Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
ABCDEF		2400609

Shortest Path Algorithms: Taxonomy and Advance in Research

[1]

As for problem 1, in order to accurately capture the effective information in the data set, we take a series of data processing methods... After that, we select potential influencing factors and establish Analytic Hierarchy Process (AHP) model to make a preliminary analysis of the influencing factors and calculate specific parameters for each factor. We find that the biggest factor is score difference. Apart from that ,whether scored in the last point, running distance, unforced error and fast win will also positively or negatively influence the momentum to a comparatively large extent. The above conclusion is verified in our later models.

In problem 2, we believe that momentum will affect the future scores of the match. So to answer the coach's question, we first perform autocorrelation test on momentum, and we find that it has strong first-oredr autocorrelation. Then we quantify the future scores and then determine the correlation of momentum the scores in the future, they are highly correlated.

Problem 3 is devided into 3 parts. To predict the swings in the match , we establish model based on Gated Recurrent Unit(GRU) algorithm. We process data in problem 1 and add more features. We define the swings and use previous information to predict future. Then compared to the result in problem 1, our accuracy is ....

To identify the most related factors, we use a novel argorithm called Permutation Feature Importance argorithm, it can determine the importance of features by calculating their prediction error after permutation. We find that ....

As for the ideas for specific atheletes, we use previous model on their matches to identify their feature importance. We find that different atheletes have different features in their match.

Problem 4

Key words: AHP, correlation analysis, GRU, Permutation Feature Importance

Indroduction

Tennis more than any other sport, is a game of momentum. The absence of a clock to do the dirty work of finishing off an opponent, and a scoring system basedon units used, makes the flow of the match much more important than any lead that has been established.–Chuck Kriese

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1.2 Restatement of the Problem

First Problem Momentum is "strength or force gained by motion or by a series of events." It directly shows the player's current performance. To assess the players' performance, it is crucial to have a clear understanding of "momentum." We will focus on the following tasks:

- determine the influencing factors of "momentum"
- Quantify the variations in "momentum"
- Visualize the "momentum" function

Second Problem "momentum's role in the match" means the level of momentum affects the future scores of the match. The coach may subscribe to the idea that each point is an independent event and governed by probability. In this view, consecutive success and momentum changes (swings) are seen as more random than influenced by previous events. To judge this autocorrelation and to use our model, we

- perform autocorrelation test on momentum
- perform correlation test between current momentum and future scores.

Third Problem The goal of the third problem is to predict the swings of the play. Considering the extra tasks, we will split the problem into five parts:

- define swing of the play
- decide what data the prediction is based on
- develop a model to predict the swings of the play
- decide what among the data are the most decisive to the prediction
- give advice to player based on the weights of the data

Fourth Problem Testing the model on other matches is a process of generalization ability test. We will split the problem into five parts:

- predict the swings on test data
- compare the prediction with the momentum function in Problem 1
- figure out reasons for the poor prediction
- generalize all our models to Women's matches or other sports

Fifth Problem The memo of advice for coaches on momentum, and players on preparation for potential momentum swings will be placed in the Appendix.

1.3 Assumptions

To simplify the problem, we made the following assumptions:

- **Assumption 1:** The `px_unf_err` column of the data only counts those unforced errors that occurred when the player was hitting in baseline.
  Justification: Usually when a player is at net, the point will end in a few strikes, and there's little probability that the player will hit an unforced error within that few strikes. What's more, the `px_net_point` and `px_net_point_won` columns of the data can predominantly reflect the player's ability at net, therefore reducing the impact of counting the unforced errors while at net.

- **Assumption 2:** The "current performance" we usually refer to on a certain aspect of a player can be reflected by the player's 3 latest shots of that aspect.
  E.g. The current performance can be reflected by a combination of, the proportion of aces in the 3 latest **serves** of the player, the proportion of winners in the 3 latest **shots** of the player, the return depth of the 3 latest **returns** of the player, etc.
  Justification: The current performance of a player consists of the average performance and the status of the player at the moment, which can be comprehensively reflected in the player's performance on recent shots. For convenience, we specified that the 3 latest shots can reflect the player's current performance.

2 Momentum Evaluation Model

2.1 Model Introduction

To determine which player is performing better at a specific time, we create a indicator "Momentum" using the Analytic Hierarchy Process (AHP) to give a quantitative and overall evaluation.

To investigate the reasons behind "momentum", we first need to provide a preliminary definition for "momentum". The magnitude of "momentum" is defined as

$$f_{ijk} = \boldsymbol{\omega} \cdot \boldsymbol{x_{ijk}}$$

where:

1. f_{ijk} represents the “momentum” of player k before the jth point number in the ith match (in the order given by the table).

2. $\boldsymbol{x_{ijk}}$ is an n-dimensional column vector representing some influencing factors at the corresponding moment. Specific details will be provided later.

3. $\boldsymbol{\omega}$ is an n-dimensional row vector indicating the specific weights of the influencing factors, which will be obtained through the Analytic Hierarchy Process (AHP).

4. In this formula, there are two different calculation methods, one representing rounds where the player serves and the other representing rounds where the opponent serves. We can express it as

$$\boldsymbol{\omega} = \boldsymbol{\omega_0} \circ \boldsymbol{\delta} = (\omega_0^{(0)}\delta^{(0)}, \omega_0^{(1)}\delta^{(1)}, \ldots, \omega_0^{(n)}\delta^{(n)})$$

   representing a vector formed by element-wise multiplication of two vectors of the same dimension. Here, $\boldsymbol{\delta}$ is a $0, 1$ vector indicating whether it is the player's serving round. In the specific calculation, we will consider two cases separately.

Out of our own interpretation of AHP, we will break down the problems into three parts:

1. Factor Normalization and Data Cleaning

2. Collinearity Detection

3. Analytic Hierarchy Process (AHP)

2.1.1 Notations

Symbols	Description
$player$	the current player we are considering (e.g. while calculating momentum)
$point_i$	the i^{th} point of the match, a vector consists of fields stated in the given dictionary
cur	the current index of the point, i.e. the match is currently at the cur^{th} point
H_i	denotes the set $\{point_{cur}, point_{cur-1}, \ldots, point_{cur-i+1}\}$
S_i	the set of latest i points where $player$ serves
R_i	the set of latest i points where $player$ returns
P_{ace}	current probability of hitting an ace by $player$
P_{df}	current probability of double-faulting by $player$
P_{1st}	current first serve goal rate by $player$
P_{fw}	current probability of $player$ winning a served point within 3 rallies
rd	current return depth of $player$
P_{win}	current probability of hitting a winner by $player$
P_{net}	current net win rate of $player$
$dist$	$player$'s running distance on the point
P_{unf}	current probability of hitting an unforced error by $player$
$scored$	whether $player$ scored the current point
$diff$	the score diffrence for $player$ in the current game (by number of points)
M	the current momentum of $player$ after a point

To access a certain field in a point, we simply use the field name stated in the given dictionary as index, i.e. for a point $point$, we use $point_{ace}$ to denote the binary variable that shows whether $player$ hits an ace ball in the point.

2.1.2 Factor Normalization and Data Cleaning

For the specific definition of $\boldsymbol{x}_{ij}^{n}$, we believe that, in addition to whether the player is serving, many other factors can have an impact, including the player's skills, fatigue level, and real-time mental state of the games. Based on these three main aspects, we have organized 12 factors as preliminary influencing factors, as follows:

$$P_{ace} = \frac{\sum_{p\in S_3} p_{ace}}{3} \tag{1}$$

$$P_{df} = -\frac{\sum_{p\in S_3} p_{double\_fault}}{3} \tag{2}$$

$$P_{1st} = \frac{\sum_{p\in S_3} [p_{serve\_no} = 1]}{3} \tag{3}$$

$$P_{fw} = \frac{\sum_{p\in S_3} [p_{rally\_count} \leq 3][p_{point\_victor} = player]}{3} \tag{4}$$

$$rd = \frac{\sum_{p\in R_3} \begin{cases} 0, & p_{return\_depth} = ND \\ 1, & p_{return\_depth} = D \\ -1, & p_{return\_depth} = NA \end{cases}}{3} \tag{5}$$

$$P_{win} = \frac{\sum_{p\in H_3} p_{winner}}{3} \tag{6}$$

$$P_{net} = \frac{\sum_{p\in H_3} p_{net\_pt\_won}}{\sum_{p\in H_3} p_{net\_pt}} \tag{7}$$

$$dist = \begin{cases} 0, & point_{cur,distance\_run} < 5 \\ -1, & point_{cur,distance\_run} > 45 \\ \dfrac{5 - point_{cur,distance\_run}}{40}, & otherwise \end{cases} \tag{8}$$

$$P_{unf} = -\frac{\sum_{p\in H_3} p_{unf\_err}}{3} \tag{9}$$

$$scored = [point_{cur,point\_victor} = player] \tag{10}$$

$$diff = \frac{\sum_{p\in point} [p_{set\_no} = point_{cur,set\_no}][p_{game\_no} = point_{cur,game\_no}](2[p_{point\_victor} = player] - 1)}{\min\{3, \sum_{p\in point} [p_{set\_no} = point_{cur,set\_no}][p_{game\_no} = point_{cur,game\_no}]\}} \tag{11}$$

In order to normalize the data processed, we convert the original data to limit them in $[-1, 1]$. For those factors that negatively influence the momentum, such as P_{df}, we made sure it's in $[-1, 0]$. For those factors that positively influence the momentum, such as P_{win}, we made sure it's in $[0, 1]$. For those factors that influence the momentum in both ways, such as $diff$, we made sure it's in $[-1, 1]$.

2.1.3 Collinearity Detection

After processing the data, considering the potential collinearity among factors within the same category, such as serving aces, first-serve scoring rate, and whether the previous point was scored may be correlated, as well as running distance and the number of strokes possibly being related, we conducted collinearity detection using Stata. The results of the detection indicate a significant variance inflation factor between running distance and the number of strokes. Therefore, we decided to exclude one of them, choosing to retain the remaining 11 variables for the Analytic Hierarchy Process (AHP).

2.1.4 Analytic Hierarchy Process

We have previously decomposed the included factors from top to bottom into several levels, where factors within the same level are subordinate to factors in the level above or influence factors in the level above. They also dominate factors in the next level or are influenced by factors in the next level. Starting from the second level of the hierarchy, we construct comparison matrices for each factor influencing the factor in the level above, until reaching the bottom level. Each element in the matrix indicates the preference level between factor i and factor j at the same level. It is essential to note that we have separately established a series of such comparison matrices for two different serving types (serving by oneself and serving by the opponent). Here, we illustrate the matrix using serving by oneself as an example:

influe	ability	degre	manta
ability	1	1	1/3
degre	1/1	1	1
manta	3	1/1	1

ability	serve_	winne	net_wi
serve_	1	5	7
winne	1/5	1	3
net_wi	1/7	1/3	1

Figure 1: Comparison matrix for influencing factors and ability

degre	distan	unforc
distan	1	3
unforc	1/3	1

manta	scored	score_
scored	1	1/5
score_	5	1

Figure 2: Comparison matrix for degree of fatigue and mantality

serve_	ace	doubl	first_s	fast_w
ace	1	1	1/3	1/3
doubl	1/1	1	1/3	1/3
first_s	3	3	1	1/3
fast_w	3	3	3	1

Figure 3: Comparison matrix for serving

We obtain the weights for each component by calculating the maximum eigenvalue and normalizing its corresponding eigenvector. Certainly, for each matrix, we first need to test consistency using the

Consistency Ratio (CR), where $CR = \frac{CI}{RI}$, $CI = \frac{\lambda_{max}-n}{n-1}$, $RI = 0.0, 0.58, 0.9$ (for matrices of size 2, 3, 4). The computed Consistency Ratios for the matrices are 0.076, 0.037, 0.0, 0.0, 0.046. Since they are all less than 0.1, it confirms the consistency of the matrices.

Therefore, the weights for our model are as follows:

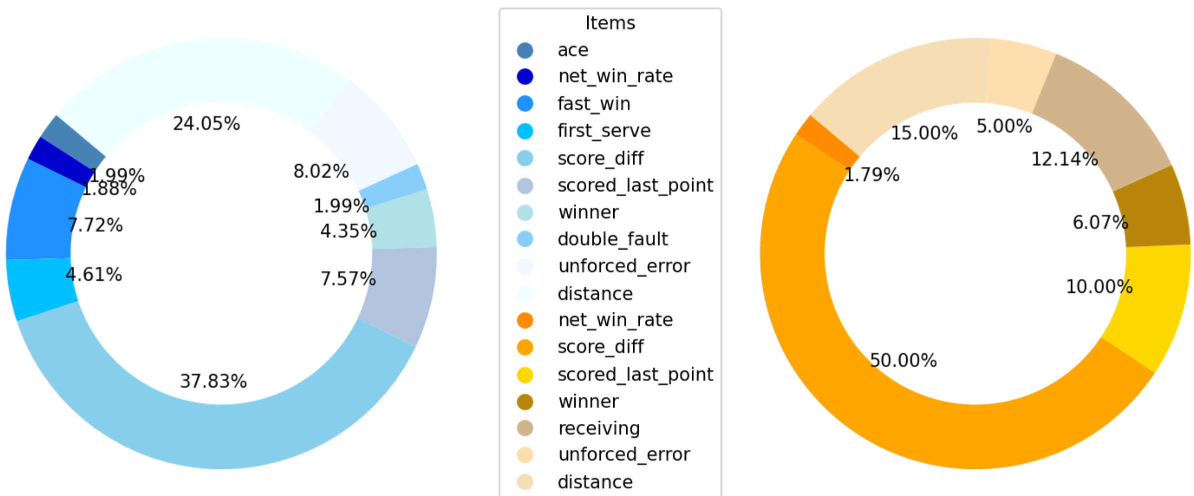


Figure 4: Weights in two different situations

Analyzing the various factors in the chart, it is evident that the most impactful factor is whether the previous point was scored. Following closely is the distance covered during the play, which aligns well with common intuition.

Thus, our final momentum is defined as:(need to change symbol)

$$momentum = \begin{cases} \sum_{n=1,n\neq 5}^{11} \omega_n x_n, & \text{if the player serves} \\ \sum_{n=5}^{11} \omega_n x_n, & \text{if the opponent serves} \end{cases}$$

Where $\omega_n (n = 1, \ldots, 11)$ represent the weight of the factors, which is listed in Figure 4.

2.2 Visualization and Analysis

Now, we illustrate the graph of the "momentum" in the first match:

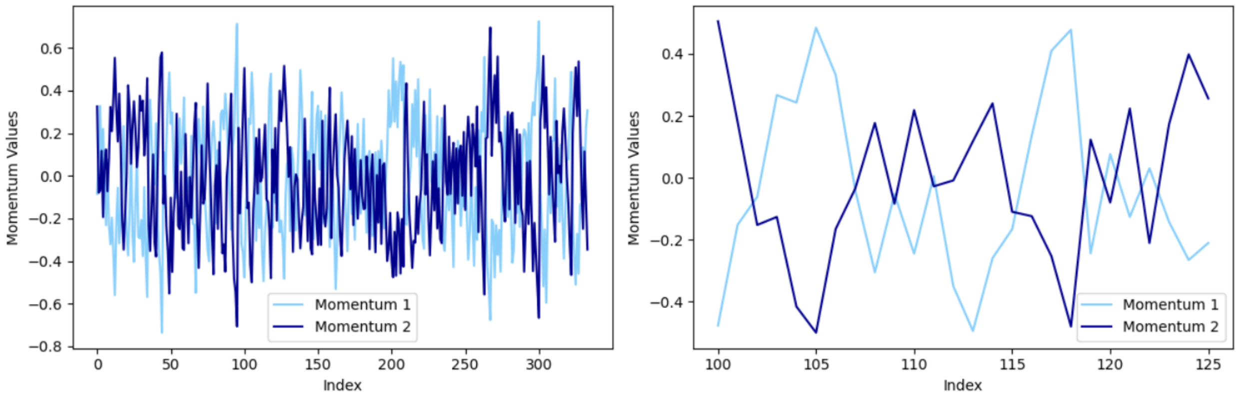


Figure 5: Momentum change in the first competition(global and local)

It can be observed that the variation in "momentum" is a process of give and take. In photo 2, when momentum 1 is above momentum 2, it means that the player 1 is performing better than the opponent.

2.3 momentum autocorrelation and correlation with runs of success

To answer the coach's doubt, we need to perform autocorrelation test on momentum, and perform correlation test between current momentum and future scores in this section.

If the momentum has a high autocorrelation, it means that the momentum at this moment has a high impact on future performance. And if the correlation between momentum and future scores is high, it means that the player with higher momentum has a higher chance to win the next multiple round.

2.3.1 momentum autocorrelation

To check if sequence of momentum is self-related, we calculate the Pearson correlation between momentum and that with a time lag.

Algorithm 1 Calculate autocorrelation function

for $i = 1$ to 31 **do**

  $time\_series \leftarrow momentum(i^{th}\ match\_index : i+1^{th}\ match\_index - 1)$

  $max\_lag \leftarrow \lfloor length(time\_series)/2 \rfloor$ ▷ Consider lags up to half of the length of the time series

  $autocorrelation \leftarrow \text{zeros}(1, max\_lag)$

  for $shift = 1$ to max_lag **do**

    $correlation \leftarrow \text{corrcoef}(time\_series(1 : \text{end} - shift), time\_series(shift + 1 : \text{end}))$

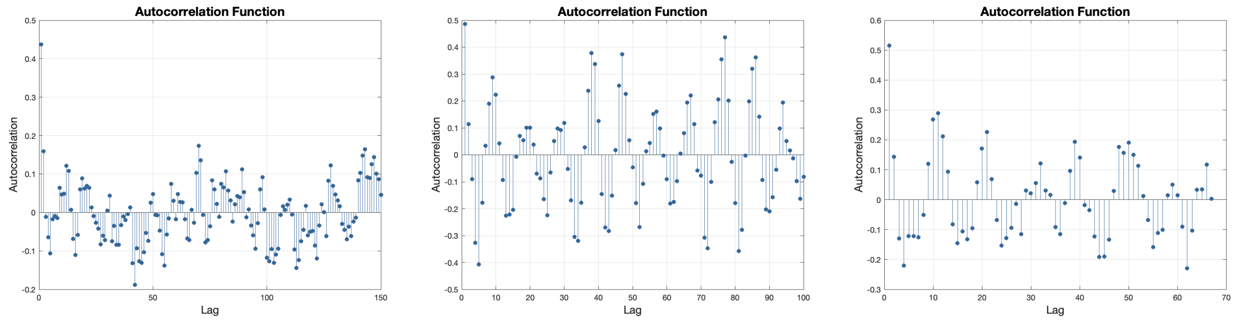
    $autocorrelation(shift) \leftarrow correlation(1, 2)$

  end for

  ▷ Further processing or visualization can be performed here

end for

Here we display the autocorrelation of momentum of the first player in first three games. There are similar results for the second player and for the momentum difference.



(a) autocorrelation in match 1 (b) autocorrelation in match 2 (c) autocorrelation in match 3

Figure 6: Momentum autocorrelation

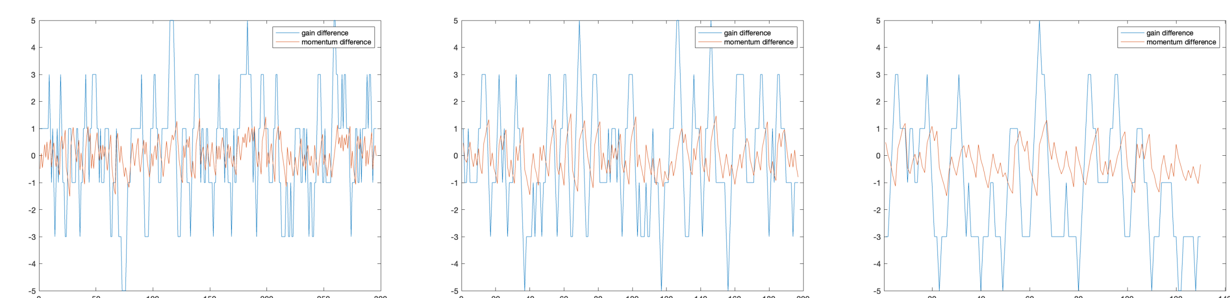
The corrcoef of lag 1 in match one is 0.4546, match two 0.5149,match three 0.5342. It can be seen that the autocorrelation of lag one is high, which means that the momentum at this moment has a strong relationship with the momentum in the next round. And the autocorrelation decreases to random as the lag increases.

2.3.2 correlation with runs of success

To give a quantitative evaluation of “future scores”, we count points gain in future multiple rounds, and derive the difference by minus that of the opponent. For example, if the player gains 3 points in the next 5 rounds, and the opponent gains 2 points, the difference is 1. The difference indicates how much better the player is performing than the opponent.

In intuition, the player with higher momentum should have a higher chance to win the next round. And momentum at this moment should have less impact on the future rounds as time extends. The correlation between momentum and future scores verifies our intuition.

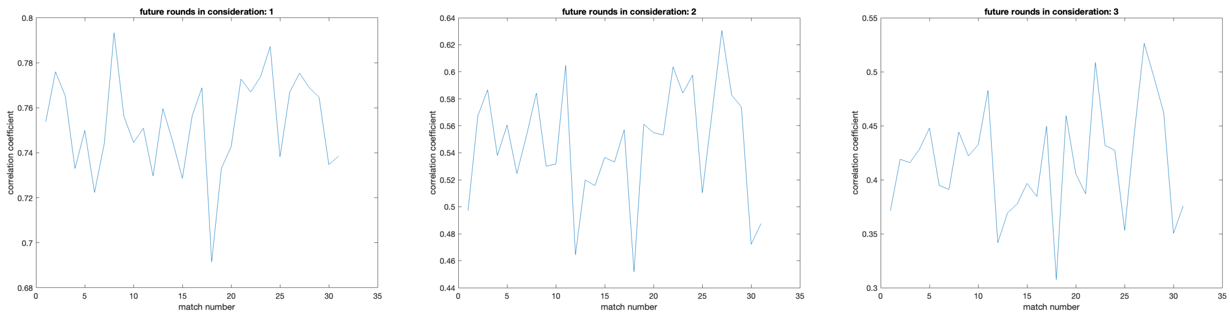
We calculate points gain difference in future one to five rounds at each time of all matches. Here we display five points gain difference and momentum difference in first three games.



(a) 2023-wimbledon-1301 (b) 2023-wimbledon-1302 (c) 2023-wimbledon-1303

Figure 7: Gain Difference and Momentum in First Three Games

And we derive the correlation between gain difference from one to five rounds and momentum difference of all matches. Here we display the three of them.



(a) gain difference in 1 round (b) gain difference in 2 rounds (c) gain difference in 3 rounds

Figure 8: Correlation Between Gain Difference and Momentum Difference

Here we display the max and min correlation in different rounds. (hint. the max and min correlation means maximum and minimum of all matches.)

rounds	1	2	3	4	5
max	0.7934	0.6307	0.5264	0.4678	0.3910
min	0.6914	0.4516	0.3074	0.1824	0.0627

Table 1: max and min Correlation of all matches in different rounds

As we can see from the table, the correlation between momentum and future scores is bigger than 0.5 considering the next 1 round, it implies that momentum has a substantial impact on the next round. And the correlation decreases as the rounds extend, which verifies our intuition, that the momentum at this moment has less impact on the future rounds as time extends.

Now, we have finished problem 1 and 2.

3 Swing Prediction Model

3.1 Model Introduction

To predict the swings, we need to find potential factors that may affect future momentum. In problem 1, we established a standard to judge the player's performance; in problem 2, we found the player's performance, "momentum", has a one order of autocorrelation, which indicates that some factors of the momentum have impact on the future momentum.

To find what and how factors affect the future momentum, we can take advantage of Machine Learning. In this section, we will primarily focus on establishing a supervised learning classification model using the Gated Recurrent Unit (GRU) network to predict momentum swings. We will break down the problems into three parts:

- definition of the swing of the play
- data (potential factors) and label used to train GRU
- Utilizing Gated Recurrent Unit

3.1.1 Definition of "Swings of the Play"

We first give the definition of the swings of the play. Based on our previous definition of "momentum," the significant changes of the game largely depend on the "momentum" of the two players. Therefore, we choose "momentum" to represent the swings of the play. The specific definition is as follows:

We use $\Delta f(t)$ to represent the difference in "momentum" between the two players., so it can be easily seen that if $\Delta f(t)$ and $\Delta f(t+1)$ have different signs, it indicates a "swing" in the game's momentum. In this way, we can define four states of "momentum" at time t:

$$states = \begin{cases} state1, & \text{if } \Delta f(t) > 0 \text{ and } \Delta f(t+1) > 0, \text{ which means stay positive} \\ state2, & \text{if } \Delta f(t) < 0 \text{ and } \Delta f(t+1) > 0, \text{ which means rise from negative to positive} \\ state3, & \text{if } \Delta f(t) < 0 \text{ and } \Delta f(t+1) < 0, \text{ which means stay negative} \\ state4, & \text{if } \Delta f(t) > 0 \text{ and } \Delta f(t+1) < 0, \text{ which means decrease from positive to negative} \end{cases}$$

Obviously, if state 2 or state 4 appears, we can determine that a swing has occurred.

To visualize the swing, we choose four shapes of lines to represent the four states of swings at time t:

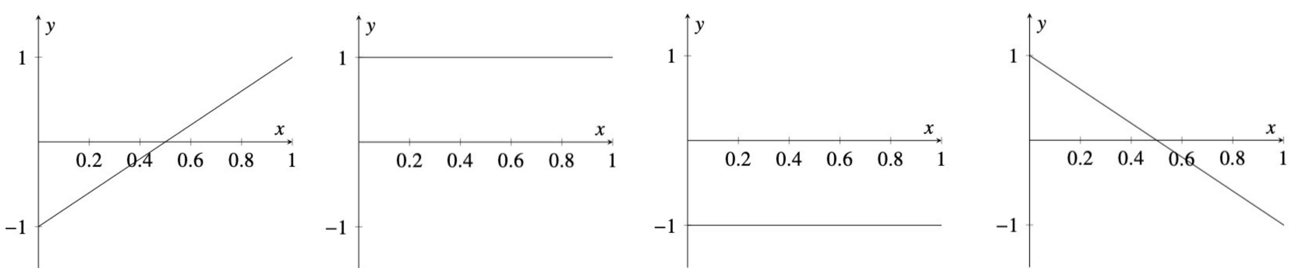
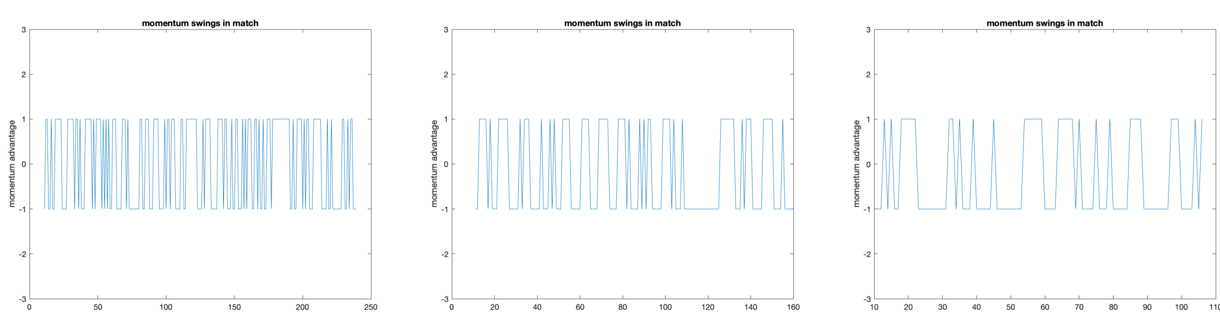


Figure 9: visualization of swings, from state1 to state4

Here we display the momentum swings drived from AHP of first three matches.



(a) swings in match 1 (b) swings in match 2 (c) swings in match 3

Figure 10: swings in first three matches

3.1.2 Data and Label used to train GRU

To verify all potential factors, meanwhile avoiding over-fitting due to the large number of features, the data's feature we select is ...

What's more, we want to find out can swings merely be predicted by the technical data, so that we can give advice to players on skills, therefore we delete `scored_last_point, score_diff, game_victor, set_v` before we train a second model.

The label we use is the state of the swing derived from model of AHP.

we use different sets of data to train and derive different models.

- First, we will train the model with data from all matches available.
- Then, we will train a new model with data from a single player.
- Finally, we will train the model with part of available match statistics

From the first model, we can find the important factors that affect all players' momentum swings. From the second model, we can find the important factors that affect especially well on that player's momentum swings. From the third model, we can examine the generalization ability to accurately predict the swings in irrelevant players and matches.

3.1.3 Utilizing the Gated Recurrent Unit (GRU) Network

Gated Recurrent Units (GRUs) are a type of recurrent neural network (RNN) architecture that has gained popularity for sequential data processing. And it is appropriate for our many(10)-to-many(5) prediction model.

We will focus on a single unit of recurrent neural network to interpret the hidden mathematical principles:

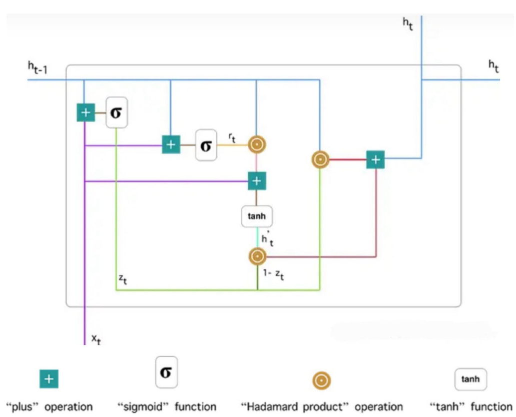


Figure 11: Structure of GRU

1. The update gate at time step t is computed using the following formula:

$$z_t = \sigma(W_z \cdot x_t + U_z \cdot h_{t-1})$$

Here, when x_t is input to the network unit, it is multiplied by its own weight W_z. Similarly, h_{t-1}, which holds the information from the previous $t-1$ units, is multiplied by its own weight U_z. These two results are then summed together and passed through a sigmoid activation function to compress the result between 0 and 1.

2. The essence of the reset gate is for the model to decide how much past information to forget. To compute it, we use:

$$r_t = \sigma(W_r \cdot x_t + U_r \cdot h_{t-1})$$

3. The computation of the new memory content using the reset gate is as follows:

$$h'_t = tanh(Wx_t + r_t \odot Uh_{t-1})$$

4. The computation for the new memory content h_t using the update gate is as follows:

$$h_t = z_t \odot h_{t-1} + (1 - z_t) \odot h'_t$$

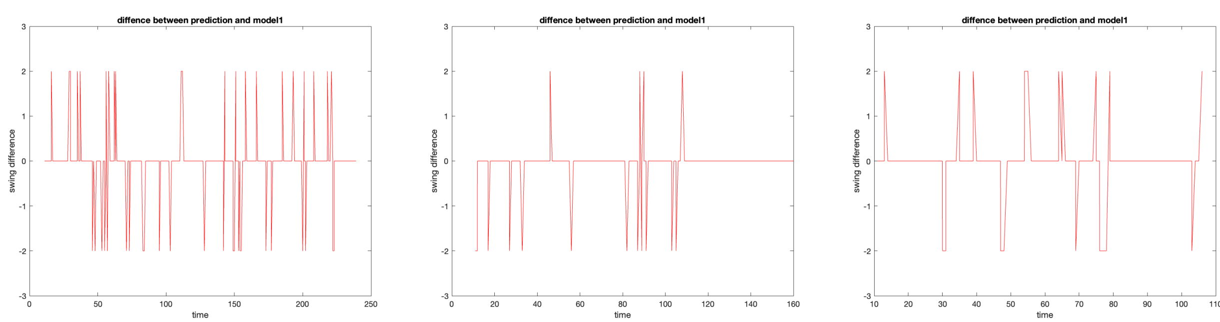
So, we can predict the momentum swings from the technical data of the past 10 points.

3.2 Visualization and Analysis

Out of simplicity, the model shown in this section is trained merely on technical data.

First, we display the model trained on all matches available to predict the swings of the play on test data. To avoid dazzling figure, we minus the swing from AHP with prediction, showing the difference.

Here's the difference of first three matches on training data.



(a) difference in match 1 (b) difference in match 2 (c) difference in match 3

Figure 12: difference between prediction and AHP model

Here's the difference of first three matches on test data.

calculate the match percentage of each match between the prediction model and the result from AHP.

3.3 Weight of Factors

Permutation Feature Importance theory is used to measure the importance of a feature by calculating the increase in the model's prediction error after permuting the feature.

Using the above argorithm, we sort FI and normalize them, then we plot the figure:

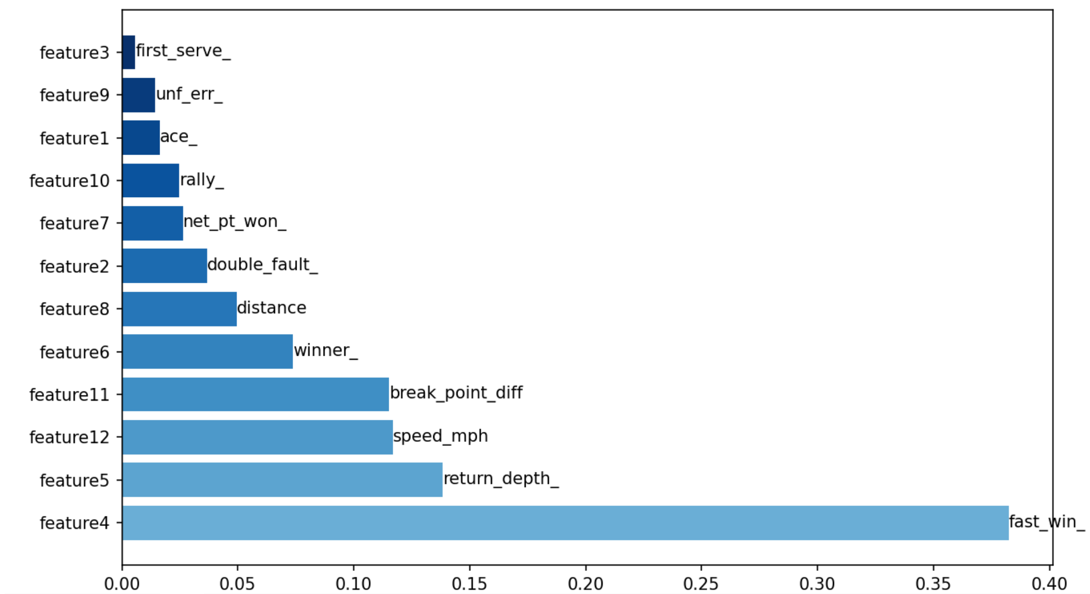


Figure 13: feature importance

We can see that the most important feature is fast win(rally count ¡ 3 and wins the point), and the second most important feature is unforced error. Compared to problem 1(main skill features are running distance, unforced error and fast win),they coincide. The running distance become insignificant, we think it is because this factor is also related to fast win.

3.3.1 Permutation Feature Importance theory

We measure the importance of a feature by calculating the increase in the model's prediction error after permuting the feature. A feature is "important" if shuffling its values increases the model error, because in this case the model relied on the feature for the prediction. A feature is "unimportant" if shuffling its values leaves the model error unchanged, because in this case the model ignored the feature for the prediction.

If using match data of multiple different players, the factor with bigger weight is effective to all players.

3.4 Advice against Opponent Based on Weight of Factors

Algorithm 2 Permutation Feature Importance

Input: Trained model $\hat{f}$, feature matrix X, target vector y, error measure $L(y, \hat{f})$.
Estimate the original model error $e_{orig} = L(y, \hat{f}(X))$ (e.g. mean squared error)
For each feature $j \in \{1, ..., p\}$ **do:**
∘ Generate feature matrix X_{perm} by permuting feature j in the data X. This breaks the association between feature j and true outcome y.
∘ Estimate error $e_{perm} = L(Y, \hat{f}(X_{perm}))$ based on the predictions of the permuted data.
∘ Calculate permutation feature importance as quotient $FI_j = e_{perm}/e_{orig}$ or difference $FI_j = e_{perm} - e_{orig}$
Sort features by descending FI.

If using match data of a single player, the factor with bigger weight is extremely effective to this player.

Now we choose three players:rublev(three matches), sinner(four matches), alcaraz(five matches), and we use the feature analysis method in problem 3 to find their features, the result is as follows:

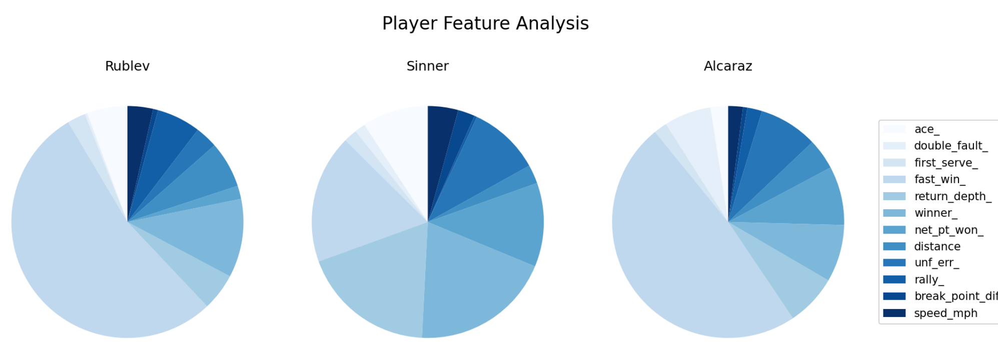


Figure 14: features of three players

1. In comparison to general features, both Rublev and Alcaraz phave significant feature fast win. Therefore, when competing against other players, we recommend them to emphasize the rhythm of seizing the early shots. For example, they can focus more on the quality of their serves, progressively achieving a fast win to enhance their momentum.

2. Additionally, Rublev is hardly affected by negative factors such as double fault and unforced error. Hence, in matches, there is no need for him to worry about occasional mistakes; instead, he can play boldly.

3. For Sinner, his technical characteristics are markedly different. Important features for him are winners and return depth. Consequently, we advise Sinner to fully exploit his strong scoring abilities of winner points and utilize his control over return depth to steer the course of the match and establish an advantage.

Now we have finished Problem 3.

3.5 Prediction Error compared with AHP model outcome

Overlap rate of the prediction model and the result from AHP is?

Factors we may fail to consider First of all, the AHP model is not considering all possible factors to momentum. For example, the. So the overlap rate's effectiveness is limited.

Then, from the conclusion of problem 2, momentum only has one order of autocorrelation. So when we try to predict the far future momentum, there's no reason to be accurate.

4 Robustness Analysis

4.1 Generalization ability

In the previous section, we have only used the data of the first three matches. In this section we will focus on the generlization of the model.

test on all given data

We use our model in problem 3 in predicting all matches to verify its accuracy. Considering the difference length of the matches, we use weighted accuracy to denote the generability of our model. The fomula is as follows:

$$P_{avg} = \frac{1}{L_{tot}} \sum_{matches} \frac{P_i}{L_i}$$

where L_i stands for the length of the match, we measure it by point number.

So the general accuracy of our model in predicting matches is //////,
following are some factors that are not included in our model, all of which we think may influence the result, some of them may be hard to quantify, some of them are not included due to lack of data. Adding them to the model can make it more complete, this is left for future work.

1. The change of the players' strategies, during the course of the game, they may become more familiar with the opponent's technical characteristics and make targeted changes to shift "momentum".

2. The possibility that the players intentionally hide their strength, this is similar to the previous item.

3. The incentive of the audience, this is more likely to have something to do with the difference of home and away, just like football. The influence of the audience on foreign and native players is totally different.

Analysis on Women's Tennis and Table Tennis

As for women's tennis, it is a pity that we did not find enough data online, but from all men's match predictions, the accuracy rate is stable, so we have every reason to believe that our data can also apply to women's matches.
For table tennis, technical statistics also includes:

1. more serve types, such asforehand, backhand, short, long, or sidespin serves.
2. more receiving techniques,such as forehand push, backhand push, flick, or topspin.

So we need to modify the influencing factors in our model, but the general framework is similar.

Now, we have finished Problem 4.

4.2 Parameters Sensitivity Analysis

5 Strength and Weaknesses

5.1 Strengths

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5.2 Weaknesses

test

References

[1] Alice Axford, Bob Birkin, Charlie Copper, and Danny Dannford. Demostration of bibliography items. *Journal of TEXperts*, 36(7):114–120, Mar 2013.

A Advice to Tennis Coaches and Players on Momentum

dear coach:

We hope this letter finds you well. We are , a team with a keen interest in the training and development of tennis athletes. We would like to share some discoveries of ”momentum” during the match that We believe could be beneficial for athlete training and play spot, using mathematical modeling as an analytical tool. After data analysis and modeling. The following are some suggestions based on our analysis:

We hope these suggestions are helpful for your coaching endeavors. If you have any questions or would like to discuss this further and learn more detailed information, you can read the full text of our model.Also, we would be more than willing to discuss additional thoughts and insights with you.

B report on Use of AI

1.

OpenAI ChatGPT (Feb 2,2024 version,ChatGPT3.5)

Query1: *"swings in play and runs of success by one player are random" can this sentence to be interpreted as swings in play and runs of success by one player are independent?*

Output: *Yes, the sentence "swings in play and runs of success by one player are random" can be interpreted as suggesting that swings in play and runs of success by one player are independent of each other. In statistical terms, independence means that the occurrence or outcome of one event does not influence the occurrence or outcome of another.*

In the context of tennis, this would imply that the fluctuations in the dynamics of the match (swings in play) and a player's success streaks (runs of success) are not connected or dependent on each other. Each event is considered random and not influenced by the occurrence of the other.

2.

OpenAI ChatGPT (Feb 5,2024 version,ChatGPT3.5)

Query2: *what is general technical statistics in table tennis games*

Output: *In table tennis, general technical statistics refer to the analysis and measurement of various technical aspects of a player's performance during a match. These statistics provide valuable insights into a player's strengths, weaknesses, and overall playing style. Here are some general technical statistics commonly used in table tennis games:*

Serve Statistics:

Serve Type: Recording the types of serves a player employs, such as forehand, backhand, short, long, or sidespin serves. Serve Effectiveness: Analyzing the success rate of different types of serves, including direct points and opponent errors. Receiving Statistics: Receiving Techniques: Documenting how a player receives various types of serves, such as forehand push, backhand push, flick, or topspin. Receiving Success Rate: Calculating the player's success rate in returning different types of serves. Stroke Execution Statistics: Forehand and Backhand Shots: Analyzing the usage and effectiveness of forehand and backhand strokes in various situations. Types of Strokes: Recording the diversity of strokes played, including topspin, backspin, drives, and smashes. Rally Statistics: Rally Length: Measuring the average length of rallies in terms of the number of strokes exchanged. Rally Outcome: Tracking the outcomes of rallies, such as points won or lost, and identifying patterns in rally progression. Placement and Direction Statistics: Ball Placement: Analyzing where a player tends to place the ball on the opponent's side of the table. Direction of Shots: Recording the direction of shots, including cross-court, down the line, and shots to the opponent's body. Point Distribution Statistics: Point Sources: Identifying the sources of points, such as winners, opponent errors, or forced errors. Point Distribution: Analyzing how points are distributed throughout the match, helping to identify key moments. Error Analysis: Unforced Errors: Documenting the number of unforced errors made by a player. Forced Errors: Identifying instances where the opponent's play led to errors by the player. These technical statistics are crucial for players, coaches, and analysts to assess performance, strategize for future matches, and tailor training programs to address specific areas of improvement. They provide a comprehensive overview of a player's game and contribute to a more informed and strategic approach to table tennis.