12

El Niño and the Southern Oscillation

Mathematics introduced:
advection equation and its finite difference approximation

12.1 Introduction

The 1997–1998 El Niño was the most severe event of its kind on record, eclipsing even the 1982–1983 "El Niño of the century." It deranged atmospheric weather patterns around the world, killed over 2,000 people, and was responsible for \$33 billion in property damage worldwide. Curt Suplee, a *Washington Post* science writer, wrote in the February 1999 issue of *National Geographic*:

> It rose out of the tropical Pacific in late 1997, bearing more energy than a million Hiroshima bombs.... Peru was where it all began, but El Niño's abnormal effects on the main components of climate—sunshine, temperature, atmospheric pressure, wind, humidity, precipitation, cloud formation, and ocean currents—changed weather patterns across the equatorial Pacific and in turn around the globe. Indonesia and surrounding regions suffered months of drought. Forest fires burned furiously in Sumatra, Borneo, and Malaysia, forcing drivers to use their headlights at noon. The haze traveled thousands of miles to the west into the ordinarily sparkling air of the Maldives Islands, limiting visibility to half a mile at times. Temperature reached 108°F in Mongolia; Kenya's rainfall was 40 inches above normal; central Europe suffered a record flooding that killed 55 in Poland and 60 in the Czech Republic; and Madagascar was battered with monsoons and cyclones. In the U.S. mudslides and flashfloods flattened communities from California to Mississippi, storms pounded the Gulf Coast, and tornadoes ripped Florida.

Some of these individual disasters may just be coincidental, but El Niño and the associated abnormal floods and droughts were actually predicted by a simple climate model months in advance.

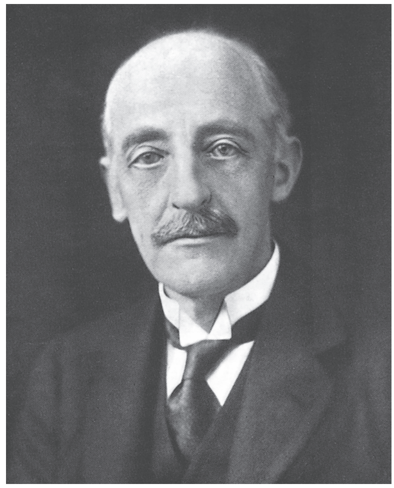

Figure 12.1. Sir Gilbert Walker. (Courtesy of Eugene M. Rasmusson, University of Maryland.)

El Niño is now defined as an anomalous warming of the surface waters of the tropical eastern Pacific Ocean—from the South American coast to the International Date Line—that persists for three or more seasons. The name actually originated in the 18th century with Spanish sea captains and, later, Peruvian fishermen, and referred to the weaker and far more benign seasonal warming of the ocean surface off the coast of Peru near Christmas—hence the name "El Niño," or "The Child."

Sir Gilbert Walker (Figure 12.1) arrived in India in 1904 as the Director General of the Observatories of India following the devastating monsoon of 1899 (an El Niño year) and the associated famine. In an attempt to predict monsoon failures in India, Walker looked for indicators and correlates all over the globe. In 1920 he discovered the "Southern Oscillation (SO)," which he defined as the pressure difference between Tahiti in the central equatorial Pacific and Darwin, Australia, in the western equatorial Pacific. Remarkably, pressure measurements from these two stations, separated by thousands of kilometers of ocean, rise and fall in a see-saw pattern. When pressure at Tahiti is high, that at Darwin is low, and vice versa. Normally, Tahiti's pressure was higher than Darwin's, but when this Southern Oscillation index became notably weak in some years, Walker found that there would be heavy rainfall in the central Pacific, drought in India, warm winters in southwestern Canada, and cold winters in the southeastern United States. Walker's work was ignored for decades until Jacob Bjerknes (Figure 12.3) advanced a conceptual model in 1969 that tied the Southern Oscillation in the atmosphere to the El Niño in the equatorial ocean. The coupled atmosphere–ocean

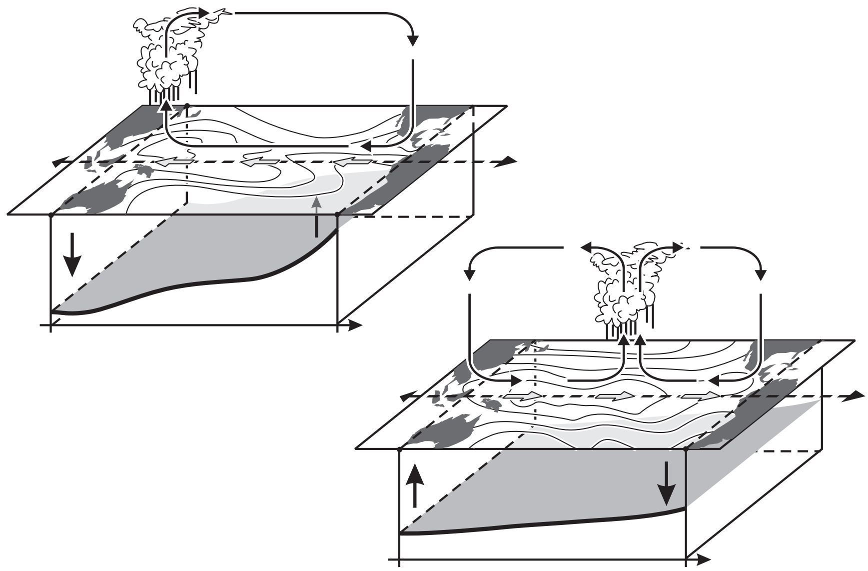

Figure 12.2. Top panel: Normal conditions in the equatorial Pacific. Lower panel: El Niño conditions. The thick line in the ocean box denotes the thermocline, separating cold deep water from the warmer ocean surface water. (Drawing by Wm. Dickerson.)

phenomenon is now referred to as El Niño–Southern Oscillation, or ENSO for short. Mark Cane and his student Stephen Zebiac constructed a simple coupled atmosphere–ocean model in the mid 1980s (Zebiac's MIT Ph.D. thesis in 1984; Zebiac and Cane, 1987). That model for the first time predicted the onset of the 1997–1998 El Niño, a remarkable achievement in climate prediction. (Recall Lorenz's conclusion that weather cannot be predicted in detail more than two weeks in advance!)

12.2 Bjerknes' Hypothesis

The waters of equatorial oceans are usually cold at depth and warm near the surface, where they are heated by the sun. The warm and cold waters are separated by a sharp transition region called the thermocline. See Figure 12.2. Under normal conditions, the trade wind over the equatorial Pacific is easterly; i.e., it blows from east to west. The easterly trade wind tends to blow the warm surface water to the western boundary of the Pacific ocean basin, where it is piled up. The sea level in the Philippines is usually 60 cm (23 inches) higher than that near Panama. The western ocean is also warmer. In fact, the western Pacific has the warmest ocean surface on earth, as high as 31.5°C (89°F). The thermocline here is about 100 to 200 m (330–660 ft) deep. Near the east coast of the ocean, the coast of Peru, the depth of the warm upper layer

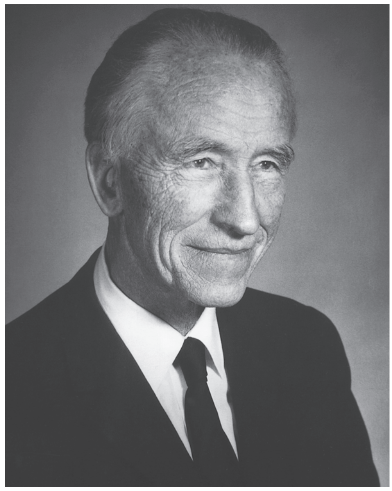

Figure 12.3. Professor Jacob Bjerknes of UCLA. (Courtesy of Eugene M. Rasmusson, University of Maryland.)

is shallower, about 40 m (130 ft) deep. Thus the cold water below the thermocline is closer to the surface (see Figure 12.2, top). Since the deep water is full of nutrients, the east coast of the Pacific supports a rich variety of fish, the birds that feed on the fish, and a thriving fertilizer industry dependent on bird droppings.

The trade winds, in turn, are driven by the sea-surface temperature patterns. In the tropics, the region of heavy precipitation and convection in the atmosphere—with the associated ascending motion—tends to form over the warmer waters, and sinking motion tends to occur over the cooler ocean waters. Consequently, there is a rising motion due to convection near Indonesia in the western Pacific and sinking motion over the eastern Pacific near Peru. Completing the atmospheric circulation is a surface wind from the east to the west. Bjerknes named this atmospheric circulation pattern the *Walker circulation*, in honor of Sir Gilbert Walker.

Bjerknes' hypothesis is that El Niño is a breakdown of these mutually reinforcing atmosphere–ocean patterns. If for some reason the easterly trade winds slacken, then the warm surface waters from the west would slosh back to the east coast, depressing the thermocline there and causing a warming of the eastern Pacific. When the east–west ocean temperature contrast is reduced, the strength of the Walker circulation is reduced, the trade winds weaken further and may even reverse, and this further acts to warm the eastern Pacific (see Figure 12.2, bottom). The warming of the eastern Pacific is called the El Niño, and the associated atmospheric connection is the *Southern Oscillation*.

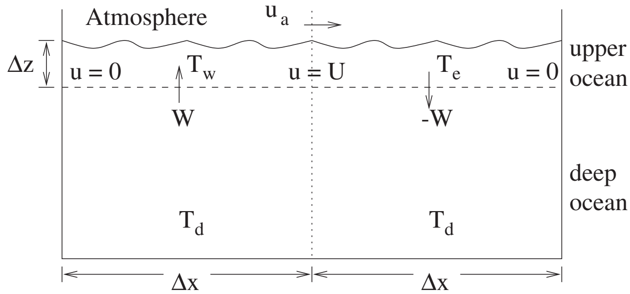

Figure 12.4. A simple model of the equatorial Pacific.

Bjerknes' model is not actually a mathematical model of ENSO but a conceptual one. Nevertheless, it has most of the ingredients for a mathematical model, but not being expressed in a mathematical form, it produces no quantitative results that can be used for verification against observation. Furthermore, the conceptual model cannot be used for prediction, as it does not explain what causes the transition from the normal state of affairs to an El Niño event and vice versa.

This "chicken and egg" problem is common to coupled oscillations. If it can be shown that the ENSO interaction *is* a coupled oscillation, then it is not necessary to enquire about an external cause for the start of an El Niño event. Nor is it necessary to attribute the eastward warming of the ocean surface to the weakening of the trade winds, or, conversely, attribute the weakening trade winds to the warming of the eastern Pacific. Furthermore, since the occurrence of ENSO is rather irregular, and probably chaotic, it would be nice if the model exhibited similar behavior in its oscillations. A simple model with all these attributes is described below. It couples the Walker circulation to the east–west sea-surface temperature difference, the latter being in turn driven by the trade winds of the Walker circulation. And, since the movement of surface waters would also involve an ocean circulation, a mass exchange with the deep water must occur.

12.3 A Simple Mathematical Model of El Niño

We discuss here a simple mathematical implementation of Bjerknes' conceptual model, by G. K. Vallis (1988). See Figure 12.4.

The Atmosphere

Let u_a be the surface wind in the atmosphere, positive if from the west. It is driven by the east–west pressure difference in the atmosphere, i.e., the SO, which is in turn driven by the east–west temperature difference, convection, and the Walker circulation. In this simple model, it is assumed that it is driven by the east–west temperature difference, $(T_e - T_w)$, and that it will relax back to some "normal" equatorial easterlies ($u_0 \leq 0$, specified from observation of "normal" conditions) in the absence of the temperature difference. Thus:

$$\boxed{\frac{du_a}{dt} = b(T_e - T_w) + r(u_0 - u_a),} \tag{12.1}$$

where r is the rate at which u_a relaxes to u_0 in the absence of coupling with the ocean temperature. The parameter b is the rate at which the ocean is influencing the atmosphere.

Since the ocean changes slowly and the atmosphere responds to oceanic changes rapidly, it is a good assumption that the atmosphere is in "quasi-equilibrium" with the ocean. Thus, at the time scale appropriate to the ocean, $\frac{d}{dt}u_a \sim 0$ (i.e., the atmosphere has already reached a steady state given an ocean temperature because the latter changes slowly):

$$0 \cong b(T_e - T_w) - r(u_a - u_0).$$

Solving for u_a:

$$\boxed{u_a = \frac{b}{r}(T_e - T_w) + u_0.} \tag{12.2}$$

So the atmosphere is "known" if the ocean temperature is known.

Air–Sea Interaction

Let U be the surface ocean current at midbasin; it is dragged by the wind stress at the air–sea interface. We assume that it is driven by the surface wind u_a:

$$\boxed{\frac{dU}{dt} = Du_a - CU.} \tag{12.3}$$

In the absence of surface wind, U will relax to 0 with a time scale of C^{-1}.

Substituting u_a from (12.2) into the above equation, we get an ordinary differential equation for the ocean current U:

$$\boxed{\frac{dU}{dt} = B(T_e - T_w) - C(U - U_0),} \tag{12.4}$$

where $B \equiv Db/r$, $U_0 \equiv Du_0/C$.

Ocean Temperature Advection

The equation describing the change in temperature due to advection of a warmer or colder temperature from elsewhere by a current u in the x-direction and a current w in the z-direction is (see the appendix to this chapter, section 12.5, for a derivation of the advection equation):

$$\frac{\partial T}{\partial t} + u\frac{\partial}{\partial x}T + w\frac{\partial}{\partial z}T = 0.$$

We use a very crude finite difference scheme to turn this partial differential equation into ordinary differential equations.

Let W be the vertical velocity in the western box across the thermocline. To conserve mass flux across the thermocline, it must be $-W$ in the eastern box (see Figure 12.4).

Consider the eastern box:

$$\frac{\partial T_e}{\partial t} + U\frac{T_e - T_w}{\Delta x} + (-W)\frac{T_e - T_d}{\Delta z} = 0;$$

and the western box:

$$\frac{\partial}{\partial t}T_w + U\frac{T_e - T_w}{\Delta x} + W\frac{T_w - T_d}{\Delta z} = 0.$$

(Recall that T_d is the temperature of the deep water; see Figure 12.4.) Mass conservation considerations relate U and W. The density of sea water is considered as approximately constant. The mass flux is the constant density times the velocity. Conservation of mass then implies $W\Delta x = U\Delta z$, or

$$W = U\Delta z/\Delta x.$$

The temperature advection equation then becomes

$$\frac{\partial T_e}{\partial t} = \frac{U}{\Delta x}(T_w - T_d),$$

$$\frac{\partial T_w}{\partial t} = \frac{U}{\Delta x}(T_d - T_e).$$

To the right-hand sides we add a relation to a prescribed temperature T_0 in the absence of temperature advection. That is, in the absence of an east–west temperature difference, the above equation will not determine the ocean surface temperature. That temperature is supposedly determined by processes not in our model. A simple way to deal with this problem is to specify that temperature T_0 to which the temperatures will relax with a rate A. T_0 is determined by heat transfers (radiative and diffusive) from the upper ocean to the atmosphere and to the deep ocean.

$$\boxed{\begin{aligned}\frac{\partial T_e}{\partial t} &= \frac{U}{\Delta x}(T_w - T_d) - A(T_e - T_0),\\ \frac{\partial T_w}{\partial t} &= \frac{U}{\Delta x}(T_d - T_e) - A(T_w - T_0).\end{aligned}} \tag{12.5}$$

Without loss of generality we set $T_d = 0$ (we can always measure temperature relative to that of the deep water). The equations are made dimensionless by letting

$$\begin{aligned} x &= U/(A\Delta x),\\ y &= (T_e - T_w)/(2T_0),\\ z &= 1 - (T_e + T_w)/(2T_0),\\ \hat{t} &= At. \end{aligned}$$

Then

$$\boxed{\text{ENSO}\begin{cases}\dfrac{dx}{d\hat{t}} = \sigma y - \rho(x - x_0),\\[2mm] \dfrac{dy}{d\hat{t}} = x - xz - y,\\[2mm] \dfrac{dz}{d\hat{t}} = xy - z,\end{cases}} \tag{12.6}$$

where

$$x_0 = U_0/(A\Delta x), \sigma = 2BT_0/(\Delta x A^2), \text{ and } \rho = C/A.$$

This set is comparable to the Lorenz equations if $x_0 = 0$:

$$\text{Lorenz} \begin{cases} \dfrac{dx}{dt} = -\sigma x + \sigma y, \\ \dfrac{dy}{dt} = rx - xz - y, \\ \dfrac{dz}{dt} = xy - bz. \end{cases} \tag{12.7}$$

In fact, the ENSO set becomes the Lorenz set under the following rescaling (for the case $x_0 = 0$):

$$x = \alpha\hat{x}, \quad y = \beta\hat{y}, \quad z = \zeta\hat{z},$$

$$\alpha\zeta/\beta = 1, \quad \beta/\alpha = \rho/\sigma, \quad \alpha = 1,$$

$$\text{rescaled ENSO} \begin{cases} \dfrac{d\hat{x}}{d\hat{t}} = \rho\hat{y} - \rho\hat{x}, \\ \dfrac{d\hat{y}}{d\hat{t}} = (\sigma/\rho)\hat{x} - \hat{x}\hat{z} - \hat{y}, \\ \dfrac{d\hat{z}}{d\hat{t}} = \hat{x}\hat{y} - \hat{z}. \end{cases}$$

We therefore identify σ/ρ with Lorenz's r and ρ with Lorenz's σ and set Lorenz's b to 1. We will, however, discuss the results below using the original (unscaled) variables x, y, z, which are more physical.

The case of $x_0 = 0$ will be considered first along the lines used previously to study the Lorenz equations. Just as in the Lorenz equations, $(x, y, z) = (0, 0, 0)$ could be the solution. This would have implied that there is no coupled atmosphere–ocean oscillation, and thus no ENSO. It is possible, however, that the parameters in the equatorial Pacific are such that the trivial solution is not attainable because it is unstable. We will look into the stability issue next.

Using analogy with the Lorenz equations, we see that for $r \equiv \sigma/\rho < 1$, the trivial solution

$$P_1 = (x_1^*, y_1^*, z_1^*) = (0, 0, 0)$$

is the only realizable equilibrium. Physically, this solution represents the case when there is no east–west temperature difference in the ocean, no ocean advection, no trade wind, and the surface temperature in the ocean equals T_0.

For $r \equiv \sigma/\rho > 1$, there are three equilibria—P_1, plus two new solutions:

$$P_2 = \left((r-1)^{1/2},\ \left(1/r\left(1-\frac{1}{r}\right)\right)^{1/2},\ 1-1/r\right),$$

$$P_3 = \left(-(r-1)^{1/2},\ -\left(\frac{1}{r}\,(1-1/r)\right)^{1/2},\ 1-1/r\right).$$

P_1 becomes unstable. P_2 and P_3 are stable until $r > r_c$. From Lorenz equation results in chapter 11, all three equilibria lose stability when

$$\sigma > \sigma_c \equiv \frac{(4+\rho)\rho^2}{\rho-2}.$$

For the Pacific Ocean, $\Delta x = 7{,}500\,\mathrm{km}$, $T_0 \sim 15^\circ$ (as measured from T_d), the frictional decay rate $C \sim 1/(2\ \text{months})$, and the temperature decay rate is $A \sim 1/(6\ \text{months})$. If $B\Delta x \sim 12\mathrm{m}^2\mathrm{s}^{-2}C^{-1}T_0^{-1}$, then $\rho \sim 3$, $\sigma_c \sim 63$, and $\sigma \sim 102$.

Therefore the system could very well be in the unstable regime. Drawing upon what we know about the Lorenz system, which possesses irregular oscillations when all three of its equilibria become unstable, we anticipate that Vallis's system also produces irregular oscillations ranging from El Niño, to normal, to perhaps La Niña (anti–El Niño) events. Whether such oscillations bear any resemblance to the observed ones can be revealed by displaying the solution numerically, using MATLAB. The MATLAB code is given in Appendix B.

Figure 12.5 displays the east–west temperature difference (in terms of $y(t) \equiv (T_e - T_w)/2T_0$) as a function of time (in terms of $\hat{t} = At$). Since A, the thermal relaxation rate of the ocean, is taken to be 1/(6 months), each two units of time plotted correspond to one year. The top panel of Figure 12.5 shows $y(t)$ for 30 years for the case of $U_0 = 0$. The bottom panel is for $U_0 = -0.45\,\mathrm{m/s}$. Both figures show an irregular oscillation, which is self-sustained (i.e., without the need for external anomalous forcing). Without a prevailing easterly pushing the warm water westward, it is equally likely to have El Niño (the extreme warming of the eastern Pacific) as La Niña (the extreme cooling of the eastern Pacific). This symmetry is broken when there is a prevailing easterly, as shown in the bottom panel of Figure 12.5. In this case, on average, T_e is colder than T_w; hence the average of $y(t)$ in time is negative. El Niño is the large positive deviation from the time average, while La Niña is the large negative deviation. El Niño is more frequent than La Niña. Furthermore, the interval between El Niño events becomes longer, and closer to the observed situation, when there is a prevailing easterly wind.

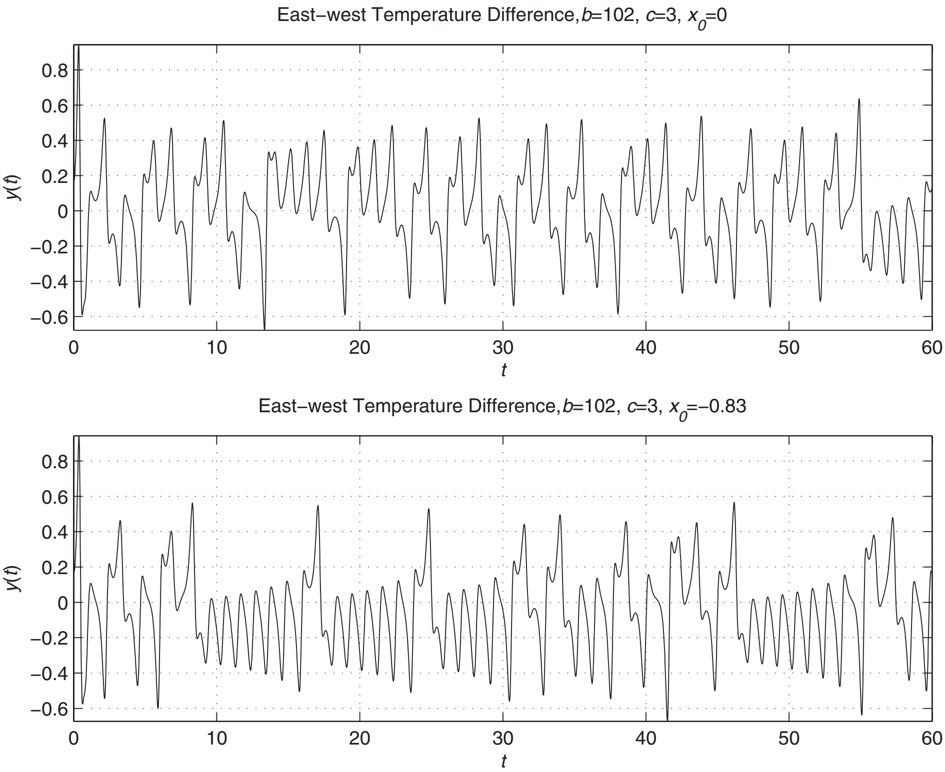

Figure 12.5. Numerical solution of the east–west temperature difference as a function of time.

12.4 Other Models of El Niño

The Vallis model presented in the previous section is not the type of model used for prediction or even quantitative analysis of ENSO. For one thing, the finite difference adopted is too coarse ($\Delta x = 7500$ km!). Also, as a result of coarse resolution, waves, which propagate signals from one ocean boundary to the other, are not resolved. The only effect incorporated is that of temperature advection by ocean currents.

These equatorial ocean waves were incorporated in the original model of Zebiac and Cane (1987), involving coupled partial differential equations. Recently there has been an attempt to model the ENSO oscillations as a delayed oscillator, taking into account the fact that the signal travels at the speed of the equatorial ocean waves.

The idea that ENSO is a self-sustaining chaotic system of relatively low order has been challenged by Penland and Sardeshmukh (1995), Thompson and Battisti (2000, 2001), and others who have shown that rapidly varying forcing (in surface wind stress and heat flux) could be the source of the irregularity of El Niño. Models involving "stochastic" (random) high-frequency forcing are being studied, even using linear models.

12.5 Appendix: The Advection Equation

A useful conservation equation is the advection equation, which describes the evolution in space and time of a quantity, F, say, which is *conserved*. That is, F does not change if the observer is moving with it; i.e.,

$$\frac{dF}{dt} = 0.$$

The coordinate system $(x(t), y(t), z(t))$, which is attached to the observer, is moving with the velocity

$$(u, v, w) = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$$

relative to a stationary frame of reference. Such a moving coordinate system is called the Lagrangian coordinates. Newtonian mechanics is usually expressed in terms of Lagrangian coordinates. The second law of motion, e.g., is expressed as

$$\frac{d}{dt}(mv) = f.$$

That is, the rate of change of momentum (mv) *following the particle* of mass m is equal to the applied force f.

Often, however, we desire to write the equations in a stationary frame of reference, called the "laboratory" frame. This frame is adopted in the field of fluid dynamics, where it is almost impossible to track every fluid particle with its own Lagrangian coordinate system. Instead, the observer simply sits in a stationary frame and lets the particles move by with velocities (u, v, w). Since the velocities and the quantity F are different at different locations and times, they are in turn functions of x, y, z, and t, where x, y, z are now fixed. This coordinate system is called the Eulerian coordinates.

We now need a transformation from a Lagrangian coordinate description to a Eulerian description:

$$\frac{d}{dt}F(x(t), y(t), z(t), t)$$
$$= \frac{dx}{dt}\frac{\partial}{\partial x}F + \frac{dy}{dt}\frac{\partial}{\partial y}F + \frac{dz}{dt}\frac{\partial}{\partial z}F + \frac{\partial}{\partial t}F.$$

The partial derivative with respect to x, $\frac{\partial}{\partial x}F$, means that the derivative is taken while holding all other variables (in this case, y, z, and t) fixed. Similarly, $\frac{\partial}{\partial t}F$ means holding x, y, z fixed, even though x, y, z also depend on t in the Lagrangian description.

Since

$$(u, v, w) = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right),$$

we have

$$\frac{d}{dt}F = u\frac{\partial}{\partial x}F + v\frac{\partial}{\partial y}F + w\frac{\partial}{\partial z}F + \frac{\partial}{\partial t}F.$$

The final step in the transformation is to treat F from now on as functions of fixed space x, y, z, and time t. Thus, the advection equation becomes

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right) F(x, y, z, t) = 0.$$

12.6 Exercises

1. One of the criticisms of the simple three-component model of ENSO presented here is the coarse resolution of the finite difference adopted for x and z derivatives in the ocean advection part of the model. The result turns out to be qualitatively sensitive to the particular type of finite difference scheme used, as pointed out by the author, Vallis, himself. In section 12.3, we derived a Lorenz-type system by using a centered differencing scheme. An equally reasonable scheme, no more or less accurate, is the upstream scheme. Under this scheme, Eq. (12.5) is replaced by

$$\frac{\partial}{\partial t}T_e = \frac{U}{\Delta x}(T_w - T_e) - A(T_e - T_0) \quad \text{if } U > 0,$$

$$\frac{\partial}{\partial t}T_e = \frac{U}{\Delta x}(T_e - T_d) - A(T_e - T_0) \quad \text{if } U < 0,$$

$$\frac{\partial T_w}{\partial t} = \frac{U}{\Delta x}(T_d - T_w) - A(T_w - T_0) \quad \text{if } U > 0,$$

$$\frac{\partial T_w}{\partial t} = \frac{U}{\Delta x}(T_w - T_e) - A(T_w - T_0) \quad \text{if } U < 0.$$

The U equation remains as in Eq. (12.4), but we will consider only the case of $U_0 \equiv 0$.

a. Show that there is an east–west symmetry. That is, reversing the sign of U and substituting T_w for T_e and T_e for T_w leads to an identical equation set.

b. Because of (a), we need to consider only one sign of U. Take $U > 0$. Define x, y, z as was done in section 12.3. Write down the system of three ordinary differential equations for these variables.

c. Find the equilibria.

d. Determine the linear stability of each equilibrium.

e. Discuss the possible steady states of this coupled atmosphere–ocean model.

f. Can you show that there is no limit cycle solution in this model?