Name: ________________________

ID: __________________________

8.5+20

Stat 421, Test 3, Fall, Dec. 15, 2015; Marzban

ONLY a half-size "cheat sheet" is allowed

Multiple choice: Circle all the correct answers; there is wrong-answer penalty

For rest, SHOW answer & work; NO CREDIT for correct answer without explanation

Points

1. We have a problem involving 3 factors; Factor 1: Fruit Type (~~4~~ 2 levels) Factor 2: Peeling Method (3 levels), and Factor 3: Operator (4 levels). There are, therefore, $2 \times 3 \times 4 = 24$ possible treatment combinations. In the following situations, which of the answers describes the performed experiment?

~ 1 - In question 1, suppose for practical reasons, only 12 runs are performed in a random fashion.
a) Completely Randomized Design (CRD)
c) Balanced Incomplete-Block Design (BIBD)
b) Randomized Complete-Block Design (RCBD)
d) A fractional design. ✓

~ 1 - In question 1, suppose operator 1 performs 6 runs in a random fashion. After operator 1 is finished, operator 2 performs another 6 runs in a random fashion. Etc.
a) CRD b) RCBD c) BIBD d) A fractional design.

This is OK, if we skip

← each block (oper.) is incomplete.

~ 1 - In question 1, suppose all 24 combinations are run in randomized fashion. And then the entire experiemnt is replicated.
a) CRD b) RCBD c) BIBD d) A fractional design.

← either one →

If replication is treated as a block factor, Then its RCBD. Else, it's CRD replicated.

1 **2.** Circle all correct statements. A 3-way interaction effect ABC
a) measures how much main effect A varies with main effect B varies with main effect C.
b) measures how much the interaction effect AB varies across the levels of C
c) measures how much the interaction effect BC varies across the levels of A
d) has no such interpretation, and is therefore often ignored.

3. In a 2^k experiment, with no replication, a full model has the following properties:
a) The estimates of errors are all zero.
1.5 b) The predictions are exactly equal to the observed y-values.
c) The k-way interaction is "confounded" with error. → $\widehat{(\alpha\beta)}_{ij} = \epsilon_{ij}$
d) Daniel's method canNOT be used, because there is no replication.

4. A 2^{3-1} experiment with defining relation ABC= 1 has been performed. The +/− table for the complementary set with ABC=−1, can be generated by performing which of the following
1 reflections on the primary set?
a) reflecting 1 of the 3 factors
c) reflecting all 3 factors
b) reflecting 2 of the 3 factors
d) none of the above.

~ 1 **5.** Consider the mixed effects model $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$, where α_i is a fixed effect, and β_j is a random effect. The sum of the **estimated** variance components is equal to
a) $V[y]$ b) $V[\alpha]$ c) $V[\beta]$ d) none of the above.

$\sigma_y^2 = \sigma_\alpha^2 + \sigma_\beta^2 + \sigma_{\alpha\beta}^2 + \sigma_\epsilon^2$ But $\sigma_y^2 \neq \hat{\sigma}_\alpha^2 + \hat{\sigma}_\beta^2 + \cdots$

~ 1 **6.** Suppose you want to see if the quality of coffee varies with the store from which one buys the coffee, and with the country of origin of the coffee. There are many stores, and so you take a sample of 3 stores. There are only 6 coffee-producing contries on the planet, and so you develop a mixed effects model. If the p-value of the factor Store is less then α, you can conclude that quality varies across
a) the three stores b) all stores c) any 3 stores d) any 3 stores, but only for a given country.

~~2~~ 1.5

7. Consider the model $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$, where $i, j = 1, 2$. Starting from the definition $SSA = \sum_{i,j}(\overline{y_{i.}} - \overline{y_{..}})^2$, show that $SSA = \frac{1}{4}(\text{contrast}_A)^2$. You must **NOT** use the relation we derived in class between contrasts and effects.

No B!

$$SS_A = \sum_{ij}^{2\,2} (\bar{Y}_{i.} - \bar{Y}_{..})^2 = 2\sum_{i}^{2} (\bar{Y}_{i.} - \bar{Y}_{..})^2 = 2\left[(\bar{Y}_{1.} - \bar{Y}_{..})^2 + (\bar{Y}_{2.} - \bar{Y}_{..})^2\right]$$

$$= 2\left[\left(\bar{Y}_{1.} - \tfrac{1}{2}(\bar{Y}_{1.} + \bar{Y}_{2.})\right)^2 + \left(\bar{Y}_{2.} - \tfrac{1}{2}(\bar{Y}_{1.} + \bar{Y}_{2.})\right)^2\right] = 2\tfrac{1}{4}\left[(\bar{Y}_{1.} - \bar{Y}_{2.})^2 + (\bar{Y}_{2.} - \bar{Y}_{1.})^2\right] = (\bar{Y}_{2.} - \bar{Y}_{1.})^2$$

$$= \tfrac{1}{4}(Y_{2.} - Y_{1.})^2 = \tfrac{1}{4}\text{Cont}_A^2$$

~~2~~ 1.5

8. In a 2^3 design, the 8 runs have been performed in the following two blocks: Block 1: $\{(1), b, ac, abc\}$ and Block 2: $\{a, ab, c, bc\}$. What effect will be confounded with block? Show work.

	A	B	C
(1)	−	−	−
a	+	−	−
b	−	+	−
ab	+	+	−
c	−	−	+
ac	+	−	+
bc	−	+	+
abc	+	+	+

for Block 1 : AC = +1

for Block 2 : AC = −1

$\Rightarrow$ AC Confounded with Block

~~2~~ 1.5

9. Consider the 2^{4-1} design with defining relation ABCD=1. For practical reasons we have been forced to perform the experiment in the following two blocks: Block 1: $\{(1), ab, ac, bc\}$ and Block 2: $\{ad, bd, cd, abcd\}$. Write down the two effects which are confounded with block. Show work. Hint: It's not necessary to look at the +/− table for the fractional design.

Block 1: { (1), ab, ac, bc } D = −1

Block 2: { ad, bd, cd, abcd } D = +1

$\Rightarrow$ D is confounded with block; ABC (= D) " " "

Technically, only 1 effect is confounded (because with 2 blocks, there is 1 block effect), and that is D + ABC

~ 2

10. Suppose in 2^{5-2} design one of the generators involves all 5 letters (i.e., ABCDE = 1). Is there any choice of the second generator that will lead to a good design (e.g., where main effects are not aliased with each other)? Show work.

without loss of generality

If 2nd gen. has 1 letter, e.g. A = 1 $\Rightarrow$ bad

2 letters AB = 1 $\Rightarrow$ A = B $\Rightarrow$ bad

3 " ABC = 1 $\Rightarrow$ DE = 1 $\Rightarrow$ D = E $\Rightarrow$ bad

4 " ABCD = 1 $\Rightarrow$ E = 1 $\Rightarrow$ bad

No!

~ 2

11. Consider a 2^{5-2} design with defining relations ABD=±1, ACE=+1. BCDE = +1

a) List the 7 effects one can estimate from the 8 runs. The 31 letter combinations are provided:

A, B, C, D, E,
AB,AC,AD,AE,
BC,BD,BE,
CD,CE,
DE
ABC, ABD, ABE,
ACD, ACE, ADE,
BCD, BCE, BDE,
CDE
ABCD, ABCE
ABDE, ACDE
BCDE,
ABCDE

Alias structure A = BD = CE = ABCDE

Etc. replace = with +

A + BD + CE + ABCDE

B + AD + CDE + ABCE

C + AE + BDE + ABCD

D + AB + BCE + ACDE

E + AC + BCD + ABDE

BC + DE + ACD + ABE

BE + CD + ABC + ADE

~ 2 **b)** Suppose we have discovered sufficient resources (i.e., money) to perform one more set of 8 runs, and we have decided to run the 8 with ABD=−1, ACE=+1 (Note the second defining relation is still +1). List the 7 effects one can estimate from **these** 8 runs. ← sufficient to reflect B

$B \to -B$.

$A - BD + CE - ABCDE$

$-B + AD + CDE - ABCE$

$C + AE - BDE - ABCD$

$D - AB - BCE + ACDE$

$E + AC - BCD - ABDE$

$-BC + DE + ACD - ABE$

$-BE + CD - ABC + ADE$

~ ~~2~~ 1 **c)** List the 14 estimable effects if one folds over the answers in part a and b.

$A + CE$, $BD + ABCDE$

$B + ABCE$, $AD + CDE$

$C + AE$, $BDE + ABCD$

$D + ACDE$, $AB + BCE$

$E + AC$, $BCD + ABDE$

$DE + ACD$, $BC + ABE$

$CD + ADE$, $BE + ABC$

Adding/Subtracting Parts a, b

~ ~~2~~ 1.5 **d)** The fold-over design in part c is based on 8+8=16 runs, which is half of the 2^5 runs in the factorial design. In other words, if we had performed all 16 runs in randomized fashion, the design woud have been 2^{5-1}. What is the defining relation of that design? Hint: look at the defining relations; which product of letters is constant across the 16 runs.

The 8 runs in primary set satisfy $ABD=+1$ $\boxed{ACE=+1}$ $BCDE=+1$

" " Complementary set " $ABD=-1$ $\boxed{ACE=+1}$ $BCDE=-1$

The 16 runs in The fold over set satisfy $\boxed{ACE=+1}$ That's The defn. rel. in 2^{5-1}

~ 2 **e)** The 2^{5-1} design in part d has 15 estimable effects. DO NOT list them; but write down the one effect that is not estimable in the fold over design of part c. **Explain.** Hint: It is sufficient to look at **all** the defining relations; you may also simply use, without derivation, any results from class.

In The soln to The "puzzle" we found that if primary set has $X=1, Y=1, XY=1$ and The compl. set has $X=-1, Y=-1, XY=+1$, Then $X+Y$ is The "missing" effect. Here, ACE is like XY (ie. Constant in fold over set). See end

So, ABD + BCDE is The effect that cannot be estimated by The fold over.

~ 2 **12.** Consider the model $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$. Suppose we treat both A and B as fixed factors, and compute the F-ratios F_A, F_B, F_{AB}. Now suppose we treat A as a fixed factor, but B as a random effect, and compute the F-ratios F'_A, F'_B, F'_{AB}. Assuming all the F-ratios are larger than 1, is F_A going to be larger or smaller than F'_A. Answer, and show work.

$$F_A = \frac{MS_A}{MS_E}, \quad F_B = \frac{MS_B}{MS_E}, \quad F_{AB} = \frac{MS_{AB}}{MS_E}$$

$$F'_A = \frac{MS_A}{MS_{AB}}, \quad F'_B = \frac{MS_B}{MS_E}, \quad F'_{AB} = \frac{MS_{AB}}{MS_E}$$

$$\therefore F'_A \cdot F'_{AB} = F_A \Rightarrow \text{If } F > 1, \text{ Then } \boxed{F_A \geq F'_A}$$

It's "harder" to reject A, if B is random ⟸ p-value$_A \leq$ p-value$'_A$

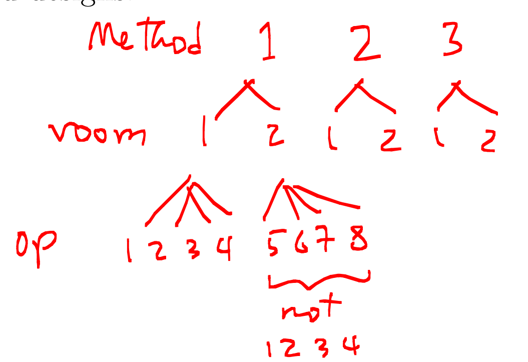
13. In an attempt to improve manufacturing speed, an industrial engineer has designed three methods and two rooms that seem promising. Operators are required to perform the assembly, and it is decided to randomly select four operators for each method-room combination. For practical reasons, the four operators chosen for room 1 are different individuals from the four operators chosen for room 2.

~ ~~2~~ 1 **a)** Is the most appropriate model for this experiment a fixed-effects, random-effects, or mixed-effects model? Explain which factors are fixed and/or random.

Mixed with method = fixed
room = fixed
oper = random

~ 2 **b)** Describe this experiment in terms of nested and/or crossed designs.

method is crossed with room
operator is nested under room



Extra space

11e) In This 2^{5-2} design, The principal fraction has ABD = 1, ACE = 1. There are $2^{5-2} - 1 = 7$ estimable effects.
It's true The ABD × ACE = BCDE = 1 also, but That's implied by The (2) generators ABD = 1 and ACE = 1. So, it does not count as a defining relation.
The alt. fraction has ABD = −1, ACE = +1. Here, too, there are 7 est. effs.
Since ACE is constant (+1) across both fractions, ACE = +1 is The defining relation of The combined design. Therefore, There are 7 + 7 = 14 est. effects in The combined design.
Now, if instead of This fold-over design (ie. 2^{5-2} in 2 blocks) we had done a 2^{5-1} experiment with ACE = +1, we would expect to have $2^{5-1} - 1 = 15$ estimable effects. Based on The alias structure, They would be:

A + CE, B + ABCE, C + AE, D + ACDE, E + AC
BD + ABCDE, AD + CDE, BDE + ABCD, AB + BCE, BCD + ABDE, $\boxed{\text{ABD + BCDE}}$
BC + ABE, DE + ACD
CD + ADE, BE + ABC

Of These 15, The one That's missing in The fold-over is This one. We know That because ACE = +1 ⟹ ABD = BCDE, and ABD is confounded with block (it changes sign across The 2 fractions).

Alternatively, we can introduce a block factor, F, and describe the fold-over experiment as a 2^{6-2} design.

A, B, C, D, E, F

block factor changing sign across the 2 fractions.

2 defining relations:

$\begin{cases} ACE = +1 \text{ and} \\ \underbrace{ABCDE}F = +1 \end{cases}$

note that ABCDE does change sign across the 2 fractions; and so does F.

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