Missing Value Imputation using Low Rank Models

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In this analysis we compare the standard imputation algorithm in Social Sciences, Multiple Imputation with imputations obtained via Low Rank matrix completion algorithms, including trace norm regularization.

1 Imputation Techniques

Given an observed dataset df_{obs}, with N_{obs} entries, we apply the following 5 imputation techniques to obtain imputed values of the missing datapoints N_{miss}:

1 GLRM (Generalized Low Rank Models) on scaled data – where the low rank 'k' is picked via 5−fold crossvalidation.

2 GLRM on unscaled data – where the low rank 'k' is picked via 5−fold crossvalidation.

3 Trace Norm regularization on the Full Rank model – where the regularization constant α is picked via 5-fold crossvalidation, and the rank of the model is kept fixed at the full rank.

4 Trace Norm regularization on the Low Rank model –where the regularization constant α is picked via 5-fold crossvalidation, and the rank of the model is kept fixed at the low rank from the unscaled GLRM.

5 MICE (Multiple Imputation via Chained Equations) – this is a popular implementation of the Multiple Imputation method. The algorithm involves iteratively fitting univariate regressions one column at a time using a subset of the remaining columns of the data.

2 Dataset 1: General Social Survey

The first dataset we apply the techniques to is a subsample of the General Social Survey (GSS) 2014 dataset. The GSS is a sociological survey which collects data on attitudes and demographic characteristics of adults from randomly selected households in the United States. As a first-pass we extracted a subsample of the data by:

– removing columns corresponding to identifying variables (eg: sampling related, interviewer characteristics, interview description);

– removing columns with non-varying entries;

– removing columns with more than 33% missing entries;

– keeping only one column from pairs of very highly correlated columns (correlation>0.70).

The final dataset used in the analysis has 122 columns and 2538 rows.

3 Dataset 2: Simulation

We also apply the technique to a simulated dataset.

- Started with 25 independent columns of length 1000– mix of positive real-valued, integer and boolean.
- Introduced columnwise dependencies by multiplying with a 25×25 matrix of random parameters.
- Introduced rowwise structure by adding a random error term to each datapoint whose variance depended on the row (increasing variance with row number).

The final dataset consists of 1000 rows and 25 columns (5 each of the types: large and positive real-valued, moderate and positive real valued, integer, boolean and categorical).

4 Evaluation Scheme

The following procedure is used to compare the different imputation techniques:

1 Of the observed datapoints N_{obs}, 10% are randomly assumed missing, these are refered to as 'induced missing', labeled $N_{miss,ind}$. Let's call the resulting dataset $df_{miss,ind}$. The true value of a data point is given by y_0 and the imputed value is given by $\widehat{y}$.

2 The 5 imputation techniques – Low Rank (Scaled), Low Rank (Unscaled), Trace Norm (Full Rank), Trace Norm (Low Rank) and MICE are applied to $df_{miss,ind}$.

3 From the resulting imputed datasets, a 'loss' is calculated over the $N_{miss,ind}$ observations by comparing imputations $\widehat{y}$ to the true values y_0.

4 To do this, we scale all the (non-categorical) columns to ensure that each column has equal weight in the overall loss function. For all non-categorical columns we calculate quadratic loss, $Loss = (\widehat{y} - y_0)^2$, whereas for all categorical columns we calculate zero-one loss, $Loss = I(\widehat{y} \neq y_0)$.

5 Overall Results and Areas for Improvement

Overall for the two datasets we consider, the GLRM techniques (particularly the Trace Norm) outperform a single imputation using MICE – (at best about 30% lower total loss). The main cause for concern is that while for most columns (about 80%) GLRM techniques lead to $30-70\%$ reduction in losses, for a handful of columns (¡ 10%) they lead to significantly *higher* losses $(500 - 2000\%)$.

Other points to explore further include

- Applying the techniques to larger subsets of the GSS data as well as other social science datasets like National Longitudnal Survey of Youth.
- Working with a more advanced version of the MICE command.
- Improving the loss metric that is used for comparisons.
- Understanding why all five techniques still produce a handful of NA values for one or two columns.
- Think about possibly extending GLRM to a multiple imputation setting?

Detailed Results - GSS Subsample

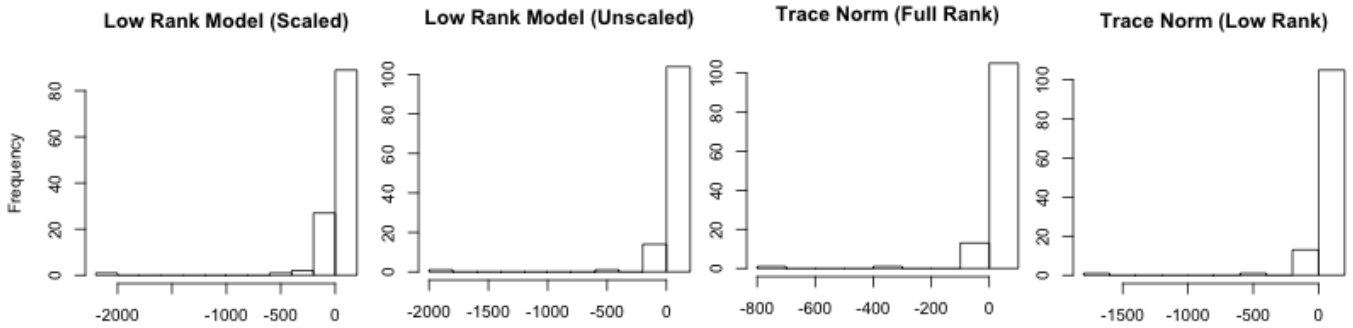
For the GSS Subsample the broad results are:

1. Overall the Trace (Full Rank) had the lowest loss, about 30% less than a single imputation using MICE. All the LowRankModels techniques did outperform MICE in terms of the total loss.
2. If we look at the column-wise losses – in general LowRankModels had lower total column loss for most columns (88% for Trace (Full Rank)) and most of these columns had 30-50% lower loss than MICE.
3. However for a few columns MICE had *much* lower losses – for one column it did 2000% better. This may be an area of concern.
4. If we look at the type of column, we note that Trace (Full Rank) frequently has the lowest loss values for non-categorical columns and Low Rank (Unscaled) for categorical columns.

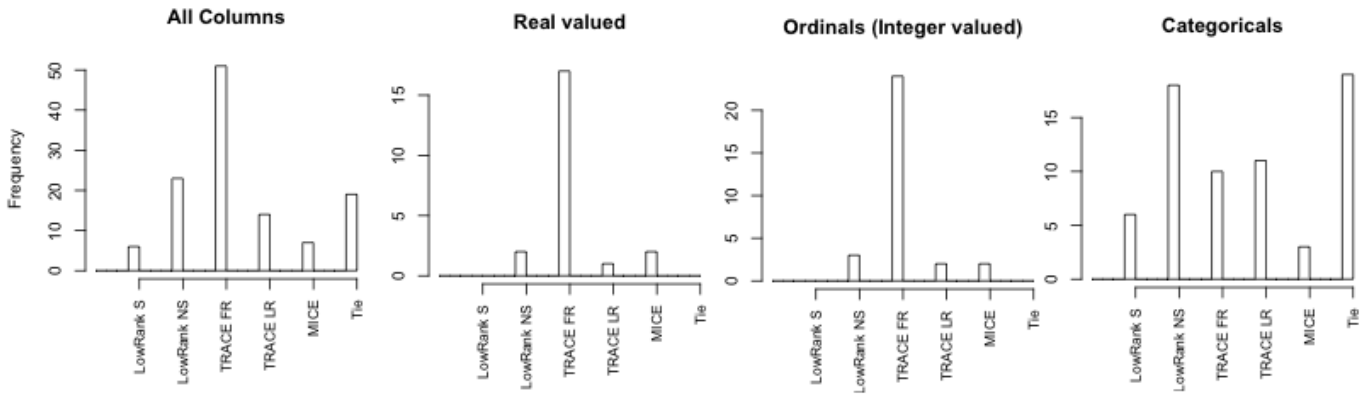
Comparison of Imputation Techniques (scaled GSS data)

	LowRank (S)	LowRank (NS)	Trace (FR)	Trace (LR)	MICE
Scaled Loss/(10^3)	18.50	15.80	14.40	15.80	20.60
%age reduction wrt MICE	10.10 %	23.40 %	30.10 %	23.00 %	–
%age cols w/ lower loss	73.50 %	84.60 %	87.20 %	84.60 %	–

Percent reduction in loss on using LowRankModels vs MICE



Column-wise lowest losses



Detailed Results – Simulation data

The results from the simulation are very similar to the GSS subsample:

1 Overall the Trace (Full Rank) had the lowest loss, about 25% less than a single imputation using MICE. All the LowRankModels techniques did outperform MICE in terms of the total loss.

2 If we look at the column-wise losses – in general LowRankModels had lower total column loss for most columns (65% for Trace (Full Rank)) and most of these columns had 30-50% lower loss than MICE.

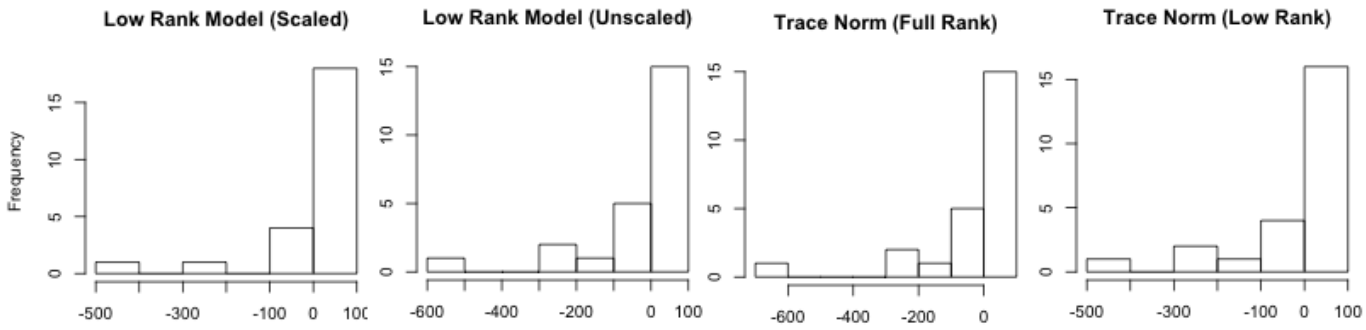
3 However again for a few columns MICE had *much* lower losses – for one column it did 500% better.

4 If we look at the type of column, we note that Trace (Full Rank) frequently has the lowest loss values for categorical columns and Low Rank (Scaled) for other columns.

Comparison of Imputation Techniques (scaled simulation data)

	LowRank (S)	LowRank (NS)	Trace (FR)	Trace (LR)	MICE
Scaled Loss/(10^3)	1.60	1.40	1.40	1.40	1.80
% reduction (wrt MICE)	9.30 %	24.60 %	25.10 %	22.70 %	–
% columns with lower loss	78.30 %	65.20 %	65.20 %	69.60 %	–

Percent reduction in loss on using LowRankModels vs MICE



Column-wise lowest losses