Further Studies on Heuristics

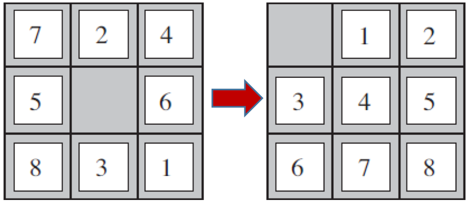
I. Search Efficiency of Heuristics

Recall: Heuristics for 8-puzzle

- $h_{mis}(s)$ = #misplaced titles $\in [0,8]$: **Admissible**.
- $h_{1stp}(s)$ = #(1-step move) to reach the goal configuration: **Admissible**.

➢ $h_{1stp}(s) \geq h_{mis}(s) \Rightarrow h_{1stp}(s)$ is '**better**' than $h_{mis}(s)$.

What does 'better' mean?



Dominance

- For **admissible** h_1 and h_2, if $h_1(s) \geq h_2(s)$ for $\forall s$

  $\Rightarrow h_1$ **dominates** h_2 and is **more efficient** for search.

- **Theorem**: For any admissible heuristics h_1 and h_2, define

$$h(s) = \max\{h_1(s), h_2(s)\}$$

$h(s)$ is admissible and dominates both h_1 and h_2.

- **'Better' heuristic = dominance = better search efficiency.**

Even Better Dominance

- **Question**: Which one to choose from a collection of admissible heuristics $h_1, \cdots, h_m$ & none dominates any other?

- **Answer**: $h(s) = \max\{h_1(s), \cdots, h_m(s)\}$ dominates all the others.

Quantify Search Efficiency

- **Effective Branching Factor b^***: For a solution from A*, calculate b^* satisfying: $N = b^* + (b^*)^2 + \cdots + (b^*)^d$
  - N: #nodes of the solution,
  - d: depth of the solution tree.
  - E.g., A* finds a solution at depth 5 using 52 nodes ⇒ $b^* = 1.92$.
- Good heuristics have b^* close to 1 ⇒ large problems solved at reasonable computational cost.
- **b^* quantifies search efficiency of heuristics.**

Empirical: Factor b^*

- **Aim**: Compare h_1 and h_2 regarding the search efficiency.
- **Setting**: Generate 1200 random problems with $d = \{2, \cdots, 24\}$ and solve them with IDS and A* with h_1 & h_2.
- **Note**: IDS – a baseline.

Empirical: Factor b^*

	Search Cost (nodes generated)			Effective Branching Factor		
d	IDS	$A^*(h_1)$	$A^*(h_2)$	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	3644035	227	73	2.78	1.42	1.24
14	–	539	113	–	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

Empirical: Factor b^*

	Search Cost (nodes generated)			Effective Branching Factor		
d	IDS	A*(h_1)	A*(h_2)	IDS	A*(h_1)	A*(h_2)
2	10	6	6	2.45	1.79	1.79
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14	–	[illegible]	[illegible]	–	[illegible]	[illegible]
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- h_2 is '**better**' than h_1 regarding **search efficiency**.
- This **goodness** is reflected by $\boldsymbol{b^*}$ **being closer to 1**.
- A$*$ with h_2 performs much better than IDS.

II. Generate Admissible Heuristics

We Know about Heuristics ...

- We know:
  - How to judge their admissibility.
  - How to compare their goodness regarding searching efficiency.

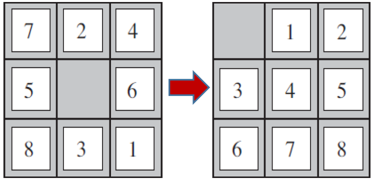
- **Question**: How to produce such ‘good’ heuristics?

(1) Generate from Relaxed Problems

Where are h_{mis} & h_{1stp} from?

For 8-puzzle problem:

- **Real Rule**: A tile can only move to the **adjacent empty** square.
- **Relaxed rules**: h_{mis} and h_{1stp} are admissible
  - R1: A tile can move **anywhere** $\Rightarrow h_{mis}(s) = \#$(misplaced titles).
  - R2: A tile can move one step in **any direction** regardless of an occupied neighbour $\Rightarrow h_{1stp}(s) = \#$(1-step move) to reach goal.
- **Optimal solutions to problems with R1, R2 are easier to find.**



Relaxed Problem

- **Relaxed problem**: a problem with **relaxed rules** on the action.
- E.g. 8-puzzle problems with R1 and R2.

- **Theorem**: The cost of an optimal solution to **a relaxed problem** is an **admissible heuristic** for the original problem.
- No wonder h_{mis} and h_{1stp} are admissible.

(2) Generate from Sub-problems

Subproblem

- Subproblem
  - **Task**: get tiles 1, 2, 3 and 4 into their correct positions.
  - **Relaxation**: move them disregarding the others.
- **Theory**: cost*(subproblem)<cost*(original).
  - cost*(subproblem): the cost of the optimal solution of this subproblem.

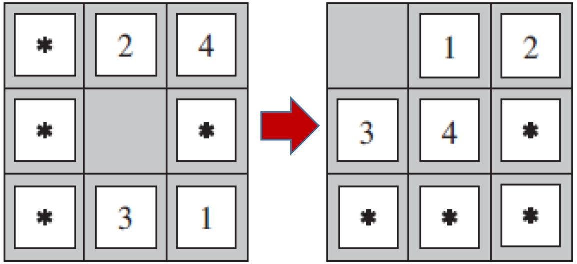


Fig.1. A subproblem of 8-puzzle.

Subproblem and Admissible Heuristics

- **Admissible $h^*_{sub}(s)$**: estimate the cost from s to the subproblem goal.
  - E.g. $h_{sub}^{(1,2,3,4)}$ is the cost to solve the 1-2-3-4 subproblem.

- **Theorem**: $h_{sub}(s)$ dominates $h_{1stp}(s)$,
  - $h_{sub}(s) = max\{h_{sub}^{(1,2,3,4)}(s), h_{sub}^{(2,3,4,5)}(s), \cdots\}$.

Disjoint Subproblems

- **Question**: Will the **addition of heuristics** from subproblem (1-2-3-4) and (5-6-7-8) give an **admissible heuristic**, considering the two subproblems are not overlapped?
- **Answer**: No, since they always **share some moves**.

- **Question**: What if **not count** those shared moves?
- **Answer**: $h_{sub}^{(1,2,3,4)}(s) + h_{sub}^{(5,6,7,8)}(s) \leq c^*(s) \Rightarrow$ admissible.
    - Disjoint pattern database

(3) Generate from Experiences

‘Experience’ Formulation

For 8-puzzle problem:

- Solve many 8-puzzles to obtain **many examples**.
- Each **example** consists of a state from the solution path and the actual cost of the solution from that point.
- These **examples** are our ‘**experience**’ for this problem.

- **Question**: How to learn $h(s)$ from these **experience**?

Learn Heuristics from Experience

- **Question**: What are the **good experience features**?
- **Answer**: **Relevant** to predicting the states' cost to Goal, e.g.
  - $x_1(s)$: #(displaced tiles).
  - $x_2(s)$: #(pairs of adjacent tiles) that are not adjacent in Goal state.
- **Question**: How to learn h from those **relevant experience features**?
- **Answer**: (e.g.) Construct model as

$$h(s) = w_1 x_1(s) + w_2 x_2(s),$$

where w_1, w_2 are model parameters to learn from training data by a learning method such as neural networks and decision trees.