Introduction: Applied Regression [1]

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[1] (based on lectures of Elena Erosheva)

Basics

- Syllabus: Text, schedule, grades.
- Lectures and lab sessions, laptops, homework.
- Project.

Course Projects

- STAT/CSSS 504 is a project-based course.
- Students identify a research question and a corresponding data set, and carry out a regression analysis to answer the research question.
- Everyone proposes a project idea. Instructor selects projects. Best projects start with a question or idea, then find data.
- If not enough viable project ideas are proposed, there will be an in-class final exam and the class schedule will be revised accordingly.
- Project groups deliver short oral presentations in class.
- Project groups present final results in poster format during the finals week.

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- To gain practical skills necessary to formulate a research question, carry out analyses, interpret results, and present findings addressing the research question from a regression study.
- To become a critical consumer of research that employs regression techniques.

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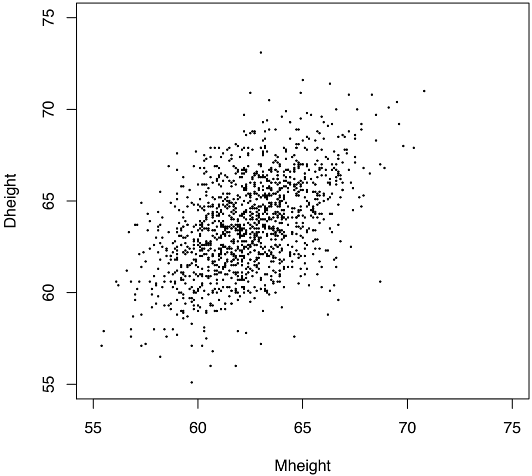
- When $p = 1$, the analysis is called **simple** regression; when $p > 1$, it is called **multiple** regression.

Inheritance of Heights: Mothers and Daughters

From R package `alr4` dataset `Heights`.



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Note: the function f is not necessarily linear in the predictors!

Linear regression

Which equation is not a linear regression function?

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$$

$$y = \beta_0 + \beta_1 \log(x_1) + \beta_2 x_2 + \beta_3 \log(x_1) x_2 + \beta_4 x_2^2 + \epsilon$$

$$y = \beta_0 + \beta_1 x_1 + x_2^{\beta_2} + \epsilon$$

$$y = \beta_0 + \beta_1 \boxed{x_1^{x_2}} + \varepsilon$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \varepsilon$$

$$y = \beta_0 + \beta_1' x_1^2 + \varepsilon$$

$$x_1' = x_1^{x_2}$$

$$x_1' = x_1 + a$$

$$y = \beta_0 + \beta_1 x_1' + \varepsilon$$

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- inherent randomness (unpredictable aspects of Y).

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   - What is the relationship between mothers' and daughters' heights?
   - What is the relationship between education and voting Democrat?

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On the other hand, **description and inference** objectives are characteristic of problems where understanding the mechanism is the key issue and predictions are by-products.

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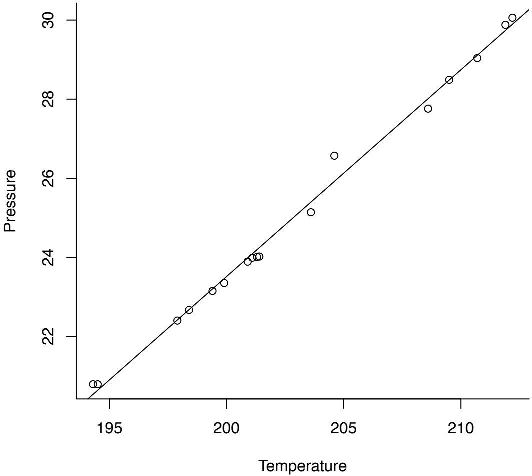
Assuming we have already examined numerical summaries and univariate plots, let us look at the scatterplots.

Data: `Forbes` in R package `alr4`.

Example: Forbes data

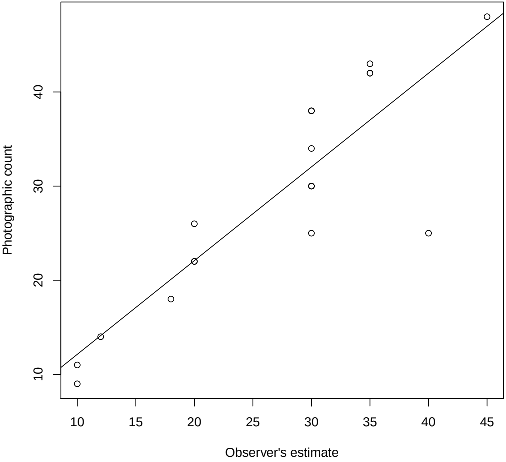


Example: Forbes data

Forbes Data: Observations from the scatterplot.

- Points appear to lie close to a line, however some curvature can be seen (by theory, log(pressure) is linearly related to temperature).
- One point does not “fit”.

Example: Snow Geese



Data: `snowgeese` in R package `alr3`.

Example: Snow Geese

Snow geese: Observations from the scatterplot.

- Small sample size; some x values have multiple y values recorded; some data points may be duplicated (we are not able to see this on the plot).
- Although a non-constant variance (heteroscedasticity) is not easily spotted on the plot due to a relatively small sample size, we expect it to be present because estimation errors by wildlife service members are likely to increase with the size of a flock.

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- For the graph, data were jittered (uniform, $U(-0.5, 0.5)$, random noise added to mothers' and daughters' heights).

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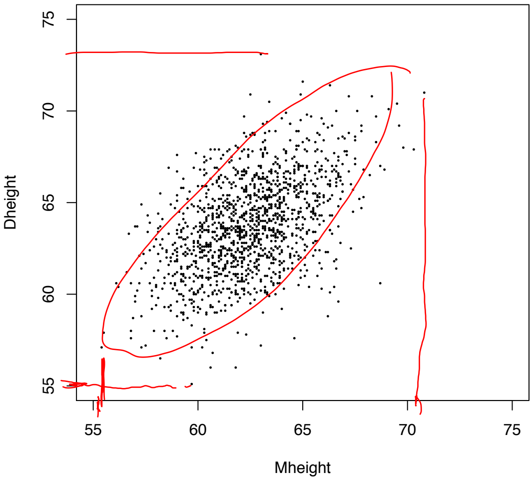

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- Mothers' and daughters' heights are clearly not independent, although the variability is high compared to the first two examples.
- The scatter appears elliptically shaped (rather typical if (X, Y) is a bivariate normal random vector).
- What about variance in the daughter's height for short, about average, and tall mothers?

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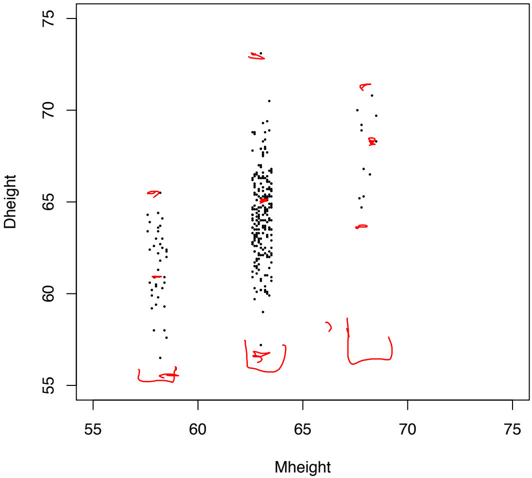

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- The next figure illustrates two possible regression lines.

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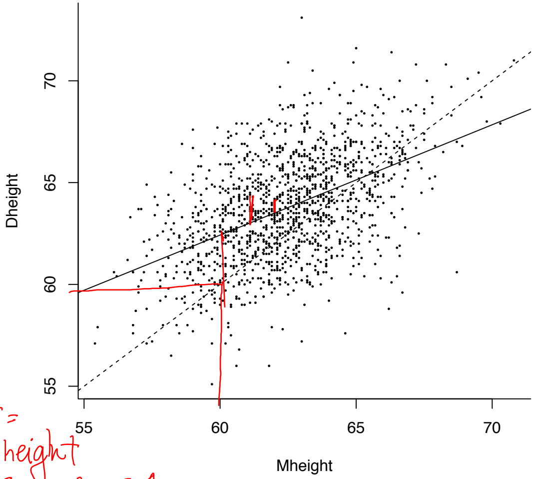

$Y = X$

Dheight = Mheight

$\beta_0 = 0$ $\beta_1 = 1$

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For multiple regression, scatterplot matrices can be useful.

Data: `fuel2001` from R package `alr4`.

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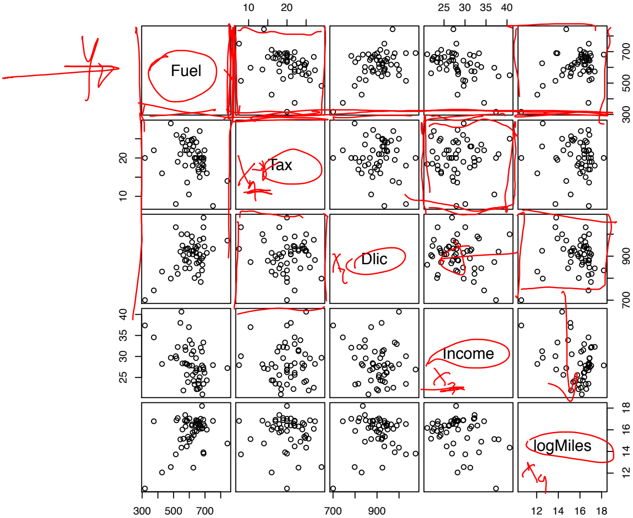

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- Because marginal relationships among the pairs of the predictors is weak, marginal plots for fuel versus the predictors are informative for the multiple regression problem.

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- rating - a rating of the cereals
- calories - calories per serving
- fiber - grams of dietary fiber
- sugars - grams of sugars
- vitamins - vitamins and minerals: 0, 25, or 100, indicating the typical percentage of FDA recommended daily intake.

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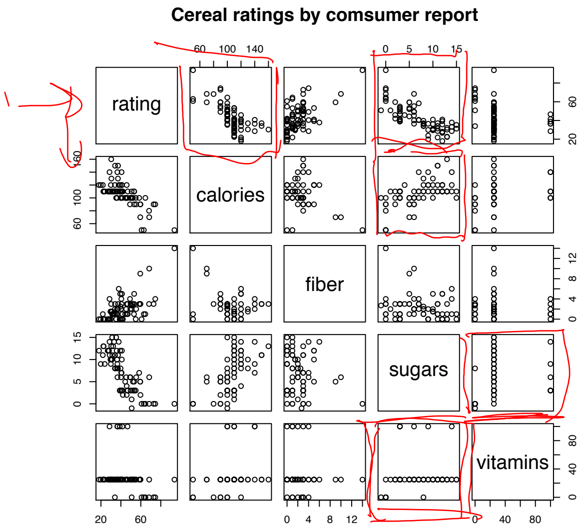

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- Note: If three or more predictors were linearly related, such as

$$X_1 + X_2 - X_3 \approx 0,$$

we would not be able to see this sort of relationship on a matrix of scatterplots.