

Few more things - can you add - - percentage return vs log returns - how can you go from one to another - one period return vs multi period return - e.g going from daily log/percentage return to calculate monthly, annual return - vice versa go back from annual / monthly to daily using both percentage and log return - next you already few things about arithmetic, geometric and harmonic return- please add when will you use arithmetic return vs geometric return vs harmonic return. - if you complete that - then please add how to measure risk - volatility standard deviation of individual securities and of portfolio of equities
 Convert daily volatility to annual and back using different return types - percentage / log return - how will you handle in case of arithmetic geometric and Harmonic return
 How is compounding continuously different than compounding discretely?

Time value of money

The time value of money is the idea that money available at the present is worth more than the same amount in the future due to its earning capacity.

$$FV = PV(1 + i/n)^{n \cdot t}$$

Where, FV=Future Value

PV=Present Value

i=interest rate

n=number of compounding periods per year

t=number of years

Discounting and compounding

Discounting is the process of determining the present value of a payment or a stream of payments that is to be received in the future.

Future Value to Present Value is discounting.

$$PV = FV / (1 + r)^n$$

Example, at the discount rate of $i=10\%$, present value of 2600 dollars in 3 years equals 1953.42.

$$PV = 2600 / [(1 + 0.1)^3] = 1953.42$$

It is the opposite of compounding where the interest rates are used in determining how an investment will grow.

Present value to future value is compounding.

$$FV = PV (1 + r)^n$$

Example, assume I've 20,000 dollars (principal) in a bank for the interest rate of 4%. After 10 years, bank will pay me

$$FV = 20,000 \cdot (1 + 0.04)^{10} = 20,000 \cdot 1.48024 = 29604.8$$

Discounted cash flow

Suppose I offered to give you either \$1000 in June 2006 or \$150 every June for the next 10 years, starting in 2007. Which offer is worth more? How would you figure this out? The answer is: by calculating discounted cash flows.

Discounted cash flow analysis essentially takes the cash flows for each period and discounts them back to the current moment. So, suppose we have cash flows of \$100 starting next year

for the next 10 years, and our discount rate is 8%. Then what we're calculating looks like the following:

$$DCF = \$100/(1+0.08) + \$100/(1+0.08)^2 + \$100/(1+0.08)^3 + \dots + \$100/(1+0.08)^{10}$$

This means that the value of receiving \$100 every year for 10 years isn't \$1000 but \$671.

Statistical arbitrage

A profit situation arising from pricing inefficiencies between securities. These strategies are supported by substantial mathematical, computational and trading platforms.

Annuity

An annuity is a financial product that is designed to accept and grow funds from an individual and then, pay out a stream of payments to the individual at a later point in time.

Before payouts- Accumulation phase

After payouts- Annuitization phase

Types of annuity-Fixed or variable

Perpetuity

Perpetuity means state or quality of lasting forever. In finance, it is a constant stream of identical cash flows with no end.

A perpetuity is a type of annuity that lasts forever.

Security

A financing or investment instrument issued by a company or government agency that denotes an ownership interest and provides evidence of a debt, right to share in the earnings of the issuer, or a right in the distribution of property.

Securities includes bonds, notes, options, shares, etc..

RETURN

Return is a profit on an investment. It is usually quoted as percentage.

A loss instead of a profit is described as negative return.

General rule- the more risk you take, greater the potential for higher returns or losses.

Three types: Return on Investment, return on equity, return on asset, return on capital employed.

Names of return concepts:

r	R	R+1
log	simple	total
Continuously compounded	net	gross

Total- It includes dividends as well as prices. A (simple) total return in this sense is:

$$R_t = (P_t + D_t - P_{t-1}) / P_{t-1}$$

where D_t is the dividend or interest that is paid between times $t-1$ and t . This concept is sometimes expressed as the “overall” return.

The terms “gross” and “net” are more commonly used to mean something like before and after tax.

Rate of Return

It is profit on an investment over a period of time expressed as a proportion of original investment.

Simple Return

Traditionally, simple returns are denoted with R . It is defined as,

$$R_t = (P_t - P_{t-1}) / P_{t-1} = P_t / P_{t-1} - 1$$

where P_t is the price of the asset at time t . We are defining the return from time $t-1$ to time t .

Compound Return

It is the growth rate that gets you from the initial investment value to the ending investment assuming that the investment has been compounding over the time period.

NOTE: When expressed in annual terms, a compound return is referred to as **Compound Annual Growth Rate**(CAGR).

$$\text{CAGR} = \left(\frac{\text{Ending Value}}{\text{Beginning Value}} \right)^{\left(\frac{1}{\# \text{ of years}} \right)} - 1$$

Effective Annual Interest rate- It is used to compare different products that calculate compounded interest differently. It is interest that is actually earned or paid on an investment.

$$= \left(1 + \frac{i}{n} \right)^n - 1$$

i = stated annual interest rate

n = number of compounding periods

Example, if investment A pays 10%, compounded monthly, and a investment B pays 10.1% compounded semi-annually, the effective annual interest rate can be used to determine which investment will actually pay more over the course of the year.

For investment A, this would be: $10.47\% = (1 + (10\% / 12))^{12} - 1$

And for investment B, it would be: $10.36\% = (1 + (10.1\% / 2))^2 - 1$

There is a limit to the compounding phenomenon. Even if compounding occurs an infinite amount of times, not just every second or microsecond, but continuously, the limit of compounding is reached. The continuous rate is calculated by raising the number “e” (approximately equal to 2.71828) to the power of the interest rate and subtracting one.

For 10%, the continuously compounded effective annual interest rate = $2.171828^{(0.1)} - 1 = 10.517\%$.

Holding Period Return

HPR is probably the most intuitive and widely used financial return calculation. It is very straightforward, simple and does not leave much room for calculation errors. All you need are three variables: the beginning and end market values of a portfolio (or a share) and the total cash flow received from the portfolio (if any) during the time period analyzed. Simply, it is the rate of return on an asset or portfolio over the whole period during which it was held.

$HPR = (\text{end value} - \text{initial value}) / \text{Initial value}$

Where end value is the entire gain which is the sum of incomes and capital gains.

Example, if we analyze the annual HPR using the twelve monthly returns and dividends received, we would use three values: \$24.85 (beginning) \$28.73 (end) and \$0.94 (total cash-flow).

$HPR = (28.73 + 0.94 - 24.85) / 24.85 = 19.4\%$

That is, an investor made 19.4% HPR on his investment held for one year.

When to use HPR?

HPR is widely used when you want to have a quick and simple overview of your investment in a particular product (stock, portfolio, etc). HPR is simple and quick to calculate so it's definitely a great tool to use when you are simply interested in a quick and general return number.

What to look out when calculating HPR?

One of the main disadvantages of HPR is the fact the the formula ignores any movement in the share price between the beginning and end value of the investment. Sometimes, the investment might have performed very well during a given period, but on the date when the HPR was calculated the ending value was negatively affected by the market forces and the overall return appeared weaker.

Arithmetic return

Formula- $FV/PV - 1$

An arithmetic average is simply the sum of all the terms (numbers) divided by the count of that sequence.

Generally, we calculate holding period return for shorter period and then take mean of these returns.

When to use arithmetic return?

We can use the Arithmetic Return to know an average return made by investing in a particular product. It's interesting to note how Arithmetic Return solves one of the main shortcomings of Holding Period Return. Since the Arithmetic Return uses more data (twelve monthly returns) instead of only one as in the case with Holding period return, we have a better understanding of the investment's performance throughout the year.

What to look out for?

The primary disadvantage of the Arithmetic Return is the fact that it ignores the effect of compounding returns. Similar to the notion of compounded interest, compounded return results when the return in one month is influenced by the return obtained in the previous month. If you would look at your investment as a series of independent monthly returns, then the effect of compounded returns would not be a concern. However, since your financial returns are all linked together and since your current return depends on your past return, the effect of compounding interest should not be ignored.

Geometric mean or time-weighted rate of return

It is a measure of the compound rate of growth. We multiply the monthly returns and then we take the twelfth root of the answer to arrive at a monthly return. The result obtained with the Geometric Return is more representative of the return the investor actually received from his investment.

Advantage: This method eliminates the distorting effects created by inflows of new money.

Note: If there is a cash inflow/outflow within a year, multiply percentage returns(don't take roots).

Arithmetic return Vs Geometric return

Example, a 50% gain and a 50% loss,

Arithmetic Average = $(.50 + (-.50)) / 2 = 0$ which is not true.

The geometric average will provide the actual results:

Geometric Average = $[(1.5 \times .50)^{(1/2)}] - 1 = -13.4\%$

Year 1: $\$100.0 - 13.4\% = \86.6

Year 2: $\$86.6 - 13.4\% = \75.0

Arithmetic return is simply the sum of all the terms. Numbers are not independent to each other (order matters!). If lost a ton of money one year, less capital to generate returns during the following years.

Arithmetic and geometric averages serve different purposes and only geometric averages will accurately reflect compounded investment returns. Arithmetic averages will always over state investment returns unless there is zero volatility. The greater the volatility the greater the difference will be between arithmetic and geometric averages. When it comes to investment returns and retirement planning it is compounded (geometric) averages that matter.

$$(1 + r_{ave})^2 - \text{StdDev}^2 = (1 + \text{CAGR})^2$$

It is accurate to describe the CAGR as what was actually earned per year on average, compounded annually. The arithmetic return represents what was earned during a typical, or average, year.

Additionally, arithmetic returns do not account for compounding. The CAGR and geometric returns take compounding into consideration.

Thus, the geometric return, is more complicated to calculate but is at the end of the day a more accurate measure of compounded average returns. It is more useful to extrapolate returns into the future, and these will usually be smaller than the arithmetic average, especially when returns are more volatile.

Logarithmic or continuously compounded return

Usually denoted with a lower-case r . It is defined as

$$r_t = \log(P_t / P_{t-1}) = \log(P_t) - \log(P_{t-1})$$

where P_t is the price of the asset at time t . We are defining the return from time $t-1$ to time t .

The \log function here is the natural logarithm.

Logarithmic rate of return $= r/t$, where t is the length of time period

It is annualised rate of return if t is measured in years.

Percentage return Vs Log return

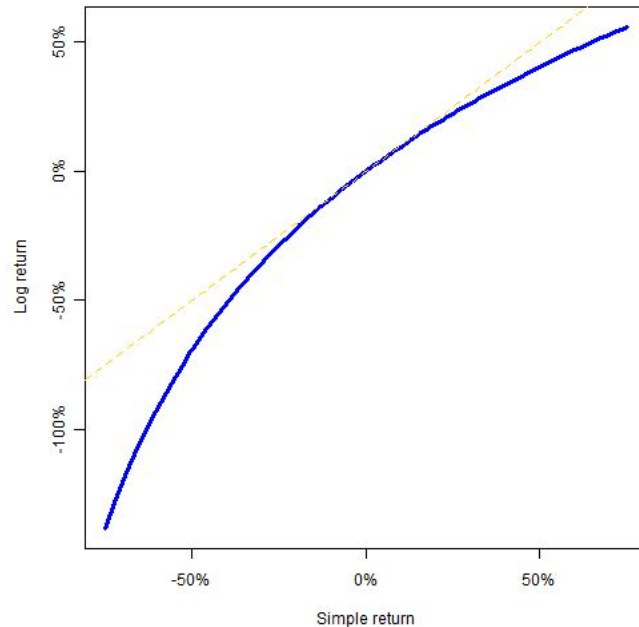
To go from simple to log returns, we can do:

$$r = \log(R + 1)$$

To go from log return to simple return, we can do:

$$R = \exp(r) - 1$$

Comparison of simple and log returns,



It shows that log returns are always smaller than simple returns. The two types act very differently when it comes to aggregation. Each has an advantage over the other:

- Simple returns aggregate across assets
- Log returns aggregate across time

The log return for a time period is the sum of the log returns of partitions of the time period. For example the log return for a year is the sum of the log returns of the days within the year.

The simple return of a portfolio is the weighted sum of the simple returns of the constituents of the portfolio.

How to convert daily log return to annual log return? How is log return easier in calculations?

Log returns are additive. Just add the daily/weekly return together to get yearly log return. If we only have one average daily return, we annualize simply by multiplying with an annualization factor. Often 252 is used but it depends on the specific use case.

How to convert annual log return to daily log return?

One period return Vs multi period return

One period return measures the growth of investment within a single period of time assuming all cash flow is only at the beginning and end of the period of time (no intermediate cash flows). In multi-period, returns are measured over regular and consecutive periods of time such as daily, monthly or quarterly. It gives a single return number measuring the investment performance of a long-term investment which may have cash flows at the intermediate points in time (periods should be short enough such that the assumption of no intermediate cash flows does not matter). Periodic returns can be averaged across time to determine the 'time-weighted' multi period return.

How to convert daily/monthly/weekly/quarterly percentage return to annual percentage return? The most useful expression of an investment's returns is on an annual basis, so it's important to know how to convert daily (and weekly, monthly, etc.) returns into annual. Here's the procedure,

$$\text{Daily return} = \frac{\text{Amount of return}}{\text{Value of investment}}$$

$$\text{Annual return} = [(\text{Daily return} + 1)^{365} - 1] * 100$$

For example, let's say that you have an investment that pays a 0.03% daily return, which in decimal format is 0.0003.

$$\text{Annual return} = [(0.0003 + 1)^{365} - 1] * 100 = 11.6\%$$

We can convert from weekly or monthly returns to annual returns in a similar way. Simply replace the 365 with the appropriate number of return periods in a year. So, for weekly returns, we would raise the daily return portion of the equation to the 52nd power. For monthly returns, we would use 12. And, for quarterly returns, you would use the fourth power.

How to convert annual percentage return to daily/weekly percentage return?

The formula for changing from an annual percentage rate to a semiannual, quarterly, or monthly,

$$\text{Effective rate for period} = (1 + \text{annual rate})^{(1 / \# \text{ of periods})} - 1$$

So for monthly, quarterly, and semiannual rates, the math becomes:

$$\text{Monthly rate} = (1 + \text{annual rate})^{(1/12)} - 1$$

$$\text{Quarterly rate} = (1 + \text{annual rate})^{(1/4)} - 1$$

$$\text{Semiannual rate} = (1 + \text{annual rate})^{(1/2)} - 1$$

Example, If someone has a loan with an annual percentage rate of 6% and want to calculate the amount you're paying each month, your effective rate each month isn't 0.5% but 0.486%.

Effective rates take the impact of compounding into account, whereas simply dividing one rate by the number of periods ignores this factor.

Sharpe Ratio

It describes how much excess return you are receiving for the extra volatility that you endure for holding a riskier asset.

$$S(x) = (r_x - R_f) / \text{StdDev}(x)$$

Where,

r_x is the average rate of return and can be of any frequency as long as they are normally distributed.

R_f is the best available rate of return of a risk-free security.

$\text{StdDev}(x)$ is the standard deviation of x .

Expected return

It is the amount of profit or loss an investor anticipates on an investment that has various known or expected rates of return. It is calculated by multiplying potential outcomes by the chances of them occurring, and summing these results.

It is dangerous to make investment decisions based on expected returns alone. Investor should also review the risk characteristics of investment. **Risk** is calculated by **standard deviation** of returns in previous years.

Harmonic Mean

It may be viewed as a special type of weighted mean in which an observations weight is inversely proportional to its magnitude. It is appropriate when averaging ratios.

A well known application is in the investment strategy known as cost averaging, which involves periodic investment of a fixed amount of money.

Correlation and covariance

Covariance is the measure of how one variable varies with another.

Correlation is the covariance normalized.

Normal distribution

It is used in stock market analysis. We generally form an assumption that the returns are normally distributed to do further analysis.

Skewness- measures the symmetry of a distribution

Kurtosis- measures the tail ends of a distribution and whether the distribution of a dataset has skinny tails or fat tails.(for standard deviation it is 3)

z-scores

It tells how many standard deviations from the mean the score is.

Volatility

Simply put, volatility is the amount of price change a security experiences over a given period of time. If price stays relatively stable, the security has low volatility. A highly volatile security is one that hits new highs and lows, moves erratically, and experiences rapid increases and dramatic falls.

Volatility is a statistical measure of the [dispersion](#) of returns for a given security or [market index](#). Volatility can either be measured by using the [standard deviation](#) or variance between returns from that same security or market index. Commonly, the higher the volatility, the riskier the security.

EQUITY

It can have different meanings, depending on the context and the type of asset. In general, it is one's ownership in any asset after all debts associated with it are paid off.

Common equity and preferred equity

Both represent a piece of ownership in a company. Basic differences among them are:

- 1) Holders of common stock have voting privileges whereas holders of preferred stock may not.
- 2) Preferred stockholders have a greater claim to a company's assets and earnings.
Preferred stockholders must be paid before common stockholders.

Preferred stocks are rated for financial strengths like bond, giving a little chance of netting a large capital gain.

Financial statements

- 1) Income statement
- 2) Balance sheet
- 3) Cash flow statement
- 4) Statement of shareholders' equity

Bond

A bond is a fixed income investment in which an investor loans money to an entity which borrows money for a definite period of time at a variable or fixed interest rate. Most bonds share common basic characteristics including:

- 1) Face value- worth at its maturity and reference amount when calculating interest payments.
- 2) Coupon rate- rate of interest
- 3) Coupon dates- dates on which the bond issuer will make interest payments.
- 4) Maturity date
- 5) Issue price- price at which the bond issuer originally sells the bonds.

Bond valuation

It includes calculating the present value of the bond's future interest payments(cash flow) and the bond's value upon maturity.

Investor uses bond valuation to determine rate of return.

Yield to maturity

It is the discount rate at which the sum of all future cash flows from the bond is equal to the current price of the bond.

Yield curve

Interest rates on various bonds often behave quite differently from one another, depending on their maturity. A yield curve is a way to visualize this difference.

A yield curve is way to measure bond investors' feelings about risk, and can have a tremendous impact on the returns.

Different types: Normal, flat and inverted yield curve