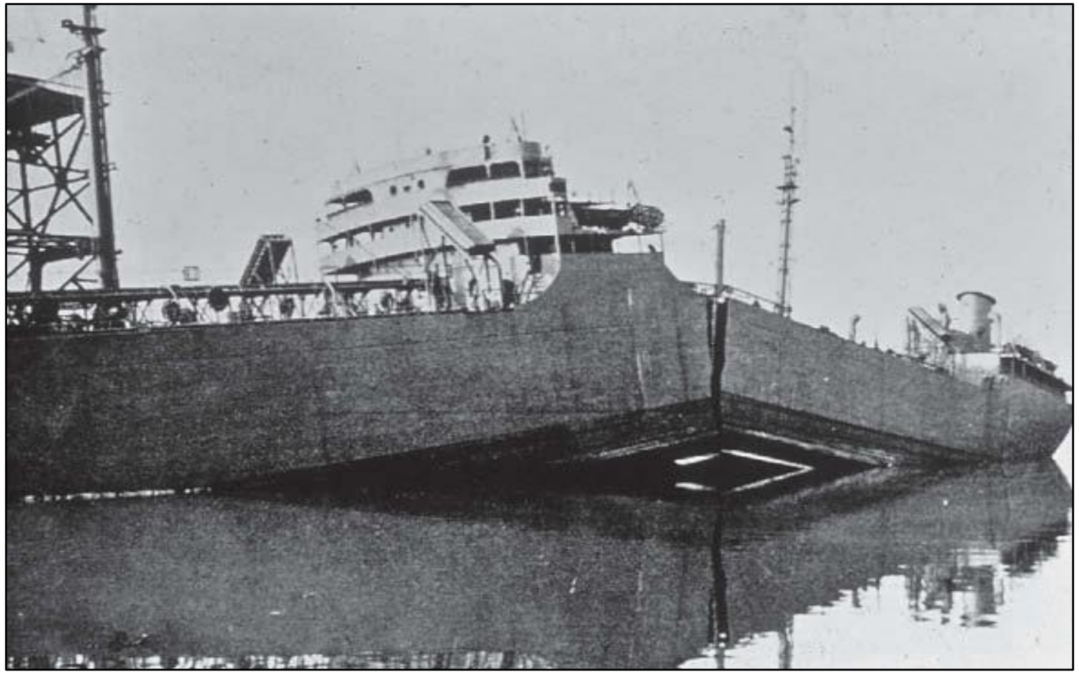
FATIGUE OF SHIPS AND OFFSHORE STRUCTURES

Source: https://www.explorermagazin.de/amstar/amaut_e.htm



Source: https://metallurgyandmaterials.wordpress.com/2015/12/25/liberty-ship-failures/

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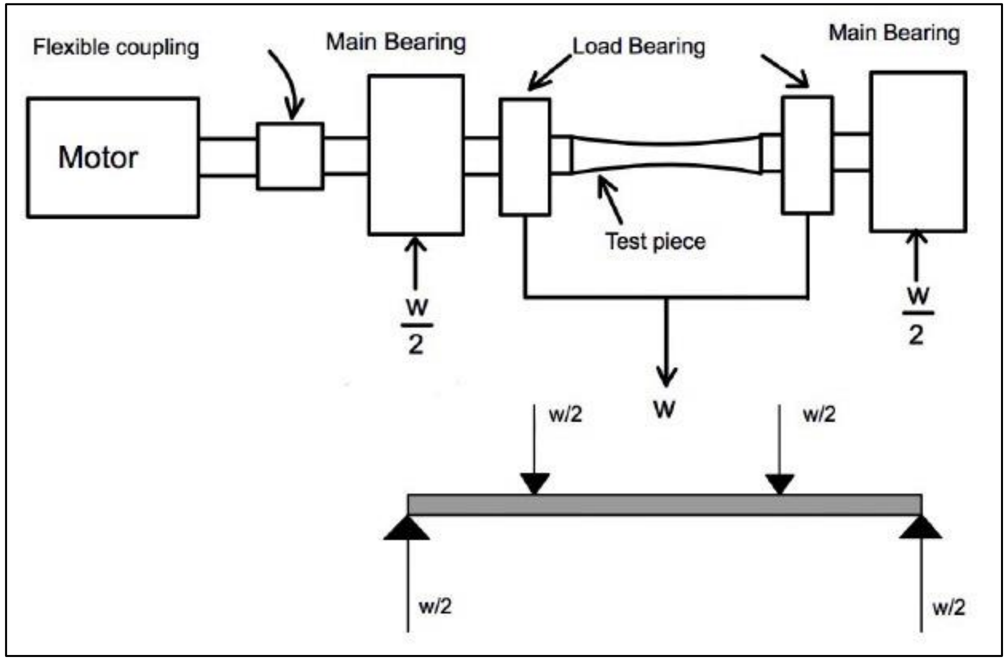
Philippe Rigo

https://github.com/Nandarhline

Repository: Fatigue_Lecture

❑ Wohler (SN test)

- fixed amplitude
- experimental, numerical



❑ Short-term fatigue assessment

- variable amplitude
- experimental, numerical
- time domain (rainflow counting)
- frequency domain (RAO + Wave spectrum)

Time-domain stress history

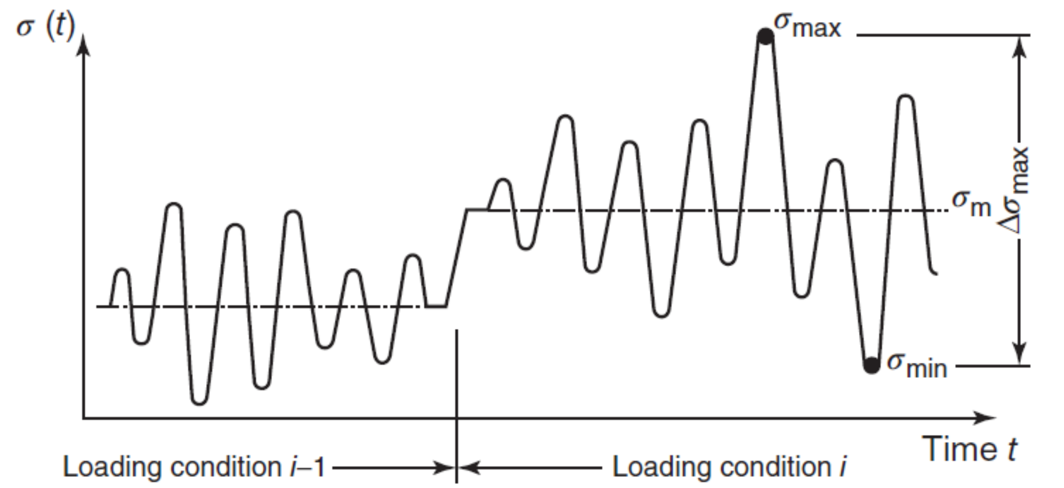
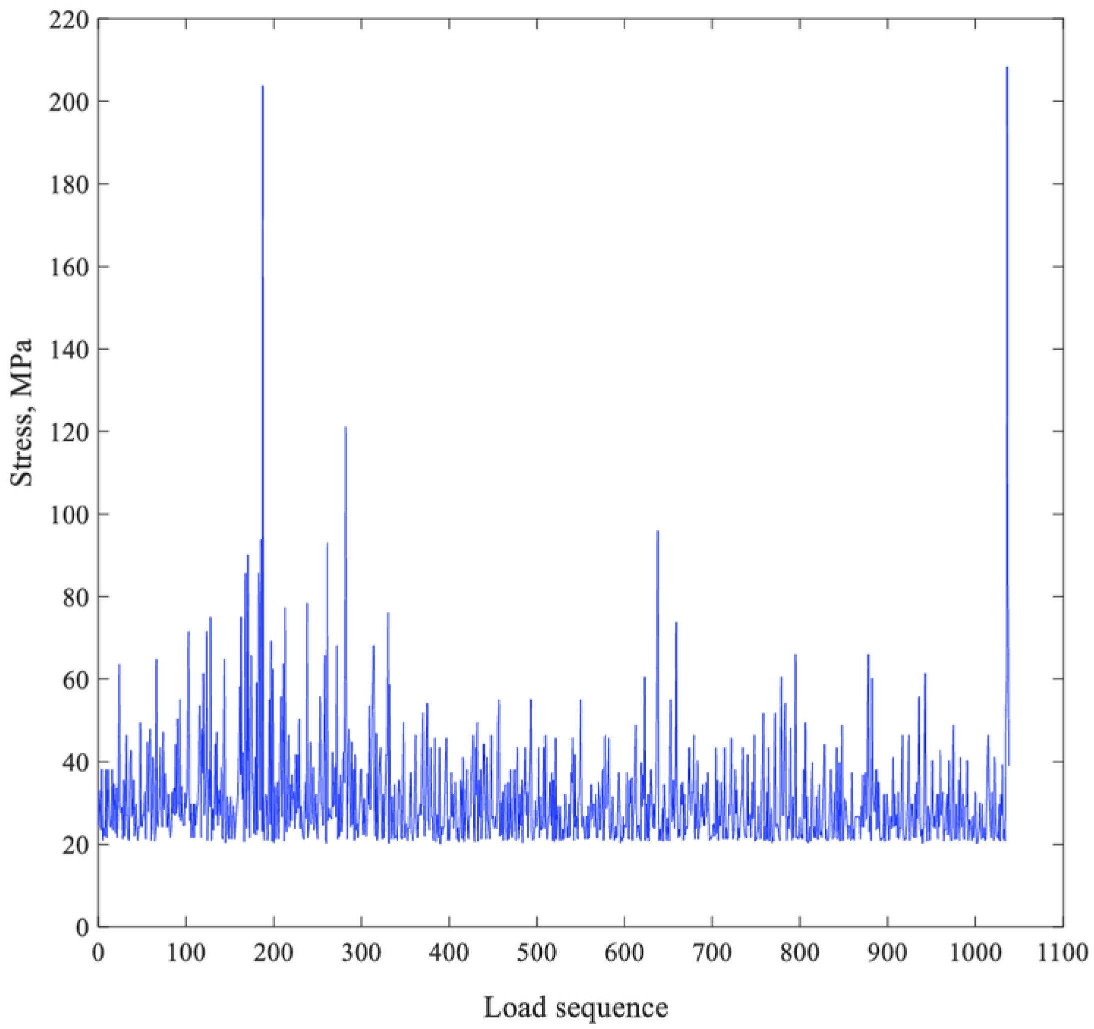
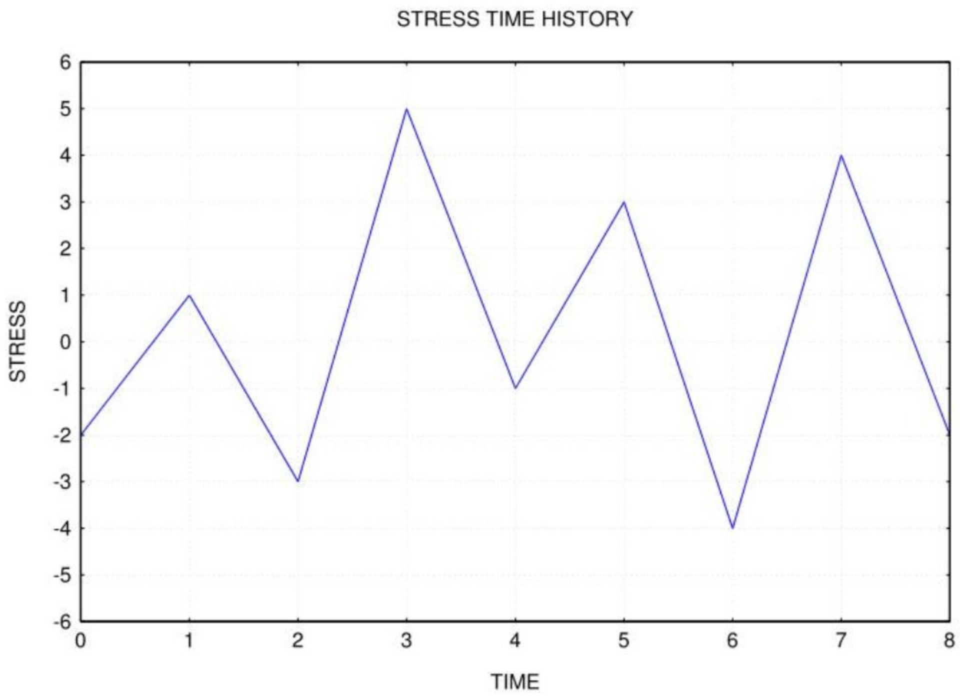
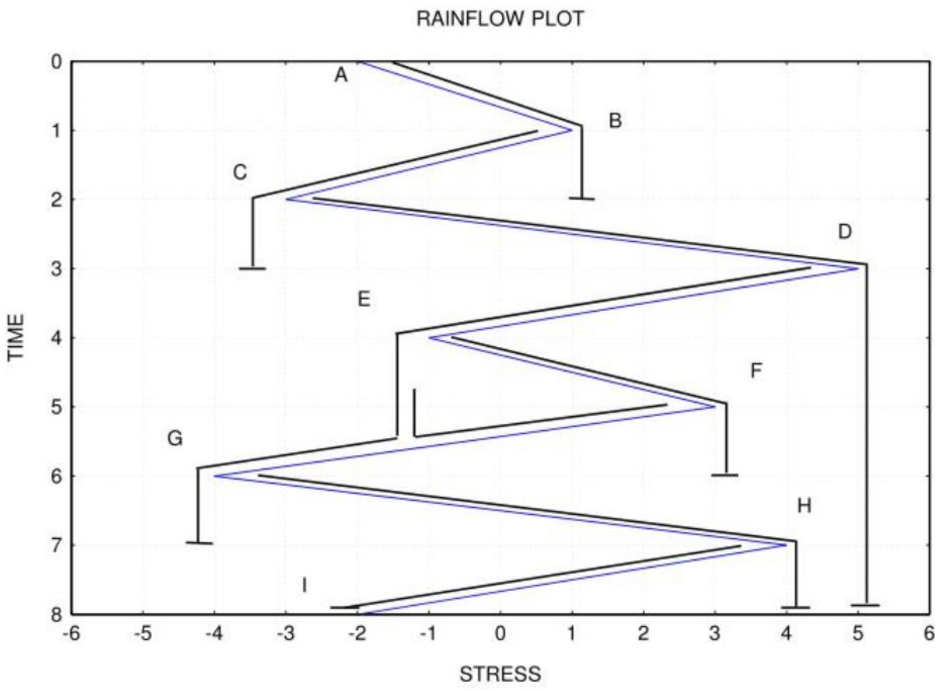


Figure 1. Stress history for superimposed stillwater and wave-induced loads (schematical).



Rainflow counting





Rainflow Cycle Counting

Rotate time history plot 90 degrees clockwise

Rainflow Cycles by Path		
Path	Cycles	Stress Range
A-B	0.5	3
B-C	0.5	4
C-D	0.5	8
D-G	0.5	9
E-F	1.0	4
G-H	0.5	8
H-I	0.5	6

http://www.maths.lth.se/matstat/wafo/

❑ Long-term fatigue assessment

- variable amplitude
- experimental, (numerical)
- time domain (rainflow counting, load cases + probabilities)
- frequency domain (RAO + Wave spectrum + probabilities)

❑ Can we do it more easily?

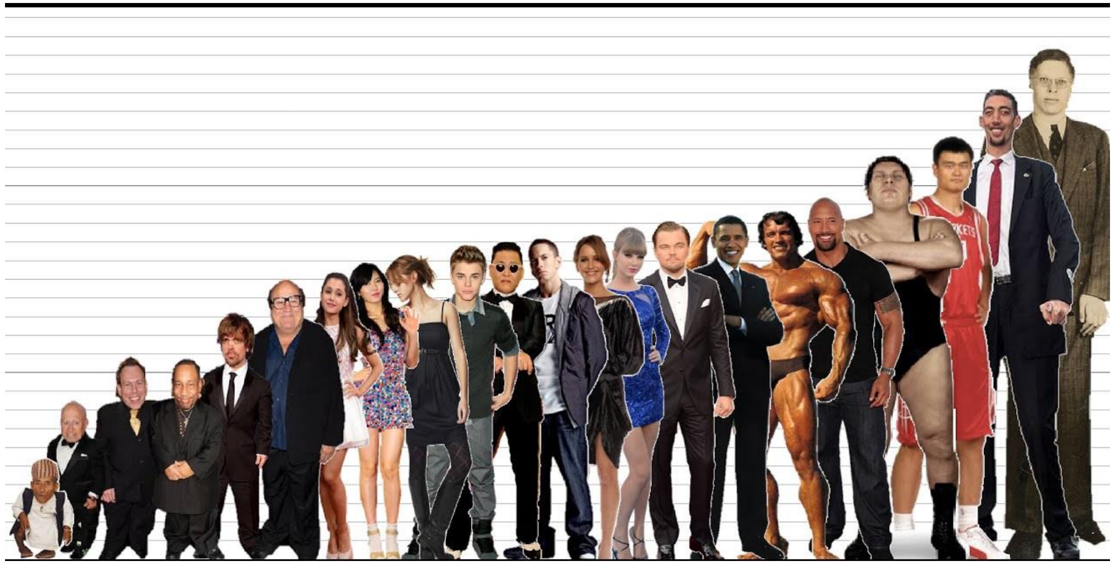
- simplified fatigue assessment

To understand the simplified fatigue assessment

$$\boldsymbol{D} = \frac{\boldsymbol{n_t}\, \boldsymbol{q^m}}{\boldsymbol{K}} \boldsymbol{\Gamma}\left(\boldsymbol{1} + \frac{\boldsymbol{m}}{\boldsymbol{h}}\right)$$

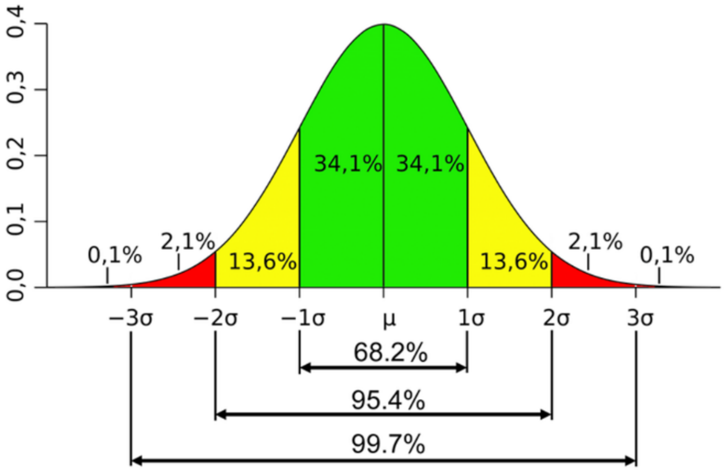
D = long-term fatigue damage
n_t = total number of stress cycles for the design lifetime
K, m = SN curve parameters
q, h = Weibull scale and shape parameters

Probability distributions

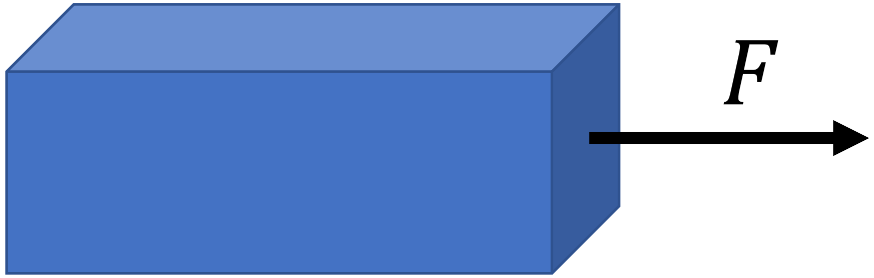


Continuous

Discrete



Probability and statistics



$L = 2\,m$

$E = 210\,GPa$

$F = 40\,kN$

$$\boldsymbol{\delta = \frac{FL}{EA}}$$

Deterministic approach

$A = wh$

$h = 10\,mm$

$w = 20\,mm$

$$\boldsymbol{\delta = 1.9\,mm}$$

Probabilistic approach

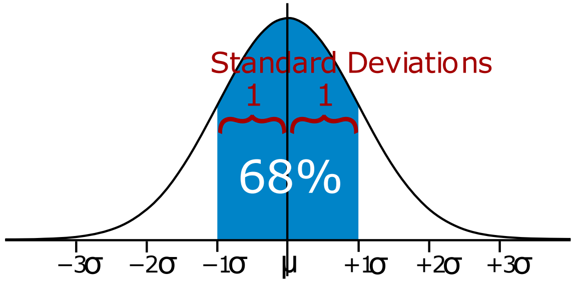
$A = wh$

$h = 10\,mm$

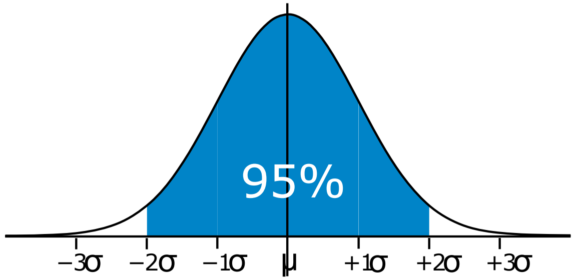
$w \sim N[\mu = 20\,mm, CoV = 15\%]$

$CoV = \frac{\sigma}{\mu}, \quad \sigma = 3mm$

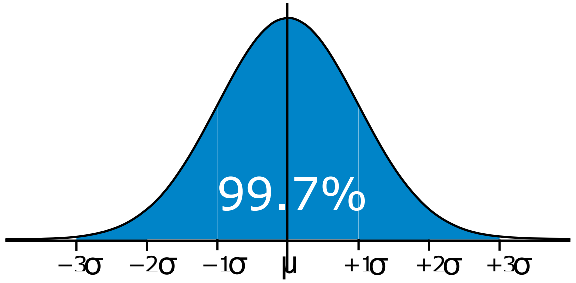
What is a normal (Gaussian) distribution?



68.2% of the samples are with 1 standard deviation of the mean.



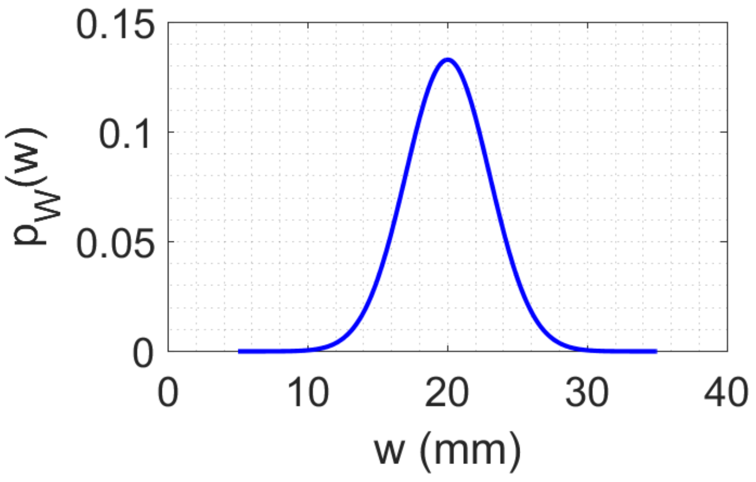
95.4% of the samples are with 2 standard deviation of the mean.

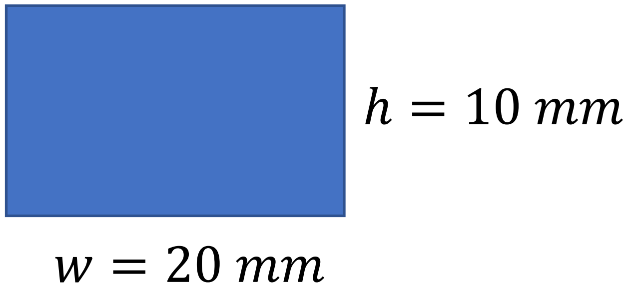


99.7% of the samples are with 3 standard deviation of the mean.

Carl Friedrich Gauss discovered the normal distribution in 1809.

$\boldsymbol{Probability\ density\ function\ (pdf)}$





$$w \sim N[\mu = 20\ mm, CoV = 15\%]$$

$\boldsymbol{Mean\ \ \mu_w}$

$$\mu_w = \int_{-\infty}^{+\infty} \boldsymbol{w}\, p_w(w)dw$$

$\boldsymbol{Variance\ \ \sigma_w^2}$

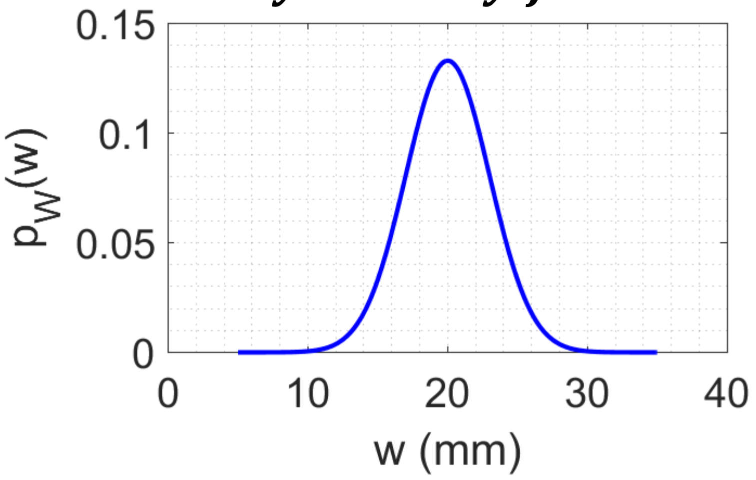
$$\sigma_w^2 = \int_{-\infty}^{+\infty} (\boldsymbol{w} - \boldsymbol{\mu})^2\, p_w(w)dw$$

$$p_w(w) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{w-\mu}{\sigma}\right)^2}$$

$\boldsymbol{Probability\ (P)\ -\ 18 < W < 20}$

$$P = \int_{18}^{20} p_w(w)dw$$

$\boldsymbol{Probability\ density\ function\ (pdf)}$

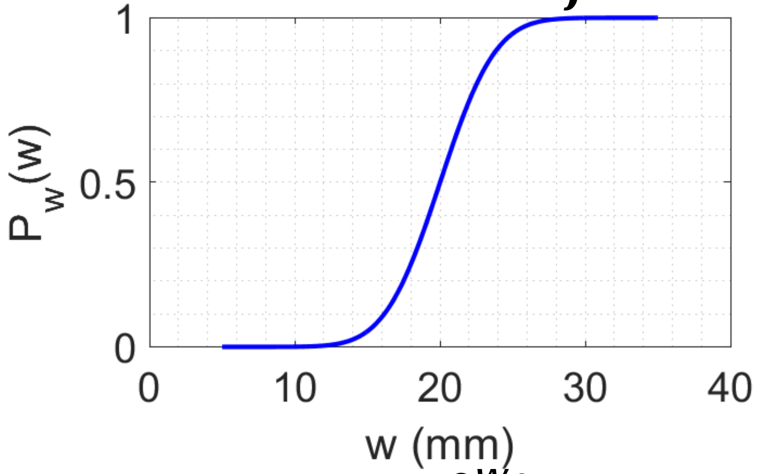


$h = 10\,mm$

$w = 20\,mm$

$w \sim N[\mu = 20\,mm, CoV = 15\%]$

$\boldsymbol{Cumulative\ distribution\ function\ (cdf)}$



$$P_w(W \le w_0) = \int_{-\infty}^{w_0} p_w(w)dw$$

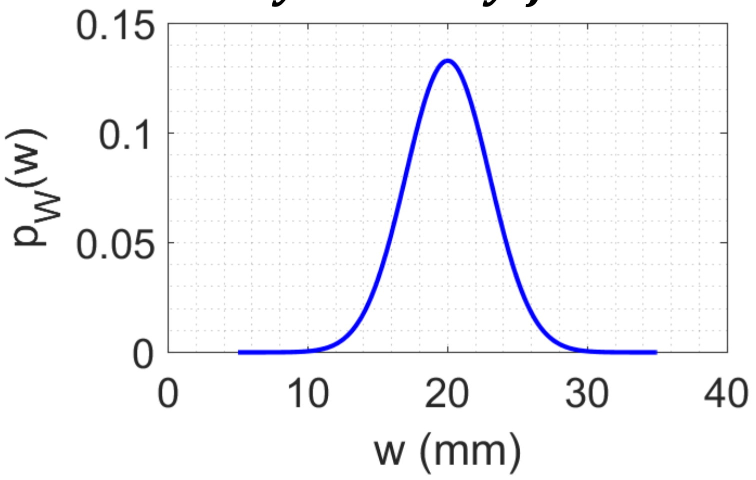
$\boldsymbol{Probability\ (P)\ -\ 18 < W < 20}$

$$P = \int_{18}^{20} p_w(w)dw$$

$$P = P_W(20) - P_W(18)$$

Probability and statistics

$$Probability\ density\ function\ (pdf)$$

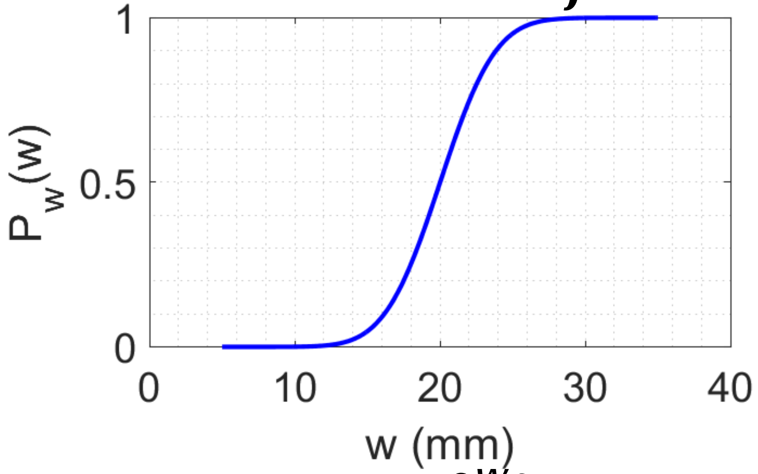


$$h = 10\ mm$$

$$w = 20\ mm$$

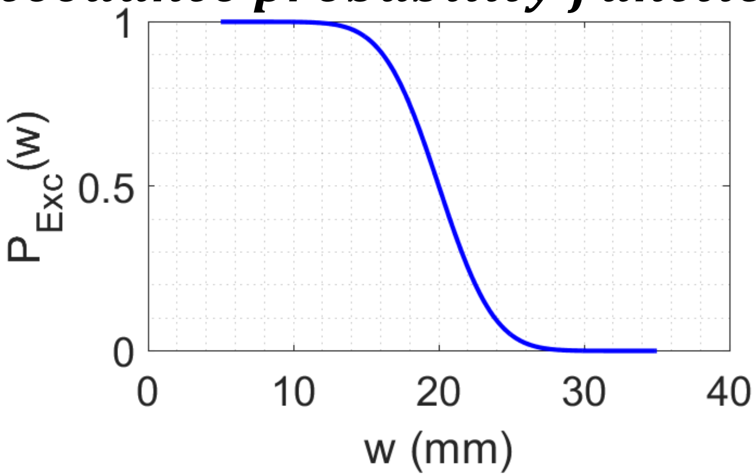
$$w \sim N[\mu = 20\ mm, CoV = 15\%]$$

$$Cumulative\ distribution\ function\ (cdf)$$



$$P_w(W \leq w_0) = \int_{-\infty}^{w_0} p_w(w)dw$$

$$Exceedance\ probability\ function$$



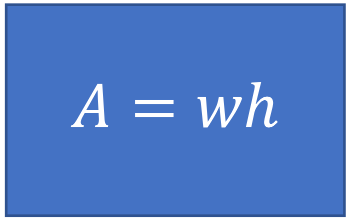
$$P_w(W \geq w_0) = 1 - P_w(W \leq w_0)$$

$$\boldsymbol{\delta = \frac{FL}{EA}}$$

$L = 2\ m$

$E = 210\ GPa$

$F = 40\ kN$



$h = 10\ mm$

$w = 20\ mm$

$w \sim N[\mu = 20\ mm, CoV = 15\%]$

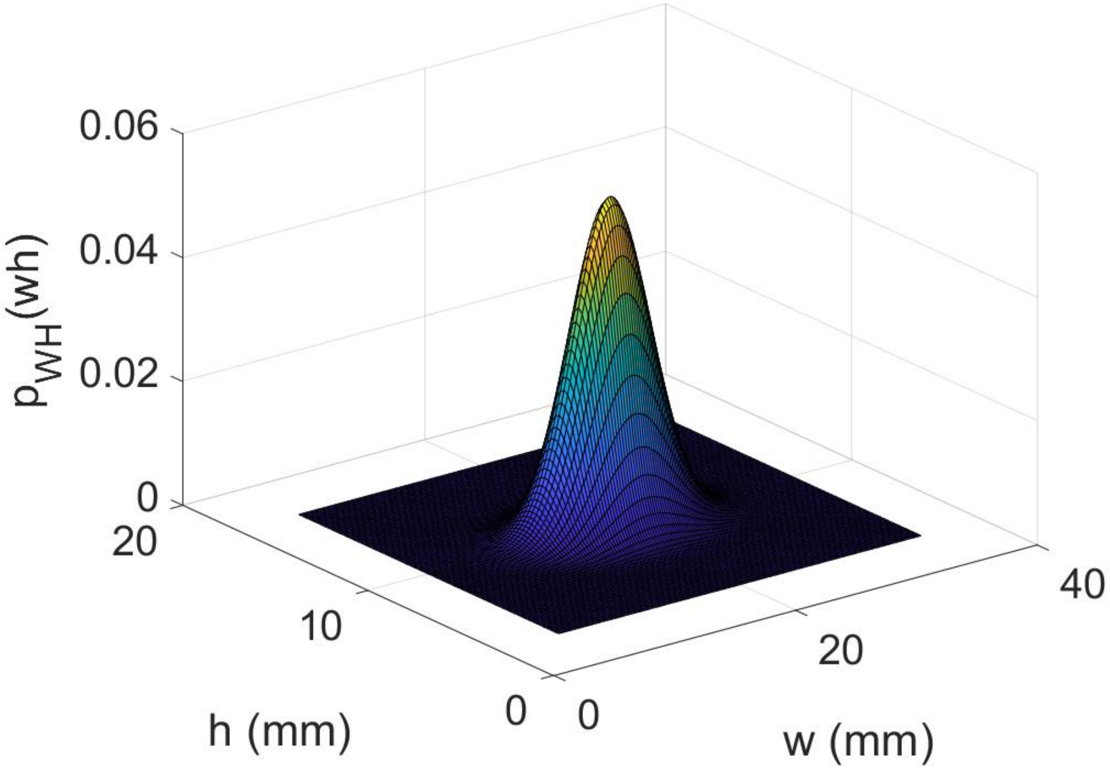
$CoV = \frac{\sigma}{\mu}$

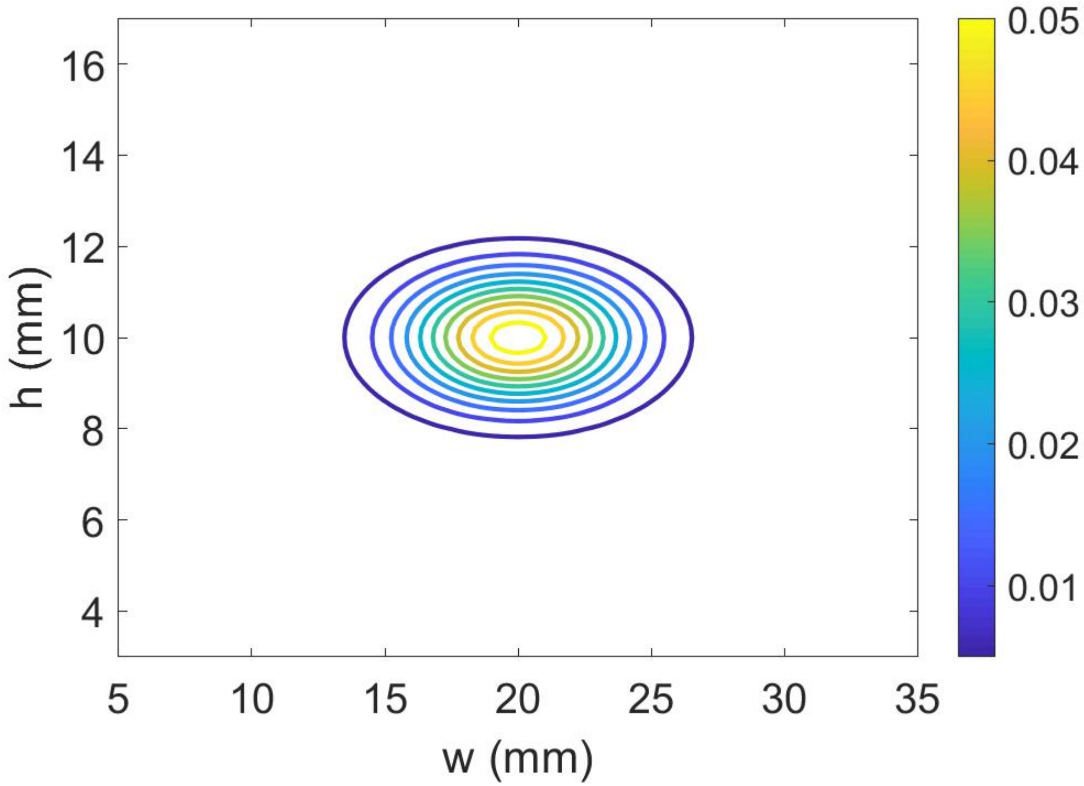
$\sigma = 3mm$

$h \sim N[\mu = 10\ mm, CoV = 10\%]$

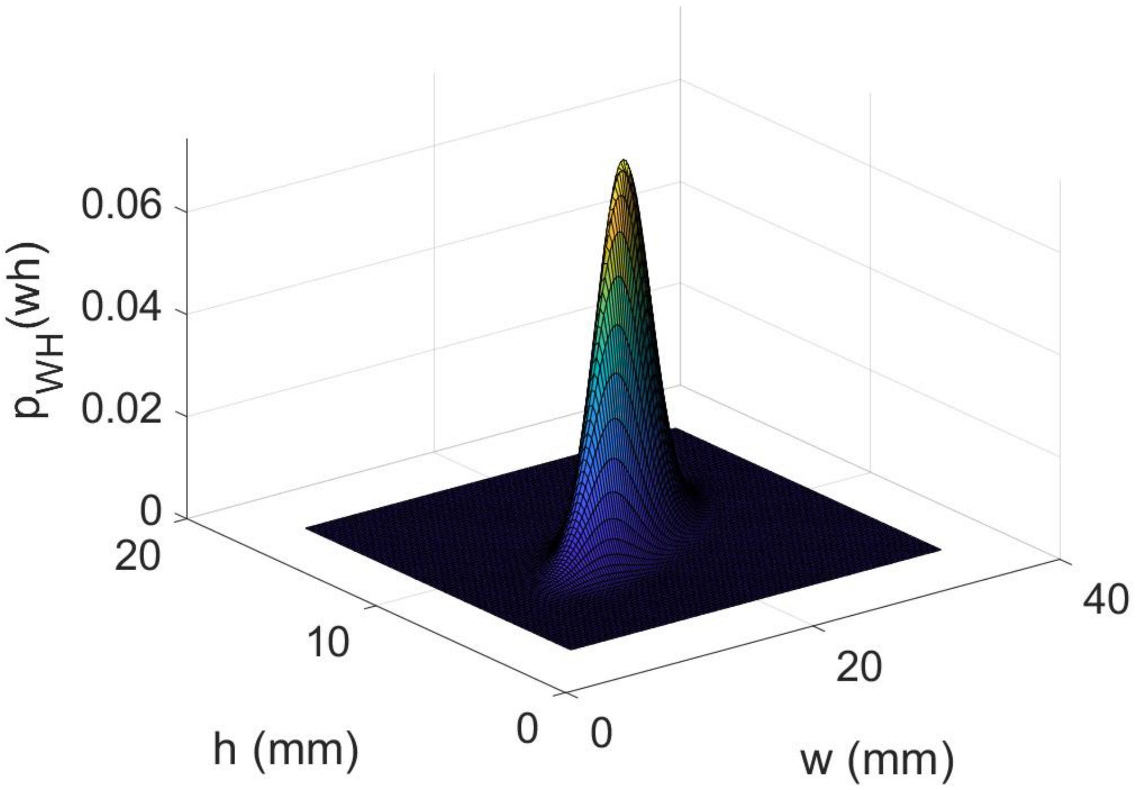
$\sigma_h = 1\ mm$

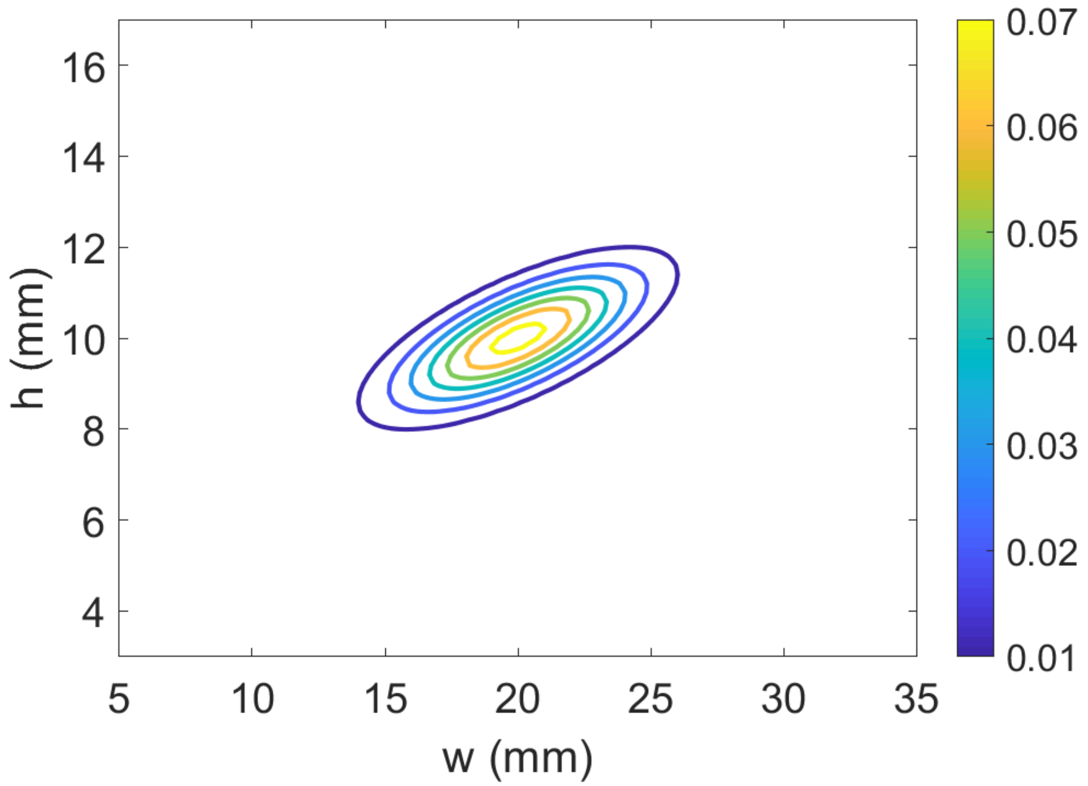
$\boldsymbol{Correlation}$: $\boldsymbol{\rho = 0}$





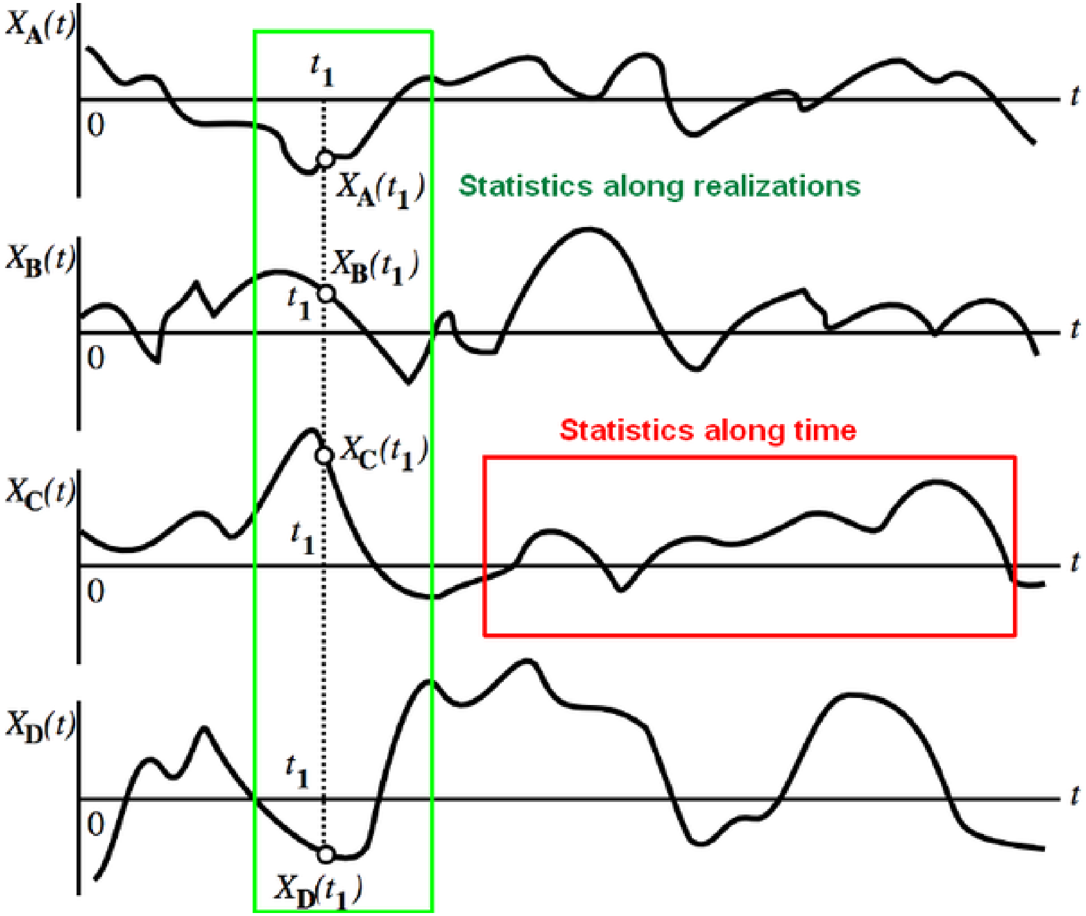
$Correlation$: $\boldsymbol{\rho = 0.7}$

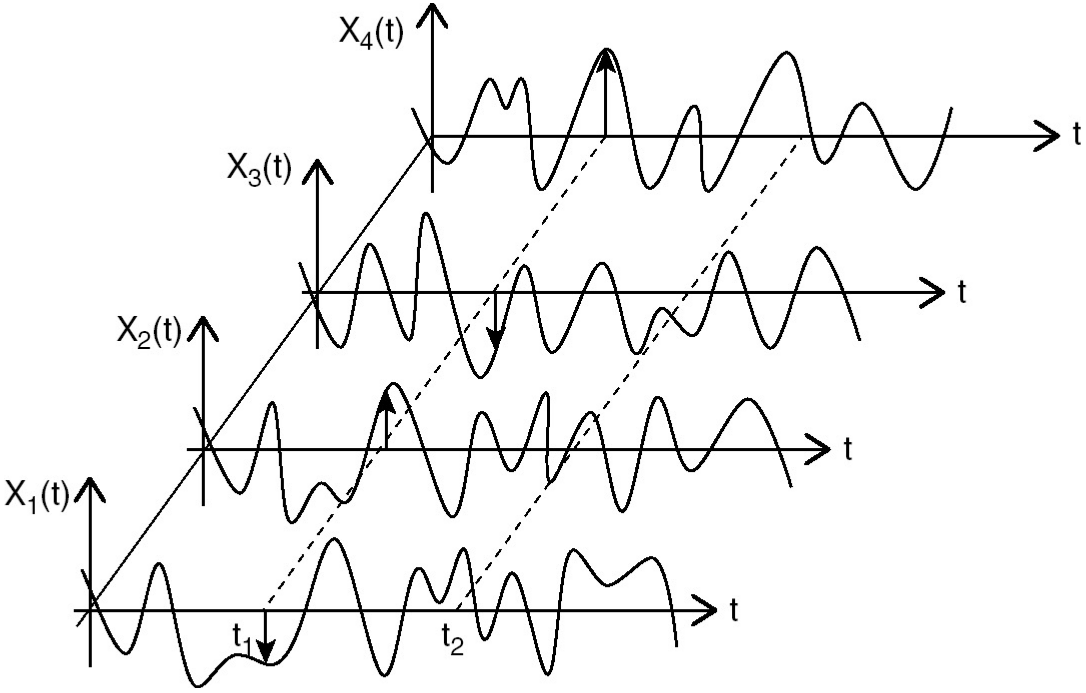




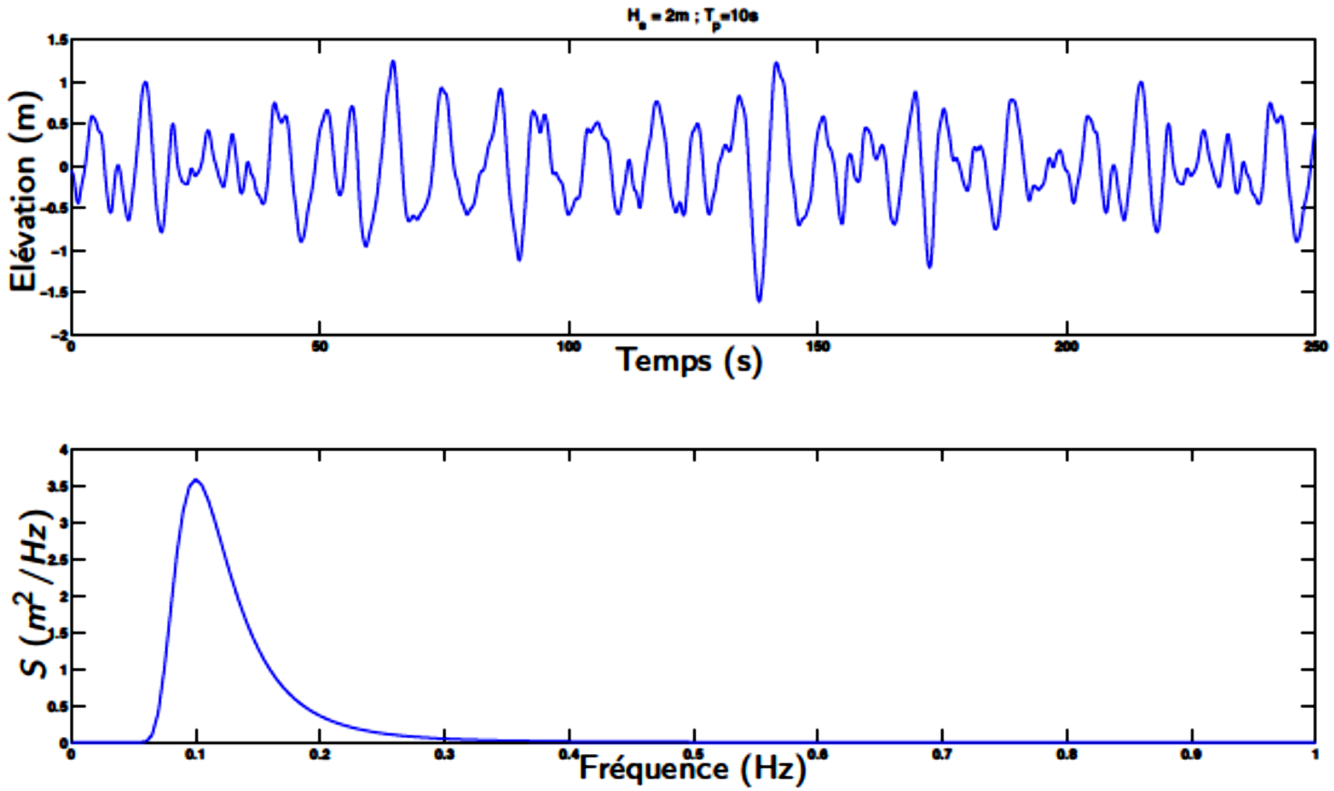
Random process

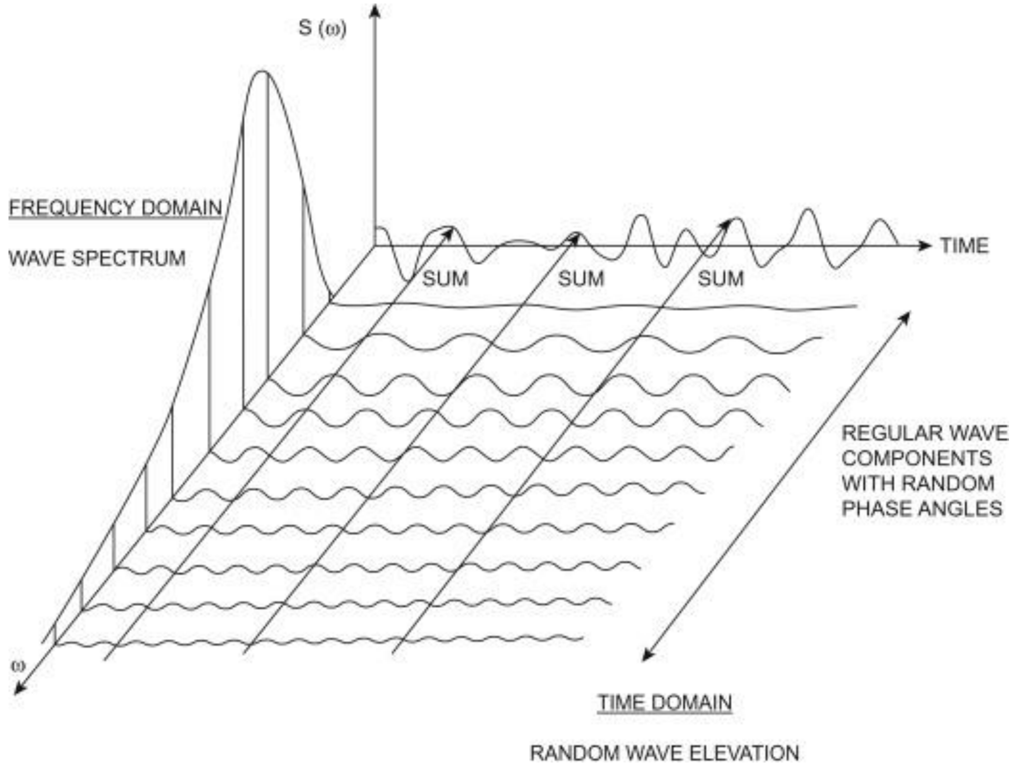
Ergodic (wide-sense stationary) – Sea state



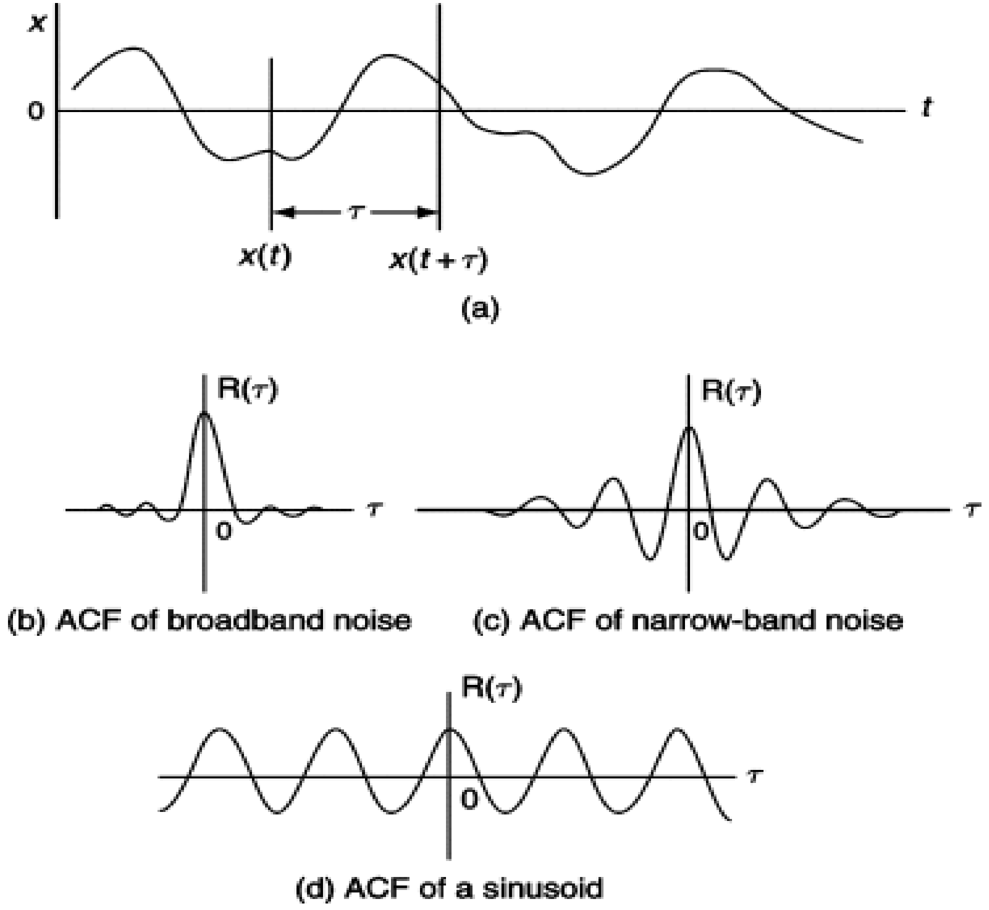


- A random sea surface is just a sum of several regular periodic waves with different wave frequencies/wavelengths.





Autocorrelation / autocovariance



Continuous signal:

$$R_{ff}(\tau) = \int_{-\infty}^{\infty} f(t+\tau)\overline{f(t)}\,\mathrm{d}t = \int_{-\infty}^{\infty} f(t)\overline{f(t-\tau)}\,\mathrm{d}t$$

Discrete signal:

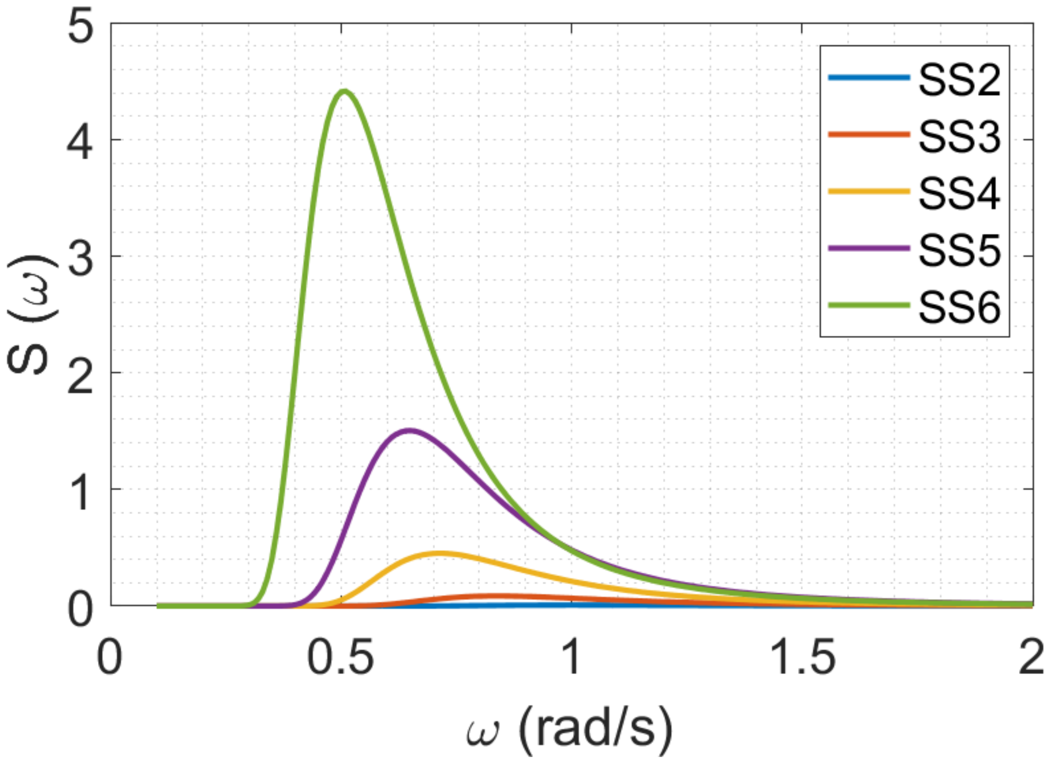
$$R_{yy}(\ell) = \sum_{n \in Z} y(n)\,\overline{y(n-\ell)}$$

The Fourier transform of the autocorrelation function gives a wave spectrum, i.e. the distribution of wave energy over various frequencies.

Example: Bretschneider Spectrum

$$S(\omega) = \frac{5}{16}\frac{w_m^4}{\omega^5} H_{1/3}^2 e^{-5\omega_m^4/4\omega^4}$$

Sea state (SS)	Peak period $T_m\ (s)$	Significant wave height $H_{1/3}\ (m)$
2	6.3	0.3
3	7.5	0.9
4	8.8	1.9
5	9.7	3.3
6	12.4	5.0



https://ocw.mit.edu/courses/mechanical-engineering/2-017j-design-of-electromechanical-robotic-systems-fall-2009/assignments/MIT2_017JF09_p04.pdf

Response spectrum

Response amplitude operator

X^2/m^2

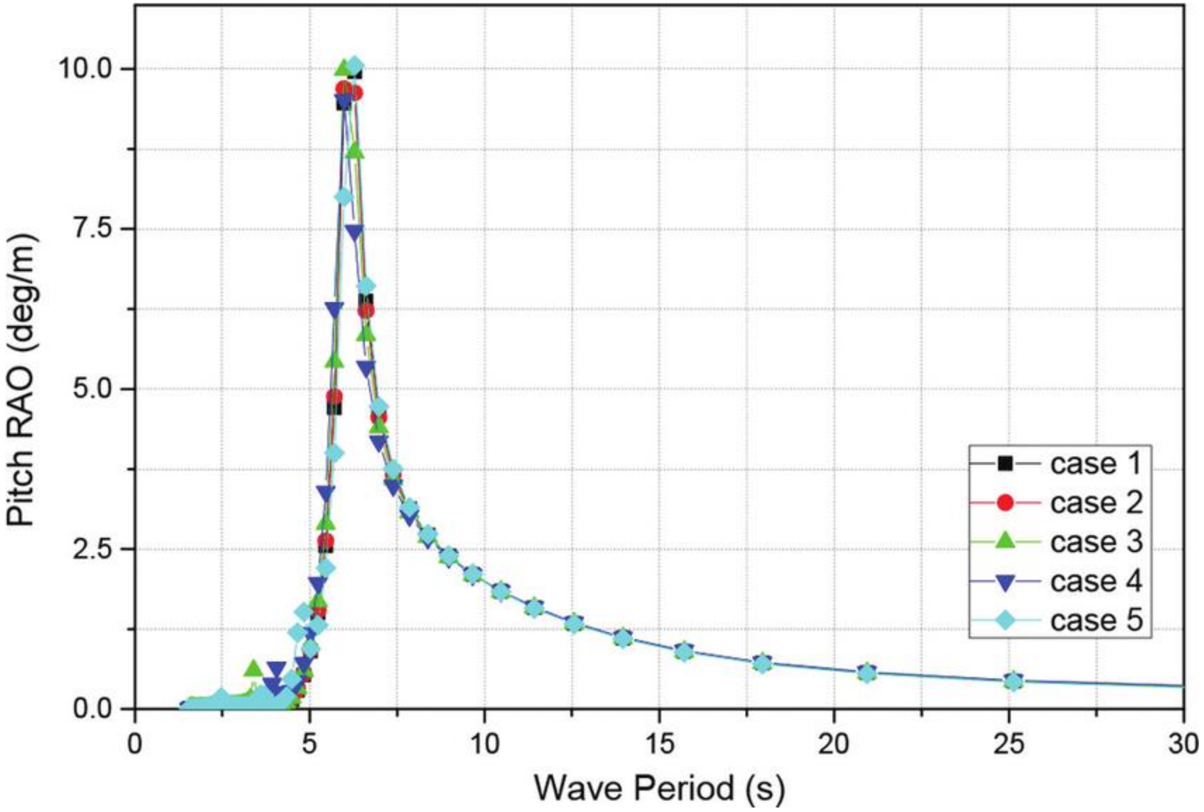
Wave spectrum

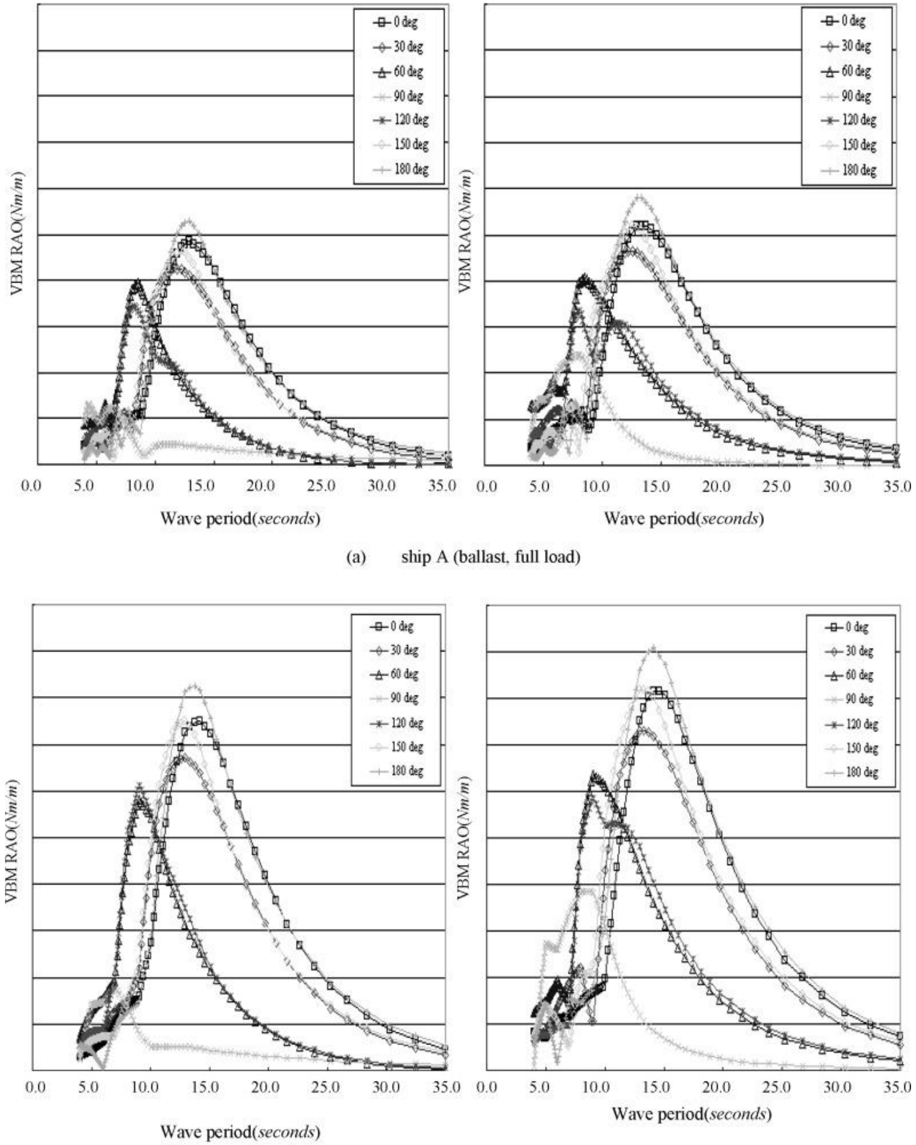
m^2/s

$$S_\Lambda(\omega, \emptyset, V) = |RAO(\omega, \emptyset, V)|^2 \cdot S_\omega(\omega)$$

$$RAO = \frac{y_0 Re\left[e^{i\omega t}\right]}{x_0 Re\left[e^{i\omega t}\right]}$$

RAOs





(a) ship A (ballast, full load)

(b) ship B (ballast, full load)

Short-term structural response

$$S_{\Lambda}(\omega, \emptyset, V) = |RAO(\omega, \emptyset, V)|^2 \cdot \ S_{\omega}(\omega)$$

Wave frequency

Heading angle

Ship speed

Long-term structural response

$$p(S) = \int\int\int\int S_{\Lambda}(\omega, \emptyset, V)\, p(\bar{H}, T_m) p(\emptyset) p(V)\, d\emptyset\, dV\, dT\, d\bar{H}$$

Probability sea state

Probability heading angle

Probability speed

Weibull distribution

Probability density function

$$p(S) = \frac{h}{q}\left(\frac{S}{q}\right)^{h-1} e^{-\left(\frac{S}{q}\right)^{h}}$$

Exceedance probability

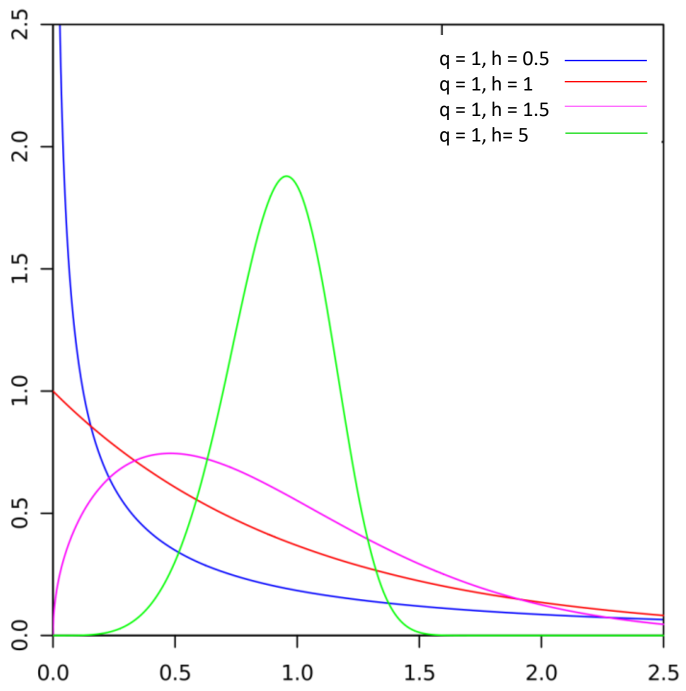
$$p(S > S_0) = e^{-\left(\frac{S_0}{q}\right)^{h}}$$

Long-term structural response

Weibull distribution(q,h)

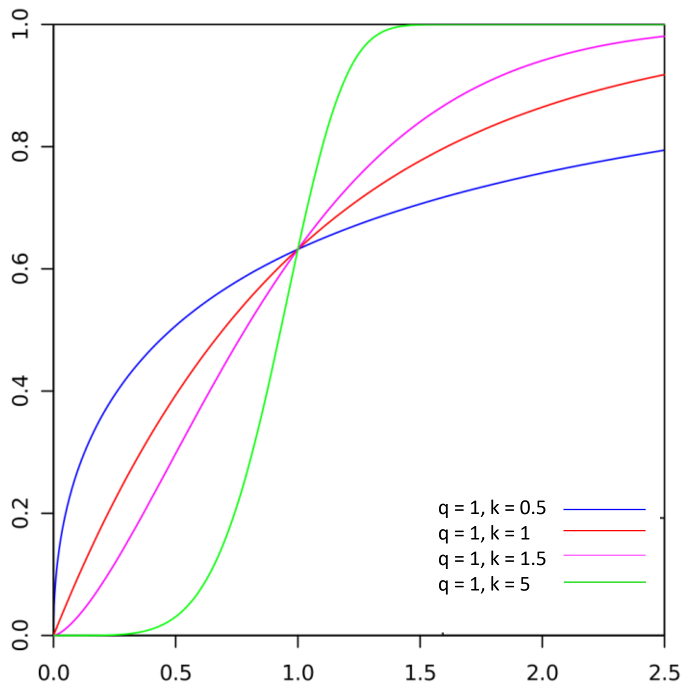
Probability density function

$$p(S) = \frac{h}{q}\left(\frac{S}{q}\right)^{h-1} e^{-\left(\frac{S}{q}\right)^{h}}$$

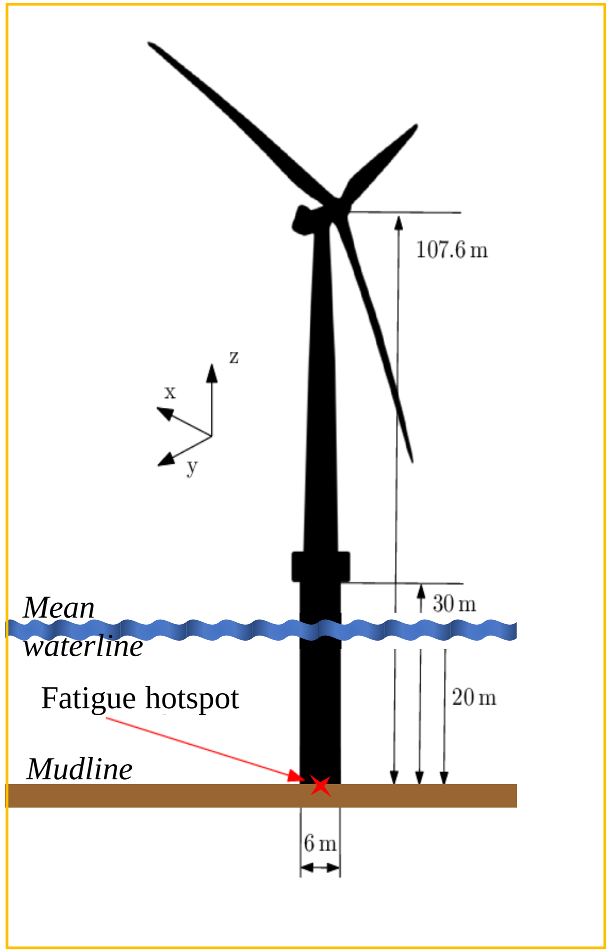


Exceedance probability

$$p(S > S_0) = e^{-\left(\frac{S_0}{q}\right)^{h}}$$

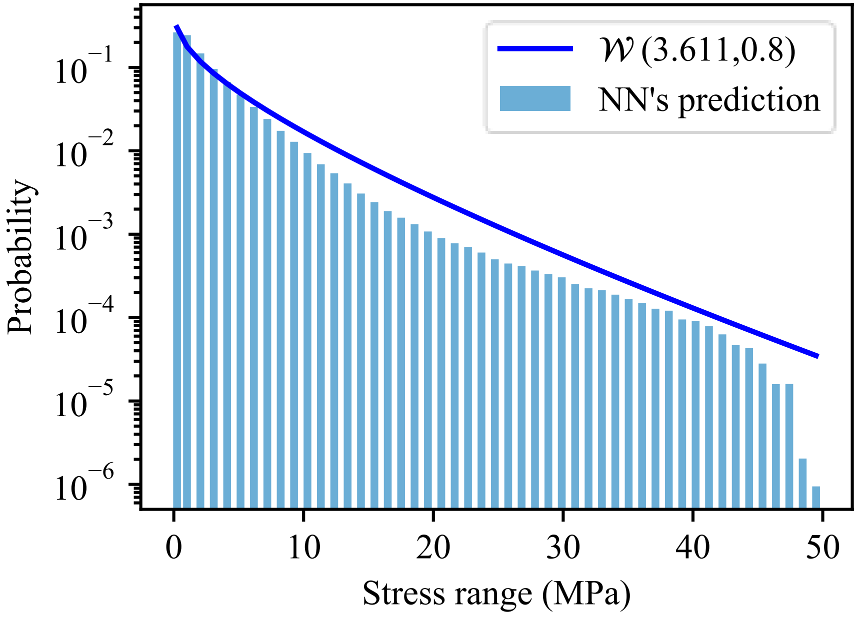


Example: Long-term structural response at the mudline of the NREL 5-MW offshore wind turbine



Wind turbine: Monopile-supported NREL 5MW
Location: Ijmuiden site (Dutch North Sea)

Variable	Description
Wind speed (V_w)	$V_w \sim$ Weibull ($scale = 10.49,\ shape = 2.08$)
Wind direction (θ_{wind})	$P(\theta_{wind}, \theta_{wave} \mid V_w)$
Turbulence intensity (I)	IEC-3 ($I_{15\,m/s} = 0.14$)
Significant wave height (H_s)	$P(H_s, T_p \mid V_w)$
Peak period (T_p)	$P(H_s, T_p \mid V_w)$
Wave direction (θ_{wave})	$P(\theta_{wind}, \theta_{wave} \mid V_w)$
Rotational speed (ω)	$f(V_w)$
Yaw error (θ_{yaw})	$\theta_{yaw} \in [-10, 10]$



Spectral Method

Wave spectrum + RAO

↓

Response spectrum

↓

Probabilities → Short-term response

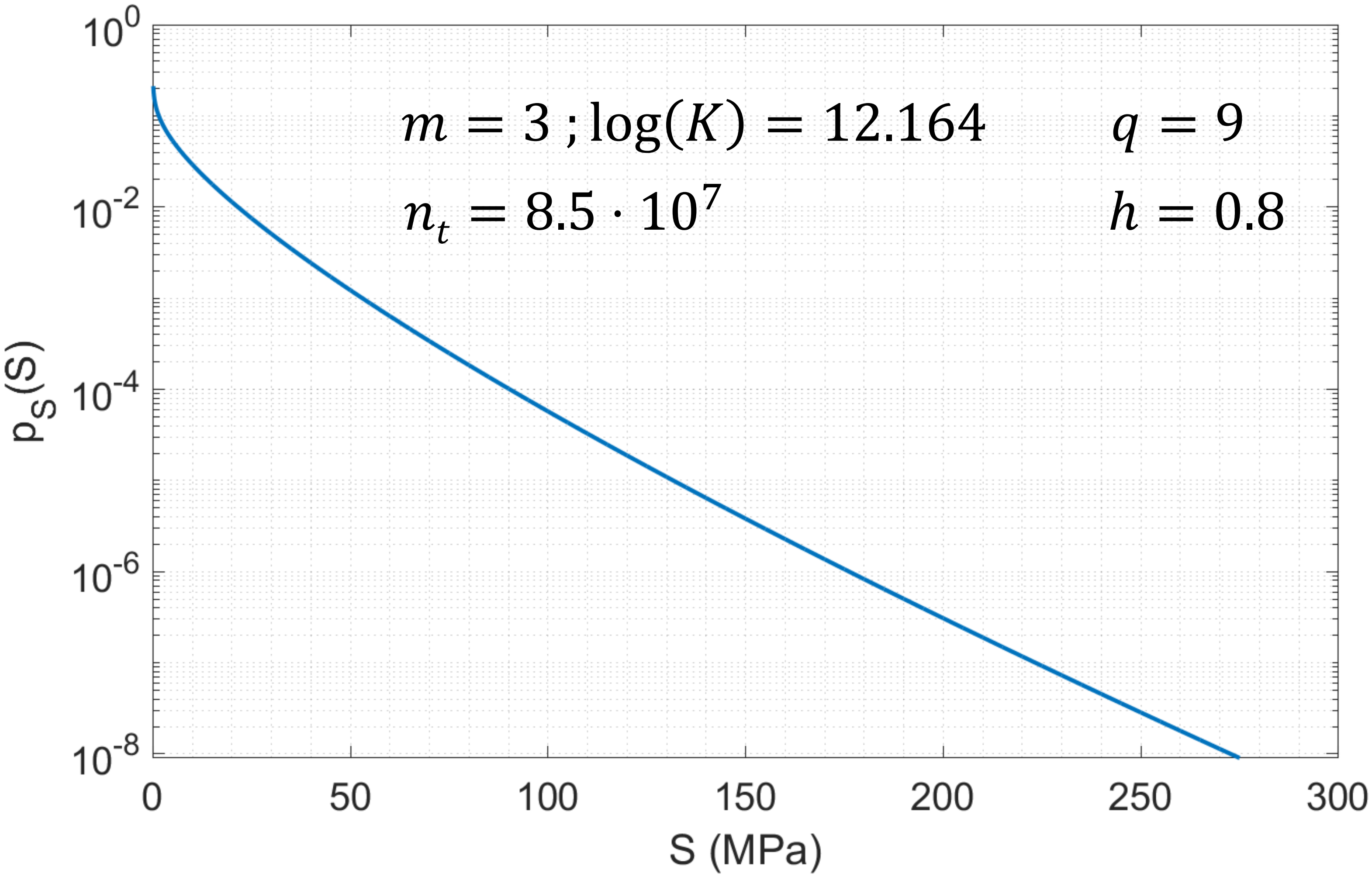
↓

S-N curve → Long-term response ~ Weibull (q,h)

↓

Fatigue damage

*Assumption: The long-term stress range is represented by a Weibull distribution.



Fatigue damage

D?

$$(1)\ N = K \cdot S^{-m}$$

$$(2)\ D = \sum_{S} \frac{n_i}{N_i}$$

If S is infinitesimal, then (2) becomes:

$$(3)\ D = \int_0^{\infty} \frac{n_t p(S) dS}{N(S)}$$

Combining (1) and (3):

$$(4)\ D = \frac{n_t}{K} \int_0^{\infty} S^m p(S) dS$$

The stress range is represented by a Weibull distribution:

$$(5)\ p(S, q, h) = \frac{h}{q} \left(\frac{S}{q}\right)^{h-1} e^{-\left(\frac{S}{q}\right)^h}$$

*Where q and h are the scale and shape parameters respectively

Replacing $f(S)$ in (4) from (5):

$$(6)\ D = \frac{n_t}{K}\int_0^{\infty} \boldsymbol{S^m} \frac{\boldsymbol{h}}{\boldsymbol{q}}\left(\frac{\boldsymbol{S}}{\boldsymbol{q}}\right)^{\boldsymbol{h-1}} e^{-\left(\frac{\boldsymbol{S}}{\boldsymbol{q}}\right)^{\boldsymbol{h}}} dS$$

Introducing a new variable ‘x’:

$$(7)\ x = \left(\frac{\boldsymbol{S}}{\boldsymbol{q}}\right)^{\boldsymbol{h}} \begin{cases} dx = \dfrac{\boldsymbol{h}}{\boldsymbol{q}}\left(\dfrac{\boldsymbol{S}}{\boldsymbol{q}}\right)^{\boldsymbol{h-1}} \boldsymbol{dS} \\ S = x^{1/h}\, q \\ \boldsymbol{S^m} = x^{m/h}\, q^m \end{cases}$$

Combining (6) and (7):

$$(8)\ D = \frac{n_t}{K}\int_0^{\infty} q^m\, x^{m/h} dx\, e^{-x}$$

$$(8)\ D = \frac{n_t\, q^m}{K}\int_0^{\infty} x^{m/h} e^{-x} dx$$

*Gamma function:

$$(9)\ \Gamma(z) = \int_0^{\infty} x^{z-1} e^{-x} dx$$

Combining (6) and (7):

$$(8)\ D = \frac{n_t}{K}\int_0^\infty q^m\, x^{m/h} dx\; e^{-x}$$

$$(8)\ D = \frac{n_t\, q^m}{K}\int_0^\infty x^{m/h} e^{-x} dx$$

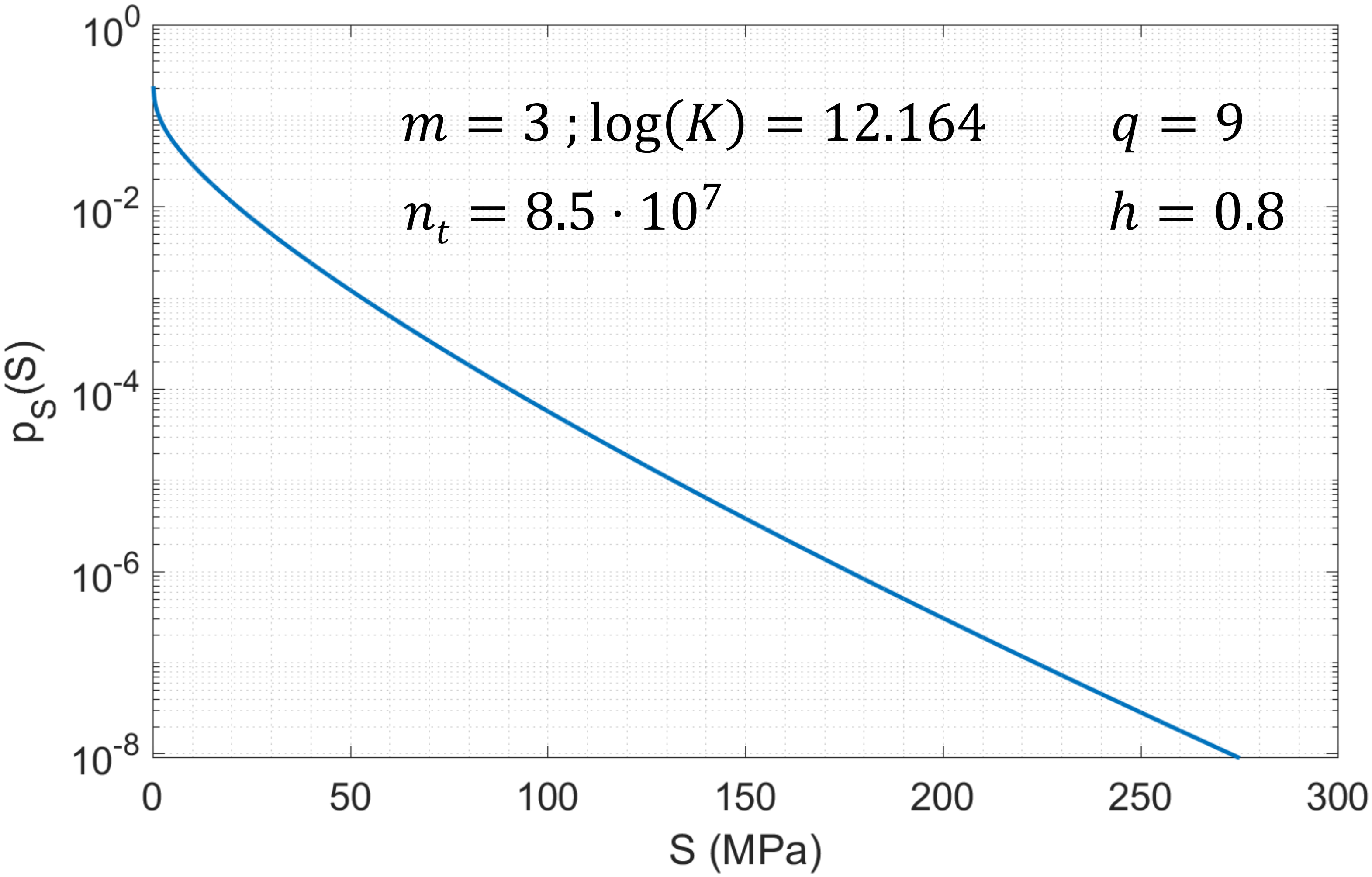
*Gamma function:

$$(9)\ \Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx$$

A **closed-form equation** is obtained for the fatigue damage:

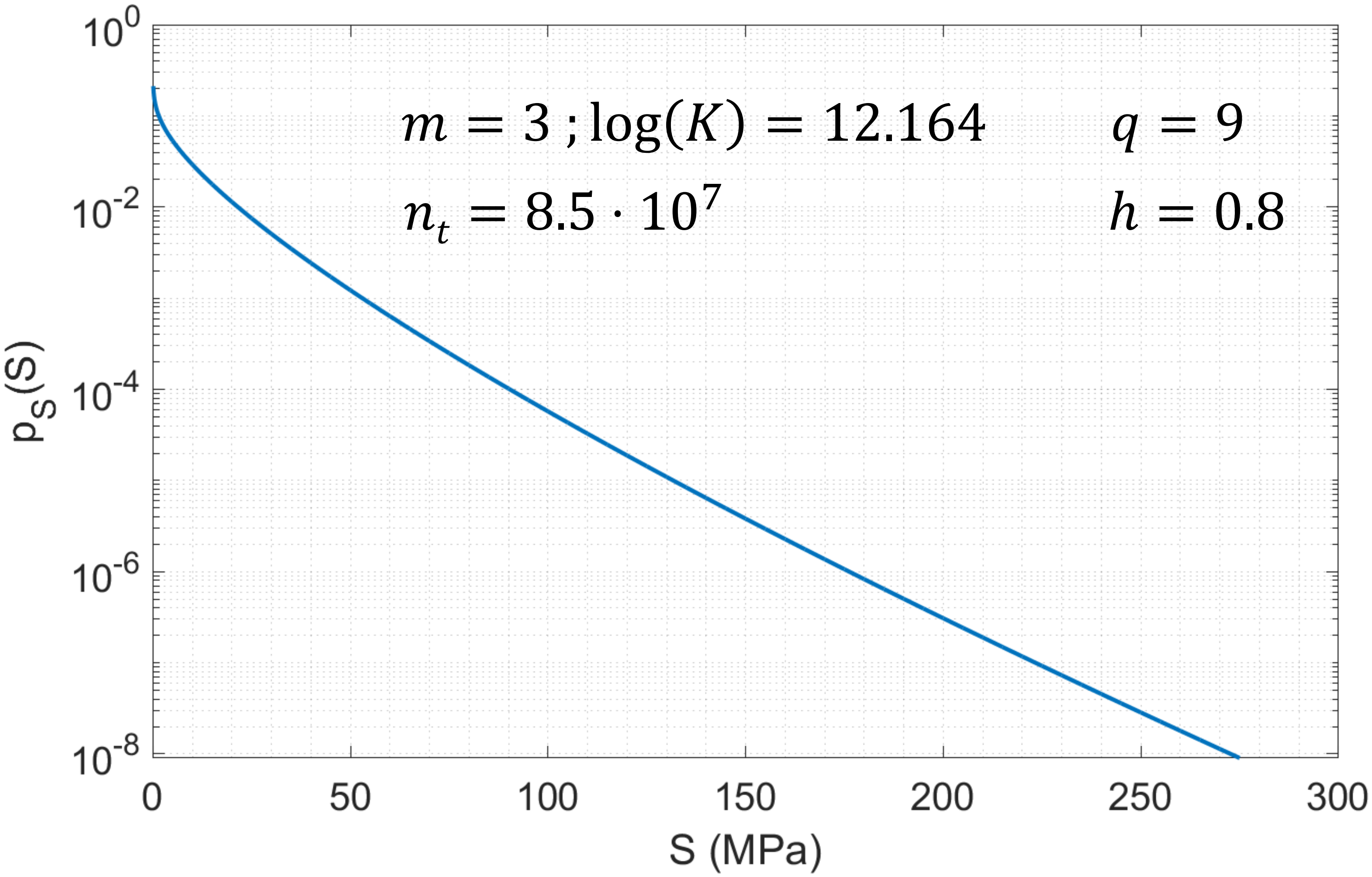
$$\mathbf{(10)}\ \boldsymbol{D = \frac{n_t\, q^m}{K}\Gamma\left(1 + \frac{m}{h}\right)}$$

*Assumption:** The long-term stress range is represented by a Weibull distribution.



Fatigue damage **D**?

*__Assumption:__ The long-term stress range is represented by a Weibull distribution.



Fatigue damage

$\mathbf{D = 0.7}$

Simplified fatigue assessment

Closed-form equation - fatigue damage:

$$(10)\ D = \frac{n_t\, q^m}{K}\Gamma\left(1 + \frac{m}{h}\right)$$

Exceedance probability – 'Weibull'

$$(11)\ p_{exc}(S_0) = exp\left(-\frac{S_0}{q}\right)^h = \frac{1}{n_0}$$

NB: if p_{exc}(30 MPa) = 0.01, 1 out of 100 stress cycles will exceed 30 MPa.

Rearranging (11):

$$(11)\ q = \frac{S_0}{[ln(n_0)]^{1/h}}$$

Replacing 'q' from (11) into (10):

$$(12)\ \boldsymbol{D} = \frac{\boldsymbol{n_t S_0^m}}{\boldsymbol{K\,[ln(n_0)]^{m/h}}}\boldsymbol{\Gamma}\left(\boldsymbol{1} + \frac{\boldsymbol{m}}{\boldsymbol{h}}\right)$$

Scale parameter:

$$(11)\ q = \frac{S_0}{[ln(n_0)]^{1/h}}$$

Exceedance probability – 'Weibull'

$$(13)\ p_{exc} = exp\left(-\frac{S}{q}\right)^{h} = \frac{1}{n}$$

Re-arranging (13):

$$(14)\ S = q\ [ln(n)]^{1/h}$$

Replacing 'q' from (11) into (13):

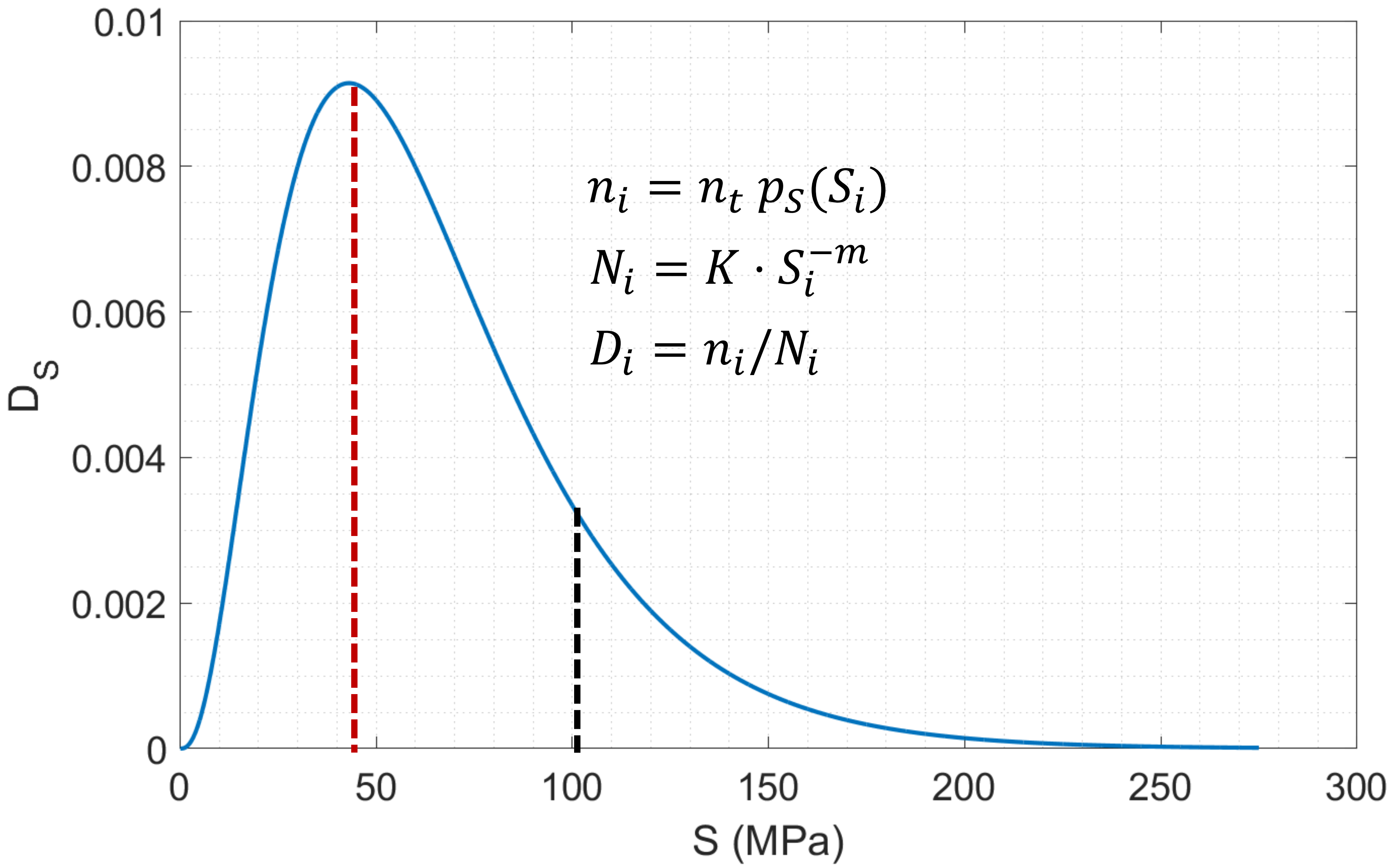
$$\mathbf{(15)}\ \boldsymbol{S = S_0 \cdot \left[\frac{ln(n)}{\ln(n_0)}\right]^{1/h}}$$

To understand the simplified fatigue assessment

$$\boldsymbol{D} = \frac{\boldsymbol{n_t\, q^m}}{\boldsymbol{K}}\boldsymbol{\Gamma}\left(\mathbf{1}+\frac{\boldsymbol{m}}{\boldsymbol{h}}\right) = \frac{\boldsymbol{n_t S_0^m}}{\boldsymbol{K\,[ln(n_0)]^{m/h}}}\boldsymbol{\Gamma}\left(\mathbf{1}+\frac{\boldsymbol{m}}{\boldsymbol{h}}\right)$$

D = long-term fatigue damage
n_t = total number of stress cycles for the design lifetime
K, m = SN curve parameters
q, h = Weibull scale and shape parameters
S_0 = Stress range at the reference probability of exceedance of 0.01 in Mpa
n_0 = number of cycles corresponding to the reference probability of exceedance of 0.01, i.e., n_0 = 100

Damage contribution – Stress Range

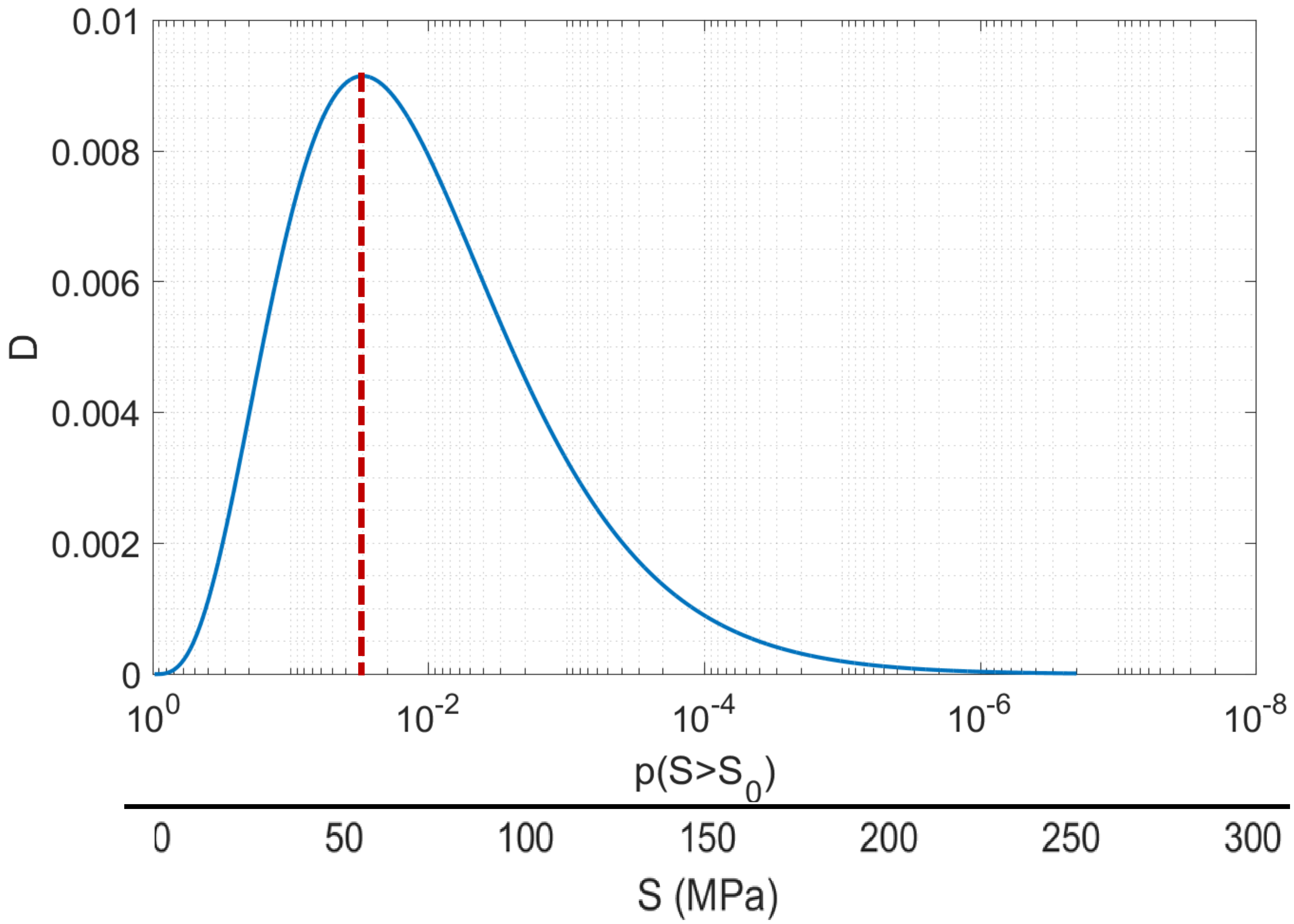


$$n_i = n_t \, p_S(S_i)$$

$$N_i = K \cdot S_i^{-m}$$

$$D_i = n_i / N_i$$

Damage contribution – Prob. of exceedance



Probabilistic Fatigue Assessment

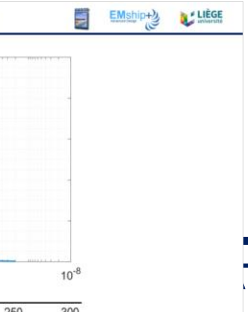
Uncertainties involved in the fatigue assessment

- Uncertainty of SN test. $\boldsymbol{logK \sim Normal(mean = 12.164 + 2std, cov = 0.2)}$

- Uncertainty of Miner's rule $\boldsymbol{\Delta \sim Lognormal(mean = 1, cov = 0.3)}$

- Uncertainties of RAO, environmental models $\boldsymbol{q \sim Normal(mean = 9, cov = 0.2)}$

Probabilistic fatigue assessment: $\boldsymbol{D = \frac{n_t\, q^m}{K} \Gamma\left(1 + \frac{m}{h}\right)}$



Exercise

Perform the sensitivity analysis of the fatigue failure probability with respect to the varying uncertainty degrees of the stress scale parameter.

$m = 3$
$n_t = 8.5 \cdot 10^7$
$logK \sim Normal(mean = 12.164 + 2std, cov = 0.2)$
$\Delta \sim Lognormal(mean = 1, cov = 0.3)$
$q \sim Normal(mean = 9, cov = [0.1, 0.15, 0.2, 0.25, 0.3])$
$h = 0.9$

Probabilistic fatigue assessment: $\boldsymbol{D} = \frac{\boldsymbol{n_t\, q^m}}{\boldsymbol{K}} \boldsymbol{\Gamma}\left(\boldsymbol{1} + \frac{\boldsymbol{m}}{\boldsymbol{h}}\right)$

$\boldsymbol{P_f} = \boldsymbol{P}(\boldsymbol{D} > \Delta)$