Name: ______________________
ID: ______________________
Quiz section or time: ______________________

6 + 10.5

Stat 421, Test 2, Fall, Nov. 14, 2012; Marzban

ONLY a half-size "cheat sheet" is allowed

Multiple choice: Circle all the correct answers; there is wrong-answer penalty; do NOT explain

The rest: SHOW answer & work; NO CREDIT for correct answer without explanation

Points

1 Ch4 p.140 **1.** Given 3 factors, each with p levels, a completely random design with no replication requires __p^3__ observations, and a latin-square design requires __p^2__ observations. Fill in the blanks.

1 Ch5, lect 11 **2.** In a 2-factor problem, if an interaction term is found to be NOT significant, then
a) the difference between the response at one level of one factor and another level of the same factor is expected to be a constant.
b) the residuals of the full model (including interaction) are expected to be comparable to the residuals of the reduced model (excluding interaction).
c) the two factors are expected to be independent (or uncorrelated).
d) none of the above.

$y_{ij} = \underset{1}{\mu} + \underset{a-1}{\alpha_i} + \underset{b-1}{\beta_j} + \epsilon_{ij}$

Example 5.5 hw-P

1 **3.** A problem deals with two quantitative variables, each observed at **3 levels**, with $n = 1$ replication. Both ANOVA and regression models are considered. Draw line(s) between the equivalent ANOVA model(s) (on left) and regression model(s) (on right). The term "equivalent" may be interpreted as comparable predictions, MSE, or goodness-of-fit.

$y \sim A + B$ → 5 pars including μ	$y \sim x_1 + x_2$ → $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$ ⟹ 3 pars. including intercept.
$y \sim A + B + A * B$ → 9 pars	$y \sim x_1 + x_2 + x_1x_2$ → 4 pars.
$y \sim A + B + A * B + A^2 + B^2$	$y \sim x_1 + x_2 + x_1x_2 + x_1^2 + x_2^2 + x_1x_2^2 + x_1^2x_2 + x_1^2x_2^2$ → $\boxed{9 \text{ pars.}}$

$1 + (a-1) + (b-1) + (a-1)(b-1)$

This makes no sense in ANOVA!

1 **4.** In a 2^k design with $n = 1$ replication, suppose for reasons beyond your control, one of the k factors has been blocked. Which of the following is/are true? And it's incomplete.
a) One of the 2^k effects will be confounded with block effects.
b) None of the main effects will be confounded with block effects.
c) Only the highest-order interaction will be confounded with block effects.
d) It is not necessary for any effect (main or interaction) to get confounded with block effects.

2 hwY **5.** Circle all the true statements. In a full 2^k model with no replication
a) $MS_E = 0$.
b) It is impossible to identify significant effects. qqnorm of effects
c) If the design is a balanced incomplete block design, then some effect will be confounded with a block effect.
d) If effect X is confounded with a block, then an estimate of the block effect is actually the sum (or difference) of the true block effect and the X effect.

~ 2 Lect 19, p.2 Ch7 **6.** In one or two sentences, define/describe a Balanced Incomplete Block Design.

2 A Design in which each block has The same # of Treatment combinations, but each block does not contain all The Treatment combinations.

Right	Wrong	points
3	0	2
2	0	1.5
1	0	1
3	1	1
2	1	1
1	1	0.5

~ 2 example p.210 **7.** In a 2^2 design with $n = 10$ replicates, we have found $(1) = 80, a = 100, b = 60, ab = 90$. Compute the contributions, to the total SS, of the effect A and the interactions effect AB. Hint: consider contrasts.

2

b 60 ab 90

(1) 80 a 100

$$SS = \frac{(\text{Contrast})^2}{2^2 \cdot n} = \frac{[2^{2-1} \cdot n \text{ effect}]^2}{2^2 \cdot n} = n \cdot [\text{effect}]^2$$

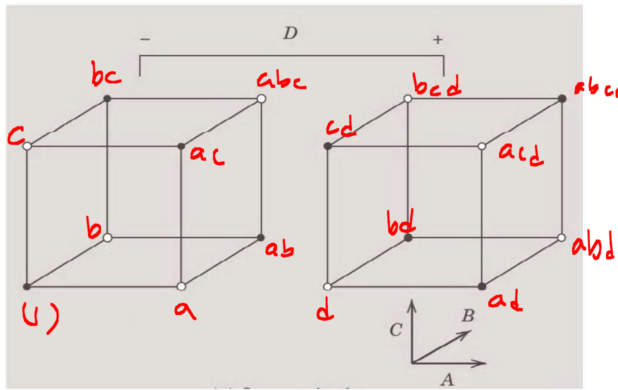
$$SS_A = 10 \cdot \left[\frac{(100-80)+(90-60)}{2 \cdot 10}\right]^2 = \frac{(50)^2}{40} = \frac{2500}{40} = \frac{125}{2}$$

$$SS_{AB} = 10\left[\frac{90+80}{2(10)} - \frac{100+60}{2(10)}\right]^2 = \frac{(10)^2}{4(10)} = \frac{10}{4} = \frac{5}{2}$$

~ 2 example 6.2 **8.** The following cubes are intended to provide a geometric view of a 2^4 design.

(1) a) Place the elements $(1), a, b, ab, ..., abcd$ on the appropriate vertices of the cubes.

(1.5) b) Write the effect ABD, in terms of $(1), a, b, ...$



$$ABD = \frac{1}{2}\left[(AB \text{ effect} \mid D=+) - (AB \text{ effect} \mid D=-)\right]$$

$$= \frac{\left(\frac{abd+d}{2} - \frac{ad+bd}{2}\right) + \left(\frac{abcd+cd}{2} - \frac{bcd+acd}{2}\right)}{2} - \frac{\left(\frac{ab+(1)}{2} - \frac{a+b}{2}\right) + \left(\frac{abc+c}{2} - \frac{bc+ac}{2}\right)}{2}$$

$$= \frac{1}{8}(a-1)(b-1)(c+1)(d-1).$$

~ 2 6.36 **9.** Often the fitted regression model from a 2^k factorial design is used to make predictions at points of interest in the design space. Find the variance of the predicted response $\hat{y}$ at a point $x_1, x_2, ..., x_k$ in the design space. Hint: Let the x's be coded variables (i.e., $-1 \le x_i \le +1$). Assume a 2^k design with an equal number of replicates n at each design point, so that the variance of a regression coefficient is $\sigma^2/(2^k n)$, and that the covariance between any pair of regression coefficients is zero.

2

$$\hat{y} = \hat{\beta}_0 + \sum_{i=1}^{p} \hat{\beta}_i x_i \quad (\text{e.g. } \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \cdots + \hat{\beta}_3 x_1 x_2 + \cdots)$$

$$V[\hat{y}] = V[\hat{\beta}_0] + \sum_i^p V[\hat{\beta}_i \cdot x_i]$$

$$= \underbrace{V[\hat{\beta}_0]}_{\sigma^2/2^k \cdot n} + \sum_i^p x_i^2 \underbrace{V[\hat{\beta}_i]}_{\sigma^2/2^k \cdot n} = \frac{\sigma^2}{2^k \cdot n}\left(1 + \sum_i^p x_i^2\right)$$

~ 2 7.4, 7.9, ... **10.** In a 2^3 design involving the 3 factors A, B, C, with $n = 1$ replicate, how should we block the data so that the effects AB and BC will be confounded with block effect? Refer to the data in the form $(1), a, b, ...$

2

	A	B	C	AB	AC
(1)	−	−	−	+	+
a	+	−	−	−	−
b	−	+	−	−	+
ab	+	+	−	+	−
c	−	−	+	+	−
ac	+	−	+	−	+
bc	−	+	+	−	−
abc	+	+	+	+	+

Block 1 : (1), abc

2 : a, bc

3 : b, ac

4 : c, ab