Chapter 5 Programming Assignments

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1 Model Review

In this section, we will primarily focus on establishing a model using the Analytic Hierarchy Process (AHP) to address problems 1 and 2. We will break down the problems into five parts:

1. Problem Analysis
2. Data Cleaning and Processing
3. Collinearity Detection
4. Analytic Hierarchy Process (AHP)

1.1 Problem Analysis

To investigate the reasons behind "momentum," we first need to provide a preliminary definition for "momentum." The magnitude of "momentum" is defined as

$$f_{ijk} = \boldsymbol{\omega} \cdot \boldsymbol{x_{ijk}}$$

where:

1. f_{ijk} represents the "momentum" of player k before the jth point number in the ith match (in the order given by the table).
2. $\boldsymbol{x_{ijk}}$ is an n-dimensional column vector representing some influencing factors at the corresponding moment. Specific details will be provided later.

3. $\boldsymbol{\omega}$ is an n-dimensional row vector indicating the specific weights of the influencing factors, which will be obtained through the Analytic Hierarchy Process (AHP).

4. In this formula, there are two different calculation methods, one representing rounds where the player serves and the other representing rounds where the opponent serves. We can express it as

$$\boldsymbol{\omega} = \boldsymbol{\omega_0} \circ \boldsymbol{\delta} = (\omega_0^{(0)}\delta^{(0)}, \omega_0^{(1)}\delta^{(1)}, \dots, \omega_0^{(n)}\delta^{(n)})$$

   representing a vector formed by element-wise multiplication of two vectors of the same dimension. Here, $\boldsymbol{\delta}$ is a $0, 1$ vector indicating whether it is the player's serving round. In the specific calculation, we will consider two cases separately.

For the specific definition of $\boldsymbol{x_{ij}^n}$, we believe that, in addition to whether the player is serving, many other factors can have an impact, including the player's skills, fatigue level, and real-time mental state of the game (here, we mainly consider these three points). Based on these three main aspects, we have organized 12 factors as preliminary influencing factors, as follows:

1.2 Data Processing and Normalization

1.3 Collinearity Detection

After processing the data, considering the potential collinearity among factors within the same category, such as serving aces, first-serve scoring rate, and whether the previous point was scored may be correlated, as well as running distance and the number of strokes possibly being related, we conducted collinearity detection using Stata. The results of the detection indicate a significant variance inflation factor between running distance and the number of strokes. Therefore, we decided to exclude one of them, choosing to retain the remaining 11 variables for the Analytic Hierarchy Process (AHP).

1.4 Analytic Hierarchy Process

We have previously decomposed the included factors from top to bottom into several levels, where factors within the same level are subordinate to factors in the level above or influence factors in the level above. They also dominate factors in the next level or are influenced by factors in the next level. Starting from the second level of the hierarchy, we construct comparison matrices for each factor influencing the factor in the level above, until reaching the bottom level. Each element in the matrix indicates the preference level between factor i and factor j at the same level. It is essential to note that we have separately established a series of such comparison matrices for two different serving types (serving by oneself and serving by the opponent). Here, we illustrate the matrix using serving by oneself as an example:

influe	ability	degre	manta
ability	1	1	1/3
degre	1/1	1	1
manta	3	1/1	1

ability	serve_	winne	net_wi
serve_	1	5	7
winne	1/5	1	3
net_wi	1/7	1/3	1

图 1: Comparison matrix for influencing factors and ability

degre	distan	unforc
distan	1	3
unforc	1/3	1

manta	scored	score_
scored	1	1/5
score_	5	1

图 2: Comparison matrix for degree of fatigue and mantality

serve_	ace	doubl	first_s	fast_w
ace	1	1	1/3	1/3
doubl	1/1	1	1/3	1/3
first_s	3	3	1	1/3
fast_w	3	3	3	1

图 3: Comparison matrix for serving

We obtain the weights for each component by calculating the maximum eigenvalue and normalizing its corresponding eigenvector. Certainly, for each matrix, we first need to test consistency using the Consistency Ratio (CR), where $CR = \frac{CI}{RI}$, $CI = \frac{\lambda_{max}-n}{n-1}$, $RI = 0.0, 0.58, 0.9$ (for matrices of size $2, 3, 4$). The computed Consistency Ratios for the matrices are 0.076, 0.037, 0.0, 0.0, 0.046. Since they are all less than 0.1, it confirms the consistency of the matrices.

Therefore, the weights for our model are as follows:

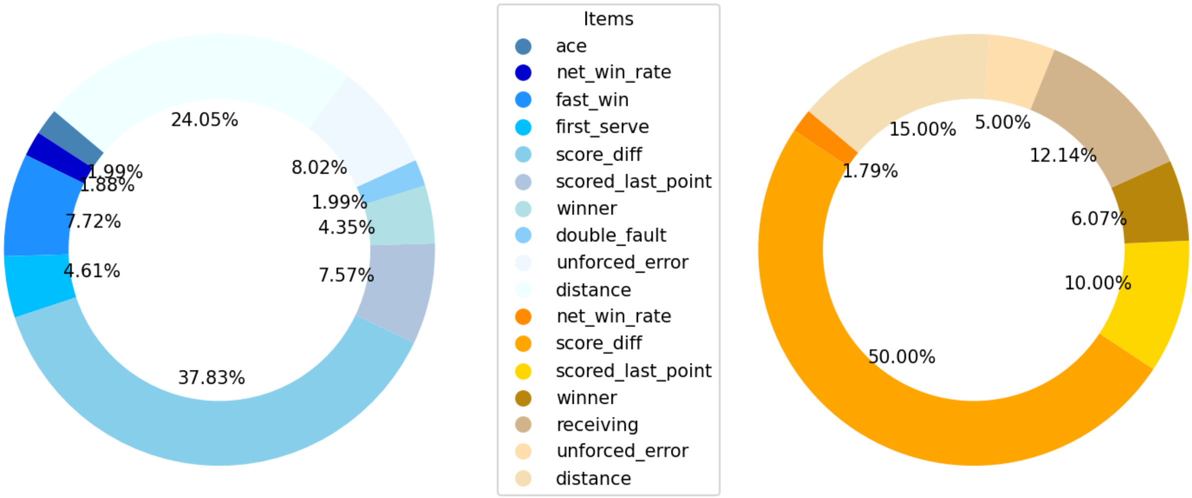


图 4: Weights in two different situations

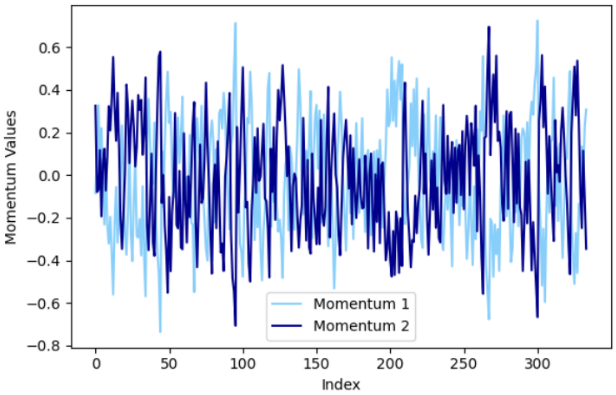
Analyzing the various factors in the chart, it is evident that the most impactful factor is whether the previous point was scored. Following closely is the distance covered during the play, which aligns well with common intuition.

Thus, our final momentum is defined as:(need to change symbol)

$$momentum = \begin{cases} \sum_{n=1, n\neq 5}^{11} \omega_n x_n, & \text{if the player serves} \\ \sum_{n=5}^{11} \omega_n x_n, & \text{if the opponent serves} \end{cases}$$

Where $\omega_n (n = 1, \ldots, 11)$ represent the weight of the factors, which is listed in Figure 4.

Now, we illustrate the graph of the "momentum" in the first match:



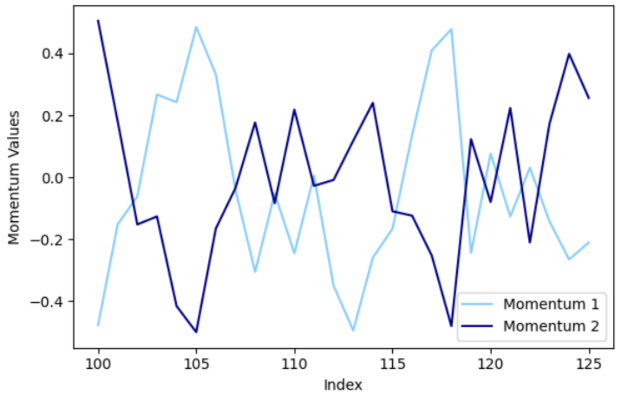


图 5: Momentum change in the first competition(global and local)

It can be observed that the variation in ”momentum” is a process of give and take.