Partial Fourier Transform for Accelerating MRI Scans

BENG 280A/ECE 207 Final Project

Benjia Zhang, Juo-Hsuan Chang, Iris Zaretzki

Outline

- Background
- Partial FT Methods
  - Zero-Padding
  - Phase Correction & Conjugate Symmetry
  - Homodyne
  - Projection onto Convex Sets
- Result Evaluation

Background

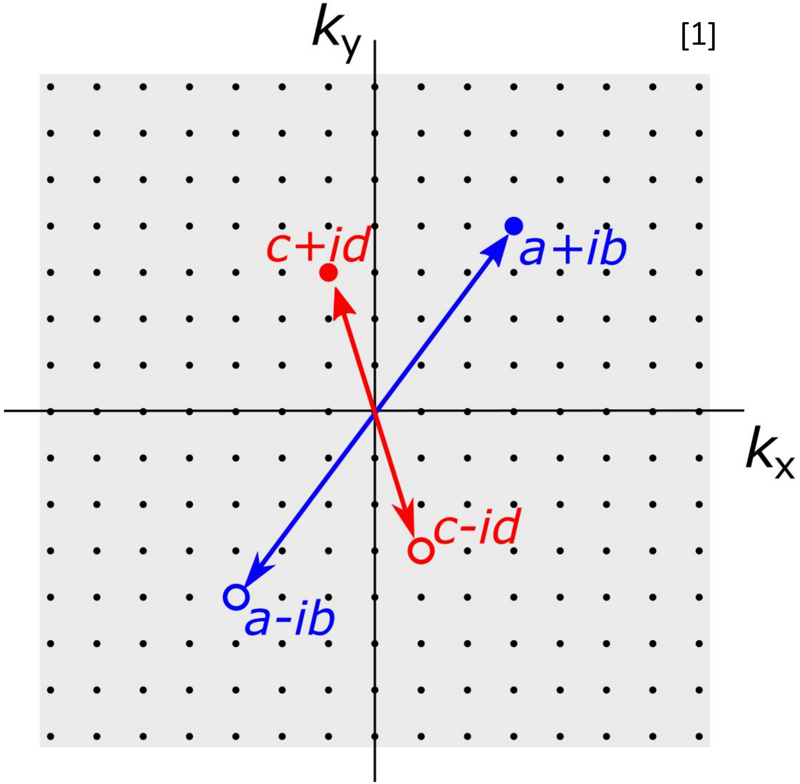
- MRI signals are real-valued, but when we take the FT we get complex values

- During acquisition, receiver coils measure

  Amplitude of M vector → “Real”

  Phase → “Complex”

- Real signals have conjugate symmetry in Fourier space

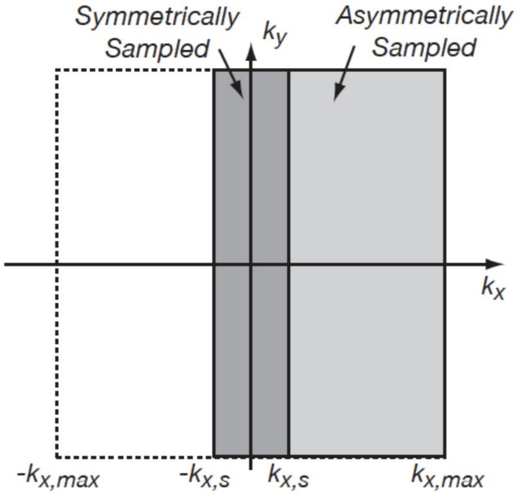


[1]

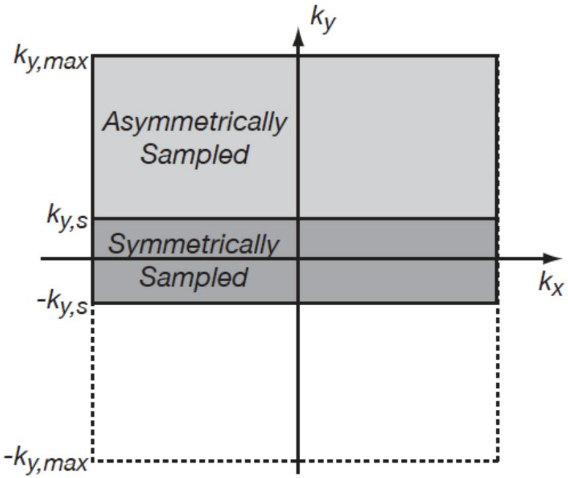
Partial Fourier Imaging

How can this conjugate symmetry be leveraged to improve MRI?

- Since the k-space data is redundant, we only *need* to collect half of it



[2]



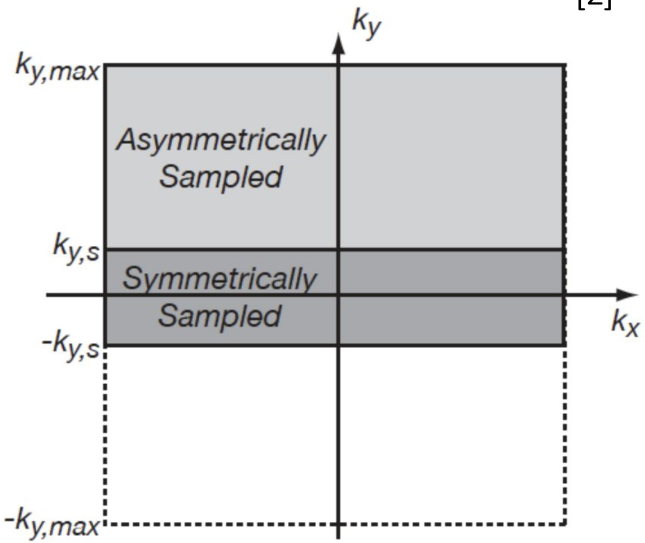
Partial Fourier Imaging

- In theory, only 50% of data is necessary
- Phase distort the symmetry
  - Patient motion
  - Distortions in B0 field
  - Magnetic susceptibility

- By collecting slightly more than half of the data, we can correct for these errors

- Benefits: reduced acquisition time and minimum TE

[2]



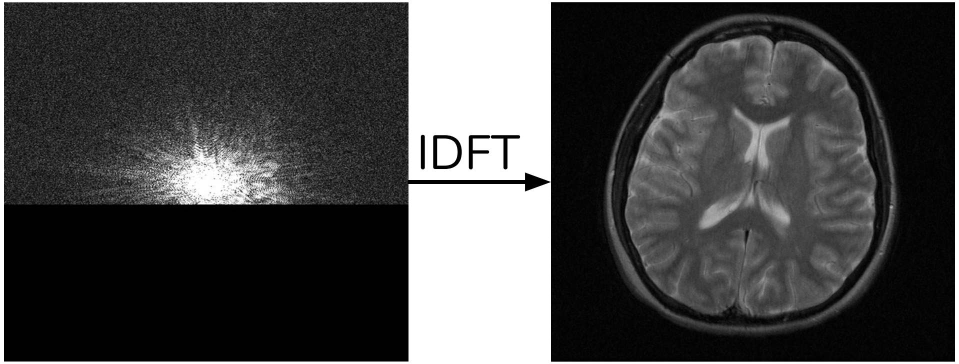
Next Steps

1. Investigate the theory behind different partial fourier reconstruction methods.
2. Simulate the methods on real MRI data.
3. Quantify the performance under different conditions (data sampled, noise)

Zero-Padding (ZP)

[3]

- Taking the Inverse Discrete Fourier Transform directly without filling in the missing k-space values.
- Requires the least computation



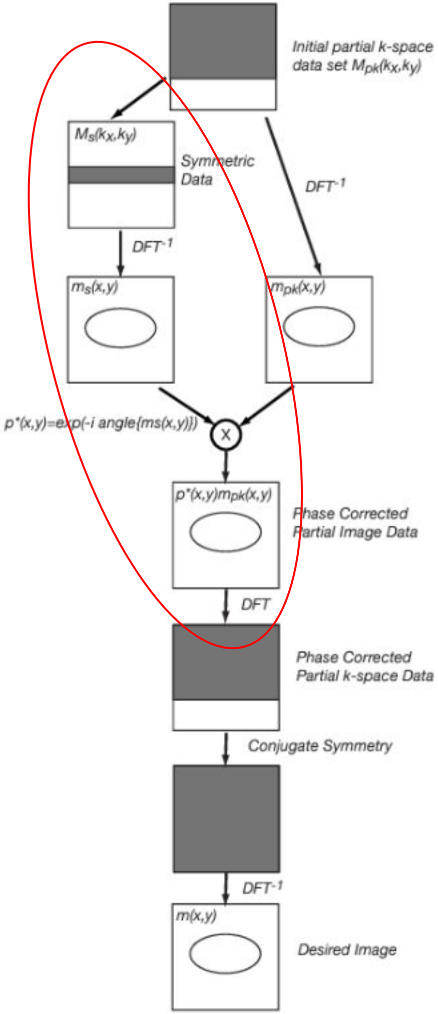
Conjugate Symmetry

- Takes advantage of the Hermitian matrix properties of Fourier Transforms of real objects

$$
\begin{bmatrix}
-1 & 1 - i & 1 + 2i \\
1 + i & 3 & -2 \\
1 - 2i & -2 & 0 \\
i & 3 + 2i & 4
\end{bmatrix}
$$

Phase Correction & Conjugate Symmetry (PCCS)

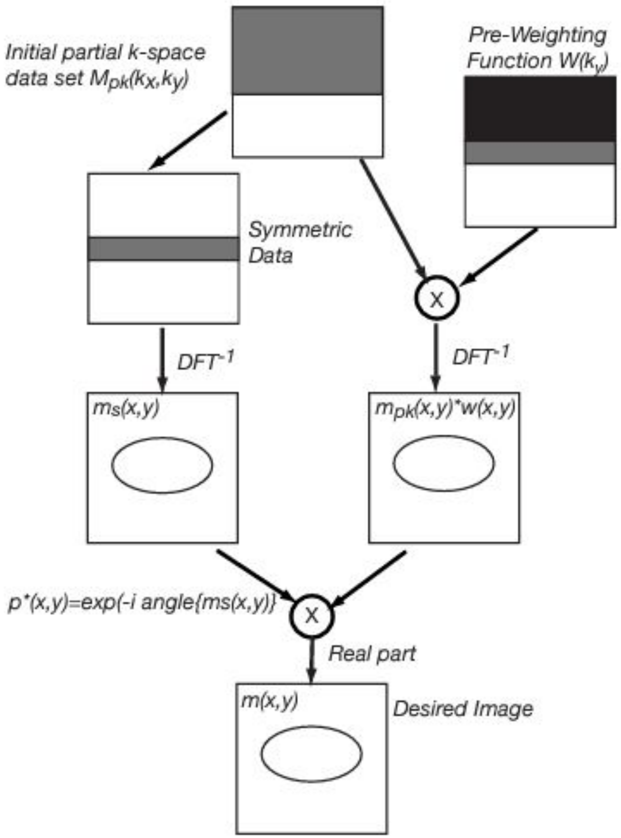
- The symmetric low frequency data points are used to estimate the phase as a picture is made up of mostly low frequency components



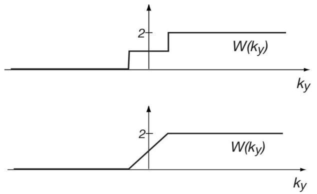
[3]

Homodyne (HOMO)

- A weighted function (high pass) is applied before being phase corrected
- Known high frequencies are scaled by 2 to emphasize the details in the image and maintain the lost energy in the bottom half
- Combined with the phase correction with the low pass filter, the image quality can be improved.

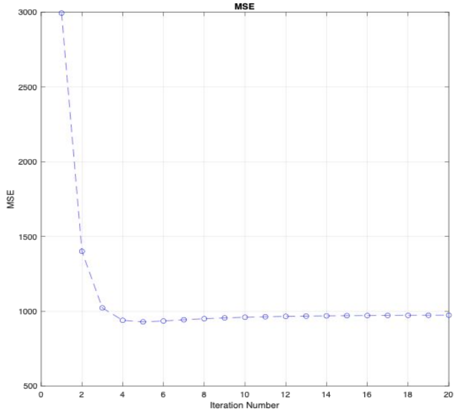


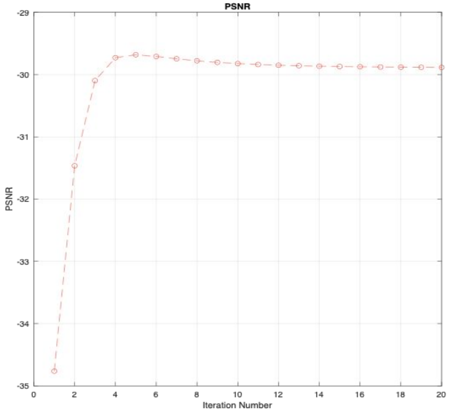
[3]

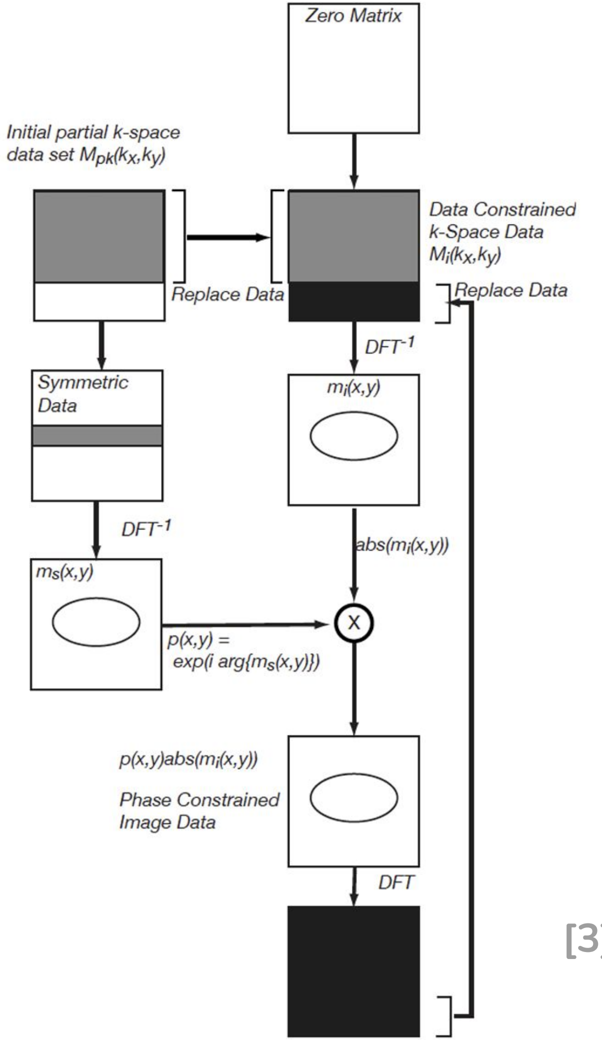


Projection onto Convex Sets (POCS)

- Iterative process of phase correction
- The phase corrected image's DFT can be used in the next iteration to fill in the empty k-space



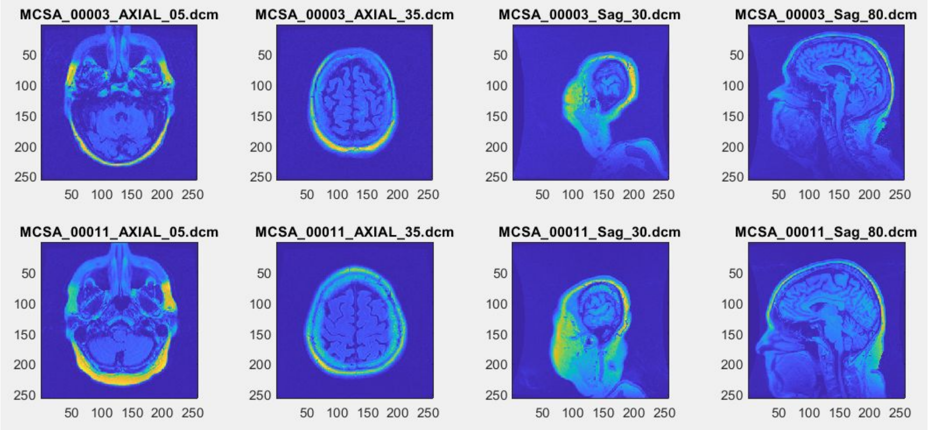




[3], [4]

Result Evaluation

Input data: 8 dicom images from **Mayo Clinic Study of Aging Dataset** [5]

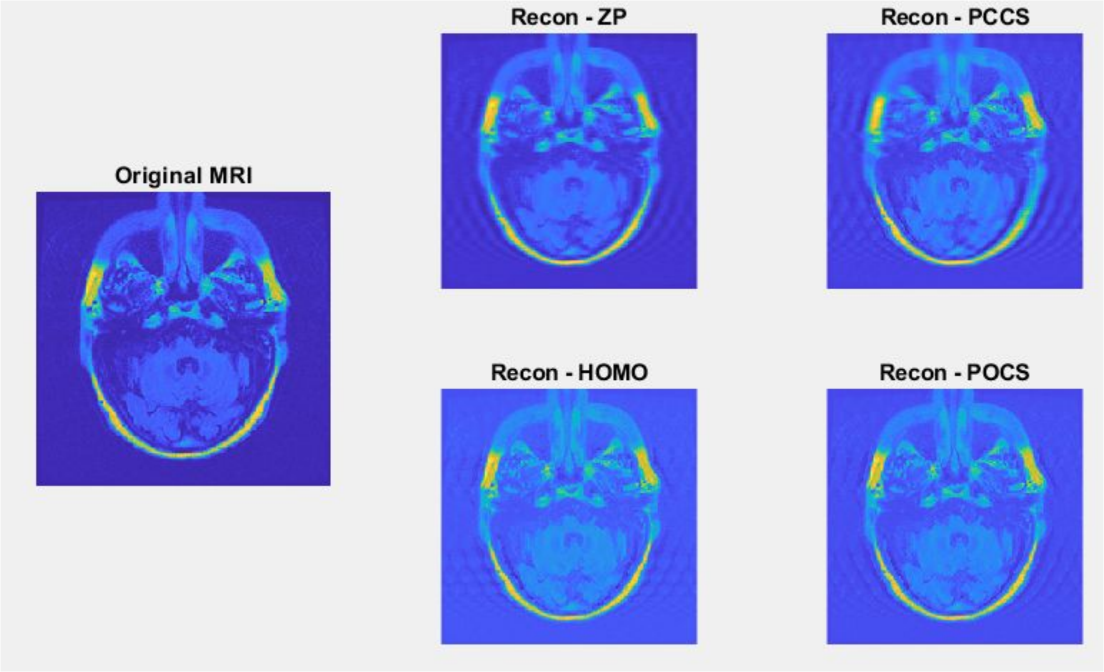


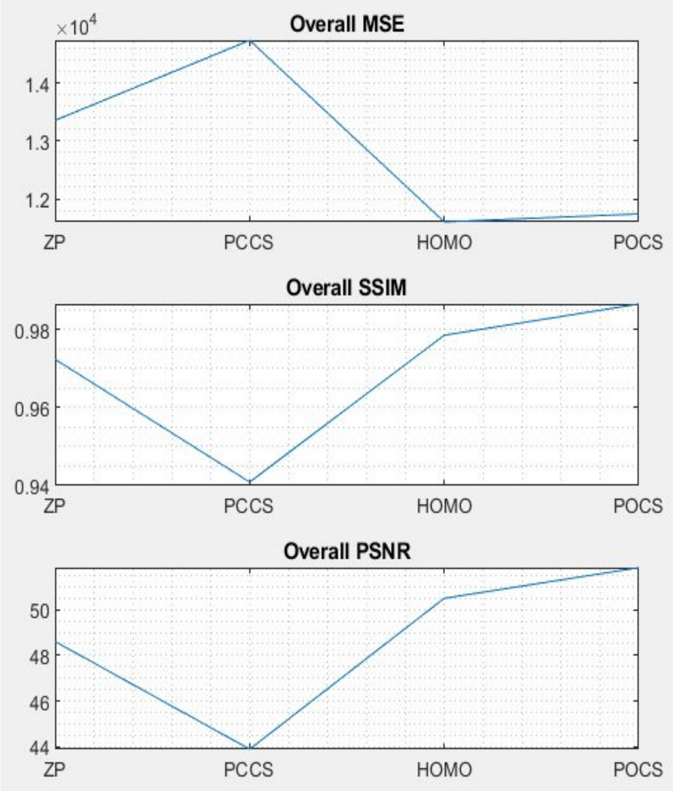
Quantitative Evaluation Metrics

- Mean Square Error (MSE)
- Structural Similarity Index (SSIM)
- Peak Signal-to-Noise Ratio (PSNR)

Result Evaluation

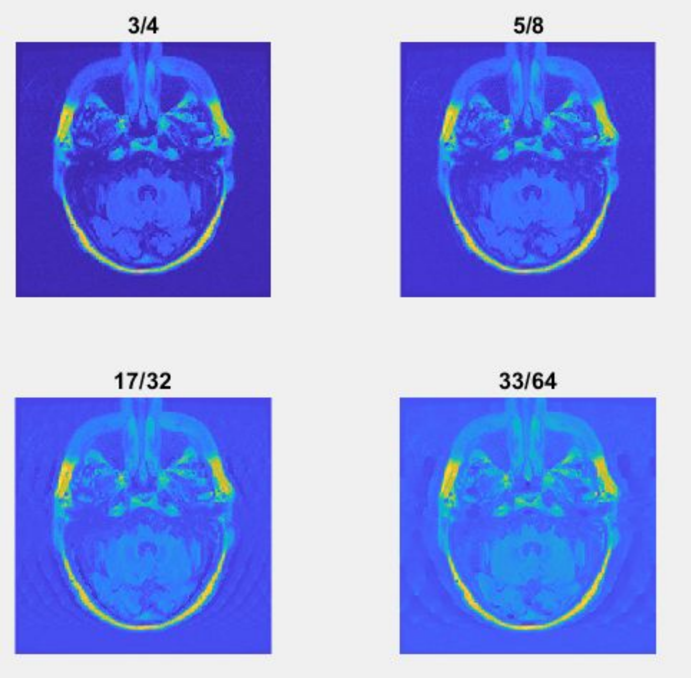
Different Reconstruction Methods



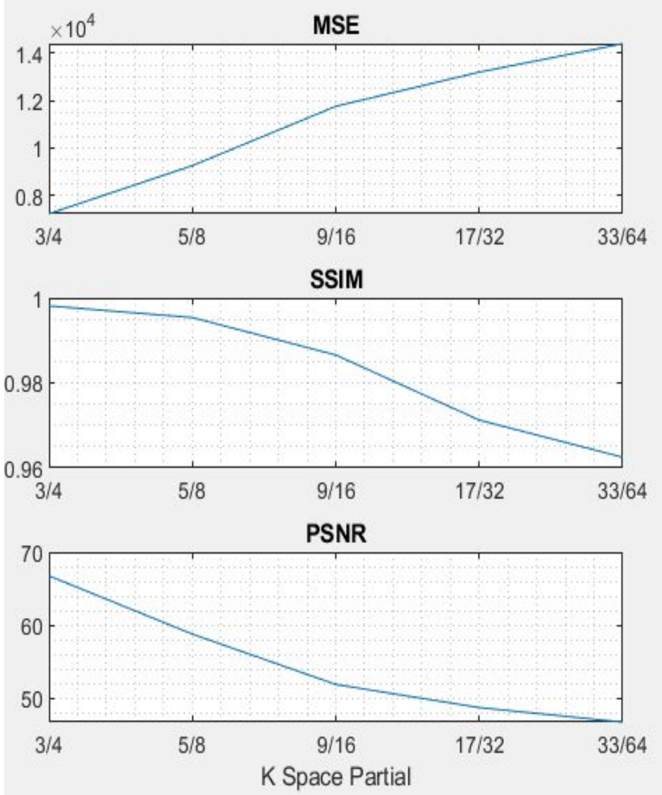


Result Evaluation

Different Sizes of K-space Partial

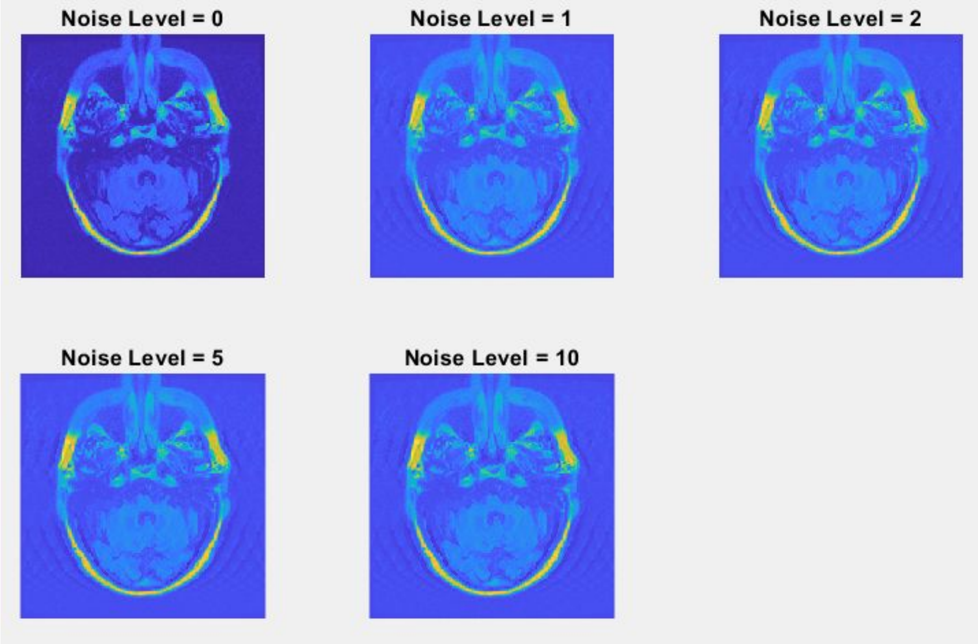


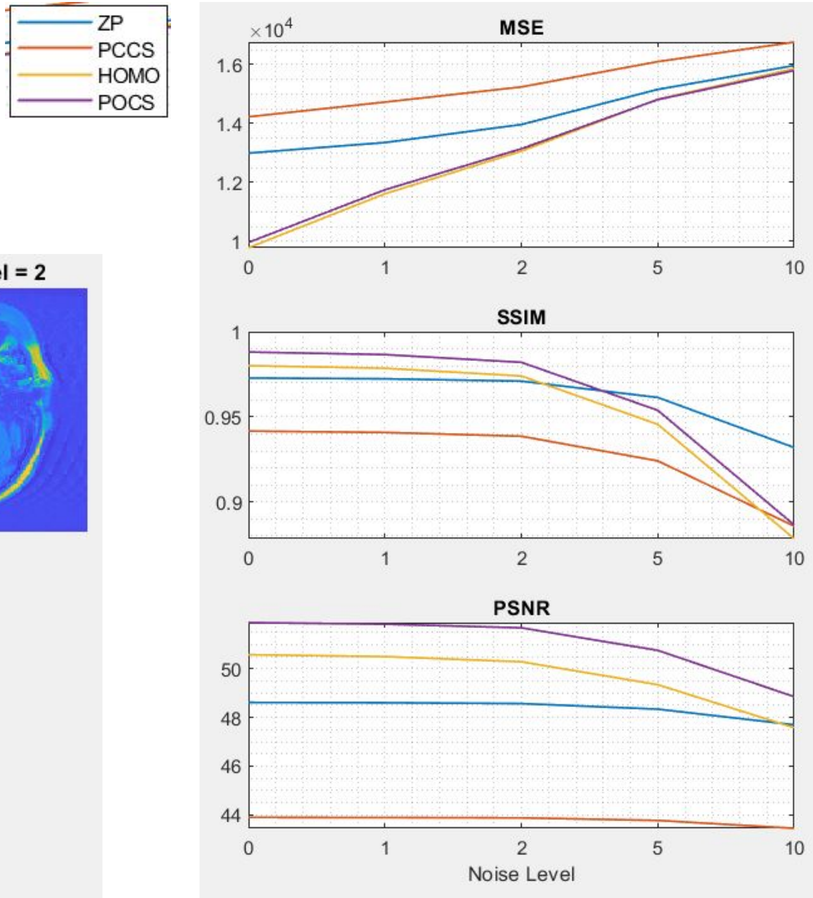
All reconstructed with POCS



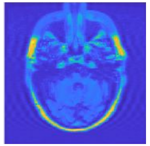
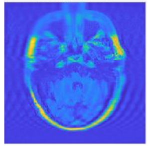
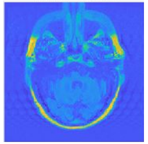
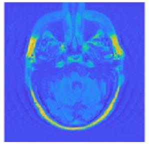
Result Evaluation

Different Levels of Noise





Result Table

	MSE Low Noise	MSE High Noise	SSIM Low Noise	SSIM High Noise	PSNR Low Noise	PSNR High Noise
Zero-Padding	13358	15977	0.9723	**0.9319**	48.6059	47.6980
PCCS	14736	16764	0.9408	0.8857	43.8911	43.4416
Homodyne	**11614**	15875	0.9785	0.8784	50.4996	47.5715
POCS	**11769**	15788	**0.9865**	0.8868	51.8381	48.8362

Reference

[1] Wikipedia Contributors, “k-space in magnetic resonance imaging,” Wikipedia, Sep. 29, 2024.

[2] K. Sung, “Image Reconstruction Partial k-space Reconstruction M229 Advanced Topics in MRI,” 2020. Accessed: Dec. 09, 2024. [Online]. Available: https://labs.dgsom.ucla.edu/mrrl/files/view/m229-2020/M229_Lecture14_PartialFourier.pdf

[3] J. Pauly. Partial k-space reconstruction [Online]. Available: https://users.fmrib.ox.ac.uk/~karla/reading_group/lecture_notes/Recon_Pauly_read.pdf

[4] Michael Völker (2024). MRI Partial Fourier reconstruction with POCS (https://www.mathworks.com/matlabcentral/fileexchange/39350-mri-partial-fourier-reconstruction-with-pocs), MATLAB Central File Exchange. Retrieved December 4, 2024.

[5] *Mayo Clinic Study of Aging Data (MCSA)* https://www.mayo.edu/research/centers-programs/alzheimers-disease-research-center/research-activities/mayo-clinic-study-aging/overview