Tutorial 4:-

Sol 1:- $T(n) = 3T(n/2) + n^2$

$a = 3 \quad b = 2 \quad f(n) = n^2$

$\because$ a & b are constant and f(n) is a +ve function ~~in~~

$\therefore$ Master's theorem is ~~applicaple~~ applicable

$c = \log_b a$

$= \log_2 3 = 1.58$.

$\Rightarrow n^c = n^{1.58}$

which is $n^2 > n^{1.58}$

$\therefore$ Case 3 is applied here

$\Rightarrow \boxed{T(n) = \theta(n^2)}$

Sol 2:- $T(n) = 4T(n/2) + n^2$.

$a = 4 \quad b = 2 \quad f(n) = n^2$.

$\because$ a & b are const. and f(n) is a positive function.

$\therefore$ Master's theorem is applicable.

$c = \log_b a$

$= \log_2 4 = \log_2 2^2 = 2 \log_2 2 = 2$.

$\therefore n^c = n^2$.

$\therefore n^c = f(n)$

$\therefore$ case 2 is applied here.

$\Rightarrow \boxed{T(n) = \theta(n^2 \log n)}$

Sol 3:- $T(n) = T(n/2) + 2^n$

$a = 1 \quad b = 2 \quad f(n) = 2^n$

$\because$ a & b are constant and f(n) is a +ve function

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_2 1$

$= 0$

$\Rightarrow n^c = n^0 = 1$.

$\because f(n) > n^c$

$\therefore$ case 3 is applied here.

$\Rightarrow \boxed{T(n) = \Theta(2^n)}$

Sol 4:- $T(n) = 2^n T(n/2) + n^n$.

$a = 2^n \quad b = 2 \quad F(n) = n^n$

$\because$ a is not constant, its value depends on n.

$\therefore$ Master's theorem is not applicable here.

Sol 5:- $T(n) = 16\,T(n/4) + n$.

$a = 16 \quad b = 4 \quad f(n) = n$.

$\because$ a & b are constant and f(n) is a +ve function

$c = \log_b a$

$= \log_4 16 \Rightarrow \log_4 4^2 = 2 \log_4 4 = 2$.

$\Rightarrow n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1 is applied here.

$\boxed{T(n) = \Theta(n^2)}$

Sol 6:- $T(n) = 2T(n/2) + n\log n$

$a = 2$ $b = 2$ $f(n) = n\log n$

$\because$ a, b are constant and $f(n)$ is a +ve function

& $c = \log_b a$

$= \log_2 2 = 1$.

$n^c = n^1 = n$

$\because f(n) > n^c$

$\therefore$ Case 3 is applied

$\Rightarrow \boxed{T(n) = \Theta(n\log n)}$

Sol 7:- $T(n) = 2T(n/2) + n/\log n$.

$a = 2$ $b = 2$ $f(n) = n/\log n$.

$\because$ a and b are constant & $f(n)$ is a +ve function

$c = \log_b a$

$= \log_2 2 = 1$.

$n^c = n = n$

& f(n) $\because$ ~~dt~~ non-polynomial difference b/w $f(n)$ & n^c

$\therefore$ Master's theorem is not applicable.

Sol 8:- $T(n) = 2T(n/4) + n^{0.51}$.

$a = 2$ $b = 4$ $f(n) = n^{0.51}$.

$\because$ a & b are constant & $f(n)$ is a +ve function

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_4 2 = 0.50$.

$n^c = n^{0.50}$

$\because f(n) > n^c$

$\therefore$ case 3 is applicable

$\Rightarrow \boxed{T(n) = \Theta(n^{0.51})}$

Sol9:- $T(n) = 0.5\,T(n/2) + 1/n$

$a = 0.5 \quad b = 2 \quad f(n) = 1/n$.

$\because a$ is < 1

$\therefore$ Master's theorem is not applicable.

Sol10:- $T(n) = 16\,T(n/4) + n!$

$a = 16 \quad b = 4 \quad f(n) = n!$

$\because a$ & b are const & $f(n)$ is a +ve function.

$\therefore$ Master's theorem is applicable

$c = \log_b a$
$= \log_4 16 = \log_4 4^2 = 2\log_4 4 = 2$

$n^c = n^2$.

$\because f(n) > n^c$

$\therefore$ case 3 is applied here.

$\Rightarrow \boxed{T(n) = \Theta(n!)}$

Sol11:- $T(n) = 4T(n/2) + \log n$.

$a = 4 \quad b = 2 \quad f(n) = \log n$.

$\because a$ & b are constant & $f(n)$ is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$.

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1 is applied

$\Rightarrow \boxed{T(n) = \theta(n^2)}$

Sol 12 $\sqrt{n}\, T(n/2) + \log n$

$a = \sqrt{n}$ $b = 2$ $f(n) = \log n$

~~$\because$ a & b are constant & f(n) is a +ve function~~

~~$\therefore$ Master's theorem is applicable~~

~~$c = \log_b a = \log$~~

$\because$ a is not constant

$\therefore$ Master's theorem is not applicable

Sol 14 :- $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3$ $b = 3$ $f(n) = \sqrt{n}$

$\because$ a & b are constant & f(n) is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_3 3 = 1.$

$n^c = n^1 = n.$

$\because f(n) < n^c$

$\therefore$ case 1 is applicable

$\Rightarrow \boxed{T(n) = \theta(n)}$

Sol 13 :- $T(n) = 3T(n/2) + n$

$a = 3$ $b = 2$ $f(n) = n$

$\because$ a & b are constant & f(n) is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 3 = 1.58$

$\therefore\ n^c = n^{1.58}$

$\because f(n) < n^c$

$\therefore$ case 1 is applied here

$\Rightarrow \boxed{T(n) = \Theta(n^{1.58})}$

Sol15:- $T(n) = 4T(n/2) + c.n$

$a = 4 \quad b = 2 \quad f(n) = c.n$

$\because$ a & b are constant and $f(n)$ is a +ve function

$\therefore$ Master's theorem is applicable here

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2.$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1 is applicable here

$\Rightarrow \boxed{T(n) = \Theta(n^2)}$

Sol16:- $T(n) = 3T(n/4) + n\log n$

$a = 3 \quad b = 4 \quad f(n) = n\log n$

$\because$ a & b are constant & $f(n)$ is a +ve function.

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_4 3 = 0.79.$

$n^c = n^{0.79}.$

$\because f(n) > n^c$

$\therefore$ case 3 is applicable here

$\Rightarrow \boxed{T(n) = \Theta(n\log n)}$

Sol17: $T(n) = 3T(n/3) + n/2$

$a = 3 \quad b = 3 \quad f(n) = n/2$

$\because$ a, b are constant & f(n) is a +ve function.

$\therefore$ Master's theorem is applicable here

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\because f(n) = n^c$

$\therefore$ Case 2 is applied here.

$\Rightarrow \boxed{T(n) = n \log n}$

Sol18:- $T(n) = 6T(n/3) + n^2 \log n$

$a = 6 \quad b = 3 \quad f(n) = n^2 \log n$

$\because$ a & b are constant & f(n) is a +ve function.

$\therefore$ Master's theorem is applicable here

$c = \log_b a = \log_3 6 = 1.63$

$n^c = n^{1.63}$

$\because f(n) > n^c$

$\Rightarrow$ case 3 is applied here

$\Rightarrow \boxed{T(n) = \Theta(n^2 \log n)}$

Sol19:- $T(n) = 4T(n/2) + n/\log n$

$a = 4 \quad b = 2 \quad f(n) = n/\log n$

$\because$ a and b are const and f(n) is a +ve function.

$\therefore$ Master's theorem is applicable here

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2 \log_2 2 = 2$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1 is applied here

$\Rightarrow \boxed{T(n) = \Theta(n^2)}$

Sol 20:- $T(n) = 64\,T(n/8) - n^2 \log n$

$\because$ a & b are constant but $f(n)$ is a -ve function

Master's theorem is not applicable here

Sol 21:- $T(n) = 7\,T(n/3) + n^2$

$a = 7 \quad b = 3 \quad f(n) = n^2$

$\because$ a, b are constant & $f(n)$ is ~~constant~~ +ve function

$\therefore$ Master's theorem is applied here

$c = \log_b a = \log_3 7 = 1.77$

$\Rightarrow n^c = n^{1.77}$

$\because f(n) > n^c$

$\therefore$ case 3 is applied here

$\Rightarrow \boxed{T(n) = \Theta(n^2)}$

Sol 22:- $T(n) = T(n/2) + n(2 - \cos n)$

$\because f(n)$ is not regular function

$\therefore$ Master's theorem doesnot applied here