Operations Research Applications Implementations, Fall 2022 (111-1) HW1

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1 (3.4-17.) Food Choices

1.1 Defining Variables

Since preparing the minimum-cost meal for 1 child is the same as for all children, only 6 variables are needed. Let I denote the set of food items. Let x_i be the amount of ingredient corresponding to the 6 food items in order listed in the problem description (x_i for bread, x_2 for peanut butter, ...x_6 for juice). Let the table column names denote the vectors to be indexed in the below program. Note that for general purpose, the below program is presented in abstracted form instead of stats used in this particular instance. Also note that the problem description is not clear about whether x_i should be integer or real number, so I try with 2 settings and present both answers.

1.2 Linear Program Formulation

$$
\begin{aligned}
\max \quad & \sum_{i \in I} Cost_i \times x_i \\
\text{s.t.} \quad & Cf = \sum_{i \in I} x_i \times CaloriesFromFat_i \\
& Ct = \sum_{i \in I} x_i \times TotalCalories_i \\
& 400 \leq Ct \leq 600 \\
& Cf \leq 0.3 C_t \\
& \sum_{i \in I} x_i \times Vitamin_i \geq 60 \\
& \sum_{i \in I} x_i \times Protein_i \geq 12 \\
& x_2 = 2 \\
& x_2 \geq 2x_3 \\
& x_5 + x_6 \geq 1 \\
& x_i \geq 0, x_i \in \mathbb{R}, \forall i \in I \\
& (\text{or} \quad x_i \geq 0, x_i \in \mathbb{Z}, \forall i \in I)
\end{aligned}
$$

1.3 Code & Solution to this Instance

1.3.1 Continuous Setting

This instance is solved by constructing a program in *Gurobi* by setting up the variables (with sign constraints), the objective function (with the optimization direction specified), and the many constraints (please refer to the code notebook). The amount of each food item s.t. the cost is cut down to the minimum is:

$$
\begin{aligned}
x_1 &= 2 && \text{// bread (slice)} \\
x_2 &= 0.575 && \text{// peanut butter (tbsp)} \\
x_3 &= 0.287 && \text{// jelly (tbsp)} \\
x_4 &= 1.039 && \text{// cracker (piece)} \\
x_5 &= 0.516 && \text{// milk (cup)} \\
x_6 &= 0.484. && \text{// juice (cup)}
\end{aligned}
$$

We can see that milk and juice are balanced out in the minimum requirement of 1 cup, since they respectively provide most protein and most vitamin C in proportion. Peanut butter provides a lot of fat in 1 tbsp, therefore its amount is leveraged down, but not too low because it is asked to be at least twice as much as jelly. Overall, each child's lunch costs only **47.31¢** provided that all the constraints are met.

1.3.2 Discrete (integer) Setting

$$
\begin{aligned}
x_1 &= 2 \quad \text{// bread (slice)}\\
x_2 &= 1 \quad \text{// peanut butter (tbsp)}\\
x_3 &= 0 \quad \text{// jelly (tbsp)}\\
x_4 &= 1 \quad \text{// cracker (piece)}\\
x_5 &= 0 \quad \text{// milk (cup)}\\
x_6 &= 1. \quad \text{// juice (cup)}
\end{aligned}
$$

Overall, each child's lunch costs only **57¢** provided that all the constraints are met.

2 (16.5-10.) Unit Operationality

Let the 4 states in order be $(U, V) = (2, 0), (1, 0), (0, 1), (1, 1)$.

i. Markov property

With reference to this document[1], a stochastic process is said to have the markov property if the condition probability distribution of future states of the process depends only upon the present state, not on the sequence of events preceding it.

ii. Probability of being inoperable after 2, 5, 10, 20 periods

The problem does not state the initial probability, so I use $M = [1, 0, 0, 0]$ for initial probability of the 4 states (i.e. at first both components are operational). With P as the transition matrix (row: present, col: future), $P^{n-1}M$ is the transitional probability for the 4 states after n periods. The probabilities are 0, 0.04, 0.038, 0.038 respectively.

iii. Steady-state probabilities

The probabilities for the 4 states are 0.615, 0.192, 0.038, and 0.154 respectively.

[1] https://www.igi-global.com/dictionary/markovian-reliability-in-multiple-agv-system/39939

iv. Long-run average cost

The only condition we need to pay the cost is when both components are down for repair ($U = 0$, i.e. state 3). Therefore, the expected long-run cost is $0.038 * 30000 = 1153.85$.