Name: ________________________
ID: ________________________

1.5 hrs

9+21

Stat 421, Test 3, Fall, Dec. 12, 2017; Marzban

ONLY a half-size "cheat sheet" is allowed

Multiple choice: Circle all the correct answers; there is wrong-answer penalty

For rest, SHOW answer & work; NO CREDIT for correct answer without explanation

Points

hw-lect 18-1

1 **1.** In a 2^k experiment performed in 8 blocks, → $df_{block} = 8-1=7$

a) A total of k effects are confounded with k different block effects.
b) A total of 7 effects are confounded with 7 different block effects.
c) A total of 7 effects are confounded with a unique block effect.
d) A total of k effects are confounded with 7 different block effects.

1 **2.** In an incomplete design involving 4 factors, suppose we find that A is aliased with ABC. Let's also assume that the true ABC effect in the complete design is negligible. Then,
a) A is negligible. b) A + ABC is negligible c) BC is not estimable d) None of the above

hw-lect 19-3

1 **3.** In a 2^{5-1} design with $ABCD = 1$, which of the following is/are TRUE?
a) E is not estimable
b) E is not aliased with anything
c) In the ± table, one can generate D from A, B, C only
d) The ± table has 2^{5-1} runs in it.

-0.5 penalty.

Apparently, this only, confused a lot of students. So, (c,d) or (d) count as 1 point.

hw-lect 20-5

1 **4.** In a 2^{k-p} design, which of the following is/are TRUE?
a) If two columns in the ± table are identical, then the corresponding factors are aliased.
b) If effects X, Y, and Z are aliased, then a linear combination of X, Y, Z effects in the complete design is estimable.
c) If effects X, Y, and Z are aliased, and Z is confounded with block, then a linear combination of X, Y, Z effects in the incomplete design is confounded with block. Ignored, because it's confusing.
d) None of the above.

hw-lect 21-1

1 **5.** We are attempting to perform a 2^{6-2} experiment. One defining relation is $ABCE = 1$. What should the second defining relation be if we want our design to have resolution III?
a) $BCDF = 1$ b) $ABCDF = 1$ c) We don't need a second defining relation.

↳ $BCDF(ABCE) = ADEF \Rightarrow$ Res IV. → $ABCE(ABCDF) = DEF \rightarrow$ Res III.

lab 6

(*) 1 **6.** In the ANOVA table printout from a 2^{k-p} analysis, if an SS entry is
a) "NA," then that effect is possibly confounded with block effect.
b) "NA," then that effect is possibly aliased with some other effect.
c) missing, then that effect is possibly confounded with block effect.
d) missing, then that effect is possibly aliased with some other effect.

(*) Even though this problem is worth 1 point, it should not have said so, because both of these are correct. So, circling either one got you full score.

hw-lect 22-2

1 **7.** Consider a 3^3 design in 3 blocks. We have learned that if we block according to the values of $L = xA + yB + zC(mod3)$, with x, y, z taking values $0, 1, 2$, then the component of the interaction term that is confounded with block is denoted as SS_{A^x,B^y,C^z}. We have also learned that the 8 dfs associated with the ABC interaction can be decomposed into the following four components. If we block according to $L = 2A + B + C$, then SS_{block} is equal to
a) SS_{ABC} b) SS_{AB^2C} c) SS_{ABC^2} d) $SS_{AB^2C^2}$

$2A+B+C = 2(2A+B+C)$
$= 4A+2B+2C$
$= A+2B+2C$

hw-lect 23-2 and 24-2

1 **8.** In a random-effects model, which of the following is/are TRUE.
a) Each of the analysis-of-variance estimators of the variance components is unbiased.
b) The sample variance of the response is an unbiased estimator of σ_y^2.
c) The sum of the estimated variance components (including the error variance) is σ_y^2.

9. The following is the table of expected values for a mixed-effects model (ignore the numerical data in there). According to this table, which of the following is/are true?

A	$\sigma^2 + bn\sigma^2_{\tau\gamma} + cn\sigma^2_{\tau\beta} + n\sigma^2_{\tau\beta\gamma} + \frac{bcn}{a-1}\sum \tau_i^2$
B	$\sigma^2 + an\sigma^2_{\beta\gamma} + acn\sigma^2_{\beta}$
C	$\sigma^2 + an\sigma^2_{\beta\gamma} + abn\sigma^2_{\gamma}$
AB	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + cn\sigma^2_{\tau\beta}$
AC	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + bn\sigma^2_{\tau\gamma}$
BC	$\sigma^2 + an\sigma^2_{\beta\gamma}$
ABC	$\sigma^2 + n\sigma^2_{\tau\beta\gamma}$
Error	σ^2

a) $\sigma^2_\tau = E[MS_A + MS_{ABC} - MS_{AB} - MS_{AC}]$

b) $\frac{MS_A+MS_{ABC}}{MS_{AB}+MS_{AC}} \sim F$ (approximately), if $\tau_i = 0$.

c) One can build an approximate CI for σ^2_τ.

d) None of the above.

τ=fixed, ie. There is no σ_τ^2!

~2 lect 17, Table 7-9

10. We are planning on performing a 2^4 experiment in 8 incomplete blocks, and we are trying to decide which effects to confound with block. We care about the main effects. Our options are I) AB, BC, AD, or II) AC, BC, ABD. Which one is the better option? Explain. I is better.

I) AB BC AD

AC ABCD

BD

$AB(BD)=AD$

$BC(BD)=CD$

$AD(BD)=AB$

II) AC BC ABD

AB ACD

BCD

$BC(BCD)=D$

↑

Bad.

~2 hw lect 17-1

11. In a 2^3 design, the runs have been performed in the following 4 blocks: $[(1), a], [b, ab], [c, ac], [bc, abc]$. Find the effects(s) that is/are confounded with block. Show work.

4 blocks ⇒ 3 block effects

	[(1), a]	[b, ab]	[c, ac]	[bc, abc]
C	−	−	+	+
B	−	+	−	+

⇒ B, C, and BC are confounded with block.

~2 ?

12. In a 2^{2-1} design with $AB = -1$, we have shown that $[A] = -[B]$ by writing $[A]$ and $[B]$ in Yates' notation. (Note that this defining relation selects the runs a and b only.) Here, we want to get that result in a different way. Consider the model $y_{ij} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ij}$. Write $\hat{\alpha}_1$ and $\hat{\beta}_1$ in terms of y_{ij}, and show that $\hat{\alpha}_1 = -\hat{\beta}_1$.

Y_{21} Y_{12}

$\hat{\alpha}_1 = \bar{Y}_{1\cdot} - \bar{Y}_{\cdot\cdot} = \bar{Y}_{1\cdot} - \frac{1}{2}(\bar{Y}_{1\cdot} + \bar{Y}_{2\cdot}) = \frac{1}{2}(\bar{Y}_{1\cdot} - \bar{Y}_{2\cdot}) = \frac{1}{2}(Y_{12} - Y_{21})$

$\hat{\beta}_1 = \bar{Y}_{\cdot 1} - \bar{Y}_{\cdot\cdot} = \bar{Y}_{\cdot 1} - \frac{1}{2}(\bar{Y}_{\cdot 1} + \bar{Y}_{\cdot 2}) = \frac{1}{2}(\bar{Y}_{\cdot 1} - \bar{Y}_{\cdot 2}) = \frac{1}{2}(Y_{21} - Y_{12})$

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hw lect 21-3

13. Consider the 2^{6-3} design with $ABD = ACE = BCF = 1$. In a complete fold-over,

~~~2~~ 1.5 a) What are the defining relations of combined design? I.e. words That don't change across The two fractions.

$(ABD)(ACE) = BCDE = 1$

$(ABD)(BCF) = ACDF = 1$

$(ACE)(BCF) = ABEF = 1$

Any 2 of These, because The Third is implied. Technically There are $\underline{\underline{2}}$ defining relations. But we gave full credit (1.5) if you wrote all 3.

~2 b) If we treat the principal fraction and the alternate fraction as two blocks, what is/are the effect(s) confounded with block? No need to include 4- (and higher-) order effects. words That do change.

ABD, ACE, BCF, and $\underline{\text{DEF}}$ change across The 2 fractions. But because There are 2 blocks, There is only 1 block effect: $\underline{ABD+ACE+BCF+DEF}$
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hw-lect20-5

14. Consider a 2^{5-2} design with defining relations $ABD = ACE = 1$. for part b

~ 2 a) Write the $\pm$ table, and label each row in Yates' notation.

	A	B	C	D=AB	E=AC	ABC	ACD	BC
de	−	−	−	+	+	−	+	+
a	+	−	−	−	−	+	+	+
be	−	+	−	−	+	+	−	−
abd	+	+	−	+	−	−	−	−
cd	−	−	+	+	−	+	−	−
ace	+	−	+	−	+	−	−	−
bc	−	+	+	−	−	−	+	+
abcde	+	+	+	+	+	+	+	+

← for part c

~ 2 b) In Yates' notation, write the elements of the 4 blocks that will lead to ABC and ACD to get confounded with block.

	B_1	B_2	B_3	B_4
	[abd, ace]	[de, bc]	[be, cd]	[a, abcde]
ABC	−	−	+	+
ACD	−	+	−	+

~~~ 2~~ 1 c) From the $\pm$ table, write the BC effect in Yates' notation; don't worry about overall coefficients.

BC effect $\sim$ (de + a + bc + abcde) − (be + abd + cd + ace)

~ 2 d) Without using the alias structure (i.e., using only the $\pm$ table, write the combination of the blocks which gives the BC effect. I.e., what is the block effect with which BC is confounded?

Block effect $= (B_4 + B_2) - (B_1 + B_3)$

lect22 example

~ 2 15. In a 3^2 design, we have blocked according to $L = A + 2B(\text{mod } 3)$ and obtained the following three blocks, where "10" denotes that $A = 1, B = 0$, etc.. What is the value of the SS_{AB^2} component of the AB interaction, if the observed response (Y) values are as given?

	Y total
[00, 11, 22] (4, −4, 0)	0
[10, 21, 02] (−2, 1, 8)	7
[20, 01, 12]. (0, 5, −5)	0
	7

Y		B		
		0	1	2
	0	4	5	8
A	1	−2	−4	−5
	2	0	1	0

$SS_{AB^2} = SS_{block}$

$= \frac{1}{3}[0^2 + 7^2 + 0] - \frac{1}{9}(7)^2 = \frac{2}{9}7^2$

hw-lect23-2

~ 2 16. In a random effects model $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$, with $i = 1 \cdots a$; $j = 1 \cdots n$, show that $V[\overline{y_{i.}}] = \sigma_y^2 - \frac{n-1}{n}\sigma_\epsilon^2$.

$\rightarrow \sigma_Y^2 = \sigma_\alpha^2 + \sigma_\epsilon^2$

use model asap

$V[\bar{Y}_{i.}] = V[\mu + \alpha_i + \bar{\epsilon}_{i.}] = \underbrace{V[\alpha_i]}_{=\sigma_\alpha^2} + \underbrace{V[\bar{\epsilon}_{i.}]}_{=\sigma_\epsilon^2/n} + \cancel{2Cov[\alpha, \epsilon]}^{\,0} = \sigma_\alpha^2 + \frac{\sigma_\epsilon^2}{n}$

$= \sigma_Y^2 - \sigma_\epsilon^2 + \frac{\sigma_\epsilon^2}{n} = \sigma_Y^2 - \frac{n-1}{n}\sigma_\epsilon^2$