Name: ______________________________

Section: ______________________________

RULES

1. This is a closed book test and should take 110 minutes. There will be no cheat sheet or distribution tables allowed. You should know the
   - PDF/PMF of the normal distribution, binomial, Poisson, and exponential;
   - the form of the MP test and LRT statistic;
2. Statistical tables will be provided as needed.
3. A calculator is necessary; however, graphing calculators are not allowed. Cell phones are not allowed either. You are NOT allowed to share calculators.
4. **Circle your final answer when it is a numerical one**. You must show your work to receive credit.
5. Students must follow a reasonable code of conduct. Cheating or other dishonest practices will not be tolerated and will result in a a quiz grade of zero. Such practices include, but are not limited to:
   - use of notes or calculator computations other than those permitted,
   - communicating with other students' during the test,
   - looking at other students' work, and
   - exposing your work purposely to the view of others.
6. Please sign the pledge below.

   I have followed a reasonable code of conduct as described in point 5 above. I have neither given or received un-authorized aid.

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Logistics

The midterm is scheduled for Friday, May 3. You will have both class periods to work on it. Please bring pen/pencil/eraser/calculator with you. Normal, t and chi square tables will be provided, as necessary.

The test will cover likelihood inference. The following is a checklist of some of the things we have learned. *All page numbers below refer to Larsen & Marx.*

- Identify if a hypothesis is simple or composite.
- Identify the rejection region of a test. Equivalently, identify the acceptance region.
- Calculate Type 1 error of a test. Shade in the region corresponding to the Type 1 error (see fig 6.4.6 on page 371).
- Calculate Type 2 error/power for a test.
- Calculate Wald test statistic and come up with size α decision rule based on it.
- Calculate most powerful test for a simple H_0 versus simple H_1 and come up with size α decision rule based on it. Decide if the test is UMP.
- Calculate LRT statistic for a problem and come up with size α decision rule based on it.
  - Decision rules for normal data with known variance can be found in Theorem 7.4.2 (page 401).
  - Decision rules for normal data with unknown variance can be found in Theorem 7.5.2 (page 415).
  - For binomial data, the decision rule for the large sample (normal-based) test can be found in Theorem 6.3.1 (page 361) and the small sample test (using the exact binomial distribution) can be found on page 364. Use a randomization sub-step for discrete random variables in order to achieve α exactly.
- Come up with size α decision rule using asymptotic chi-square distribution of the LRT.
- Calculate a one and two sided P-values for continuous and discrete distributions.
- Calculate an exact P-value using Fisher's exact test in a hypergeometric model.
- Be able to invert an acceptance region for a test to obtain confidence intervals for the parameter.
- Be able to classify the data structure as binomial, hypergeometric, negative binomial.

Some Problems

1. True or false? Explain your thinking.
   (a) If the significance level α of a test is increased, the power would be expected to increase. ____________
   (b) The power of a test is determined by the null distribution of a statistic. ____________
   (c) The two-sided z test for the difference of means is a uniformly most powerful test. ____________
   (d) The likelihood ratio is a random variable. ____________
   (e) The significance level of a statistical test is equal to the probability that the null hypothesis is true. ____________
   (f) A type I error occurs anytime the test statistic falls in the rejection region of the test. ____________
   (g) If the significance level of a test is decreased, the power would be expected to increase. ____________
   (h) If the P-value is less than .03, the corresponding test will reject at the significance level .02. ____________
   (i) If a chi-square test statistic with 4 degrees of freedom has a value of 8.5, the P-value is less than .05. (You may assume the test rejects for large values of the statistic) ____________
   (j) A size α hypothesis test rejects $H_0 : \theta = \theta_0$ iff the θ_0 does not lie in the $100(1-\alpha)\%$ confidence interval obtained by inverting the hypothesis test. ____________
   (k) The degrees of freedom for the chi-square test of $\sigma^2 = \sigma_0^2$ using a sample of size n from a normal population with known mean is $(n-1)$. ____________

2. Let YBinomial(n, θ). Give the form of the MP test for $H_0 : \theta = \theta_0$ versus $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$. Is the test UMP?

3. Consider the random variable Y which has a binomial distribution with parameters $n = 5$ and $p = \theta$. Let $p(y;\theta)$ denote the mass function of Y at a particular value y and let $H_0 : \theta = 1/2$ versus $H_1 : \theta = 3/4$. The following tabulation gives, at points of positive probability the values of $p(y; \frac{1}{2})$ and $p(y; \frac{3}{4})$, and the ratio $p(y; \frac{3}{4})/p(y; \frac{1}{2})$.

y	0	1	2	3	4	5
$p(y, \frac{1}{2})$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$
$p(y, \frac{3}{4})$	$\frac{1}{1024}$	$\frac{15}{1024}$	$\frac{90}{1024}$	$\frac{270}{1024}$	$\frac{405}{1024}$	$\frac{243}{1024}$
$\frac{p(y,\frac{3}{4})}{p(y,\frac{1}{2})}$	$\frac{1}{32}$	$\frac{3}{32}$	$\frac{9}{32}$	$\frac{27}{32}$	$\frac{81}{32}$	$\frac{243}{32}$

(a) Give the most powerful test of $H_0 : \theta = 1/2$ versus $H_1 : \theta = 3/4$ of size $\alpha = 1/32$.

(b) What is the power of this MP test?

(c) Is this test a size $\alpha = 1/32$ UMP test for testing $H_0 : \theta = 1/2$ versus $H_1 : \theta > 1/2$?

(d) Modify the decision rule of the MP test in part (a) with a randomization sub-step so it achieves an $\alpha = 0.05$ exactly.

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from an exponential distribution with the density function

$$f(x, \theta) \quad = \quad \theta\, exp(-\theta\, x),\ x > 0,\ \theta > 0$$

(a) Derive a LRT of

$$H_0 : \theta = \theta_0$$
$$\text{vs}$$
$$H_1 : \theta \neq \theta_0$$

and show that the rejection region is of the form $\left\{\overline{X}\, exp(-\theta_0 \overline{X}) \leq c\right\}$ where c is a constant chosen to ensure a Type 1 error of a given size.

(b) Does the test from the previous part reject for both large and small values of $\overline{X}$? Or only small values of $\overline{X}$? Or only large values of $\overline{X}$? Explain your thinking.

5. Let $Y_1, Y_2, \ldots, Y_{10}$ be a random i.i.d. sample of size 10 from a Poisson distribution with mean θ. Let $L(\theta|\mathbf{y})$ be the joint probability mass function of $Y_1, Y_2, \ldots, Y_{10}$. The problem is to test $H_0 : \theta = \frac{1}{2}$ against $H_1 : \theta = 1$.

(a) Show that $L(1|\mathbf{y})/L(\frac{1}{2}|\mathbf{y}) \geq k$ is equivalent to $x = \sum_{i=1}^{10} y_i \geq c$ where k and c are some constants.

(b) In order to make α exactly equal to 0.05, show that H_0 is rejected if and only if $x > 9$ and, if $x = 9$, reject H_0 with probability $\frac{1}{2}$ (using an auxillary random sub-step).

6. In testing $H_0 : \sigma^2 \leq \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$ for a $N(\mu, \sigma^2)$ suppose we use the critical region defined by $(n-1)S^2/\sigma_0^2 \geq k$ where $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is the sample variance.

(a) If $n = 13$ and we want the test to have size $\alpha = 0.025$, what should we use for k?

(b) If you invert the hypothesis test, will you get an upper or lower confidence bound for σ^2? Go ahead and give an expression for the confidence bound for σ^2.

7. Suppose $X_1, X_2, \ldots, X_n \sim N(\theta, \sigma^2)$ with σ^2 unknown. Invert the size α LRT test for testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ to obtain a $100(1-\alpha)\%$ confidence interval for θ.

8. The following is R output for testing the mean of a normal distribution with variance unknown.

```
> t.test(x,mu=70,alternative="two.sided",conf.level=0.95)
data:  x
t = 1.3383, df = 5, p-value = 0.2384
alternative hypothesis: true mean is not equal to 70
95 percent confidence interval:
66.31687 81.68313
```

(a) What is the sample size?

(b) What is the value for the sample mean?

(c) What is the value for the sample SD $s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$?

(d) Calculate a 95% lower confidence bound for σ^2.

9. A certain genetic model suggests that the probabilities of a particular trinomial distribution are, respectively, $p_1 = p^2$, $p_2 = 2p(1-p)$, and $p_3 = (1-p)^2$, where $0 < p < 1$. If X_1, X_2, X_3 represent the respective frequencies in n independent trials, explain how we could check the adequacy of the genetic model using a size α LRT test.

10. Identify the data structure described in the problems below. This means define the random variable(s) of interest in this context and a plausible distribution for it.

(a) The Park Service in a city is experimenting with two different forms of dandelion control. They apply it to 14 lawns in city parks by dividing each lawn into two equal parts and randomly assigning each weed killer to one of the two halves. After three weeks they inspect the lawns, and find that weed killer A (which is more expensive) has been more effective on 10 of the lawns.

(b) An undesirable effect of some antihistamines (cold medicine) is drowsiness which can be measured by the flicker frequency of patients (number of flicks of the eyelids per minute). One study reported data on 9 patients, each given meclastine (A), or a placebo (B) in random order.

Patient	Drug A	Drug B
1	31.25	33.12
2	26.63	26.00
3	24.87	26.13
4	28.75	29.63
5	28.63	28.37
6	30.63	31.25
7	24.00	25.50
8	30.12	28.50
9	25.13	27.00

(c) Is there an association between gender and bathing? Students in a statistics class surveyed a random sample of 50 female students and 50 male students, asking each if they preferred a bath or a shower. The table below shows the data.

	Male	Female	Total
Bath	6	21	27
Shower	44	29	73
Total	50	50	100

11. Suppose a retrospective study is done among men ages 50-54 in a specific county who died over a 1 month period. The investigators tried to include an approximately equal number of men who died from CVD (the cases) and men who died from other causes (the controls).

Of the 35 who died from CVD, 5 were on a high salt diet before they died. Of the 25 people who died from other causes, only 2 were on a high salt diet.

The data are presented below.

	Type of diet		
Cause of death	High Salt	Low salt	Total
Non CVD	2	23	25
CVD	5	30	35
Total	7	53	60

Does the data provide evidence that salt intake increases the risk of CVD? Provide an answer to this question using

(a) A p-value based on a two sample (normal based) test of binomial proportions.

(b) An exact p-value based on Fisher's exact test

For each method, clearly STATE/DRAW the model you are assuming. Also comment on whether each model is a reasonable choice for this problem or not.