	2 hvs. ONLY a half-size "cheat sheet" is allowed			
	Multiple choice: Circle all the correct answers; there is wrong-answer	penalt	у	
m Points	For rest, SHOW answer & work; NO CREDIT for correct answer without	explai	nation	
	$\frac{1 - \ln a k(>3)}{1 - \ln a k(>3)}$ -factor problem, the sum-of-ys, abe, appears in the (Circle all c	orrect	answe	ers)
1)	(a) A effect in a full 2^k model. (c) ABC effect of an incomplete			
	(b) A effect in an additive 2^k model. (d) ABC effect of an incomplete			
	I ignored These. In These Cas	es, A	4BC is	not estimable
)	2. Which is TRUE? In an unreplicated 2^3 experiment performed in 4 blocks, if	f a 2-v	vay inte	eraction
	is confounded with block, then it follows that effect must also be confounded.			
	a) no other description of a conformation of the description of the de	dw	不り	och. One is X
	b) some main (1-way) e) Insufficient info.			UVW
_	c) some 1- or 2-way If The other is $Z(1-way) \Rightarrow xyz$ also confined $xz(2-way) \Rightarrow yz$		+	
)	xZ(2-way) => YZ 11 "			
	3. In a certain design with the adjacent $\pm \tanh[e_3-v_{sy}] \Rightarrow \frac{2}{3}$	• • •		•
	if $XYZ=+1$ and $XYZ=-1$ are run in 2 blocks, respectively, then	• • •	+	_
	a) XYZ and UVW are each confounded with block. 1 block effect.	• • •	_	T .
	b) XYZ + UVW is confounded with block.	• • •		•
2	c) Neither A nor B	• • •	_	+
	4. We have performed a 2^{k-p} experiment, and are now planning on a fold-over.	The i	number	of runs
	in the combined experiment is 2 for a single-factor fold-over, and 2 for a	a com	plete fo	old-over.
7	let 18-2, and 19-2) The fold-over toubles The runs I		_	
(har-	5 In a 2^k design in 4 incomplete blocks, we choose to confound X and Y with	block	a. As w	e know,
	XY is then also confounded with block. Note that these statements refer t	o the	effects	. Now,
	suppose we block in a way that X and XY are each confounded with block, th	en the	e runs i	in
	(a) the new blocks will be the same as the old ones.			
	b) the new blocks will not necessarily be the same as the old ones.			
	c) whether or not the new blocks will be the same as the old ones depends on	the a	lias str	ucture.
		1 .	1 3/3///	7 (T)
075)	6. In an incomplete design involving 3 factors, suppose we find that X is aliased as a suppose we find that X is a suppose we find that	ed wit	h XYZ	L. Then,
	$(a)[X] = X + XYZ \qquad (b)[XYZ] = X + XYZ \qquad (c)[X] = [XYZ]$	ı) nor	ie or tn	le above
\sim	7 A	_ :		ADEF DEF
(1)	7. An article uses an unreplicated 2^{6-2} design, and reports one of the alias characteristic that the defining relations are (single all correct engages)	ams a	s A = 1	BCE = VEF
	ABCE = BCDF = 1 c) $ABCE = BDEF = 1$		ABC	E BCDF
	b) $ABCE = ADEF = 1$ d) Cannot be determined from only one of	ahain		- IDCUP
	d) Cannot be determined from only one of	onam.		
J 2	8. Suppose you are planning on performing a 2^k experiment but there is	ച സ്വവ	d chan	oce that
<i>5</i> <u>2</u>	your measuring device will heat-up and stop working about a quarter of the	_		
	experiment. Although the device will work again after it is allowed to cool do		·	9
	break permanantly after about half of the 2^k runs are performed. Briefly expl		_	
	we have learned) what is the "best" design for this experiment	,		
	k-1 here is the sest design for this experiment.	s Se	povat	ed by
	2 pin 2 blocks. Decause each of the quarter runs is a cooling down phase, ic. restriction	1 000	V0-1-	
	a country again, process, for registron		7 4443	001 600 1 801
	because at The vory end we have done only half The 2	ru	ns,	
	150			
	(or A fold-over of 2k-2)			
	(A told-over of 2"-L)			

Stat 421, Test 3, Fall, Dec. 11, 2018; Marzban

Name: ______ 1D: _____ 9. For a 2^3 design performed in 4 incomplete blocks, in a lecture we showed that the 3 (zero-sum) block effects/contrasts $(B_1 + B_2) - (B_3 + B_4)$, $(B_1 + B_3) - (B_2 + B_4)$, and $(B_1 + B_4) - (B_2 + B_3)$, are proportional to the 3 effects (say, X, Y, Z) confounded with block. In a hw you showed that the block effect $(B_1 - \bar{B}_1)$ is confounded with X + Y + Z.

a) What about the block effect $(B_2 - \bar{B}_1)$, or $(B_3 - \bar{B}_1)$ - are any effects confounded with them? Circle Yes or No. Because each of The 4-1=3 block effects must be confounded. b) If No, explain why not. If Yes, write one of the effects that is confounded with either $(B_2 - \bar{B}_1)$ or $(B_3 - \bar{B}_1)$; no explanations necessary. Feel free to guess (yes, guess), based on your experience.

X+Y-Z is confounded with B_2-B . Z-B. Z-B. Z-B. Z-B. Z-B. Z-B. Z-B. Z-B.

c) Finally, show that any block effect of the type $(B_i - \bar{B}_i)$, with $i = 1 \cdots n$, where n is the total number of blocks is a zero-sum contrast in B_i . Hint: find the contrast vector. This doesn't have to be a mathematically rigorous proof, but do show some math.

$$B_i - \overline{B}_i = B_i - \frac{1}{n} (B_1 + B_2 + \dots + B_i' + \dots + B_n) = \frac{1}{n} (-B_1 - B_2 + \dots + (n-1)B_i' - \dots - B_n)$$

contrast vector = $\frac{1}{n} (-1, -1, \dots, (n-1), \dots, -1)$

10. In a 23 experiment, the 8 runs are performed in the following four blocks: [(1), abc], [a, bc], [b, ac], [c, ab]. What is/are the effect(s) selected to confound with block? Do NOT use the ± table, but show your work thinking. In each block, A = +, B = +, C = +; 50 no 2 of [(1), abc] (+ + Those can be used. Similarly, ABC = + in all 4 blocks;

Factorial Effect

In a 2^3 experiment, the 8 runs were performed in the following two blocks: [(1), a, b, c] and [ab, ac, bc, abc]. Can you determine the effect that is confounded with the block effect? If so, find it. If not, explain why? Consult the \pm table provided.

_	31och	1	A	В	AB	C	AC	BC	ABC
This blocking	1	(I) +	_	_	+	_	+	+	_
abeg not	+	a +	+	_	_	-	_	+	+
	+	b +	2200	+	-	_	+	122	+
correspond to	_	ab +	+	+	+		_		_
any effect	+	C +	_	_	+	+	_	1	+
1		a(+	+	-	_	+	+	-	-
l		bc+	<u></u>	+	_	+	_	+	(<u></u>
116/027		abc+	+	+	+	+	+	+	+

12. Is the following argument correct? If so, say YES. If not point out which part(s) is/are wrong, and explain why not. A complete 2^k experiment is performed in 2^p incomplete blocks.

- a) To construct the 2^p blocks, we need to choose p effects to confound with block effects.
- b) But we know that in a design with 2^p blocks, there are $2^p 1$ block effects.
- c) Therefore, $2^p 1 p$ block effects are not confounded with anything. \leftarrow WRONG.

But Then all products of These are also confounded with block, ic.

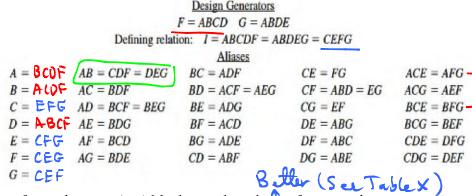
XIX2, (2-way)

XIX2, (3-way)

The sum of all These, and these is (2k-1),

and That is exactly the number of block effects (b

- 13. The following is the alias structure for a 2^{7-2} design.
- a) Write-in the lowest-order term (just 1 term) on the right-hand-side of the chains involving the main effects.



b) We want to perform the runs in 4 blocks, and we know for sure that we want to confound ACE ~ 2 with block. Suppose we don't care much about the BCE and the BCG effect. Which one should we confound with block? Explain.

c) What is/are the defining relation(s) for the complete fold-over? Explain. It is not necessary to include all aliased effects.

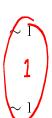
d) If we treat the two fractions as blocks, what is/are the effect(s) confounded with block? It necessary to include all aliased effects.

Effets That change between prince & all. are ABCDF, ABDEG. But There is is only I block effect, so The effect confi with block is ABCDF+ ABDEG.

Consider a 2^{6-2} design with defining relations ABCE = 1 and BCDF = 1. Show that a complete fold-over does not allow any de-aliasing to occur at all. Hint: You do not need to work out the alias structure, nor the \pm table.

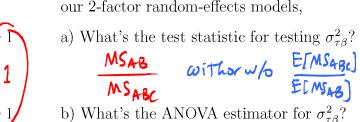
 ~ 2

15. Consider a Resolution IV design. The lowest-order defining relation will consist of a 4-letter word. Now suppose we perform a fold-over such that each of those 4 letters is flipped simultaneously. As such, the defining relation of the alternative fraction will include the same 4-letter word. Show that in the combined design each of the letters in the 4-letter word cannot be de-aliased from 3-way interactions involving the other 3 letters. Hint: Use X, Y, Z, W as the 4 letters.



16. The adjacent table shows E[MS] for all the effects in a 3-factor random-effects model. Based on what we have learned from 6 our 2-factor random-effect

s models,
c for testing $\sigma_{\tau\beta}^2$?
W/O E[MSABE]
FIME 2



Model Term
 Factor
 Expected Mean Squares

$$\sigma_i^2$$
 A_i main effect
 σ_i^2
 $en\sigma_{eg}^2 + bn\sigma_{eg}^2 - n\sigma_{eg}^2 + ben\sigma_{e}^2$
 B_i
 B_i main effect
 σ_i^2
 $en\sigma_{eg}^2 + an\sigma_{gg}^2 - an\sigma_{eg}^2$
 γ_i
 C_i main effect
 σ_i^2
 $en\sigma_{eg}^2 + an\sigma_{eg}^2 + an\sigma_{eg}^2$
 $(r\beta)_{ij}$
 AB_i two-factor interaction
 σ_i^2
 $en\sigma_{eg}^2 - an\sigma_{eg}^2$
 $(r\beta\gamma)_{ij}$
 AC_i two-factor interaction
 σ_i^2
 $en\sigma_{eg}^2 - an\sigma_{eg}^2$
 $(r\beta\gamma)_{ijk}$
 ABC_i three-factor interaction
 σ_i^2
 $en\sigma_{eg}^2$
 e_{ijk}
 ens_{eg}^2
 ens_{eg}^2

c) For which variance component(s) can we build CIs? Only

17. For a single-factor random-effects model $y_{ij} = \mu + \tau_i + \epsilon_{ij}$, with the usual normality and iid assumptions, show that $E[SST] = (an-1)\sigma_{\epsilon}^2 + (a-1)n\sigma_{\tau}^2$. You may use $E[(\epsilon_{ij} - \overline{\epsilon_{..}})^2] = (1 - \frac{1}{an})\sigma_{\epsilon}^2$.

$$E[SST] = E[\underbrace{S}_{ij} (Y_{ij} - \overline{Y}_{i})^{2}] = \underbrace{S}_{ij} E[(M+T_{i} + E_{ij} - M - \overline{T}_{i} - \overline{E}_{i})^{2}]$$

$$= \underbrace{S}_{ij} \left[E[(T_{i} - \overline{T}_{i})^{2}] + E[(E_{ij} - \overline{E}_{i})^{2}] + E[2(T_{i} - \overline{T}_{i}) (E_{ij} - \overline{E}_{i})]\right]}_{\text{and } E \text{ ave indep.}}$$

$$V[T_{i} - \overline{T}_{i}] + \underbrace{E^{2}[T_{i} - \overline{T}_{i}]}_{\text{O}} = \underbrace{E[T_{i} - \overline{T}_{i}]}_{\text{O}} = \underbrace{E[T_{i} - \overline{T}_{i}]}_{\text{O}} = \underbrace{E[G_{ij}] = 0}_{\text{O}}$$

$$E(SST) = \frac{1}{(1-\frac{1}{a})} \left(\frac{1-\frac{1}{a}}{1-\frac{1}{a}} \right) \frac{1}{\sigma_{r}^{2}} + \left(\frac{1-\frac{1}{a}}{1-\frac{1}{a}} \right) \frac{1}{\sigma_{e}^{2}}$$

$$= (a-1)n \sigma_{r}^{2} + (an-1)\sigma_{e}^{2}$$

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