

Write clearly and justify ALL your answers briefly.

Be sure to write your name on every page, and number questions correctly.

1. 16 points total: 4 each part

The jointly continuous random variables  $X$  and  $Y$  have joint density function

$$f_{X,Y}(x,y) = 2 \text{ on } 0 \leq x, 0 \leq y, (x+y) \leq 1 \text{ and } f_{X,Y}(x,y) = 0 \text{ for all other } (x,y).$$

(a) Find the marginal pdf  $f_X(x)$  of  $X$ .

(b) Show how you **would show** that  $E(X) = 1/3$  and  $\text{var}(X) = 1/18$ .

(You need not work out the integrals, but should specify them fully.).

(c) Show how you **would show** that  $E(XY) = 1/12$ .

(You need not work out this integral, but should specify it fully).

(d) Find a numerical expression for the correlation between  $X$  and  $Y$ .

(Hint: by symmetry,  $X$  and  $Y$  have the same marginal pdf)

2. 16 points total; 3 each part

Suppose  $Z_1$ ,  $Z_2$  and  $Z_3$  are independent Normal random variables each with mean 0 and variance 1.

Let  $X = 3 + 3Z_1 + Z_2$  and  $Y = 6 - 2Z_1 + 3Z_3$ .

(a) Find the means and variances of  $X$  and of  $Y$ .

(b) Find the covariance of  $X$  and  $Y$ .

(c) Find the mean and variance of  $Y - 3X$ .

(Note for (c): You need not work out the variance, but should show a clear expression for it.)

(d) Find the probability that  $Y + 3$  is larger than  $3X$ .

(Hint for (d): use part (c), but remember to justify your answer.)

3. (16 points: 4 each part.) Coffee mugs for a student union promotion are made in two colors, either blue or white. Since the white mugs are more popular, the manufacturer produces twice as many white mugs as blue. They are packed into large crates randomly, so that each mug has probability  $2/3$  of being white and  $1/3$  of being blue, independently of other mugs in the crate.

The students decide to unpack mugs for display until they have at least 8 white mugs and at least 4 blue mugs.

(a) What is the probability the students unpack 6 white mugs, followed by 2 blue mugs?

(b) What is the probability the students unpack only 12 mugs in total?

(c) What is the probability the students unpack 8 white mugs and 8 blue mugs (16 in total)?

(d) Suppose the students do unpack 16 mugs, 8 of each color. An assistant randomly puts 8 mugs on each of two display shelves. What is the probability there are 4 white mugs and 4 blue mugs on each shelf?

**Note 1:** You do not need to work out the numbers, but should give and justify a clear numerical expression.

**Note 2:** The parts (a) to (d) are four **separate** unrelated possibilities.