A Matrix Factorization Approach to Multiple Imputation

Knowledge Lab Team Presentation

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February 10, 2016

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- 'R' Packages: Amelia, MICE, MI.

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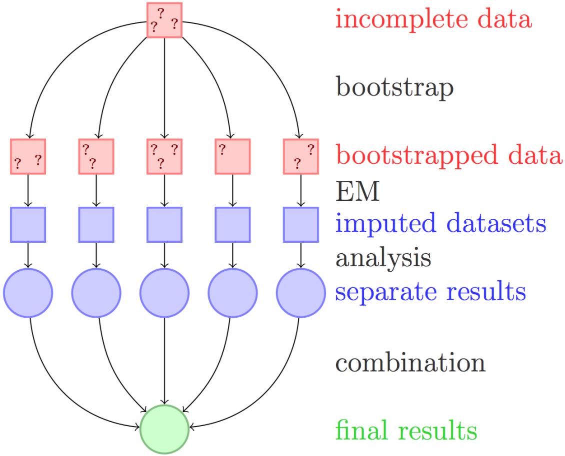

Image credit: https://cran.r-project.org/web/packages/Amelia/vignettes/amelia.pdf

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  - Summary Table

	LowRank (S)	LowRank (NS)	Trace (FR)	Trace (LR)	MICE
Loss/(10^3)	18.50	15.80	14.40	15.80	20.60
%age reduction over MICE	10.10 %	23.40 %	30.10 %	23.00 %	–
%age cols w/ lower loss	73.50 %	84.60 %	87.20 %	84.60 %	–

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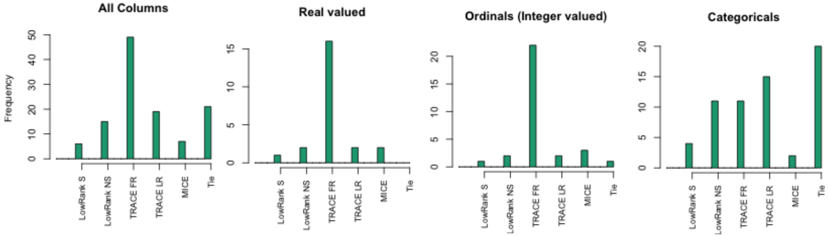

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%age cols w/ lower loss	75.70%	92.90 %	94.30 %	94.30 %	–

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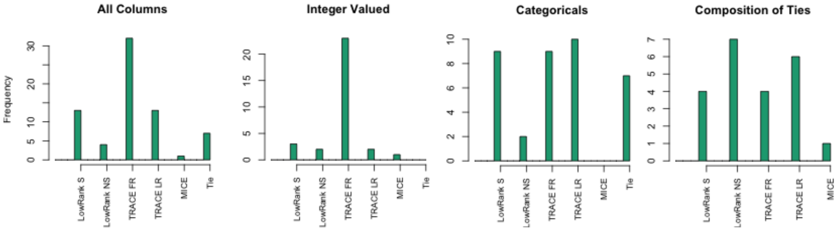

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Thank you!
(Comments and Suggestions Welcome)