Name: ________________________
ID: ___________________________

Stat 421, Test 3, Fall, Dec. 11, 2018; Marzban

6.75 + 21

2 hrs. ONLY a half-size "cheat sheet" is allowed

Multiple choice: Circle all the correct answers; there is wrong-answer penalty

For rest, SHOW answer & work; NO CREDIT for correct answer without explanation

Points

2017/test2/#6

1 **1.** In a $k(>3)$ -factor problem, the sum-of-ys, abc, appears in the (Circle all correct answers)

a) A effect in a full 2^k model.
b) A effect in an additive 2^k model.
c) ABC effect of an incomplete 2^{k-p} with ABC=+1
d) ABC effect of an incomplete 2^{k-p} with ABC=−1

I ignored these. In these cases, ABC is not estimable.

1 **2.** Which is TRUE? In an unreplicated 2^3 experiment performed in 4 blocks, if a 2-way interaction is confounded with block, then it follows that ______ effect must also be confounded with block.

a) no other
b) some main (1-way)
c) some 1- or 2-way
d) some 1- or 2- or 3-way
e) Insufficient info.

2 effects to confound with block. One is XY

If the other is Z (1-way) ⇒ XYZ also conf.
" " XZ (2-way) ⇒ YZ " "
" " XYZ (3-way) ⇒ Z " "

1 **3.** In a certain design with the adjacent ± table
if XYZ=+1 and XYZ=−1 are run in 2 blocks, respectively, then

a) XYZ and UVW are each confounded with block.
b) XYZ + UVW is confounded with block.
c) Neither A nor B

1 block effect.

⋯	XYZ	UVW
⋯	+	−
⋯	⋮	⋮
⋯	+	−
⋯	−	+
⋯	⋮	⋮
⋯	−	+

1 **4.** We have performed a 2^{k-p} experiment, and are now planning on a fold-over. The number of runs in the combined experiment is 2^{k-p+1} for a single-factor fold-over, and 2^{k-p+1} for a complete fold-over.

fold-over doubles the runs

hw-lect18-2, and 19-2

1 **5.** In a 2^k design in 4 incomplete blocks, we choose to confound X and Y with block. As we know, XY is then also confounded with block. Note that these statements refer to the effects. Now, suppose we block in a way that X and XY are each confounded with block, then the runs in

a) the new blocks will be the same as the old ones.
b) the new blocks will not necessarily be the same as the old ones.
c) whether or not the new blocks will be the same as the old ones depends on the alias structure.

0.75 **6.** In an incomplete design involving 3 factors, suppose we find that X is aliased with XYZ. Then,

a) [X] = X + XYZ   b) [XYZ] = X + XYZ   c) [X] = [XYZ]   d) none of the above

1 **7.** An article uses an unreplicated 2^{6-2} design, and reports one of the alias chains as $A = BCE = DEF$. It follows that the defining relations are (circle all correct answers)

ADEF, ABCE, BCDF

a) $ABCE = BCDF = 1$
b) $ABCE = ADEF = 1$
c) $ABCE = BDEF = 1$
d) Cannot be determined from only one chain.

~2 **8.** Suppose you are planning on performing a 2^k experiment but there is a good chance that your measuring device will heat-up and stop working about a quarter of the way through the experiment. Although the device will work again after it is allowed to cool down, we expect it to break permanantly after about half of the 2^k runs are performed. Briefly explain (in the language we have learned) what is the "best" design for this experiment.

2^{k-1} in 2 blocks. because each of the quarter runs is separated by a cooling down phase, i.e. restriction on randomization.

because at the very end we have done only half the 2^k runs.

(or
A fold-over of 2^{k-2})

hw-lect 8-3

9. For a 2^3 design performed in 4 incomplete blocks, in a lecture we showed that the 3 (zero-sum) block effects/contrasts $(B_1 + B_2) - (B_3 + B_4)$, $(B_1 + B_3) - (B_2 + B_4)$, and $(B_1 + B_4) - (B_2 + B_3)$, are proportional to the 3 effects (say, X, Y, Z) confounded with block. In a hw you showed that the block effect $(B_1 - \bar{B}_.)$ is confounded with $X + Y + Z$.

~1 a) What about the block effect $(B_2 - \bar{B}_.)$, or $(B_3 - \bar{B}_.)$ - are any effects confounded with them? Circle Yes or No . Because each of the 4−1=3 block effects must be confounded.

~1 b) If No, explain why not. If Yes, write one of the effects that is confounded with either $(B_2 - \bar{B}_.)$ or $(B_3 - \bar{B}_.)$; no explanations necessary. Feel free to guess (yes, guess), based on your experience.

$X+Y-Z$ is confounded with $B_2 - \bar{B}_.$
$X-Y+Z$ " " " $B_3 - \bar{B}_.$
} e.g. for the hw, $AB + AC - BC = B_2 - \bar{B}_.$ etc.

~2 c) Finally, show that any block effect of the type $(B_i - \bar{B}_.)$, with $i = 1 \cdots n$, where n is the total number of blocks is a zero-sum contrast in B_i. Hint: find the contrast vector. This doesn't have to be a mathematically rigorous proof, but do show some math.

$$B_i - \bar{B}_. = B_i - \frac{1}{n}(B_1 + B_2 + \cdots + B_i + \cdots + B_n) = \frac{1}{n}(-B_1 - B_2 + \cdots + (n-1)B_i - \cdots - B_n)$$

$$\text{contrast vector} = \frac{1}{n}\underbrace{(-1, -1, \cdots, (n-1), \cdots, -1)}_{\text{There are } (n-1) \text{ negative 1's, and 1 positive } (n-1) \Rightarrow \text{0-sum.}}$$

lect16/p.4, lect18/p.9

1.5 **10.** In a 2^3 experiment, the 8 runs are performed in the following four blocks: [(1), abc], [a, bc], [b, ac], [c, ab]. What is/are the effect(s) selected to confound with block? Do NOT use the ± table, but show your work/thinking. In each block, A=+, B=+, C=+; so no 2 of those can be used. Similarly, ABC=+ in all 4 blocks; so ABC cannot be used. That leaves us with AB, AC, or BC, and any 2 of them will work, e.g. AB, AC etc.

	AB	AC
[(1), abc]	+	+
[a, bc]		
[b, ac]		+
[c, ab]	+	−

c means C=+, A=−, B=− ⇒ AC = −

lect16/p.4

1 **11.** In a 2^3 experiment, the 8 runs were performed in the following two blocks: [(1), a, b, c] and [ab, ac, bc, abc]. Can you determine the effect that is confounded with the block effect? If so, find it. If not, explain why? Consult the ± table provided.

This blocking does not correspond to any effect

Block		Factorial Effect: *I*	*A*	*B*	*AB*	*C*	*AC*	*BC*	*ABC*
+	(1)	+	−	−	+	−	+	+	−
+	a	+	+	−	−	−	−	+	+
+	b	+	−	+	−	−	+	−	+
−	ab	+	+	+	+	−	−	−	−
+	c	+	−	−	+	+	−	−	+
−	ac	+	+	−	−	+	+	−	−
−	bc	+	−	+	−	+	−	+	−
−	abc	+	+	+	+	+	+	+	+

lect19/p.3

~~~2~~ 1 **12.** Is the following argument correct? If so, say YES. If not, point out which part(s) is/are wrong, and explain why not. A complete 2^k experiment is performed in 2^p incomplete blocks.

a) To construct the 2^p blocks, we need to choose p effects to confound with block effects.

b) But we know that in a design with 2^p blocks, there are $2^p - 1$ block effects.

c) Therefore, $2^p - 1 - p$ block effects are not confounded with anything. ← WRONG.

a) we select p effects to generate 2^p blocks, e.g. $\underbrace{X_1, X_2, \cdots X_p}$.

But then all products of these are also confounded with block, i.e.

X_1X_2, ... (2-way)
$X_1X_2X_3$, ... (3-way)
⋮
$X_1X_2 \cdots X_p$ (p-way)

} The sum of all these, and these is $(2^{k-p} - 1)$, and that is exactly the number of block effects (b
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$\sim$ ~~2~~ 1 **13.** The following is the alias structure for a 2^{7-2} design.

a) Write-in the lowest-order term (just 1 term) on the right-hand-side of the chains involving the main effects.

Design Generators

$F = ABCD \quad G = ABDE$

Defining relation: $I = ABCDF = ABDEG = CEFG$

Aliases

A = BCDF	$AB = CDF = DEG$	$BC = ADF$	$CE = FG$	$ACE = AFG$
B = ACDF	$AC = BDF$	$BD = ACF = AEG$	$CF = ABD = EG$	$ACG = AEF$
C = EFG	$AD = BCF = BEG$	$BE = ADG$	$CG = EF$	$BCE = BFG$
D = ABCF	$AE = BDG$	$BF = ACD$	$DE = ABG$	$BCG = BEF$
E = CFG	$AF = BCD$	$BG = ADE$	$DF = ABC$	$CDE = DFG$
F = CEG	$AG = BDE$	$CD = ABF$	$DG = ABE$	$CDG = DEF$
G = CEF				

Better (See Table X)

$\sim$ 2 b) We want to perform the runs in 4 blocks, and we know for sure that we want to confound ACE with block. Suppose we don't care much about the BCE and the BCG effect. Which one should we confound with block? Explain.

ACE and BCE confounded $\Rightarrow$ AB Confounded $\Rightarrow$ CDF and DEG confounded Better

ACE and BCG " $\Rightarrow$ ABEG " $\Rightarrow$ D = confounded Bad

$\hookrightarrow$ ABEG = D

$\sim$ ~~2~~ 1 c) What is/are the defining relation(s) for the complete fold-over? Explain. It is not necessary to include all aliased effects.

principle fraction : ABCDF = ABDEG = 1 (CEFG = 1)

altern. " : " " = −1 (" = 1)

defn. rel. doesn't change, ie. it's CEFG = 1

$\sim$ 2 d) If we treat the two fractions as blocks, what is/are the effect(s) confounded with block? It is not necessary to include all aliased effects.

Effects That change between princ. & alt. are ABCDF, ABDEG. But There is is only 1 block effect, so The effect conf. with block is ABCDF + ABDEG .

hw-left 23-2(a)

$\sim$ ~~2~~ 1 **14.** Consider a 2^{6-2} design with defining relations $ABCE = 1$ and $BCDF = 1$. Show that a complete fold-over does not allow any de-aliasing to occur at all. Hint: You do not need to work out the alias structure, nor the $\pm$ table.

Princ. fraction : ABCE = 1 BCDF = 1 (ADEF = 1)

Alt. fraction: same $\Rightarrow$ No de-aliasing.

$\sim$ ~~2~~ 1 **15.** Consider a Resolution IV design. The lowest-order defining relation will consist of a 4-letter word. Now suppose we perform a fold-over such that each of those 4 letters is flipped simultaneously. As such, the defining relation of the alternative fraction will include the same 4-letter word. Show that in the combined design each of the letters in the 4-letter word cannot be de-aliased from 3-way interactions involving the other 3 letters. Hint: Use X, Y, Z, W as the 4 letters.

One defn rel. XYZW = 1 , other defn. relations will be 4, 5, ... way

in alt. frac. same same −1

$\hookrightarrow$ X = YZW, etc.

$\hookrightarrow$ X = YZW, etc. $\Rightarrow$ cannot dealias main from 3-way .

16. The adjacent table shows $E[MS]$ for all the effects in a 3-factor random-effects model. Based on what we have learned from our 2-factor random-effects models,

Model Term	Factor	Expected Mean Squares
τ_i	A, main effect	$\sigma^2 + cn\sigma^2_{\tau\beta} + bn\sigma^2_{\tau\gamma} + n\sigma^2_{\tau\beta\gamma} + bcn\sigma^2_{\tau}$
β_j	B, main effect	$\sigma^2 + cn\sigma^2_{\tau\beta} + an\sigma^2_{\beta\gamma} + n\sigma^2_{\tau\beta\gamma} + acn\sigma^2_{\beta}$
γ_k	C, main effect	$\sigma^2 + bn\sigma^2_{\tau\gamma} + an\sigma^2_{\beta\gamma} + n\sigma^2_{\tau\beta\gamma} + abn\sigma^2_{\gamma}$
$(\tau\beta)_{ij}$	AB, two-factor interaction	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + cn\sigma^2_{\tau\beta}$
$(\tau\gamma)_{ik}$	AC, two-factor interaction	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + bn\sigma^2_{\tau\gamma}$
$(\beta\gamma)_{jk}$	BC, two-factor interaction	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + an\sigma^2_{\beta\gamma}$
$(\tau\beta\gamma)_{ijk}$	ABC, three-factor interaction	$\sigma^2 + n\sigma^2_{\tau\beta\gamma}$
ϵ_{ijkl}	Error	σ^2

~ 1 1

a) What's the test statistic for testing $\sigma^2_{\tau\beta}$?

$$\frac{MS_{AB}}{MS_{ABC}} \quad \text{with or w/o} \quad \frac{E[MS_{ABC}]}{E[MS_{AB}]}$$

~ 1

b) What's the ANOVA estimator for $\sigma^2_{\tau\beta}$?

$$\hat{\sigma}^2_{\tau\beta} = \frac{1}{nc}\left(MS_{AB} - MS_{ABC}\right)$$

~~~ 1~~ 1.5

c) For which variance component(s) can we build CIs? Only σ_ϵ^2!

hw-lect24-2

~ 2

17. For a single-factor random-effects model $y_{ij} = \mu + \tau_i + \epsilon_{ij}$, with the usual normality and iid assumptions, show that $E[SST] = (an-1)\sigma^2_\epsilon + (a-1)n\sigma^2_\tau$. You may use $E[(\epsilon_{ij} - \overline{\epsilon_{..}})^2] = (1 - \frac{1}{an})\sigma^2_\epsilon$.

$$E[SST] = E\Big[\sum_{ij} (y_{ij} - \bar{y}_{..})^2\Big] = \sum_{ij} E\big[(\cancel{\mu} + \tau_i + \epsilon_{ij} - \cancel{\mu} - \bar{\tau}_{\cdot} - \bar{\epsilon}_{..})^2\big]$$

$$= \sum_{ij}\Big(E[(\tau_i - \bar{\tau}_{\cdot})^2] + E[(\epsilon_{ij} - \bar{\epsilon}_{..})^2] + E[2(\tau_i - \bar{\tau}_{\cdot})(\epsilon_{ij} - \bar{\epsilon}_{..})]\Big)$$

$\boxed{\tau \text{ and } \epsilon \text{ are indep.}}$

$$V[\tau_i - \bar{\tau}_{\cdot}] + \underbrace{E^2[\tau_i - \bar{\tau}_{\cdot}]}_{0} \qquad \boxed{E[\tau_i] = 0}$$

$$2\underbrace{E[(\tau_i - \bar{\tau}_{\cdot})]}_{0}\,\underbrace{E[\epsilon_{ij} - \bar{\epsilon}_{..}]}_{0} \qquad \boxed{E[\tau_i] = E[\epsilon_{ij}] = 0}$$

$$V[\tau_i] + V[\bar{\tau}_{\cdot}] - 2Cov[\tau_i, \bar{\tau}_{\cdot}] = \underline{\left(1 - \frac{1}{a}\right)\sigma_\tau^2}$$

$\boxed{iid}$

$$V[\tau_i] = \sigma_\tau^2, \quad V[\bar{\tau}_{\cdot}] = \frac{\sigma_\tau^2}{a}, \quad Cov[\tau_i, \bar{\tau}_{\cdot}] = \frac{1}{a}\underbrace{\sum_k Cov[\tau_i, \tau_k]}$$

$\boxed{iid}$ $Cov[\tau_i, \tau_i] = V[\tau_i] = \sigma_\tau^2$

$$E[SST] = \underbrace{\sum_{ij}}_{an}\left[\left(1 - \frac{1}{a}\right)\sigma_\tau^2 + \left(1 - \frac{1}{an}\right)\sigma_\epsilon^2\right]$$

$$= (a-1)n\,\sigma_\tau^2 + (an-1)\,\sigma_\epsilon^2$$