Write clearly and justify ALL your answers briefly.
All numerical answers may be left as numerical expressions (not worked out).
Be sure to write your name on the cover page, and number questions correctly.

1. (*12 points: 4 each part*) **Note the Hint**
(a) The jointly continuous random variables X and Y have joint density function (p.d.f.)

$$f_{X,Y}(x,y) \;=\; \frac{2}{x^2y^3} \text{ on } 1 \le x < \infty,\;\; 1 \le y < \infty, \quad \text{and} \;\; f_{X,Y}(x,y) \;=\; 0 \text{ for all other } (x,y).$$

Are X and Y independent? Find the marginal p.d.f of Y.
(b) The jointly continuous random variables X and Y have joint density function (p.d.f.)

$$f_{X,Y}(x,y) \;=\; \frac{3}{x^2y^3} \text{ on } 1 \le y \le x < \infty, \quad \text{and} \;\; f_{X,Y}(x,y) \;=\; 0 \text{ for all other } (x,y).$$

Are X and Y independent? Find the marginal p.d.f of Y.
(c) **in the case of (a)**, show how you *would* show that $\mathrm{P}(Y < X) \;=\; 2/3$.
(*Either:* Specify the integral clearly; You need not work it out to the end.
Or: Deduce this result directly from the p.d.f's given in (a) and (b), but justify this clearly.)
Hint: You are reminded that the indefinite integral of $(1/x^k)$ **is** $-1/((k-1)x^{k-1})$.

2. *(12 points: 3 each part.)* There are 8 red/green stop lights on Fred's bus route to school. The lights are not synchronized and are far enough apart that each will be independently red (stop) with probability 1/3 when the bus arrives at it, and will be green (go) with probability 2/3. Fred will be late for school if the bus must stop at 3 or more of the 8 lights.
(a) Write down an expression for the probability Fred is late to school. (Leave it as a sum of terms; do not try to simplify.)
(b) On one morning, Fred was hopeful: of the first 6 lights, only the third light was red. What is the probability Fred is late to school on this day?
(c) On another morning, Fred was less lucky. The third red light was at stop number 5. What is the probability of this?
(d) In the case of (c), when stop number 5 is the third red light, what is the expected total number of red lights Fred encounters on his journey to school that day.

3. (*12 points total*) The numerical answer to the probability in 2(a) above, that Fred is late to school on any given day, is 0.53, and the days are independent.
(a) (*5 points*) Fred takes the bus every day for an entire school year of 180 days. Show how you would use a Normal approximation to find the probabiity that he is late on more than 100 days. (You should include a continuity correction in your answer.)

(b) Sam and Sarah take the same school bus as Fred, but not every day. In fact, in the school year, Sam takes the bus 40 days, and Sarah takes the bus 30 days. There were just 5 of these days when both Sam and Sarah were taking the bus.
Let X be the number of days Sam is late, considering only the 40 days he takes the bus.
Let Y be the number of days Sarah is late for school, considering only the 30 days she takes the bus.
(i) (*2 points*) What are the mean and variance of Y?
(ii) (*2 points*) What is the expected value (mean) of $(X - Y)$?
(iii) (*3 points*) What is the covariance of X and Y?