CS201 DISCRETE MATHEMATICS FOR COMPUTER SCIENCE

Dr. QI WANG

Department of Computer Science and Engineering
Office: Room903, Nanshan iPark A7 Building
Email: wangqi@sustech.edu.cn

Graph Concepts

- $G = (V, E)$, *simple* graph, *multigraph*, *pseudograph*
- *Undirected*, *directed* graph
- Special graphs

  K_n, C_n, W_n, Q_n, $K_{m,n}$

  Hall's Marriage Theorem on *bipartite* graphs

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  Hall's Marriage Theorem on *bipartite* graphs
- Representation of graphs

  adjacency list, *adjacency matrix*, *incidence matrix*

Isomorphism of Graphs

- **Definition** The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are *isomorphic* if there is a one-to-one and onto function from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1. Such a function is called an *isomorphism*.

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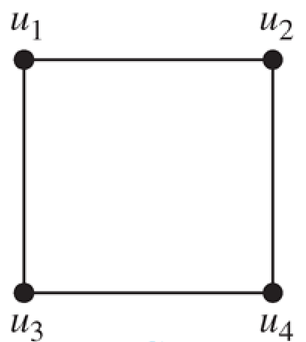
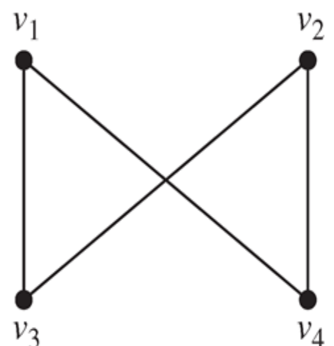


G



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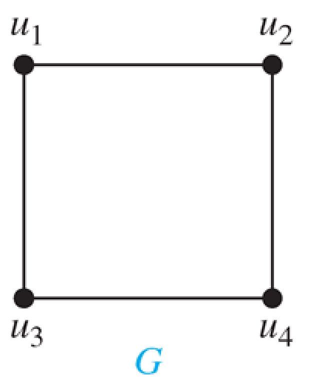

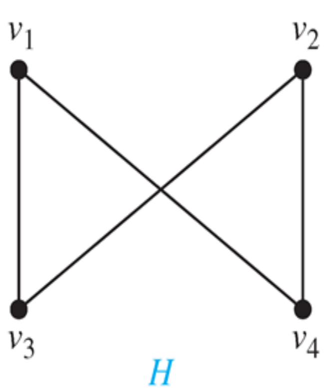

Are the two graphs isomorphic?

Define a one-to-one correspondence:
$f(u_1) = v_1$, $f(u_2) = v_4$, $f(u_3) = v_3$, and $f(u_4) = v_2$

Isomorphism of Graphs

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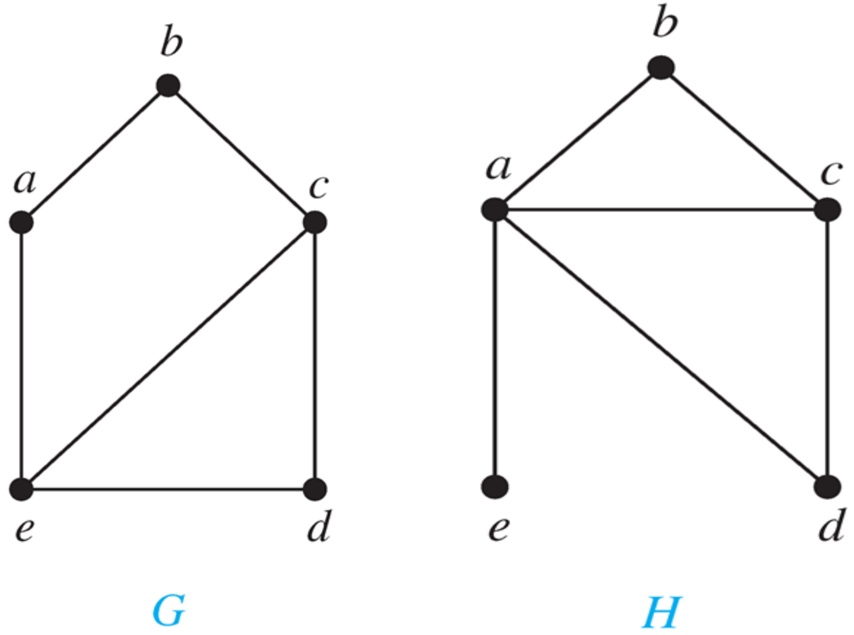
- Useful graph invariants include the number of vertices, number of edges, degree sequence, etc.

Isomorphism of Graphs

- **Example** Determine whether these two graphs are isomorphic.



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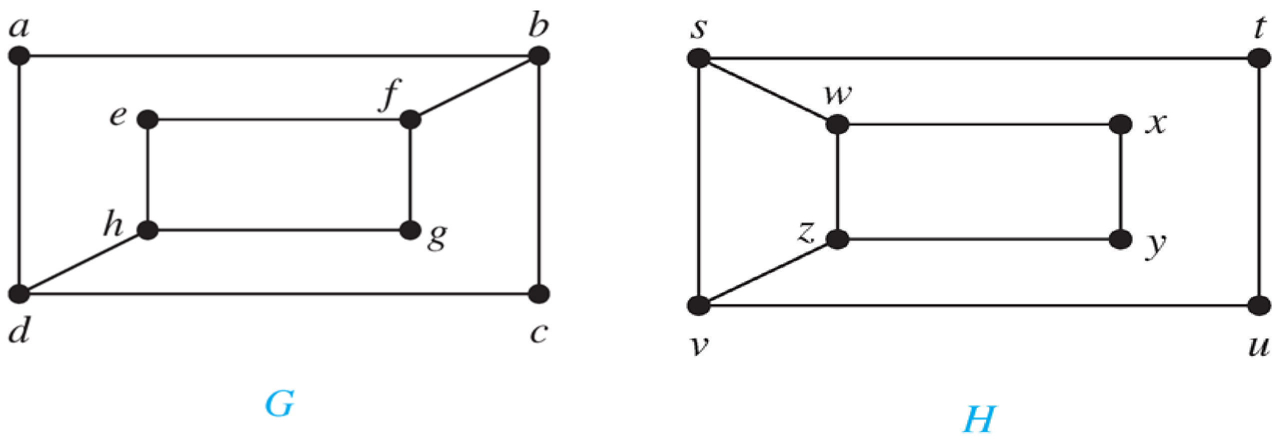

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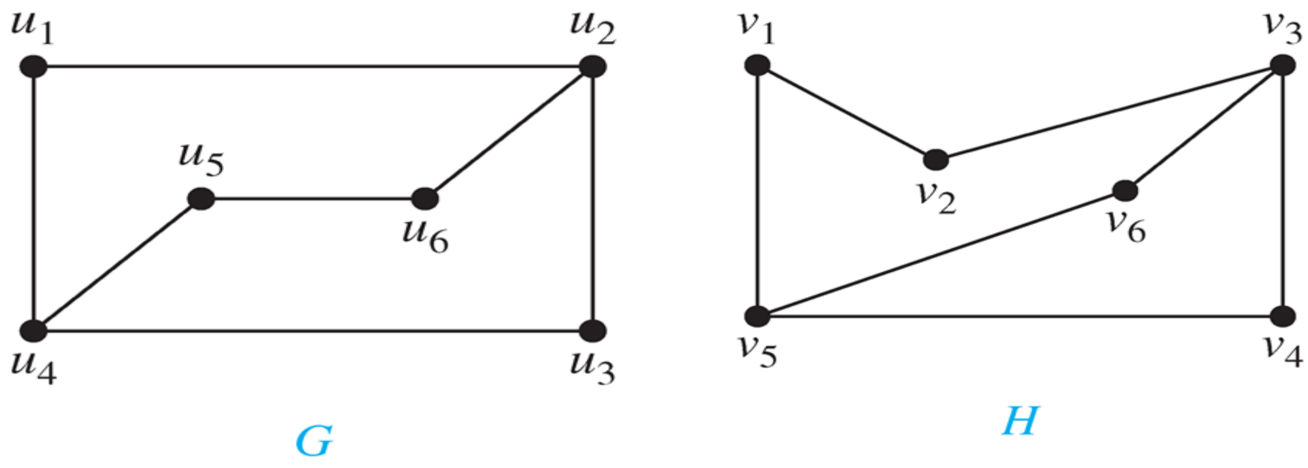

Path

- **Definition** Let n be a nonnegative integer and G an undirected graph. A *path of length n* from u to v in G is a sequence of n edges $e_1, e_2, \ldots, e_n$ of G for which there exists a sequence $x_0 = u, x_1, \ldots, x_{n-1}, x_n = v$ of vertices such that e_i has the endpoints x_{i-1} and x_i for $i = 1, \ldots, n$. The path is a *circuit* if it begins and ends at the same vertex, i.e., if $u = v$ and has length greater than zero. A path or circuit is *simple* if it does not contain *repeating vertices*.

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Length of a path $=$ # of edges on path

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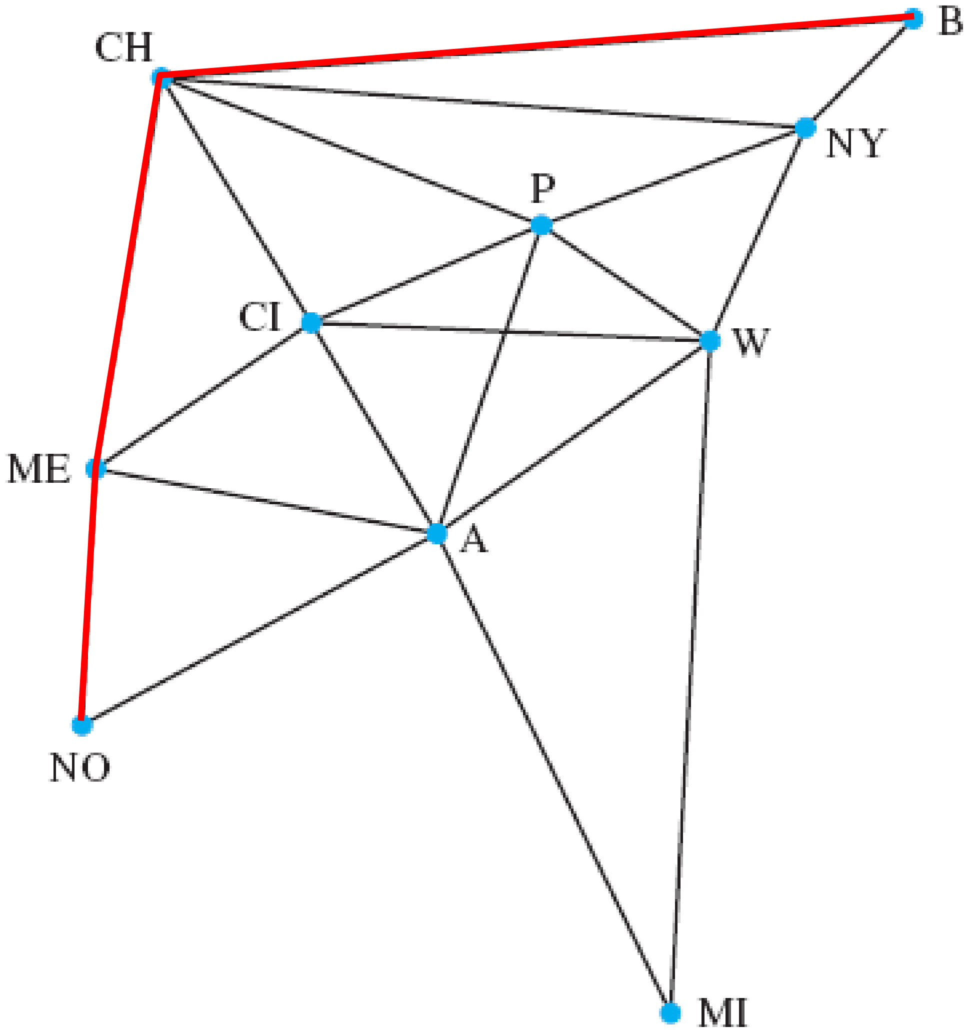

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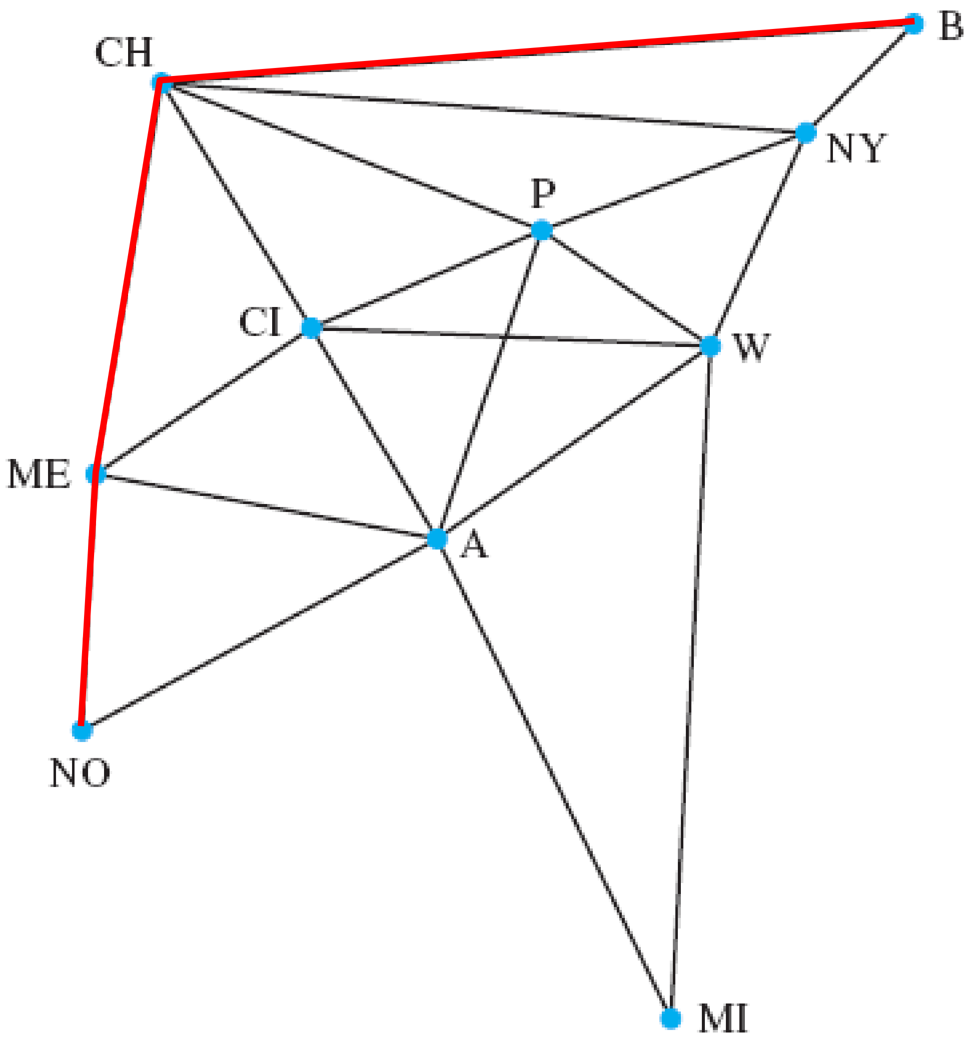
Path from Boston to New Orleans is B, CH, ME, NO



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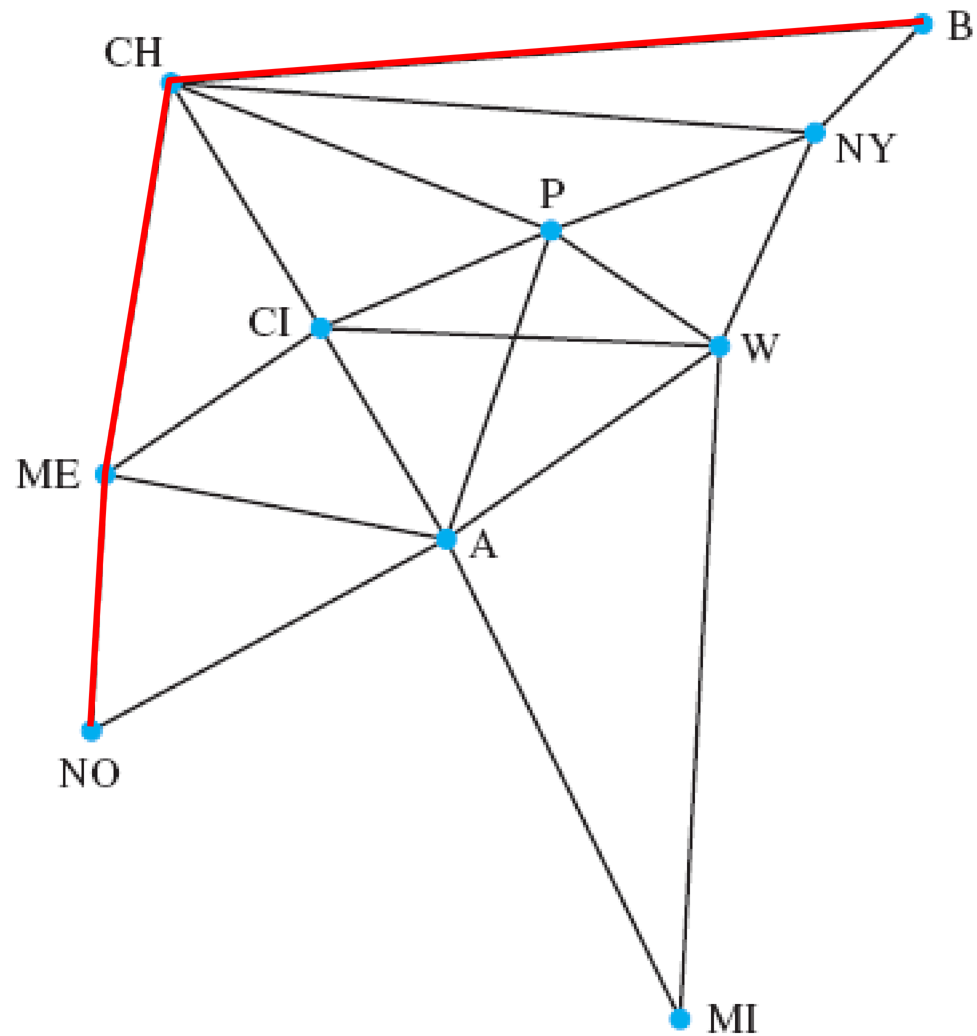
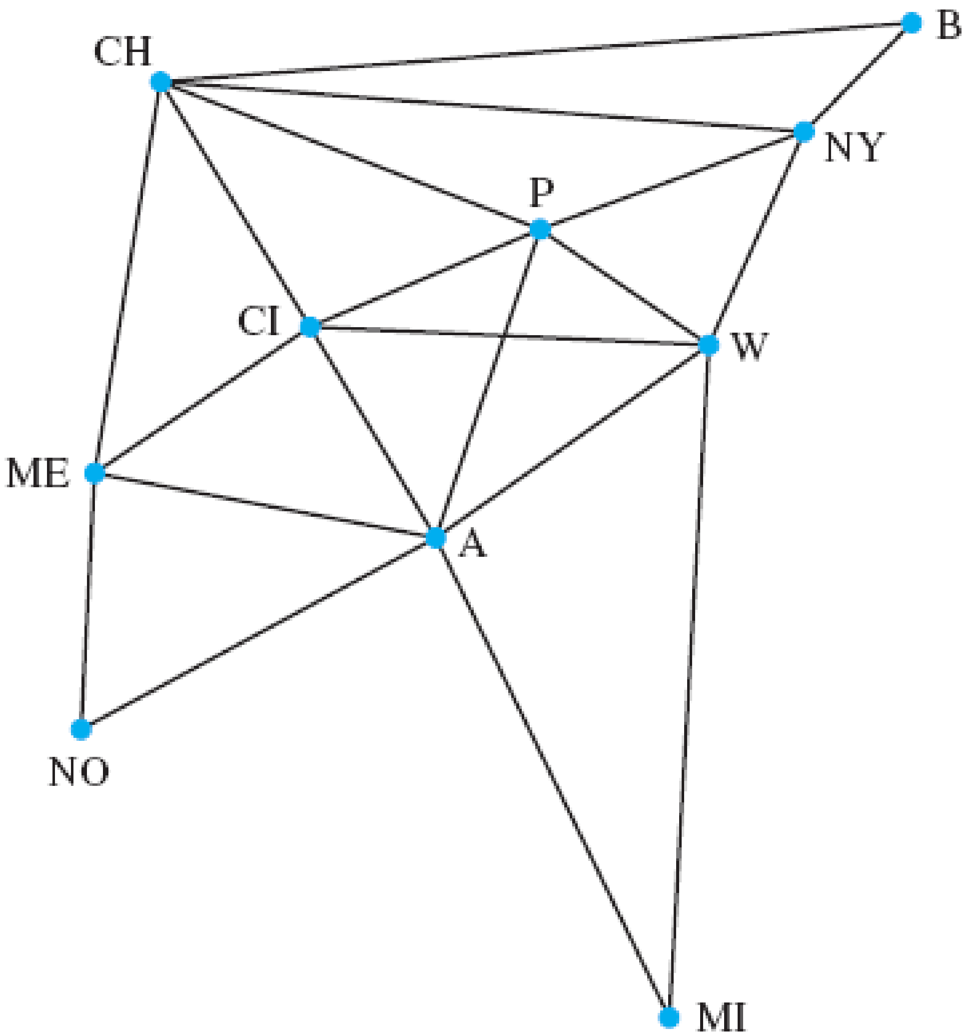
This path has length 3.



Connectivity



Company decides to lease only minimum number of communication lines it needs to be able to send a message from any city to any other city by using any number of intermediate cities.

What is the minimum number of lines it needs to lease?

Connectivity

- Choosing 10 edges?

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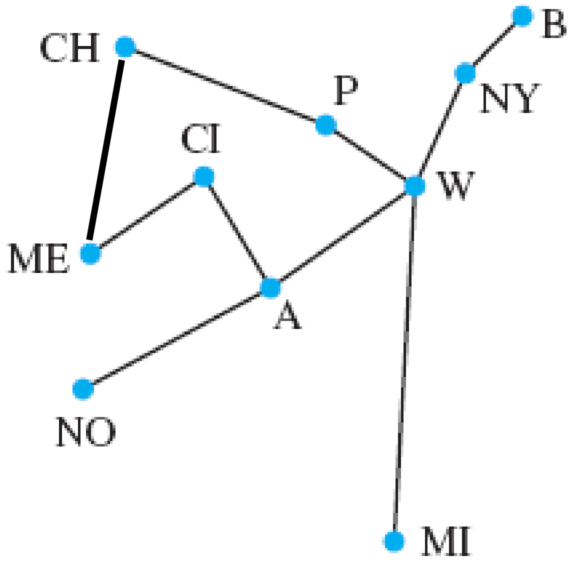

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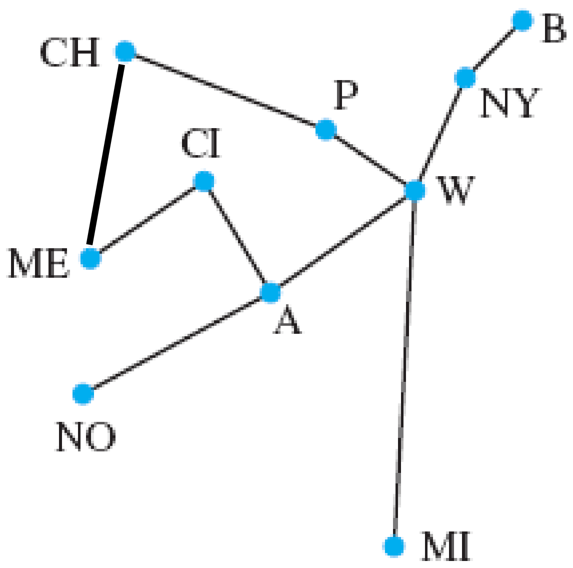

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Could throw away edge CI, A, and still have a solution.

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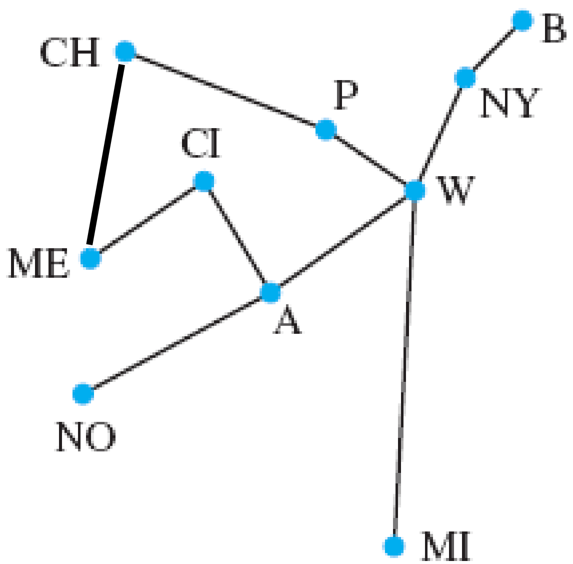

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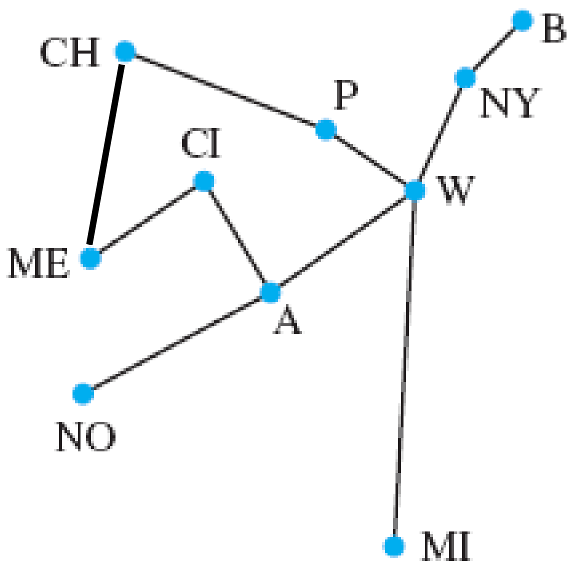

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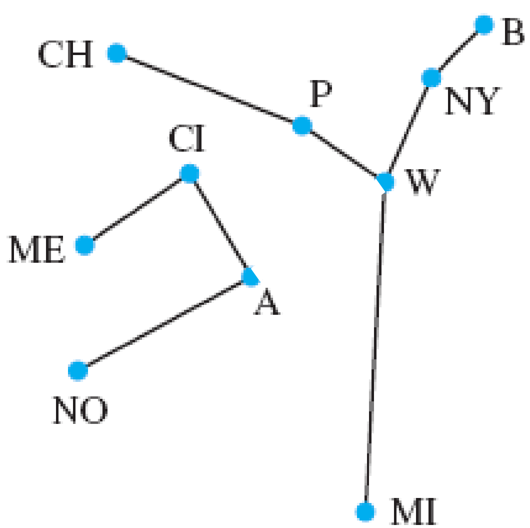

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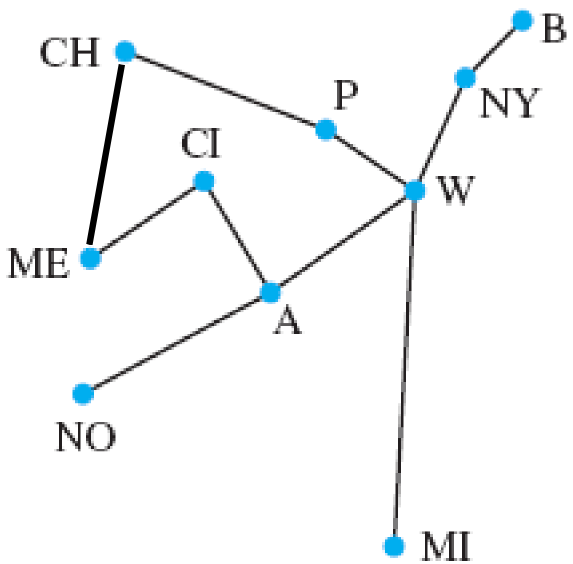

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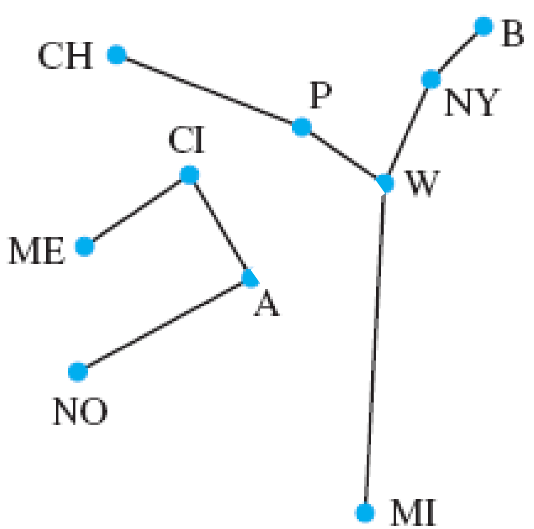

Not enough.

There is no path from, e.g., NO to B.

Connectivity

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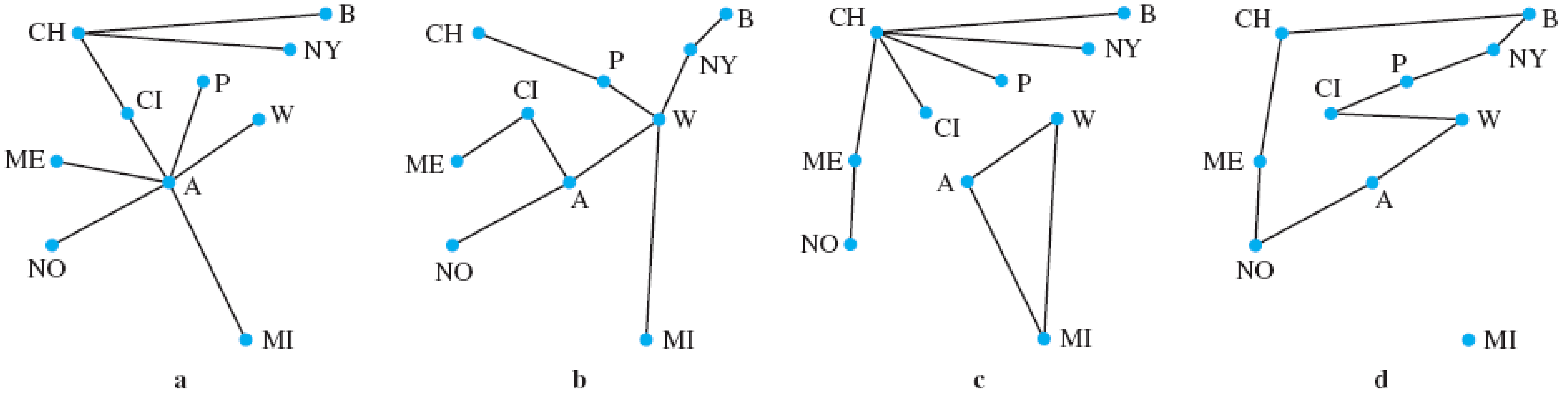

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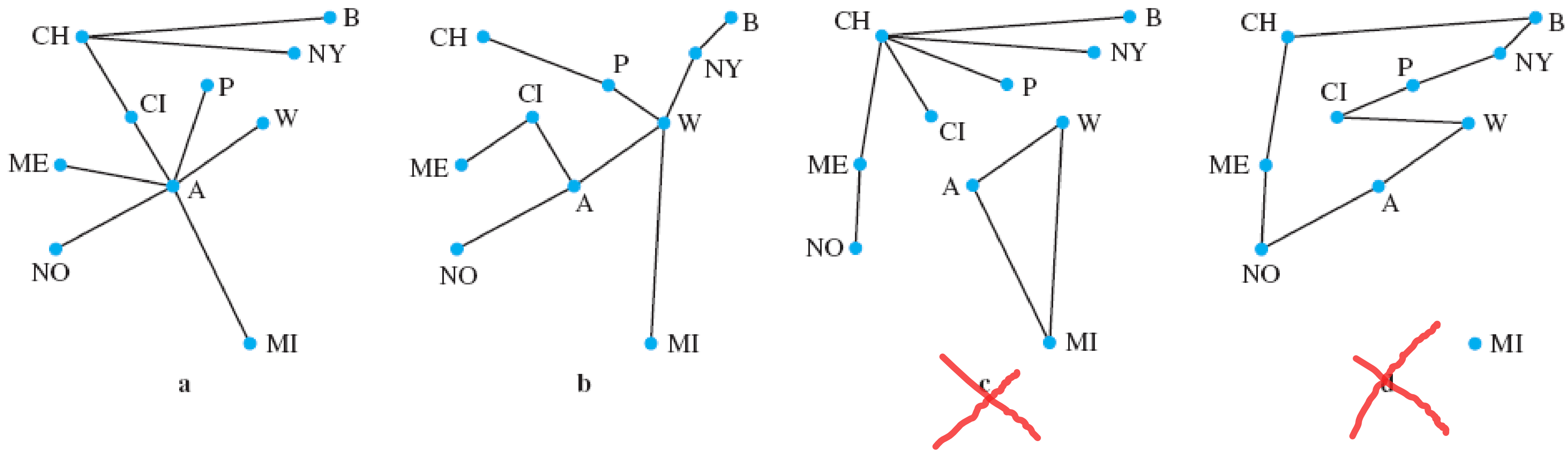

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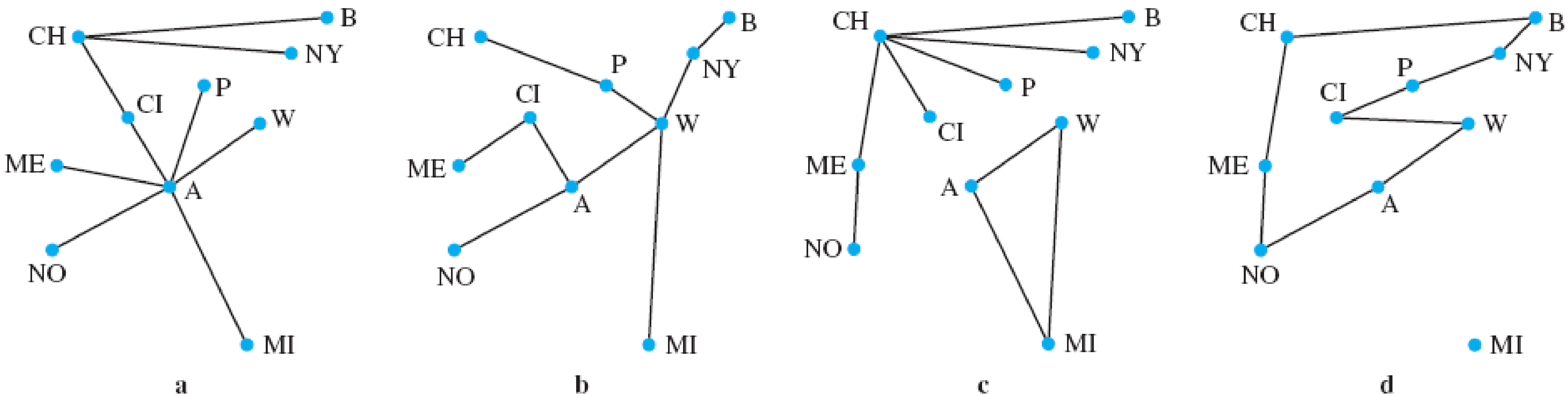

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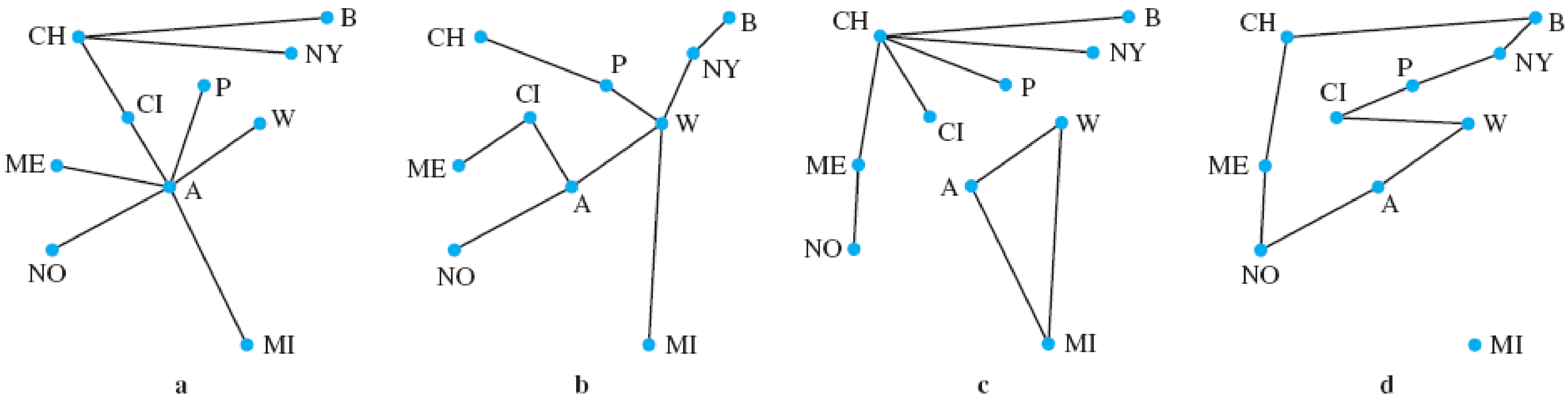

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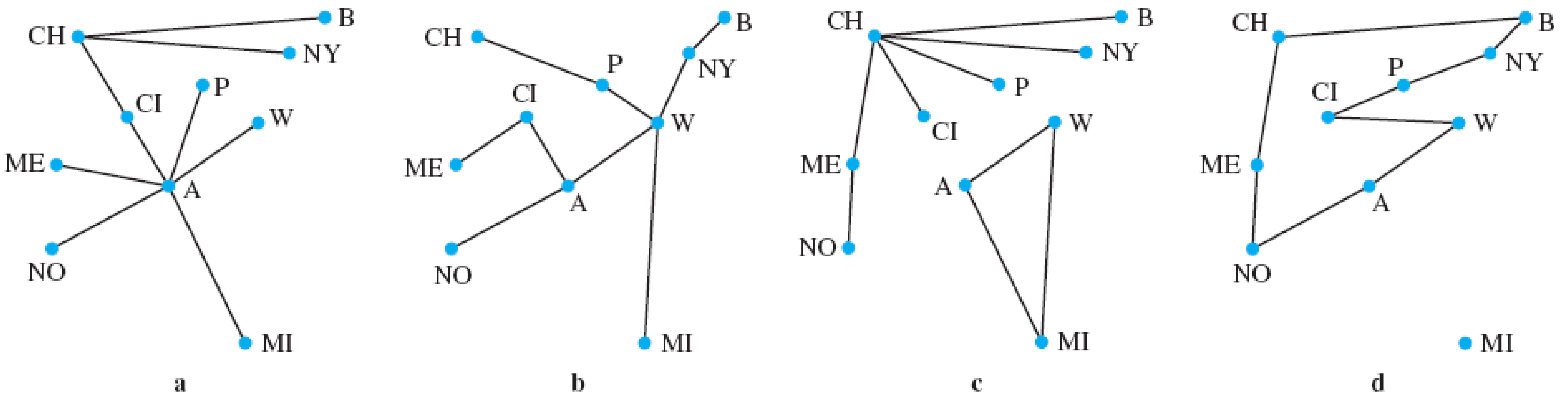

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Example: (a) and (b) are connected, (c) and (d) are disconnected.

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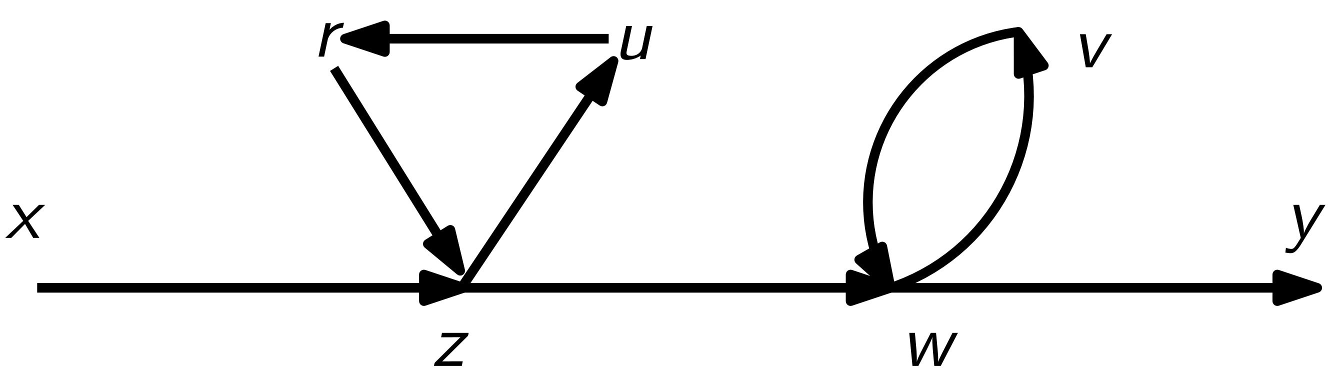

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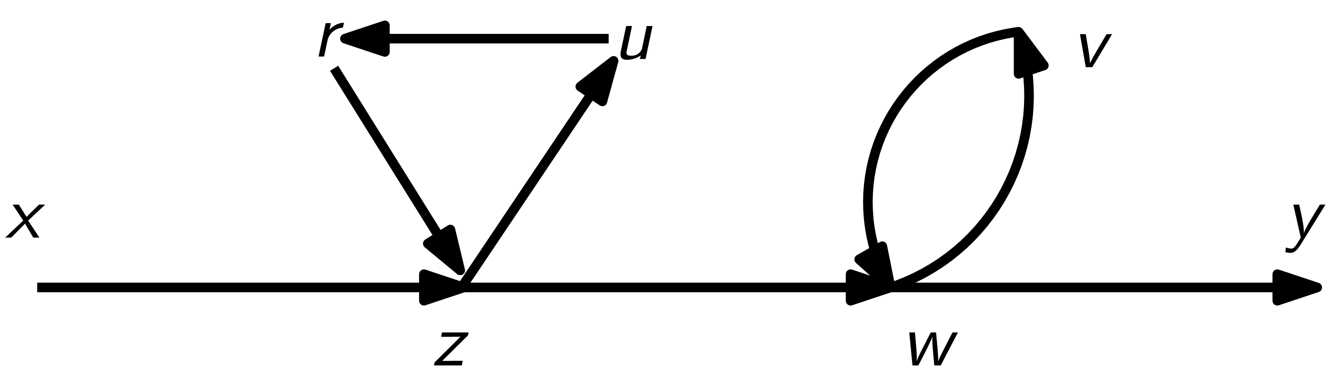

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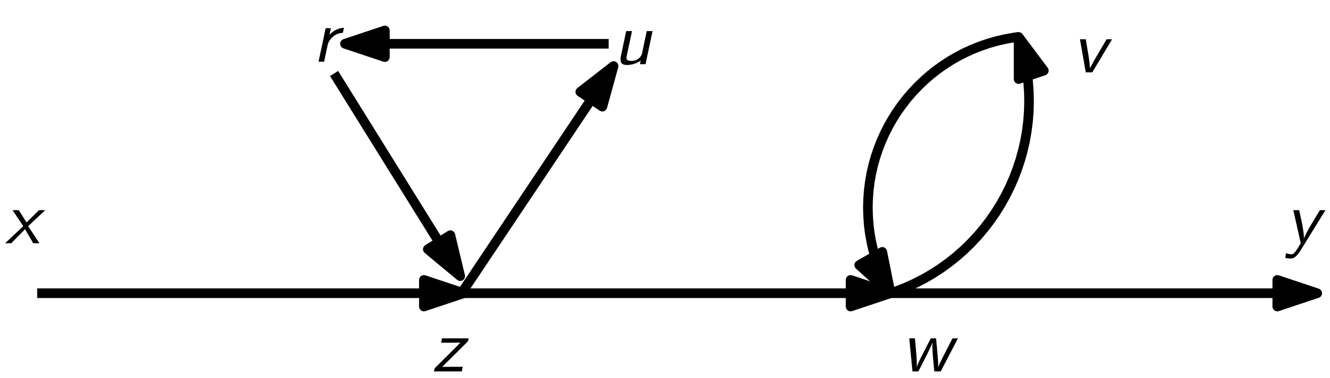

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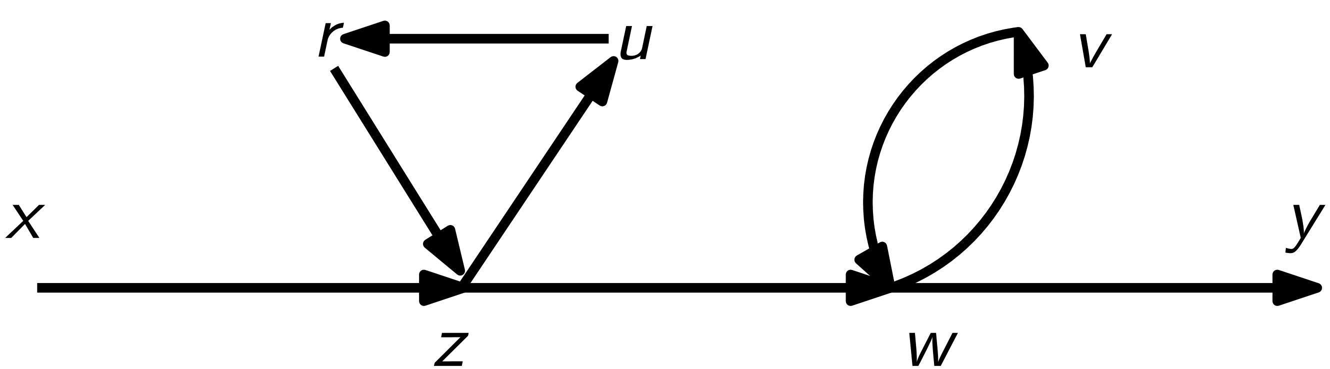

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Path from x to y
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Theorem There is a simple path between every pair of distinct vertices of a connected undirected graph.

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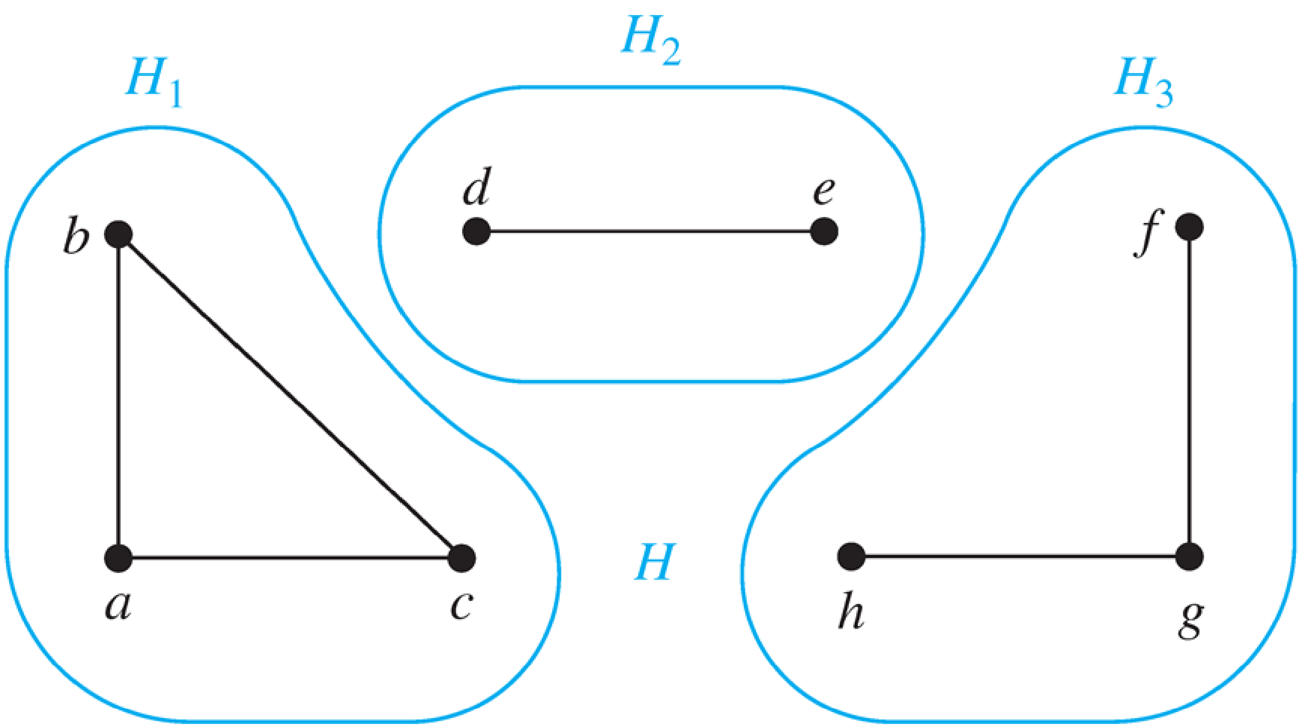

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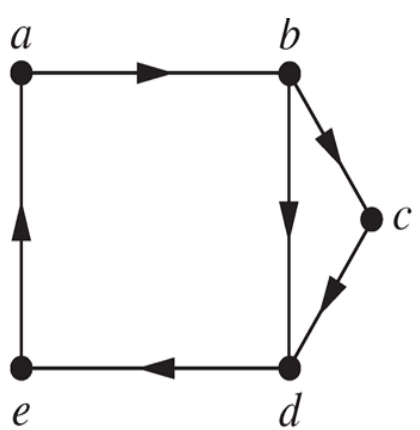

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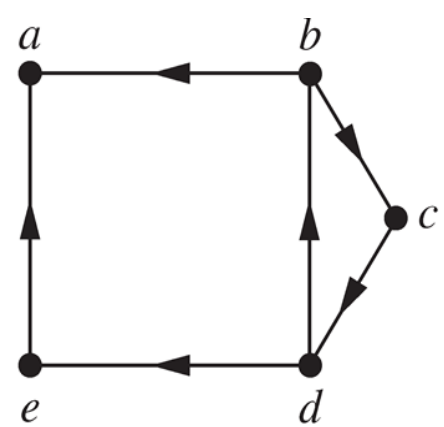

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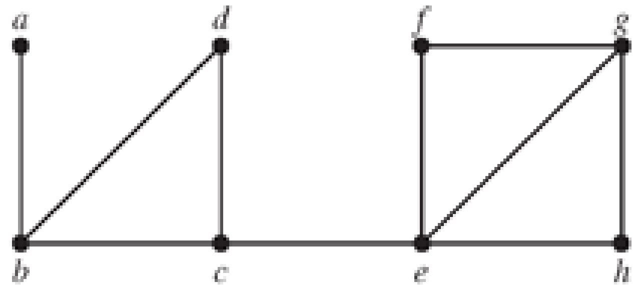

G_1

Paths and Isomorphism

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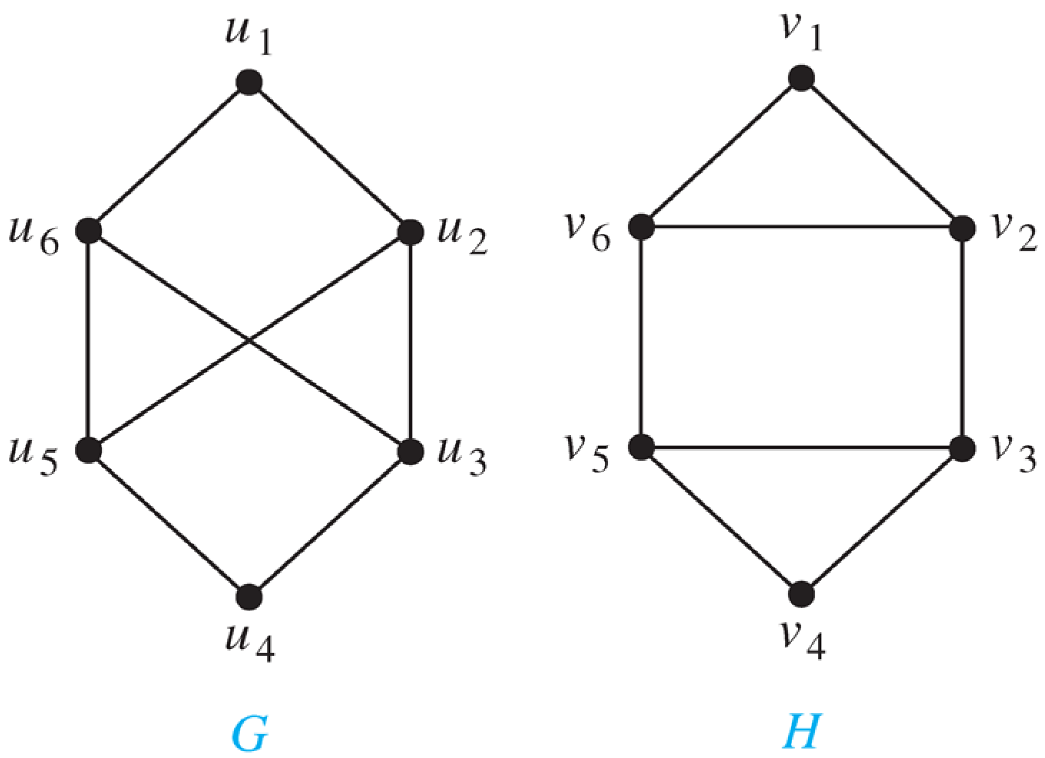

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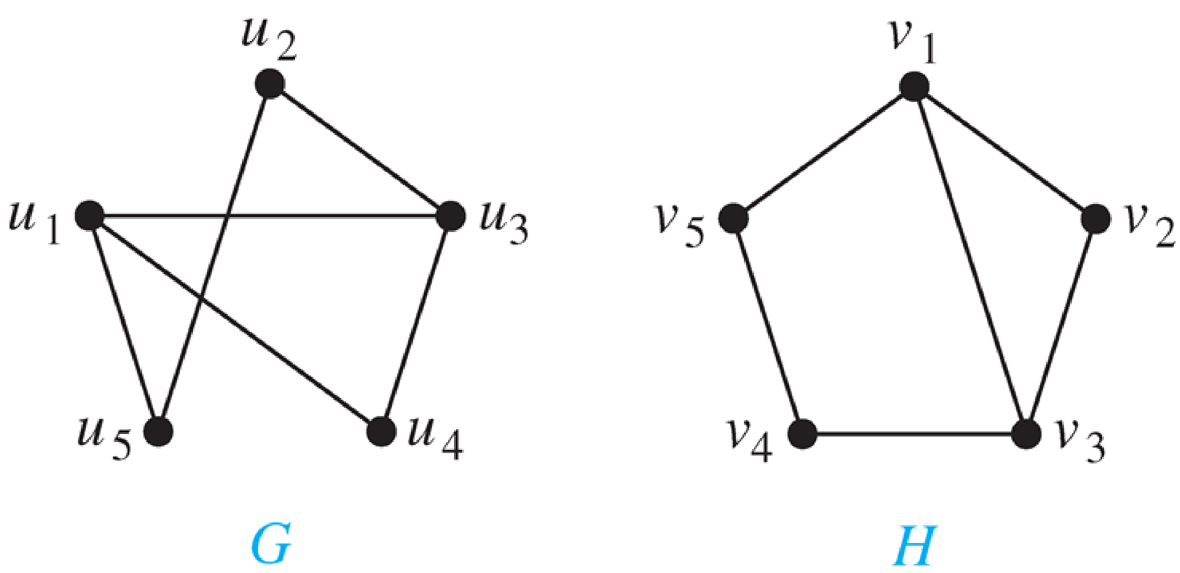

Counting Paths between Vertices

- **Theorem** Let G be a graph with adjacency matrix $\mathbf{A}$ with respect to the ordering $v_1, v_2, \ldots, v_n$ of vertices. The number of different paths of length r from v_i to v_j, where $r > 0$ is positive, equals the (i, j)-th entry of $\mathbf{A}^r$.

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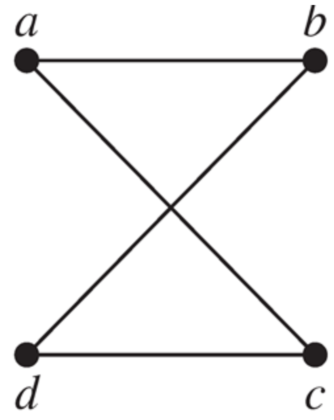
Proof (by **induction**)

$\mathbf{A}^{r+1} = \mathbf{A}^r\mathbf{A}$, the (i, j)-th entry of $\mathbf{A}^{r+1}$ equals $b_{i1}a_{1j} + b_{i2}a_{2j} + \cdots + b_{in}a_{nj}$, where b_{ik} is the (i, k)-th entry of $\mathbf{A}^r$.

Counting Paths between Vertices

- **Example** How many paths of length 4 are there from a to d in the graph G?



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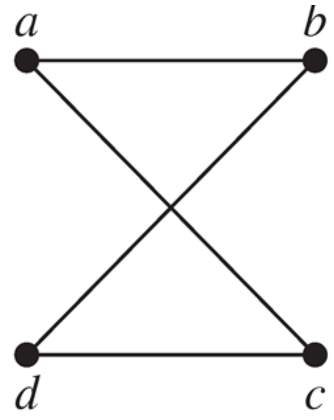

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

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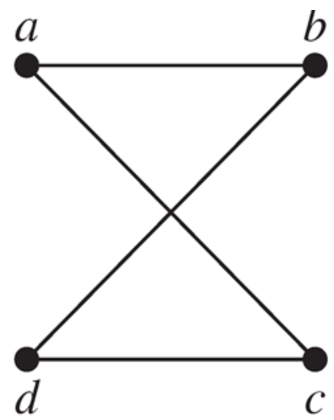

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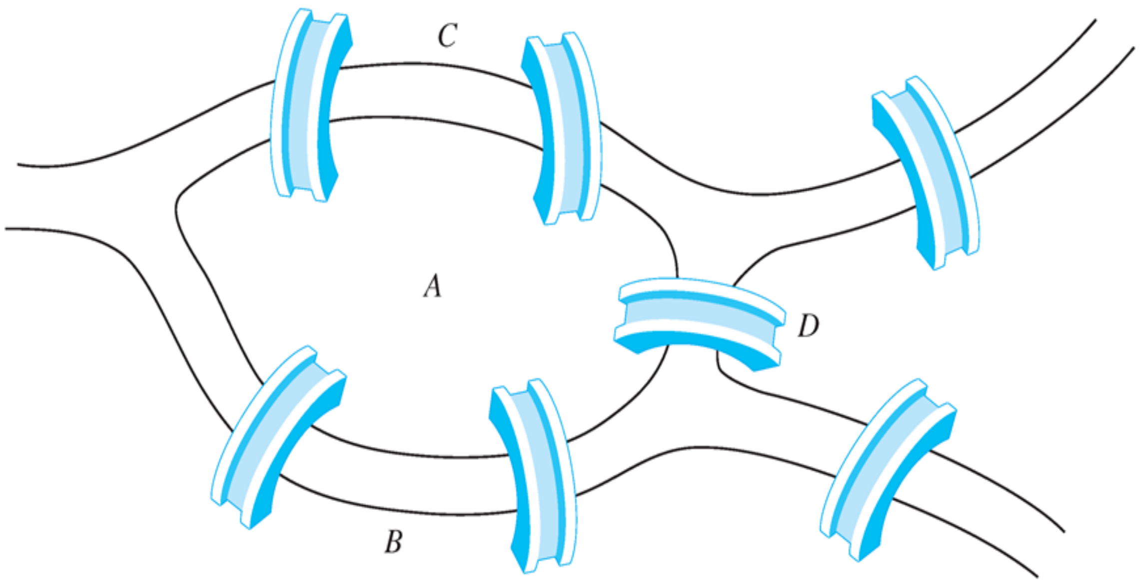
- **Königsberg seven-bridge problem**

  People wondered whether it was possible to start at some location in the town, travel across all the bridges once without crossing any bridge twice, and return to the starting point.



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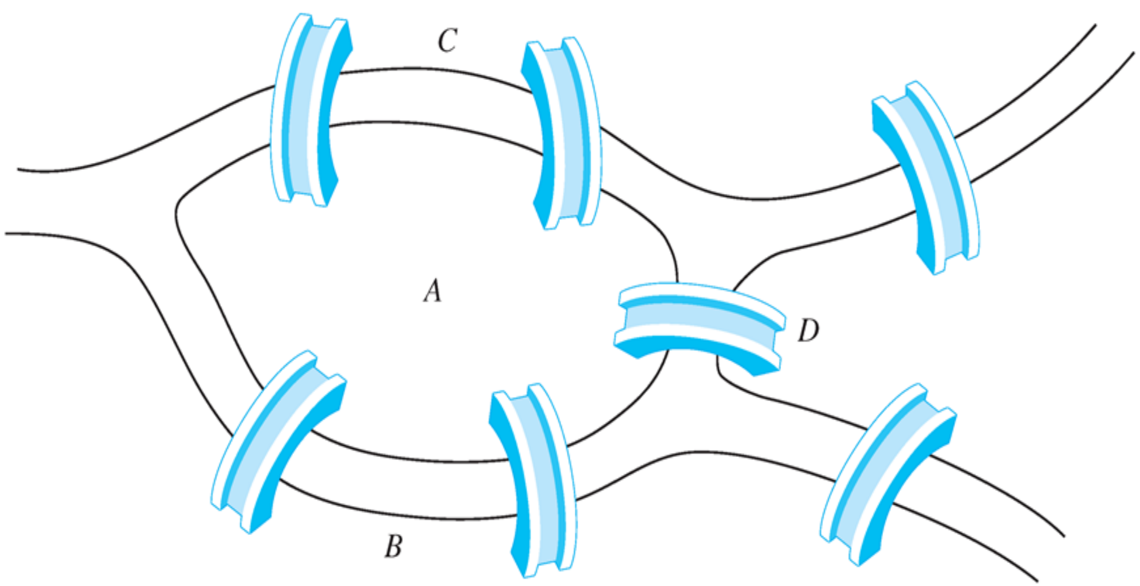

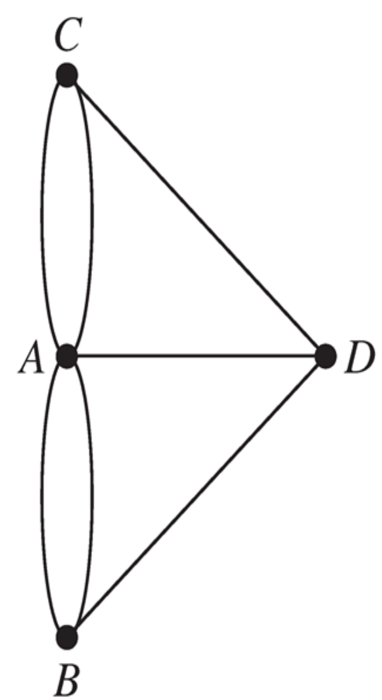

Euler Paths and Circuits

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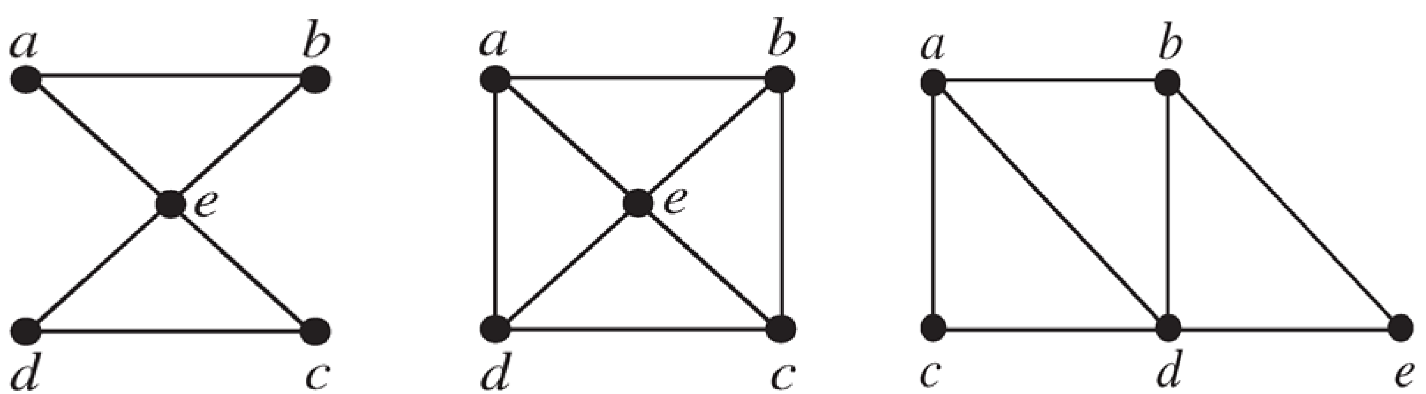

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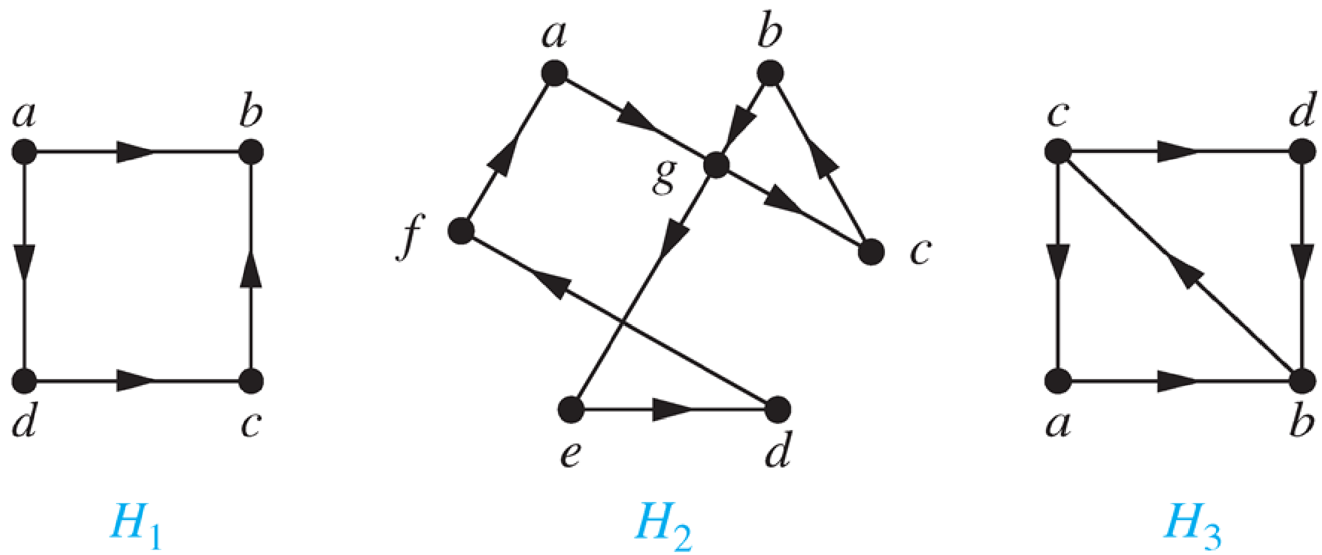

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- The initial vertex and the final vertex of an Euler path have odd degree.

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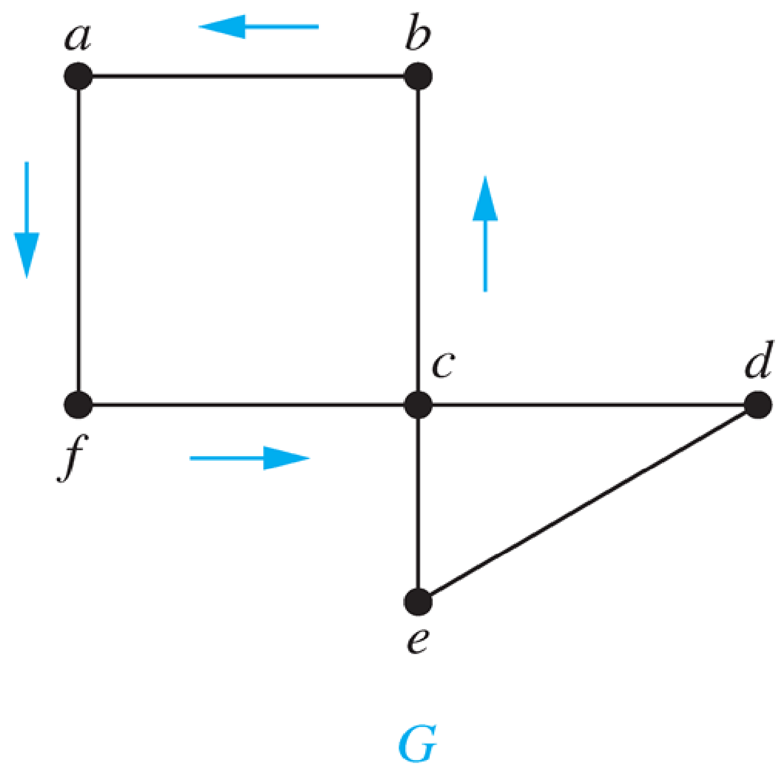

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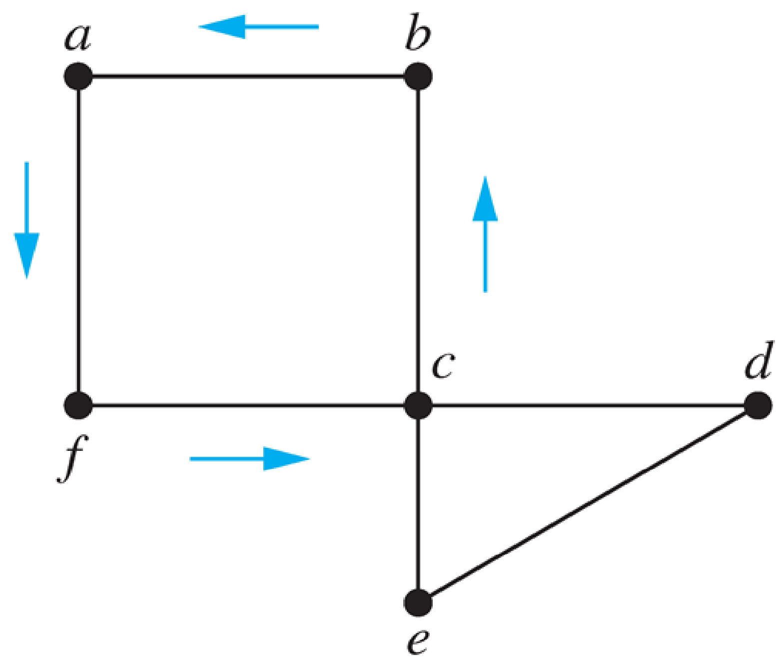
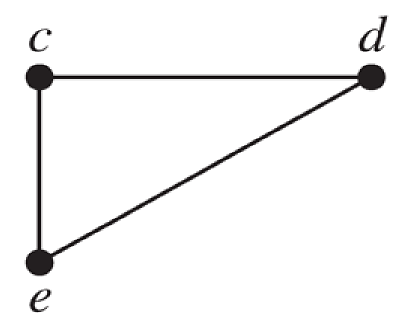


G



H

Algorithm for Constructing an Euler Circuit

- **procedure** *Euler*(G: connected multigraph with all vertices of even degree)
  circuit := a circuit in G beginning at an arbitrarily chosen vertex with edges
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  H := G with the edges of this circuit removed
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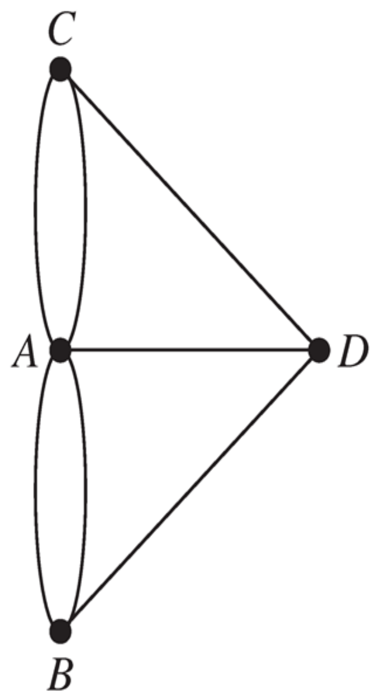

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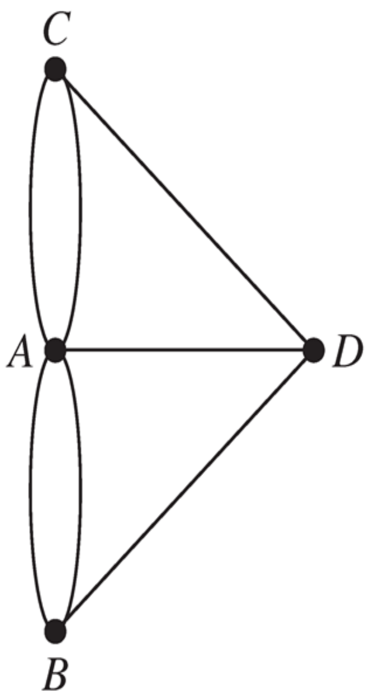


No Euler circuit

Euler Circuits and Paths

- **Example**

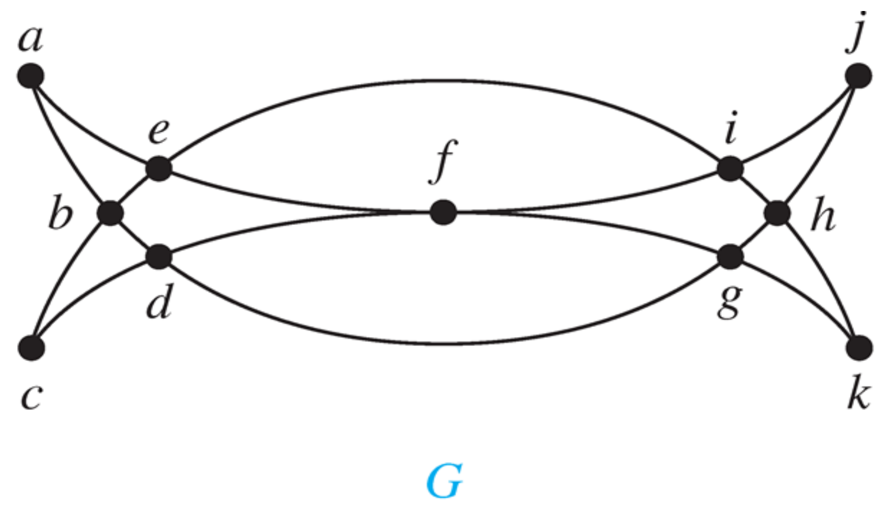
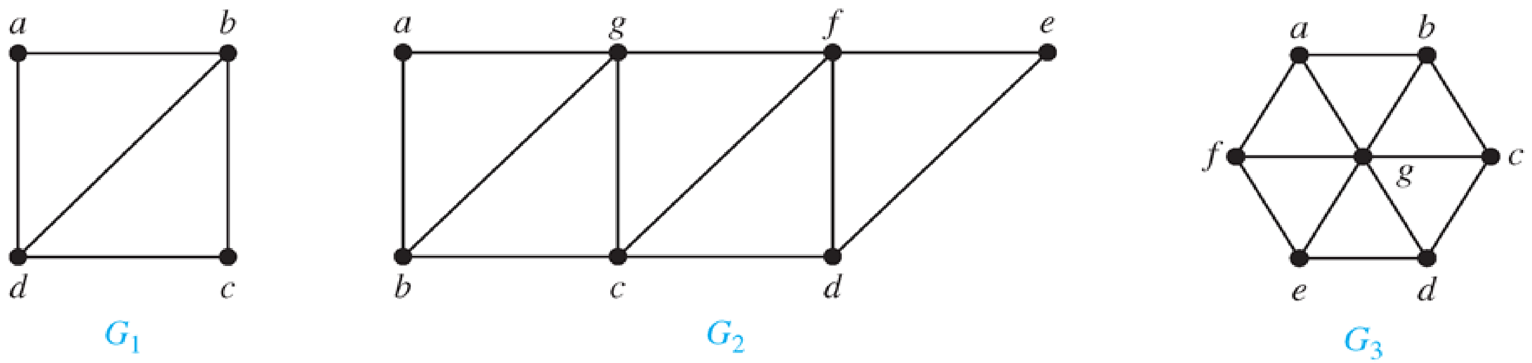


FIGURE 6 **Mohammed's Scimitars.**

- **Example**



Applications of Euler Paths and Circuits

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  - ◇ road in a transportation network
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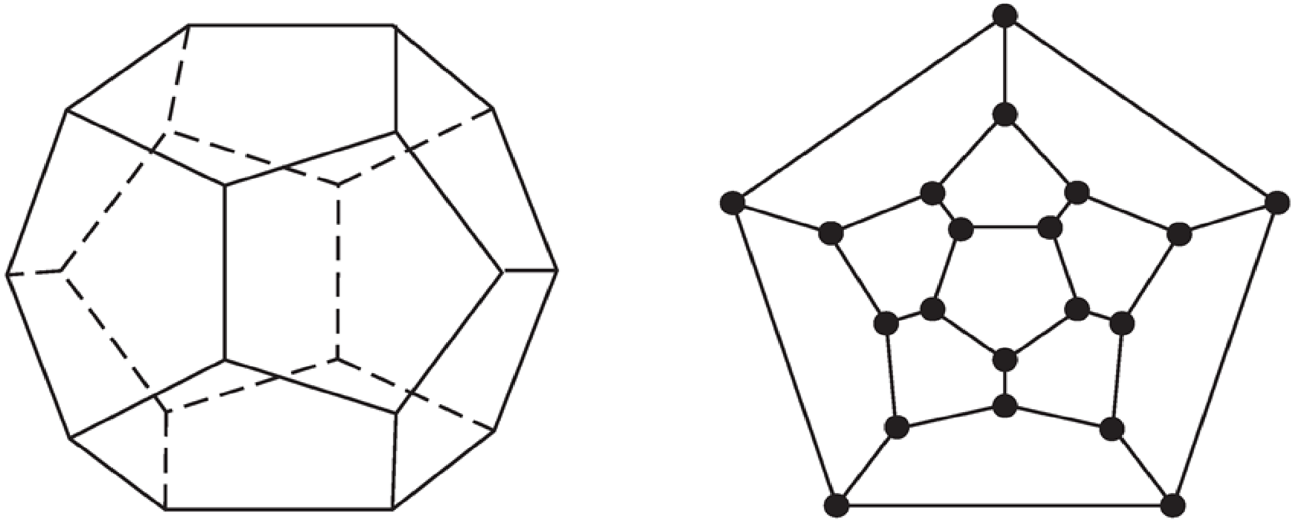

(a) (b)

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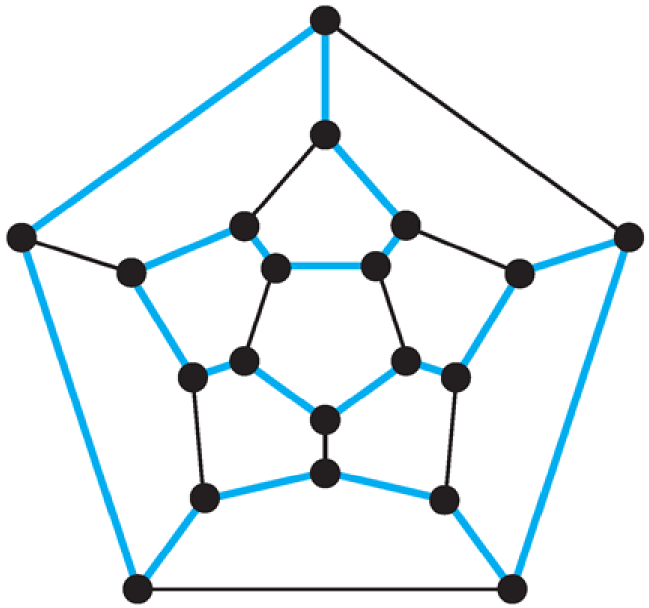

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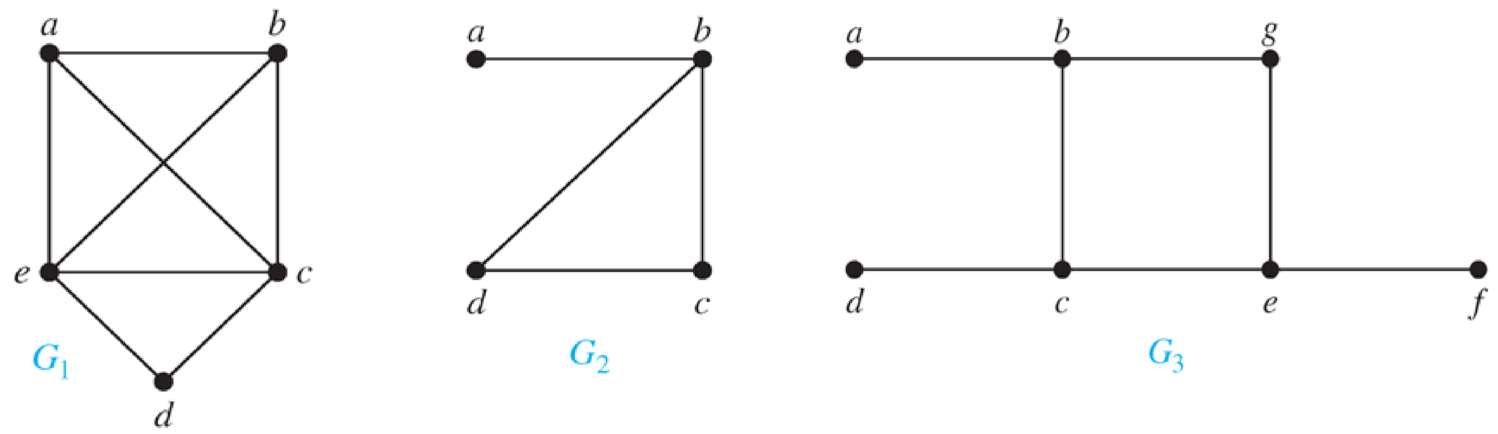

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  Hamilton path problem $\in$ NPC

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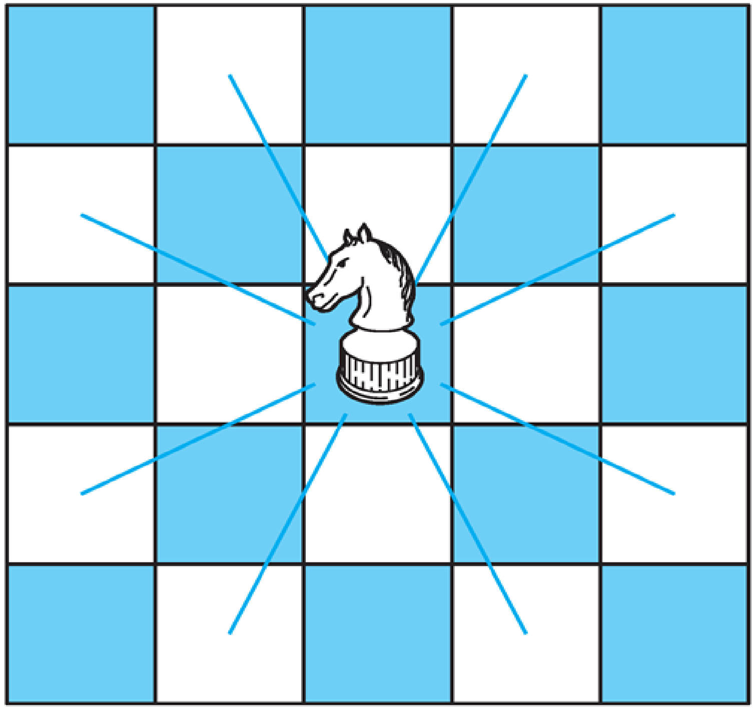

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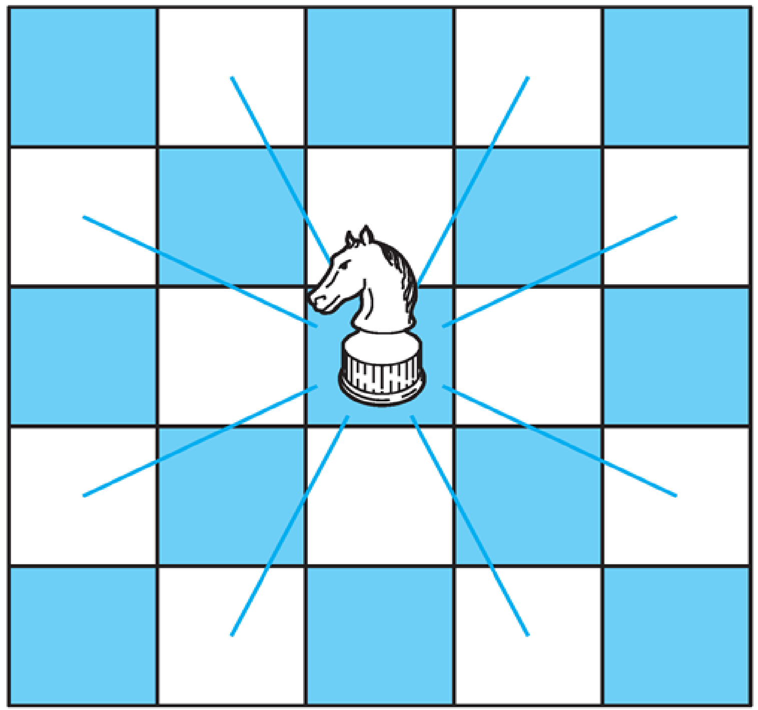

What about in 6×6 chessboard?

Next Lecture

- Graph theory III ...