

STAT 340: FALL 2018: Some examples about sums, integrals, and sets

1. Summing series

(i) What is $(1 + x + x^2 + \dots + x^n) = \sum_{j=1}^n x^j$?

(ii) What is $(x^k + x^{k+1} + x^{k+2} + \dots + x^n) = \sum_{j=k}^n x^j$?

(iii) How do we define $\sum_{j=1}^{\infty} x^j$?

If $|x| < 1$, what is $\sum_{j=1}^{\infty} x^j$?

(iv) If $|x| < 1$ what is $(1 + 2x + 3x^2 + 4x^3 + \dots) = \sum_j j x^{j-1}$?

(Do this two ways: (a) If the sum is S , what is xS , what is $(1 - x)S$.

(b) Integrate each term, sum, and differentiate the result.

Do you get the same answer? What justifies this?)

2. The series for $e^x = \exp(x) = \sum_j x^j/j!$.

(i) For what values of x does this sum have a finite value?

(ii) Show $\sum_{j \text{ odd}} x^j/j! = (1/2)(e^x - e^{-x})$

(iii) Show $\sum_{j \text{ even}} x^j/j! = (1/2)(e^x + e^{-x})$

3. Sums and integrals.

(i) What is $\int_1^N (1/x) dx$? What happens as N gets large?

(ii) About how large is $\sum_{j=1}^N (1/j)$? What happens as N gets large?

(iii) What is $\int_1^N (1/x^2) dx$? What happens as N gets large?

(iv) About how large is $\sum_{j=1}^N (1/j^2)$? What happens as N gets large?

Or should we rather consider $\int_{\frac{1}{2}}^{N+\frac{1}{2}} 1/x^2 dx$?

(Draw a picture.)

4. Another integral

(i) What is $\int_0^A x/(1+x^2) dx$? What happens as A gets large?

(ii) What is $\int_{-A}^A x/(1+x^2) dx$? What happens as A gets large? (Draw a picture.)

(iii) What is $\int_{-A}^A |x|/(1+x^2) dx$? What happens as A gets large? (Draw a picture.)

5. Countable unions and intersections of sets (May be new to some)

Round brackets denote open intervals: $(a, b) = \{x : a < x < b\}$.

Square brackets denote closed intervals: $[a, b] = \{x : a \leq x \leq b\}$.

(i) Let $A_n = (0, (1 - (1/n)))$. What is $\bigcup_1^{\infty} A_n$?

(ii) Let $B_n = (0, (1 + (1/n)))$. What is $\bigcap_1^{\infty} B_n$?

(iii) Let $C_n = ((1/2) - (1/n), (1/2) + (1/n))$. What is $\bigcap_1^{\infty} C_n$?

(iv) Let $C_n = ((1/2) - (1/n), (1/2) + (1/n))$. What is $\bigcup_1^{\infty} C_n$?

6. Another series to sum (Harder)

(i) What is $\sum_{j=2}^N (1/j(j-1))$: Hint $1/j(j-1) = (1/(j-1)) - (1/j)$

(ii) What happens to $\sum_{j=2}^N (-1)^j/j = (1/2) - (1/3) + (1/4) - (1/5) + (1/6) - (1/7) + \dots$ as N becomes large?

(iii) What happens to $\sum_{j=1}^N 1/(2j) = (1/2) + (1/4) + (1/6) + \dots$ as N becomes large?

(iv) What happens to $\sum_{j=1}^N 1/(2j+1) = (1/3) + (1/5) + (1/7) + \dots$ as N becomes large?

(v) What is going on here? Compare with # 2, where the order of terms does not matter. See also # 4, for a similar example with integrals.