Name: ________________________
ID: __________________________

Stat 421, Test 2, Fall, Nov. 15, 2017; Marzban

7.5+16

ONLY a half-size "cheat sheet" is allowed

Multiple choice: Circle all the correct answers; there is wrong-answer penalty

For rest, SHOW answer & work; NO CREDIT for correct answer without explanation

Points

$E[MS] = \sigma_\epsilon^2 + \cdots$ (e.g. lect 13)

1 **1.** Suppose the **true model** for a certain problem is known to be $y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}\ i = 1 \cdots a, j = 1 \cdots b$, where τ_i is the treatment effect, and β_j is the block effect. Then $E[\underbrace{\sum_j (\overline{y_{.j}} - \overline{y_{..}})^2}_{SS_{block}}]$ is proportional to

$E[SS_{block}] = \sigma_\epsilon^2 + \Sigma\beta^2 \cdots$

a) 0 b) σ_ϵ^2 c) none of the above.

lect 11/p.4

1 **2.** In an RCBD involving one treatment and one block factor, the predicted response (or the fitted value) depends on (or varies with)

$\hat{Y}_{ij} = \hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j$

a) The level of the treatment factor
b) The level of the block factor
c) Both a and b
d) none of the above

1 **3.** Suppose we are interested in performing a 1-sided test of whether a specific contrast is greater than zero. The most appropriate test(s) is/are

a) t-test b) chi-squared test c) F test

lect 13/p.8

1 **4.** In a replicated factorial design involving 3 treatments A, B, C, which of the following is true?
a) An F-test of A effect cannot be performed, if there exists an AB interaction term in the model.
b) An F-test of A effect cannot be performed, if there exists an ABC interaction term in the model.
c) An F-test of ABC effect cannot be performed.
d) None of the above.

lect 14/p7

1.5 **5.** We have two treatment factors A, B, and two block factors C, D, respectively with 3, 3, 9, and 9 levels. Possible models/designs are
a) lm(y ~ A + B + C + D) on $3 \times 3 \times 9 \times 9$ runs from a factorial design.
b) lm(y ~ A * B * C * D) on $3 \times 3 \times 9 \times 9 \times n$ runs from a replicated factorial design.
c) lm(y ~ A + B + C + D) on 9×9 runs from an LSD, with A and B combined into a 9-level factor.
d) lm(y ~ E + C + D) on 9×9 runs from an LSD, with E defined as a 9-level factor combining A and B.

1 **6.** In a 2^3 design, involving factors A, B, and C. Which statement is FALSE?
a) In a model without an ABC interaction, the numerical estimate of the A effect will not depend on *abc* sum of y's. Just look at the expression for the A effect. You will see (abc).
b) In a model without an ABC interaction, the numerical estimate of the A effect will be equal to that from a model with ABC interaction.
c) In a model without a BC interaction, the numerical estimate of the A effect will be equal to that from a model with BC interaction.
d) None of the above.

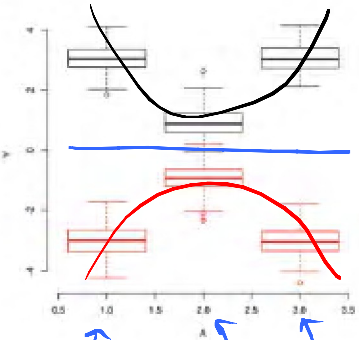
The effect of A does not change as we add things to model. SSA does not change either. SSE does. And therefore, p-value of $H_0: \alpha_i = 0$ changes.

lect 17/p.6

1 **7.** In 2^k, if there are only two blocks, then it's natural to confound _____ with the block effect
a) a main effect b) a 2-way interaction c) the highest-order interaction d) None of the above.

2016/7

1 **8.** Consider a problem involving a response Y observed at each of 3 levels of factor A. The data are shown in the adjacent figure. The black (red) boxplot denotes the data collected on day 1 (2).



diff. shape

Numeric; so order matters and so, shape matters.

a) Without doing any tests (i.e., based on the figure only), is factor A useful? Explain in 1 sentence.

No, because The mean of y does not vary with A.

Yes, if you pay attention to "within Day."

1.5

1 **b)** If you run a test of $\alpha_i = 0$ in the model $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$ what kind of p-value would you expect? Circle: Small Large

1 **c)** If you run a test of $\alpha_i = 0$ in the model $y_{ijk} = \mu + \alpha_i + \beta_j + \epsilon_{ijk}$ with β corresponding to the Day factor, what kind of p-value would you expect? Circle: Small Large for each level of B, 1 box plot is different from The other 2.

1 **d)** If you run a test of $(\alpha\beta)_{ij} = 0$ in the model $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$, what kind of p-value would you expect? Circle: Small Large

The effect of A on Y is different across the levels of B

2013/11 & lect-hw12_4

~~2~~ 1.5 **9.** The following values of the response have been observed in a Standard LSD. Compute $SS_{\text{treatment}}$. Hint: recall there is a "quick" way and a "long" way.

	B_1	B_2	B_3
A_1	$+1$ C_1	-1 C_2	-1 C_3
A_2	0 C_2	-1 C_3	0 C_1
A_3	-1 C_3	$+2$ C_1	$+1$ C_2

$$SS_c = \frac{1}{3}\Big[(+1+0+2)^2 \leftarrow C_1 \quad +(-1+0+1)^2 \leftarrow C_2 \quad +(-1-1-1)^2\Big] \leftarrow C_3 \quad -\frac{1}{9}(0)^2$$

$$= \frac{1}{3}[9+0+9] - 0 = \boxed{6}$$

title 2016/11

10. Consider the model $y_{ijk} = (\alpha\beta)_{ij} + \epsilon_{ijk}$ $, i = 1-a, j = 1-b, k = 1-n$.

~~2~~ 1 a) **Starting from the expression for SSE**, find the least-squares equations, and estimate the interaction term.

$$SSE = \sum_{ijk}\left(Y_{ijk} - (\alpha\beta)_{ij}\right)^2, \quad \frac{\partial}{\partial(\alpha\beta)_{ij}} \Rightarrow \sum_k \left(Y_{ijk} - (\widehat{\alpha\beta})_{ij}\right) = 0 \Rightarrow \underline{(\widehat{\alpha\beta})_{ij} = \overline{Y}_{ij.}}$$

~1 b) How many independent parameters does this model have, and which are they? ab, $(\alpha\beta)_{ij}$

~1 2 c) How many independent least-squares equations are there for this problem? ab

~1 d) How may constraints are necessary? 0 Graded according to consistency with b, c.

hw-lect10-1

~2 **11.** For the model $y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}$, the estimated SSE is $\sum_{ij}(y_{ij} - \overline{y_{i.}} - \overline{y_{.j}} + \overline{y_{..}})^2$. Similarly, for the model $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$, the estimated SSE is $\sum_{ij}(y_{ij} - \overline{y_{i.}})^2$. Show that the difference between these two SSE's is related to the SS of another effect; what is that effect?

$a^2 - b^2 = (a-b)(a+b)$

$$SSE_{red.} - SSE_{full} = \sum_{ij}\left[(Y_{ij} - \overline{Y}_{i.})^2 - (Y_{ij} - \overline{Y}_{i.} - \overline{Y}_{.j} + \overline{Y}_{..})^2\right]$$

$$= \sum_{ij}\left[\cancel{Y_{ij}} - \cancel{\overline{Y}_{i.}} - \cancel{Y_{ij}} + \cancel{\overline{Y}_{i.}} + \overline{Y}_{.j} - \overline{Y}_{..}\right]\left[Y_{ij} - \overline{Y}_{i.} + Y_{ij} - \overline{Y}_{i.} - \overline{Y}_{.j} + \overline{Y}_{..}\right]$$

$$= \sum_j (\overline{Y}_{.j} - \overline{Y}_{..}) \underbrace{\sum_i^a (2Y_{ij} - 2\overline{Y}_{i.} - \overline{Y}_{.j} + \overline{Y}_{..})}_{2a\overline{Y}_{.j} - 2a\overline{Y}_{..} - a\overline{Y}_{.j} + a\overline{Y}_{..} = a(\overline{Y}_{.j} - \overline{Y}_{..})} = a\sum_j (\overline{Y}_{.j} - \overline{Y}_{..})^2 = SSB.$$

Many students did
$SST = SSA + SSB + SSE_f$
$SST = SSA + SSE_r$
$\therefore SSE_r - SSE_f = SSB$
This is OK, IF you show The 2 SSAs are equal.

~2 2 lect15/magic

12. In a 2^4 design involving factros A, B, C, D, what is the contrast vector for the AB effect, in the Yates order (i.e., A changes fastest).

(1) a ... $\overbrace{\qquad\qquad}^{2^4=16 \text{ runs}}$

A − + − + − + − + − + − + − + − +

B − − + + − − + + − − + + − − + +

AB + − − + + − − + + − − + + − − + ⟸ Answer.

C ...

D ... } not necessary.

~~2~~ 1.5 lect16/p4

13. In 2^k design with n replications, every individual effect can be tested with a t-test. If there are p terms in the model (including μ, but excluding ϵ), what is the *df* of that test (i.e., the df of SSE)? Show work.

$$y: \quad Y_{ij\ldots} = \overbrace{\mu + \alpha_i + \beta_j + \cdots}^{p \text{ terms.}} + \epsilon_{ij\ldots}$$

$$SS: \quad SST = \underbrace{SSA + SSB + \cdots}_{} + SSE$$

$$df: \quad 2^k n - 1 = p - 1 + \boxed{2^k \cdot n - p}$$

each SS has df = 1

~2 2 lect17/p.8

14. A scientist who does not know much about experimental design performed the 8 runs in a 2^3 design in the following 4 blocks: $[(1), a], [b, ab], [c, ac], [bc, abc]$. Which effects are going to be confounded with block? Show work.

The runs in [(1), a] have B and C low

" [b, ab] have B high

[c, ac] have C high

[bc, abc] have B and C high

⟹ B and C

∴ B and C (and consequently BC) are confounded with block

FYI

	A	B	C
(1)	−	−	−
a	+	−	−
b	−	+	−
ab	+	+	−
c	−	−	+
ac	+	−	+
bc	−	+	+
abc	+	+	+

~~2~~ 1.5

15. Consider a 2^4 design performed in 8 blocks. What is the **total** number of effects that will be confounded with blocks? Show work.

Blocking the 2^4 runs according to the levels of 1 factor gives 2 blocks.

" " 2 factors " 4 " .

" " 3 " " 8 " .

Denote these 3 factors X, Y, Z, which are confounded with block.

But then, XY, XZ, YZ, and XYZ are also confounded. Total = 7