Write clearly and justify ALL your answers briefly.

Answers may be left as numerical expressions.

Be sure to write your name clearly, and number questions correctly.

1. (*12 points total; 4 points each part*)

In a very popular Massive Open Online Course (MOOC), there are two tests that students can use to check their progress. The tests aim to test quite different skills and knowledge, and all students continue in the MOOC regardless of their test outcomes.

In test-1, 60% of the students achieve a passing score.

In test-2, 70% of the students achieve a passing score.

(a) If the outcomes on the two tests are *independent*,

what percentage of students get failing scores on *both* tests?

(b) In fact, 50% of students pass both tests. What percentage of students get failing scores on *both* tests?

(c) What is the probability that a student who passes the first test will also pass the second test?

What is the probability that a student who passes the second test also passed the first test?

2. (*12 points: 4 points each part*)

In a certain large population, there is a new flu virus. Each individual is susceptible to the virus with probability 1/3, and not susceptible with probability 2/3. In any given winter, a susceptible individual gets this type of flu with probability 3/4, and a non-susceptible individual gets this flu with probability 1/4. The susceptibility of an individual *does not change* from year to year. Also, for any individual, getting this flu one year is independent of what happened to the same individual in previous years.

Fred is a random member of the population.

(a) What is the probability that Fred gets flu this winter?

(b) Fred gets flu. Show that the probability that Fred is a susceptible individual is 3/5.

(c) What is the probability Fred will **not** get flu next winter?

3. (*12 points total; 4 points each part*)

(a) There are 8 balls in an urn: 3 are blue, and 5 are red.

Three balls are drawn from the urn *without replacement*. What is the probability all three balls are red?

(b) The urn initially has 8 balls; 3 blue and 5 red, as in (a).

Now when a ball is drawn it is replaced, and an *additional* ball of the same color is added also to the urn. Three balls are drawn from the urn, following this process. What is the probability all three balls are red?

(c) If the balls were drawn *with replacement*, the probability of a red ball would be (5/8) on every draw, and the probabilty of three red balls would be $(5/8)^3 = 0.244$.

Will your answer in (a) be larger than 0.244? Explain briefly.

Will your answer in (b) be larger than 0.244? Explain briefly.

4. (12 points: 4 each part)

(a) An exponential random variable with parameter λ has pdf

$$
\begin{aligned}
f_X(x) &= \lambda \exp(-\lambda x) \quad \text{for } 0 \le x < \infty \\
&= \quad\quad 0 \quad\quad \text{for } -\infty < x < 0
\end{aligned}
$$

Find the cumulative distribution function $F_X(x) = P(X \le x)$.

(b) For an exponential random variable with parameter $\lambda = 2$, find $P(X > 6 \mid X > 2)$.

(c) For an exponential random variable with parameter $\lambda = 2$, find $P(X \le 2 \mid X \le 6)$.