STRUCTURAL RELIABILITY

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Design

Past

Present / Future

Experience

Lessons learned

Conservatism

Innovative designs/materials

Uncertain loading

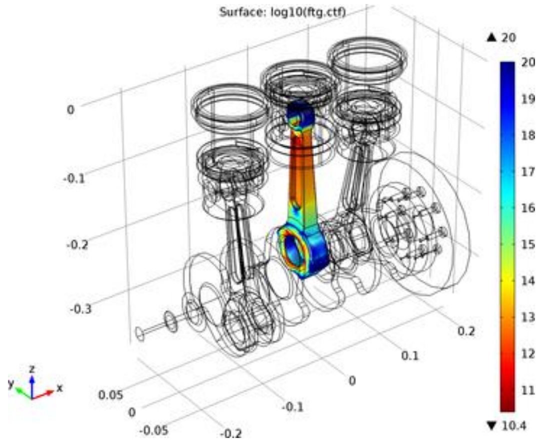
Deterioration

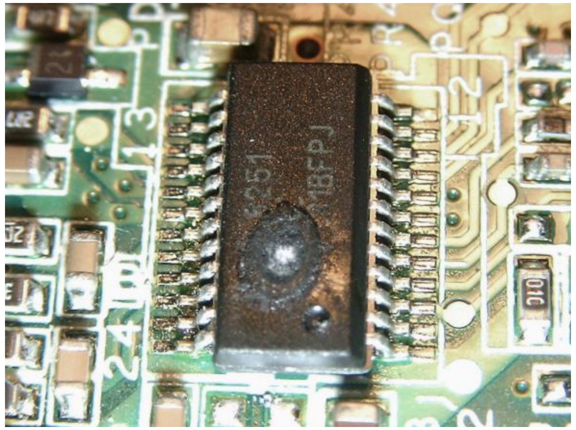
Optimization

Frequentist

Classical

Bayesian





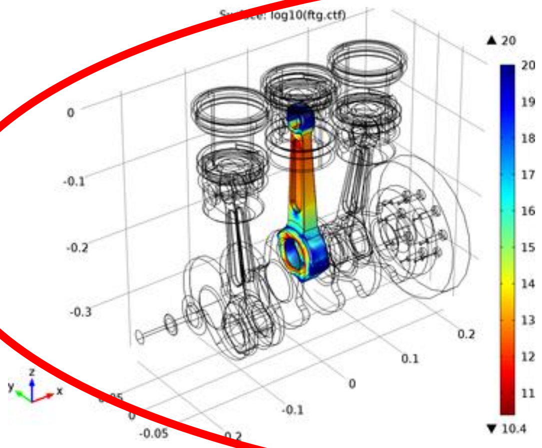
VS

Probability

Frequentist

Classical

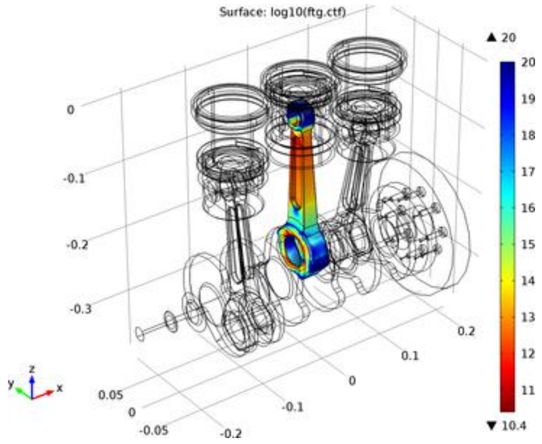
Bayesian

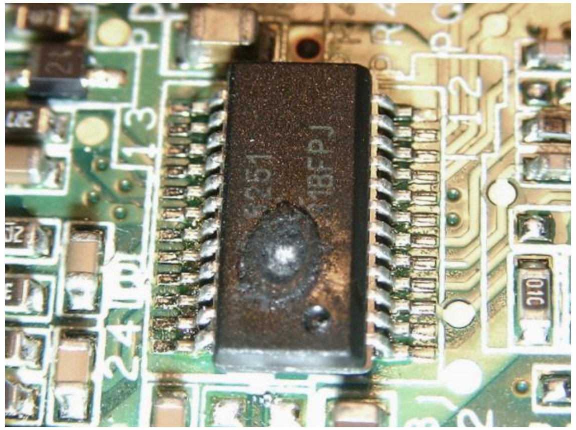


Frequentist

Classical

Bayesian





Structural reliability

Why uncertainty quantification?

Example: penalization due to late arrival (1000 €) and the meeting is taking place in 60 minutes.

- Option 1 (fast taxi): average 40 mins
- Option 2 (taxi with gps): average 50 mins

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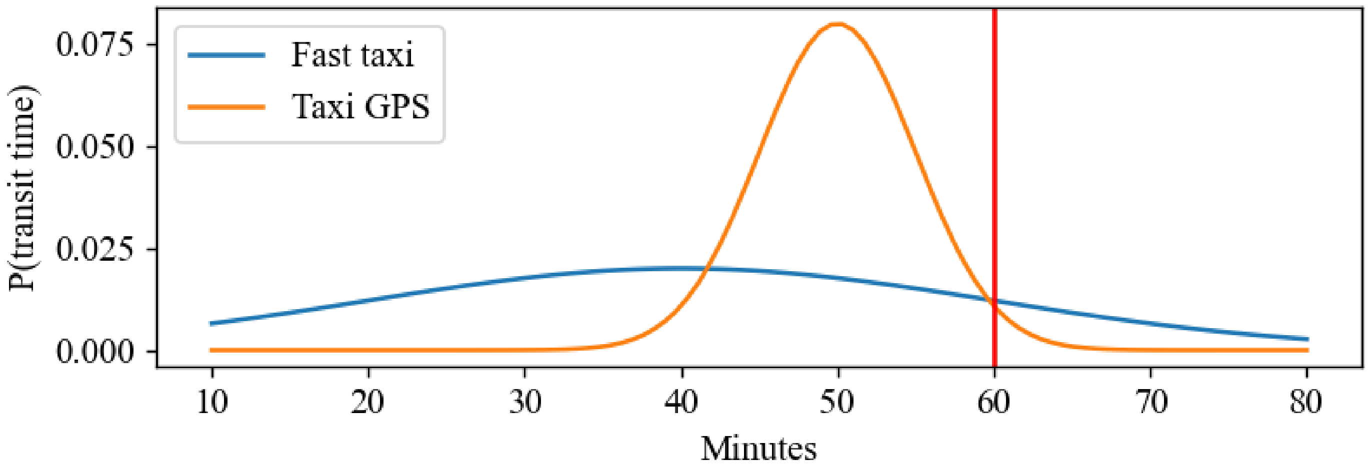
- Option 1 (fast taxi): average 40 mins, std 20 mins, CoV=0.50;
- Option 2 (taxi with gps): average 50 mins, std 5 mins, CoV=0.10.

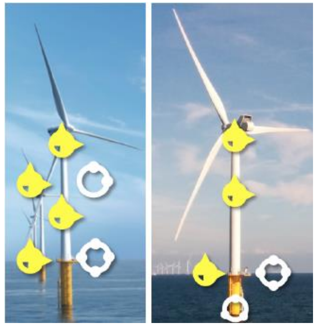


Uncertainty quantification

Physical

Model

Statistical



Measurement

Aleatory

Epistemic

Bayesian Inference

KNOWLEDGE

KNOWLEDGE

KNOWLEDGE

Design

Production

Operation

Decommissioning

Reliability assessment levels

- **Level I methods**

  - Uncertainty → one value
  - Partial Safety Factor

- **Level II methods**

  - Gaussian (Mean & std)
  - 'Reliability index method'

- **Level III methods**

  - Joint distribution functions
  - 'Probability of failure'

- **Level IV methods**

  - Consequence (cost) of failure
  - Risk = Probability of failure * cost

https://ascelibrary.org/doi/10.1061/AJRUA6.0001104

Level II and III methods

- First order reliability method (FORM)
  - Cornell’s reliability index for linear failure function (Level II & independent variables)
  - Hasofer & Lind’s reliability index for non-linear failure function (Level II & independent variables)
  - …
- Second order reliability method (SORM)
- Simulation techniques
  - Monte Carlo simulations
  - Importance sampling
  - …

1. Formulate limit state function.

2. Identify random variables (uncertainties) and deterministic parameters.

3. Specify distribution types and statistical parameters for random variables.

4. Estimate the reliability (probability of failure).

Limit state functions

- Fatigue limit state
- Serviceability limit state
- Ultimate limit state
- Accidental limit state

Note: A combination of more than one limit state is also possible (e.g., the failure function of a system can be a combination (series or parallel system) of those of its components.

Failure probability estimation

Let’s consider a fundamental limit state:
failure is expected when the load exceeds the resistance of the structure.

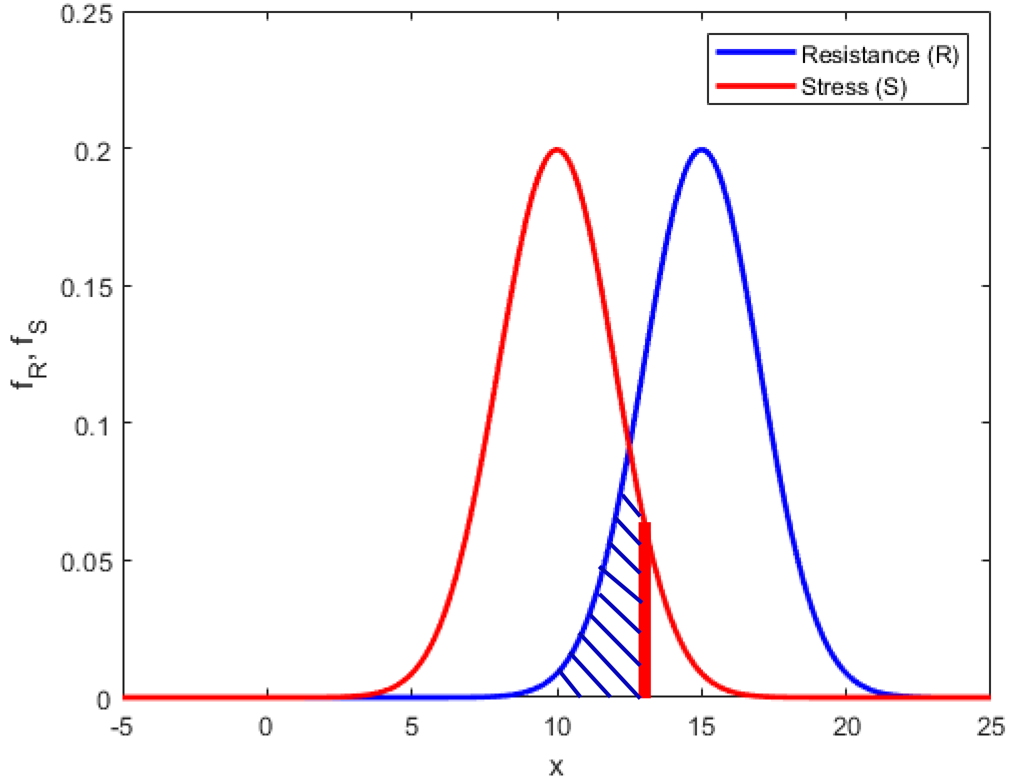
$$g(x) = \boldsymbol{R} - \boldsymbol{S}$$

Failure event → $R \leq S \; or \; g(x) \leq 0$

$$P_F = \int_{-\infty}^{+\infty} P(R \leq S)$$

$$= \int_{-\infty}^{+\infty} P(R \leq x) P(x \leq S \leq x + dx) dx$$

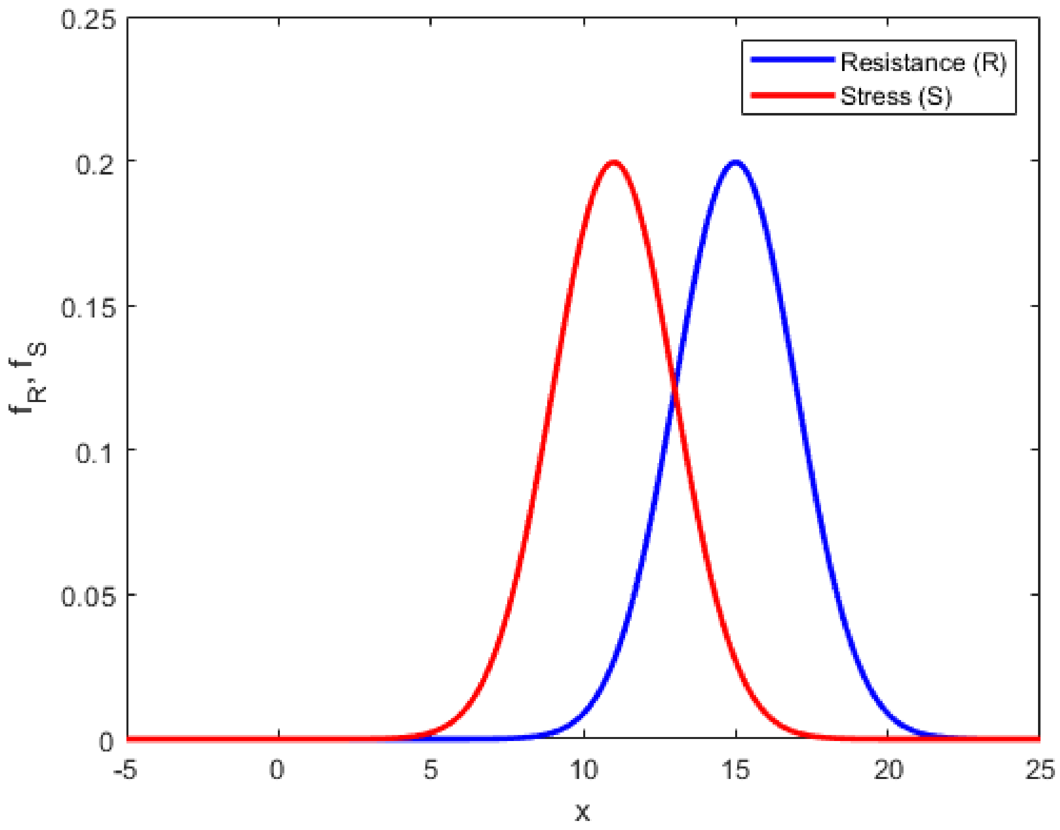
$$= \int_{-\infty}^{+\infty} F_R(x)\, f_S(x)\, dx$$

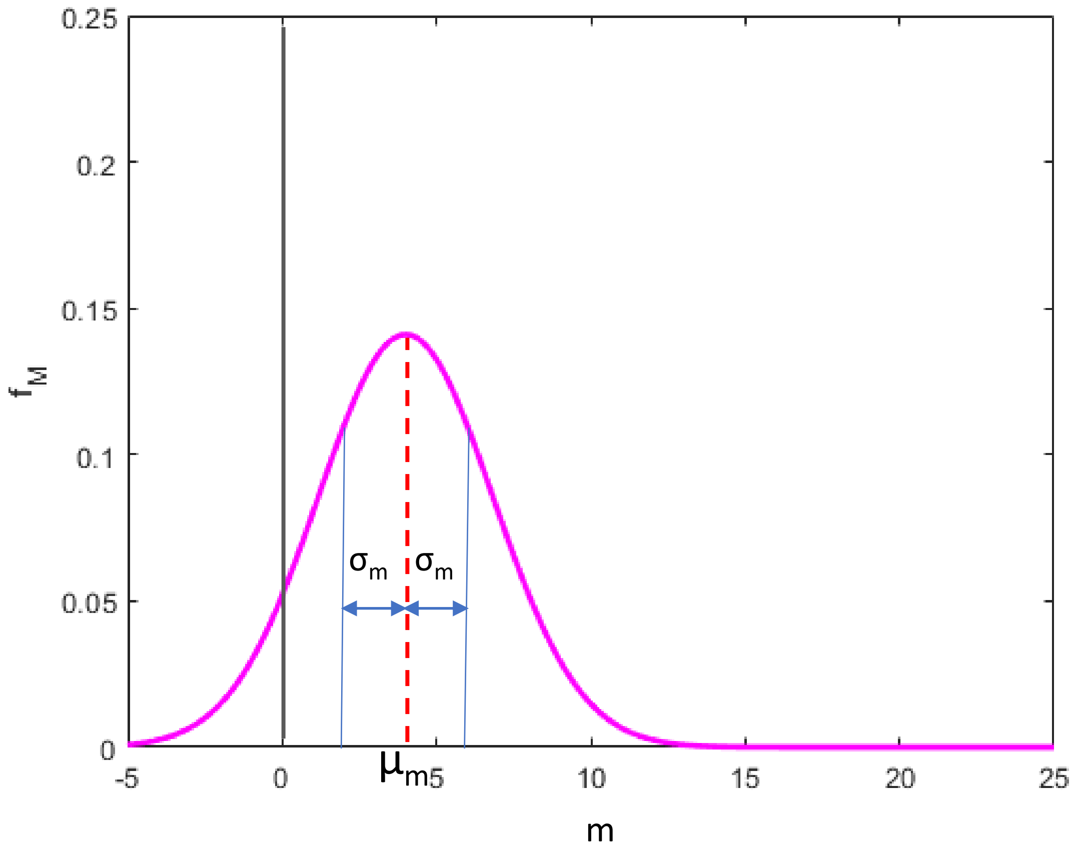


Failure probability estimation (Level II method)

Linear safety margin for fundamental limit state: $\boldsymbol{M} = \boldsymbol{R} - \boldsymbol{S}$

If $\boldsymbol{R}$ and $\boldsymbol{S}$ are normally distributed , $\boldsymbol{M}$ is also normally distributed.





Mean: $\mu_M = \mu_R - \mu_S$

Variance: ${\sigma_M}^2 = \sigma_R^2 + \sigma_S^2$

Probability of failure:

$$P_F = P(R \leq S) = P(M \leq 0) = \int_{-\infty}^{+0} f_M dm$$

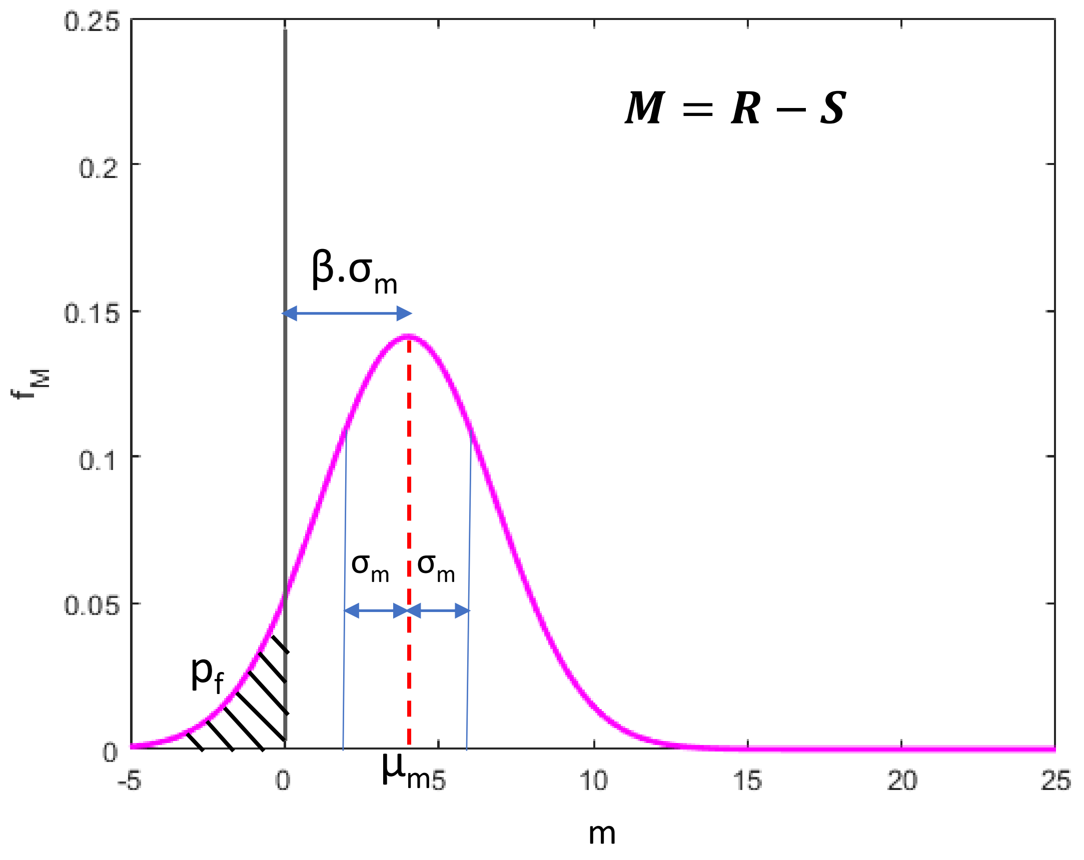
Reduce to standard normal distribution function N(0,1)

$$P_F = \Phi(\frac{0 - \mu_m}{\sigma_m})$$

$$P_F = \Phi(-\beta) \Leftrightarrow \beta = -\Phi^{-1}(P_F)$$

CDF of N(0,1)

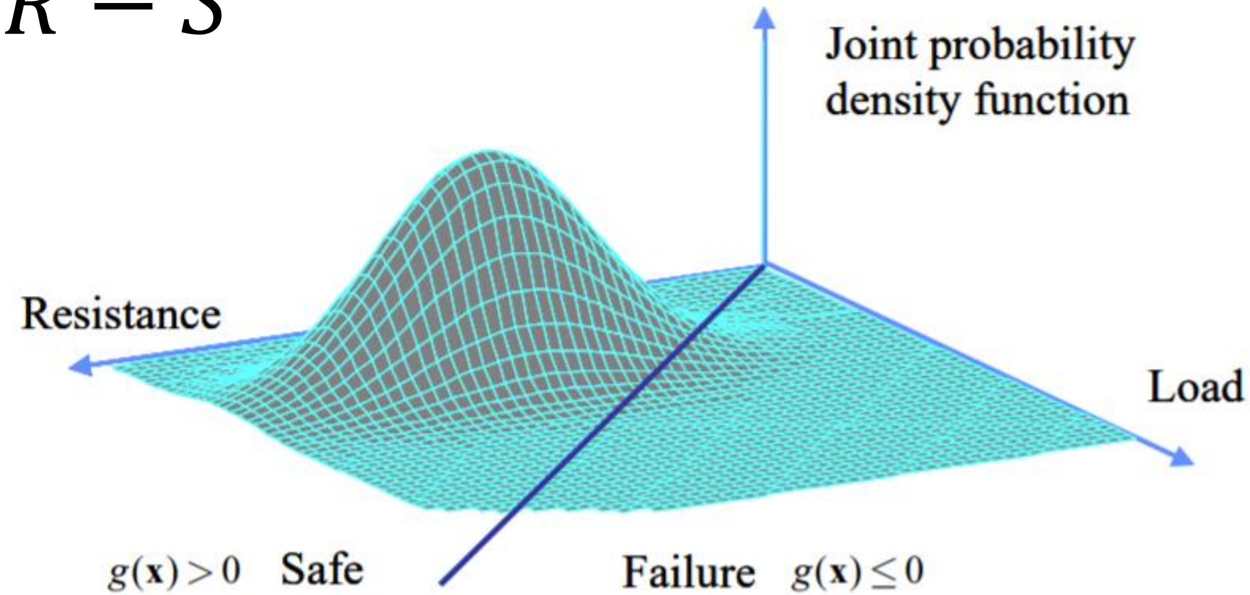
Reliability index



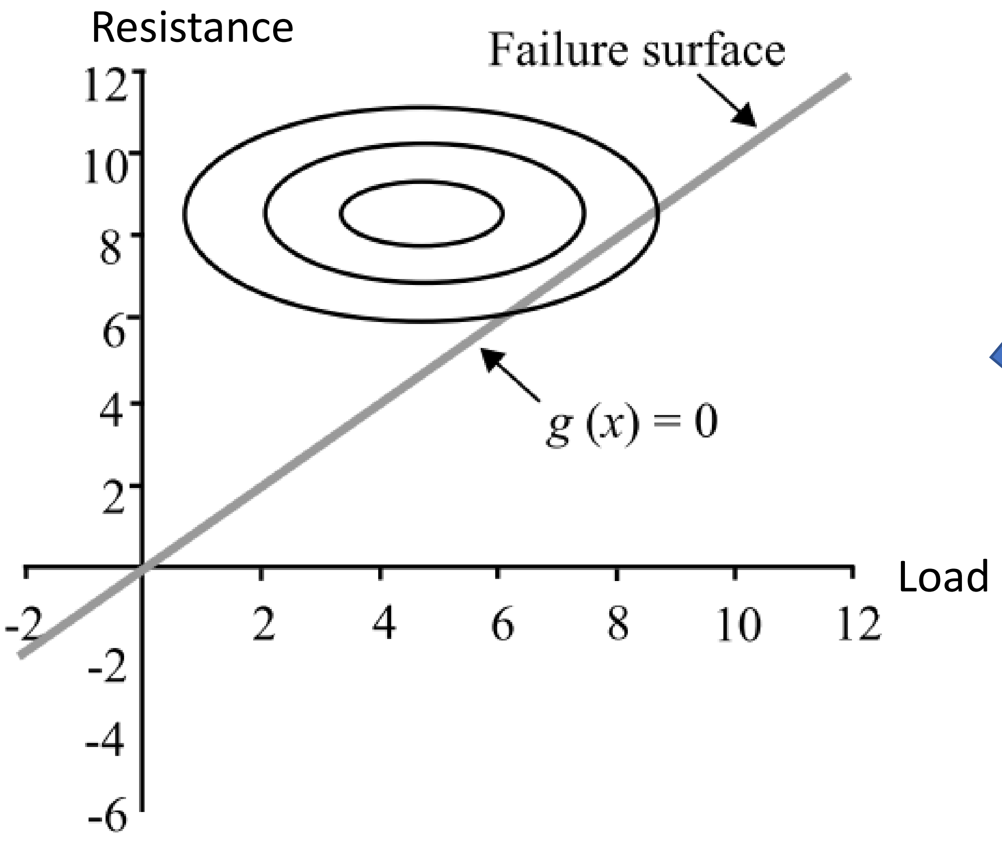
Joint probability distribution function $f(R, S)$:

$$P_F = \int_{g(\boldsymbol{x})\leq 0} f(\boldsymbol{x})dx\,, \qquad \boldsymbol{x} = [x_1, x_2, ..x_N]^T$$

$$g(x) = R - S$$

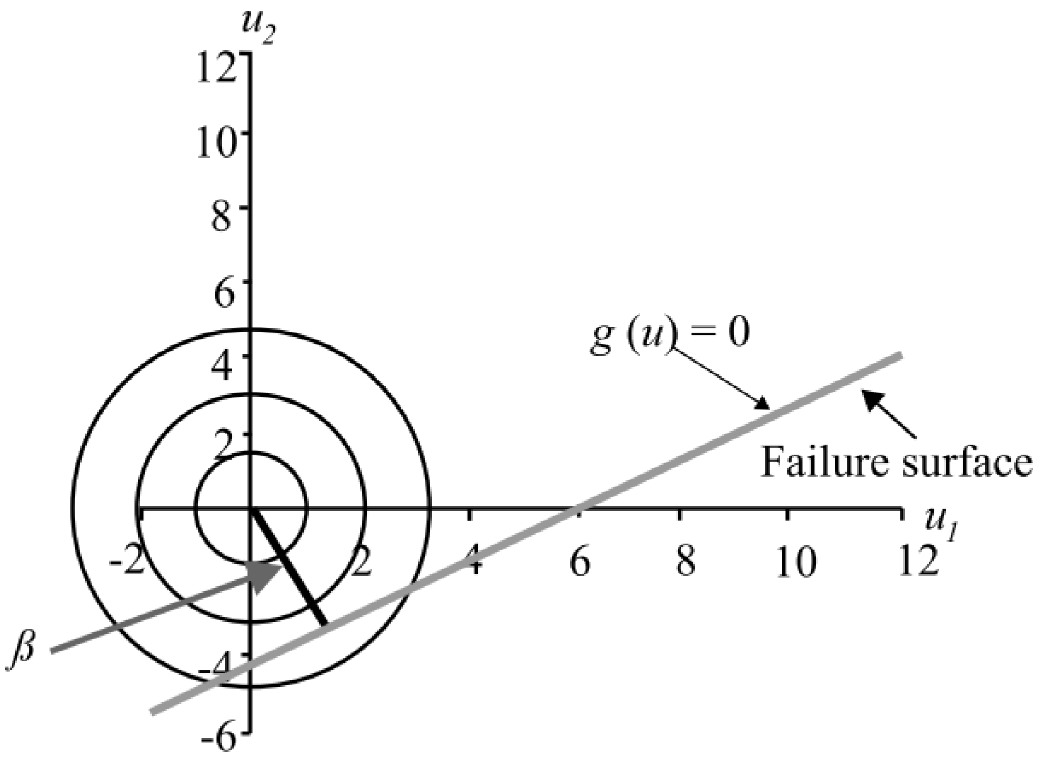


Geometrical interpretation of reliability index



Transformation to standard normal space:

$$\boldsymbol{u}_i = \frac{\boldsymbol{x}_i - \mu_{x_i}}{\sigma_{x_i}}$$

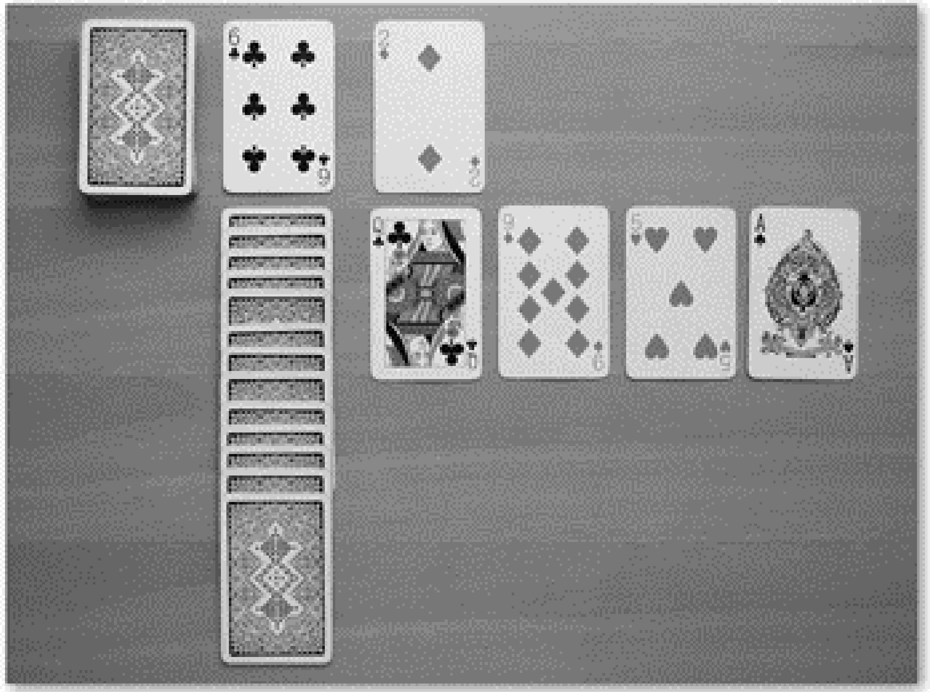


Ship structures – Safety level

	Serviceability		Collapse/Ultimate	
	P_f	β	P_f	β
Extreme	$1.0 \cdot 10^{-3}$	3.1	$5.1 \cdot 10^{-6}$	4.5
Severe	$5.2 \cdot 10^{-3}$	2.5	$3.1 \cdot 10^{-5}$	4
Moderate	$2.3 \cdot 10^{-2}$	2.0	$1.2 \cdot 10^{-4}$	3.7

Monte Carlo Simulations – named after Monte Carlo casino (Monaco)

Stanislaw Ulam
(1909-1984)



John Von Neumann
(1903-1957)

Law of large numbers – rolling a die

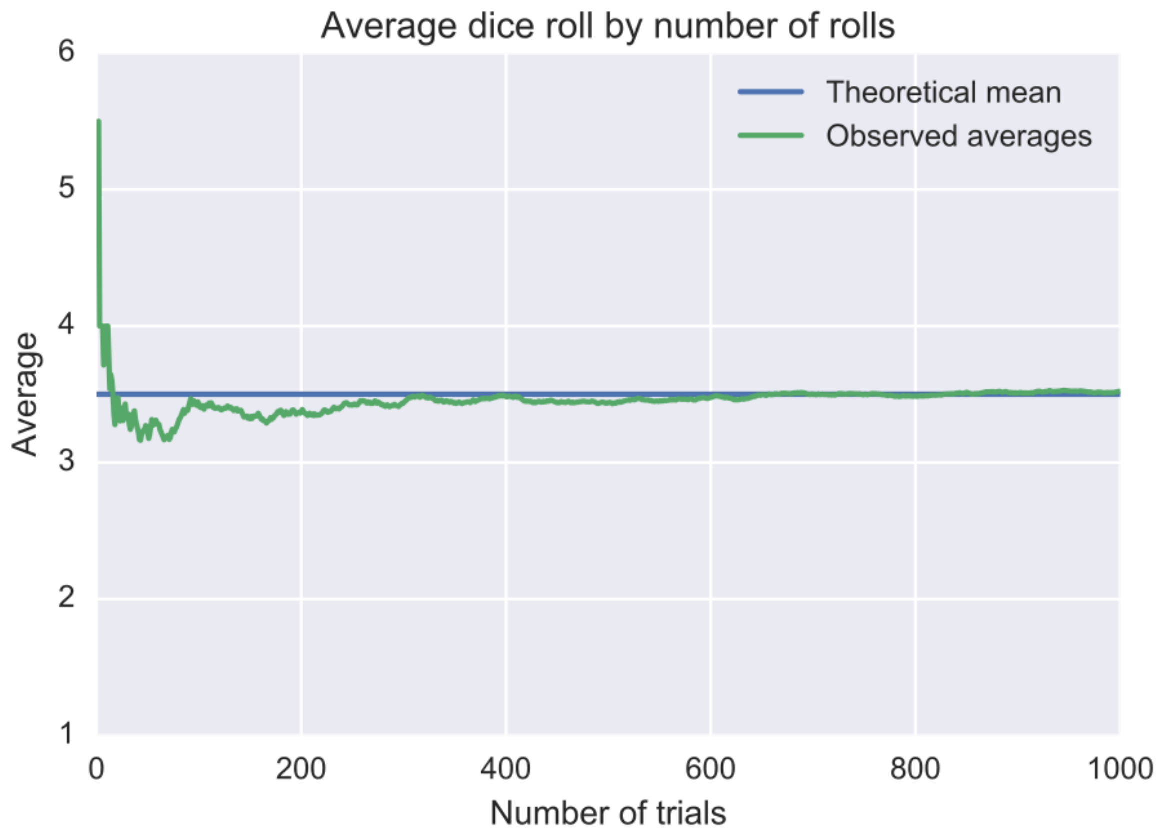
Example: Rolling a die

i	1	2	3	4	5	6	7	8	9
x_i	5	6	1	6	4	1	2	4	6
$E[X]$	5	5.5	4	4.5	4.4	3.8	3.5	3.6	3.88

Theoretical mean:

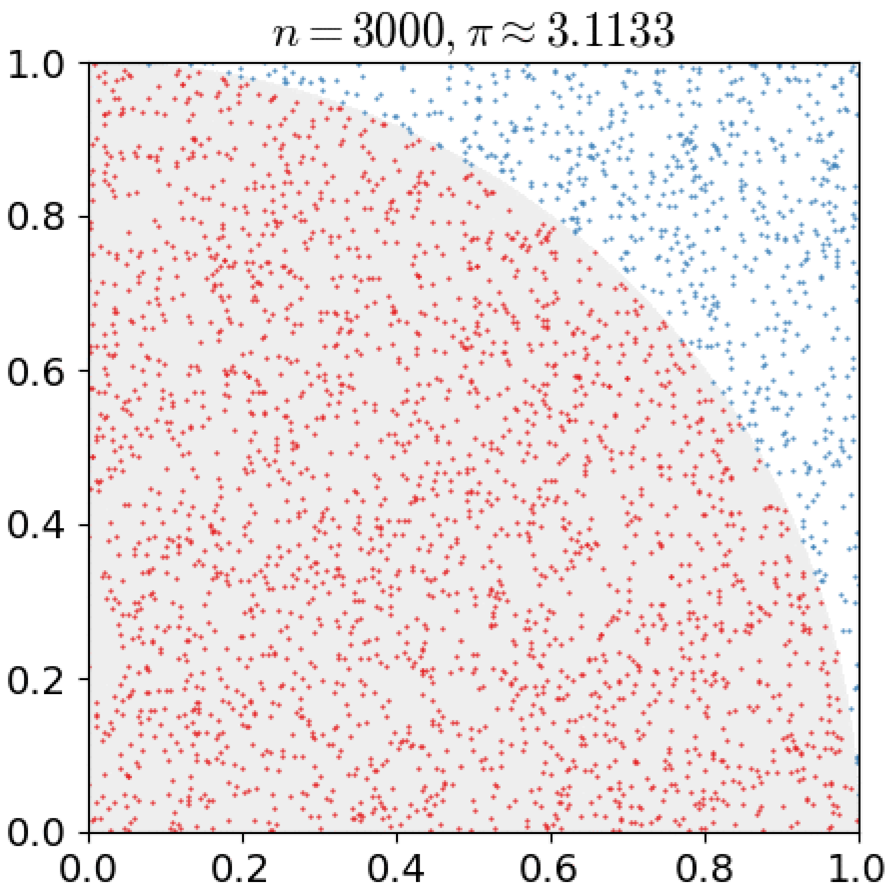
$$E[X] = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$E[X] = 1\frac{1}{6} + 2\frac{1}{6} + 3\frac{1}{6} + 4\frac{1}{6} + 5\frac{1}{6} + 6\frac{1}{6} = 3.5$$



Example: Approximating the value of pi

$$\frac{A_{pie}}{A_{square}} = \frac{\pi}{4}$$



Two important points:

- Must be random (uniformly distributed).
- Enough number of samples (n).

Procedures

(1) Collect independent random samples from the input random variables

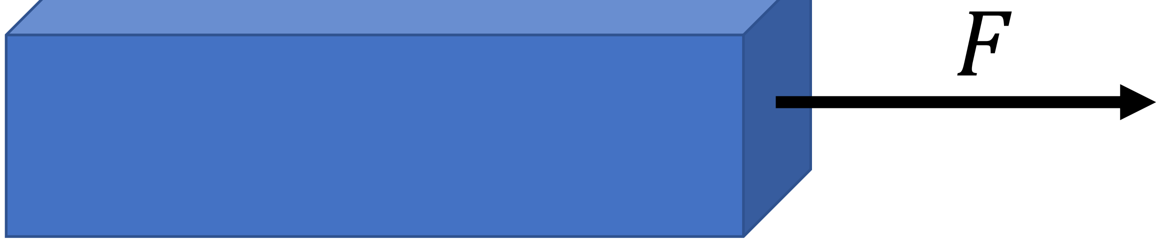
(2) Evaluate the function of interest

(3) Assess the statistical properties of the output function

Monte Carlo approach

$$g(F, A, S_y) = S_y - \frac{F}{A} = S_y - S$$

Limit state function



$h = 10\ mm$

$w = 20\ mm$

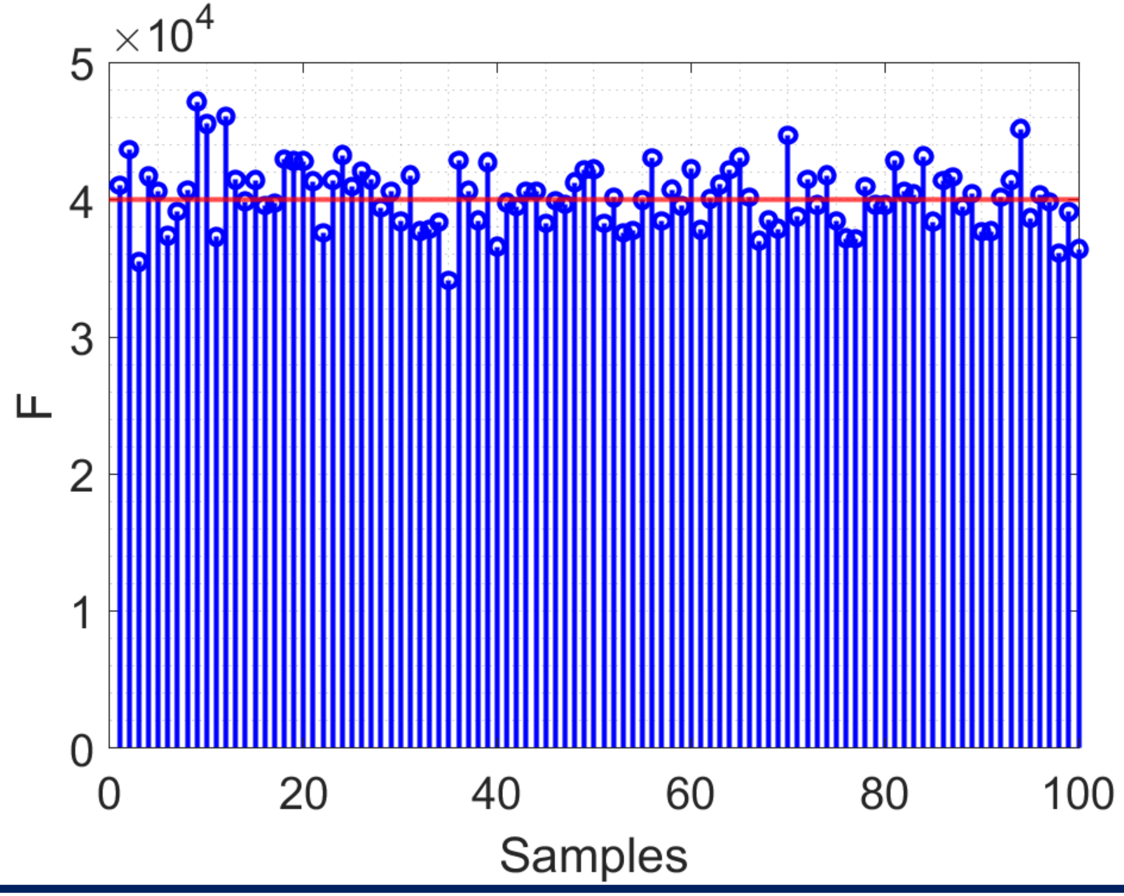
$$F \sim N[\mu_F = 40\ kN; CoV = 5\%]$$

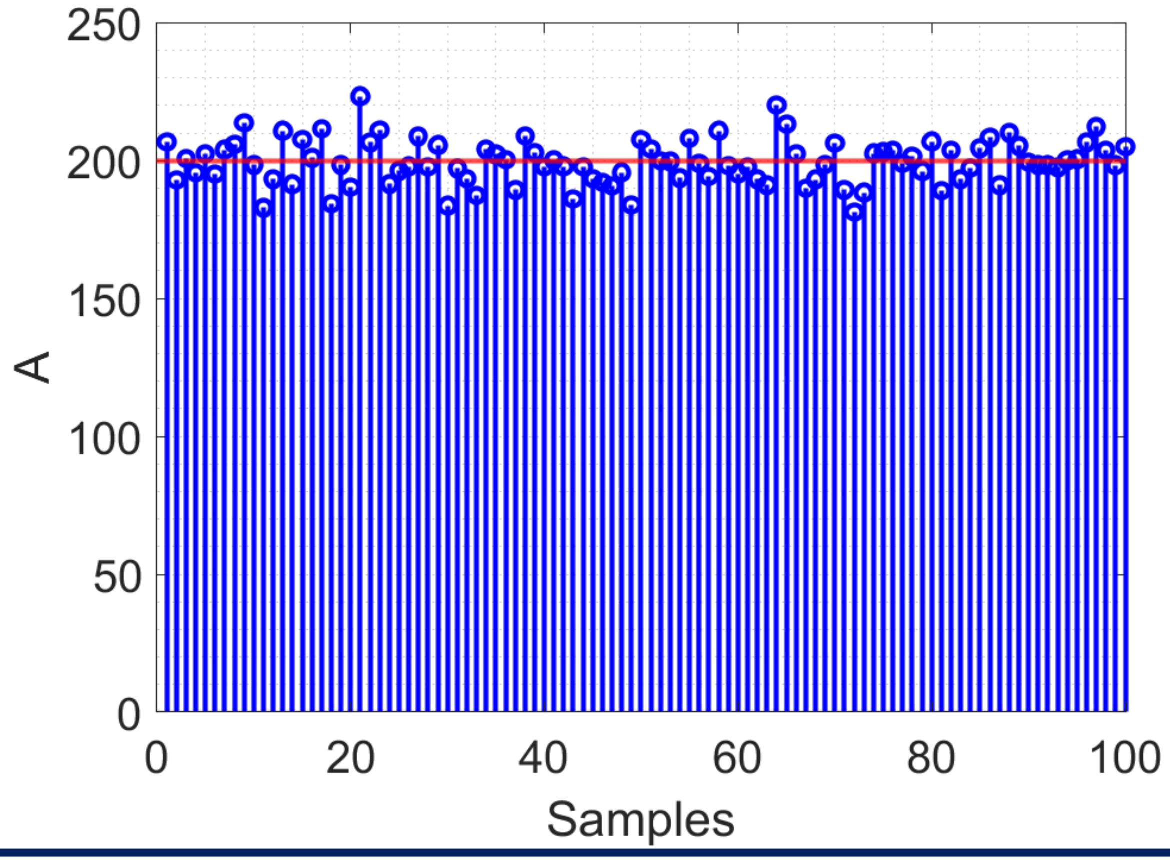
$$A \sim N[\mu_A = 200\ mm^2; CoV = 4\%]$$

$$S_y \sim N[\mu_{S_y} = 235\ N/mm^2; CoV = 10\%]$$

$$S = \frac{F}{A}$$

(1) Collect independent random samples from the input random variables





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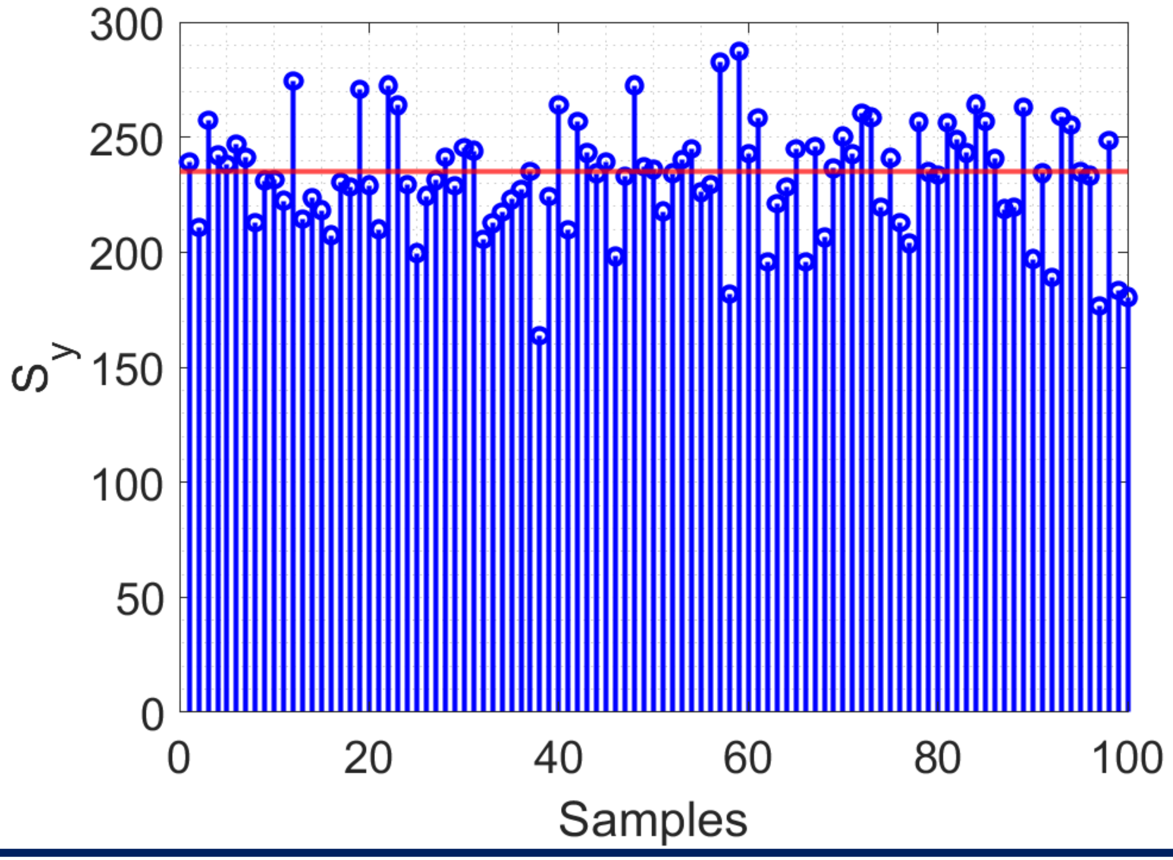
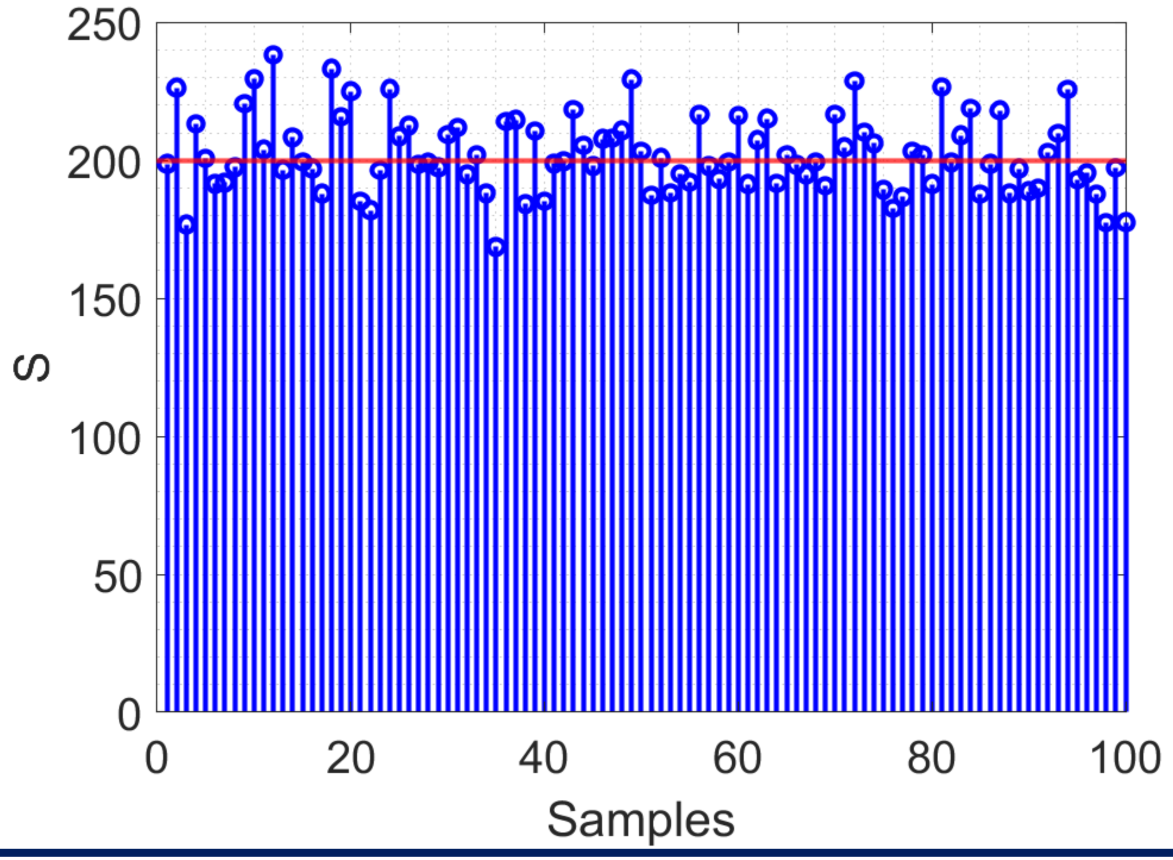


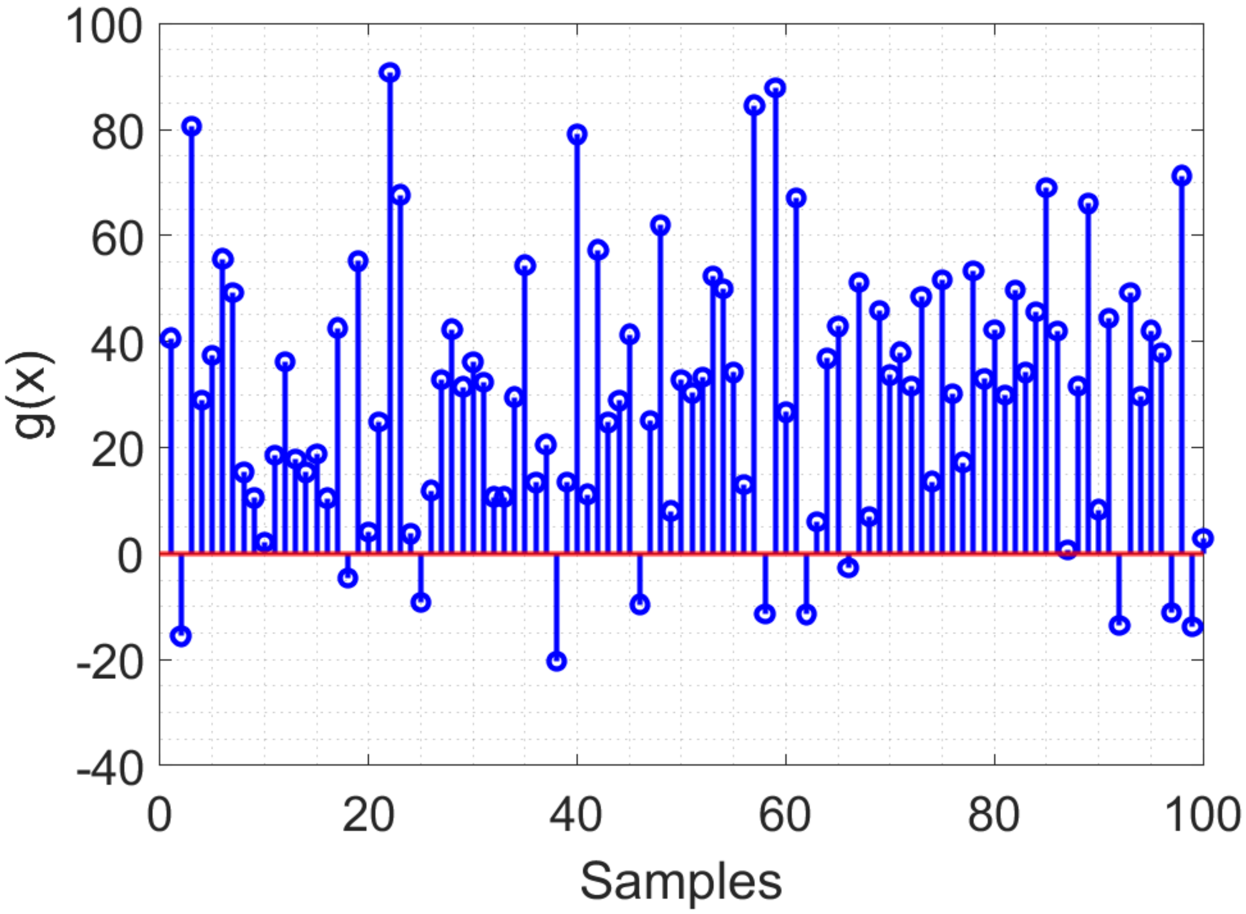


(2) Evaluate the function of interest

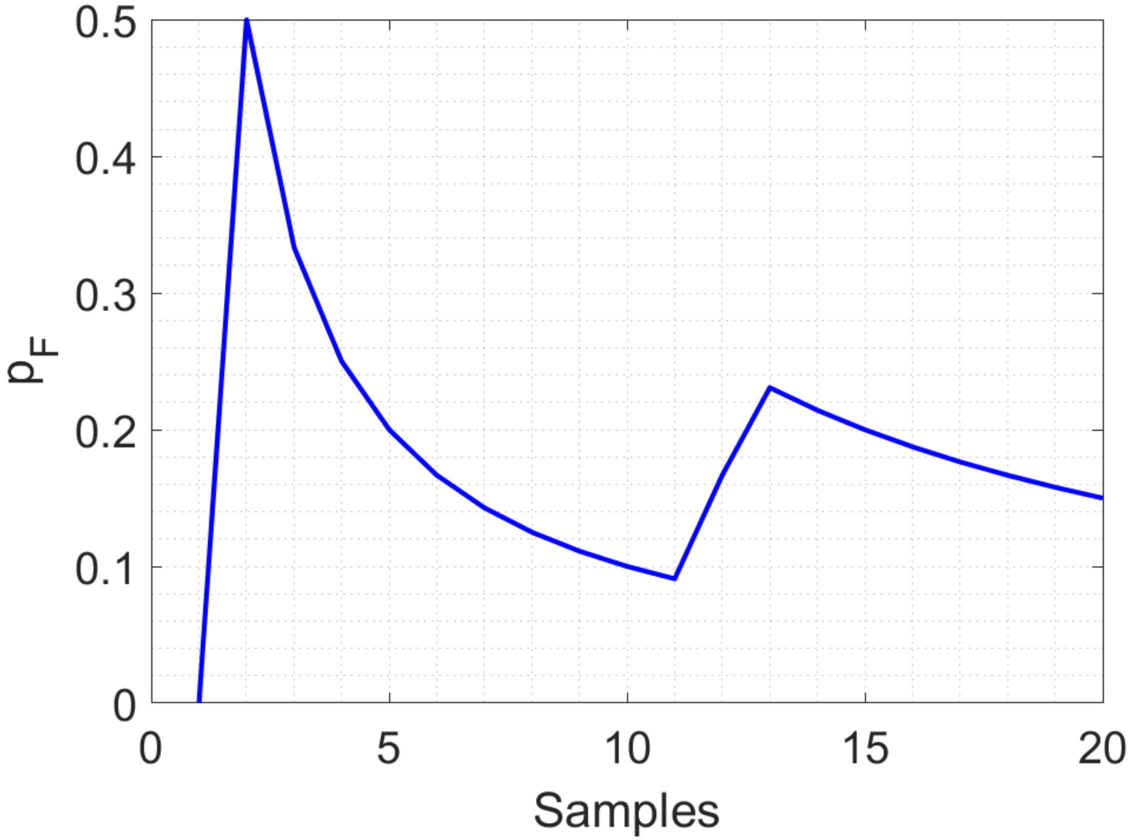
$$g(F, A, S_y) = S_y - \frac{F}{A} = S_y - S$$

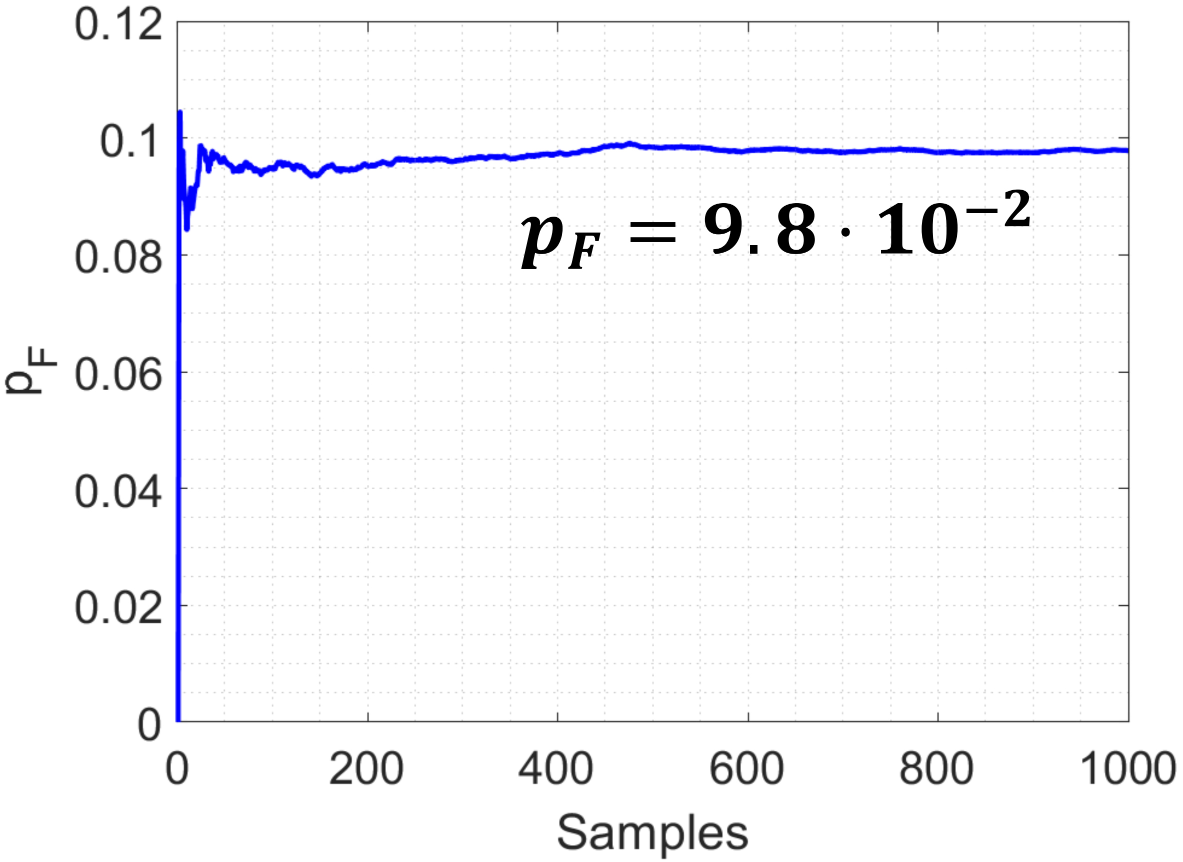
$$p_F = \frac{1}{N}\sum_{i=1}^{N} I[g(\boldsymbol{x})] \begin{cases} 1 \; if \; g(\boldsymbol{x}) \leq \mathbf{0} \\ 0 \; if \; g(\boldsymbol{x}) > \mathbf{0} \end{cases}$$

$$p_F = \frac{1}{100} 11 = 0.11$$



(3) Assess the statistical properties of the output function



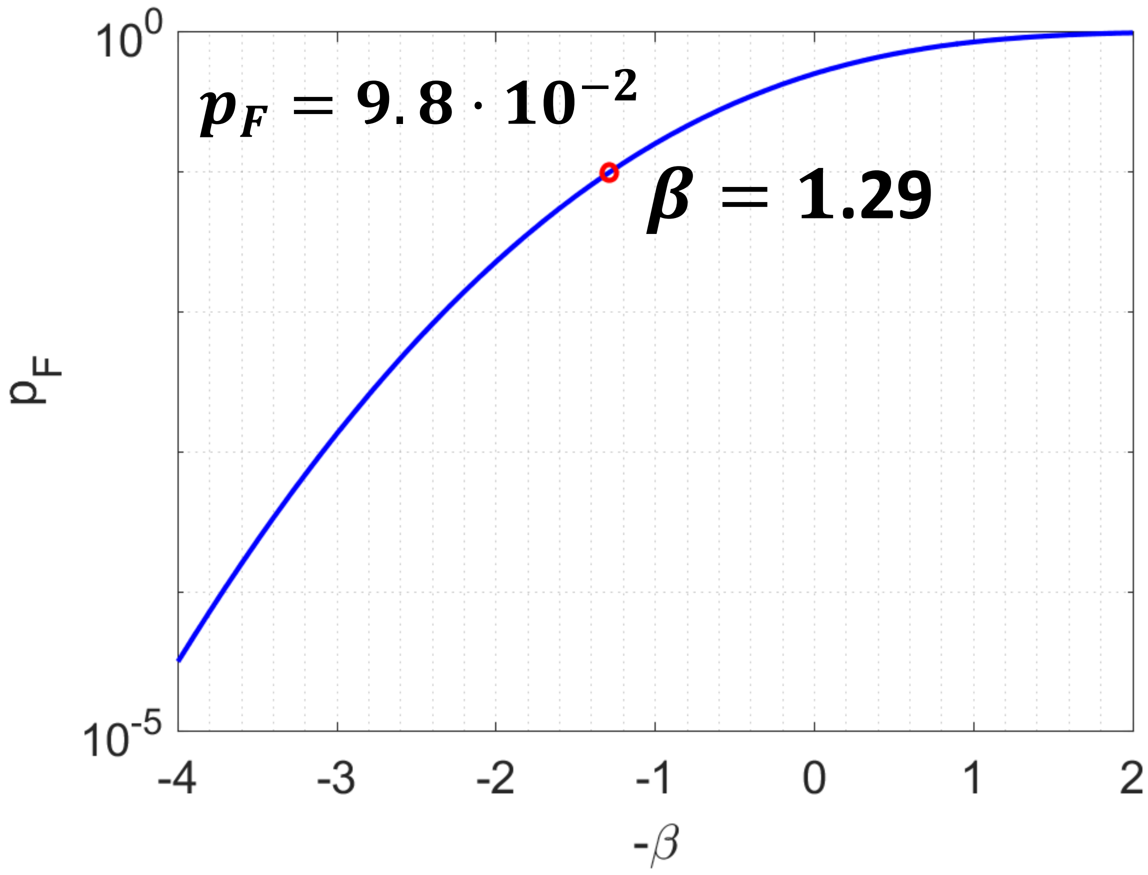


$S_y \sim N[\mu_{S_y} = 235\ N/mm^2; CoV = \mathbf{10\%}]$

$$\boldsymbol{\beta = -\Phi^{-1}[p_F]}$$

$$\boldsymbol{p_F = \Phi[-\beta]}$$

	Serviceability	
	P_f	β
Extreme	$1.0 \cdot 10^{-3}$	3.1
Severe	$5.2 \cdot 10^{-3}$	2.5
Moderate	$2.3 \cdot 10^{-2}$	2.0



$$S_y \sim N[\mu_{S_y} = 235\ N/mm^2; CoV = \mathbf{3}\%]$$

$$\boldsymbol{\beta = -\Phi^{-1}[p_F]}$$

$$\boldsymbol{p_F = \Phi[-\beta]}$$

	Serviceability	
	P_f	β
Extreme	$1.0 \cdot 10^{-3}$	3.1
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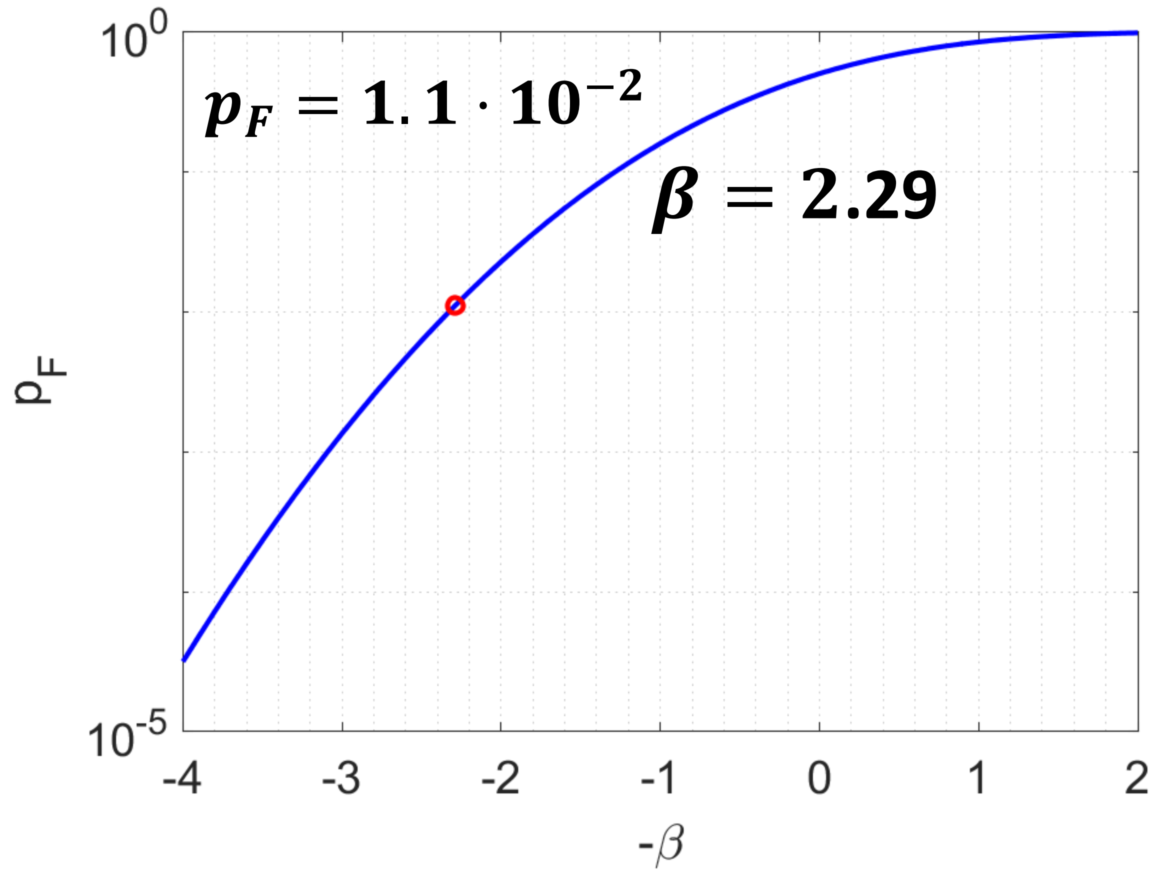
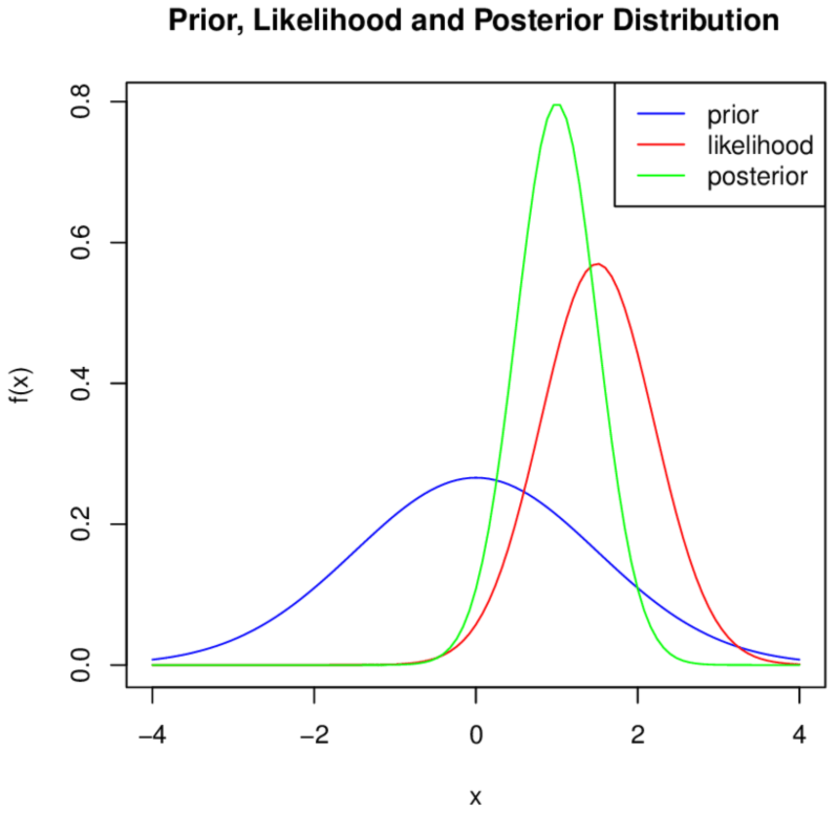


- Bayes' theorem is used to update the probability of a hypothesis/event when more information is available.

$$P(\theta|\, D) = \frac{P(D|\, \theta)P(\theta)}{P(D)}$$



Monty Hall Problem

2 goats | 1 Tesla

?

Question

2 goats | 1 Tesla

? Shall we change our first choice after one door has been opened?

- The host knows where the tesla is.
- The host only reveals a goat.

Solution (mathematical)

$p(T_1) = 1/3$

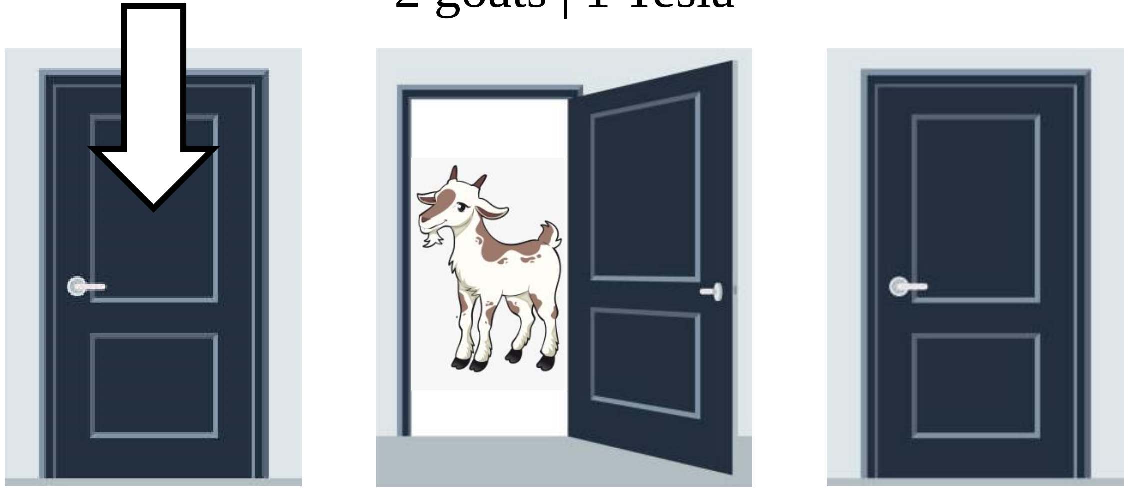
$p(T_2) = 1/3$

$p(T_3) = 1/3$

- T_x: Tesla is behind door ‘x’
- D_k: door ‘k’ is open

Solution (mathematical)

2 goats | 1 Tesla



$p(T_1) = 1/3$

$p(T_3) = 1/3$

$p(T_1|D_2)?$

$p(T_3|D_2)?$

$$p(T_1|D_2) = \frac{p(D_2|T_1)p(T_1)}{p(D_2)}$$

$$p(T_3|D_2) = \frac{p(D_2|T_3)p(T_3)}{p(D_2)}$$

Solution (mathematical)

2 goats | 1 Tesla

$p(T_1) = 1/3$

$p(T_1|D_2)?$

$p(D_2|T_1) = 0.5$

$$p(T_1|D_2) = \frac{p(D_2|T_1)p(T_1)}{p(D_2)}$$

Solution (mathematical)

2 goats | 1 Tesla

$p(T_3) = 1/3$

$p(D_2|T_3) = 1$

$p(T_3|D_2)?$

$$p(T_3|D_2) = \frac{p(D_2|T_3)p(T_3)}{p(D_2)}$$

Solution (mathematical)

2 goats | 1 Tesla

$$p(T_1|D_2) = \frac{p(D_2|T_1)p(T_1)}{p(D_2|T_1)p(T_1) + p(D_2|T_3)p(T_3)} = \frac{\frac{1}{3}\cdot 0.5}{\frac{1}{3}\cdot 0.5 + \frac{1}{3}\cdot 1} = \mathbf{1/3}$$

Solution (mathematical)

2 goats | 1 Tesla

$$p(T_1|D_2) = \frac{p(D_2|T_1)p(T_1)}{p(D_2|T_1)p(T_1) + p(D_2|T_3)p(T_3)} = \frac{\frac{1}{3}\cdot 0.5}{\frac{1}{3}\cdot 0.5 + \frac{1}{3}\cdot 1} = \mathbf{1/3}$$