Chapter 5 Programming Assignments

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restatement of the problem

First Problem momentum is “strength or force gained by motion or by a series of events.” It directly impacts the player’s performance in subsequent points. To assess the players’ performance, it is crucial to have a clear understanding of ”momentum.” We will focus on the following tasks:

1. determine the influencing factors of ”momentum”
2. Quantify the variations in ”momentum” using fluctuating data.
3. Visualize the process of ”momentum” changes.

1 Model Review

In this section, we will primarily focus on establishing a model using the Analytic Hierarchy Process (AHP) to address problems 1 and 2. We will break down the problems into five parts:

1. Problem Analysis
2. Data Cleaning and Processing
3. Collinearity Detection
4. Analytic Hierarchy Process (AHP)

1.1 Problem Analysis

To investigate the reasons behind ”momentum,” we first need to provide a preliminary definition for ”momentum.” The magnitude of ”momentum” is defined as

$$f_{ijk} = \boldsymbol{\omega} \cdot \boldsymbol{x_{ijk}}$$

where:

1. f_{ijk} represents the ”momentum” of player k before the jth point number in the ith match (in the order given by the table).

2. $\boldsymbol{x_{ijk}}$ is an n-dimensional column vector representing some influencing factors at the corresponding moment. Specific details will be provided later.

3. $\boldsymbol{\omega}$ is an n-dimensional row vector indicating the specific weights of the influencing factors, which will be obtained through the Analytic Hierarchy Process (AHP).

4. In this formula, there are two different calculation methods, one representing rounds where the player serves and the other representing rounds where the opponent serves. We can express it as

   $$\boldsymbol{\omega} = \boldsymbol{\omega_0} \circ \boldsymbol{\delta} = (\omega_0^{(0)}\delta^{(0)}, \omega_0^{(1)}\delta^{(1)}, \ldots, \omega_0^{(n)}\delta^{(n)})$$

   representing a vector formed by element-wise multiplication of two vectors of the same dimension. Here, $\boldsymbol{\delta}$ is a $0, 1$ vector indicating whether it is the player’s serving round. In the specific calculation, we will consider two cases separately.

For the specific definition of $\boldsymbol{x_{ij}^{n}}$, we believe that, in addition to whether the player is serving, many other factors can have an impact, including the player’s skills, fatigue level, and real-time mental state of the game (here, we mainly consider these three points). Based on these three main aspects, we have organized 12 factors as preliminary influencing factors, as follows:

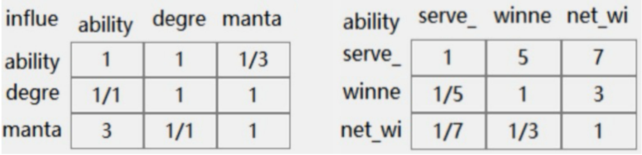
1.2 Data Processing and Normalization

1.3 Collinearity Detection

After processing the data, considering the potential collinearity among factors within the same category, such as serving aces, first-serve scoring rate, and whether the previous point was scored may be correlated, as well as running distance and the number of strokes possibly being related, we conducted collinearity detection using Stata. The results of the detection indicate a significant variance inflation factor between running distance and the number of strokes. Therefore, we decided to exclude one of them, choosing to retain the remaining 11 variables for the Analytic Hierarchy Process (AHP).

1.4 Analytic Hierarchy Process

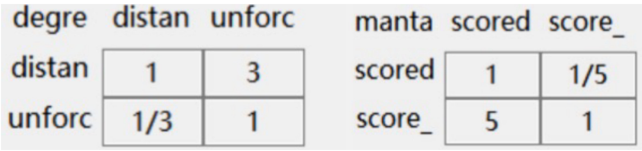
We have previously decomposed the included factors from top to bottom into several levels, where factors within the same level are subordinate to factors in the level above or influence factors in the level above. They also dominate factors in the next level or are influenced by factors in the next level. Starting from the second level of the hierarchy, we construct comparison matrices for each factor influencing the factor in the level above, until reaching the bottom level. Each element in the matrix indicates the preference level between factor i and factor j at the same level. It is essential to note that we have separately established a series of such comparison matrices for two different serving types (serving by oneself and serving by the opponent). Here, we illustrate the matrix using serving by oneself as an example:



influe	ability	degre	manta
ability	1	1	1/3
degre	1/1	1	1
manta	3	1/1	1

ability	serve_	winne	net_wi
serve_	1	5	7
winne	1/5	1	3
net_wi	1/7	1/3	1

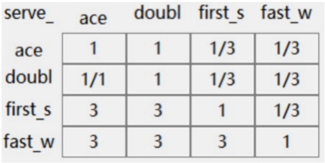
图 1: Comparison matrix for influencing factors and ability



degre	distan	unforc
distan	1	3
unforc	1/3	1

manta	scored	score_
scored	1	1/5
score_	5	1

图 2: Comparison matrix for degree of fatigue and mantality



serve_	ace	doubl	first_s	fast_w
ace	1	1	1/3	1/3
doubl	1/1	1	1/3	1/3
first_s	3	3	1	1/3
fast_w	3	3	3	1

图 3: Comparison matrix for serving

We obtain the weights for each component by calculating the maximum eigenvalue and normalizing its corresponding eigenvector. Certainly, for each matrix, we first need to test consistency using the Consistency Ratio (CR), where $CR = \frac{CI}{RI}$, $CI = \frac{\lambda_{max}-n}{n-1}$, $RI = 0.0, 0.58, 0.9$ (for matrices of size $2, 3, 4$). The computed Consistency Ratios for the matrices are 0.076, 0.037, 0.0, 0.0, 0.046. Since they are all less than 0.1, it confirms the consistency of the matrices.

Therefore, the weights for our model are as follows:

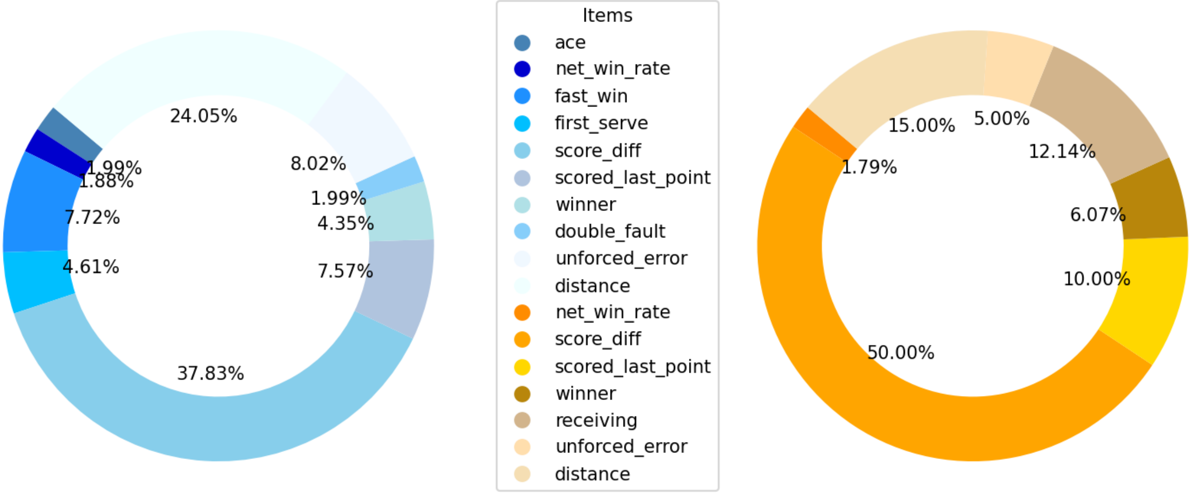


图 4: Weights in two different situations

Analyzing the various factors in the chart, it is evident that the most impactful factor is whether the previous point was scored. Following closely is the distance covered during the play, which aligns well with common intuition.

Thus, our final momentum is defined as:(need to change symbol)

$$momentum = \begin{cases} \sum\limits_{n=1,n\neq5}^{11} \omega_n x_n, & \text{if the player serves} \\ \sum\limits_{n=5}^{11} \omega_n x_n, & \text{if the opponent serves} \end{cases}$$

Where $\omega_n (n = 1, \ldots, 11)$ represent the weight of the factors, which is listed in Figure 4.

Now, we illustrate the graph of the "momentum" in the first match:

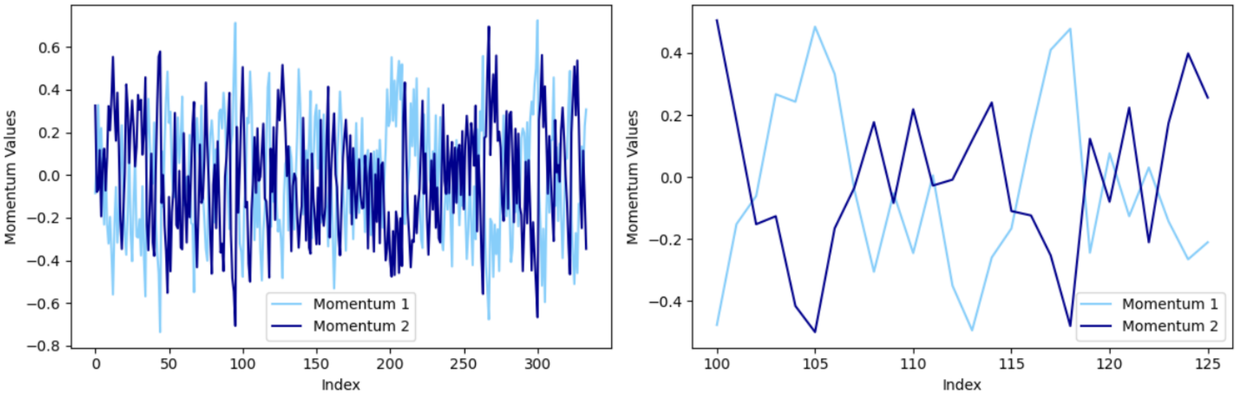


图 5: Momentum change in the first competition(global and local)

It can be observed that the variation in "momentum" is a process of give and take.

Third Problem model overview In this section, we will primarily focus on establishing a model using the Gated Recurrent Unit(GRU) algorithm to address problem 3 . We will break down the problems into five parts:

1. definition of the swing of the play
2. Gated Recurrent Unit(GRU) algorithm
3. Permutation Feature Importance theory
4. code
5. second part analysis

1.5 definition of the swing of the play

We first give the definition of the swings of the play. Based on our previous definition of "momentum", the significant changes of the game largely depend on the "momentum" of the two players. Therefore, we choose "momentum" to represent the swings of the play. The specific definition is as follows:

We use $\Delta f(t)$ to represent difference in "momentum" between the two players., so it can be easily seen that if $\Delta f(t)$ and $\Delta f(t+1)$ has different signs, it indicates a "swing" in the game's momentum. In this way, we can define four states of "momentum" at time t:

$$states = \begin{cases} state1, & \text{if } \Delta f(t) < 0 \text{ and } \Delta f(t+1) > 0, \text{ which means rise from negative to positive} \\ state2, & \text{if } \Delta f(t) > 0 \text{ and } \Delta f(t+1) > 0, \text{ which means stay positive} \\ state3, & \text{if } \Delta f(t) < 0 \text{ and } \Delta f(t+1) < 0, \text{ which means stay negative} \\ state4, & \text{if } \Delta f(t) < 0 \text{ and } \Delta f(t+1) > 0, \text{ which means decrease from positive to negative} \end{cases}$$

（这里可以画一张图展示四种变化）Obviously, if state 1 or state 4 appears, we can determine that a swing has occurred.

1.6 Indroduction of Gated Recurrent Unit(GRU) algorithm

GRU (Gated Recurrent Unit) is a type of recurrent neural network (RNN). It is similar to the LSTM (Long Short-Term Memory). But compared to LSTM, GRU is more effective and easier to train.

We will focus on a single unit of recurrent neural network to interpret the hidden mathematical principles:

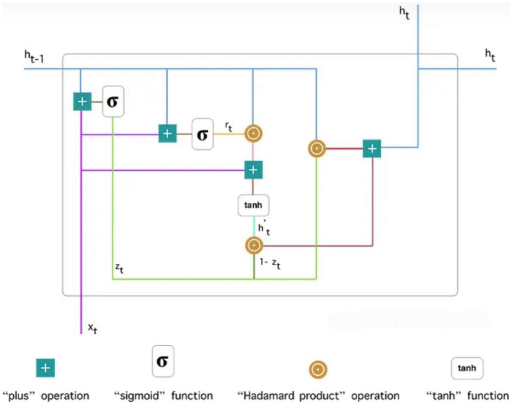


图 6: Structure of GRU

1. The update gate at time step t is computed using the following formula:

$$z_t = \sigma(W_z \cdot x_t + U_z \cdot h_{t-1})$$

   Here, when x_t is input to the network unit, it is multiplied by its own weight W_z. Similarly, h_{t-1}, which holds the information from the previous $t-1$ units, is multiplied by its own weight U_z. These two results are then summed together and passed through a sigmoid activation function to compress the result between 0 and 1.

2. The essence of the reset gate is for the model to decide how much past information to forget. To compute it, we use:

$$r_t = \sigma(W_r \cdot x_t + U_r \cdot h_{t-1})$$

3. The computation of the new memory content using the reset gate is as follows:

$$h'_t = tanh(Wx_t + r_t \odot Uh_{t-1})$$

4. The computation for the new memory content h_t using the update gate is as follows:

$$h_t = z_t \odot h_{t-1} + (1 - z_t) \odot h'_t$$

1.7 Permutation Feature Importance theory

We measure the importance of a feature by calculating the increase in the model’s prediction error after permuting the feature. A feature is “important” if shuffling its values increases the model error, because in this case the model relied on the feature for the prediction. A feature is "unimportant" if shuffling its values leaves the model error unchanged, because in this case the model ignored the feature for the prediction.

Algorithm 1: Permutation Feature Importance

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Input: Trained model $\hat{f}$, feature matrix $X$, target vector $y$, error measure $L(y, \hat{f})$.
Estimate the original model error $e_{orig} = L(y, \hat{f}(X))$ (e.g. mean squared error)
For each feature $j \in \{1, ..., p\}$ do:
    ○ Generate feature matrix $X_{perm}$ by permuting feature $j$ in the data $X$. This breaks the association between feature $j$ and true outcome $y$.
    ○ Estimate error $e_{perm} = L(Y, \hat{f}(X_{perm}))$ based on the predictions of the permuted data.
    ○ Calculate permutation feature importance as quotient $FI_j = e_{perm}/e_{orig}$ or difference $FI_j = e_{perm} - e_{orig}$
Sort features by descending FI.
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Fourth Problem In the previous section, we have only used the data of the first three matches. In this section we will focus on the generlization of the model.

1.8 test on all given data

We use our model in problem 3 in predicting all matches to verify its accuracy. Considering the difference length of the matches, we use weighted accuracy to denote the generability of our model. The fomula is as follows:

$$P_{avg} = \frac{1}{L_{tot}} \sum_{matches} \frac{P_i}{L_i}$$

where L_i stands for the length of the match, we measure it by point number.

So the general accuracy of our model in predicting matches is //////,
following are some factors that are not included in our model, all of which we think may influence the result, some of them may be hard to quantify, some of them are not included due to lack of data. Adding them to the model can make it more complete, this is left for future work.

1. The change of the players' strategies, during the course of the game, they may become more familiar with the opponent's technical characteristics and make targeted changes to shift ”momentum”.
2. The possibility that the players intentionally hide their strength, this is similar to the previous item.
3. The incentive of the audience, this is more likely to have something to do with the difference of home and away, just like football. The influence of the audience on foreign and native players is totally different.

As for women's tennis, it is a pity that we did not find enough data online, but from all men's match predictions, the accuracy rate is stable, so we have every reason to believe that our data can also apply to women's matches. For table tennis,

letter

dear coach:

We hope this letter finds you well. We are , a team with a keen interest in the training and development of tennis athletes. We would like to share some discoveries of ”momentum” during the match that We believe could be beneficial for athlete training and play spot, using mathematical modeling as an analytical tool. After data analysis and modeling. The following are syome suggestions based on our analysis:

First, we use

We hope these suggestions are helpful for your coaching endeavors. If you have any questions or would like to discuss this further and learn more detailed information, you can read the full text of our model.Also, we would be more than willing to discuss additional thoughts and insights with you.

summary

(一段) As for problem 1, in order to accurately capture the effective information in the data set, we take a series of data processing methods... After that, we select potential influencing factors and establish Analytic Hierarchy Process (AHP) model to make a preliminary analysis of the influencing factors and calculate specific parameters for each factor. We find that the biggest factor is score difference. Apart from that ,whether scored in the last point, running distance, unforced error and fast win will also positively or negatively influence the momentum to a comparatively large extent. The above conclusion is verified in our later models.

In problem 2, we believe that momentum will affect the future scores of the match. So to answer the coach's question, we first perform autocorrelation test on momentum, and we find that it has strong first-oredr autocorrelation. Then we quantify the future scores and then determine the correlation of momentum the scores in the future, they are highly correlated.

Problem 3 is devided into 3 parts. To predict the swings in the match , we establish model based on Gated Recurrent Unit(GRU) algorithm. We process data in problem 1 and add more features. We define the swings and use previous information to predict future. Then compared to the result in problem 1, our accuracy is ....

To identify the most related factors, we use a novel argorithm called Permutation Feature Importance argorithm, it can determine the importance of features by calculating their prediction error after permutation. We find that ....

As for the ideas for specific atheletes, we use previous model on their matches to identify their feature importance. We find that different atheletes have different features in their match.

Problem 4

Key words: AHP, correlation analysis, GRU, Permutation Feature Importance

Indroduction

Tennis more than any other sport, is a game of momentum. The absence of a clock to do the dirty work of finishing off an opponent, and a scoring system basedon units used, makes the flow of the match much more important than any lead that has been established.–Chuck Kriese

problem3

Using the above argorithm, we sort FI and normalize them, then we plot the figure:

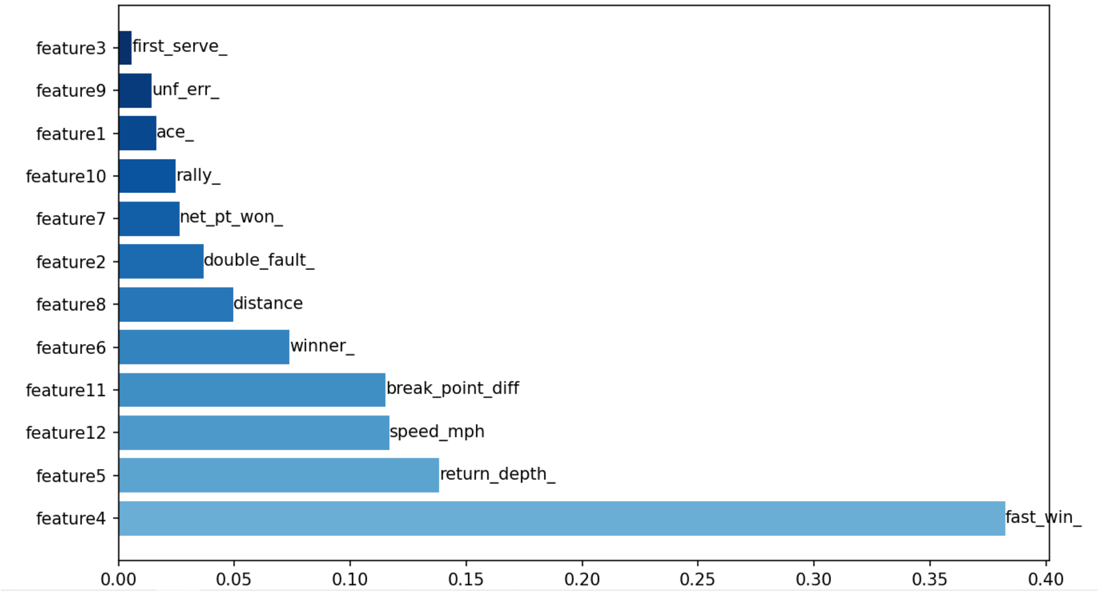

图 7: feature importance

We can see that the most important feature is fast win(rally count < 3 and wins the point), and the second most important feature is unforced error. Compared to problem 1(main skill features are running distance, unforced error and fast win),they coincide. The running distance become insignificant, we think it is because this factor is also related to fast win.

As for women's tennis, it is a pity that we did not find enough data online, but from all men's match predictions, the accuracy rate is stable, so we have every reason to believe that our data can also apply to women's matches.

For table tennis, technical statistics also includes:

1. more serve types, such asforehand, backhand, short, long, or sidespin serves.
2. more receiving techniques,such as forehand push, backhand push, flick, or topspin.

So we need to modify the influencing factors in our model, but the general framework is similar.

User what is general technical statistics in table tennis games ChatGPT In table tennis, general technical statistics refer to the analysis and measurement of various technical aspects of a player's performance during a match. These statistics provide valuable insights into a player's strengths, weaknesses, and overall playing style. Here are some general technical statistics commonly used in table tennis games:

Serve Statistics:

Serve Type: Recording the types of serves a player employs, such as forehand, backhand, short, long, or sidespin serves. Serve Effectiveness: Analyzing the success rate of different types of serves, including direct points and opponent errors. Receiving Statistics:

Receiving Techniques: Documenting how a player receives various types of serves, such as forehand push, backhand push, flick, or topspin. Receiving Success Rate: Calculating the player's success rate in returning different types of serves. Stroke Execution Statistics:

Forehand and Backhand Shots: Analyzing the usage and effectiveness of forehand and backhand strokes in various situations. Types of Strokes: Recording the diversity of strokes played, including topspin, backspin, drives, and smashes. Rally Statistics:

Rally Length: Measuring the average length of rallies in terms of the number of strokes exchanged. Rally Outcome: Tracking the outcomes of rallies, such as points won or lost, and identifying patterns in rally progression. Placement and Direction Statistics:

Ball Placement: Analyzing where a player tends to place the ball on the opponent's side of the table. Direction of Shots: Recording the direction of shots, including cross-court, down the line, and shots to the opponent's body. Point Distribution Statistics:

Point Sources: Identifying the sources of points, such as winners, opponent errors, or forced errors. Point Distribution: Analyzing how points are distributed throughout the match, helping to identify key moments. Error Analysis:

Unforced Errors: Documenting the number of unforced errors made by a player. Forced Errors: Identifying instances where the opponent's play led to errors by the player.

These technical statistics are crucial for players, coaches, and analysts to assess performance, strategize for future matches, and tailor training programs to address specific areas of improvement. They provide a comprehensive overview of a player's game and contribute to a more informed and strategic approach to table tennis.