Q2

Let p_1, p_2, p_3, p_4 denote multinomial probabilities for each interval based on presumed model. σ, θ are unknown

$$
\begin{aligned}
p_1 &= \int_{-\infty}^{600} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\theta)^2}{2\sigma^2}} dx \\
&= \Phi(\frac{600-\theta}{\sigma}), \text{ where } \Phi \text{ is cdf of standard normal} \\
p_2 &= \Phi(\frac{1200-\theta}{\sigma}) - \Phi(\frac{600-\theta}{\sigma}) \\
p_3 &= \Phi(\frac{1800-\theta}{\sigma}) - \Phi(\frac{1200-\theta}{\sigma}) \\
p_4 &= 1 - \Phi(\frac{1800-\theta}{\sigma})
\end{aligned}
$$

Let x_1, x_2, x_3, x_4 denote number of observed data that fall in each interval.

$x_1 = 9, x_2 = 20, x_3 = 7, x_4 = 2$

Then the likelihood function based on observed data and presumed multinomial probability can be represented as

$$
\begin{aligned}
L(\theta|x_1, x_2, x_3, x_4) &\propto \prod (p_i)^{x_i} \\
&\propto \Phi(\frac{600-\theta}{\sigma})^9 \times [\Phi(\frac{1200-\theta}{\sigma}) - \Phi(\frac{600-\theta}{\sigma})]^{20} \\
&\times [\Phi(\frac{1800-\theta}{\sigma}) - \Phi(\frac{1200-\theta}{\sigma})]^7 \times [1 - \Phi(\frac{1800-\theta}{\sigma})]^2
\end{aligned}
$$

MLE of θ and σ are 917.6799 477.9240.

Based on MLE's, values in each interval are 9.618508 17.842061 9.306897 1.232534

$$
\begin{aligned}
D(x) &= \sum_{i=1}^{4} \frac{(x_i - np_i)^2}{np_i} \\
&= 1.350458
\end{aligned}
$$

There are four categories and two unknown parameters, thus, degree of freedom is 1.

$$
\begin{aligned}
p - value &= P(\chi_1^2 \geq D(x)) \\
&= 0.245198
\end{aligned}
$$

chi-square test p-value of observed data is 0.245198, therefore, under test size 0.05, observed data failed to provide significant evidence that normality model is incorrect.

(R code in Appendix)