**CS201: Discrete Math for Computer Science**
**2020 Fall Semester    Written Assignment #1**
**Due: Oct. 13th, 2020, please submit at the beginning of class**

Q.1 Let $p$, $q$ be the propositions
$p$: You get 100 marks on the final.
$q$: You get an A in this course.
Write these propositions using $p$ and $q$ and logical connectives (including negations).

(a) You do not get 100 marks on the final.

(b) You get 100 marks on the final, but you do not get an A in this course.

(c) You will get an A in this course if you get 100 marks on the final.

(d) If you do not get 100 marks on the final, then you will not get an A in this course.

(e) Getting 100 marks on the final is sufficient for getting an A in this course.

(f) You get an A in this course, but you do not get 100 marks on the final.

(g) Whenever you get an A in this course, you got 100 marks on the final.

Q.2 Construct a truth table for each of these compound propositions.

(a) $p \oplus \neg p$

(b) $\neg p \oplus \neg q$

(c) $(p \oplus q) \wedge (p \oplus \neg q)$

Q.3 Use truth tables to decide whether or not the following two propositions are equivalent.

(a) $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$

(b) $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$

(c) $(p \vee q) \to r$ and $(p \to r) \wedge (q \to r)$

(d) $(p \to \neg q) \leftrightarrow (r \to (p \vee \neg q))$ and $q \vee (\neg p \wedge \neg r)$

Q.4 Use logical equivalences to prove the following statements.

(a) $(p \wedge \neg q) \to r$ and $p \to (q \vee r)$ are equivalent.

(b) $(p \to q) \to ((r \to p) \to (r \to q))$ is a tautology.

Q.5

(a) Use a truth table to decide whether or not the following implication is a tautology:

$$(p \wedge (p \to q)) \to q.$$

(b) Using rules of logical equivalences, prove that

$$\neg p \to (q \to r) \equiv q \to (p \vee r).$$

Q.6 Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.

Q.7 Explain, without using a truth table, why $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true, when $p$, $q$, and $r$ have the same truth value and it is false otherwise.

Q.8 Prove that if $p \wedge q$, $p \to \neg(q \wedge r)$, $s \to r$, then $\neg s$.

Q.9 Let $P(x)$ be the statement "$x$ can speak Russian" and let $Q(x)$ be the statement "$x$ knows the computer language C++". Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

(a) There is a student at your school who can speak Russian and who knows C++.

(b) There is a student at your school who can speak Russian but who doesn't know C++.

(c) Every student at your school either can speak Russian or knows C++.

(d) No student at your school can speak Russian or knows C++.

Q.10 Let $F(x, y)$ be the statement "$x$ can fool $y$", where the domain consists of all people in the world. Use quantifiers to express each of these statement.

(a) Everybody can fool Fred.

(b) Evelyn can fool everybody.

(c) Everybody can fool somebody.

(d) There is no one who can fool everybody.

(e) Everyone can be fooled by somebody.

(f) No one can fool both Fred and Jerry.

(g) Nancy can fool exactly two people.

(h) There is exactly one person whom everybody can fool.

(i) No one can fool himself or herself.

(j) There is someone who can fool exactly one person besides himself or herself.

Q.11 Express the negations of each of these statements so that all negation symbols immediately precede predicates.

(a) $\exists x \exists y (Q(x, y) \leftrightarrow Q(y, x))$

(b) $\forall y \exists x \exists z (T(x, y, z) \vee Q(x, y))$

(c) $\forall x \exists y \forall z T(x, y, z)$

(d) $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$

Q.12

(a) Give the negation of the statement

$$\forall n \in \mathbb{N} \ (n^3 + 6n + 5 \text{ is odd} \Rightarrow n \text{ is even}).$$

(b) Either the original statement in (a) or its negation is true. Which one is it and explain why?

Q.13 For the following argument, explain which rules of inference are used for each step.

"Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program."

Q.14 Prove the **triangle inequality**, which states that if $x$ and $y$ are real numbers, then $|x| + |y| \geq |x + y|$ (where $|x|$ represents the absolute value of $x$, which equals $x$ if $x \geq 0$ and equals $-x$ if $x < 0$.

Q.15 Prove or disprove that there is a rational number $x$ and an irrational number $y$ such that $x^y$ is irrational.

Q.16 Prove or disprove that if $a$ and $b$ are rational numbers, then $a^b$ is also rational. .

Q.17 Prove that $\sqrt[3]{2}$ is irrational.

Q.18 Give a direct proof that: Let $a$ and $b$ be integers. If $a^2 + b^2$ is even, then $a + b$ is even.