.4560
9	635047.189	635047.1371
10	1330333.959	1330333.9493
11	11970.508	11970.5472
12	47452.152	47452.3067
13	183053.487	183054.0625
14	631688.210	631689.8806
15	1329662.856	1329663.2066
16	14641.492	14640.0974
17	57922.848	57917.3922
18	221626.223	221606.2958
19	739150.343	739097.1321
20	1317571.683	1317594.8163
21	62301.312	62210.9653
22	237560.887	237233.2485
23	780938.022	780094.1514
24	1294159.505	1294735.9504
25	152091.546	149920.2578
26	538970.668	532252.7801

27	1284414.529	1279132.0546
28	188496.069	207991.7790
29	647391.971	702185.3756
30	1332218.791	1329548.5972
31	4454.442	15095.9718
32	17758.241	59700.2221
33	70086.898	228108.5388
34	265611.072	756333.6387
35	850796.563	1309212.8357
36	1231621.877	94736.5953
37	375810.164	352021.3137
38	1079540.817	1036328.2389
39	821938.141	923384.2993
40	1261005.641	1135621.5046
41	273616.884	673577.4132
42	869868.938	1333190.0580
43	1209459.844	573.0397
44	449460.033	2291.1736
45	1191797.168	9148.9459

R code for reproducing chaotic bank balance

```
yr <- 45
K <- 1
p0 <- 0.058
rt <- 3
p_set_9 <- numeric(yr)
p_set_9[1] <- p0
p_set_10 <- numeric(yr)
p_set_10[1] <- p0

# keep decimal to given level
keep_decimal <- function(origin_num, decimal_place) {
  temp_num <- floor(origin_num * 10^(decimal_place))
  return(temp_num / 10^(decimal_place))
}

# calculate balance at t + delta t
interest_cal <- function(curr_p, decimal_place) {
  new_p <- curr_p + rt * curr_p * (1 - curr_p)
  return(keep_decimal(new_p, decimal_place))
```

```
}

for (i in 1:yr) {
  new_p_9 <- interest_cal(p_set_9[i], 9)
  p_set_9[i+1] <- new_p_9

  new_p_10 <- interest_cal(p_set_10[i], 10)
  p_set_10[i+1] <- new_p_10
}

result <- data.frame(0:45, p_set_9 * 1000000, p_set_10 * 1000000)
names(result) <- c('Year', 'Balance (9 decimals)', 'Balance (10 decimals)')
print(result, row.names = F)
```

Exercise 2.4

a

Plug in equations $m = m_c N_c$, $Y = y_0 m^{\frac{3}{4}}$ into differential equation.

$$\frac{dN_c}{dt} = \frac{dN_c}{dm} \cdot \frac{dm}{dt} = \frac{1}{m_c}\frac{dm}{dt}$$

$$\begin{aligned} Y &= Y_c N_c + E_c \frac{dN_c}{dt} \\ y_0 m^{\frac{3}{4}} &= Y_c N_c + E_c \frac{1}{m_c}\frac{dm}{dt} \\ \frac{dm}{dt} &= (\frac{y_0 m_c}{E_c}) m^{\frac{3}{4}} - (\frac{Y_c}{E_c}) m \end{aligned}$$

b

Find M by setting $\frac{dm}{dt} = 0$

$$\begin{aligned} \frac{dm}{dt} &= aM^{\frac{3}{4}} - bM = 0 \\ aM^{\frac{3}{4}} &= bM \\ M &= (\frac{a}{b})^4 \end{aligned}$$

Then $b = \frac{a}{M^{\frac{1}{4}}}$, plug in equation of b into differential equation.

$$\frac{dm}{dt} = am^{\frac{3}{4}} - \frac{a}{M^{\frac{1}{4}}} m = am^{\frac{3}{4}}(1 - (\frac{m}{M})^{\frac{1}{4}})$$

c

Solve the separable differential equation.

$$\begin{aligned}
\frac{dR}{dt} &= -(\frac{a}{M^{1/4}})R \\
\int_0^t \frac{1}{R}dR &= \int -(\frac{a}{M^{1/4}})dt \\
ln(R(t)) - ln(R(0)) &= -(\frac{a}{M^{1/4}})t \\
ln(R(t)/R(0)) &= -(\frac{at}{M^{1/4}}) \\
ln(\frac{R(t)}{R(0)})/(\frac{at}{M^{1/4}}) &= -1
\end{aligned}$$

This shows slope of of $ln(\frac{R(t)}{R(0)})$ vs $(\frac{at}{M^{1/4}})$ is equal to -1

d

Previous problem shows that $ln(\frac{R(t)}{R(0)}) = -\frac{at}{M^{1/4}}$, where a is a constant. Therefore, for some given time t, the interval between heartbeats depends on $M^{\frac{1}{4}}$.