hw lect 19-1

This is a revision of problems 6.26 and 7.8.
Consider the data in exercise 6.26. Write R code to generate the anova table for
a) the full model, and
b) a full model involving 4 incomplete blocks. Arrange the blocks so that the effects ABC and CDE (and consequently, ABDE) are confounded with block.
c) Compare the two anova tables; and write down all of your observations.

Hints for part b: This is one way to make a block factor L with the desired levels:

```
L1 = A*B*C  # if A, B, C are +1/-1
L2 = C*D*E
L = numeric(16)
L[L1==-1 & L2==-1] = 1
L[L1==+1 & L2==-1] = 2
L[L1==-1 & L2==+1] = 3
L[L1==+1 & L2==+1] = 4
```

a)

```
rm(list=ls(all=TRUE))
library(AlgDesign)          # for gen.factorial()

design = gen.factorial(2,5,varNames=c("A","B","C","D","E")) # -1/+1
attach(design)
y = c(7,9,34,55,16,20,40,60,
      8,10,32,50,18,21,44,61,
      8,12,35,52,15,22,45,65,
      6,10,30,53,15,20,41,63)

lm.1 = lm(y~A*B*C*D*E)   # Because each SS has only 1 df, it doesn't matter if A,B... are factors. The effects that lm() returns will depend on whether AB,... are factors, but the SSs will not
summary.aov(lm.1)
```

b)

```
L1 = A*B*C
L2 = C*D*E
L = numeric(16)      # Make a single 4-level factor for the 4 block levels.
L[L1==-1 & L2==-1] = 1
L[L1==+1 & L2==-1] = 2
L[L1==-1 & L2==+1] = 3
L[L1==+1 & L2==+1] = 4

lm.2 = lm(y~ as.factor(L) + A*B*C*D*E)
summary.aov(lm.2)
```

c) We know that the effects ABC, CDE, and ABDE are confounded with block. But here we can see what that confounding means in terms of SSs. In the blocked case, SS_ABC, SS_CDE, and SS_ABDE do not appear at all, precisely because the corresponding effects are confounded with block. But the value of SS_block (14) is equal to the sum of the SS's of those effects in the unblocked design:
SS_ABC = 2
SS_CDE = 5
SS_ABDE = 7

hw-lect19-2

An experiment involving 4 binary treatment factors is to be replicated twice, but we have been unable to replicate all $2^4 = 16$ runs. a) Suppose each block contains only 8 of the 16 runs, and we want to confound the ABCD effect with Block. In (1), a, b... notation, write the elements of the blocks. Use Table 6.11

b) Now suppose each block contains only 4 runs, and we choose to confound ABC and ABD with blocks. Write the elements of the blocks.

c) According to what we have learned, the CD effect will also be confounded with blocks, because $(ABC)(ABD) = A^2B^2CD = CD$. Which block effect is confounded with the CD effect? E.g. The last block (B_4) minus the 1st block (B_1)?

d) Which block effect is confounded with the ABC effect?

e) Prove that a block effect of the type $\underbrace{B_1 + B_2 + B_3 - B_4}_{\text{non-zero-sum}}$ cannot be confounded with any effect.

a) This is a 2^4 experiment in 2 blocks. Table 6.11 tells us that the 2 blocks should be [(1), ab, ac, bc, ad, bd, cd, abcd] (ABCD=+)
[a, b, c, abc, d, abd, acd, bcd] (ABCD=−)

b) This is a 2^4 experiment in 4 blocks. Table 6.11 tells us ~~that~~ the 4 blocks should be

B_1:	[(1), ab, acd, bcd]	ABC=−	ABD=−
B_2:	[ac, bc, d, abd]	ABC=−	ABD=+
B_3:	[c, abc, ad, bd]	ABC=+	ABD=−
B_4:	[a, b, cd, abcd]	ABC=+	ABD=+

Table 6.11

c) CD effect $\overset{\downarrow}{\sim}$ $[(1)+a+b+ab-c-ac-bc-abc$
$-d-ad-bd-abd+cd+acd+bcd+abcd]$

$\sim [(B_1+B_4)-(B_2+B_3)]$ This numbering will vary across students.

$\therefore$ CD $\sim$ The Avg of 2 blocks minus The Avg. of The other 2 blocks.

Table 6.11

d) ABC effect $\overset{\downarrow}{\sim}$ $[a+b+c+abc+ad+bd+cd+abcd$
$-(1)-ab-ac-bc-d-abd-acd-bcd]$

$\sim (B_3+B_4)-(B_1+B_2)$

Note That eventhough ABC and CD are both confounded with Block, They are confounded with different block effects.

e) Every column in The +/- Table has an equal number of plus's and minus's. But a block effect like $B_1+B_2+B_3-B_4$ will have a different # of plus's and minus's, and so, no effect in The +/- Table can be confounded with That block effect.

hw-lect 19-3

Consider a 2^3 design with 2 blocks. In the (1), a, b, ... notation, a) write down all possible principal blocks, and b) for each possibility specify the effect that gets confounded with blocks. Hint for part a): There are 7 possibilities. Hint for part b): find a column that has a constant value (either + or −) for all the elements in the principal block.

Important: Define a "principal block" as a block that
1) includes the (1) element, AND
2) is a group under multiplication.

a) [(1), a, b, ab] $\Rightarrow$ C = −1   Table 7.4   b) i.e. C is confounded with block.

start with (1), and then include one other letter, e.g. a, and then one more letter, e.g. b. Then the last element will be determined by group property. Etc.

[(1), a, c, ac] $\Rightarrow$ B = −1   B " "

[(1), a, bc, abc] $\Rightarrow$ BC = +1   BC " "

[(1), b, c, bc] $\Rightarrow$ A = −1   A " "

[(1), b, ac, abc] $\Rightarrow$ AC = +1   AC " "

[(1), c, ab, abc] $\Rightarrow$ AB = +1   AB " "

[(1), ab, ac, bc] $\Rightarrow$ ABC = −1   ABC " "

In the hw problem based on the 2^4 design in exercise 6.40 (and 7.18), generate the anova table for the full model,
a) in two blocks, confounding ABC with blocks, with the block factor taking +/- values; (Recall that this can be done by deleting the argument factors="all" in gen.factorial()).
b) in two blocks, confounding ABC with blocks, with the block factor taking 0/1 values using the contrast method; (Recall that this can be done by including the argument factors="all" in gen.factorial, and then using as.numeric() on the factors, and more ...) .
c) in 4 blocks, confounding AB and CD with blocks, using the contrast method.

hw-lect 19-4

```
  rm(list=ls(all=TRUE))
  library(AlgDesign)          # for gen.factorial()
  y = c(23,15, 16, 18, 25, 16, 17, 26, 28, 16, 18, 21, 36, 24, 33, 34)
# a)
  design = gen.factorial(2,4,varNames=c("A","B","C","D"))     # , factors="all")
  attach(design)
  cbind(A,B,C,D,y)
  L = as.factor(A*B*C)
  summary.aov(lm(y~L + A*B*C*D ))
# L           1   2.25    2.25
# A           1  42.25   42.25
# B           1   0.00    0.00
# C           1 196.00  196.00
# D           1 182.25  182.25
# A:B         1 196.00  196.00
# A:C         1   1.00    1.00
# B:C         1  20.25   20.25
# A:D         1  12.25   12.25
# B:D         1   1.00    1.00
# C:D         1  64.00   64.00
# A:B:D       1   0.00    0.00
# A:C:D       1   4.00    4.00
# B:C:D       1   2.25    2.25
# A:B:C:D     1   6.25    6.25

# b)
  design = gen.factorial(2,4,varNames=c("A","B","C","D"), factors="all")
  attach(design)
  A = as.numeric(A) - 1       # to convert 1/2 to 0/1
  B = as.numeric(B) - 1
  C = as.numeric(C) - 1
  D = as.numeric(D) - 1       # Don't need this, but convert anyway.
  L = (A + B + C ) %% 2
  summary.aov((lm(y~ L + A*B*C*D ))
  summary.aov(lm.2)

# Exactly the same as in part a.

# c)
  L1 = (A + B ) %% 2
  L2 = (C + D ) %% 2
  L = numeric(16)
  L[L1==0 & L2==0] = 1
  L[L1==1 & L2==0] = 2
  L[L1==0 & L2==1] = 3
  L[L1==1 & L2==1] = 4
  summary.aov(lm(y~ as.factor(L) + A*B*C*D ))   # as.factor(L) is important.

#              Df Sum Sq Mean Sq
# as.factor(L)  3 266.25   88.75
# A             1  42.25   42.25
# B             1   0.00    0.00
# C             1 196.00  196.00
# D             1 182.25  182.25
# A:C           1   1.00    1.00
# B:C           1  20.25   20.25
# A:D           1  12.25   12.25
# B:D           1   1.00    1.00
# A:B:C         1   2.25    2.25
# A:B:D         1   0.00    0.00
# A:C:D         1   4.00    4.00
# B:C:D         1   2.25    2.25

# Note that the effects AB and CD (and ABCD) do not appear in the table, but SS_block is equal to the sum of their SS's in the unblocked design.
```

hw-lect20-1

a) Consider the 2^5 design.

Given that there are 5 factors (A, B, C, D, E), there are $(\overbrace{2^5 - 1}^{31})$ effects (excluding the grand mean). Write out all of them in groups of 1-factor effects, 2-factor effects, etc.

$\underbrace{A \quad B \quad C \quad D \quad E}_{5}$

AB AC AD AE
BC BD BE
CD CE
DE
10

ABC ABD ABE
ACD ACE ADE
BCD BCE BDE
CDE
10

ABCD ABCE
ABDE ACDE
BCDE
5

ABCDE 1

b) Now consider the 2^{5-1} design with defn. relation ABCDE = 1

Write out the alias structure

(Note that you should get a total of $df = 2^{5-1} - 1 = 15$ aliasing relationships relating the 31 effects).

A = BCDE
B = ACDE
C = ABDE
D = ABCE
E = ABCD

All 31 effects appear here.

But only $2^{5-1} - 1 = 15$ = df ✓.
effects are estimable

AB = CDE
AC = BDE
AD = BCE
AE = BCD

BC = ADE
BD = ACE
BE = ACD

CD = ABE
CE = ABD

DE = ABC

hw_lect 20-2

write code to reproduce the aliased effects on the bottom left of p.332 (e.g. [A]' = 24.25, ...)

1

```
# example 8.1 and 8.3  2^{4-1} with ABCD=1

  rm(list=ls(all=TRUE))
  library(AlgDesign)          # for gen.factorial()

  # Make +/- table for THREE (Not 4) factors:
  design = gen.factorial(2,4-1,varNames=c("A","B","C")) # , factors="all")

  attach(design)
  design
#   D = A*B*C    # generate the dropped factor by "solving" for D in ABCD=1 (example 8.1)
  D = -A*B*C   # Same, but for ABCD = -1   (example 8.3)

#   A = as.factor((A+3)/2)              # No need to scale back to 1,2.
#   B = as.factor((B+3)/2)
#   C = as.factor((C+3)/2)
#   D = as.factor((D+3)/2)
#   E = as.factor((E+3)/2)

  A = as.factor(A)
  B = as.factor(B)
  C = as.factor(C)
  D = as.factor(D)

#   y = c(45,100,45,65,75,60,80,96) # data for ABCD = 1  (page 325; example 8.1)
  y = c(43,71,48,104,68,86,70,65) # data for ABCD = -1 (page 332; example 8.3)

  contr = as.character("contr.helmert")
  lm.1 = lm(y~A*B*C*D,  contrasts = list(A=contr,B=contr,C=contr,D=contr))

  eff = as.matrix(2*lm.1$coefficients)
  eff

# (Intercept) 141.5   # Table 8.4, p.296 ; example 8.1
# A1          19.0
# B1           1.5
# C1          14.0
# D1          16.5
# A1:B1        -1.0
# A1:C1       -18.5
# B1:C1        19.0
# A1:D1          NA   because AD = BC
# B1:D1          NA
# C1:D1          NA
# A1:B1:C1       NA
# A1:B1:D1       NA
# A1:C1:D1       NA
# B1:C1:D1       NA
# A1:B1:C1:D1    NA
```

2

Note that one can explicitly implement the alias structure into the model, but you will have to know which effects are estimable. That information is contained in Table X of the book. For this problem, you can see that the terms you need to include in the model are A, B, C, D, AB, AC, AD (which, of course, are the ones returned by R in the above).

```
lm.2 = lm(y~ A + B + C + D + A*B + A*C + A*D,  contrasts = list(A=contr,B=contr,C=contr,D=contr))

eff = as.matrix(2*lm.2$coefficients)
eff

# (Intercept) 141.5
# A1          19.0
# B1           1.5
# C1          14.0
# D1          16.5
# A1:B1       -1.0
# A1:C1      -18.5
# A1:D1       19.0

#######################################
```

Now, example 8.3:
Running the data on page 332, requires changing changing the y values (above) and changing D=A*B*C to D = -A*B*C. The result is

```
# (Intercept) 138.75
# A1          24.25
# B1           4.75
# C1           5.75
# D1          12.75
# A1:B1        1.25
# A1:C1      -17.75
# B1:C1      -14.25
# A1:D1          NA
# B1:D1          NA
# C1:D1          NA
# A1:B1:C1       NA
# A1:B1:D1       NA
# A1:C1:D1       NA
# B1:C1:D1       NA
# A1:B1:C1:D1    NA
```

Note: It may be tempting to run the code with D = A*B*C and just changing the y values. The reasoning may be that D is
entered into lm() as a factor anyway, so it's sign should be irrelevant. BUT, that would be wrong, because if you use D = A*B*C
all the effects that involve D will have the incorrect sign. That's because the values of D=A*B*C, even as factor are
-1 1 1 -1 1 -1 -1 1
While the values of D=-A*B*C, as factor are
1 -1 -1 1 -1 1 1 -1
So that switches what's "High" and what's "Low," giving the wrong sign.

hw-lect 20-3

For the data shown in problem 8.10

a) What type of design is this (ie. $2^{?-?}$), and what is the defining relation? What effect is E aliased with?

Hint: Note that the A, B, C, columns look "normal," but the D column does not follow the normal pattern.

b) Write out the alias structure by hand. Work out the following counts, so that you won't miss anything in the alias structure.

how many main effects? $\binom{5}{1} = 5$

" " 2-way interactions? $\binom{5}{2} = 10$

3-way effects? $\binom{5}{3} = 10$

4-way effects? $\binom{5}{4} = 5$

5-way effects? $\binom{5}{5} = 1$

c) Write R code to generate the ANOVA Table. At $\alpha = .01$, what are the significant effects? By R

d) Estimate all the estimable effects, and use Daniel's method to determine the significant effects. Do these results agree with those in part c?

a) It looks like D is generated from A, B, and C.

A bit of examination of the Table shows that D = ABC.

So, the design is 2^{5-1}, with the defining relation ABCD = 1.

The E factor does not enter that defining relation.

But that does not mean that E is not aliased with anything.

In fact, because ABCD = 1, it follows that ABCDE = E,

ie. The E effect is aliased with the very high order term ABCDE.

5 ✓

10 ✓

10 ✓

b)

$A = \boxed{BCD}$

$B = \boxed{ACD}$

$C = \boxed{ABD}$

$D = \boxed{ABC}$

$E = \boxed{ABCDE}$

1 ✓

$AB = CD$

$AC = BD$

$AD = BC$

$AE = BCDE$

$BE = ACDE$

$CE = ABDE$

$DE = ABCE$

$ABCD = 1$

5 ✓

$ABE = CDE$

$ACE = BDE$

$ADE = BCE$

c)

```
rm(list=ls(all=TRUE))
library(AlgDesign) # for gen.factorial()

design = gen.factorial(2,4,varNames=c("A","B","C","E")) # Shorter +- table (w/o D)
design = rbind(design,design,design)                     # 3 replicates
attach(design)

D = A*B*C            # generate the dropped factor by "solving" for D in ABCD=1

A = as.factor(A); B = as.factor(B); C = as.factor(C); D = as.factor(D); E = as.factor(E)

y = c(7.78,8.15,7.50,7.59,7.54,7.69,7.56,7.56,7.50,7.88,7.50,7.63,7.32,7.56,7.18,7.81,
      7.78,8.18,7.56,7.56,8.00,8.09,7.52,7.81,7.25,7.88,7.56,7.75,7.44,7.69,7.18,7.50,
      7.81,7.88,7.50,7.75,7.88,8.06,7.44,7.69,7.12,7.44,7.50,7.56,7.44,7.62,7.25,7.59)

contr = as.character("contr.helmert")
lm.1 = lm(y~A*B*C*D*E, contrasts = list(A=contr,B=contr,C=contr,D=contr,E=contr))
summary.aov(lm.1)
```

0

2

```
summary.aov(lm.1)

#            Df Sum Sq Mean Sq F value   Pr(>F)
# A           1 0.7033  0.7033  35.888 1.12e-06 ***
# B           1 0.3218  0.3218  16.420 0.000302 ***
# C           1 0.0295  0.0295   1.506 0.228774
# D           1 0.0999  0.0999   5.099 0.030893 *
# E           1 0.6840  0.6840  34.906 1.42e-06 ***
# A:B         1 0.0105  0.0105   0.536 0.469451
# A:C         1 0.0000  0.0000   0.001 0.975515
# B:C         1 0.0063  0.0063   0.322 0.574603
# A:E         1 0.0488  0.0488   2.489 0.124500
# B:E         1 0.2806  0.2806  14.319 0.000640 ***
# C:E         1 0.0130  0.0130   0.664 0.421343
# D:E         1 0.0188  0.0188   0.959 0.334662
# A:B:E       1 0.0001  0.0001   0.003 0.959204
# A:C:E       1 0.0046  0.0046   0.235 0.631251
# B:C:E       1 0.0426  0.0426   2.174 0.150128
# Residuals  32 0.6271  0.0196

# The significant effect at alpha = 0.01 are: A, B, E, and BE.
# d)
  eff = as.matrix(2*lm.1$coefficients)
  eff
# (Intercept)    15.251250000
# A1              0.242083333
# B1             -0.163750000
# C1             -0.049583333
# D1              0.091250000
# E1             -0.238750000
# A1:B1          -0.029583333
# A1:C1           0.001250000
# B1:C1          -0.022916667
# A1:D1                    NA        Note: all the NAs are effects aliased with something else.
# B1:D1                    NA
# C1:D1                    NA
# A1:E1           0.063750000
# B1:E1           0.152916667
# C1:E1          -0.032916667
# D1:E1           0.039583333
# A1:B1:C1                 NA
# A1:B1:D1                 NA
# A1:C1:D1                 NA
# B1:C1:D1                 NA
# A1:B1:E1        0.002083333
# A1:C1:E1        0.019583333
# B1:C1:E1       -0.059583333
# A1:D1:E1                 NA
# B1:D1:E1                 NA
# C1:D1:E1                 NA
# A1:B1:C1:D1              NA
# A1:B1:C1:E1              NA
# A1:B1:D1:E1              NA
# A1:C1:D1:E1              NA
# B1:C1:D1:E1              NA
# A1:B1:C1:D1:E1           NA

 qqnorm(eff[-1])        # exclude the mu/intercept
 abline(0,.08)
# The anomalous effects are A and BE (above the line), and B and E (below the line). These results are consistent with those in part c.
```