Lect 25-2

a

Let A denotes factor operation, B denotes factor part.

```
library(AlgDesign)
design1 <- gen.factorial(c(3,2,10), varNames = c('Rep', 'Ope', 'Part'), factors = 'all')
attach(design1)
y <- c(50, 49, 50, 50, 48, 51, 52, 52, 51, 51, 51, 51, 53, 50, 50, 54, 52, 51,
         49, 51, 50, 48, 50, 51, 48, 49, 48, 48, 49, 48, 52, 50, 50, 52, 50, 50,
         51, 51, 51, 51, 50, 50, 52, 50, 49, 53, 48, 50, 50, 51, 50, 51, 48, 49,
         47, 46, 49, 46, 47, 48)
lm1 <- lm(y~Ope*Part)
MSA <- summary.aov(lm1)[[1]][1,3]
MSB <- summary.aov(lm1)[[1]][2,3]
MSAB <- summary.aov(lm1)[[1]][3,3]
MSE <- summary.aov(lm1)[[1]][4,3]

> summary.aov(lm1)
            Df Sum Sq Mean Sq F value   Pr(>F)
Ope          1   0.42   0.417   0.278    0.601
Part         9  99.02  11.002   7.335 3.22e-06 ***
Ope:Part     9   5.42   0.602   0.401    0.927
Residuals   40  60.00   1.500

> c(MSA, MSB, MSAB, MSE)
[1]  0.4166667 11.0018519  0.6018519  1.5000000
```

b

```
a <- 2
b <- 10
n <- 3
F_A <- MSA/MSAB
F_B <- MSB/MSAB
F_AB <- MSAB/MSE
p_A <- pf(F_A, df1=a-1, df2=(a-1)*(b-1), lower.tail = F)
p_B <- pf(F_B, df1=b-1, df2=(a-1)*(b-1), lower.tail = F)
p_AB <- pf(F_AB, df1=(a-1)*(b-1), df2=a*b*n-a*b, lower.tail = F)

> c(F_A, F_B, F_AB)
[1]  0.6923077 18.2800000  0.4012346
> c(p_A, p_B, p_AB)
[1] 4.269057e-01 9.381063e-05 9.270089e-01
```

For $H_0 : \sigma_A^2 = 0$, F-ration is 0.6923077. With the null hypothesis, the p-value is $4.269057e-01$.

This test result fails to provide significant evidence against null hypothesis.

For $H_0 : \sigma_B^2 = 0$, F-ration is 18.2800000, p-value is $9.381063e-05$, which provides significant evidence against the null hypothesis that $sigma_B^2 = 0$.

For $H_) : \sigma_{AB}^2 = 0$, F-ration is 0.4012346, and p-value is $9.270089e-01$, failed to provide strong evidence against the null.

c

```
est_sigmaA <- 1/(n*b) * (MSA - MSAB)
est_sigmaB <- 1/(n*a) * (MSB - MSAB)
est_sigmaAB <- 1/n * (MSAB - MSE)
est_sigmaE <- MSE

> c(est_sigmaA, est_sigmaB, est_sigmaAB, est_sigmaE)
[1] -0.00617284  1.73333333 -0.29938272  1.50000000
```

$\hat{\sigma_A}^2 = -0.00617284$
$\hat{\sigma_B}^2 = 1.73333333$
$\sigma\hat{_{AB}}^2 = -0.29938272$
$\hat{\sigma_\epsilon}^2 = 1.5$

d

```
 sigmaY <- var(y)
 > sigmaY
[1] 2.794068
> sum(est_sigmaA, est_sigmaB, est_sigmaAB, est_sigmaE)
[1] 2.927778
```

Sum of estimated variances is approximately equal to variance of data.