

# Practical Machine Learning

## Day 12: Sep22 DBDA

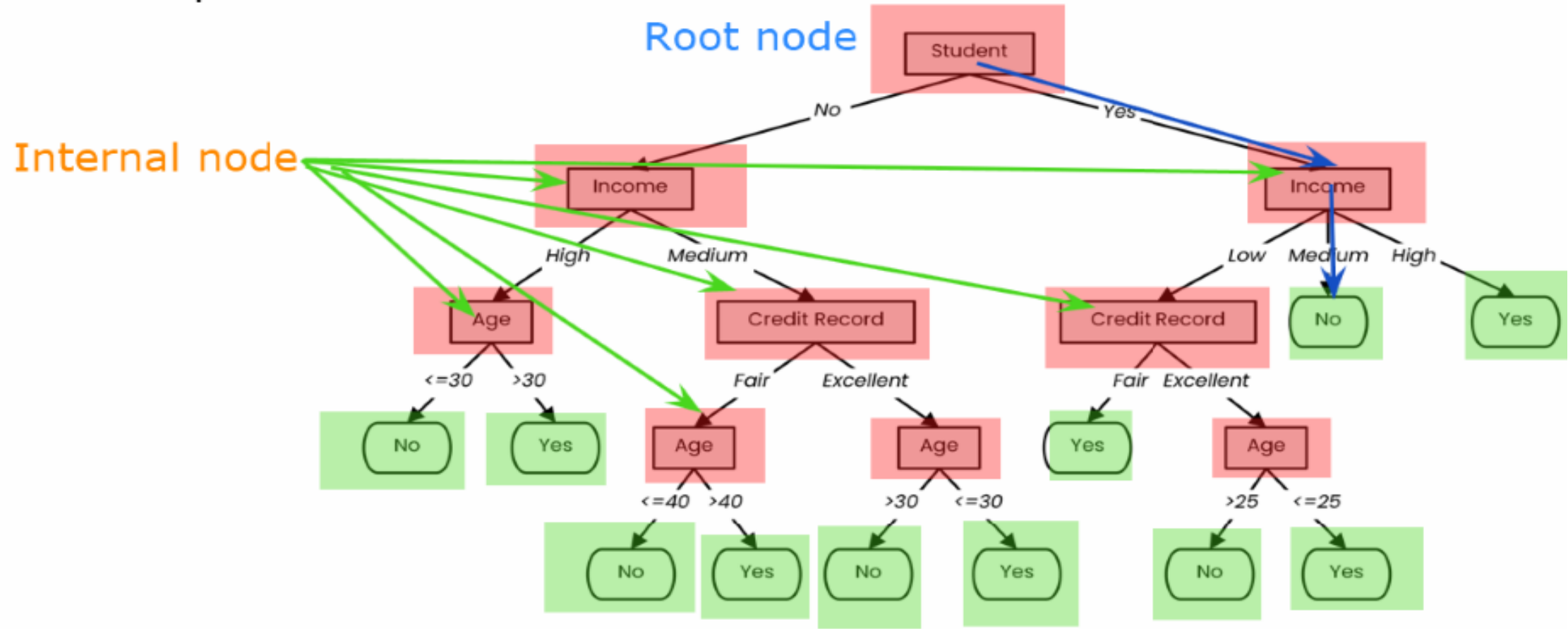
Kiran Waghmare

# Agenda

- Decision Tree
- Random Forest

# Definition

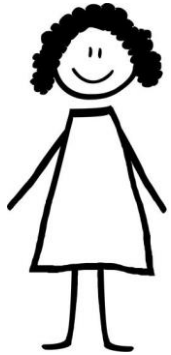
- A tree-like model that illustrates series of events leading to certain decisions
- Each node represents a test on an attribute and each branch is an outcome of that test



# Definition

- A tree-like model that illustrates series of events leading to certain decisions
- Each node represents a test on an attribute and each branch is an outcome of that test

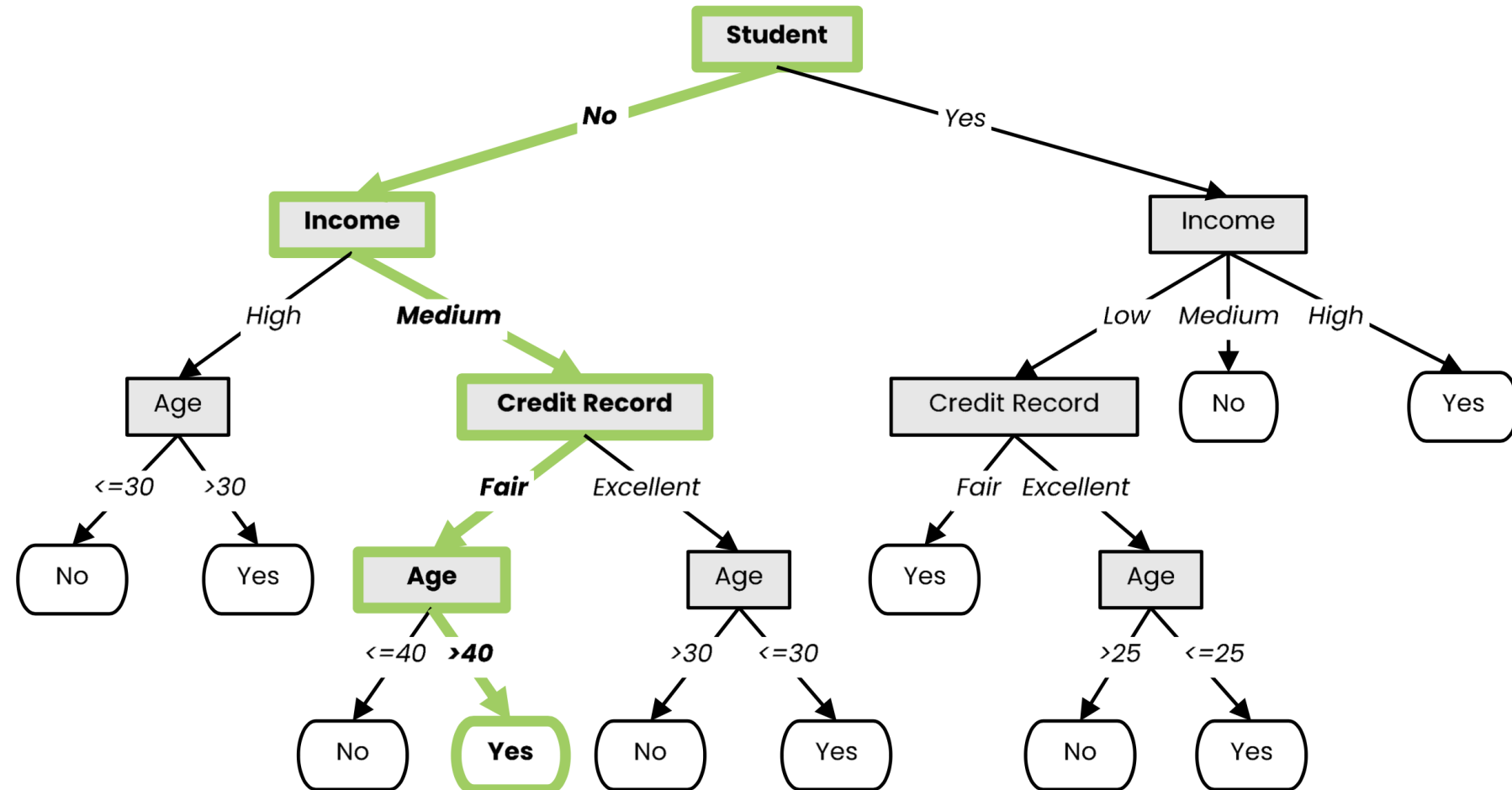
## Who to loan?



- Not a student
- 45 years old
- Medium income
- Fair credit



- Student
- 27 years old
- Low income
- Excellent credit



# Decision Tree Learning

- Basic step: choose an attribute and, based on its values, split the data into smaller sets
  - Recursively repeat this step until we can surely decide the label

Outlook = Sunny

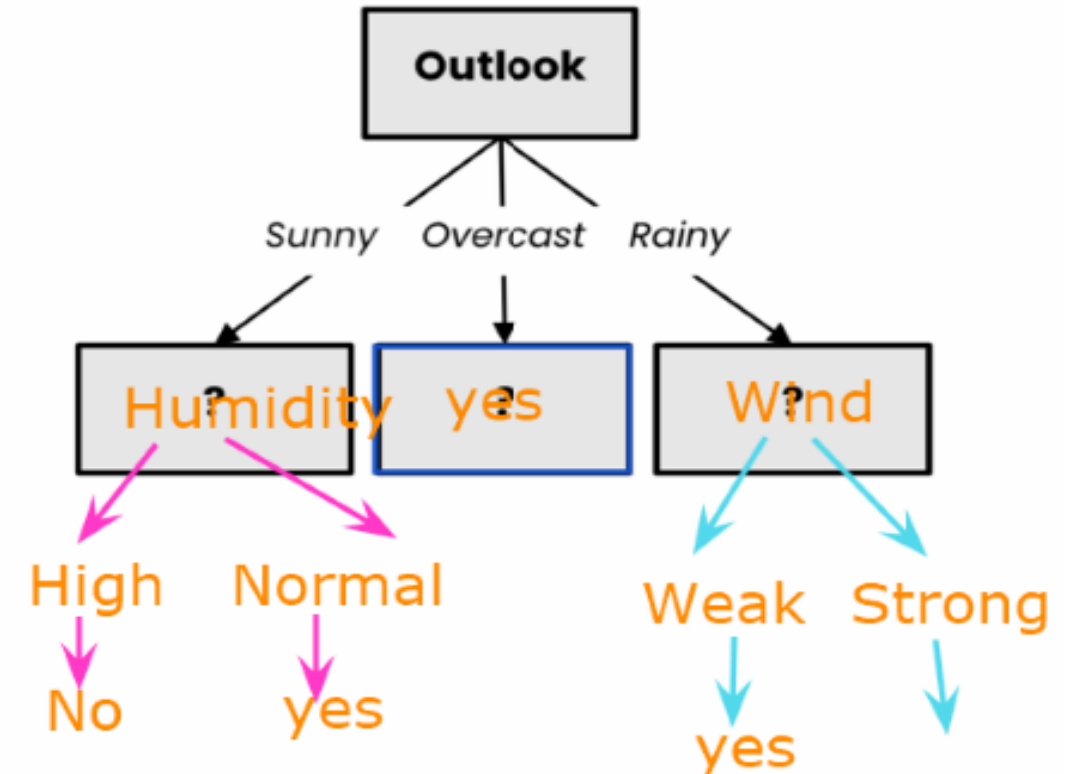
Temperature	Humidity	Wind	Play Tennis?
Hot	High	Weak	No
Hot	High	Strong	No
Mild	High	Weak	No
Cool	Normal	Weak	Yes
Mild	Normal	Strong	Yes

Outlook = Overcast

Temperature	Humidity	Wind	Play Tennis?
Hot	High	Weak	Yes
Cool	Normal	Strong	Yes
Mild	High	Strong	Yes
Hot	Normal	Weak	Yes

Outlook = Rainy

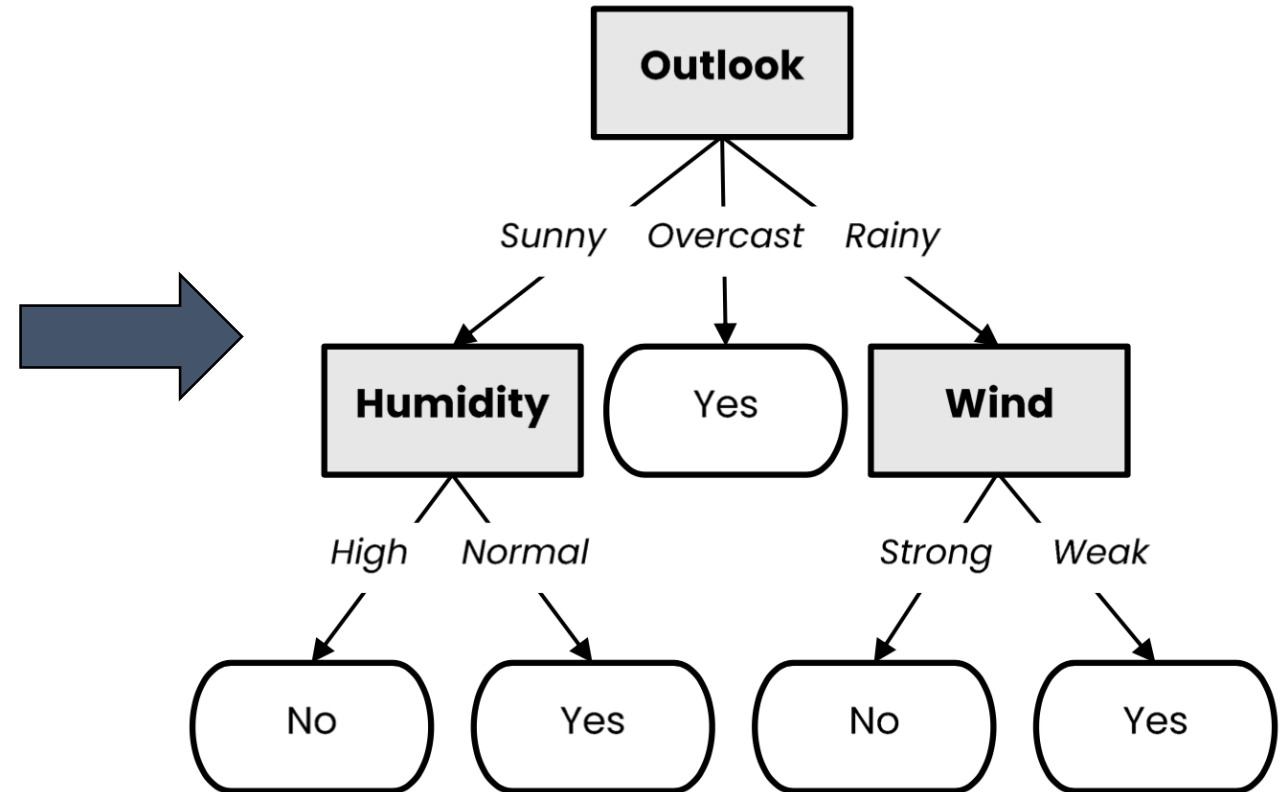
Temperature	Humidity	Wind	Play Tennis?
Mild	High	Weak	Yes
Cool	Normal	Weak	Yes
Cool	Normal	Strong	No
Mild	Normal	Weak	Yes
Mild	High	Strong	No



# Decision Tree Learning

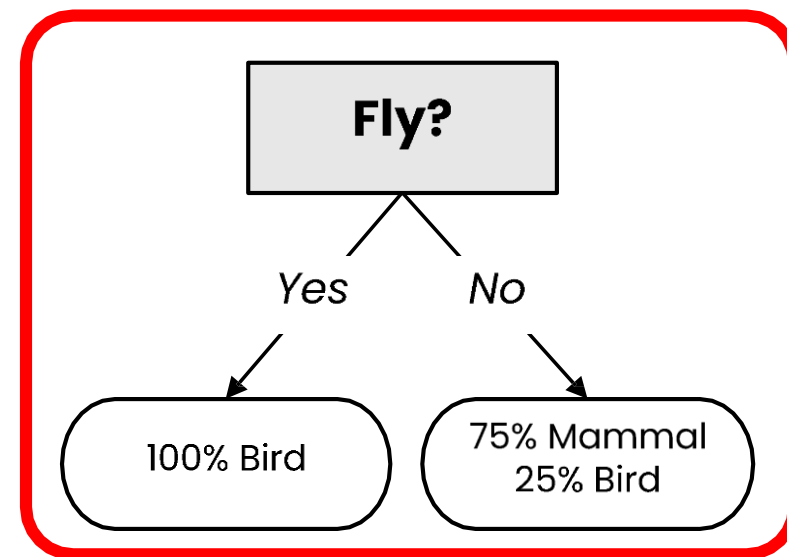
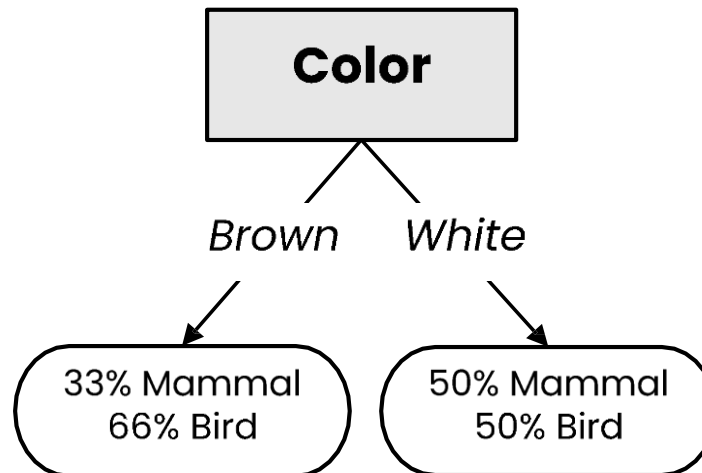
- We use labeled data to obtain a suitable decision tree for future predictions
  - We want a decision tree that works well on unseen data, while asking as few questions as possible

Outlook	Temperature	Humidity	Wind	Play Tennis?
Sunny	Hot	High	Weak	No
Sunny	Hot	High	Strong	No
Overcast	Hot	High	Weak	Yes
Rainy	Mild	High	Weak	Yes
Rainy	Cool	Normal	Weak	Yes
Rainy	Cool	Normal	Strong	No
Overcast	Cool	Normal	Strong	Yes
Sunny	Mild	High	Weak	No
Sunny	Cool	Normal	Weak	Yes
Rainy	Mild	Normal	Weak	Yes
Sunny	Mild	Normal	Strong	Yes
Overcast	Mild	High	Strong	Yes
Overcast	Hot	Normal	Weak	Yes
Rainy	Mild	High	Strong	No



# What is a good attribute?

Does it fly?	Color	Class
No	Brown	Mammal
No	White	Mammal
Yes	Brown	Bird
Yes	White	Bird
No	White	Mammal
No	Brown	Bird
Yes	White	Bird



- Which attribute provides **better** splitting?
- Why?
  - Because the resulting subsets are more **pure**
  - Knowing the value of this attribute gives us **more information** about the label  
(the entropy of the subsets is lower)



# Entropy

- Entropy measures the degree of randomness in data

Low entropy



High entropy

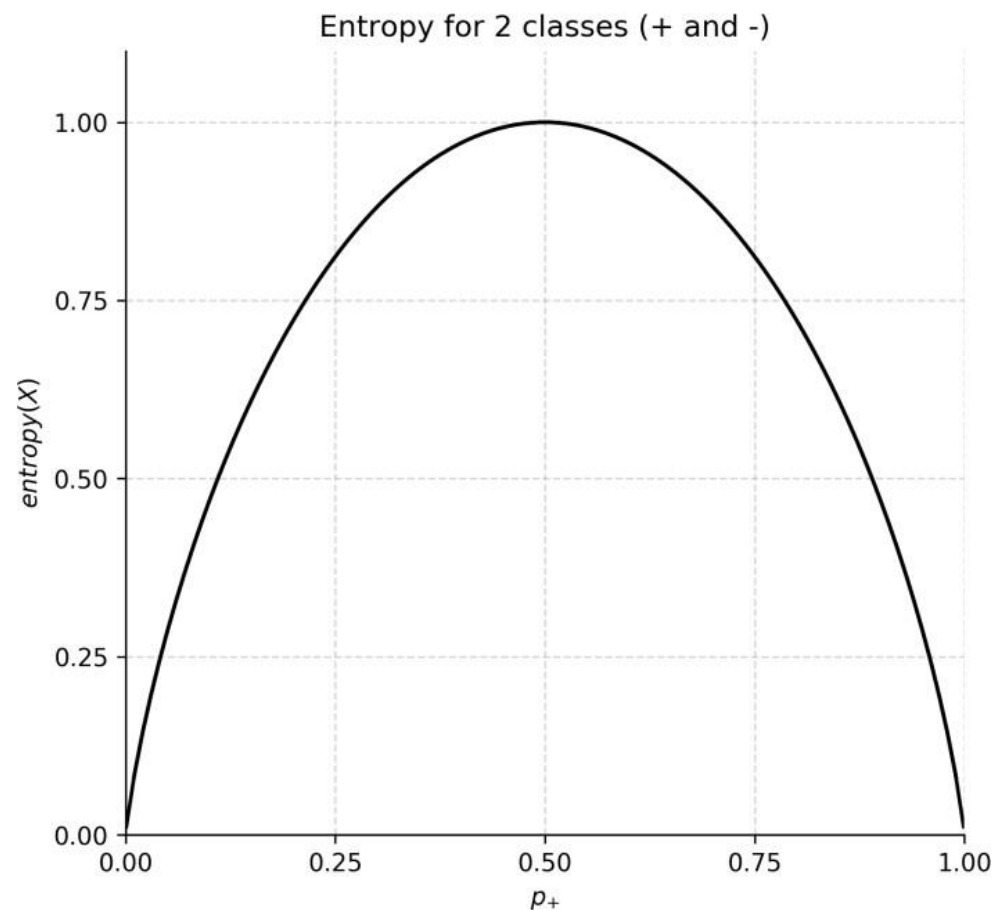


- For a set of samples  $X$  with  $k$  classes:

$$\text{entropy}(X) = - \sum_{i=1}^k p_i \log_2(p_i)$$

where  $p_i$  is the proportion of elements of class  $i$

- Lower entropy implies greater predictability!





# Information Gain

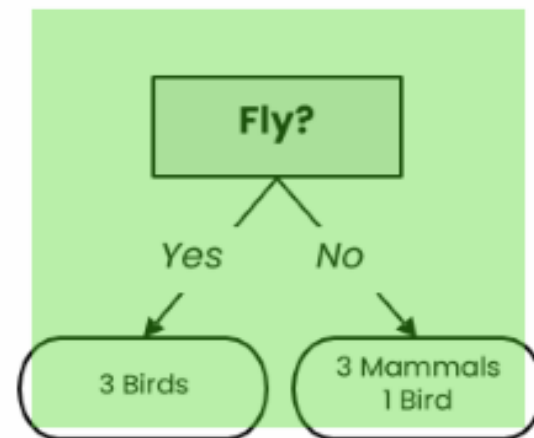
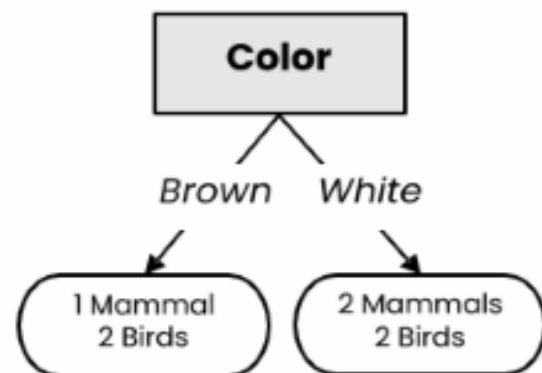
- The information gain of an attribute  $a$  is the expected reduction in entropy due to splitting on values of  $a$ :

$$gain(X, a) = \overset{y}{\boxed{entropy(X)}} - \sum_{v \in Values(a)} \frac{|X_v|}{|X|} entropy(X_v)$$

where  $X_v$  is the subset of  $X$  for which  $a = v$

# Best attribute = highest information gain

Does it fly?	Color	Class
No	Brown	Mammal
No	White	Mammal
Yes	Brown	Bird
Yes	White	Bird
No	White	Mammal
No	Brown	Bird
Yes	White	Bird



$$\text{entropy}(X) = -p_{\text{mammal}} \log_2 p_{\text{mammal}} - p_{\text{bird}} \log_2 p_{\text{bird}} = -\frac{3}{7} \log_2 \frac{3}{7} - \frac{4}{7} \log_2 \frac{4}{7} \approx 0.985$$

$$\text{entropy}(X_{\text{color}=\text{brown}}) = -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} \approx 0.918$$

$$\text{entropy}(X_{\text{color}=\text{white}}) = 1$$

$$\text{gain}(X, \text{color}) = 0.985 - \frac{3}{7} \cdot 0.918 - \frac{4}{7} \cdot 1 \approx 0.020$$

$$\text{entropy}(X_{\text{fly}=\text{yes}}) = 0$$

$$\text{entropy}(X_{\text{fly}=\text{no}}) = -\frac{3}{4} \log_2 \frac{3}{4} - \frac{1}{4} \log_2 \frac{1}{4} \approx 0.811$$

$$\text{gain}(X, \text{fly}) = 0.985 - \frac{3}{7} \cdot 0 - \frac{4}{7} \cdot 0.811 \approx 0.521$$

# Gini Impurity

- Gini impurity measures how often a randomly chosen example would be incorrectly labeled if it was randomly labeled according to the label distribution



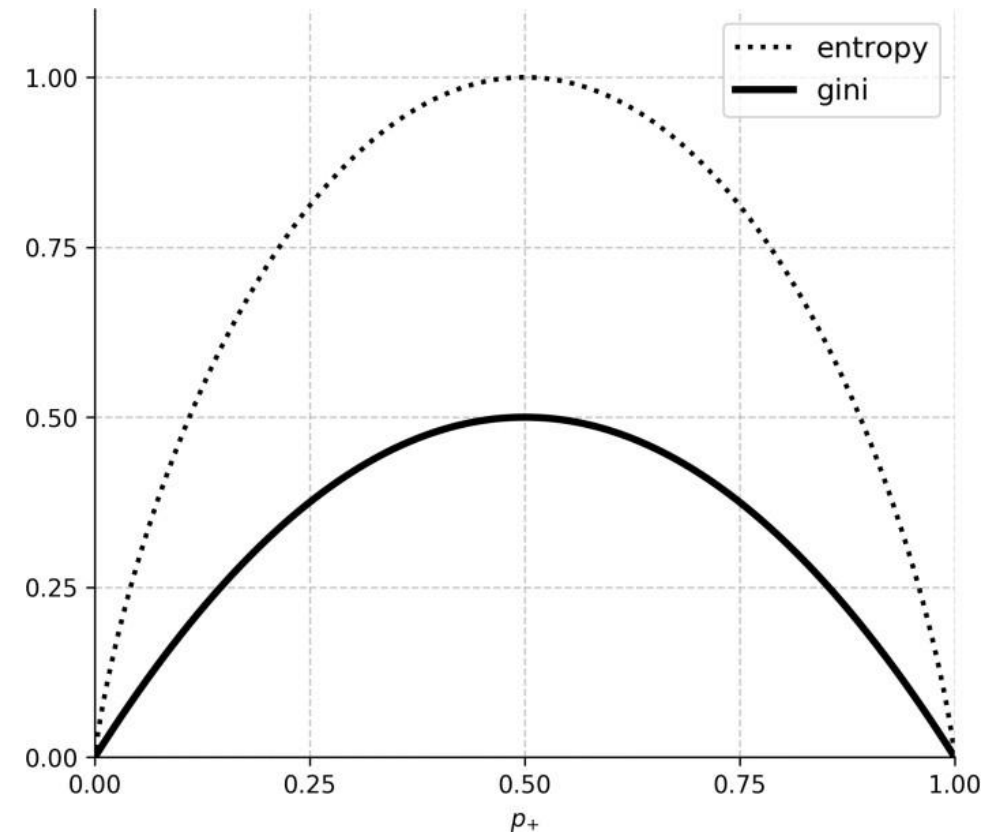
Error of classifying  
randomly picked  
fruit with randomly  
picked label



- For a set of samples  $X$  with  $k$  classes:

$$gini(X) = 1 - \sum_{i=1}^k p_i^2$$

where  $p_i$  is the proportion of elements of class  $i$

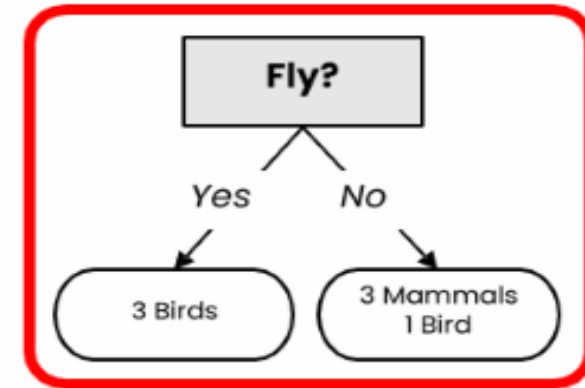
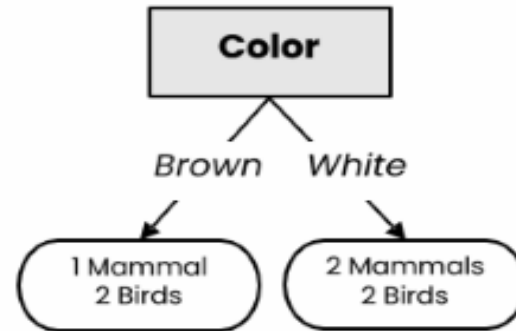


- Can be used as an alternative to entropy for selecting attributes!

# Best attribute = highest impurity decrease

In practice, we compute  $gini(X)$  only once!

Does it fly?	Color	Class
No	Brown	Mammal
No	White	Mammal
Yes	Brown	Bird
Yes	White	Bird
No	White	Mammal
No	Brown	Bird
Yes	White	Bird



$$gini(X) = 1 - \left(\frac{3}{7}\right)^2 - \left(\frac{4}{7}\right)^2 \approx 0.489$$

$$gini(X_{color=brown}) = 1 - \left(\frac{1}{3}\right)^2 - \left(\frac{2}{3}\right)^2 \approx 0.444$$

$$gini(X_{color=white}) = 0.5$$

$$\Delta gini(X, color) = 0.489 - \frac{3}{7} \cdot 0.444 - \frac{4}{7} \cdot 0.5 \approx 0.013$$

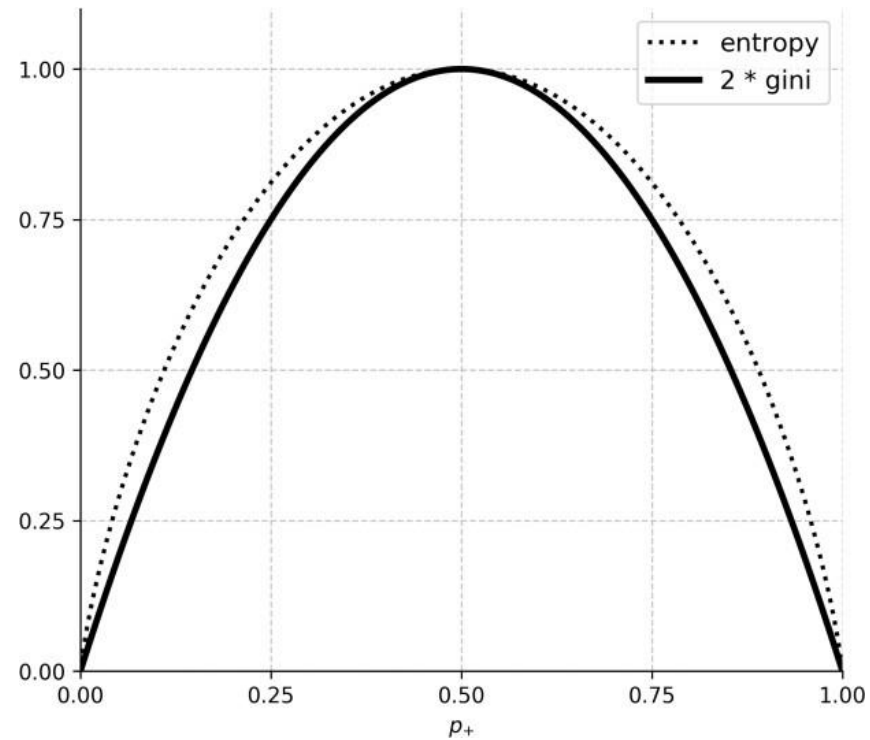
$$gini(X_{fly=yes}) = 0$$

$$gini(X_{fly=no}) = 1 - \left(\frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2 \approx 0.375$$

$$\Delta gini(X, fly) = 0.489 - \frac{3}{7} \cdot 0 - \frac{4}{7} \cdot 0.375 \approx 0.274$$

# Entropy versus Gini Impurity

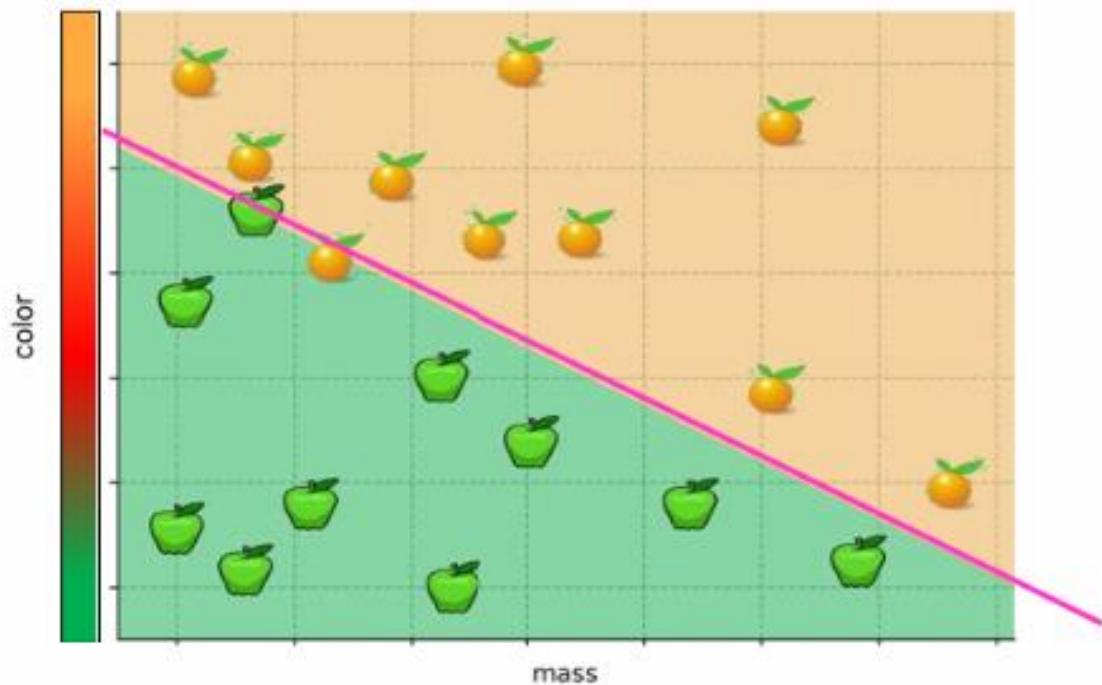
- Entropy and Gini Impurity give similar results in practice
  - They only disagree in about 2% of cases
    - “Theoretical Comparison between the Gini Index and Information Gain Criteria”  
[Răileanu & Stoffel, AMAI 2004]
  - Entropy might be slower to compute, because of the log



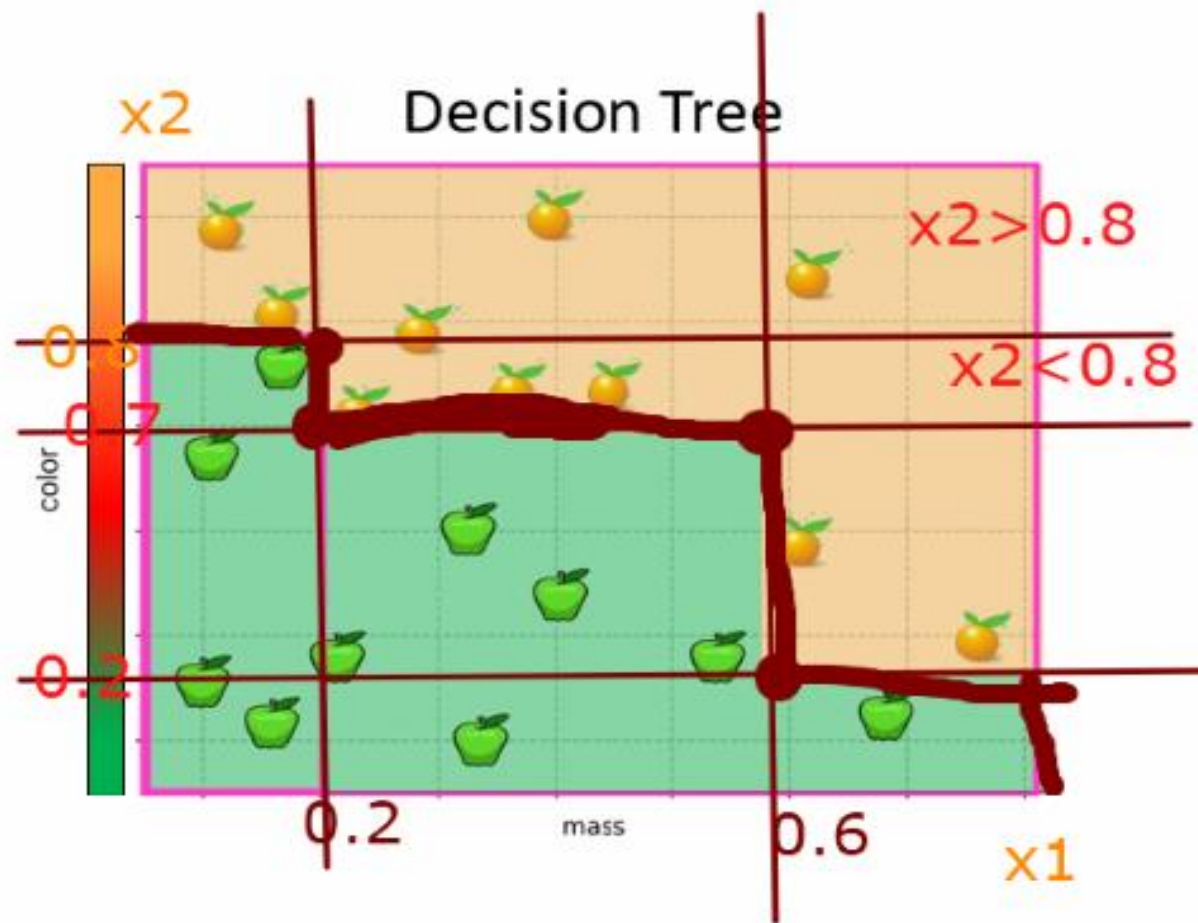
# Decision Boundaries

- Decision trees produce non-linear decision boundaries

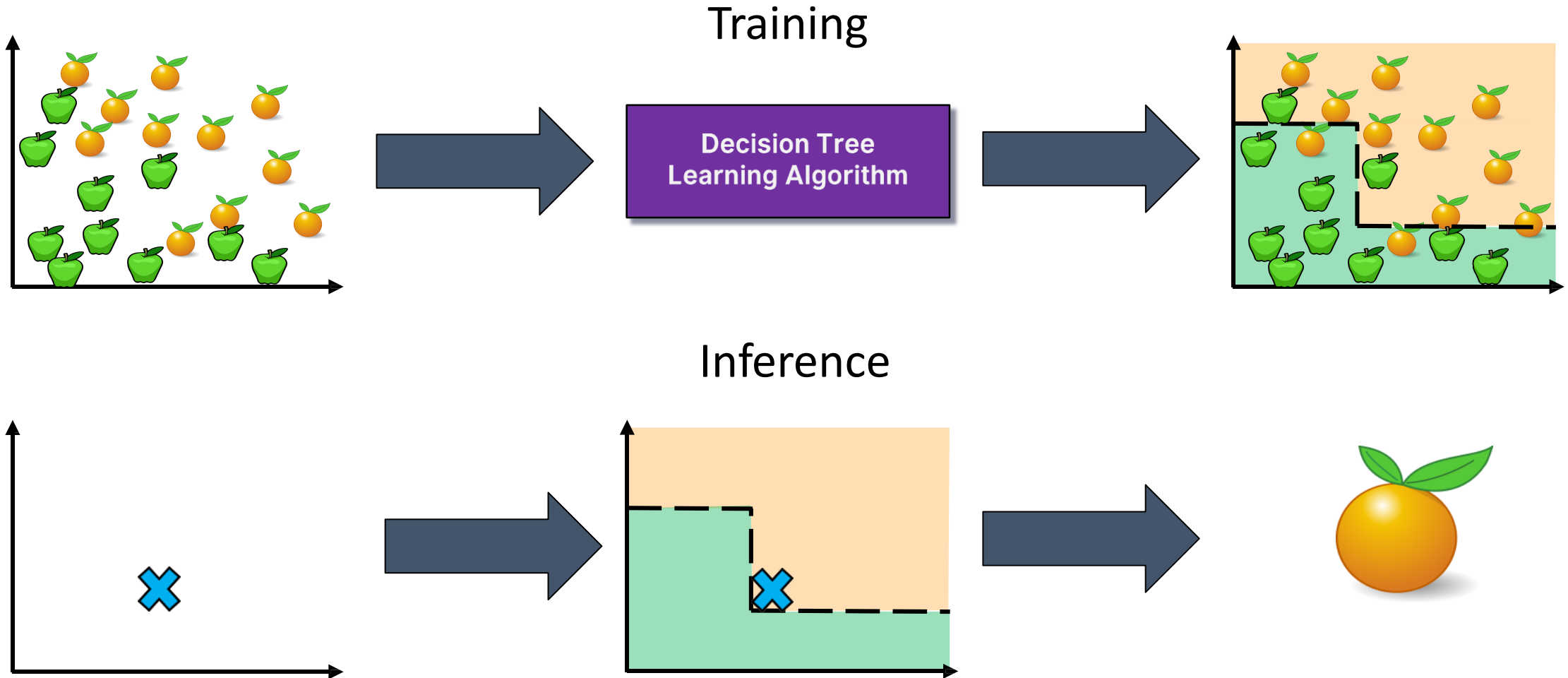
Support Vector Machines



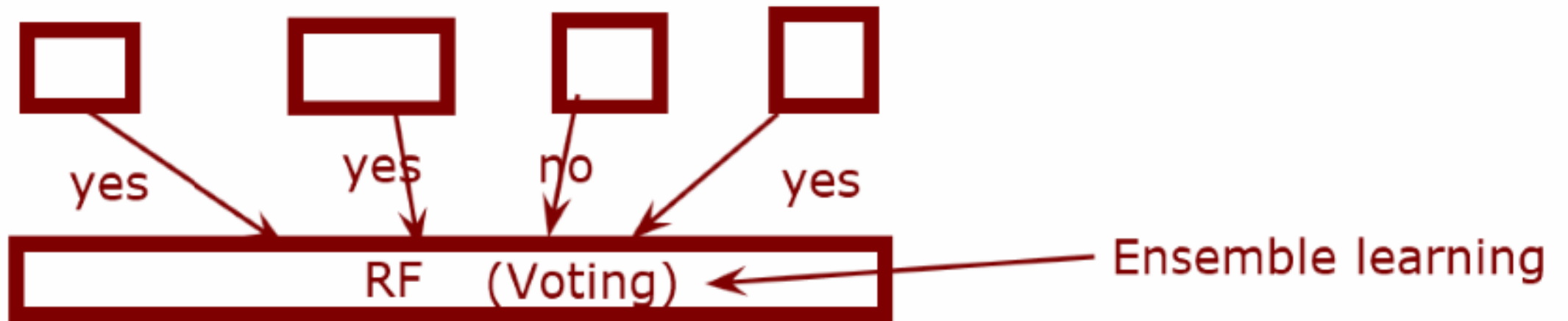
Decision Tree



# Decision Trees: Training and Inference



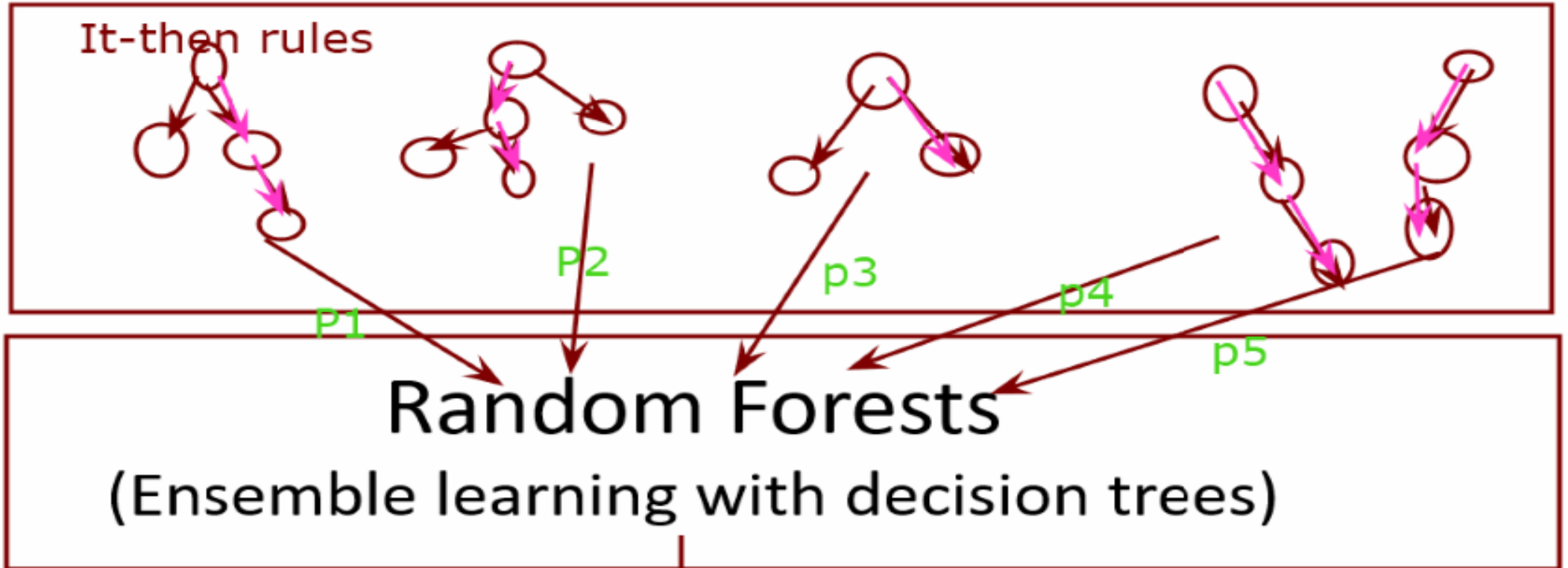




# Random Forests

(Ensemble learning with decision trees)





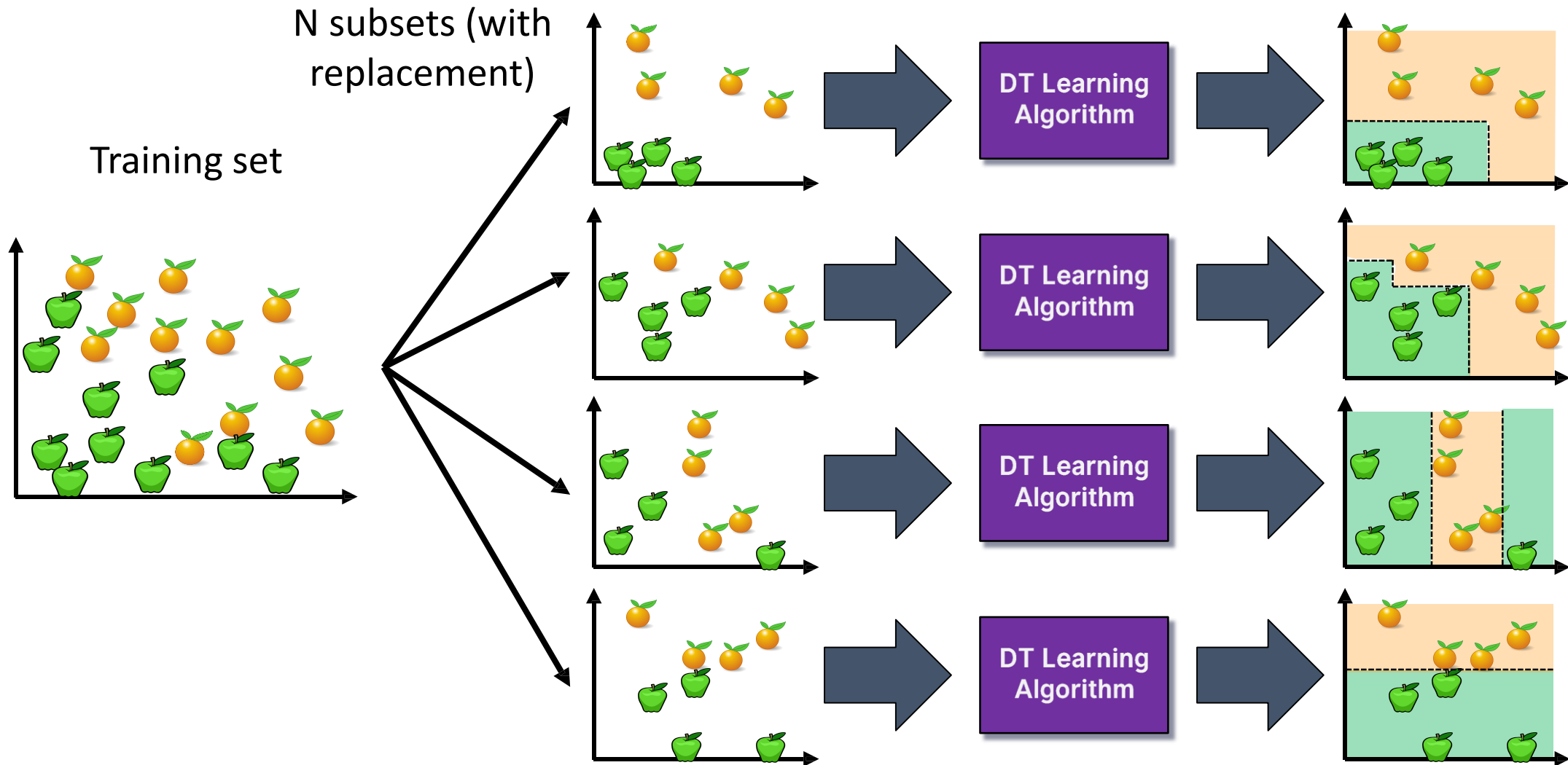
1. Bagging  
(Bootstrap Aggregating)

2. Random Subspace Method  
(Feature Bagging)

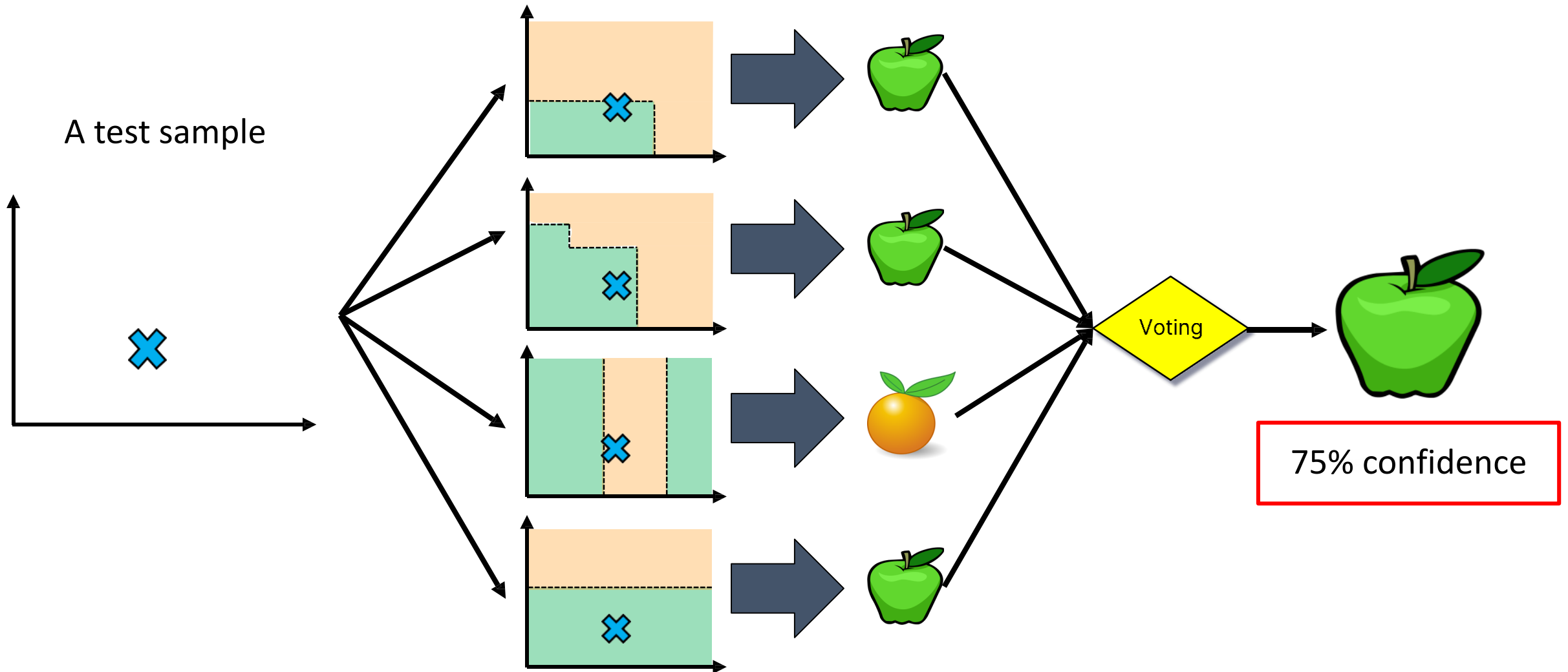
# Random Forests

- Random Forests:
  - Instead of building a single decision tree and use it to make predictions, build many slightly different trees and combine their predictions
- We have a single data set, so how do we obtain slightly different trees?
  1. Bagging (**B**ootstrap **A**ggregating):
    - Take random subsets of data points from the training set to create N smaller data sets
    - Fit a decision tree on each subset
  2. Random Subspace Method (also known as Feature Bagging):
    - Fit N different decision trees by constraining each one to operate on a random subset of features

# Bagging at training time



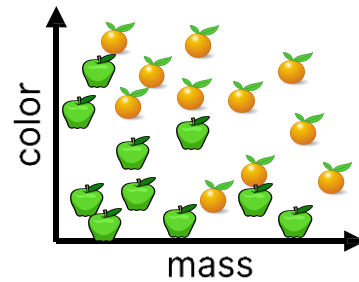
# Bagging at inference time



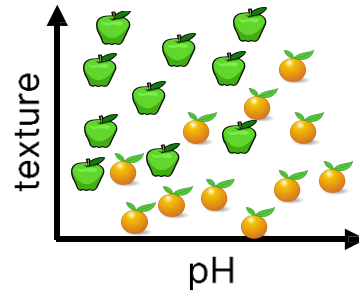
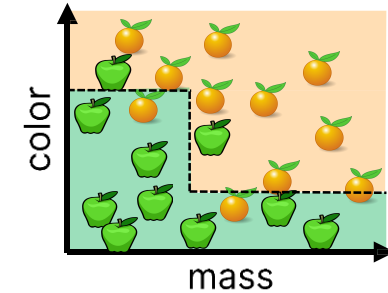
# Random Subspace Method at training time

Training data

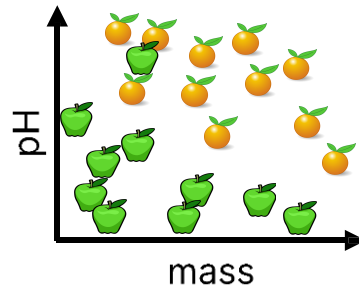
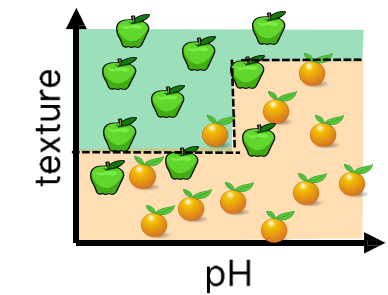
Mass (g)	Color	Texture	pH	Label
84	Green	Smooth	3.5	Apple
121	Orange	Rough	3.9	Orange
85	Red	Smooth	3.3	Apple
101	Orange	Smooth	3.7	Orange
111	Green	Rough	3.5	Apple
...				
117	Red	Rough	3.4	Orange



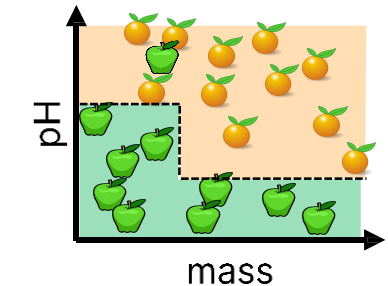
DT Learning  
Algorithm



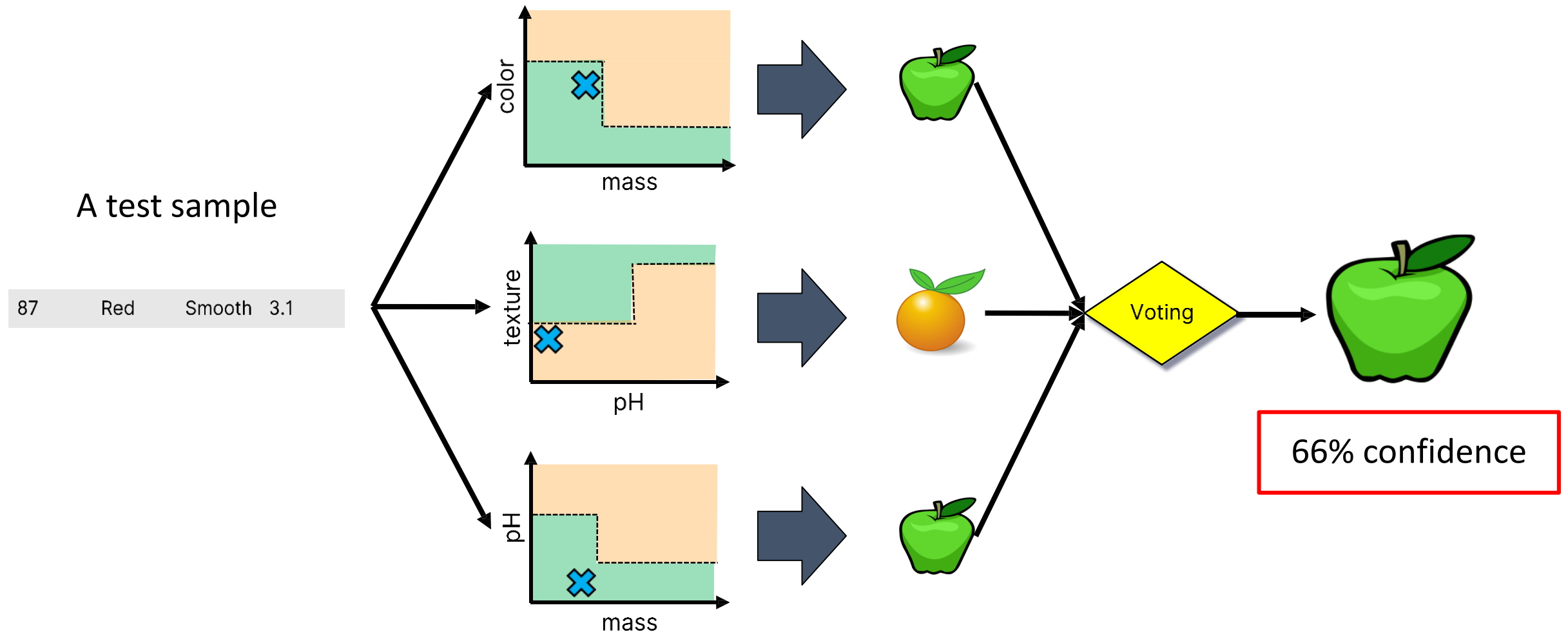
DT Learning  
Algorithm



DT Learning  
Algorithm



# Random Subspace Method at inference time





# Random Forests

Mass (g)	Color	Texture	pH	Label
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...				
117	Red	Rough	3.4	Orange



Bagging +  
Random Subspace Method +  
Decision Tree Learning Algorithm



# Ensemble Learning

- Ensemble Learning:
  - Method that combines multiple learning algorithms to obtain performance improvements over its components
- **Random Forests** are one of the most common examples of ensemble learning
- Other commonly-used ensemble methods:
  - **Bagging:** multiple models on random subsets of data samples
  - **Random Subspace Method:** multiple models on random subsets of features
  - **Boosting:** train models iteratively, while making the current model focus on the mistakes of the previous ones by increasing the weight of misclassified samples

# Boosting

