Binomial: $B(n,\ p)$ Discrete

Describes number of success in n trials with replacement, while each trial is either success or failure with probability of success p. Each single trial is called Bernoulli process. **Mean:** np **Variance:** $np(1-p)$

PMF: $P(X=k)=\binom{n}{k}p^k(1-p)^{n-k};\ k=0,1,2,\dots n$

CDF: $P(x\le m)=\sum_{k=0}^{m}\binom{n}{k}p^k(1-p)^{n-k}$

Bernoulli: $Bern(p)$ Discrete

Probability distribution of any single trial that asks a yes-no question, or a Boolean-value. **Mean:** p **Variance:** $p(1-p)$

PMF: $1-p\ if\ k=0; p\ if\ k=1$

CDF: $1-p\ if\ 0\le k<1$

Normal: $N(\mu,\sigma^2)$ Continuous

Sample mean distributed converges to Normal distribution.

Standard Normal distribution $N(0,\ 1)$ can be used as approximation of binomial by Central Limit Theorem.

$B(n,\ p)\approx P\left(\frac{np-\mu}{np(1-p)}\right)$ **Mean:** μ **Variance:** σ^2

PDF: $f_X(x)=\frac{1}{\sqrt{2\pi\sigma^2}}\mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}}$

CDF: $F_X(m)=\int_{-\infty}^{m}f_X\,(t)dt$

Hypergeometric: (N,k,n) Discrete

Describes the probability of k successes in n draws without replacement, from finite population of size N.

If N,k large enough, it can be considered as binomial distribution. **Mean:** $\frac{nk}{N}$ **Variance:** $np(1-p)\frac{N-n}{N-1}$

PMF: $P(X=x)=\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}; x=\min(0,\ k+n-N),\dots,\max(n,k)$

CDF: $P(X\le m)=\sum_{i=0}^{m}\frac{\binom{k}{i}\binom{N-k}{n-i}}{\binom{N}{n}}$

Geometric: $G(p)$ Discrete

Describe number of Bernoulli(p) trials to get first success. $G(p)=NegB(1,p)$. **Mean:** $\frac{1}{p}$ **Variance:** $\frac{1-p}{p^2}$

PMF: $P(X=k)=P(1-P)^{k-1};\ k=1,2,3,\dots$

CDF: $P(X\le m)=\sum_{k=1}^{m-1}(1-p)^k$

Negative Binomial: $NegB(r,p)$ Discrete

Describes number of Bernoulli(p) trials to get rth success. **Mean:** $\frac{r}{p}$ **Variance:** $\frac{r(1-p)}{p^2}$

PMF: $P(X=k)=\binom{k-1}{r-1}p^r(1-p)^{k-r};\ r=1,2,3,\dots;\ k\ge r$

CDF: $P(X\le m)=\sum_{k=r}^{m}\binom{k-1}{r-1}p^r(1-p)^{k-r}$

Poisson: $Po(\mu)$ Continuous

μ denotes number of events in fixed interval.

Describes the probability of given number of events in fixed time interval.

Can be approached by Poisson process, where $\mu=\lambda h$ (h is length of interval).

When p in Binomial is small (np < 10), binomial distribution could be approximated by Poisson distribution where $\mu = np$.
Mean: μ **Variance:** μ

PDF: $P(X = k) = \frac{e^{-\mu}\mu^k}{k!}$; $k = 0,1,2,\ldots$

CDF: $P(X \leq m) = \sum_{k=0}^{m} \frac{e^{-\mu}\mu^k}{k!}$

Exponential: $\epsilon(\lambda)$ Continuous

λ denotes number of events in fixed interval. Exponential distribution describes the time between events in Poisson distribution. **Mean:** $\frac{1}{\lambda}$ **Variance:** $\frac{1}{\lambda^2}$

PDF: $f_X(x) = \lambda e^{-\lambda x}$; $0 \leq x < \infty$

CDF: $F_X(x) = 1 - e^{-\lambda x}$; $0 \leq x < \infty$

Uniform: $U(a, b)$ Continuous

Characterized as uniform density for all values in interval [a, b]. In time interval, the time of event happening is in uniform distribution. **Mean:** $\frac{b+a}{2}$ **Variance:** $\frac{(b+a)^2}{12}$

PDF: $f_X(x) = \frac{1}{(b-a)}$; $a < x < b$

CDF: $F_X(x) = \frac{x-a}{b-a}$; $a < x < b$

Gamma: $\Gamma(r, \lambda)$ Continuous

Time to the r th event in fixed interval. $X_i = \epsilon(\lambda)$, $\sum^r X_i = \Gamma(r, \lambda)$; $Z^2 = \chi_1^2 = \Gamma(\frac{1}{2}, \frac{1}{2})$; $\sum Z\text{^}2 = \Gamma(\frac{n}{2}, \frac{1}{2})$;

$if\ Y_1, Y_2 \sim \Gamma(r1, \lambda)\ and\ \Gamma(r2, \lambda), then\ Y_1 + Y_2 \sim \Gamma(r1 + r2, \lambda)$

PDF: $f_Y(y) = \frac{\lambda^r}{\Gamma(k)} y^{r-1} e^{-\lambda y} = \frac{\lambda^r}{(r-1)!} y^{r-1} e^{-\lambda y}$; $\Gamma(r+1) = r\Gamma(r) = r!$; $\Gamma\left(\frac{1}{2}\right) = \pi$

Chi-Square: χ_n^2 Continuous

Sum of n independent squared standard normal deviates with degree of freegom n.

$if\ Y_1, Y_2 \sim \chi_n^2\ and\ \chi_m^2, independent, then\ Y_1 + Y_2 \sim \chi_{n+m}^2$

$\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2, with\ df = n - 1$. **Mean:** n **Variance:** $2n$

PDF: $f_Y(y) = \frac{1}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} y^{\frac{n}{2}-1} e^{-\frac{y}{2}}, for\ y > 0$

F distribution: $F_{n,m}$ Continuous

U, V are independent r.v. with χ_n^2, χ_m^2, then $F = \left(\frac{U}{n}\right) / \left(\frac{V}{m}\right)\ with\ df = n, m$. If $S1^2\ and\ S2^2$ are two independent sample variances, then $F = \left(\frac{S1^2}{\sigma 1^2}\right) / \left(\frac{S2^2}{\sigma 2^2}\right) = \left(\frac{\chi_{n-1}^2}{n-1}\right) / \left(\frac{\chi_{m-1}^2}{m-1}\right) \sim F_{n-1,m-1}$ with df n-1 and m-1. F can be used to estimate ratio of two variances. **Mean:** $\frac{n}{n-2}\ for\ n > 2$ **Variance:** $\frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$

T: distribution Continuous

Degree of freedom is sample size -1. $Z \sim Normal(0,1)$ $V \sim \chi_{n-1}^2$ Z and V are independent, then $Y = \frac{Z}{\sqrt{\frac{V}{n}}} = \frac{Z}{\frac{s}{\sqrt{n}}}$ follows T distribution with degree of freedom n-1. **Mean:** 0 **Variance:** $\frac{\text{df}}{\text{df}-2}$ PDF: $f_T(t) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\Gamma\left(\frac{n}{2}\right)\left(1+\frac{t^2}{n}\right)^{\frac{n+1}{2}}}$