Probability Distributions

Monday, January 14, 2019 2:51 PM

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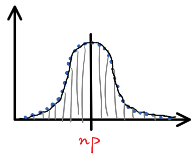
- **Binomial:** $B(n,p)$

Discrete distribution.
Describes number of success in n trials with replacement, while each trial is either success or failure with probability of success p.
Each single trial is called Bernoulli process.
If $np > 10$, Binomial can be approached by Normal approximation.
If $np < 10$, Binomial can be approximated by Poisson distribution.

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$$
$$k=0,1,2,\dots n$$

$$P(x\le m)=\sum_{k=0}^{m}\binom{n}{k}p^k(1-p)^{n-k}$$



Mean: np
Variance:

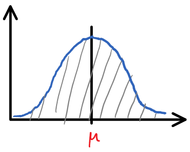
- **Normal:** $N(\mu,\sigma^2)$

Continuous distribution.
With mean μ and variance σ^2.
Sample mean distributed converges to Normal distribution.
Standard Normal distribution $N(0,1)$ can be used as approximation of binomial by Central Limit Theorem.

$$B(n,p)\approx P\left(\frac{np-\mu}{np(1-p)}\right)$$

$$f_X(x)=\frac{1}{\sqrt{2\pi\sigma^2}}\mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$F_X(m)=\int_{-\infty}^{m}f_X(t)dt$$



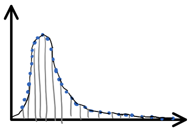
Mean: μ
Variance: σ^2

- **Hypergeometric:** (N,k,n)

Discrete distribution.
Describes the probability of k successes in n draws without replacement, from finite population of size N.
Each draw is either success or failure, without replacement, the probability of drawing success in each trial is different.
If N,k large enough, it can be considered as binomial distribution.

$$P(X=x)=\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$
$$x=\min(0,k+n-N),\dots,\max(n,k)$$

$$P(X\le m)=\sum_{i=0}^{m}\frac{\binom{k}{i}\binom{N-k}{n-i}}{\binom{N}{n}}$$



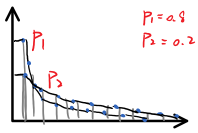
Mean: $\frac{nk}{N}$
Variance: $np(1-p)\frac{N-n}{N-1}$

- **Geometric:** $G(p)$

Discrete distribution.
Describe number of Bernoulli(p) trials before first success.
$G(p)=NegB(1,p)$

$$P(X=k)=P(1-P)^{k-1}$$
$$k=1,2,3,\dots$$

$$P(X\le m)=\sum_{k=1}^{m-1}(1-p)^k$$



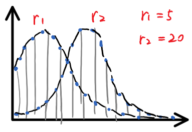
Mean: $\frac{1}{p}$
Variance: $\frac{1-p}{p^2}$

- **Negative Binomial:** $NegB(r,p)$

Discrete distribution.
Describes number of Bernoulli(p) trials before rth success.
Similar to Geometric distribution *when r* is small.

$$P(X=k)=\binom{k-1}{r-1}p^r(1-p)^{k-r}$$
$$r=1,2,3,\dots$$
$$k\ge r$$

$$P(X\le m)=\sum_{k=r}^{m}\binom{k-1}{r-1}p^r(1-p)^{k-r}$$



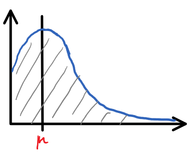
Mean: $\frac{r}{p}$
Variance: $\frac{r(1-p)}{p^2}$

- **Poisson:** $Po(\mu)$

Continuous distribution.
μ denotes number of events in fixed interval.
Describes the probability of given number of events in fixed time interval.
Can be approached by Poisson process, where $\mu=\lambda h$ (h is length of interval).
When p in Binomial is small (np < 10), binomial distribution could be approximated by Poisson distribution where $\mu=np$.

$$P(X=k)=\frac{\mathrm{e}^{-\mu}\mu^k}{k!}$$
$$k=0,1,2,\dots$$

$$P(X\le m)=\sum_{k=0}^{m}\frac{\mathrm{e}^{-\mu}\mu^k}{k!}$$



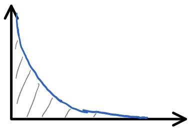
Mean: μ
Variance: μ

- **Exponential:** $\varepsilon(\lambda)$

Continuous distribution.
λ denotes number of events in fixed interval.
Exponential distribution describes the time between events in Poisson distribution.

$f_X(x) = \lambda e^{-\lambda x}$
$0 \le x < \infty$

$F_X(x) = 1 - e^{-\lambda x}$
$0 \le x < \infty$



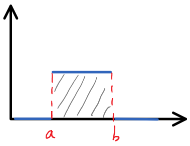
Mean: $\frac{1}{\lambda}$
Variance: $\frac{1}{\lambda^2}$

- **Uniform:** $U(a, b)$

Continuous distribution.
Characterized as uniform density for all values in interval [a, b] .
In time interval, the time of event happening is in uniform distribution.

$$f_X(x) = \frac{1}{(b-a)}$$
$a < x < b$

$$F_X(x) = \frac{x-a}{b-a}$$
$a < x < b$



Mean: $\frac{b+a}{2}$
Variance: $\frac{(b+a)^2}{12}$