Operations Research Applications HW2, Fall 2022 (111-1)

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Problem 1: Stochastic Programming (45%)

(a) Defining variables and Formulation

Let x_1 be the number of acres to plant wheat; x_2 be the number of acres to plant corn; x_3 be the number of acres to plant sugar beet. w_1 be the sold wheat in tons; w_2 be the sold corn in tons. w_3 be the sold sugar beets in tons at \$36/ton; w_4 be the sold sugar beets in tons at \$10/ton. y_1 be the purchased wheat in tons; y_2 be the purchased corn in tons.

$$
\begin{aligned}
\max \quad & (170w_1 - 150x_1 - 238y_1) + (150w_2 - 230x_2 - 210y_2) + (36w_3 + 10w_4 - 260x_3) \\
\text{s.t.} \quad & x_1 + x_2 + x_3 \leq 500 \\
& 2.5x_1 - y_1 - w_1 \geq 200 \\
& 3x_2 - y_2 - w_2 \geq 240 \\
& w_3 \leq 6000 \\
& 20x_3 - w_3 - w_4 \geq 0 \\
& x_i \geq 0 \quad \forall i = 1, 2, 3 \\
& w_i \geq 0 \quad \forall i = 1, ..., 4 \\
& y_i \geq 0 \quad \forall i = 1, 2
\end{aligned}
$$

(b) Solve the Scenario Analysis and EV solution

Scenario Analysis

(1) Low: We grow 100 acres of wheat, 25 acres of corn, 375 acres of sugar beets. Objective value: \$ 59,950.

(2) Medium: We grow 120 acres of wheat, 80 acres of corn, 300 acres of sugar beets. Objective value: \$118,600.
(3) High: We grow 183.33 acres of wheat, 66,6 acres of corn, 250 acres of sugar beets. Objective value: \$167,666,66.
The weighted mean of the objective value over the 3 scenarios is \$115,405.5.

EV Solution

is the medium solution in Scenario Analysis/Wait-n-see, because the probability of each of the 3 scenarios are equal. We grow 120 acres of wheat, 80 acres of corn, 300 acres of sugar beets. Objective value: \$118,600.

(c) Define new decision variables for RP-DEP.
Let x_i be the number of acres to plant the crop defined in (a). w_i^ω be the sold crop defined in (a) under scenario $\omega \in \Omega$. Similarly, y_i^ω be the purchased wheat and corn in tons under scenario $\omega \in \Omega$. The formulation aims to maximize the total profit.

$$
\begin{aligned}
\max \quad & -(150x_1 + 230x_2 + 260x_3) + \Sigma_{\omega\in\Omega} 170w_1^\omega - 238y_1^\omega + 150w_2^\omega - 210y_2^\omega + 36w_3^\omega + 10w_4^\omega \\
\text{s.t.} \quad & x_1 + x_2 + x_3 \leq 500 \\
& d_1^\omega x_1 - y_1^\omega - w_1^\omega \geq 200 \quad \omega \in \Omega \\
& d_2^\omega x_2 - y_2^\omega - w_2^\omega \geq 240 \quad \omega \in \Omega \\
& w_3^\omega \leq 6000 \quad \omega \in \Omega \\
& d_3^\omega x_3 - w_3^\omega - w_4^\omega \geq 0 \quad \omega \in \Omega \\
& x_i \geq 0 \quad \forall i = 1,2,3 \quad \omega \in \Omega \\
& w_i^\omega \geq 0 \quad \forall i = 1,...,4 \quad \omega \in \Omega \\
& y_i^\omega \geq 0 \quad \forall i = 1,2 \quad \omega \in \Omega
\end{aligned}
$$

(d) Solve RP.
The RP optimal value is \$108,390, and for this we grow 170 acres of wheat, 80 acres of corn and 250 acres of sugar beets.

(e) Calculate the EVPI and VSS.
$EVPI = RPObj - WSObj = 7015.6$

$VSS = RPObj - EEV = 5500$

EEV means the objective value by plugging into RP objective function the optimal X ($x_i \forall i = 1, 2, 3$) obtained in EV solution, leaving other stochastic variables (y_i, w_i) optimal in terms of RP.

(f) Do you think RP providing a good solution in this study? Why?

Yes. Although $EVPI$ shows that RP is still not good enough because it has a $7015 gap from the value of perfect information, it could be observed that VSS is 5500, meaning the profit value of stochastic solution is $5500; ignoring the randomness/uncertainty and using EV solution instead is likely to lead to a loss of $5500, which proves that RP is still valuable in this study.

(g) For continuous scenarios, calculate the objective function's CI of lower bound and upper bound.

For the lower bound, we do $N = 30$ sampling of scenarios and optimize once, and then repeat the sampling + optimization process for $M = 15$ times, the resulting 15 optimal values $\hat{z}_N^j$ are: [1096275.45, 1214004.26, 1135037.49, 1108265.08, 1166365.27, 1121675.29, 1166806.31, 1113645.22, 1133085.4, 1099287.67, 1122861.44, 1155390.26, 1112180.73, 1129131.47, 1121738.34]. And we save the sampled scenarios (the $N * M = 450$ sampled yield rates) and the $M = 15$ optimal solutions $(x_N^*)^j \forall j \in 1...M$ for the upper bound calculation. For the upper bound, since $N = \overline{N} = 30$ and $M = T = 15$, I do not do optimization and only plug in the results saved in lower-bound part. For each batch j within M, we have a $(x_N^*)^j$ and a set of 30 sampled yield rates; pairing the $(x_N^*)^j$ with 1 sampled yield rate give us 1 optimal value (see `XDgetObj()` in the program, where `X` corresponds to $(x_N^*)^j$ and `D` corresponds to the tons per acre stats given the sampled yield-rate). For each batch, 30 optimal values are generated and averaged as $\hat{z}_{\bar{N}}^t$; and the resulting 15 optimal values $\hat{z}_{\bar{N}}^t$ are: [125260.72, 147009.66, 131054.53, 130200.1, 138258.61, 129572.14, 138057.32, 128708.2, 130803.12, 126935.4, 132856.75, 137515.51, 129252.96, 130671.51, 129591.84]. By now we get 2 distributions for each bound. Calculate its mean and sample variance gives us the confidence intervals for both bounds.

Lower Bound CI:[1133049.98 - 15773.73,1133049.98 + 15773.73]

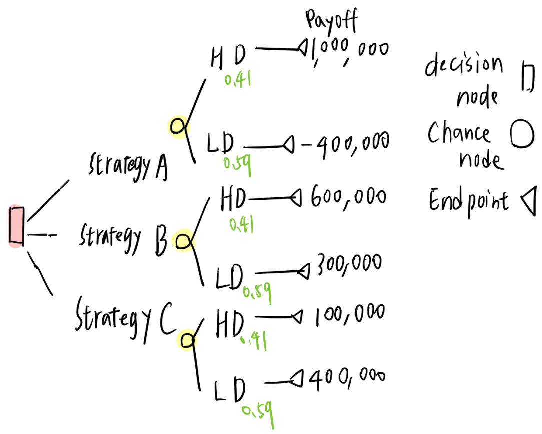
Upper Bound CI:[132383.23 - 2832.50,132383.23 + 2832.50]

Problem 2: Decision Analysis and Value of Information (55%)

(a) **1** decision node; **3** branches.

(b) **3** chance nodes; **2** branches (low/high demand).

(c) Draw the decision tree.



(d) Solve the decision tree.

Strategy A: $P(\theta = 1) \times P(A, \theta = 1) + P(\theta = 0) \times P(A, \theta = 0) = 0.41 \times 1,000,000 + 0.59 \times (-400,000) = 174,000$

Strategy B: $0.41 \times 600,000 + 0.59 \times 300,000 = 423,000$

Strategy C: $0.41 \times 100,000 + 0.59 \times 400,000 = 277,000$

$\therefore$ Strategy B is the best production strategy given no extra information.

(e) Calculate the joint probabilities.

$$P(X = 1, \theta = 1) = P(\theta = 1) \times P(X = 1|\theta = 1) = 0.328$$
$$P(X = 0, \theta = 1) = P(\theta = 1) \times P(X = 0|\theta = 1) = 0.082$$
$$P(X = 0, \theta = 0) = 0.413$$
$$P(X = 1, \theta = 0) = 0.177$$

(f) Calculate the marginal probabilities.

$$P(X = 0) = \sum_{i=1}^{2} P(X = 0|\theta = i)(\theta = i) = 0.2 \times 0.41 + 0.7 \times 0.59 = 0.495$$
$$P(X = 1) = \sum_{i=1}^{2} P(X = 1|\theta = i)(\theta = i) = 0.8 \times 0.41 + 0.3 \times 0.59 = 0.505$$

(g) Calculate the posterior probabilities.

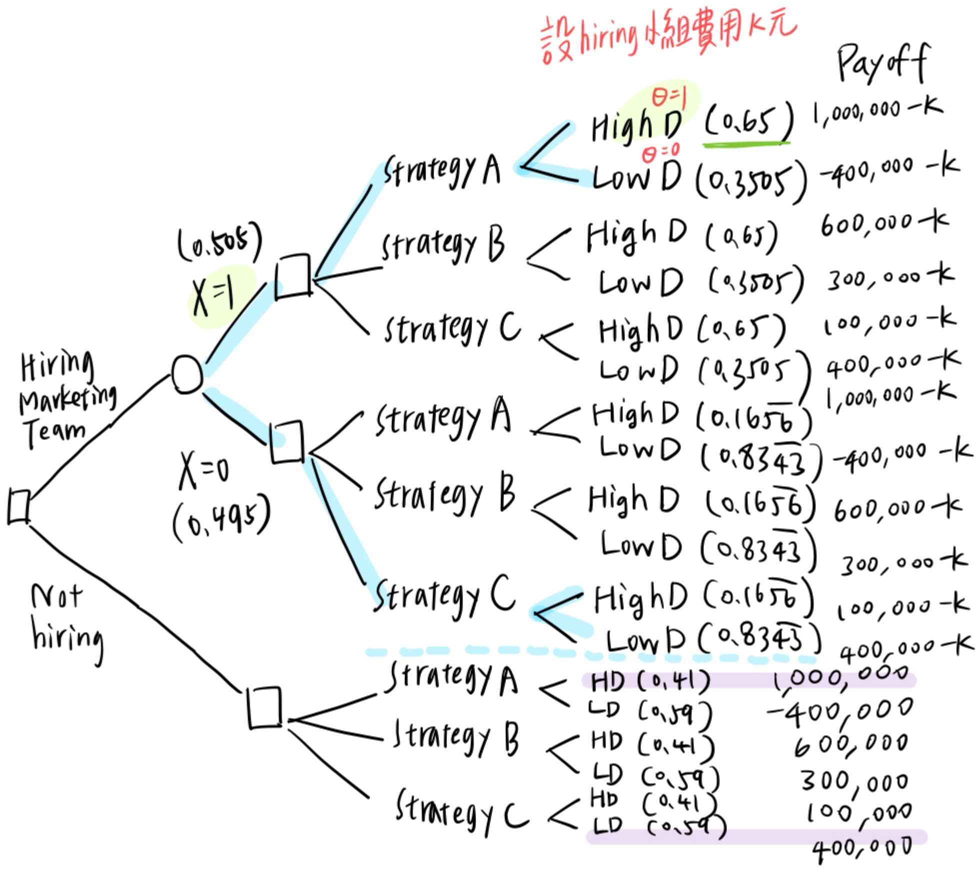
$$P(\theta = 0|X = 0) = P(\theta = 0, X = 0)/P(X = 0) = \frac{0.413}{0.495} = 0.83\overline{43}$$

$$P(\theta = 1|X = 0) = 0.16\overline{56}$$

$$P(\theta = 0|X = 1) \approx 0.3505$$

$$P(\theta = 1|X = 1) \approx 0.65$$

(h) Redraw the revised decision tree.



(i) Calculate the EVPI (Expected Value of Perfect Information).

$$ERofstratA : 1,000,000 \times 0.41 + (-400,000) \times 0.59 = 174,000$$
$$ERofstratB : 600,000 \times 0.41 + 300,000 \times 0.59 = 423,000$$
$$ERofstratC : 100,000 \times 0.41 + 400,000 \times 0.59 = 277,000$$
$$ERw/o = \max\{ER_A, ER_B, ER_C\} = 423,000$$
$$ERPI = 1,000,000 \times 0.41 + 400,000 \times 0.59 = 646,000$$
$$EVPI = ERPI - ERw/o = 223,000$$

(j) Calculate the EVE when we directly use the marketing results with the assumption that if $X = 1$ we go for strategy A, otherwise strategy C. Note that although the revised decision tree in (h) marks the hiring fee K in payoffs, the below EVE does not take K into account.

$$EMV(StrategyA|X = 1) \approx 0.65 \times 1,000,000 + 0.3505 \times (-400,000) = 510,000$$
$$EMV(StrategyC|X = 0) \approx 0.1656 \times 1,000,000 + 0.8343 \times 400,000 = 348,560$$
$$\begin{aligned} ERE &= \Sigma_{x \in X} P(X = x) \times EMV(DecisionRule|X = x) \\ &= P(X = 1) \times EMV(StrategyA|X = 1) + P(X = 0) \times EMV(StrategyC|X = 0) = 430,087. \end{aligned}$$
$$EVE = ERPI - ERw/o = 430,087 - 423,000 = 7,087$$

(k) If the marketing team costs NTD 50,000, should we hire the team?
No, because $EVE = 7,087 < 50,000$.