1.a. P(Failure | Test 1) = 1 - .6 = .4
P(Failure | Test 2) = 1 - .7 = .3

Independence

P((Failure | Test 1) ∩ (Failure | Test 2)) = P(Failure | Test 1) P(Failure | Test 2)
= .4 · .3 = .12 ⇒ 12% fail both

b.

		Test 1 Pass	Test 1 Fail	Total
Test 2	Pass	.5	.2*	.7
	Fail	.1	.2***	.3
		.6	.4**	1

underlined are given

20% fail both tests

* = .7 - .5 = .2
** = 1 - .6 = .4
*** = ** - * = .4 - .2 = .2

c. P(Pass test 2 | Pass test 1) = P(Pass both | Pass test 1) = .5/.6 = 5/6
P(Pass test 1 | Pass test 2) = P(Pass both | Pass test 2) = .5/.7 = 5/7

2. a. Fred is unknown,

$$\frac{1}{3} \cdot \frac{3}{4} + \frac{2}{3} \cdot \frac{1}{4} = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$$

susceptible, gets flu, not susceptible, gets flu

b. $P(\text{susceptible} \mid \text{gets flu}) = \dfrac{\frac{1}{3} \cdot \frac{3}{4}}{\frac{5}{12}} = \frac{1/4}{5/12} = \frac{3/12}{5/12} = \frac{3}{5}$

$\frac{1}{3}$ = P(susceptible) $\frac{3}{4}$ = P(gets flu | susceptible)
$\frac{5}{12}$ = P(gets flu)

c. The probabilities change for Fred.

P(gets flu | doesn't get flu tested) $= \frac{3}{5} \cdot \frac{1}{4} + \frac{2}{5} \cdot \frac{3}{4} = \frac{3}{20} + \frac{6}{20} = \frac{9}{20}$

susceptible, doesn't get flu, not susceptible, doesn't get flu

3. a. $\frac{\binom{5}{3}}{\binom{8}{3}} = \frac{5!}{3!2!} \Big/ \frac{8!}{3!5!} = {}^{5!}/_{2!} \Big/ {}^{8!}/_{5!} = \frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6}$ The Hypergeom.

b. $\left(\frac{5}{8}\cdot\frac{6}{9}\cdot\frac{7}{10}\right)$

c. My answer in c will be smaller than 0.244. Since we are removing red balls when we draw them, [illegible] the probability of drawing them is lower.

My answer in d will be larger than 0.244. Since we are adding red balls when we draw them, the probability of drawing them is higher

4. a. $F_X(x) = \int_0^x f_X(x)\,dx = \int_0^x \lambda e^{-\lambda x}\,dx = -e^{-\lambda x}\Big|_0^x = -e^{-\lambda x} - (-e^{-\lambda 0})$

$= 1 - e^{-\lambda x}$

So $F_X(x) = \begin{cases} 1 - e^{-\lambda x} & 0 \le x < \infty \\ 0 & x < 0 \end{cases}$

b. $\lambda = 2 \rightarrow F_X(x) = \begin{cases} 1 - e^{-2x} & 0 \le x < \infty \\ 0 & x < 0 \end{cases}$

$P(X > 5 \mid X > 2) = \frac{P(X > 5)}{P(X > 2)} = \frac{1 - P(X \le 5)}{1 - P(X \le 2)} = \frac{1 - (1 - e^{-10})}{1 - (1 - e^{-4})} = \frac{e^{-10}}{e^{-4}}$

c. $\lambda = 2 \rightarrow F_X(x) = \begin{cases} 1 - e^{-2x} & 0 \le x < \infty \\ 0 & x < 0 \end{cases}$

$P(X \le 2 \mid X \le 5) = \frac{P(X \le 2)}{P(X \le 5)} = \frac{1 - e^{-4}}{1 - e^{-10}}$