hw5

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Problem 1

(a)

$$
\begin{aligned}
p(X) &= \sum_y p(x|Y)p(Y) \\
&= p(x|Y=1)p(Y=1) + p(x|Y=2)p(Y=2) \\
&= \frac{1}{4} \cdot I(-4 < x < -2) + \frac{1}{4} \cdot I(2 < x < 4)
\end{aligned}
$$

Marginal distribution of X follows uniform distribution on seperate intervals [-4, -2] and [2, 4].

$$
\begin{aligned}
p(Y=1|X \in [-4,-2]) &= [p(X|Y=1) \cdot p(Y=1)]/[p(X)] \\
&= (\frac{1}{2} \cdot 1 \cdot \frac{1}{2})/(\frac{1}{4}) \\
&= 1 \\
p(Y=2|X \in [-4,-2]) &= 0, \text{ since } p(x \in [-4,-2]|Y=2) = 0
\end{aligned}
$$

similiarly, $p(Y=2|X \in [2,4]) = 1$, $p(Y=1|X \in [2,4]) = 0$

(b)

Since the conditional probability distribution of Y base on X can be represented as

$$
\begin{aligned}
p(Y=1|X \in [-4,-2]) &= 1 \\
p(Y=2|X \in [-4,-2]) &= 0 \\
p(Y=1|X \in [2,4]) &= 0 \\
p(Y=2|X \in [2,4]) &= 1
\end{aligned}
$$

$$
\begin{aligned}
p(Y=1|X \in [-4,-2]) &> p(Y=2|X \in [-4,-2]) \\
p(Y=2|X \in [2,4]) &> p(Y=1|X \in [2,4])
\end{aligned}
$$

Therefore, estimator of bayes rule for y is $f_B(x \in [-4,-2]) = 1$, $f_B(x \in [2,4]) = 2$.
In this case, risk is zero, since $p(Y=2|X \in [-4,-2]) = p(Y=1|X \in [2,4]) = 0$, their sum is zero as well.

(c)