Temperature estimates using O III lines

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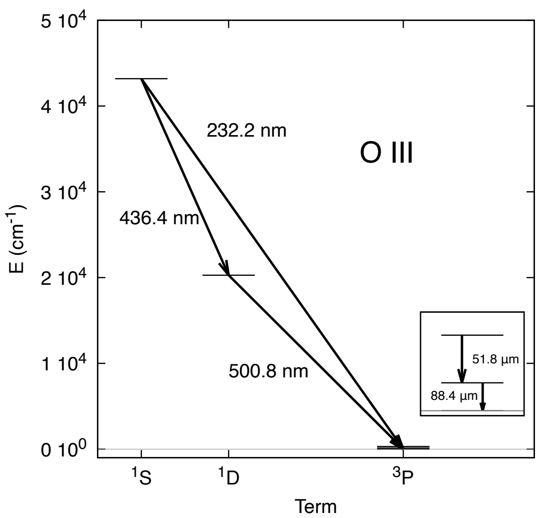
Contents

This exercise is taken from a discussion in Draine (2011, Section 18.1).

1 Nebulium

In 1864, the British astronomer William Huggins observed green spectral lines in NGC 6543 that he could not explain. He postulated that they came from an unknown new element that he named "Nebulium". In 1927, US astronomer Ira Bowen showed that these lines came from doubly ionized oxygen, O III, and that no new element was needed to explain them[1]. Below: NGC 6543 (Cat's eye), as seen in a composite X-ray (Chandra) and visible (HST) image. © NASA.

O III has the same electronic structure as C I: $1s^2\,2s^2\,2p^2$. The outer electrons angular momentum may combine to give 3 terms: ^3P, ^1D and ^1S, and the triplet has a fine structure splitting giving a total of 5 energy levels: $^3P_0$, $^3P_1$, $^3P_2$, $^1D_2$, $^1S_0$, with statistical weight $g = 2J + 1$. Table 1 gives the characteristics of these levels, and Table 2 lists all possible transitions between them. One can see that their strengths vary by several orders of magnitude. The fine structure transitions are in the mid-infrared range and can be see only from space. In the visible The strongest lines (by far) couples the two excited states ^1D and ^1S to the ^3P term and between them. The green lines are those coupling the ^1D and ^3P states. The last line between ^1S and ^3P is in the ultraviolet range and requires also observation from space (e.g. with SOFIA, see Rho et al. (2019)). All those lines are "forbidden" lines with M1 transitions being typically stronger than E2 transition if there is a change of spin[2].



[1] See https://en.wikipedia.org/wiki/Nebulium for some details.
[2] E2 transition probabilities are not given in Table 2 if the M1 transition is possible.

Table 1: O III energy levels. Data from NIST.

Configuration	i	Term	J	g_i	Energy (cm^{-1})
$2s^2\,2p^2$	0	^3P	0	1	0.000
	1		1	3	113.178
	2		2	5	306.174
	3	^1D	2	5	20273.27
	4	^1S	0	1	43185.74

Table 2: O III radiative transitions. Data from NIST.

Terms	λ (nm)	A_{ij} $\left(\mathrm{s}^{-1}\right)$	Type
$^3P_0 - {}^3P_1$	88356.4	$2.61\,10^{-5}$	M1
$^3P_1 - {}^3P_2$	51814.5	$9.76\,10^{-5}$	M1
$^3P_0 - {}^3P_2$	32661.2	$3.17\,10^{-11}$	E2
$^3P_2 - {}^1D_2$	500.8240	$1.81\,10^{-2}$	M1
$^3P_1 - {}^1D_2$	496.0295	$6.21\,10^{-3}$	M1
$^3P_0 - {}^1D_2$	493.2603	$2.41\,10^{-6}$	E2
$^3P_2 - {}^1S_0$	233.2113	$6.34\,10^{-4}$	E2
$^3P_1 - {}^1S_0$	232.1664	$2.15\,10^{-1}$	M1
$^1D_2 - {}^1S_0$	436.4436	$1.71\,10^{0}$	E2

2 Excitation in a low density medium

In low density regions, most collisional excitations are followed by radiative decay. If these collisions are the main pumping mechanism towards excited states, then the line intensity is a direct measure of the number of collisions.

At fixed density and gas composition, collisional excitation is a pure function of temperature. Hence there is an opportunity to use these lines to derive the gas temperature, ...if we can get rid of any other interfering physical process. O III offers such a possibility, and this is the subject of the exercise.

In an ionized gas (H II region, planetary nebulae or in the vicinity of an AGN), collisions with electrons dominate. For an electron density n_e (m^3), the collision rate from level l ("low") to level u ("up") is:

$$k_{lu} = \left(\frac{2\pi\,\hbar^4}{k\,m_e^3}\right)^{1/2} T^{-1/2}\,\frac{\Omega_{lu}}{g_l}\,\exp\left(-\frac{E_{ul}}{k\,T}\right) \tag{1}$$

where T is the gas temperature in K, E_{ul} is the transition energy difference, g_l is the statistical weight of level l. Ω_{lu} is called the "collision strength. It is a dimensionless quantity which incorporates the quantum mechanical effects on collisions beyond the classical expression. The pre-factor constant is:

$$\left(\frac{2\pi\,\hbar^4}{k\,m_e^3}\right)^{1/2} = 8.629\,10^{-12}\,\mathrm{m}^3\,\mathrm{s}^{-1}\,\mathrm{K}^{1/2}$$

where $\hbar$ is the reduced Planck constant, k is Boltzmann constant and m_e is the electron mass. The resulting collision rate k_{lu} is in $\mathrm{m^3\,s^{-1}}$.

The collision strength is symmetrical, so by detailed balance, we have:

$$k_{ul} = \frac{g_l}{g_u}\, k_{lu}\, \exp\left(\frac{E_{ul}}{k\,T}\right) = \left(\frac{2\pi\,\hbar^4}{k\,m_e^3}\right)^{1/2} T^{-1/2}\,\frac{\Omega_{ul}}{g_u} \tag{2}$$

2.1 Collision strengths

Collision strengths for transitions between the 5 lowest levels of O III have been computed by Storey et al. (2014). Transitions between and towards ^3P are shown on Figure 1. The quantity plotted is $\dfrac{\Omega_{ul}}{g_l}$. We see that collisions from ^1S and ^1D towards ^3P_J (Eq 1) do not depend on the J of the lower level. That property will be useful later. Ω_{ul} is almost constant in the temperature range of interest.

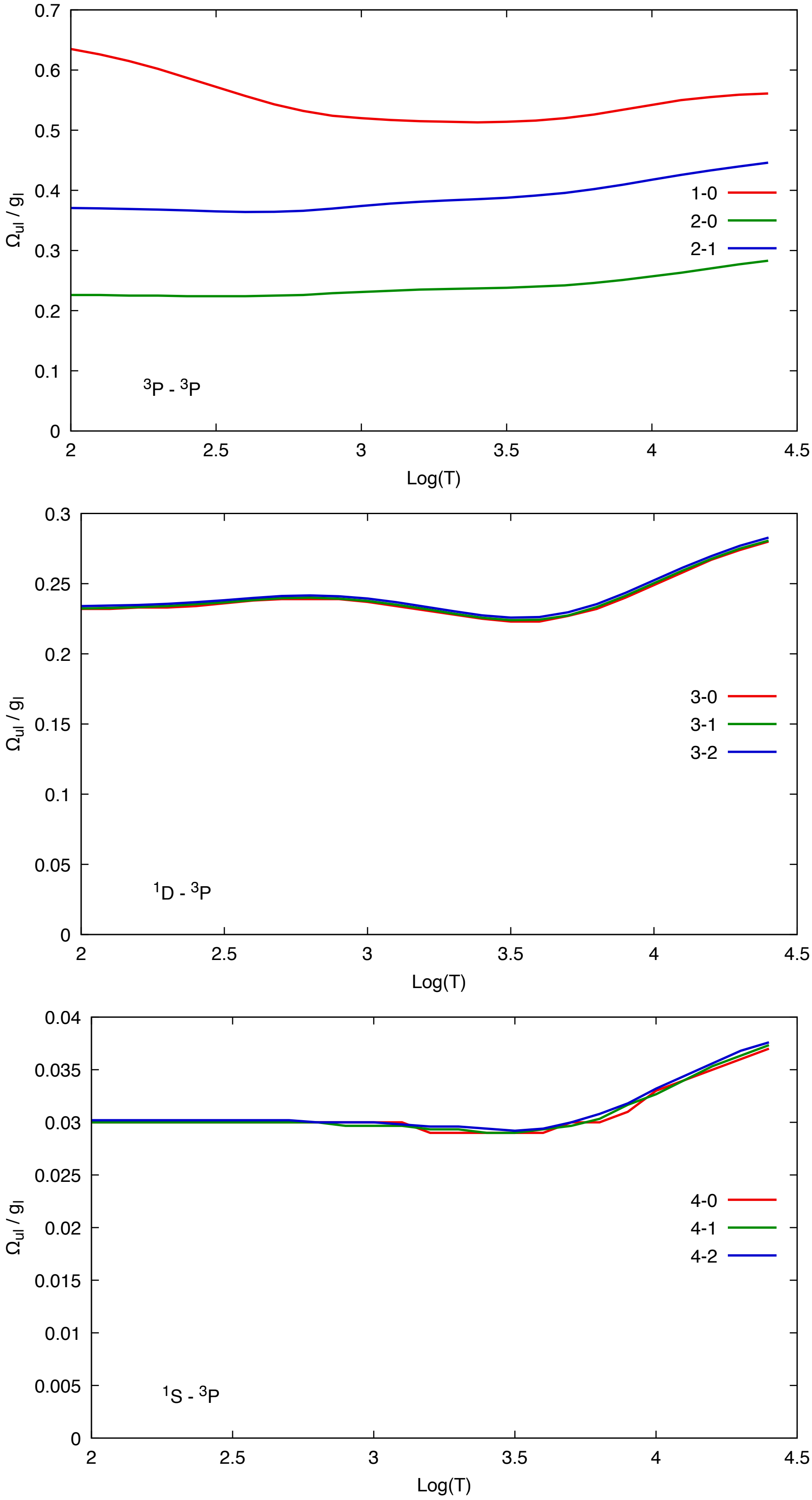


Figure 1: Collision strength for O III, from Storey et al. (2014). Top: within ^3P, middle: from ^1D, bottom: from ^1S.

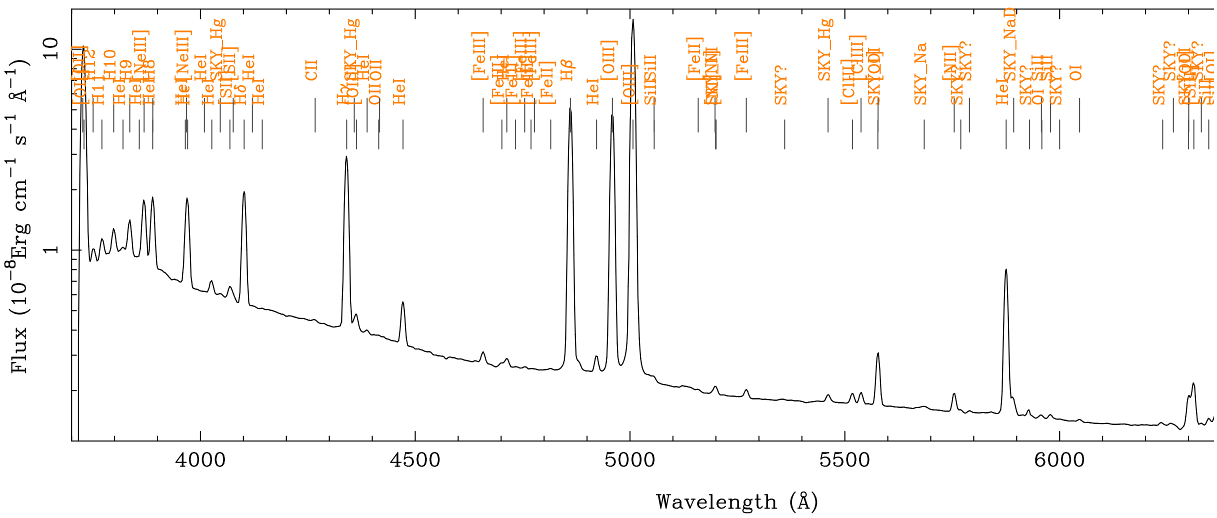


Figure 2: Low resolution spectrum of Orion nebulae from Sánchez et al. (2007, Fig. 1).

3 Exercise

We aim at estimating the electron temperature in the Orion Nebulae. A full spectrum, taken from Sánchez et al. (2007), is shown on Figure 2. OIII levels are numbered from 0 to 4 (see Table 1). Where necessary, the abundances are noted from n_0 to n_4 in m^3.

1. Compute the critical density associated with mid-infrared transitions within ^3P. If the gas density n_H is well below these densities, what can we deduce concerning the populations of levels 0, 1 and 2?

2. Compute the critical densities for transitions from ^1D and ^1S levels.

3. In all the following, we suppose that the gas density is much lower than the $^1D - {}^3P$ and $^1S - {}^3P$ critical densities. If an ion has just been excited to the $^1S_0$ level, what is the probability $p\,(4 \to 3)$ that it will decay radiatively towards $^1D_2$?

4. From level $^1D_2$ what is the probability $p\,(3 \to 2)$ to reach $^3P_2$?

5. Considering collisional transitions from $^3P_0$ with an abundance of n_0 in m^{-3}, what is the rate of population of level $^1S_0$, in $\mathrm{m}^{-3}\,\mathrm{s}^{-1}$?

6. What is the total rate of population of level $^1D_2$, considering all significant processes?

7. What are the power P_{43} radiated in the $^1S_0 \to {}^1D_2$ line at 436.44 nm and the power P_{32} radiated in the $^1D_2 \to {}^3P_2$ line at 500.82 nm?

8. What can you say of the ratio of these radiated powers? Show that it can be used to evaluate the gas temperature.

9. Plot the ratio $\frac{P_{43}}{P_{32}}$ as a function of T and discuss the result.

10. Derive a numerical expression for the electron temperature from the ratio $\frac{P_{43}}{P_{32}}$. Compare your expression to Eq (5) of Rubin et al. (2003)

11. Derive the uncertainty on T_e from variations in $R = \frac{P_{32}}{P_{43}}$. If the S/N is equal to 10, what is the uncertainty on T_e for $R = 300$?

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