hw-let 21-1: Produce the alias structure for 2^{5-2} with $ABC = ADE = 1$

Hint: recall that in a previous hw, you have already listed all $2^5 - 1 = 31$ effects. consult that hw to assure that your alias structure here includes all 31 effects.

$ABC = ADE = 1 \Rightarrow BCDE = 1$

$A = BC = DE = ABCDE$

$B = AC = CDE = ABDE$

$C = AB = BDE = ACDE$

$D = AE = BCE = ABCD$

$E = AD = BCD = ABCE$

$BD = CE = ABE = ACD$

$CD = BE = ACE = ABD$

All 31 effects present ✓.

hw-let 21-2 Sometimes 2 alias structures will look different, but only because of relabeling of the factors. For example, show that a 2^{6-3} with $ABD = ACE = BCF = 1$ is the same design as a 2^{6-3} with $ABC = ADE = CDF = 1$. Hint: no alias structure necessary.

$ABD = 1 \quad ACE = 1 \quad BCF = 1$

note the pattern then change labels

$ABC \quad ADE \quad CDF$

So, the relabeling is $B \to C \to D$.

handout 21-3

Suppose in a 2^{5-2} design, one of the generators is $ABCD = 1$

what are the possible choices for the 2^{nd} generator that lead to a good design (where all main effects are estimable, and none of the main effects are aliased with other main effects)?

Hint: You do not need to work out the alias structures.

And you do not need Table X either.

Hint: Consider generators that have only 2 letters.

Then, only 3 letters. Then, 4 letters, ...

If the 2^{nd} gen. has

→ 1 letter, then that effect is not even estimable. Bad!

→ 2 letters, then those 2 effects are aliased. Bad!

→ 3 letters, then there are 2 possibilities:

a) 3 letters common with ABCD.

w/o loss of generality, consider $ABC = 1$

Then, the product of the 2 gens is $D = 1$,

i.e. D is not even estimable. Bad!

The only possibility.

b) 2 letters in common with ABCD.

w/o loss of generality, consider $ABE = 1$.

Then, the product of the 2 gens is $CDE = 1$

which is OK.

↳ See comment below.

→ 4 letters, Then The only possibility is for 1 letter to be not common with ABCD. w/o loss of generality, Consider ABCE = 1. Then The product of The 2 gens is. DE = 1, ie. D and E effects are aliased. Bad!

→ 5 letters, ie. ABCDE = 1. Then product ⇒ E = 1
E is not estimable at all. Bad!

So, The only acceptable possibility is

$$ABCD = 1 \quad , \quad ABE = 1 \ (\text{or } CDE = 1)$$

Note that by re-labeling The letters here, you get The Same gens given in Table X. Exchange $A \leftrightarrow E$ and $B \leftrightarrow C$.

Then

$$ECBD = 1 \quad , \quad ECA = 1 \ (\text{or } BDA = 1)$$

ie.

$$BCDE = 1 \ , \quad \underbrace{ACE = 1 \ (\text{or } ABD = 1)}_{\text{in Table X.}}$$

This is essentially problem 8.4 (It refers to 6.24, but it should be referring to 6.26).
Using the 2^(5-2) defining relations in Table X,
a) produce the anova table. Hint: Make the shorter table, generate the missing factors, identify the necessary run, and select the corresponding y values from problem 6.26 .
b) What are the 7 estimable effects? Estimate them.

hw_lect21-4

```
# Soln a)
   rm(list=ls(all=TRUE))
   library(AlgDesign)          # for gen.factorial()
   design = gen.factorial(2,3,varNames=c("A","B","C")) # start with shorter +- table.
   attach(design)
   D = A*B  # Using the defining relations from Table X.
   E = A*C
   cbind(A,B,C,D,E)
#   A  B  C  D  E
#  -1 -1 -1  1  1   de
#   1 -1 -1 -1 -1   a
#  -1  1 -1 -1  1   be
#   1  1 -1  1 -1   abd
#  -1 -1  1  1 -1   cd
#   1 -1  1 -1  1   ace
#  -1  1  1 -1 -1   bc
#   1  1  1  1  1   abcde
   y = c(6, 9, 35, 50, 18, 22, 40, 63 )
   A = as.factor(A);  B = as.factor(B);  C = as.factor(C);  D = as.factor(D); E = as.factor(E)
   contr = as.character("contr.helmert")
   lm.1 = lm(y~A*B*C*D*E, contrasts = list(A=contr,B=contr,C=contr,D=contr,E=contr))
   summary.aov(lm.1)     # Full model gives SSE = 0
#            Df Sum Sq Mean Sq
# A           1  253.1   253.1
# B           1 2211.1  2211.1
# C           1  231.1   231.1
# D           1  120.1   120.1
# E           1   10.1    10.1
# B:C         1    6.1     6.1
# C:D         1    6.1     6.1

# b)
   eff = 2*lm.1$coef
   eff = eff[2:length(eff)]       # Exclude the grand mean.
   as.matrix(eff,col=1)
   sort(eff)
#               Estimate Estimable effects
# A1             11.25    A + BD + CE
# B1             33.25    B + AD + CDE
# C1             10.75    C + AE + BDE
# D1              7.75    D + AB + BCE
# E1              2.25    E + AC + BCD
# A1:B1             NA
# A1:C1             NA
# B1:C1          -1.75    BC + DE + ACD + ABE
# A1:D1             NA
# B1:D1             NA
# C1:D1           1.75    CD + BE + ABC + ADE
# A1:E1             NA
# ...
```

hw-lect22-1

Consider the 2^{6-2} design with defining relations $ABCE = BCDF = I$.

a) Find the 4 blocks that will lead to ABD and ACD to get confounded with block. See Table X to understand why I picked these 2 effects to confound with block.

b) According to the alias structure in Table X, CDE is aliased with ABD. Using the results in part a, find the block effect (written in terms of the 4 block sums) with which CDE is confounded. Hint: find CDE from +/− table.

a)

	A	B	C	D	E = ABC	F = BCD	ABD	ACD	CDE (= ABD)
(1)	−	−	−	−	−	−	−	−	−
ae	+	−	−	−	+	−	+	+	+
bef	−	+	−	−	+	+	+	−	+
abf	+	+	−	−	−	+	−	+	−
cef	−	−	+	−	+	+	−	+	−
acf	+	−	+	−	−	+	+	−	+
bc	−	+	+	−	−	−	+	+	+
abce	+	+	+	−	+	−	−	−	−
df	−	−	−	+	−	+	+	+	+
adef	+	−	−	+	+	+	−	−	−
bde	−	+	−	+	+	−	−	+	−
abd	+	+	−	+	−	−	+	−	+
cde	−	−	+	+	+	−	+	−	+
acd	+	−	+	+	−	−	−	+	−
bcdf	−	+	+	+	−	+	−	−	−
abcdef	+	+	+	+	+	+	+	+	+

	ABD	ACD	
Block 1	−	−	[(1), abce, adef, bcdf]
2	+	−	[bef, acf, abd, cde]
3	−	+	[abf, cef, bde, acd]
4	+	+	[ae, bc, df, abcdef]

b) $CDE = ae + bef + acf + bc + df + abd + cde + abcdef$
$\quad - (1) - abf - cef - abce - adef - bde - acd - bcdf$

$$= \underbrace{(\text{Block 4} + \text{Block 2})}_{\text{High } ABD} - \underbrace{(\text{Block 3} + \text{Block 1})}_{\text{Low } ABD}$$

Note that this makes perfect sense, because CDE is aliased with ABD, i.e. one of the 2 factors confounded with block.

1

(This is 6.7, but used to play with incomplete designs with and w/o blocks.)
Consider the data in problem 6.7. For each of the following designs, write code to produce the anova table for the full model.
Recall that the anova table for binary factors is unaffected by whether the factors are entered into lm() as factors or as numeric.
Later, check the soln to see what the moral of this problem is.
a) Replicated 2^4 design in 32 runs.

hw/lect22-2

```
   rm(list=ls(all=TRUE))
   library(AlgDesign)          # for gen.factorial()
   design = gen.factorial(2,5,varNames=c("A","B","C","D","R")) # R for replication
   attach(design)
   y = c(90,74,81,83,77,81,88,73,98,72,87,85,99,79,87,80,
         93,78,85,80,78,80,82,70,95,76,83,86,90,75,84,80)
   summary.aov(lm(y~A*B*C*D))  # No R factor.
 #             Df Sum Sq Mean Sq F value   Pr(>F)
# A             1  657.0   657.0  85.816 7.87e-08 ***
# B             1   13.8    13.8   1.800 0.198445
# C             1   57.8    57.8   7.547 0.014317 *
# D             1  124.0   124.0  16.200 0.000979 ***
# A:B           1  132.0   132.0  17.245 0.000749 ***
# A:C           1    3.8     3.8   0.494 0.492302
# B:C           1    2.5     2.5   0.331 0.573296
# A:D           1   38.3    38.3   5.000 0.039945 *
# B:D           1    0.3     0.3   0.037 0.850417
# C:D           1   22.8    22.8   2.976 0.103793
# A:B:C         1  215.3   215.3  28.118 7.15e-05 ***
# A:B:D         1  175.8   175.8  22.959 0.000200 ***
# A:C:D         1    7.0     7.0   0.918 0.352162
# B:C:D         1    7.0     7.0   0.918 0.352162
# A:B:C:D       1   47.5    47.5   6.208 0.024077 *
# Residuals    16  122.5     7.7
```

b) Replicated 2^4 design in 32 runs, with replication blocked. (Technically replication should be blocked anyway).

```
   summary.aov(lm(y ~ R + A*B*C*D))

#              Df Sum Sq Mean Sq F value   Pr(>F)
# R             1   11.3    11.3   1.521 0.236373
# A             1  657.0   657.0  88.613 1.10e-07 ***
# B             1   13.8    13.8   1.859 0.192893
# C             1   57.8    57.8   7.793 0.013690 *
# D             1  124.0   124.0  16.728 0.000966 ***
# A:B           1  132.0   132.0  17.807 0.000743 ***
# A:C           1    3.8     3.8   0.510 0.486115
# B:C           1    2.5     2.5   0.341 0.567713
# A:D           1   38.3    38.3   5.163 0.038219 *
# B:D           1    0.3     0.3   0.038 0.848193
# C:D           1   22.8    22.8   3.072 0.100035
# A:B:C         1  215.3   215.3  29.035 7.53e-05 ***
# A:B:D         1  175.8   175.8  23.708 0.000204 ***
# A:C:D         1    7.0     7.0   0.948 0.345596
# B:C:D         1    7.0     7.0   0.948 0.345596
# A:B:C:D       1   47.5    47.5   6.411 0.023008 *
# Residuals    15  111.2     7.4
```

2

c) Unreplicated 2^4 in 16 runs. Use only replicate I.

```
A = A[R==-1] ; B = B[R==-1] ; C = C[R==-1] ; D = D[R==-1]
y = y[R==-1]
summary.aov(lm(y ~ A*B*C*D))

#            Df Sum Sq Mean Sq
# A           1  400.0   400.0
# B           1    2.3     2.3
# C           1    2.2     2.2
# D           1  100.0   100.0
# A:B         1   81.0    81.0
# A:C         1    1.0     1.0
# B:C         1    6.2     6.2
# A:D         1   56.2    56.2
# B:D         1    9.0     9.0
# C:D         1    9.0     9.0
# A:B:C       1  144.0   144.0
# A:B:D       1   90.2    90.2
# A:C:D       1    0.3     0.3
# B:C:D       1   16.0    16.0
# A:B:C:D     1   42.3    42.3
```

d) Replicated incomplete 2^(4-1) design with defining relation ABCD = 1, with replication blocked. In the anova table that R produces, write-in the alias structure; for example, on the line where the SS/MS/... values for A are reported, write-in "=BCD".
(Also, it may be a good idea to start with a clean R session.)

```
rm(list=ls(all=TRUE))
library(AlgDesign)

# I start with the big +- table, only to allow easy selection of the y values. In practice,
# this step won't be necessary because we won't have all 2^k runs to pick from:
design = gen.factorial(2,5,varNames=c("A","B","C","D","R"))
attach(design)
y = c(90,74,81,83,77,81,88,73,98,72,87,85,99,79,87,80,
      93,78,85,80,78,80,82,70,95,76,83,86,90,75,84,80)
y = y[A*B*C*D==1]

design = gen.factorial(2,4,varNames=c("A","B","C","R")) # shorter +- table (but with R)
attach(design)
D = A*B*C
summary.aov(lm(y ~ R + A*B*C*D )) # with R blocked

#            Df Sum Sq Mean Sq F value   Pr(>F)
# R           1   16.0    16.0   1.647 0.240210
# A           1   20.3    20.3   2.085 0.192017  = BCD  from lecture notes
# B           1   16.0    16.0   1.647 0.240210  = ACD
# C           1    6.3     6.3   0.643 0.448858  = ABD
# D           1  400.0   400.0  41.176 0.000361  = ABC
# A:B         1   30.2    30.2   3.114 0.120979  = CD
# A:C         1    1.0     1.0   0.103 0.757695  = BD
# B:C         1  132.2   132.2  13.614 0.007760  = AD
# Residuals   7   68.0     9.7
```

3

e) Unreplicated incomplete 2^(4-1) design with ABCD=1 in 8 runs. Use only replicate I.

```
  rm(list=ls(all=TRUE))
library(AlgDesign)

# Again, I'm doing this only to select the y values easily:
design = gen.factorial(2,4,varNames=c("A","B","C","D"))
attach(design)
y = c(90,74,81,83,77,81,88,73,98,72,87,85,99,79,87,80)
y = y[A*B*C*D==1]

design = gen.factorial(2,3,varNames=c("A","B","C")) # shorter +- table (but with R)
attach(design)
D = A*B*C
summary.aov(lm(y ~ A*B*C*D ))

#          Df Sum Sq Mean Sq
# A         1      2       2
# B         1     32      32
# C         1      2       2
# D         1    288     288
# A:B       1     50      50
# A:C       1      2       2
# B:C       1     72      72
```

f) Unreplicated incomplete 2^(4-1) design with ABCD=1, in 8 runs, in 2 blocks such that AC is confounded with block.

```
L = numeric(8)
L[A*C==-1] = -1
L[A*C==+1] = +1
summary.aov(lm(y~ as.factor(L) + A*B*C*D))

#              Df Sum Sq Mean Sq
# as.factor(L)  1      2       2
# A             1      2       2
# B             1     32      32
# C             1      2       2
# D             1    288     288
# A:B           1     50      50
# B:C           1     72      72
```

Clearly, the design in part a is "best" because it allows for estimating all of the effects, and testing them. In part b, we see that in this problem, blocking the replications does not improve things.

Parts c and d both require 16 runs; but there are pros and cons to both designs. The unreplicated 2^4 design in part c allows for the estimation of all of the effects, but it does not have enough df for testing the effects. The replicated incomplete design in part d does allow for testing, but because of aliasing we cannot estimate all of the effects.

The designs in part e and f both require 8 runs. They are both incomplete, so you know that you're not going to be able to estimate all of the effects. But in part f, there is one less effect that can be estimated because it's confounded with the block effect.

hw_lect22_3

a) Consider the 2^{5-2} design with defn. rel. $ABD = ACE = I$. Write out the alias structure. Make sure all of the 31 effects are accounted for.

$A = BD = CE = ABCDE$

$B = AD = CDE = ABCE$

$C = AE = BDE = ABCD$

$D = AB = BCE = ACDE$

$E = AC = BCD = ABDE$ ← not shown in Table X(c).

All 31 effects are shown

$BC = DE = ACD = ABE$

$CD = BE = ADE = ABC$

b) Suppose after we ran the 2^{5-2} runs with $ABD = ACE = I$, we found that we have the resources (money) to run the 2^{5-2} runs with $ABD = ACE = -I$. In Yates' notation, write down the runs in the alternative fraction. Hint: you don't have to derive the alias structure in the alternative fraction. All you have to do is to write the +/– table for the principal fraction, and then flip the sign of 1 factor. Which factor? What kind of flip causes $ABD = ACE = +I$ to become $ABD = ACE = -I$?

Flipping the sign of A is sufficient to take $ABD = ACE = +I$ to $ABD = ACE = -I$. Note: in both cases we have $BCDE = +I$

$ABD = ACE = +I$

	A	B	C	D=AB	E=AC
de	−	−	−	+	+
a	+	−	−	−	−
be	−	+	−	−	+
abd	+	+	−	+	−
cd	−	−	+	+	−
ace	+	−	+	−	+
bc	−	+	+	−	−
abcde	+	+	+	+	+

The Answer →

$ABD = ACE = -I$

	A	B	C	D=AB	E=AC
ade	+	−	−	+	+
(1)	−	−	−	−	−
abe	+	+	−	−	+
bd	−	+	−	+	−
acd	+	−	+	+	−
ce	−	−	+	−	+
abc	+	+	+	−	−
bcde	−	+	+	+	+

Incorrect page! (see next page).

c) what are the estimable effects in each fraction, and those in the combined (fold-over) design.

We can estimate $\downarrow$ from one fraction, and $\downarrow$ from the reflected fraction.

$A + BD + CDE + ABCE$	$-A + BD - CDE + ABCE$
$B + AD + CE + ABCDE$	$-B + AD + CE - ABCDE$
$C + BE + ADE + ABCD$	$-C + BE - ADE + ABCD$
$D + AB + ACE + BCDE$	$-D + AB - ACE + BCDE$
$E + BC + ACD + ABDE$	$-E + BC - ACD + ABDE$
$AC + DE + BCD + ABE$	$AC + DE - BCD - ABE$
$CD + AE + BDE + ABC$	$CD + AE - BDE - ABC$

estimable effects in the combined design:

Note main effects are de-aliased from 2-factor effects.

$A + CDE$, $BD + ABCE$

$B + ABCDE$, $AD + CE$

$C + ADE$, $BE + ABCD$

$D + ACE$, $AB + BCDE$, $AC + DE$, $BCD + ABE$

$E + ACD$, $BC + ABDE$, $CD + AE$, $BDE + ABC$

To see what happened to ABD and ACE (the effects in the defn. rel.), see later (they are confounded with the block effect; the blocks are the 2 fractions).

Correct page.

c) what are the estimable effects in each fraction, and those in the combined (fold-over) design.

We can estimate from one fraction, and from the reflected fraction.

A + BD + CE + ABCDE	−A + BD + CE − ABCDE
B + AD + CDE + ABCE	B − AD + CDE − ABCE
C + AE + BDE + ABCD	C − AE + BDE − ABCD
D + AB + BCE + ACDE	D − AB + BCE − ACDE
E + AC + BCD + ABDE	E − AC + BCD − ABDE
BC + DE + ACD + ABE	BC + DE − ACD − ABE
CD + BE + ABC + ADE	CD + BE − ABC − ADE

estimable effects in the combined design:

Note main effects are de-aliased from 2-factor effects.

A + ABCDE , BD + CE
B + CDE , AD + ABCE
C + BDE , AE + ABCD
D + BCE , AB + ACDE
E + BCD , AC + ABDE

BC + DE , ACD + ABE
CD + BE , ABC + ADE

To see what happened to ABD and ACE (the effects in the defn. rel.) see later (they are confounded with the block effect; the blocks are the 2 fractions).

hw-lect23_1

The Resolution of a design is defined as the <u>smallest</u> number of letters among all of the generators. For example the 2^{6-3} design with $ABD = ACE = BCF = 1$ (listed in Table x) is a resolution $\underline{III}$ design. Note that the products of these generators give generators that have more than 3 letters. By contrast, the 2^{7-3} design with $ABCE = BCDF = ACDG = 1$ is a resolution $\underline{IV}$ design. Now, consider the 2^{6-2} design. What is the resolution if the defining relation is:

a) $ABCE = BCDF = 1$

b) $ABCE = ABCDF = 1$

a) The generators have 4 letters, and their product $ADEF = 1$ also has 4 letters. So, the resolution is $\underline{IV}$.

b) The "shortest" generator listed is ABCE, i.e. with 4 letters. But the product of the gens is $DEF = 1$. So, the resolution of this design is 3.

Higher res. designs are generally more desirable.

hw-lect23-2

Consider a full fold-over of a 2^{6-2} design with $ABCE = BCDF = 1$.

a) By looking at the defining relations (all of them), show that the alternative fraction is the same as the principal fraction.

Hint: claim B (in lecture) does not apply to Resolution IV designs.

$ABCE = 1 \longrightarrow ABCE = 1$; $BCDF = 1 \longrightarrow BCDF = 1$.

This is because there are 4 letters in the generators. Designs which involve generators with 4 or more letters are called Resolution IV designs.

b) Look at the +/- Table for the primary 2^{6-2} design (example 8.4). What happens to it after a full fold-over? I.e. what is the +/- Table in the alternate fraction? Hint: Recall part a!

The same +/- is generated; only the order of the runs is changed.

So, flipping signs in the defn. rel. of the principal fraction, and flipping signs in the +/- Table, give to the same alternative fraction.

■ **TABLE 8.9**

Construction of the 2^{6-2}_{IV} Design with the Generators $I = ABCE$ and $I = BCDF$

	Basic Design					
Run	A	B	C	D	$E = ABC$	$F = BCD$
1	− +	− +	− +	− +	− +	− +
2	+ −	− +	− +	− +	+ −	− +
3	−	+	−	−	+	+
4	+	+	−	−	−	+
5	−	−	+	−	+	+
6	+	−	+	−	−	+
7	−	+	+	−	−	−
8	+	+	+	−	+	−
9	−	−	−	+	−	+
10	+	−	−	+	+	+
11	−	+	−	+	+	−
12	+	+	−	+	−	−
13	−	−	+	+	+	−
14	+	−	+	+	−	−
15	− +	+ −	+ −	+ −	− +	+ −
16	+ −	+ −	+ −	+ −	+ −	+ −

c) Now consider a single-factor fold-over on A. By examining the alias structure for the principal fraction (one that shows all interactions, example Table 8-8), write down the estimable effects that involve main effects, in both fractions and in the combined set. Hint: claim A (in text) holds, for Res IV.

TABLE 8.8
Alias Structure for the 2^{6-2}_{IV} Design with $I = ABCE = BCDF = ADEF$

$A = BCE = DEF = ABCDF$	$AB = CE = ACDF = BDEF$
$B = ACE = CDF = ABDEF$	$AC = BE = ABDF = CDEF$
$C = ABE = BDF = ACDEF$	$AD = EF = BCDE = ABCF$
$D = BCF = AEF = ABCDE$	$AE = BC = DF = ABCDEF$
$E = ABC = ADF = BCDEF$	$AF = DE = BCEF = ABCD$
$F = BCD = ADE = ABCEF$	$BD = CF = ACDE = ABEF$
	$BF = CD = ACEF = ABDE$
$ABD = CDE = ACF = BEF$	
$ACD = BDE = ABF = CEF$	

Grace, do you see why one would consider single-factor fold-over?!

Estimable effects involving main effects in principal fraction

$[A] = A + BCE + DEF + ABCDF$

$[B] = B + ACE + CDF + ABDEF$

$[C] = C + ABE + BDF + ACDEF$

$[D] = D + BCF + AEF + ABCDE$

$[E] = E + ABC + ADF + BCDEF$

$[F] = F + BCD + ADE + ABCEF$

Estimable effects ... in alt. fraction:

$[-A]' = -A + BCE + DEF - ABCDF$

$[B]' = B - ACE + CDF - ABDEF$

$[C]' = C - ABE + BDF - ACDEF$

$[D]' = D + BCF - AEF - ABCDE$

$[E]' = E - ABC - ADF + BCDEF$

$[F]' = F + BCD - ADE - ABCEF$

$$\left.\begin{array}{l} A + ABCDF \\ B + CDF \\ C + BDF \\ D + BCF \\ E + BCDEF \\ F + BCD \end{array}\right\} \text{estimable effects}$$

All main effects are free of 2-way interactions.

d) Suppose we are interested in the main effect of A <u>and</u> its 2-way interactions with other factors. In that case the alias chain $AB = CE$ is problematic, because if the CE effect is large, then our estimate of the AB effect based on the principal fraction will be wrong. In the combined design what are the estimable 2-way interactions involving the factor A? Hint: In the alias structure, look at the alias chains that involve 2-way interactions with A.

Estimable 2-way effects in Alt. fraction	Est. effect in combined design	
$[-AB]' = -AB + CE - ACDF + BDEF$	$AB + ACDF$	
$[-AC]' = -AC + BE - ABDF + CDEF$	$AC + ABDF$	
$[-AD]' = -AD + EF + BCDE - ABCF$	$AD + ABCF$	
$[-AE]' = -AE + BC + DF - ABCDEF$	$AE + ABCDEF$	
$[-AF]' = -AF + DE + BCEF - ABCD$	$AF + ABCD$	
$[BD]' = BD + CF - ACDE - ABEF$	$BD + CF$	FYI. For completion.
$[BF]' = BF + CD - ACEF - ABDE$	$BF + CD$	

Note that none of the 2-way interactions involving A are aliased with another such effect. E.g. The $AB = CE$ is now broken in fold-over.

This is a property of single factor fold-over in Resolution <u>IV</u> designs.

e) What is the defining relation of the fold-over design in part c.

principal fraction defn. rel : ABCE = +1 BCDF = +1

altern. " " " : ABCE = −1 BCDF = +1

One relation that does not change across the 2 sets is BCDF=+1

There is only 1 defn. rel in the fold-over of 2^{6-2} (because it's 2^{6-1} in 2 blocks); so the defn. rel. is BCDF = +1.

f) If the 2 fractions are blocked, what is the effect that is confounded with the block effect?

We see that ABCE changes signs across the 2 sets. So, that effect is part of the confounded effect. BCDF does not change signs, and so (ABCE)(BCDF) = ADEF does. So, the effect confounded with block is ABCE + ADEF.