Name: ____________________

ID: ______________________

Quiz section or time: ______________ 21.5

Stat 421, Test 1, Fall, Oct. 15, 2012; Marzban

ONLY a half-size "cheat sheet" is allowed

Multiple choice: Circle all the correct answers; there is wrong-answer penalty; do NOT explain

The rest: SHOW answer & work; NO CREDIT for correct answer without explanation

Points

~ 1 3.1, P.69 **1.** An F-test of $H_0 : \mu_1 = \mu_2 = \mu_3 = ... = \mu_a$ versus $H_1 : \mu_i \neq \mu_j$ for at least one pair (i, j) involves
a) a 1-sided upper-tail test, b) a 1-sided lower-tail test, c) a 2-sided test.

~ 1 lect3 **2.** In computing the power of a 1-sample 2-sided t-test for a population mean, power increases
a) when the true mean gets farther from the mean under the null hypothesis.
b) when the variance of the sampling distribution of sample means decreases.

~ 1 3.41 **3.** In the effects model $y_{ij} = \mu + \tau_i + \epsilon_{ij}$, two different teams have used different constraints on τ_i to estimate the parameters. In general, the two teams will get the same estimate
a) of all the parameters. b) only for some combination of the parameters.

~ 1 lect4 **4.** 1-way ANOVA is appropriate only for responses that are monotonically increasing or decreasing with level. a) True b) False

~ 2 2.11 **5.** The breaking strength of a fiber is required to be at least 150 psi. In terms of μ = population mean strength, write down the appropriate H_0, H_1. $H_0: \mu \leq 150$ $H_1: \mu > 150$

~ 1/~~2~~ 2.20 **6.** Two types of plastic are suitable for use by an electronic calculator manufacturer. The company will not adopt plastic 1 unless its breaking strength exceeds that of plastic 2 by at least 10 psi. In terms of μ_i = population mean strength for plastic i, write down the appropriate H_0, H_1.

$H_0: \mu_1 - \mu_2 \leq 10$ $H_1: \mu_1 - \mu_2 > 10$

~ 2 4.1 **7.** In order to assess the effectiveness of a bone-growth drug, two different formulations of the drug are prepared. 6 cages with 5 mice in each cage are obtained. Each of the formulations is applied to 3 randomly selected cages. What is a) the treatment, b) the experimental unit, c) the number of replicates, and/or d) blocks (if any). Formulation Cage 3

None

~ 3 2.35 lect 3 **8.** A theorem states that if $X_1 \sim \chi^2_u$ and $X_2 \sim \chi^2_v$, then $F = \frac{X_1/u}{X_2/v} \sim F_{u,v}$. We also know that $\frac{(n_i-1)s_i^2}{\sigma_i^2} \sim \chi^2_{n_i-1}$ where $i = 1, 2$. a) Find a quantity involving σ_i and s_i that has an F distribution, and then b) starting from a probabilistic statement, derive the CI for σ_2^2/σ_1^2.

① a) $$F = \frac{\frac{\cancel{(n_1-1)}}{\sigma_1^2} s_1^2 \Big/ \cancel{(n_1-1)}}{\frac{\cancel{(n_2-1)}}{\sigma_2^2} s_2^2 \Big/ \cancel{(n_2-1)}} = \frac{s_1^2}{s_2^2}\frac{\sigma_2^2}{\sigma_1^2} \sim F_{n_1-1,\, n_2-1}$$

② b) $$\text{prob}\left(F_{\frac{\alpha}{2}, n_1-1, n_2-1} < F < F_{1-\frac{\alpha}{2}, n_1-1, n_2-1} \right) = 1-\alpha$$

$$\text{"} < \frac{s_1^2}{s_2^2}\frac{\sigma_2^2}{\sigma_1^2} < \text{"}$$

C.I. $\frac{\sigma_2^2}{\sigma_1^2}$: $$\boxed{\frac{s_2^2}{s_1^2} F_{\frac{\alpha}{2}, n_1-1, n_2-1} < \frac{\sigma_2^2}{\sigma_1^2} < \frac{s_2^2}{s_1^2} F_{1-\frac{\alpha}{2}, n_1-1, n_2-1}}$$

~4 3.24 **9.** Three brands of batteries are under study. It is suspected that the lives (in weeks) of the three brands are different. Five randomly selected batteries of each brand are tested. The following ANOVA table and estimates of the treatment means are produced:

(1) a) Are the lives of these brands of batteries different? Why? Yes. Because p-value $< \alpha$

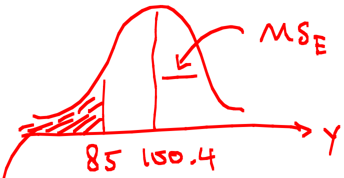
(1) b) Which brand would you select for use? Why? A_3, because longest life (100.4)

(1.5) c) Suppose you have made the selection in part b. Assuming the estimates reported in the Tables are true (i.e., refer to the population), if the manufacturer will replace without charge any battery that fails in less than 85 weeks, what percentage would the company expect to replace? (No need for a single numerical answer, but write an expression involving numbers.)

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
A	2	1196.1	598.1	38.34	6.14e-06
Residuals	12	187.2	15.6		

	Estimate	Std. Error	t value	Pr(> |t|)
A1	95.200	1.766	53.90	1.10e-15
A2	79.400	1.766	44.95	9.58e-15
A3	100.400	1.766	56.84	5.80e-16

c)



$$\Phi\left(\frac{85-100.4}{\sqrt{15.6}}\right)$$

No $/\sqrt{n}$ because This is y, not $\bar{y}$.

~ 2 3.30 **10.** In 1-way ANOVA with 3 treatments, we are interested in comparing $\mu_1 + 2\mu_2$ with $3\mu_3$. But we also want to have a complete decomposition of $SS_{Treatment}$. Write down one **set** of contrasts that will accomplish this task. ↳ we need orthogonal contrast.

For $a=3$, there are $a-1=2$ orthogonal contrasts. One is $\begin{pmatrix}1\\2\\-3\end{pmatrix}$

call the other $\begin{pmatrix}x\\y\\z\end{pmatrix}$. Must have $\underbrace{x+y+z=0}_{\text{sum to zero}} = x+2y-3z$

Solve: $x=-2y+3z \Rightarrow (-2y+3z)+y+z=0 \Rightarrow y=4z$

One soln: Set $z=1$. Then $y=4$, and $x=-5 \Rightarrow \begin{pmatrix}-5\\4\\1\end{pmatrix}$ and $\begin{pmatrix}1\\2\\3\end{pmatrix}$

~ 4 hw-I **11.** Consider the means model $y_{ij} = \mu_i + \epsilon_{ij}, i = 1, ..., a;\ j = 1, ..., n$. **a)** Find the least-squares estimate of μ_i. **b)** What is the reduction in the sum of squares from fitting this model? Write your answer in terms of a, n, and estimates of treatment means.

a) $SS_E = \sum_{i,j} (y_{ij} - \mu_i)^2$

(2)

$\frac{\partial}{\partial \mu_i}\Big|_{\hat{\mu}_i} \sim \sum_j (y_{ij} - \hat{\mu}_i) = 0$ No $\sum_i$!

$\therefore \underbrace{\sum_{j} y_{ij}}_{y_{i.}} - \hat{\mu}_i \underbrace{\sum_j 1}_{n} = 0$

$\therefore \boxed{\hat{\mu}_i = \bar{y}_{i.}}$

b) $R(\mu) = \sum_{i,j} y_{ij}^2 - SS_E(\mu)$

(2)

$= \sum_{i,j} y_{ij}^2 - \sum_{i,j} (y_{ij} - \bar{y}_{i.})^2$

$= \cancel{\sum_{i,j} y_{ij}^2} - \cancel{\sum_{i,j} y_{ij}^2} + 2\sum_{i,j} y_{ij}\bar{y}_{i.} - \sum_{i,j} \bar{y}_{i.}^2$

$= 2\sum_i \bar{y}_{i.} \underbrace{\sum_j y_{ij}}_{y_{i.} = n\bar{y}_{i.}} - \sum_i (\bar{y}_{i.})^2 \cdot \underbrace{\sum_j^n 1}_{n}$

$\boxed{R(\mu) = n\sum_i (\bar{y}_{i.})^2}$