# Distribution Summary

Monday, January 21, 2019 9:41 PM

**PDF / PMF** | **CDF** | **Sketch**

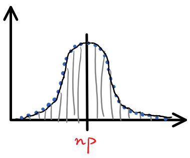
- **Binomial:** $B(n,p)$

Discrete distribution.
Describes number of success in $n$ trials with replacement, while each trial is either success or failure with probability of success $p$.
Each single trial is called Bernoulli process.
If $np > 10$, Binomial can be approached by Normal approximation.
If $np < 10$, Binomial can be approximated by Poisson distribution.

**Mean:** $np$
**Variance:**

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$
$$k = 0,1,2,\dots n$$

$$P(x \le m) = \sum_{k=0}^{m} \binom{n}{k} p^k (1-p)^{n-k}$$




- **Normal:** $N(\mu, \sigma^2)$

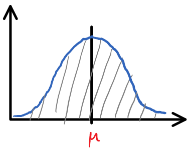
Continuous distribution.
With mean $\mu$ and variance $\sigma^2$.
Sample mean distributed converges to Normal distribution.
Standard Normal distribution $N(0,1)$ can be used as approximation of binomial by Central Limit Theorem.

$$B(n,p) \approx P\left(\frac{np-\mu}{np(1-p)}\right)$$

**Mean:** $\mu$
**Variance:** $\sigma^2$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$F_X(m) = \int_{-\infty}^{m} f_X(t)dt$$



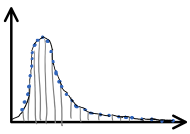

- **Hypergeometric:** $(N,k,n)$

Discrete distribution.
Describes the probability of $k$ successes in $n$ draws without replacement, from finite population of size $N$.
Each draw is either success or failure, without replacement, the probability of drawing success in each trial is different.
If $N, k$ large enough, it can be considered as binomial distribution.

**Mean:** $\frac{nk}{N}$
**Variance:** $np(1-p)\frac{N-n}{N-1}$

$$P(X=x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$
$$x = \min(0, k+n-N), \dots, \max(n,k)$$

$$P(X \le m) = \sum_{i=0}^{m} \frac{\binom{k}{i}\binom{N-k}{n-i}}{\binom{N}{n}}$$



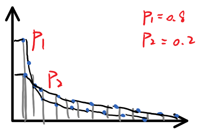
- **Geometric:** $G(p)$

Discrete distribution.
Describe number of Bernoulli($p$) trials before first success.
$G(p) = NegB(1,p)$

**Mean:** $\frac{1}{p}$
**Variance:** $\frac{1-p}{p^2}$

$$P(X=k) = P(1-P)^{k-1}$$
$$k = 1,2,3,\dots$$

$$P(X \le m) = \sum_{k=1}^{m-1} (1-p)^k$$



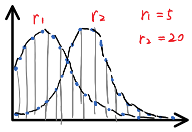

- **Negative Binomial:** $NegB(r,p)$

Discrete distribution.
Describes number of Bernoulli($p$) trials before $rth$ success.
Similar to Geometric distribution *when r* is small.

**Mean:** $\frac{r}{p}$
**Variance:** $\frac{r(1-p)}{p^2}$

$$P(X=k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$$
$$r = 1,2,3,\dots$$
$$k \ge r$$

$$P(X \le m) = \sum_{k=r}^{m} \binom{k-1}{r-1} p^r (1-p)^{k-r}$$

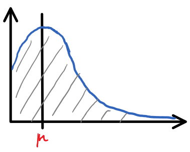



- **Poisson:** $Po(\mu)$

Continuous distribution.
$\mu$ denotes number of events in fixed interval.
Describes the probability of given number of events in fixed time interval.
Can be approached by Poisson process, where $\mu = \lambda h$ ($h$ is length of interval).
When $p$ in Binomial is small (np < 10), binomial distribution could be approximated

$$P(X=k) = \frac{e^{-\mu}\mu^k}{k!}$$
$$k = 0,1,2,\dots$$

$$P(X \le m) = \sum_{k=0}^{m} \frac{e^{-\mu}\mu^k}{k!}$$



binomial distribution could be approximated by Poisson distribution where $\mu = np$.

$\mu$

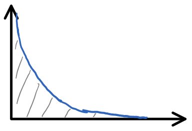
**Mean:** $\mu$
**Variance:** $\mu$

- **Exponential:** $\varepsilon(\lambda)$

Continuous distribution.
λ denotes number of events in fixed interval.
Exponential distribution describes the time between events in Poisson distribution.

$$f_X(x) = \lambda e^{-\lambda x}$$
$$0 \le x < \infty$$

$$F_X(x) = 1 - e^{-\lambda x}$$
$$0 \le x < \infty$$



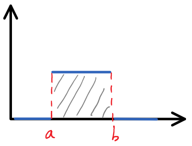
**Mean:** $\frac{1}{\lambda}$
**Variance:** $\frac{1}{\lambda^2}$

- **Uniform:** $U(a, b)$

Continuous distribution.
Characterized as uniform density for all values in interval [a, b] .
In time interval, the time of event happening is in uniform distribution.

$$f_X(x) = \frac{1}{(b-a)}$$
$$a < x < b$$

$$F_X(x) = \frac{x-a}{b-a}$$
$$a < x < b$$




**Mean:** $\frac{b+a}{2}$
**Variance:** $\frac{(b+a)^2}{12}$

- **Multinomial:** $Mn\left(n, (p_1 \dots p_k)\right)$

$$P(X_i = n_i, i = 1, \dots k) = \left(\frac{n!}{n_1!\, n_2! \dots n_k!}\right) p_1^{n_1} \dots p_k^{n_k}$$

**Mean:** $X_i \sim Bin(n, p_i)$

- **Gamma:** $\Gamma(\alpha, \beta)$ or $\Gamma(k, \theta)$

Shape parameter $\alpha = k$ and inverse scale parameter $\beta = \frac{1}{\theta}$
Is exponential when $\alpha = 1$ or $k = 1$

$$f_X(x) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}}$$

**Mean:** $k\theta$ or $\frac{\alpha}{\beta}$
**Variance:** $k\theta^2$ or $\frac{\alpha}{\beta^2}$