STAT 403 Spring 2018
HW06

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Q1

Q1-1

Q1-2

$$
\begin{aligned}
Var(\bar{X^\star}) &= Var(\frac{1}{n}\sum_{i=1}^{n} X_i), \text{ since } \bar{X^\star} = \frac{1}{n}\sum_{i=1}^{n} X_i \\
&= \frac{Var(\sum_{i=1}^{n} X_i)}{n^2}, \text{ by property of variance} \\
&= \frac{\sum_{i=1}^{n} Var(X_i)}{n^2} \\
&= \frac{Var(X)}{n} \\
\text{Note that } Var(X) &= \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2 \\
Var(\bar{X^\star}) &= \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n^2}
\end{aligned}
$$

$$
\begin{aligned}
S_n^2 &= \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 \\
(n-1)S_n^2 &= \sum_{i=1}^{n}(X_i - \bar{X})^2 \\
Var(\bar{X^\star}) &= \frac{(n-1)S_n^2}{n^2}
\end{aligned}
$$

As $n \to \infty$, $\frac{n-1}{n^2} \to n$. Therefore, when bootstrap sample size is large enough, variance of bootstrap sample mean is equal to $\frac{S_n^2}{n}$.

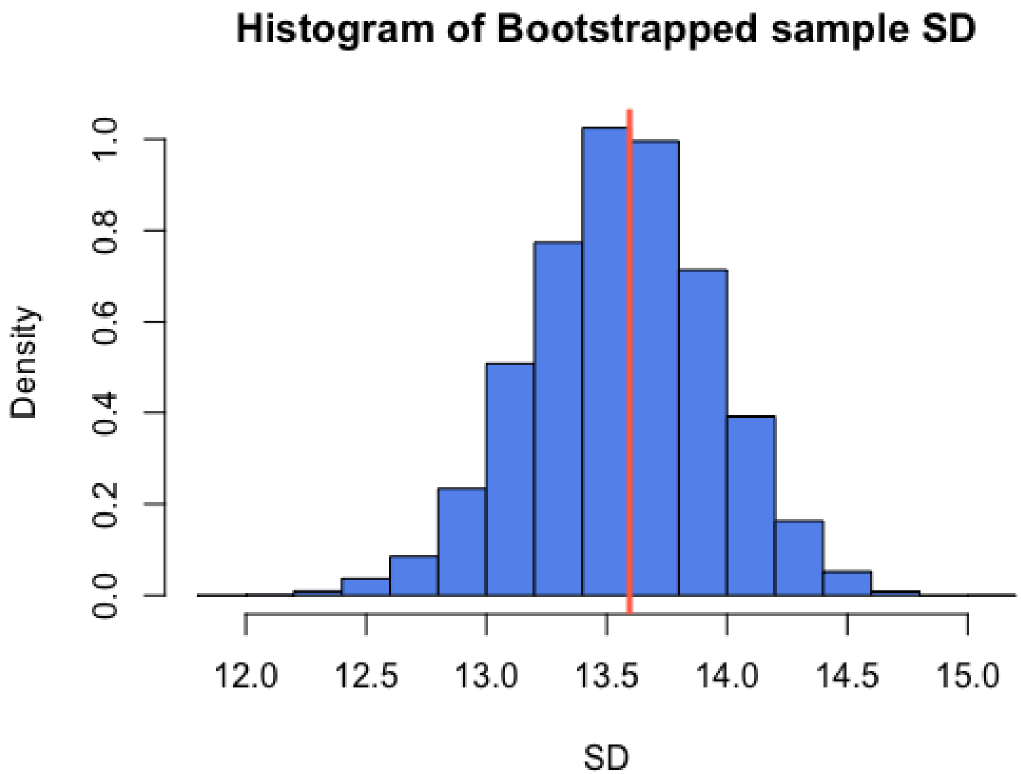
Q2

Q2-a

```
origin_sd <- sd(faithful$waiting)
bt_size <- 10000
sample_size <- length(faithful$waiting)
bt_result <- rep(NA, bt_size)

for (ii in 1:bt_size) {
  index <- sample(sample_size, sample_size, replace=T)
  bt_dt <- faithful$waiting[index]
  bt_result[ii] <- sd(bt_dt)
}

hist(bt_result, probability=T, col='cornflowerblue', xlab='SD',
     ylab='Density', main='Histogram of Bootstrapped sample SD')
abline(v=origin_sd, lwd=3, lty=1, col='coral1')
```



Q2-b

```
> sd(bt_result)^2
[1] 0.1472021
> mean((bt_result - origin_sd)^2)
[1] 0.1483897
```

Variance of sample SD is 0.1472021, and MSE is 0.1483897.

Q2-c

```
bt_sd <- sd(bt_result)
lower_bd <- origin_sd + qnorm(0.975) * bt_sd
upper_bd <- origin_sd - qnorm(0.975) * bt_sd
> lower_bd
[1] 14.34695
> upper_bd
[1] 12.843
```

```
> quantile(bt_result, c(0.025, 0.975))
    2.5%    97.5%
12.78773 14.29303
```

By theorem, the 95% confidence interval is $[12.843, 14.34695]$. In bootstrap samples, 95% interquantile is $[12.78773, 14.29303]$.

Q2-d

$$\text{p-value} = \text{P(getting value more extreme than sample SD}|H0)$$

```
bt_var <- sd(bt_result)^2
p_value <- 2 * pnorm(origin_sd, 15, bt_var)
> p_value
[1] 1.362628e-21
```

Q2-e

The variance of bootstrapped samples is $\frac{\sigma^2}{n}$, indicating the value will decrease as sample size increases. Larger sample size minimizes the margin of error, of which small margin of error make the estimation more accurate.

Q3

Q3-a

```
admission_dt <- UCBAdmissions[,,1]

origin_or <- (admission_dt[1,1] * admission_dt[2,2]) /
  (admission_dt[2,1] * admission_dt[1,2])

## generate dataset based on given information
n_male <- admission_dt[1,1] + admission_dt[2,1]
n_female <- admission_dt[1,2] + admission_dt[2,2]
n <- n_male + n_female

data_pull <- c(rep(1, admission_dt[1,1] + admission_dt[1,2]),
               rep(0, admission_dt[2,1] + admission_dt[2,2]))

N <- 10000
bt_or <- rep(NA, N)
for (ii in 1:N) {
```

```
  sp_index <- sample(n, n, replace=T)
  sp_dt <- data_pull[sp_index]
  sp_male <- sp_dt[1:n_male]
  sp_female <- sp_dt[(n_male+1):n]
  male_admit <- sum(sp_male)
  male_reject <- length(sp_male) - male_admit
  female_admit <- sum(sp_female)
  female_reject <- length(sp_female) - female_admit
  sp_or <- (male_admit * female_reject) / (male_reject * female_admit)
  bt_or[ii] <- sp_or
}

bt_mse <- mean((bt_or - mean(bt_or))^2)

> bt_mse
[1] 0.04906438
```

Under bootstrap size of 10000, I get the MSE of OR is 0.04906438.

Q3-b

```
bt_sd <- sd(bt_or)
p_value <- pnorm(origin_or, 1, bt_sd)

> p_value
[1] 0.001652304
```

Q3-c