STAT 403 Spring 2018
HW01

Nan Tang
1662478

April 6, 2019

1 Q1

1.1 Q1-1

$$\begin{aligned}
Bias(\bar{X}) &= \mathbb{E}(\bar{X}) - \beta \\
&= \mathbb{E}(\frac{1}{n}\sum_{i=1}^{n} X_i) - \beta \\
&= \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}(X_i) - \beta \\
&= \mathbb{E}(X_i) - \beta
\end{aligned}$$

Note that X follows an exponential distribution, the expected value for any sample from X, X_i is equal to β.

$$Bias(\bar{X}) = \beta - \beta = 0$$

The sample average $\bar{X}$ is an unbiased estimator.

$$\begin{aligned}
Var(\bar{X}) &= Var(\frac{1}{n}\sum_{i=1}^{n} X_i)) \\
&= \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i)
\end{aligned}$$

The variance of X_i from exponential distribution is equal to β^2.

$$
\begin{aligned}
Var(\bar{X}) &= \frac{1}{n^2}\sum_{i=1}^{n}\beta^2 \\
&= \frac{\beta}{n}
\end{aligned}
$$

1.2 Q1-2

2 Q2

3 Q3