Name: ______________________
ID: ________________________
Quiz section or time: ________________________

Stat 421, Test 3, Fall, Dec. 11, 2012; Marzban

6 + 18.5

ONLY a half-size "cheat sheet" is allowed

Multiple choice: Circle all the correct answers; there is wrong-answer penalty; do NOT explain

The rest: SHOW answer & work; NO CREDIT for correct answer without explanation

Points

1 **1.** Consider a problem with a 2-level factor A, and a 6-level factor B. When one says "I will block the factor B, and test for the effect of A," then one is essentially performing which test?
a) an unpaired t-test on the 2 levels of A b) a paired t-test on the 6 levels of B
c) a paired t-test on the 2 levels of A d) an unpaired t-test on the 6 levels of B e) None of

1 **2.** A 2^7 design in 4 blocks has been adopted. How many effects will be confounded with blocks?
a) 1 b) 2 c) 3 d) 4 e) 5 f) 6

2+1 generalized

1 **3.** In a 2^{6-2} design, which of the following pair of generators is "best" in terms of assuring that low-order interactions are not aliased.
a) $E = AB,\ F = CD$ b) $E = ABC,\ F = ABCDE$ c) $E = ABC,\ F = BCD$
d) $E = ABCD,\ F = ABCDE$ e) $E = ABC,\ F = ABCD$

1 **4.** In a 3^2 design, the interaction term has df=4. Which of the following statements is NOT true. The interaction term can be decomposed into
a) four single-df effects, if the 2 factors are quantitative.
b) two 2-df effects, if the 2 factors are qualitative.
c) four single-df effects, if the 2 factors are qualitative.
d) two 2-df effects, if the 2 factors are quantitative.

1 **5.** An experiment is designed to study pigment dispersion in paint. Four different mixes of a particular pigment are studied. The procedure consists of preparing a particular mix and then applying that mix to a panel by each of the three commercial paint application methods possible (brushing, spraying, and rolling). Three days are required to run the experiment. What is the most appropriate design for this experiment?
a) A completely randomized factorial design c) A nested design
b) A randomized complete block factorial design d) A split-plot design

6. In the previous problem, which of the following model(s) is/are NOT appropriate?
a) Mix and Method treated as fixed effects b) Mix and Method treated as random effects
c) Mix treated as a fixed effect, but Method treated as a random effect
d) Mix treated as a random effect, but Method treated as a fixed effect

2 **7.** A 2^3 design in two blocks has been adopted. Block 1 contains the runs $(1), ab, ac, bc$, while the second block contains the runs a, b, c, abc. Which effect(s) is/are confounded with block?

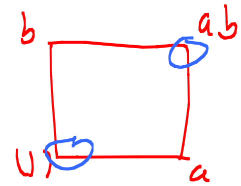
	A	B	C	ABC
(1)	−	−	−	−
a	+	−	−	
b	−	+	−	
ab	+	+	−	−
c	−	−	+	
ac	+	−	+	−
bc	−	+	+	−
abc	+	+	+	

We note that ABC = −1 for Block 1
and = +1 " " 2

So ABC is the effect confounded with block.

2 **8.** Consider a 2^{2-1} design. **a)** If the defining relation is $AB = 1$, write the effects A, B, and AB in terms of $(1), a, b, ab$, if/when possible?

$AB = 1 \implies$ The only runs are (1) and ab.



$$\therefore \begin{cases} A \text{ effect} = ab - (1) \\ B \text{ effect} = ab - (1) \\ AB \text{ effect} \sim [(\text{Avg } A \text{ effect} \mid B=+) - (\text{Avg } A \text{ effect} \mid B=-)] = \text{Not estimable} \end{cases}$$

not estimable

[Consistent with $AB = 1$]

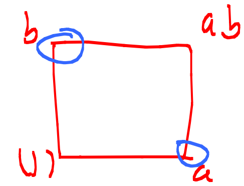
~~1~~ 1.5 **b)** Write the alias structure, and specify the estimable effect(s).

Alias structure : $A = B$ [from $AB = 1$] [Consistent with A, B above]

Estimable effect : $A + B$

~~1~~ 1.5 **c)** Repeat parts a and b, but with defining relation given by $AB = -1$.

$AB = -1 \implies$ The only runs are a, b.



$$\therefore \begin{cases} A \text{ effect} = a - b \\ B \text{ effect} = b - a \\ AB \text{ effect} \sim [(\text{Avg } A \text{ effect} \mid B=+) - (\text{Avg } A \text{ effect} \mid B=-)] = \text{Not estimable} \end{cases}$$

not estimable

$$\begin{cases} \text{Alias structure} : & A = -B \\ \text{Estimable effect} & A - B \end{cases}$$

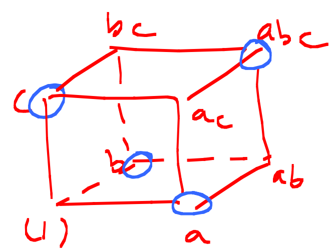
[Consistent with A, B effects]

d) Suppose we "combine" the two results, while introducing a blocking factor C. Write the effects
2 C and AB in terms of $(1), a, b, c, ab, \ldots$, if/when possible?

	A	B	C ← Block factor	
(1)	−	−	+	} Block 1
ab	+	+	+	
a	+	−	−	} Block 2
b	−	+	−	

$$C \text{ effect} = \frac{abc + c}{2} - \frac{a + b}{2}$$

$$AB \text{ effect} = \frac{1}{2}[(abc - b) - (a - c)]$$



Note: $AB = C$ Consistent with AB being confounded with C.

e) Write the defining relation AND the alias structure for the "combined" design, and specify the
2 estimable effect(s).

The defining relation : $ABC = +1$

$$\text{Alias structure} : \begin{cases} A = BC \\ B = AC \\ C = AB \end{cases}$$

estimable effects $A + BC$, $B + AC$, $C + AB$

9. In a 3^2 design, the factors A, B have been encoded as x_1, x_2, respectively, with values 0,1,2. The values of the contrast $L = x_1 + x_2 \pmod 3$ have been used to block the runs into 3 blocks.

2 **a)** Show that blocking according to $L = x_1 + 2x_2 \pmod 3$ gives a different set of 3 blocks, but that blocking according to $L = 2x_1 + x_2 \pmod 3$, reproduces the same blocks generated by one of the previous contrasts.

A B	x_1+x_2 mod 3	x_1+2x_2 mod 3	$2x_1+x_2$ mod 3
0 0	0	0	0
1 0	1	1	2
2 0	2	2	1
0 1	1	2	1
1 1	2	0	0
2 1	0	1	2
0 2	2	1	2
1 2	0	2	1
2 2	1	0	0

Block 1: $\boxed{00, 21, 12}$, Block 2: $\boxed{10, 01, 22}$, Block 3: $\boxed{20, 11, 02}$

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||

Block 1: $\boxed{00, 11, 22}$, Block 2: $\boxed{20, 01, 12}$, Block 3: $\boxed{10, 21, 02}$

The Top 2 rows are different Blockings.
But the 2^{nd} and 3^{rd} rows are the same Blockings.

2 **b)** $n = 1$ replication has led to the following estimate of the interaction term. What is the value of SS corresponding to this interaction term (i.e., $SSAB$)?

$$\begin{pmatrix} 1 & 2 & -3 \\ 3 & 4 & -7 \\ -4 & -6 & 10 \end{pmatrix} \quad SSAB = \sum_{i,j} (\widehat{\alpha\beta})^2_{ij} = 1 + 4 + 9 + \cdots + 100 = 240$$

Alternatively (long way):

$y_{..1} = 1 - 7 - 6 = -12$

$y_{..2} = 2 + 3 + 10 = 15$

$y_{..3} = -3 + 4 - 4 = -3$

$SS_{AB} = \frac{1}{3}\left(-(12)^2 + 15^2 + (-3)^2\right) - \frac{0^2}{9} = 126$

$SS_{AB^2} = \frac{1}{3}\left(15^2 + (-9)^2 + (-6)^2\right) - \frac{0^2}{9} = 114$

$\therefore SSAB = SS_{AB} + SS_{AB^2} = \boxed{240}$

~~2~~ 1.5 **c)** If the blocking according to $L = x_1 + x_2 \pmod 3$ has led to an SS of 7.0 for the AB component of the interaction term, what is the value of SS_{Blocks}? Explain.

$SS_{Blocks} = SS_{AB} = 7$, because AB is confounded with Blocks.

2 13.28 hw-AB **10.** Consider a 2-factor random-effects model with interaction and with n replicates: $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$, with $i = 1 - a, j = 1 - b, k = 1 - n$. Show that the estimate of σ^2_α according to the analysis of variance method is unbiased.

$$\hat{\sigma}^2_\alpha = \frac{1}{bn} MS_A - MS_{AB}$$

$$E[\hat{\sigma}^2_\alpha] = \frac{1}{bn}\left(E[MS_A] - E[MS_{AB}]\right)$$

$$= \frac{1}{bn}\left(\cancel{\sigma^2_\epsilon} + \cancel{n\sigma^2_{\alpha\beta}} + bn\,\sigma^2_\alpha - \cancel{\sigma^2_\epsilon} - \cancel{n\sigma^2_{\alpha\beta}}\right)$$

$$\underline{E[\hat{\sigma}^2_\alpha] = \sigma^2_\alpha}$$