	blocked data	unblocked data
blocked model	0.002	0.044
unblocked model	0.023	0.023

Name: ______________________

ID: _________________________

Stat 421, Test 2, Fall, Nov. 13, 2013; Marzban

7 + 13 +3 extra

ONLY a half-size "cheat sheet" is allowed

Multiple choice: Circle all the correct answers; there is wrong-answer penalty

For questions above horizontal line, do not explain.

For questions below horizontal line,

SHOW answer & work; NO CREDIT for correct answer without explanation

Points

1 **1.** Circle the correct answer. The t-test (or F-test) in the contrast-based approach to hypothesis testing allows one to perform

a) only pairwise comparisons.

b) any comparison, controlling family-wise error rate.

c) any comparison, without controlling family-wise error rate.

d) any comparison, controlling family-wise error rate when contrasts are orthogonal.

This model is "blind" to blocks in data.

1 Lab3 **2.** To test for a treatment effect, suppose we plan to develop the model $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$. If the design of the experiment truly does not involve blocks, then the p-value of the model will be generally _______ the p-value of the model if the design truly involves blocks. 0.023 vs. 0.023

a) lower than b) higher than c) comparable to d) unrelated to

1 hw_W Lect13 p-7 Book p.182 **3.** For the additive model $y_{ijk} = \mu + \alpha_i + \beta_j + \epsilon_{ijk}$, an estimate of the residuals is given by

a) $\bar{y}_{ij.} - \bar{y}_{...}$ b) $y_{ijk} - \bar{y}_{...}$ c) $y_{ijk} - \bar{y}_{ij.}$ d) $y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}$

$y_{ijk} = \bar{y}_{...} + (\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...})$

~~2~~ hw_W **4.** In assessing the goodness of a model, one examines the scatterplot of the residuals versus the predictions (i.e., fitted values). For the full model one should expect No corr and for the additive Ignored ~~model one should expect~~ $\boxed{\text{a corr}}$. Fill the blanks with "a correlation" or "no correlation."

For sample. But in E[ ], it depends on if ∃ interaction in pop. For sample and in E[ ].

1 Lect14 p.2 **5.** In standard notation, suppose we have used the model $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$, to represent data from a factorial problem involving two factors (A,B), and n replicates. Further, suppose we have found that α_i, and $(\alpha\beta)_{ij}$ are significant, but β_j is not. Then we can conclude that the A treatment effect is due to

a) factor A b) the interaction with B c) Both a and b.

1 hw_AC **6.** In a 2^k problem, the product of the A effect and B effect is equal to the AB interaction effect if

a) it is significant b) it is not significant c) None of the above.

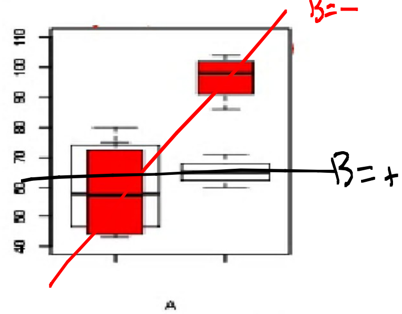
1 Lect17 example 6-2 **7.** In the following interaction plot, suppose the shaded (unshaded) boxplot corresponds to the low (high) level of a factor B. The sign of the interaction term in the model will then be

a) plus

b) minus

c) insufficient info to tell

$AB \sim \underbrace{(\text{Avg A effect} \mid B=+)}_{\sim 0} - \underbrace{\text{Avg A effect} \mid B=-)}_{>0}$



~~2~~ 1 Lect14 p.5 **8.** For a problem involving one treatment (A) and one block (B), write the most appropriate model.

$y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}$

$i = 1 --- a$

$j = 1 --- b$

OR

$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

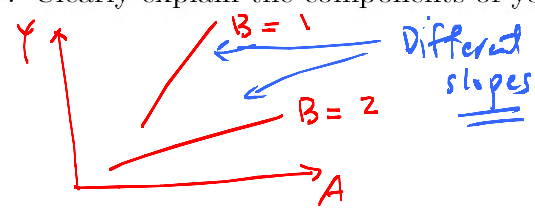
$i = 1 -- a$

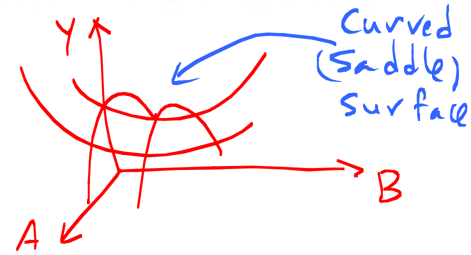
$j = 1 -- b$

$k = 1 --- n$ ← replicates.

Lect 12, P.2 Lect 13 p.6 Lect 15 p.2 Lect 17 p.5

3 **9.** In a problem involving a response (y), and two quantitative factors (A and B), how will an interaction between A and B manifest itself in **a)** a 2d plot of y vs. A, and in **b)** a 3d plot of y vs. A and B? Clearly explain the components of your diagrams.





1 Lect 17 p.8 **10.** In a 2^k problem, what *kind of design* will cause an effect to get confounded with block?

In complete Block

hw-V lect 17 **11** In a Latin Square Design involving 3 factors (A, B, C), each with 3 levels, the following data have been collected. The numbers in parantheses are y_{ijk} corresponding to the i^{th}, j^{th}, k^{th} level of

2 A,B,C, respectively. **a)** Estimate the effect of C for each of its 3 levels.

	B_1	B_2	B_3
A_1	$C_1(+1)$	$C_2(-1)$	$C_3(-1)$
A_2	$C_2(0)$	$C_3(-1)$	$C_1(0)$
A_3	$C_3(-1)$	$C_1(+2)$	$C_2(+1)$

Effect at C_1: $\bar{Y}_{..1} - \cancel{\bar{Y}_{...}}^{0} = \frac{1}{3}(+1+0+2) = +1$ restricted

Effect at C_2: $\bar{Y}_{..2} - \bar{Y}_{...} = \frac{1}{3}(-1+0+1) = 0$

Effect at C_3: $\bar{Y}_{..3} - \bar{Y}_{...} = \frac{1}{3}(-1-1-1) = -1$

$\bar{Y}_{...} = \frac{1}{9}(1-1-1+0-1+0-1+2+1) = 0$

restricted over 9, not 27

2 **b)** Compute SS_C using nothing more than $SS_C = \sum'_{ijk}(\overline{y}_{..k} - \overline{y}_{...})^2$, where the prime on the sum indicates a restricted sum over 9 elements.

$$SS_C = \sum_{ijk}{}'(\bar{Y}_{..k} - \cancel{\bar{Y}_{...}}^{0})^2 = \left[\bar{Y}^2_{..1} + \bar{Y}^2_{..1} + \bar{Y}^2_{..1} + \bar{Y}^2_{..2} + \bar{Y}^2_{..2} + \bar{Y}^2_{..2} + \bar{Y}^2_{..3} + \bar{Y}^2_{..3} + \bar{Y}^2_{..3}\right]$$

restricted over 9

$$= 3(\bar{Y}^2_{..1} + \bar{Y}^2_{..2} + \bar{Y}^2_{..3}) = 3(1+0+1) = 6$$

2 Lect 16 p.4 Book p.226 **12.** In a 2^k design, an interaction effect has been estimated to be $[(1) - a - b + ab]/2$. Under the null hypothesis of no interaction, what is the expected value of the estimate? Show work.

effect

$$E[(1) - a - b + ab] = E[(1)] - E[a] - E[b] + E[ab] = 0$$

$$\underbrace{\text{Sum of } y\text{'s}}_{\text{all equal under } H_0\text{: No effect}}$$

~~3~~ 2 Lect 16 p.2 **13.** For a 2^2 factorial problem, show that SS_A, as defined by $\frac{1}{2n}\sum_i y_{i..}^2 - \frac{1}{4n}y_{...}^2$ is equal to $\frac{1}{4n}(\text{contrast}_A)^2$, where $\text{contrast}_A = [-(1) + a - b + ab]$.

$$SS_A = \frac{1}{2n}\sum_i^2 Y_{i..}^2 - \frac{1}{4n}Y_{...}^2 = \frac{1}{2n}(Y_{1..}^2 + Y_{2..}^2) - \frac{1}{4n}(Y_{1..} + Y_{2..})^2$$

$$= \frac{1}{2n}\left[Y_{1..}^2 + Y_{2..}^2 - \frac{1}{2}Y_{1..}^2 - \frac{1}{2}Y_{2..}^2 - Y_{1..}Y_{2..}\right]$$

$$= \frac{1}{4n}\left[Y_{1..}^2 + Y_{2..}^2 - 2Y_{1..}Y_{2..}\right] = \frac{1}{4n}(Y_{1..} - Y_{2..})^2$$

$$= \frac{1}{4n}\left[\underbrace{((1)+b)}_{\text{low } A} - \underbrace{(a+ab)}_{\text{high } A}\right]^2$$

3 **14.** Consider the model $y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}$, with $i = 1, ..., a$, $j = 1, ..., b$, for a treatment factor A (with a-levels), and a block factor B (with b levels). For $a = 2$, show that the statistic $F = MS_A/MS_E$ is equal to the square of the t statistic $t = \frac{\bar{d}}{s_d/\sqrt{b}}$, where $d_j = y_{1j} - y_{2j}$. You may use the fact that $SS_A = \sum_{ij}(\overline{y}_{i.} - \overline{y}_{..})^2$ and $MS_E = \frac{1}{2}s_d^2$. (This shows that an F-test of $\alpha_i = 0$ is equivalent to a paired t-test of the equality of two means.)

$$SS_A = \sum_{ij}^{ab} (\bar{y}_{i.} - \bar{y}_{..})^2 = b \sum_{i}^{a} (\bar{y}_{i.} - \bar{y}_{..})^2 = b\left[(\underbrace{\bar{y}_{1.} - \bar{y}_{..}}_{\downarrow})^2 + (\underbrace{\bar{y}_{2.} - \bar{y}_{..}}_{\text{similarly}})^2\right]$$

$$\frac{1}{b} y_{1.} - \frac{1}{2b} y_{..}$$

$$\frac{1}{b} y_{1.} - \frac{1}{2b}(y_{1.} + y_{2.})$$

$$\frac{1}{2b}(y_{1.} - y_{2.})$$

$$\frac{1}{2}(\bar{y}_{1.} - \bar{y}_{2.})$$

similarly

$$\frac{1}{2}(\bar{y}_{2.} - \bar{y}_{1.})$$

$$= b\left[\frac{1}{4}(\bar{y}_{1.} - \bar{y}_{2.})^2 + \frac{1}{4}(\bar{y}_{2.} - \bar{y}_{1.})^2\right]$$

$$SS_A = \frac{1}{2} b (\bar{y}_{1.} - \bar{y}_{2.})^2 = \frac{1}{2} b \bar{d}^2$$

$$\therefore F = \frac{MS_A}{MS_E} = \frac{\frac{1}{2} b \bar{d}^2}{\frac{1}{2} s_d^2} = \left[\frac{\bar{d}}{s_d/\sqrt{b}}\right]^2$$