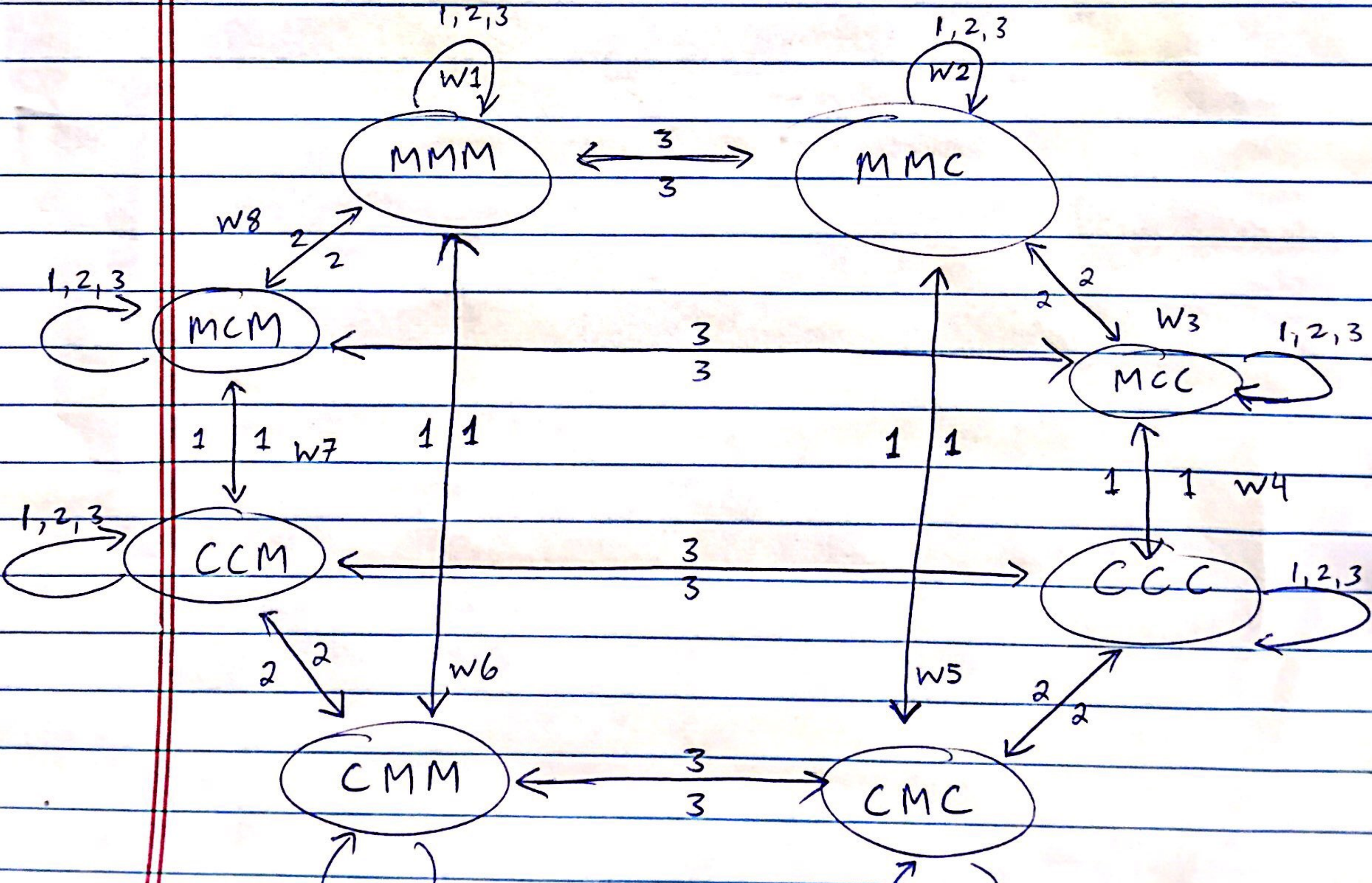
a) The set of all possible worlds is

$\Omega = (w_1 \ldots w_8)$

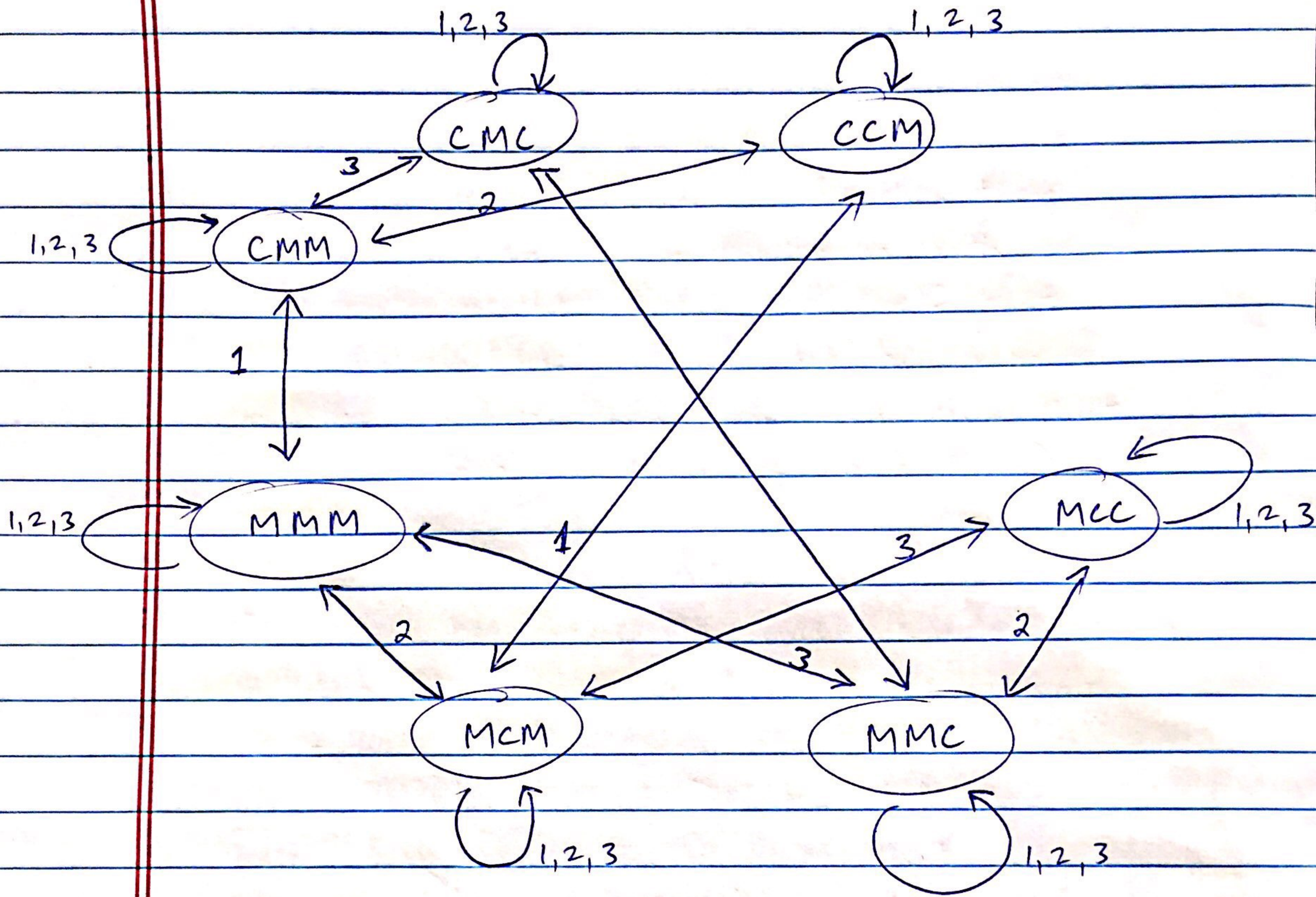
The outcome space is

$\{M, C\}^3$



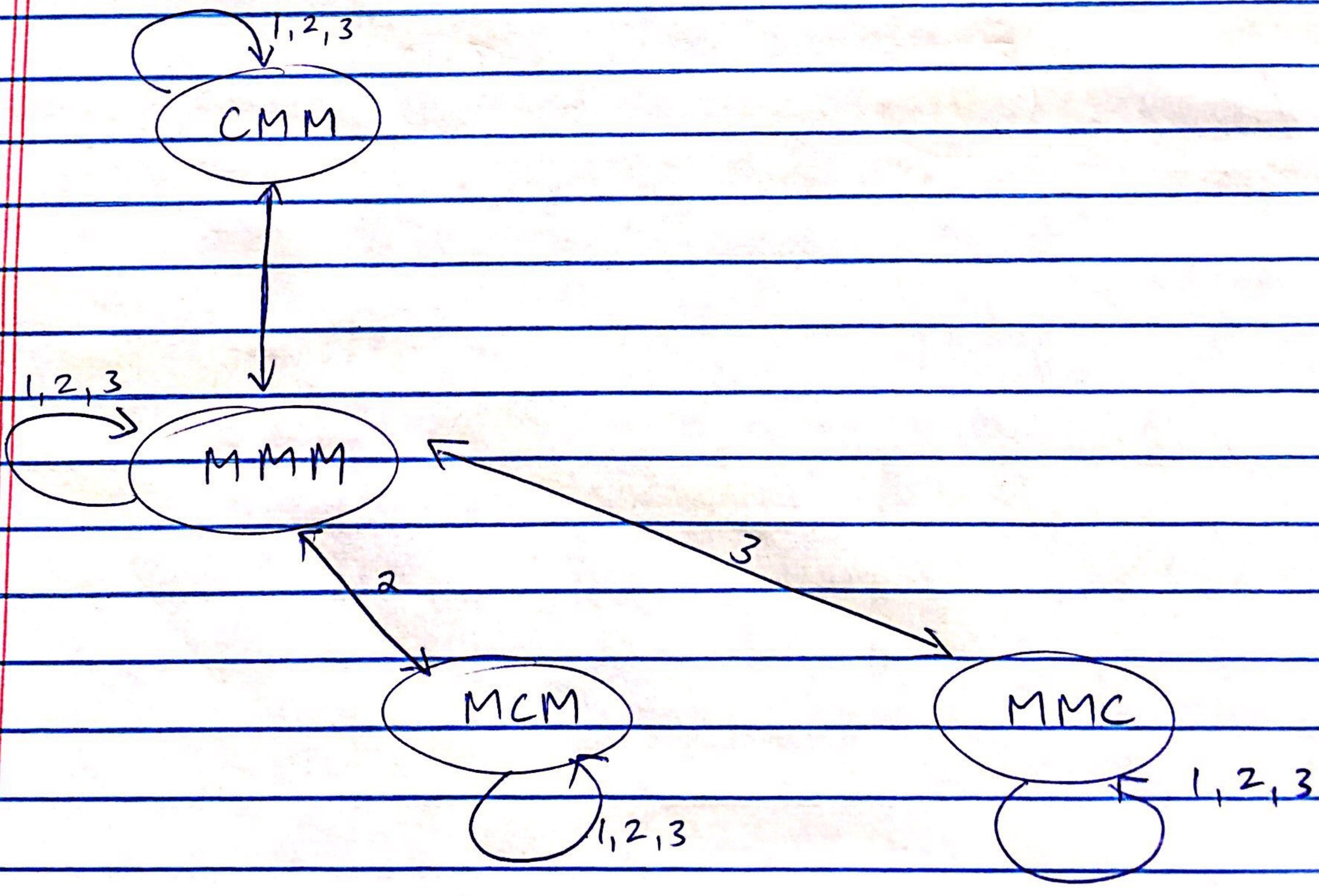
b) First, we show P(1). Since the father mentions that there are some muddy children, if there is only one muddy child, then they will see nobody else in the room with mud and know in the first round itself that they are muddy. If there are two or more muddy children, then they only know that some children (possibly including them) are muddy.

As soon as the father announces "there are muddy children", this destroys CCC, since someone must have mud on their face. For 1 muddy child, they would observe that all other kids have clear faces, which means that they would know they have muddy faces and would want to clean up.

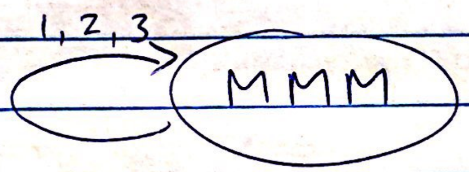


1,2,3
1,2,3
CMC
CCM
3
2
1,2,3
CMM
1
1,2,3
MMM
1
3
MCC
1,2,3
2
3
2
MCM
MMC
1,2,3
1,2,3

For P(2), Since there are ~~more~~ is than 1 muddy child, noone will say "yes" before the second round. In the second round, each muddy child sees 1 other muddy child, and therefore knows that there are either 1 or 2 muddy children total. They do know that if there were only 1 muddy child, someone would have said "yes" in the previous round. Since no one has spoken yet, each muddy child is able to say that there are 2 muddy children, including themselves. Therefore, we have the only world with at least 2 muddy children left:



For P(3), we have: Since there are > 2 muddy children, no one will say "yes" before round 3. In the third round, each muddy child sees 2 other muddy children, and knows that there are either 2 or 3 muddy children in total. However, they know that if there truly were only 2 muddy children, someone would have said "yes" in the second round. Since no one has spoken yet, each muddy child knows that there are in fact 3 muddy children, including themselves. So, the only world with 3 muddy children left is:



Let's assume that P(k) is true for $0 \le k \le n$. We will show the case for P(k+1). Suppose there are exactly k+1 muddy children. Since there are more than k muddy children, no one will say "yes" before round k+1 by the induction hypothesis. In that round, each muddy child sees k other muddy children, and knows that there are either k or k+1 muddy children total. By the induction hypothesis, they are able to infer that if there were only n muddy children, someone would have said "yes" in the previous round. Since no one has spoken yet, each muddy child is able to infer that there are indeed k+1 muddy children, including themselves. If there are strictly more than $k+1$ children, however, then all children can tell that there at least $k+1$ muddy children simply by looking at the others. Therefore, by the induction hypothesis, they can infer from the start that nobody will say "yes" in round k. So, they will have no more information than they did initially in round $k+1$, and will be unable to tell whether they are muddy as a result.