

1. Permutations and combinations

There are $n! = \prod_{i=1}^n i = 1.2.3.4. \dots n$ permutations of n objects.

There are $\binom{n}{k} = n!/(k!(n-k)!)$ ways of choosing a given k objects from n .

2. Joint and conditional probabilities

If C and D are any events: $P(C \cup D) = P(C) + P(D) - P(C \cap D)$.

The conditional probability of C given D is $P(C | D) = P(C \cap D) / P(D)$.

C and D are independent if $P(C \cap D) = P(C).P(D)$.

3. Laws and theorems

Suppose E_1, \dots, E_k is a partition of Ω . That is $E_i \cap E_j$ is empty, and $E_1 \cup E_2 \cup \dots \cup E_k = \Omega$.

The law of total probability states that: $P(D) = \sum_{j=1}^k P(D \cap E_j) = \sum_{j=1}^k P(D | E_j) P(E_j)$

Bayes' Theorem states that: $P(E_i | D) = P(D | E_i) P(E_i) / P(D)$

4. Random variables and distributions	discrete (mass)	continuous (density)
Probability mass/density function	pmf: $P(X = x) = p_X(x)$	pdf: $f_X(x)$
defined for	all x with $p_X(x) > 0$	$-\infty < x < \infty$
Probability $P(X \in B)$	$\sum_{x \in B} p_X(x)$	$\int_{x \in B} f_X(x) dx$
Cumulative distrib. func. CDF, $P(X \leq x)$	$F_X(x) = \sum_{y \leq x} p_X(y)$	$F_X(x) = \int_{-\infty}^x f_X(y) dy$

5. Standard distributions:	p_X or f_X	values
Discrete		
(a) Binomial; $B(n, p)$; index n , parameter p ;	$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$x = 0, 1, 2, \dots, n$
(b) Hypergeometric (N, k, n) ;	$P(X = x) = \binom{k}{x} \binom{N-k}{n-x} / \binom{N}{n}$	$x = \min(0, k+n-N), \dots, \max(n, k)$
(c) Poisson with parameter μ , $Po(\mu)$	$P(X = x) = \exp(-\mu) \mu^x / x!$	$x = 0, 1, 2, 3, 4, \dots$
(d) Geometric $Geom(p)$	$P(X = x) = (1-p)^{x-1} p$	$x = 1, 2, \dots$
Continuous		
(e) Uniform $U(a, b)$; on interval (a, b)	$f_X(x) = 1/(b-a)$	$a < x < b$
(f) Exponential rate λ ; $\mathcal{E}(\lambda)$	$f_X(x) = \lambda \exp(-\lambda x)$	$x > 0$