Lect 21-4

a

2^{5-2} design with defined relation $ABD = ACE = 1$

```
y <- c(7,9,34,55,16,20,40,60,
       8,10,32,50,18,21,44,61,
       8,12,35,52,15,22,45,65,
       6,10,30,53,15,20,41,63)
design <- gen.factorial(c(2,2,2,2,2),varNames = c('A','B','C','D','E'))
attach(design)

ABD <- A*B*D
ACE <- A*C*E

y2 <- y[ABD == 1 & ACE == 1]
A <- as.factor(A[ABD == 1 & ACE == 1])
B <- as.factor(B[ABD == 1 & ACE == 1])
C <- as.factor(C[ABD == 1 & ACE == 1])
D <- as.factor(D[ABD == 1 & ACE == 1])
E <- as.factor(E[ABD == 1 & ACE == 1])
lm1 <- lm(y2~A*B*C*D*E)

> summary.aov(lm1)
            Df Sum Sq Mean Sq
A            1  253.1   253.1
B            1 2211.1  2211.1
C            1  231.1   231.1
D            1  120.1   120.1
E            1   10.1    10.1
B:C          1    6.1     6.1
C:D          1    6.1     6.1
```

b

```
contr <- as.character("contr.helmert")
lm2 <- lm(y2~A*B*C*D*E, contrasts = list(A=contr,B=contr,C=contr,D=contr,E=contr))
eff <- 2 * lm2$coefficients[-1]

> na.omit(eff)
   A1    B1    C1    D1    E1 B1:C1 C1:D1
11.25 33.25 10.75  7.75  2.25 -1.75  1.75
```

From table X, seven estimable effects are
A + BD + CE = 11.25
B + AD + CDE = 33.25

C + AE + BDE = 10.75
D + AB + BCE = 7.75
E + AC + BCD = 2.25
BC + DE + ACD + ABE = -1.75
CD + BE + ABC + ADE = 1.75

Lect 22-2

a

```
rm(list=ls(all=T))
rep1 <- c(90,74,81,83,77,81,88,73,98,72,87,85,99,79,87,80)
rep2 <- c(93,78,85,80,78,80,82,70,95,76,83,86,90,75,84,80)
y <- c(rep1, rep2)
design1 <- gen.factorial(c(2,2,2,2,2), varNames = c('A','B','C','D','Rep'))
attach(design1)
lm1 <- lm(y~A*B*C*D)

summary.aov(lm1)
            Df Sum Sq Mean Sq F value   Pr(>F)
A            1  657.0   657.0  85.816 7.87e-08 ***
B            1   13.8    13.8   1.800 0.198445
C            1   57.8    57.8   7.547 0.014317 *
D            1  124.0   124.0  16.200 0.000979 ***
A:B          1  132.0   132.0  17.245 0.000749 ***
A:C          1    3.8     3.8   0.494 0.492302
B:C          1    2.5     2.5   0.331 0.573296
A:D          1   38.3    38.3   5.000 0.039945 *
B:D          1    0.3     0.3   0.037 0.850417
C:D          1   22.8    22.8   2.976 0.103793
A:B:C        1  215.3   215.3  28.118 7.15e-05 ***
A:B:D        1  175.8   175.8  22.959 0.000200 ***
A:C:D        1    7.0     7.0   0.918 0.352162
B:C:D        1    7.0     7.0   0.918 0.352162
A:B:C:D      1   47.5    47.5   6.208 0.024077 *
Residuals   16  122.5     7.7
```

Base on the ANOVA table, if replication is not considered as a block in the model, effect of factors A, C, D, AB, AD, ABC, ABD are significant.

b

```
lm2 <- lm(y~A*B*C*D + Rep)
summary.aov(lm2)
            Df Sum Sq Mean Sq F value   Pr(>F)
A            1  657.0   657.0  88.613 1.10e-07 ***
B            1   13.8    13.8   1.859 0.192893
C            1   57.8    57.8   7.793 0.013690 *
D            1  124.0   124.0  16.728 0.000966 ***
Rep          1   11.3    11.3   1.521 0.236373
A:B          1  132.0   132.0  17.807 0.000743 ***
A:C          1    3.8     3.8   0.510 0.486115
B:C          1    2.5     2.5   0.341 0.567713
```

```
A:D          1   38.3    38.3   5.163 0.038219 *
B:D          1    0.3     0.3   0.038 0.848193
C:D          1   22.8    22.8   3.072 0.100035
A:B:C        1  215.3   215.3  29.035 7.53e-05 ***
A:B:D        1  175.8   175.8  23.708 0.000204 ***
A:C:D        1    7.0     7.0   0.948 0.345596
B:C:D        1    7.0     7.0   0.948 0.345596
A:B:C:D      1   47.5    47.5   6.411 0.023008 *
Residuals   15  111.2     7.4
```

If replication is considered as block, then effect of factors A, C, D, AB, AD, ABC, ABD, ABCD are significant.

c

```
rm(list=ls(all=T))
rep1 <- c(90,74,81,83,77,81,88,73,98,72,87,85,99,79,87,80)
y <- rep1
design1 <- gen.factorial(c(2,2,2,2), varNames = c('A','B','C','D'))
attach(design1)
lm1 <- lm(y~A*B*C*D)
summary.aov(lm1)
            Df Sum Sq Mean Sq
A            1  400.0   400.0
B            1    2.3     2.3
C            1    2.2     2.2
D            1  100.0   100.0
A:B          1   81.0    81.0
A:C          1    1.0     1.0
B:C          1    6.2     6.2
A:D          1   56.2    56.2
B:D          1    9.0     9.0
C:D          1    9.0     9.0
A:B:C        1  144.0   144.0
A:B:D        1   90.2    90.2
A:C:D        1    0.3     0.3
B:C:D        1   16.0    16.0
A:B:C:D      1   42.3    42.3
```

Since degree of freedom is exactly equal to number of parameters, there is no residuals.

d

```
rm(list=ls(all=T))
rep1 <- c(90,74,81,83,77,81,88,73,98,72,87,85,99,79,87,80)
rep2 <- c(93,78,85,80,78,80,82,70,95,76,83,86,90,75,84,80)
y <- c(rep1, rep2)
```

```
design1 <- gen.factorial(c(2,2,2,2,2), varNames = c('A','B','C','D','Rep'))
attach(design1)
ABCD <- A*B*C*D
y <- y[ABCD==1]
A <- A[ABCD==1]
B <- B[ABCD==1]
C <- C[ABCD==1]
D <- D[ABCD==1]
Rep <- Rep[ABCD==1]
lm1 <- lm(y~A*B*C*D + Rep)

summary.aov(lm1)
            Df Sum Sq Mean Sq F value   Pr(>F)
A            1  400.0   400.0  41.176 0.000361 *** = BCD
B            1   20.3    20.3   2.085 0.192017     = ACD
C            1   16.0    16.0   1.647 0.240210    = ABD
D            1    6.2     6.2   0.643 0.448858    = ABC
Rep          1   16.0    16.0   1.647 0.240210
A:B          1  132.3   132.3  13.614 0.007760 ** = CD
A:C          1    1.0     1.0   0.103 0.757695    = BD
B:C          1   30.2    30.2   3.114 0.120979    = AD
Residuals    7   68.0     9.7
```

In the 2^{4-1} design with defined relation $ABCD = 1$, effect of estimable factors A+BCD and AB+CD are significant.

e

```
rm(list=ls(all=T))
rep1 <- c(90,74,81,83,77,81,88,73,98,72,87,85,99,79,87,80)
y <- rep1
design1 <- gen.factorial(c(2,2,2,2), varNames = c('A','B','C','D'))
attach(design1)
ABCD <- A*B*C*D
y <- y[ABCD==1]
A <- A[ABCD==1]
B <- B[ABCD==1]
C <- C[ABCD==1]
D <- D[ABCD==1]
lm1 <- lm(y~A*B*C*D)

summary.aov(lm1)
            Df Sum Sq Mean Sq
A            1    288     288
B            1      2       2
C            1     32      32
```

```
D               1       2        2
A:B             1      72       72
A:C             1       2        2
B:C             1      50       50
```

f

```
BL <- A*C
lm2 <- lm(y~A*B*C*D + BL)
summary.aov(lm2)
            Df Sum Sq Mean Sq
A            1    288     288
B            1      2       2
C            1     32      32
D            1      2       2
BL           1      2       2
A:B          1     72      72
B:C          1     50      50
```

The solutions to different kinds of factorial design showed that in either complete 2^k design or incomplete 2^{k-p} design, there exists no degree of freedom left for residuals if there is no replication. Furthermore, using incomplete design, even though we can run the F-test and test the significance of each effect, we are unable to estimate single main factor, since it is aliased with other factor.