1.a. P(Failure | Test 1) = 1 − .6 = .4
P(Failure | Test 2) = 1 − .7 = .3

Independence

P((Failure | Test 1) ∩ (Failure | Test 2)) = P(Failure | Test 1) P(Failure | Test 2)
= .4 · .3 = .12 ⇒ 12% fail both

b

		Test 1 Pass	Fail	Total
Test 2	Pass	.5	.2*	.7
	Fail	.1	.2***	.3
		.6	.4**	1

underlined are given

20% fail both tests

* = .7 − .5 = .2
** = 1 − .6 = .4
*** = ** − * = .4 − .2 = .2

c. P(Pass test 2 | Pass test 1) = P(Pass both | Pass test 1) = .5/.6 = 5/6
P(Pass test 1 | Pass test 2) = P(Pass both | Pass test 2) = .5/.7 = 5/7

2. a. Fred is unknown.

$\frac{1}{3} \cdot \frac{3}{4} + \frac{2}{3} \cdot \frac{1}{4} = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$

susceptible, gets flu, not susceptible, gets flu

b. P(susceptible | gets flu) = $\frac{\frac{1}{3} \cdot \frac{3}{4}}{\frac{5}{12}} = \frac{1/4}{5/12} = \frac{3/12}{5/12} = \frac{3}{5}$

$\frac{1}{3}$ = P(susceptible) $\frac{3}{4}$ = P(gets flu | susceptible)
$\frac{5}{12}$ = P(gets flu)

c. The probabilities change for Fred.

P(gets flu | got flu last winter) = $\frac{3}{5} \cdot \frac{1}{4} + \frac{2}{5} \cdot \frac{3}{4} = \frac{3}{20} + \frac{6}{20} = \frac{9}{20}$

susceptible, does not get flu, not susceptible, does not get flu

3.a. $\frac{\binom{5}{3}}{\binom{8}{3}} = \frac{5!}{3!2!} \Big/ \frac{8!}{3!5!} = \frac{5!}{2!} \Big/ \frac{8!}{5!} = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6}$

It's Hypergeome

b. $\frac{5}{8} \cdot \frac{6}{9} \cdot \frac{7}{10}$

c. My answer in a will be smaller than 0.244. Since we are removing red balls when we draw them, the probability of drawing them is lower.

My answer in b will be larger than 0.244. Since we are adding red balls when we draw them, the probability of drawing them is higher.

4.a. $F_X(x) = \int_0^x f_X(z)\,dz = \int_0^x \lambda e^{-\lambda z}\,dz = -e^{-\lambda z}\Big|_0^x = -e^{-\lambda x} - (-e^{-\lambda 0})$

$= 1 - e^{-\lambda x}$

So $F_X(x) = \begin{cases} 1 - e^{-\lambda x} & 0 \le x < \infty \\ 0 & x < 0 \end{cases}$

b. $\lambda = 2 \rightarrow F_X(x) = \begin{cases} 1 - e^{-2x} & 0 \le x < \infty \\ 0 & x < 0 \end{cases}$

$P(X>5 \mid X>2) = \frac{P(X>5)}{P(X>2)} = \frac{1 - P(X \le 5)}{1 - P(X \le 2)} = \frac{1 - (1 - e^{-10})}{1 - (1 - e^{-4})} = \frac{e^{-10}}{e^{-4}}$

c. $\lambda = 2 \rightarrow F_X(x) = \begin{cases} 1 - e^{-2x} & 0 \le x < \infty \\ 0 & x < 0 \end{cases}$

$P(X \le 2 \mid X \le 5) = \frac{P(X \le 2)}{P(X \le 5)} = \frac{1 - e^{-4}}{1 - e^{-10}}$