Operations Research Applications

Assignment 4

Due Date: Dec. 19, 2022, 5pm

Please zip your files, including MS Word, Excel, code files or others, with the file name: **ORA_Assignment4_ID_NAME.zip**, and upload your homework to **NCKU MOODLE** by due.

Questions (100%)

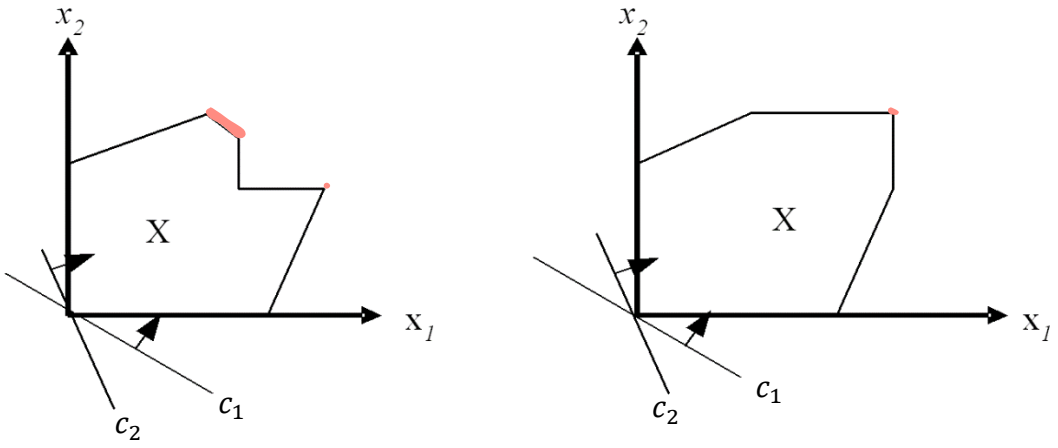
Please answer following questions and justify your answer. Show all your works in details.

1. **(30%)** Consider a production company who owns 3 plants to produce 2 consumer goods. If the available capacity and its uses are given in the following table, try to find an Efficient Set of product mixes if maximal profit and minimal pollution penalty are considered by using (Wang, 2004)
   (a) (10%) Graphical solution by the "Concept of Dominance";
   (b) (10%) Weighting method with weights 2:1 w.r.t. objectives of profit and penalty;
   (c) (10%) ε −Constraint Method with r = 2.

Plant \ Product	Capacity used per unit production rate		Capacity Available
	1	2	
1	1	0	4
2	0	2	12
3	3	2	18
Unit Profit	\$3	\$5	
Pollution Unit Penalty	\$2	\$4	

Ref: Wang, H.F., Multicriteria Decision Analysis– From Certainty to Uncertainty, ISBN 986-7777-55-7, Ting Lung Book Co., 2004.

2. **(20%)** Discuss the efficient sets from the following graph of two criteria: (Wang, 2004)



3. **(35%)** Data Envelopment Analysis (DEA)

   Please use “input-oriented” DEA approach to assess the efficiency of 41 Dept. in NCKU. There are three inputs (Personnel, Expenses, and Space) and three outputs (Teaching, Publications, and Grants) for consideration. The dataset is shown as follows. Use optimization solver to solve the example. (Kao and Hung, 2008)

School	Department	Personnel	Expenses (unit:1000)	Space	Teaching	Publications	Grants (unit:1000)
Liberal Arts							
1	Chinese	33.5	6079.5	5376	26131	5	4250.9
2	Foreign Languages	36.5	8387.4	4574	40105	0	1543.5
3	History	26	5104.8	3795	13678	0	1729.1
4	Arts	6	3420.6	1610	4926	0	3562.2
Sciences							
5	Mathematics	29.5	6426.2	4175	23647	27	10518.6
6	Physics	44.8	9360.1	8184	27613	66	60912.4
7	Chemistry	32.3	10099.4	9992	23419	49	36191.4
8	Earth Sciences	16	5875.8	4128	7700	12	45590.3
9	Biology	18.5	7136.1	5610	11981	13.5	19169.8
10	Biotechnology	5	3123.2	1168	1710	6.5	14465.4
Engineering							
11	Mechanical Eng.	56.3	14364.5	27562	38611	88	77659.7
12	Electrical Eng.	56.8	17607.4	25770	46582	161	160768.2
13	Information Eng.	16.5	7152.2	5419	11597	50	41388.9
14	Chemical Eng.	46	11161.6	27410	28664	120	58183.2
15	Resources Eng.	23	6854.5	5221	11359	12.5	30911.3
16	Materials Eng.	26	8975.2	6326	16284	95.3	101281.4
17	Civil Eng.	35	10955.1	15774	26230	29.5	52691.4
18	Hydraulic Eng.	27.5	8506.4	9435	13059	17.5	165046.9
19	Architecture	28.5	12093.7	9930	21872	4.5	44909.5
20	Engineering Science	27	11358.3	9049	18907	28.5	43943.7
21	Naval Architecture	18.5	6468.3	11804	9246	28.5	32156.8
22	Urban Planning	17.5	7493.1	6478	11551	0	40404.7
23	Industrial Design	12.5	7438.8	4584	13296	1	6143.3
24	Aeronautical Eng.	57.5	8718.2	12550	18029	70	153755.8
25	Environmental Eng.	19	6593.6	3548	8609	35	91704.7
26	Surveying Eng.	17.5	5537.1	4495	8590	2.5	40462.3
27	Manufacturing Eng.	5.3	3191.9	1695	1648	12	19752.6
28	Biomedical Eng.	10	4490.1	3937	6263	37	28470.8
Management							
29	Industrial Mgmt.	26.3	7191	5346	17792	21.5	15481.7
30	Transportation Mgmt.	20.5	5142.5	4409	12150	10.5	16728.4
31	Business Admin.	25.8	7795.6	5815	26923	1.5	32889
32	Accountancy	15	4909.1	4085	16391	2	2703.6
33	Statistics	20.8	4180.5	4501	12525	4.5	7123.8
Medicine							
34	Medicine	206.3	40418.3	29222	52817	413.5	303271
35	Nursing	17	5212.2	2175	6500	5.5	5399.1
36	Medical Technology	12.5	3892.9	2937	3668	35.5	17710.3
37	Occupational Therapy	9.5	3107.8	1521	4299	3.5	1195.8
38	Physical Therapy	11	2344.1	1515	4293	6	5102.3
Social Sciences							
39	Political Economy	19	3950.7	1682	16964	0	1322.2
40	Education	9	1825.3	1221	4857	6.5	9597.4
41	Law	7.5	2404	1016	5976	0	1309.1

(a) (5%) Define the decision variables and LP formulation

(b) (10%) Show the results of overall efficiency (OE, CRS model), technical efficiency (TE, VRS model), scale efficiency (SE)

(c) (5%) i) Which dept. does show a better performance? Why?
(5%) ii) Which dept. does show a poor performance? Why? Any suggestion to improve productivity?

(d) (5%) i) Which school does show a better average performance? Why?
(5%) ii) Which school does show a poor average performance? Why? Any suggestion to improve productivity?

Ref: Kao, C. and H.-T. Hung. 2008. Efficiency analysis of university departments: An empirical study. Omega 36, 653-664.

4. **(15%)** Problem 17.4. in Russell and Norvig (2010)
For this problem, please refer to the textbook. Russell, S. and Norvig, P. (2010). Artificial intelligence: A Modern Approach. 3rd edition, Prentice Hall, Pearson Education, Inc.
You may google it and download the book from…
https://zoo.cs.yale.edu/classes/cs470/materials/aima2010.pdf

Sometimes MDPs are formulated with a reward function R(s, a) that depends on the action taken or with a reward function R(s, a, s') that also depends on the outcome state.
   i. Write the Bellman equations for these formulations.
   ii. Show how an MDP with reward function R(s, a, s') can be transformed into a different MDP with reward function R(s, a), such that optimal policies in the new MDP correspond exactly to optimal policies in the original MDP.
   iii. Now do the same to convert MDPs with R(s, a) into MDPs with R(s).

Note

1. Show all your work in detail. **Innovative** idea is encouraged.
2. If your answer refers to any external source, please "must" give an academic citation. Any "plagiarism" is not allowed.