Name: ____________________
ID: ____________________
Quiz section or time: ____________________

Stat 421, Test 3, Fall, Dec. 10, 2013; Marzban

9.5 + 18.0

ONLY a half-size "cheat sheet" is allowed

Multiple choice: Circle all the correct answers; there is wrong-answer penalty; do NOT explain

The rest: SHOW answer & work; NO CREDIT for correct answer without explanation

Points

1 **1.** Daniel's qq plot of the effects, for the purpose of identifying significant effects, works
a) only in 2^k designs, only when $SS_E = 0$
b) only in 2^k designs, even when $SS_E \neq 0$
c) for all designs, only when $SS_E = 0$
d) for all designs, even when $SS_E \neq 0$

7.17, example 9.2

1.5 **2.** Mark all correct statements. When an effect X is confounded with blocks, then
a) effect X = block effect, for any factorial design
b) effect X = block effect, for 2^k designs
c) $SS_X = SS_{\text{block}}$, for any factorial design
d) $SS_X = SS_{\text{block}}$, for 2^k designs +0.5 −0.5

1 **3.** In a 2^{6-1} 2 design, the alias structure implies AB=CDEF and ACD=BEF. If blocking is done to confound AB with blocks, and ACD with blocks, then the number of resulting blocks is
a) 2 b) 4 c) 8 d) 16

Confounding AB ⇒ 2 blocks. Confounding AB and ACD ⇒ 4 blocks. (2 cols in +/− Table)

1 **4.** Consult the 2^{5-2} alias structure shown in **problem 12**. According to that table, it is recommended that one assigns the label "A" to a
a) less interesting factor b) more interesting factor c) any factor, because labels do not matter

A+BD+CE+⋯+BCDEF
This will change only sign, not the rest.

2^{7-4}; 1/16 fraction of
7 factors in 8 runs

Design Generators

D = AB E = AC F = BC G = ABC

Defining relation: I = ABD = ACE = BCDE = BCF = ACDF = ABEF = DEF = ABCG
= CDG = BEG = ADEG = AFG = BDFG = CEFG = ABCDEFG

Aliases

A = BD = CE = FG	E = AC = DF = BG
B = AD = CF = EG	F = BC = DE = AG
C = AE = BF = DG	G = CD = BE = AF
D = AB = EF = CG	

1 **5.** Consider the 2^{7-4} relations as shown. If we want to estimate the effect A, then the complementary set is constructed
a) by switching sign of A in principal set.
b) with the defining relation $BCDEF = -1$
c) with the defining relation $ABD = ACE = BCDE = BCF = ACDF = ABEF = DEF = ABCG = -1$
d) none of the above, because A cannot be estimated.

ABD = −1
ACE = −1 } ⇒ BCDF = +1 #

1 lect 24 **6.** In a problem involving two 3-level factors, the orthogonal Latin Square Design (LSD) technique
a) decomposes the total SS.
b) decomposes the SS of interaction between the two factors
c) decomposes the interaction effect itself.
d) decomposes only the quadratic component of the SS of interaction between the two factors
e) cannot be applied, because the LSD requires 3 factors

1 lect 24 **7.** In a 3×3 problem, the orthogonal Latin square decomposition has yielded SS values of 2.1 and 8.2. Meanwhile, a regression-based decomposition has yielded 1.0, 0.1 for $SS_{L\times L}$ and $SS_{L\times Q}$, respectively. Then $SS_{Q\times L} + SS_{Q\times Q}$ is equal to
a) 10.3 - 1.1 b) 10.3 - 0.1 c) 8.2 - 1.1 d) 8.2 - 0.1
e) cannot be computed, because the two decomposition methods are unrelated.

Test 23

8. Circle all correct answers. In a random effects model $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$, one can

a) estimate α_i

1 b) estimate σ_ϵ +0.5

c) build a closed form confidence interval (CI) for α_i −0.5

d) build a closed form CI for σ_ϵ

e) build a closed form CI for σ_α

1 Table 13.11 **9.** In a problem involving A=fixed, B=random, C=random, a test of the effect of A

a) can be done using the $E[MS]$ table for any design involving one random and two fixed factors, as long as A = fixed.

b) can be done using the $E[MS]$ table for any design involving one fixed and two random factors, as long as the design is factorial.

c) can be done using the $E[MS]$ table for any design involving one fixed and two random factors, but only for a split-plot (or nested) design.

d) cannot be done using any other table of $E[MS]$.

3 **10.** In a 2^k experiment with $k > 3$, and $n = 1$ replication, the main effects of the k factors have been estimated. Suppose the estimates for the first two factors (A and B) are -1, and 3, respectively. Compute SS_B.

$\hat{\alpha}_1 = -1 \rightsquigarrow \hat{\alpha}_2 = +1$

$\hat{\beta}_1 = 3 \rightarrow \hat{\beta}_2 = -3$

$$SS_B = \underbrace{\sum_i \sum_j \cdots \sum}_{k \text{ sums}} (\hat{\beta}_{\underbrace{\cdot j \cdots\cdots}_{k \text{ indices}}})^2 = 2^{k-1} \cdot \sum_j (\hat{\beta}_{\cdot j \cdots})^2$$

$$= 2^{k-1}\left(3^2 + (-3)^2\right) = 2^k 3^2$$

In 2019, Hanwen Zhang pointed out that if we use $SS_B = 2^{k-2} \cdot n \cdot (\text{effect})^2$, then we get $SS_B = 2^{k-2}(-3)^2$. But recall that the ch. 6 defn of effect is $(2\hat{\beta})$

example p.313 and how

2 **11.** Consider a 2^{5-1} design with defining relation $ABCDE = 1$. Suppose we want to block into two blocks of 8 runs each, in a way that confounds the AB interaction with blocks.

a) Which runs must be included in each of the two blocks? Note: there are different ways of answering this question, varying from "long" (listing all the runs), to "short" (writing down a "defining relation"); give your answer in whichever method you prefer.

Block 1: runs with $AB = +1$

2 " " -1

~~2~~ 1 **b)** With the blocks given in the previous part, are there any other effects confounded with blocks? If so, which and WHY?

Yes, CDE. Because $ABCDE = 1 \Rightarrow AB = CDE$ (AB aliased with CDE)

Technically, the single effect conf. with the single block effect is $AB + CDE$.

1.5 **c)** After the above experiment was completed, we decide to perform ~~a full fold-over~~. The combined experiment is equivalent to another 2^k factorial design. What is the value of k? And what is/are the defining relation(s) of this combined design? Show some work.

A	B	C --- D	E	F	
−	−	- - -		+	2^{5-1} runs with $\underline{ABCDE = +1}$ *
+	−			⋮	
⋮				+	
+	+	- - -		−	2^{5-1} runs with $\underline{ABCDE = -1}$ *
−	+			⋮	
⋮				−	

Every factor reversed *

$2^{\overbrace{6-1}^{k}}$ with $ABCDEF = +1$

or −1 if F is reversed.

example on p.313

12. The alias structure for a 2^{5-2} design is as follows (ignoring higher order interactions):

Design Generators
$D = AB \quad E = AC$
Defining relation: $I = ABD = ACE = BCDE$
Aliases
$A = BD = CE$
$B = AD = CDE$
$C = AE = BDE$
$D = AB = BCE$
$E = AC = BCD$
$BC = DE = ACD = ABE$
$CD = BE = ABC = ADE$

2 **a)** What are the estimable effects?

7 effects for $(2^{5-2}-1)$ dfs

$A + BD + CE$
$B + AD + CDE$
$C + AE + BDE$
$D + AB + BCE$
$E + AC + BCD$
$BC + DE + ACD + ABE$
$CD + BE + ABC + ADE$

b) We have enough money to observe another 8 runs. We make these new runs with the same treatment combinations as in part a, but with the signs of B reversed. In the combined experiment what are the estimable effects?

~~2~~ 2.5

The complementary set will estimate:

$A - BD + CE$
$-B + AD + CDE$
$C + AE - BDE$
$D - AB - BCE$
$E + AC - BCD$
$-BC + DE + ACD - ABE$
$CD - BE - ABC + ADE$

Combined runs will estimate

$A + CE$, BD
B, $AD + CDE$
$C + AE$, BDE
D, $AB + BCE$
$E + AC$, BCD
$BC + ABE$, $DE + ACD$
$CD + ADE$, $BE + ABC$

~~2~~ 1.5 hw 14.21 **13.** The table of $E[MS]$ for a factorial problem involving three random factors is as follows. In terms of the MS values for each of the lines in the table, write the appropriate F-ratio for testing σ_τ^2. If it cannot be done, explain why.

Model Term	Factor	Expected Mean Squares	estimate
τ_i	A, main effect	$\sigma^2 + cn\sigma^2_{\tau\beta} + bn\sigma^2_{\tau\gamma} + n\sigma^2_{\tau\beta\gamma} + bcn\sigma^2_{\tau}$	$\sim MS_A$
β_j	B, main effect	$\sigma^2 + cn\sigma^2_{\tau\beta} + an\sigma^2_{\beta\gamma} + n\sigma^2_{\tau\beta\gamma} + acn\sigma^2_{\beta}$	
γ_k	C, main effect	$\sigma^2 + bn\sigma^2_{\tau\gamma} + an\sigma^2_{\beta\gamma} + n\sigma^2_{\tau\beta\gamma} + abn\sigma^2_{\gamma}$	
$(\tau\beta)_{ij}$	AB, two-factor interaction	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + cn\sigma^2_{\tau\beta}$	$\sim MS_{AB}$
$(\tau\gamma)_{ik}$	AC, two-factor interaction	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + bn\sigma^2_{\tau\gamma}$	$\sim MS_{AC}$
$(\beta\gamma)_{jk}$	BC, two-factor interaction	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + an\sigma^2_{\beta\gamma}$	
$(\tau\beta\gamma)_{ijk}$	ABC, three-factor interaction	$\sigma^2 + n\sigma^2_{\tau\beta\gamma}$	$\sim MS_{ABC}$
ϵ_{ijkl}	Error	σ^2	

$$F = \frac{MS_A + MS_{ABC}}{MS_{AB} + MS_{AC}}$$

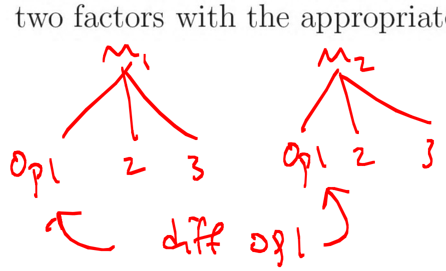
$$\sim \frac{2\sigma^2 + cn\sigma^2_{\tau\beta} + bn\sigma^2_{\tau\gamma} + 2n\sigma^2_{\tau\beta\gamma} + bc\sigma^2_\tau}{2\sigma^2 + cn\sigma^2_{\tau\beta} + bn\sigma^2_{\tau\gamma} + 2n\sigma^2_{\tau\beta\gamma}}$$

14.2 **14.** The surface finish of metal parts made on four machines is being studied. An experiment is conducted in which each machine is run by three different operators and two specimens from each operator are collected and tested. Because of the location of the machines, different operators are used on each machine.

1 **a)** Identify the factors as random or fixed.

machine = fixed
operator = random

~~2~~ 1.5 **b)** Is the design of this experiment factorial, nested, or split-plot? For your answer, associate the two factors with the appropriate parts of the design (e.g., whole-plot, subplot, etc.)



$\Longrightarrow$ Operator nested under machine.

hw AQ **15.** In a problem involving a single random factor with a levels, and with n replicates, derive an expression for $prob(\hat{\sigma}_\alpha^2 < 0)$ which allows one to compute it given $a, n, \sigma_\alpha/\sigma_\epsilon$, and tables of distributions.

$$Pr(\hat{\sigma}_\tau^2 < 0) = Pr\left(\frac{MSTr - MS_E}{n} < 0\right) = Pr(MSTr < MS_E)$$

$$= Pr\left(\frac{MSTr}{MS_E} < 1\right)$$

$$= Pr\left(\underbrace{\frac{MSTr/E[MSTr]}{MS_E/E[MS_E]}}_{F_{a-1,\,a(n-1)}} < \frac{E[MS_E]}{E[MSTr]}\right)$$

$$= Pr\left(F_{a-1,\,a(n-1)} < \frac{\sigma_E^2}{\sigma_E^2 + n\sigma_\tau^2}\right)$$
