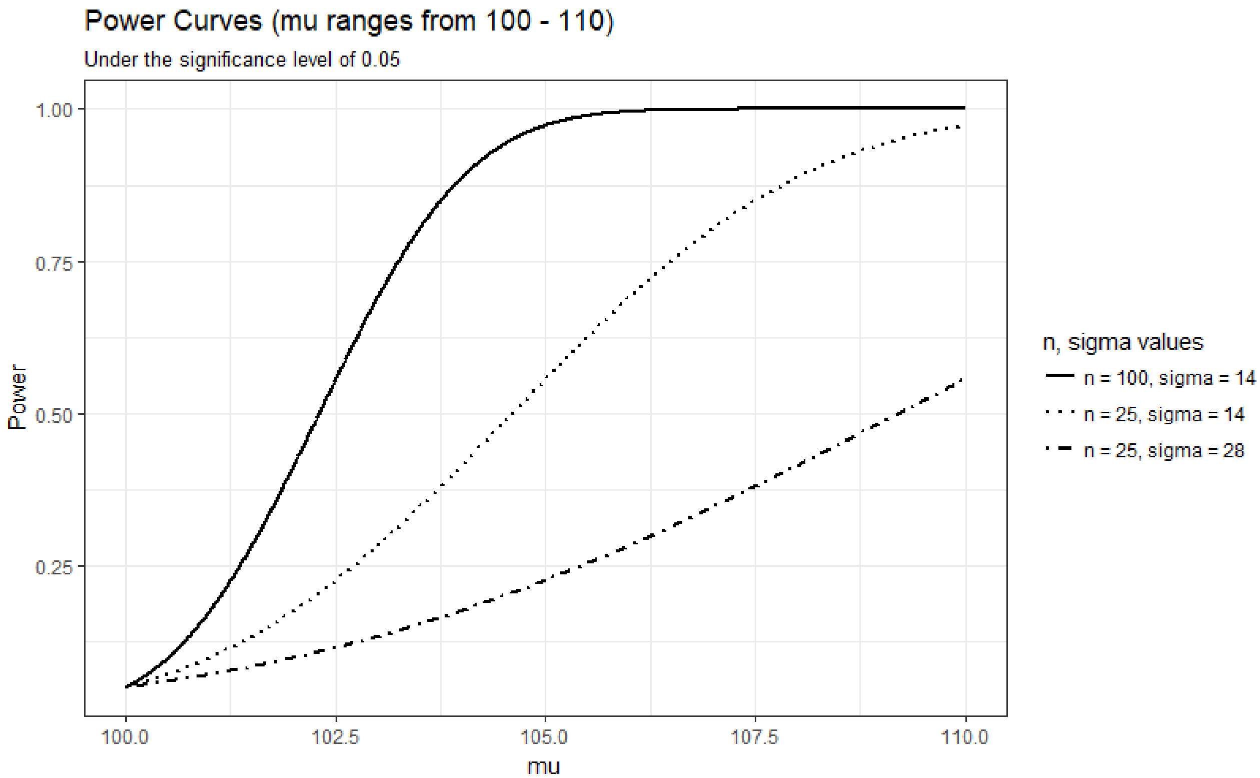
Q4

For the one-sided test of the mean μ of a normal with known variance σ^2:
H0 : $\mu \leq 100$, H1 : $\mu > 100$.
The power curve for $\alpha = 0.05$ is $P_H 1(\bar{X} \geq 100 + 1.645 * \frac{\sigma}{\sqrt{n}})$.

(a)

```
library(ggplot2)
# for mu range from 100 to 110
mu.values <- seq(100, 110, 0.01)

# calculate power curve for given n, sigma, mu
# under alpha level 0.05
get.power.curve <- function(n1, sigma1, mu1) {
  k <- 100 + 1.645 * sigma1 / sqrt(n1)
  return(1 - pnorm(sqrt(n1) * (k - mu1) / sigma1))
}

# power curve when n = 25, sigma = 14
pc1 <- get.power.curve(25, 14, mu.values)

# power curve when n = 100, sigma = 14
pc2 <- get.power.curve(100, 14, mu.values)

# power curve when n = 25, sigma = 28
pc3 <- get.power.curve(25, 28, mu.values)

pc.combine <- data.frame(mu.values, pc1, pc2, pc3)

powercurve <- ggplot(data = pc.combine) +
  geom_line(aes(x=mu.values, y=pc1, linetype='n = 25, sigma = 14'), size=1) +
  geom_line(aes(x=mu.values, y=pc2, linetype='n = 100, sigma = 14'), size=1) +
  geom_line(aes(x=mu.values, y=pc3, linetype='n = 25, sigma = 28'), size=1) +
  labs(y="Power",
       x="mu",
       title="Power Curves (mu ranges from 100 - 110)",
       subtitle="Under the significance level of 0.05",
       linetype='n, sigma values'
       ) +
  scale_linetype_manual(values=c("solid", "dotted", "dotdash"))
theme_set(theme_bw())
plot(powercurve)
```



(b)

By comparing these three power curves with different sample size and population variance, we can see that as μ getting farther away from μ_0, power of sample size 100 and population standard deviation 14 grows the fastest, and converges to 100% power at $\mu = 106$. Power curve of sample size 25, population standard deviation 28 grows slowest, since big standard error makes test statistic harder to reject the null hypothesis, even if μ is much bigger than μ_0.

We may conclude that as μ getting farther away from μ_0, the power of the test converges to 100% faster in sample with large size n, and in population with relatively small variance σ^2.