Your Name (scribe): _____

Partner Name(s): _____

1. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the pdf
   $$f_X(x;\alpha) = \frac{\alpha}{x^{\alpha+1}}, \quad x \geq 1, \alpha > 1.$$

   a. Find a sufficient statistic for α.

   b. Find the MLE of α.

   c. Is the MLE sufficient for α? Why or why not?

2. Suppose we have n M&M's packages, each comprised of 18 individual M&M's. Assume that each package is a random sample of all M&M's made. We want to estimate p, the proportion of green M&M's for all packages of M&M's.

  a. What is the pdf for the number of green M&M's in one bag? Identify which components of the pdf are fixed and which are random.

  b. Find a sufficient statistic for p.

  c. Find the method of moments estimator of p.

  d. Find the MLE of p.

e. Find the mean square error of the MLE.

f. Is the MLE efficient? If not, what is its efficiency relative to the Cramer-Rao lower bound?

3. Find the method of moments estimators for each parameter of the gamma distribution,

$$f_Y(y;r,\lambda)=\frac{\lambda^r}{\Gamma(r)}y^{r-1}e^{-\lambda},\ y\geq 0.\quad \text{Recall}\quad E(Y)=r/\lambda,\ Var(Y)=r/\lambda^2.$$

4. Let $Y_1,Y_2,...,Y_n$ be a random sample of size n from the pdf $f_Y(y;\theta)=\frac{1}{\theta}e^{-y/\theta},\ y\geq 0.$

Consider the estimators $\widehat{\theta}_1=\bar{Y},\widehat{\theta}_2=Y_1$ and $\widehat{\theta}_3=nY_{min}.$ Calculate the variances of these estimators and then calculate the relative efficiencies of $\widehat{\theta}_1$ to $\widehat{\theta}_2$ and $\widehat{\theta}_1$ to $\widehat{\theta}_3.$