Problem Number : Chapter 4 – Conceptual 7 from ITSL book

Problem Description :

Suppose that we wish to predict whether a given stock will issue a dividend this year ("Yes" or "No") based on X, last year's percent profit. We examine a large number of companies and discover that the mean value of X for companies that issued a dividend was $\bar{X} = 10$, while the mean for those that didn't was $\bar{X} = 0$. In addition, the variance of X for these two sets of companies was $\sigma^{\wedge 2} = 36$. Finally, 80 % of companies issued dividends. Assuming that X follows a nor- mal distribution, predict the probability that a company will issue a dividend this year given that its percentage profit was X = 4 last year.

Solution:

Conceptual : 7 :

Define Events and Variables :

X : Last year's percent profit (normally distributed)
$X = 4$

D : Event that the company issues a dividend this year ("Yes")

$\bar{D}$: Event that the company does not issues a divided this year ("No")

$\bar{X}_D$: mean value of X for companies issuing dividends = 10

$\bar{X}_{\bar{D}} = 0$;

$\sigma^2 = 36$;

$P(D) = 80\% = 0.8$

$P(\bar{D}) = 20\% = 0.2$

$P(D \mid X = 4) = ?$

$$P(D \mid X = 4) = \frac{P(X=4 \mid D) * P(D)}{P(X=4 \mid D) * P(D) + P(X=4 \mid \bar{D}) * P(\bar{D})}$$

pdf of the normal distribution :

$$P(x \mid y_k) = \frac{1}{\sqrt{2\pi\sigma_k^2}} e^{-\frac{(x-\mu_k)^2}{2\sigma_k^2}}$$

$$P(X=4 \mid D) = \frac{1}{\sqrt{2\pi \times 36}} \times e^{-\frac{(4-10)^2}{2\times 36}}$$

$$= \frac{1}{6\sqrt{2\pi}} \times e^{-1/2}$$

$$P(X=4 \mid \bar{D}) = \frac{1}{6\sqrt{2\pi}} \times e^{-\frac{(4-0)^2}{2\times 36}}$$

$$= \frac{1}{6\sqrt{2\pi}} \times e^{-2/9}$$

$$P(X=4) = P(D) \times P(X=4 \mid D) + P(X=4 \mid \bar{D}) \times P(\bar{D})$$

$$= \frac{0.8}{6\sqrt{2\pi}} e^{-1/2} + \frac{0.2}{6\sqrt{2\pi}} e^{-2/9}$$

$$= \frac{1}{6\sqrt{2\pi}} \left(0.8 e^{-1/2} + 0.2 e^{-2/9}\right)$$

$$P(D \mid X=4) = \frac{\frac{1}{6\sqrt{2\pi}} e^{-1/2} \times 0.8}{\frac{1}{6\sqrt{2\pi}} \left(0.8 e^{-1/2} + 0.2 e^{-2/9}\right)}$$

$$= \frac{0.8 e^{-1/2}}{0.8 e^{-1/2} + 0.2 e^{-2/9}}$$

$$= \frac{1}{1 + \frac{1}{4}\left(\frac{e^{-2/9}}{e^{-1/2}}\right)} \approx 75.78\%$$