南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

CS201 DISCRETE MATHEMATICS FOR COMPUTER SCIENCE

Dr. Qi Wang

Department of Computer Science and Engineering
Office: Room903, Nanshan iPark A7 Building
Email: wangqi@sustech.edu.cn

Review

01. Propositional Logic
02. Predicate Logic
03. Mathematical Proofs
04. Sets
05. Functions
06. Complexity of Algorithms
07. Number Theory
08. Cryptography
09. Mathematical Induction
10. Recursion
11. Counting
12. Relation
13. Graphs
14. Tree

Review

01. Propositional Logic
02. Predicate Logic
03. Mathematical Proofs
04. Sets
05. Functions
06. Complexity of Algorithms
07. Number Theory
08. Cryptography
09. Mathematical Induction
10. Recursion
11. Counting
12. Relation
13. Graphs
14. Tree

Discrete Probability
Groups, Rings and Fields

Logic

- Logical connectives

Logic

- Logical connectives

  $\neg p,\ p \vee q,\ p \wedge q,\ p \oplus q,\ p \rightarrow q,\ p \leftrightarrow q$

Logic

- Logical connectives

  $\neg p,\ p \vee q,\ p \wedge q,\ p \oplus q,\ p \rightarrow q,\ p \leftrightarrow q$

- Logical equivalence

Logic

- Logical connectives

  $\neg p,\ p \vee q,\ p \wedge q,\ p \oplus q,\ p \rightarrow q,\ p \leftrightarrow q$

- Logical equivalence

  De Morgan's laws, communtative laws, distributive laws, ...

Logic

- Logical connectives

  $\neg p,\ p \vee q,\ p \wedge q,\ p \oplus q,\ p \rightarrow q,\ p \leftrightarrow q$

- Logical equivalence

  De Morgan's laws, communtative laws, distributive laws, ...

- Predicate logic

  contains variables

Logic

- Logical connectives

  $\neg p$, $p \vee q$, $p \wedge q$, $p \oplus q$, $p \to q$, $p \leftrightarrow q$

- Logical equivalence

  De Morgan's laws, communtative laws, distributive laws, ...

- Predicate logic

  contains variables

- Quantified statements

  universal, existential, equivalence

Methods of Proving Theorems

- Basic methods to prove theorems:
  - ◇ *direct proof*
    - – $p \to q$ is proved by showing that if p is true then q follows
  - ◇ *proof by contrapositive*
    - – show the contrapositive $\neg q \to \neg p$
  - ◇ *proof by contradiction*
    - – show that $(p \land \neg q)$ contradicts the assumptions
  - ◇ *proof by cases*
    - – give proofs for all possible cases
  - ◇ *proof of equivalence*
    - – $p \leftrightarrow q$ is replaced with $(p \to q) \land (q \to p)$

Function

- function?

Function

- function?

  one-to-one (injective) function?

Function

- function?

  one-to-one (injective) function?

  onto (surjective) function?

Function

- function?

  one-to-one (injective) function?

  onto (surjective) function?

  bijective function (one-to-one correspondence)?

Function

- function?

  one-to-one (injective) function?

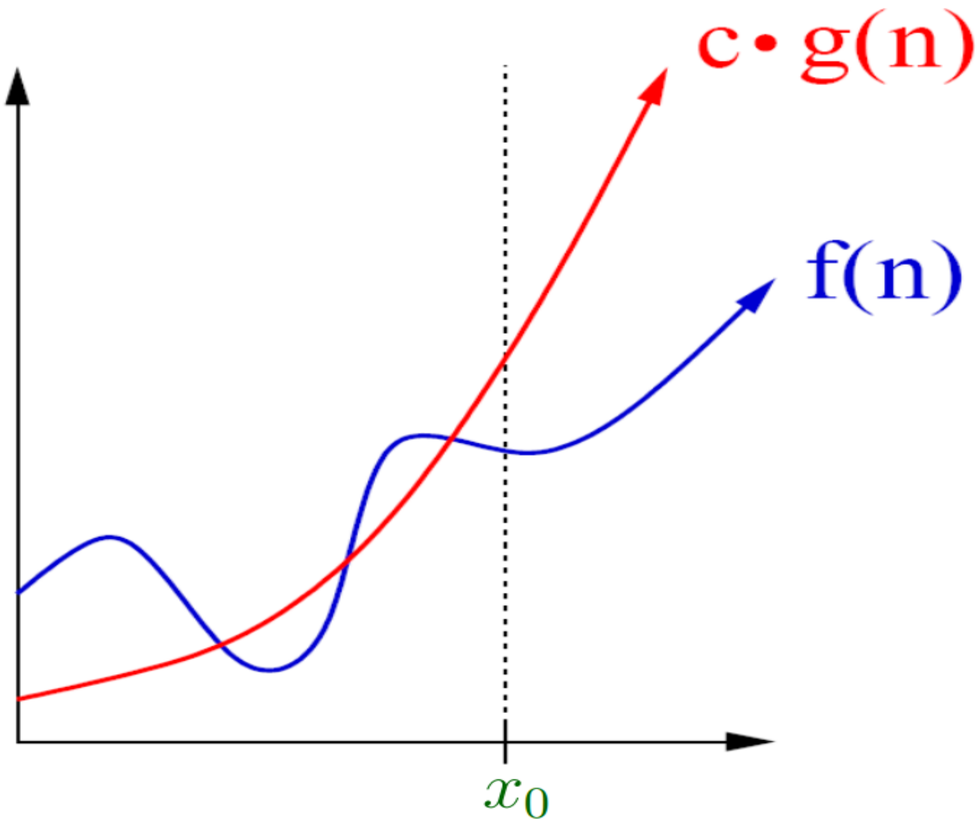
  onto (surjective) function?

  bijective function (one-to-one correspondence)?

- counting the number of such functions?

Big-O Notation

- Let f and g be functions from the set of integers or the set of real numbers to the set of real numbers. We say that $f(n) = O(g(n))$ (reads: $f(n)$ is O of $g(n)$), if there exist some positive constants C and k such that $|f(n)| \leq C|g(n)|$, whenever $n > k$.



Number Theory

- Divisibility

Number Theory

- Divisibility

  Congruence relation

Number Theory

- Divisibility

  Congruence relation

  Primes

Number Theory

- Divisibility

  Congruence relation

  Primes

  GCD and Euclidean Algorithm

Number Theory

- Divisibility

  Congruence relation

  Primes

  GCD and Euclidean Algorithm

  Modular Inverse

Number Theory

- Divisibility

  Congruence relation

  Primes

  GCD and Euclidean Algorithm

  Modular Inverse

  When does an inverse of a modulo m exist?

  How to find inverses?

Number Theory

- Divisibility

  Congruence relation

  Primes

  GCD and Euclidean Algorithm

  Modular Inverse

  When does an inverse of a modulo m exist?

  How to find inverses?

  Chinese Remainder Theorem

Number Theory

- Divisibility

  Congruence relation

  Primes

  GCD and Euclidean Algorithm

  Modular Inverse

    When does an inverse of a modulo m exist?

    How to find inverses?

  Chinese Remainder Theorem

  Back substitution

Number Theory

- Divisibility

  Congruence relation

  Primes

  GCD and Euclidean Algorithm

  Modular Inverse

  When does an inverse of a modulo m exist?

  How to find inverses?

  Chinese Remainder Theorem

  Back substitution

$$
\begin{aligned}
x &\equiv 2 \pmod{3} \\
x &\equiv 3 \pmod{5} \\
x &\equiv 2 \pmod{7}
\end{aligned}
$$

- Fermat's Little Theorem

Cryptography

- Fermat's Little Theorem

  Euler's Theorem

  Primitive roots, multiplicative order

Cryptography

- Fermat's Little Theorem

  Euler's Theorem

  Primitive roots, multiplicative order

  RSA cryptosystem

  DLP, Diffie-Hellman protocol

Mathematical Induction

- A *typical* proof by mathematical induction, showing that a statement $P(n)$ is true for all integers $n \geq b$ consists of three steps:

Mathematical Induction

- A *typical* proof by mathematical induction, showing that a statement $P(n)$ is true for all integers $n \geq b$ consists of three steps:

  1. We show that $P(b)$ is true. – Base Step

Mathematical Induction

- A *typical* proof by mathematical induction, showing that a statement $P(n)$ is true for all integers $n \geq b$ consists of three steps:

  1. We show that $P(b)$ is true. – Base Step
  2. We then, $\forall n > b$, show either

$$(*) \qquad P(n-1) \rightarrow P(n)$$

or

$$(**) \qquad P(b) \wedge P(b+1) \wedge \cdots \wedge P(n-1) \rightarrow P(n)$$

Mathematical Induction

- A *typical* proof by mathematical induction, showing that a statement $P(n)$ is true for all integers $n \geq b$ consists of three steps:

1. We show that $P(b)$ is true. – Base Step
2. We then, $\forall n > b$, show either

$$(*) \qquad P(n-1) \rightarrow P(n)$$

or

$$(**) \qquad P(b) \wedge P(b+1) \wedge \cdots \wedge P(n-1) \rightarrow P(n)$$

We need to make the inductive hypothesis of either $P(n-1)$ or $P(b) \wedge P(b+1) \wedge \cdots \wedge P(n-1)$.We then use $(*)$ or $(**)$ to derive $P(n)$.

Mathematical Induction

- A *typical* proof by mathematical induction, showing that a statement $P(n)$ is true for all integers $n \geq b$ consists of three steps:

1. We show that $P(b)$ is true. – Base Step

2. We then, $\forall n > b$, show either

$$(*) \qquad P(n-1) \rightarrow P(n)$$

or

$$(**) \qquad P(b) \wedge P(b+1) \wedge \cdots \wedge P(n-1) \rightarrow P(n)$$

   We need to make the inductive hypothesis of either $P(n-1)$ or $P(b) \wedge P(b+1) \wedge \cdots \wedge P(n-1)$.We then use $(*)$ or $(**)$ to derive $P(n)$.

3. We conclude on the basis of the principle of mathematical induction that $P(n)$ is true for all $n \geq b$.

Recurrence

- Iterating a recurrence

Recurrence

- Iterating a recurrence

  bottom up or top down

Recurrence

- Iterating a recurrence

  bottom up or top down

  prove by induction, complexity, ...

Counting

- The sum rule and product rule

Counting

- The sum rule and product rule

  The Inclusion-Exclusion Principle

Counting

- The sum rule and product rule

  The Inclusion-Exclusion Principle

  The Pigeonhole Principle

Counting

- The sum rule and product rule

  The Inclusion-Exclusion Principle

  The Pigeonhole Principle

  Theorem If N is a positive integer and k is an integer with $1 \leq k \leq n$, then there are
  $P(n, k) = n(n-1)(n-2)\cdots(n-k+1)$
  k-element permutations with n distinct elements.

Counting

- The sum rule and product rule

  The Inclusion-Exclusion Principle

  The Pigeonhole Principle

  Theorem If N is a positive integer and k is an integer with $1 \leq k \leq n$, then there are
  $P(n,k) = n(n-1)(n-2)\cdots(n-k+1)$
  k-element permutations with n distinct elements.

  $$P(n,3) = 3! \cdot C(n,3)$$

Counting

- The sum rule and product rule

  The Inclusion-Exclusion Principle

  The Pigeonhole Principle

  Theorem If N is a positive integer and k is an integer with $1 \leq k \leq n$, then there are
  $P(n, k) = n(n-1)(n-2)\cdots(n-k+1)$
  k-element permutations with n distinct elements.

  $$P(n, 3) = 3! \cdot C(n, 3)$$

  Pascal's Triangle, Identity

Counting

- The sum rule and product rule

  The Inclusion-Exclusion Principle

  The Pigeonhole Principle

  Theorem If N is a positive integer and k is an integer with $1 \leq k \leq n$, then there are
  $P(n, k) = n(n-1)(n-2)\cdots(n-k+1)$
  k-element permutations with n distinct elements.

  $P(n, 3) = 3! \cdot C(n, 3)$

  Pascal's Triangle, Identity

  The Binomial Theorem, Trinomial

Binary Relations

- Properties of relations

Binary Relations

- Properties of relations
- Representing relations

Binary Relations

- Properties of relations

  Representing relations

  Closures on relations

Binary Relations

- Properties of relations

  Representing relations

  Closures on relations

  Equivalence relation

  Definition A relation R on a set A is called an *equivalence relation* if it is reflexive, symmetric, and transitive.

Binary Relations

- Properties of relations

  Representing relations

  Closures on relations

  Equivalence relation

  Definition A relation R on a set A is called an *equivalence relation* if it is reflexive, symmetric, and transitive.

  Partial ordering

Binary Relations

- Properties of relations

  Representing relations

  Closures on relations

  Equivalence relation

  Definition A relation R on a set A is called an *equivalence relation* if it is reflexive, symmetric, and transitive.

  Partial ordering

  Definition A relation R on a set A is called a *partial ordering* if it is reflexive, antisymmetric, and transitive.

Graphs & Trees

- Basic concepts

Graphs & Trees

- Basic concepts

  connected graph, simple graph, isomophism, chromatic number, Euler circuit, Hamilton circuit, shortest path, bipartite graph, complete graph, special graphs (K_n, $K_{m,n}$, C_n, W_n), m-ary tree, tree traversal, spanning tree ...

Next Lecture

- Good Luck!