Introduction to Knock-off Filters

"Controlling False Discovery Rates via Knock-offs "
Authors: Rina Barber and Emmanuel Candes

Nandana Sengupta

October 19, 2015

Introduction

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- Consider the simple linear regession model:

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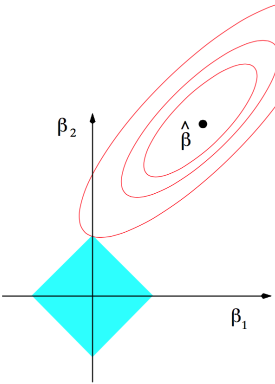

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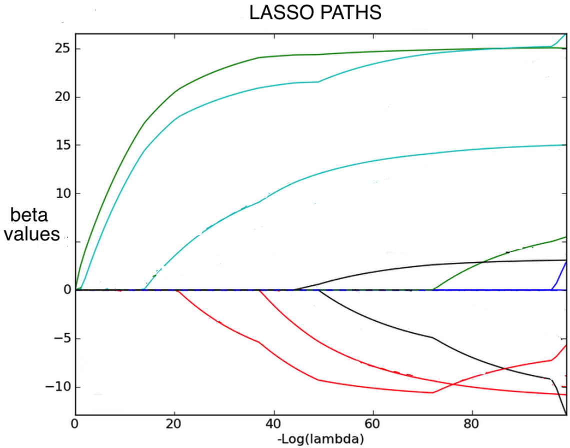

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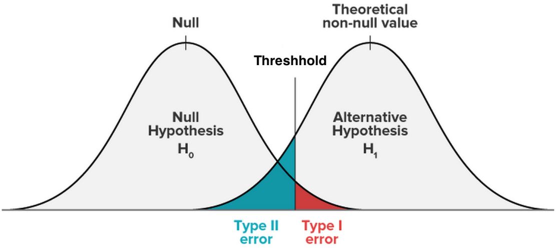

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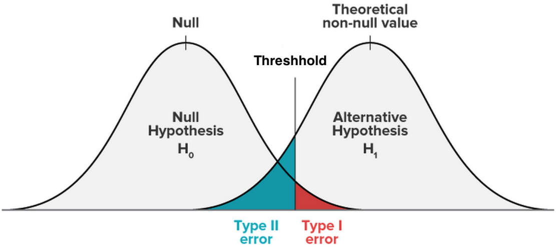
- 

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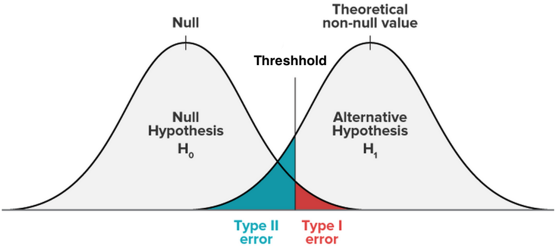
- 

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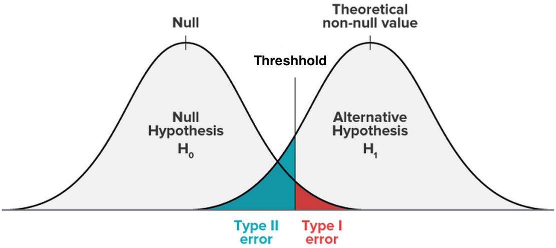
  - 

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- Work by Benjamini-Hochberg (1995, 2000)

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$$\beta_\lambda = \arg\min_{\beta \in \mathbb{R}^{2p}} \left\{ \frac{1}{2} \left\| y - \left[X \ \ \widetilde{X}\right] \cdot \beta \right\|_2^2 + \lambda \|\beta\|_1 \right\}$$

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$$\lambda_j = \sup \left\{ \lambda \ : \ \beta_j^\lambda \neq 0 \right\} = \text{ first time } X_j \text{ enters Lasso path}$$

$$\widetilde{\lambda}_j = \sup \left\{ \lambda \ : \ \widetilde{\beta}_j^\lambda \neq 0 \right\} = \text{ first time } \widetilde{X}_j \text{ enters Lasso path}$$

- $$W_j = \max\{\lambda_j, \widetilde{\lambda}_j\} \cdot \text{sign}(\lambda_j - \widetilde{\lambda}_j)$$

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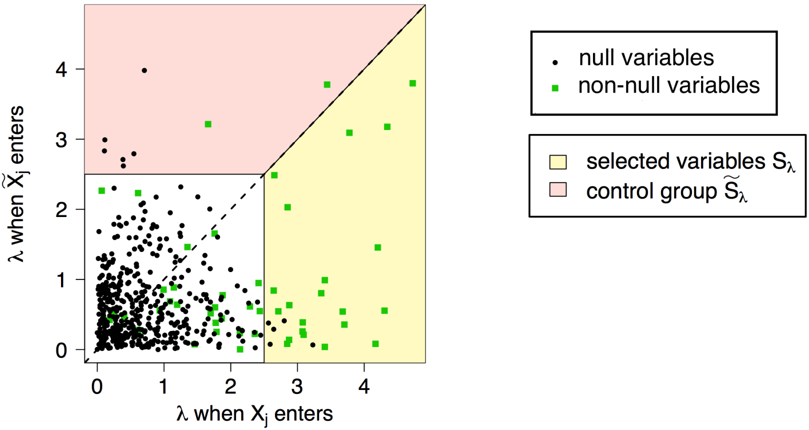

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  - model drug resistance of HIV-1 (y) on genetic mutations (X)

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  - Effect of a particular covariate on response but not sure about others covariates (?)

Thanks!