

Writing Assignment 2

Group18: Eason Tang, Peiyang Xu, Sizhe Xu, Yuqing Weng

1.

Let T be the weight of two randomly selected 5 lb bags, $T = X_1 + X_2$.

Since $X_i \sim N(5.36, 0.14)$ and two bags are independent, we know that $T \sim N(5.36 + 5.36, \sqrt{0.14*0.14 + 0.14*0.14})$. Therefore, $T \sim N$ is $(10.72, 0.198)$.

$P(T > Y) = P(T - Y > 0)$. Let the weight difference (D) between two 5 lb bags and one 10 lb bag. We know that $D \sim N(10.72-10.22, \sqrt{0.198*0.198 + 0.18*0.18})$ so that $D \sim N$ is $(0.50, 0.268)$.

$$P(D > 0) = P(Z > (0 - 0.50)/0.268) = P(Z > -1.87) = 1 - 0.031 = 0.969$$

Therefore, the probability that the sum of the weights of the two 5 lb bags exceeds the weight of one 10 lb bag is 0.969.

2.

Let X be the number of occurrence of accidents in given month

$$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - (e^{-0.4} * (0.4^0)/0!) = 0.330$$

The probability of at least 1 accident in next 8 of 12 months is $\text{choose}(12, 8) * (0.330^8) * (0.67^4) = 0.0140$.

3.

(a)

6 ways to roll double through total 36 ways to roll.

$$X \sim \text{Geometric}(p=1/6)$$

$$P(X=1) = 1/6$$

$$P(X=2) = 1/6 * 5/6 = 5/36$$

$$P(X=3) = 1/6 * 5/6 * 5/6 = 25/216$$

$$P(\text{success within 3}) = P(X=1) + P(X=2) + P(X=3) = 0.4213$$

(b) expected value = $\mu = 6$

4.

(a) $S = \{0, 1, 2, 3, 4, 5, 6\}$

(b) Binomial Distribution.

The coin was tossed 6 times, so $N = 6$

A coin has a probability of 0.5 of coming up heads. Therefore, $p = 0.5$

(c) $\mu = np = 0.5 * 6 = 3$. It is the mean of the binomial distribution. The standard deviation (σ)
 $= \sqrt{np(1-p)} = 1.2247$. It is the square root of the variance (σ^2).

(d) $P(X > 2)$

$$= 1 - P(X \leq 2)$$

$$= 1 - (P(X=2) + P(X=1) + P(X=0))$$

$$= 1 - ((6,2) * (0.5)^2 * (0.5)^4 + (6,1) * (0.5) * (0.5)^5 + (6,0) * (0.5)^0 * (0.5)^6)$$

$$= 0.6563$$

(e)

Normal distribution:

$$E(x) = np = 1200 * 0.525 = 630$$

$$\sigma = \sqrt{630 * (1 - 0.525)} = \sqrt{299.25}$$

Consider continuity correction

$$P(X \leq 595)$$

$$= P(Z \leq (595.5 - 630) / \sqrt{299.25})$$

$$= P(Z \leq -1.99435293)$$

$$= 0.0231$$

5.

a. $P(\text{no A in English or no A in Chemistry}) = P(\text{no A in English}) + P(\text{no A in Chemistry}) - P(\text{no A in both English and Chemistry})$
 $= 0.8 + 0.9 - 0.95$
 $= 0.75$

b. Events “Earning an A in Chemistry” and “Earning an A in English” are not disjoint, because $P(\text{A in English and A in Chemistry}) = 0.05 \neq 0$

c. $P(\text{A in Chemistry} \mid \text{A in English}) = 0.05 / 0.2 = 0.25 \neq 0.1$

$$P(\text{A in English} \mid \text{A in Chemistry}) = 0.05 / 0.1 = 0.5 \neq 0.2$$

Therefore, “Earning an A in English”, and “Earning an A in Chemistry” are not independent events, because $P(A) \neq P(A \mid B)$.

6.

a. It's a hypergeometric probability distribution.

b. $X \sim \text{Hypergeometric}(n=200, M=50, N=1100)$

$$P(X = 25) = [(50, 25) * (1050, 175)] / (1100, 200) = 1.1999 \times 10^{-7}$$

Here is the R code:

```
> hypergeometric <- function (x, n, N, M) {  
+   return ((choose (M, x) * choose (N - M, n - x) / choose  
+     (N, n)))  
+ }  
> hypergeometric (25, 200, 1100, 50)  
[1] 1.199866e-07
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7.

Let NV = a crime that is nonviolent

Let RNV = a crime that is reported given that it's a nonviolent crime

Let RV = a crime that is reported given that it's a violent crime

$$\begin{aligned} \text{a. } P(R) &= P(NV) \times P(RNV) + P(V) \times P(RV) \\ &= 0.83 \times 0.62 + 0.17 \times 0.88 \\ &= 0.6642 \end{aligned}$$

$$\begin{aligned} \text{b. } P(NV | R) &= P(R | NV) \times P(NV) / P(R) \\ &= (0.62 \times 0.83) / 0.6642 \\ &= 0.7748 \end{aligned}$$