Name: ____________________________
ID: ______________________________

Stat 421, Test 3, Fall, Dec. 13, 2016; Marzban

6+18

ONLY a half-size "cheat sheet" is allowed

Multiple choice: Circle all the correct answers; there is wrong-answer penalty

For rest, SHOW answer & work; NO CREDIT for correct answer without explanation

Points

hw-Summary

1 **1.** A filter comes in 3 types, and we want to know if the type has an effect on some response. In our Lab we have 3 operators. I give the 3 filters to operator 1 and ask him to perform one measurement of y on each filter in random order. Then, I give the three filters to operator 2, with the same instructions, and then to operator 3 with the same instructions. What is the design?

restriction on randomization

a) CRD with 1 treatment factor
b) RCBD with 1 treatment and 1 block factor
c) CRD with 2 treatment factors

hw-Summary

1 **2.** In the previous problem, I still have three filter types, but I also wonder if the highest college degree of the operator also has an effect. So, I select 3 operators with a BS degree, 3 with a MS degree, and 3 with a PhD. Then I randomly assign the 9 treatment combinations to the 9 operators. What kind of design is this?

a) CRD with 1 treatment factor
b) RCBD with 1 treatment and 1 block factor
c) CRD with 2 treatment factors

1 **3.** In a factorial experiment with fixed factors, and with n replications, we have estimated all of the effects, main and interaction, (e.g., by the corresponding parameters in the model). Then, we decide to treat the replication as a block factor. Circle all of the quantities that may change.

a) The effects b) The SS of the treatment factors c) SSE.

Blocking doesn't change The effect; it just makes The test more powerful by reducing SSE

hw-BJ

1 **4.** In a 2^3 design with 2 blocks, the principal block is $[(1), a, b, ab]$. Which effect is confounded with block?

C=low ⇒ other block = high C.

a) A b) B c) C d) AB e) AC f) BC g) ABC

lect 18

1 **5.** In a **complete** 2^k experiment we have arranged for two effects, say X and Y, to get confounded with block. Then (circle all correct statements)

a) X is aliased with Y b) X is confounded with Y c) XY is confounded with block

No alias in complete design

Confound is with block

lect 23

1 **6.** In a 2-factor random effects model, the ratio MSA/MSE is an appropriate test statistic for a 2-sided test of (See last page)

a) $H0 : \sigma^2_\alpha = 0$ b) $H0 : \sigma^2_{\alpha\beta} = 0$ c) $H0 : b\sigma^2_\alpha + \sigma^2_{\alpha\beta} = 0$ b) $H0 : \sigma^2_\epsilon = 0$

~~~2~~ 1 **7.** A 2^3 experiment is supposed to be done across 2 days, with half the runs on each day. Joe (who knows only a little bit of statistics) decides to do "systematic sampling" by assigning alternating runs in Yates' standard order ((1), a, b, ab, c, ac, bc, abc) to different days. What's the worse thing about this design? Explain your answer.

Day 1 : [(1), b, c, bc] ← A=low
Day 2 : [a, ab, ac, abc] ← A=high
⇒ A effect confounded with The Block.

hw-BM

8. Consider the +/− table for the wine-tasting experiment given at the end of the test. Answer these questions WITHOUT explanation.

~1 a) What kind of design is this experiment (what k and p in 2^{k-p})? 2^{8-4}

~1 b) What effect is G aliased with? G = ABC (and more)

2

This is one of The 3 generators.

The A, B, C, D cols follow The usual +/− pattern, but E, F, G, H don't. So They are generated from 3 defn. relations.
~~~

hw **9.** Consider a 2^{6-3} design, with defining relations ABD=ACE=BCF=1; the alias structure is given on the last page.

~ 2 a) Write the +/− table and identify each of the runs in Yates' notation.

for part d

	A	B	C	AB = D	AC = E	BC = F	CD	BF
def	−	−	−	+	+	+	−	−
af	+	−	−	−	−	+	+	−
be	−	+	−	−	+	−	+	−
abd	+	+	−	+	−	−	−	−
cd	−	−	+	+	−	−	+	+
ace	+	−	+	−	+	−	−	+
bcf	−	+	+	−	−	+	−	+
abcdef	+	+	+	+	+	+	+	+

~ 1 b) Write the C effect in Yates' notation; constants of proportionality are not important.

C ~ − def − af − be − abd + cd + ace + bcf + abcdef

need 2 cols in +/− Table

~ 2 c) Suppose we need to perform the runs in 4 blocks. Give an example of the effect(s) you would choose to confound with block, if you don't care too much about about the D effect. Don't explain.

CEF and ACF ← This was the answer I had originally intended. But one student convinced me that this is wrong. See last page for a thorough discussion.

~ 2 d) Now, suppose you have decided to confound the CD and BF effects with block. Write the elements (in Yates' notation) of each of the blocks. Denote the blocks with Z_i, $i = 1, 2, \cdots$

Z_1 : [def, abd] − −

Z_2 : [ace, bcf] − +

Z_3 : [af, be] + −

Z_4 : [cd, abcdef] + +

~ 2 e) In terms of the Z_i in part c, write the block effect that is confounded with the C effect? Hint: This does not require a lot of writing; just look at the blocks and ask yourself which ones have some effect at high level and which ones have that effect at low level.

$C \sim (Z_2 + Z_4) - (Z_1 + Z_3)$

Z_1 and Z_3 have low C
Z_2 and Z_4 have high C

~ 2 hw, B(1) **10.** Find the expression for the probability of obtaining a negative value for the estimate of σ_α^2 in a problem involving two random factors. See Table on last page.

$$\sigma_\alpha^2 = \frac{1}{nb} E[MSA - MSAB] \implies \hat{\sigma}_\alpha^2 = \frac{1}{nb}(MSA - MSAB)$$

$$\text{prob}(\hat{\sigma}_\alpha^2 < 0) = pr(MSA < MSAB) = pr\left(\frac{MSA}{MSAB} < 1\right)$$

$$= pr\left(\frac{MSA/E[MSA]}{MSAB/E[MSAB]} < \frac{E[MSAB]}{E[MSA]}\right) = pr\left(F_{a-1,(a-1)(b-1)} < \underline{\frac{\sigma_\epsilon^2 + n\sigma_{\alpha\beta}^2}{\sigma_\epsilon^2 + nb\sigma_\alpha^2 + n\sigma_{\alpha\beta}^2}}\right)$$

$$= pr\left(F_{a-1,(a-1)(b-1)} < \frac{1}{1 + \frac{nb\sigma_\alpha^2}{\sigma_\epsilon^2 + n\sigma_{\alpha\beta}^2}}\right) \leftarrow \text{This form can help in looking at limits, e.g. } n \to \infty.$$

11. We have data y_{ijk} on two factors A and B, with n replications, where $i = 1, \cdots a, j = 1, \cdots b, k = 1, \cdots n$. Suppose we estimate the A effect **at each level of the B factor, separately**. The model is then $y_{ijk} = \mu + \alpha_i + \epsilon_{ijk}$ **for fixed** j.

~ 2 a) Find the least-squares estimate $\hat{\alpha}_i$, after imposing some natural constraint(s). Hint: Since j is fixed, the expression for SSE will not have a sum over j.

$Y_{ijk} = \mu + \alpha_i + \epsilon_{ijk}$ for fixed j (suppressing a j index on μ, α_i, ...)

$$SSE = \sum_{ik}^{an} (Y_{ijk} - \mu - \alpha_i)^2$$

impose $\underline{\hat{\alpha}_. = 0 \text{ Constraint.}}$

$$\frac{\partial}{\partial \mu}: \sum_{ik} (Y_{ijk} - \mu - \alpha_i)^2 \implies \overline{Y}_{.j.} - \hat{\mu} - \overline{\hat{\alpha}}_. = 0 \implies \hat{\mu} = \overline{Y}_{.j.}$$

$$\frac{\partial}{\partial \alpha_i}: \sum_{k} (Y_{ijk} - \mu - \alpha_i)^2 \implies \overline{Y}_{ij.} - \hat{\mu} - \hat{\alpha}_i = 0 \implies \boxed{\hat{\alpha}_i = \overline{Y}_{ij.} - \overline{Y}_{.j.}}$$

~ 2 b) Now, consider the full model $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$, and recall the following estimates (The ˆ symbol has been neglected for convenience): $\mu = \overline{y_{...}}, \alpha_i = \overline{y_{i..}} - \overline{y_{...}}, \beta_j = \overline{y_{.j.}} - \overline{y_{...}}, (\alpha\beta)_{ij} = \overline{y_{ij.}} - \overline{y_{i..}} - \overline{y_{.j.}} + \overline{y_{...}}$. Is there a relationship between the A effect estimated in part a, and the estimates in the full model? If so, what is it? If not, explain why not?

There are infact 2 relationships (either one is sufficient)

1) α_i (full model) = Sample avg (over j) of $\hat{\alpha}_i$ (part a)

2) $$\widehat{(\alpha\beta)}_{ij} = \overline{Y}_{ij.} - \overline{Y}_{.j.} - \overline{Y}_{i..} + \overline{Y}_{...}$$

$$\underline{\widehat{(\alpha\beta)}_{ij} + \hat{\alpha}_i} = \overline{Y}_{ij.} - \overline{Y}_{.j.} = \underline{\hat{\alpha}_i \text{ above.}}$$

Since $\widehat{(\alpha\beta)}_{i.} = 0$, taking avg of both sides gives

Design and Results for Wine Tasting Experiment

Run	Variable A	B	C	D	E	F	G	H
1	−	−	−	−	−	−	−	−
2	+	−	−	−	−	+	+	+
3	−	+	−	−	+	−	+	+
4	+	+	−	−	+	+	−	−
5	−	−	+	−	+	+	+	−
6	+	−	+	−	+	−	−	+
7	−	+	+	−	−	+	−	+
8	+	+	+	−	−	−	+	−
9	−	−	−	+	+	+	−	+
10	+	−	−	+	+	−	+	−
11	−	+	−	+	−	+	+	−
12	+	+	−	+	−	−	−	+
13	−	−	+	+	−	−	+	+
14	+	−	+	+	−	+	−	−
15	−	+	+	+	+	−	−	−
16	+	+	+	+	+	+	+	+

(e) 2^{6-3}; 1/8 fraction of
6 factors in 8 runs

Resolution III

Design Generators

$D = AB \quad E = AC \quad F = BC$

Defining relation: $I = ABD = ACE = BCDE = BCF = ACDF = ABEF = DEF$

Aliases

$A = BD = CE = CDF = BEF$ $\qquad$ $E = AC = DF = BCD = ABF$

$B = AD = CF = CDE = AEF$ $\qquad$ $F = BC = DE = ACD = ABE$

$C = AE = BF = BDE = ADF$ $\qquad$ $CD = BE = AF = ABC = ADE = BDF = CEF$

$D = AB = EF = BCE = ACF$

Table of expected values for 2 random factors:

$$
\begin{aligned}
E[MSA] &= \sigma_\epsilon^2 + nb\sigma_\alpha^2 + n\sigma_{\alpha\beta}^2 \\
E[MSB] &= \sigma_\epsilon^2 + na\sigma_\beta^2 + n\sigma_{\alpha\beta}^2 \\
E[MSAB] &= \sigma_\epsilon^2 + n\sigma_{\alpha\beta}^2 \\
E[MSE] &= \sigma_\epsilon^2
\end{aligned}
$$

ACF ~ Block.

9c) : We need to pick 2 words/effects and confound each one with block. Given the alias structure, one good choice is a high-order word/effect from the CD = ... line, e.g CEF. We should not take the other word from the same line, e.g. CD, because then CD x CEF = DEF will also be confounded with block. But DEF = I is one of the defining relations (ie. it cannot be confounded with block, because it's not even estimable).

For the 2nd one, we could take a high-order effect from the D line, e.g. ACF because according to the problem, we don't care if the D effect is aliased with something which is itself confounded with block. Of course, then CEF x ACF = AE is also confounded with block. But note that AE is aliased with C, and so C gets confounded with block, too. So that's bad!

We can try confounding D itself with block. In that case CEF x D = CDEF is also confounded with block; but then CDEF is aliased with C again. So, again C will be aliased with something that is confounded with block.

At the end, it looks like choosing CEF as one of the effects to confound with blocks leads to undesirable results!
So, this was just a bad question!