Stat 311 Written Assignment 2 Solutions

1. Your local grocery store sells 5 lb bags of potatoes. However, the 5 lb bags don't weight exactly 5 lbs. If we let X_i be the weight of a randomly selected 5 lb bag of potatoes, historical data indicates that $X_i \sim N(5.36,\ 0.14)$. The local warehouse store sells 10 lb bags of potatoes, which also do not weight exactly 10 lbs. If Y is the weight of a randomly selected 10 lb bag, historical data indicates that $Y \sim N(10.22,\ 0.18)$. If we randomly select two 5 lb bags of potatoes and one 10 lb bag of potatoes, what is the probability that the sum of the weights of the two 5 lb bags exceeds the weight of one 10 lb bag?

   Let T be the weight of two randomly selected 5 lb bags; $T = X_1 + X_2$
   Since $X_i \sim N(5.36, 0.14)$ and X_1 and X_2 should be independent, we know that
   $T \sim N(5.36 + 5.36, \sqrt{0.14^2 + 0.14^2})$. Or, $T \sim N(10.72, 0.198)$.

   We are asked to find $P(T > Y) = P(T - Y > 0)$. Let $W = T - Y$.
   Then $W \sim N(10.72 - 10.22, \sqrt{0.198^2 + 0.18^2})$ ➔ $W \sim N(0.5, 0.268)$.

   $P(T - Y > 0) = P(W > 0) = P\left(Z > \frac{0-0.5}{0.268}\right) = P(Z > -1.87) = 1 - P(Z \leq -1.87) =$
   $1 - 0.0307 = 0.9693.$

   Thus, there is high likelihood (about 97%) that two 5 lb bags of potatoes will weigh more than one 10 lb bag of potatoes. You gain by buying two of the smaller bags!

   NOTE: this problem could be solved in one step without creating two new RVs, T and W.

2. The number of injury accidents at a dangerous intersection is modeled by a Poisson distribution with $\mu = 0.4$ per month. Calculate the probability that there is at least 1 accident at the intersection in eight of the next 12 months. Hint: this problem requires using two discrete probability distributions to get the final answer.

   Let X = the number of accidents at the intersection per month. Then, $X \sim \text{Poisson}(0.4)$.
   $P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{0.4^0 e^{-0.4}}{0!} = 1 - 0.6703 = 0.3297$. Thus, the probability of at least one accident per month at this intersection is about 0.3297.

   Let Y = the number of months out of 12 with at least one accident. Then, $Y \sim \text{Binomial}(12, 0.3297)$.
   $P(X = 8) = \binom{12}{8} \times (0.3297)^8 \times (0.6703)^4 = 0.0139.$

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3. One way to get out of jail in MONOPOLY is to roll a double.

   a. What is the probability that you get out of jail within 3 rolls of the dice?

   P(throwing a double) = 6/36 = 1/6. Let X = number of rolls until hitting a double. Then $X \sim \text{Geometric}(\frac{1}{6})$. $P(X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3) = \frac{1}{6} + (5/6)(1/6) + (5/6)^2(1/6) = 0.4213$

   b. What is the expected number of rolls before the roll for getting out of jail?
   $E(X) = \frac{1}{p} = \frac{1}{\frac{1}{6}} = 6$. Therefore, you would expect 6 rolls to get a double ➔ you would expect 5 rolls before getting out of jail.

4. Let X be a random variable that denotes the number of heads in a set of 6 tosses.

   a) What is the sample space for X?

   The sample space consists of the set $\{0, 1, 2, 3, 4, 5, 6\}$. In each set of six tosses of a coin, you can observe anywhere from 0 to 6 heads.

   b) Assuming a fair coin and independent toss outcomes, what is the theoretical distribution of X? Make sure to name the distribution and the values of any parameters.

   $X \sim Binomial(n = 6, p = 0.5)$

   c) What are $E(X) = \mu_X$ and σ_X based on the distribution named in part (b) above

   $\mu_X = np = 6 \times \frac{1}{2} = 3$ heads

   $\sigma_X = \sqrt{np(1-p)} = \sqrt{6 \times \frac{1}{2} \times \frac{1}{2}} = 1.225$ heads

   d) What is the probability that $X > 2$ in any given case?

   $$P(X > 2) = \sum_{i=3}^{6} P(X = i) = \sum_{i=3}^{6} \frac{6!}{i!(6-i)!} p^i (1-p)^{6-i}$$
   $$= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$$
   $$= 1 - \left[\binom{6}{0} 0.5^0 0.5^6 + \binom{6}{1} 0.5^1 0.5^5 + \binom{6}{2} 0.5^2 0.5^4\right]$$

   e) Assume that the tossed coin was an unfair coin with P(Heads) = 0.525 and that it was tossed 1200 times. Define $Y \sim \text{Binomial}(1200, 0.525)$. Use normal approximation to the binomial distribution to find the probability of observing **no more than** 595 heads in 1200 tosses of the coin.

   First, we see that $np = 1200 \times 0.525 = 630$ and $n(1-p) = 1200 \times 0.475 = 570$ are large enough for us to use the normal approximation to the binomial distribution. We want to calculate $P(Y \leq 595)$, so the continuity correction is +0.5. If $Z \sim Normal(0, 1)$, the desired probability is approximately

   $$P\left(Z < \frac{595.5 - np}{\sqrt{np(1-p)}}\right) = P\left(Z < \frac{595.5 - 1200(0.525)}{\sqrt{1200(0.525)(0.475)}}\right) = P(Z < -1.994) = 0.0231$$

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5. You believe that there is a 20% chance that you will earn an A in your English class, a 10% chance that you will earn an A in your Chemistry class, and a 5% chance that you will earn an A in both classes. Let E = A in English, C = A in Chemistry, and B = A in both. **Use the defined events and proper notation** to complete parts A through C below. Justify your answers to parts B and C with an appropriate equation (not just words).

   a) Find the probability that you do not get an A in either English or Chemistry.

   $P(E^C \cap C^C) = 1 - P(E \cup C) = 1 - [0.2 + 0.1 - 0.05] = 0.75$

   b) Are "earning an A in English" and "earning an A in Chemistry" disjoint events? Explain.

   $P(E \cap C) = 0.05 > 0$ ➔ the events E and C are not mutually exclusive (disjoint)

   c) Are "earning an A in English" and "earning an A in Chemistry" independent events? Explain.

   $P(C|E) = \frac{P(C \cap E)}{P(E)} = \frac{0.05}{0.2} = 0.25 \neq 0.10 = P(C)$; therefore, the events C and E are not independent.

6. It is thought that there are 1100 moose in the Yellowstone Park moose population. Last year 50 moose were captured and tagged. Six months later 200 moose are captured. Define the RV, X, to be the number of tagged moose in the group of 200 most recently captured moose. We will assume that all moose are still living and that the population total has not changed.

   a) Name the probability distribution that can be appropriately used to find probabilities of X.

   $$X \sim \text{Hypergeometric}(N = 1100, M = 50, n = 200)$$

   b) What is the probability that 25 of the most recently captured moose are tagged?

   $$P(X = 25) = \frac{\binom{50}{25}\binom{1100-50}{200-25}}{\binom{1100}{200}} = \frac{\binom{50}{25}\binom{1050}{175}}{\binom{1100}{200}} = 1.1997 \times 10^{-7}$$

   Okay to leave answer in formula form if your calculator could not compute the final answer, but get extra credit (2 pts) if you show R code and indicate final probability.

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7. City crime records show that 17% of all crimes are violent and 83% are nonviolent, involving theft, forgery, and so on. Eighty-eight percent of violent crimes are reported versus 62% of nonviolent crimes.

   Let V = a crime is violent **Given: $P(V) = 0.17$, $P(V^C) = 0.83$, $P(R \mid V) = 0.88$, $P(R \mid V^C) = 0.62$**
   Let R = a crime is reported

   a) What is the overall reporting rate for crimes in the city?

   $$P(R) = P(R \cap V) + P(R \cap V^C) = P(R|V)P(V) + P(R|V^C)P(V^C)$$
   $$= 0.88(0.17) + 0.62(0.83) = 0.6642$$

   b) If a crime is reported to the police, what is the probability that the crime is nonviolent?

   $$P(V^C|R) = \frac{P(V^C \cap R)}{P(R)} = \frac{P(R|V^C)P(V^C)}{P(R)} = \frac{0.62(0.83)}{0.6642} = 0.7748$$