STAT/Q SCI 403: Introduction to Resampling Method
Spring 2019
Homework 01

Instructions:

- You have to submit all your answers in a single PDF file generated by either LaTeX or *Rmarkdown*.
- You may use the LaTeXtemplate `HW_template.tex` to submit your answer.
- For questions using R, you have to attach your code in the PDF file. If the question ask you to plot something, you need to attach the plot in the PDF as well.
- If the question asks you to show a figure, the clarity of the figure will also be graded.
- The total score of this homework is 8 points.
- Questions with ♠ will be difficult questions.

Questions:

1. Let $X_1, \cdots, X_n$ be IID random points from $\mathsf{Exp}(1/\beta)$. The PDF of $\mathsf{Exp}(1/\beta)$ is

$$p(x) = \frac{1}{\beta} e^{-x/\beta}$$

for $x \geq 0$. Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ be the sample average. Let β be the parameter of interest that we want to estimate.

   (a) **(1 pt)** What is the bias and variance of using the sample average $\bar{X}_n$ as the estimator of β?

   (b) **(0.5 pt)** What is the mean square error of using $\bar{X}_n$ as the estimator of β?

   (c) **(0.5 pt)** Does $\bar{X}_n$ converges to β? Why?

   (d) **(0.5 pt)** ♠ Now consider a new estimator $\widehat{\beta} = a \times \bar{X}_n$, where $a \in \mathbb{R}$ is a real number. What is the mean square error of $\widehat{\beta}$?

   (e) **(0.5 pt)** ♠ To minimize the mean square error, which value of a should we take? Does this give us an estimator that has a lower mean square error than the sample mean $\bar{X}_n$? (Note: this new estimator is related to the *Stein's Shrinkage estimator*.)

2. **(1 pt)** Use R to plot the function $f(x) = e^{-\sin(x)} \cdot \cos(\pi \cdot x)$ for $x \in [-10, 10]$. You need to show the function as a curve.

3. **(2 pt)** Use R to generate 5000 data points from $N(2, 2^2)$. Note that $N(2, 2^2)$ is the normal distribution with mean 2 and variance $2^2 = 4$. Plot the density histogram (Y-axis is density) with 50 bins. Attach a density curve of $N(2, 2^2)$ to the histogram.

4. Let $X_1, X_2 \sim \mathsf{Uni}[2, 4]$. Let $\bar{X}_2$ be the sample mean. We will use R to check the distribution of $\bar{X}_2$.

   (a) **(1.5 pt)** Use the `for` loop to generate at least 100000 realizations of $\bar{X}_2$ and plot the corresponding histogram.

   (b) **(0.5 pt)** ♠ The density curve of $\bar{X}_2$ is

$$p(x) = \begin{cases} x - 2, & \text{when } 2 \leq x \leq 3 \\ 4 - x, & \text{when } 3 < x \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

   Attach the corresponding density curve of $\bar{X}_2$ to the histogram. (Note: if you have time, think about why this is the density curve.)