

test2-2018

November 10, 2019

1

We have come across a dataset regarding the amount of vibration inside buses. The company that collected the data did not have any experts in experimental design, and so this is what they did. They asked a driver to drive one bus while they randomly changed two factors: one 2-level factor (A) and one 3-level factor (B). For each of these 6 combinations, they recorded the vibration. Then, they asked the same operator to drive another bus, and again took 6 measurement in random order. Finally, someone commented that the driver may have an effect on the vibrations, and so the company repeated everything using the same procedure but now with a different driver. Based on all info available to you, it's not even clear why the company did this experimnt. What is the design of this experiment?

- (A)CRD (B)RCBD with 1 block factor.
- (C)RCBD with 2 block factors (D) RCBD with 3 block factors.

2

In the previous question, given the design, circle the facotrs that can be considered as treatment factors, and hence tested.

- (A)A (B)B (C) Driver (D)Bus (E)None above

3

In an RCBD, the sample variance of block-conditional-means (i.e. means within blocks)

- (a) is 0 (b) is equal to that in a CRD (c) measurement the effect of blocking (d) none of the above

4

Which of the following statements is/are False?

- (A) There is only 1 standard LS of order 3 (B) Two standard LS can not be orthogonal
 (C) There are no orthogonal LSs of order 6 (D) One can have a LS with no orthogonal counterpart
 (E) None above

5

For an additive model $y_{ijk} = \mu + \alpha_i + \beta_j + \epsilon_{ijk}$, which of the following is FALSE?

- (A) $E[\hat{y}_{ijk}] = \mu + \alpha_i + \beta_j$ (B) $E[y_{ijk}] = \mu + \alpha_i + \beta_j$ (C) $E[\epsilon_{ij.}] = 0$ (D) none of the above

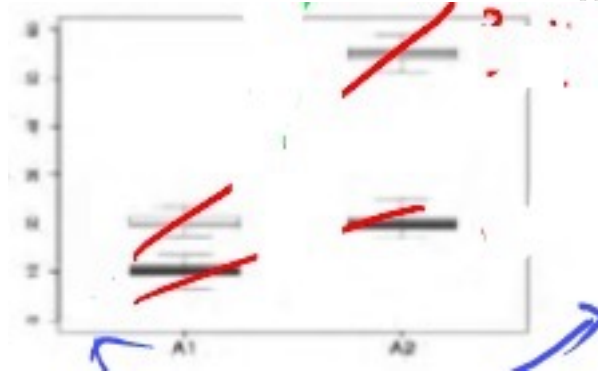
6

Which of the following is False? The residual plot (residuals VS predictions) should

- (a) display no correlation for a full model (b) be centered around the horizontal line for a full model.
 (c) display no correlation for an additive model (d) be centered around the horizontal line for an additive model.

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The adjacent figure shows the boxplots of data y involving two binary factors A and B. The clear(filled) boxplots correspond to the high (low) level of the B factor. Based on this data circle all effects that appear to exist.



- (a) A effect (B) B effect (C) AB effect (D) BA effect (e) None of above

8

Fill in the blanks: $ln2^2$, give the _____ for A and the _____ for B, one can compute the _____ for AB.

9

For data from a replicated CRD with two factors A and B, we have the following two models:

Model 1: $y_{ijk} = \mu + \alpha_i + (\alpha\beta)_{ij} + \epsilon_{ijk}; (\hat{\alpha}\hat{\beta})_{ij} = \bar{y}_{ij} - \bar{y}_{i..}; \hat{y}_{ijk} = \bar{y}_{ij}.$

Model 2: $y_{ijk} = \mu + \alpha_i + \epsilon_{ijk}; \hat{y}_{ijk} = \bar{y}_{i..}$

Show that at the least-square estimates, $SSE2 - SSE1 = \sum_{ijk} (\hat{\alpha}\hat{\beta})_{ij}^2$

10

Consider the model $y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \epsilon_{ijk}$ for the LSD shown. Using $\sum'_{ijk} y_{.j} = 3y_{...}$ (and its analog for $\sum'_{ijk} y_{i..}$ and $\sum'_{ijk} y_{..k}$), show that \sum'_{ijk} of the predictions $y_{ijk} = \bar{y}_{i..} + \bar{y}_{.j.} + \bar{y}_{..k} - 2\bar{y}_{...}$ is equal to \sum'_{ijk} of the observations.

	C_1	C_2	C_3
A_1	B_1	B_2	B_3
A_2	B_2	B_3	B_1
A_3	B_3	B_1	B_2

11

In the previous problem, suppose we are now supposed to design an experiment that includes another 3-level nuisance factor. Specify the necessary runs if we want to be able to perform tests.

12

Here is one interpretation of the interaction term: Consider the model $y_{ijk} = \mu + \alpha_i + (\alpha\beta)_{ij} + \epsilon_{ijk}$. To simplify the language, let's say that $\hat{\alpha}_i$ is the effect of A; etc. Show that $(\alpha\hat{\beta})_{ij}$ can be written in terms of the conditional effect of A (given $B = j$), and the effect of A. This takes 1 line of algebra.

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Consider the model $y_{ijk} = \alpha_i + (\alpha\beta)_{ij} + \epsilon_{ijk}$ with $i, j, k = 1 \dots a, b, n$. Note that there is no μ .

(a) Starting from the expression for SSE, derive the least square equations, and write them in dot-bar notation. (i.e. conditional means). DO NOT impose/introduce any constraints

(b) Recall that "uniquely estimable effects" are those that can be estimated from data without/before imposing constraints. Are there any such effects involving both $\hat{\alpha}_i$ and $(\hat{\alpha}\beta)_{ij}$ (and/or their sums)? If so, write at least one. If not, explain why not.

(c) Are there any such effects involving only $(\alpha\beta)_{ij}$ (and/or its sums)? If so, write at least one. If not, explain why not. (If you don't know, go to part d).

(d) The number of parameters in the model (excluding σ_ϵ) is _____.

(e) The number of independent least-squares equations in part a is _____.

(f) Specify a natural set of constraints and estimate all the effects.

(g) Perform the ANOVA decomposition of $SST = \sum_{ijk} y_{ijk}^2$ into SSA, SSAB, and SSE. You may assume the cross-terms are zero.

(h) Write the corresponding df for each term (SST, SSA, SSAB, SSE). Don't explain.