Q3-a

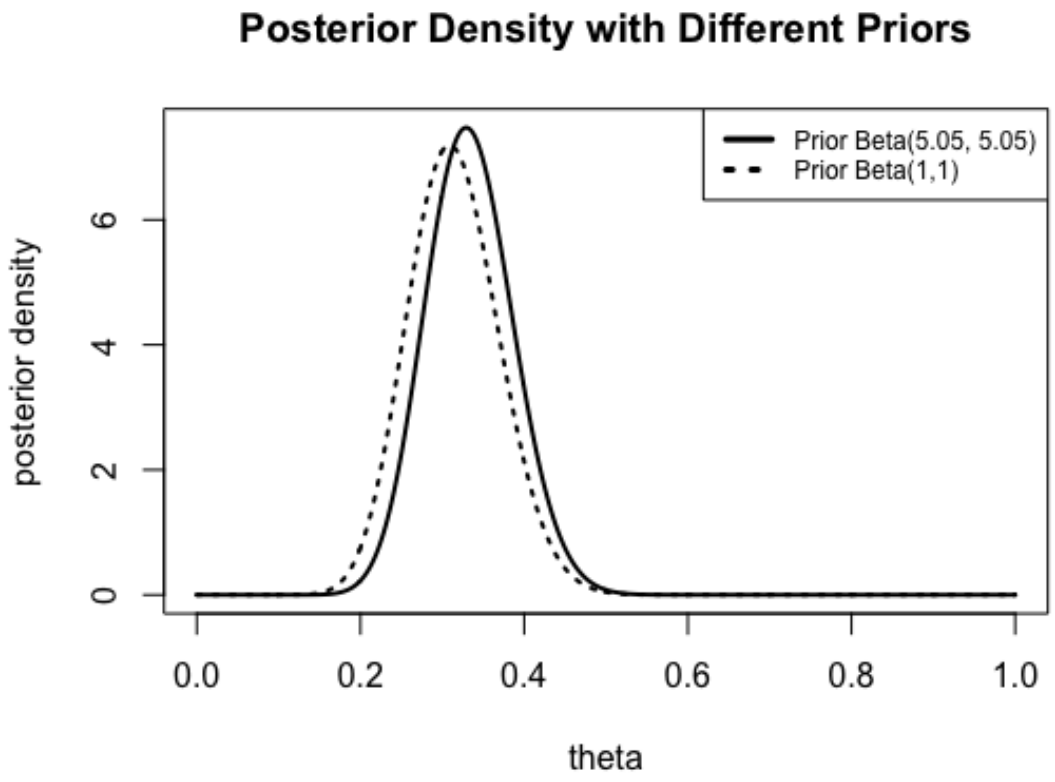
Posterior function on prior $Beta(1,1)$ can be written as

$$\begin{aligned}\pi(\theta|x) &\propto L(\theta|x)\pi(\theta) \\ &\propto \theta^{21}(1-\theta)^{68-21}\theta^{1-1}(1-\theta)^{1-1} \\ &\propto \theta^{22-1}(1-\theta)^{48-1}\end{aligned}$$

The posterior follows $Beta(22, 48)$

```
theta <- seq(0, 1, 0.001)
post1 <- dbeta(theta, 469/18, 937/18)
post2 <- dbeta(theta, 22, 48)

plot(x=theta, y=post1, type='l', lwd=2, xlab='theta', ylab='posterior density',
     main='Posterior Density with Different Priors')
lines(x=theta, y=post2, lty=3, lwd=2)
legend('topright', legend=c('Prior Beta(5.05, 5.05)', 'Prior Beta(1,1)'),
       lty=c(1,3), lwd=3, cex=0.75)
```



Two posterior distributions share identical general shape, since they are both Beta distributions. The variance of posterior with prior $Beta(1,1)$ is slightly bigger and its median is smaller than the posterior with prior $Beta(5.05, 5.05)$.

Q3-b

```
upper <- qbeta(0.975, 22, 48)
lower <- qbeta(0.025, 22, 48)
> upper
[1] 0.4268747
> lower
[1] 0.2117479
```

The 95% credible interval is [0.2117479, 0.4268747].
Compare to the credible interval calculated in previous problem, this interval has lower values in both lower and upper bounds.