# NONLINEAR FINITE ELEMENT METHODS

Assignment for summer term 2022

Documentation

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Matriculation number:66994

Coded in Matlab (v2022)

Theory:-

- From the given assignment, it is known that the given one-dimensional Cahn-Hilliard equation is of the strong form

$$\dot{c} = \left[ M(c) \left[ \partial_c \Psi(c) - \lambda c'' \right]' \right]'$$

*Figure 1. Strong form of Cahn-Hilliard equation*

- Since the above equation contains higher order derivatives,it becomes difficult to solve the problem.So,the above equation has been written in weak form

$$0 = \delta W = \int_0^L \left[ \dot{c}\delta c + M(c)\, \partial^2_{cc}\Psi(c)\, c'\delta c' + \partial_c M(c)\, \lambda c''c'\delta c' + M(c)\, \lambda c''\delta c'' \right] \mathrm{d}x + \left[ \bar{H}^c \delta c + \left[ M(c)\, \lambda c' - \bar{H}^\xi \right] \delta c'' - r\delta c' \right]_{x=0}^{L}$$

*Figure 2. Weak form of Cahn-Hilliard equation*

- The above weak form contains element routine and material routine.The two nodes in an element will be approximated using cubic Hermite polynomials.

$$[\mathbf{N}]^{\mathrm{Herm}}(\xi) = \left[ \frac{1}{4}(2+\xi)(\xi-1)^2, \frac{h^e}{8}(\xi+1)(\xi-1)^2, \frac{1}{4}(2-\xi)(\xi+1)^2, \frac{h^e}{8}(\xi-1)(\xi+1)^2 \right] \quad (4)$$

$$\text{in } \Omega_\square = \{\xi \in [-1,1]\} \ ,$$

*Figure3 Hermite Shape function*

- The global coordinates is approximated by employing linear lagrange polynomials.

$$[\mathbf{N}]^{\mathrm{Lagr}}(\xi) = \left[ \tfrac{1}{2}(1-\xi), \tfrac{1}{2}(1+\xi) \right] \text{ in } \Omega_\square = \{\xi \in [-1,1]\}$$

Figure4 Hermite Shape function

- With the shape functions using above,we will be able to get the element routine.The given mobility and free energy function will give the material routine.Here four

Gauss quadrature points per elements are substituted in element routine.The Euler implicit method will be induced to solve the non-linear system of equations.In order to get the accurate values,we will use Newton-Raphson method with convergence criteria

- Condition 1:-

$$\|\blacktriangle\hat{\underline{\mathbf{c}}}_k\|_\infty < 10^{-5} * \|\hat{\underline{\mathbf{c}}}\|_\infty$$

- Condition 2:-

$$\left\|\hat{\underline{\mathbf{R}}}\right\|_\infty < 0.005 * \max\left(\left\|\hat{\underline{\mathbf{F}}}_{\text{int}}|_{k=0}\right\|_\infty, 10^{-8}\right)$$

- After solving the above criteria,we will be able to plot the evolved concenteration against time over the initial concenteration against time.

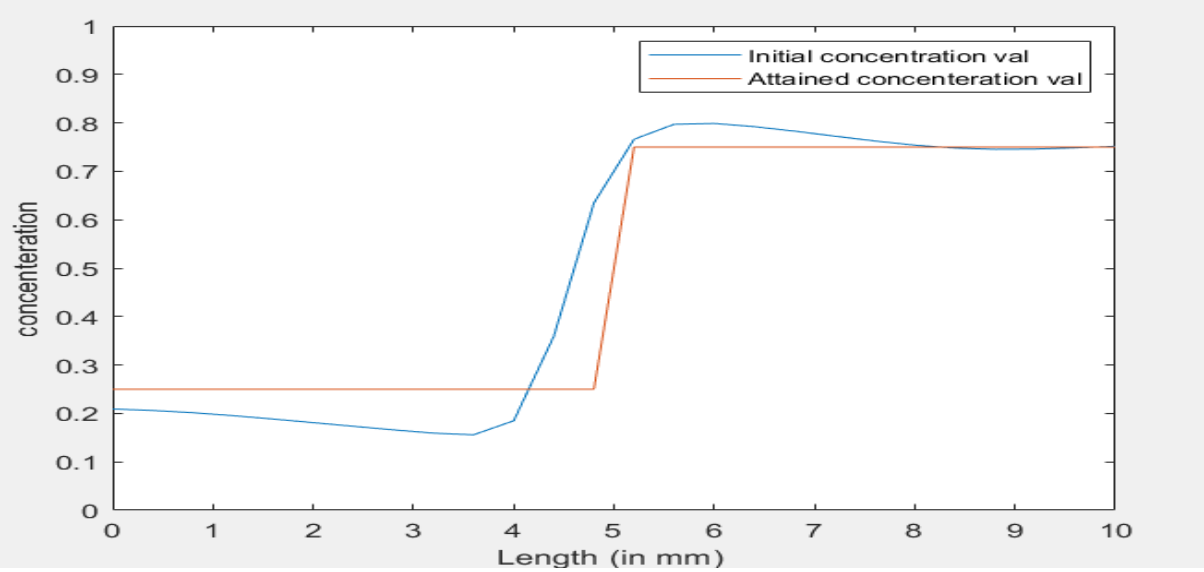
## Program Structure :-

- At the first,we will get inputs from user for the size of the element and the time.With the time only,we will get the exact solution for the respective assigned values(t>6000).
- Then the constants will be declared that will be used in the element and material routine.
- Now the main program consists of two “FOR” loops,one for the Global element Assembly(FOR loop2) and the other for the time step assembly(FOR loop1).The Global element assembly (FOR loop2) will be under the while loop.The while loop will be broken once the convergence criteria is satisfied which will be written in the “IF” statement.If it is not broken,again the ccap values(“Variable used in matlab code”) will be updated and runs FOR loop2
- Once the while loop converges,it comes to the FOR loop 1 which is initialised for time.So it runs for the next time step.Again the while and the Global element assembly will run.The program will end till the time loop(FOR loop1) reaches the end value.
- The material routine(free energy function and its derivatives and mobility function and its derivatives) has been declared as separate functions which will be used inside the main program.
- The output will get saved in the Cglobnew(“Variable used in matlab code”) which comprises the values of evolved concenterations.Then it will be plotted in the graph as concenterations against the Total Length of the element.
- To verify the stability of the concenteration,the free energy function has to be computed to check how the concenteration reaches the convergence.
- To plot the time history of the number of required Newton-raphson iterations,it is plotted in the graph.

Results:-

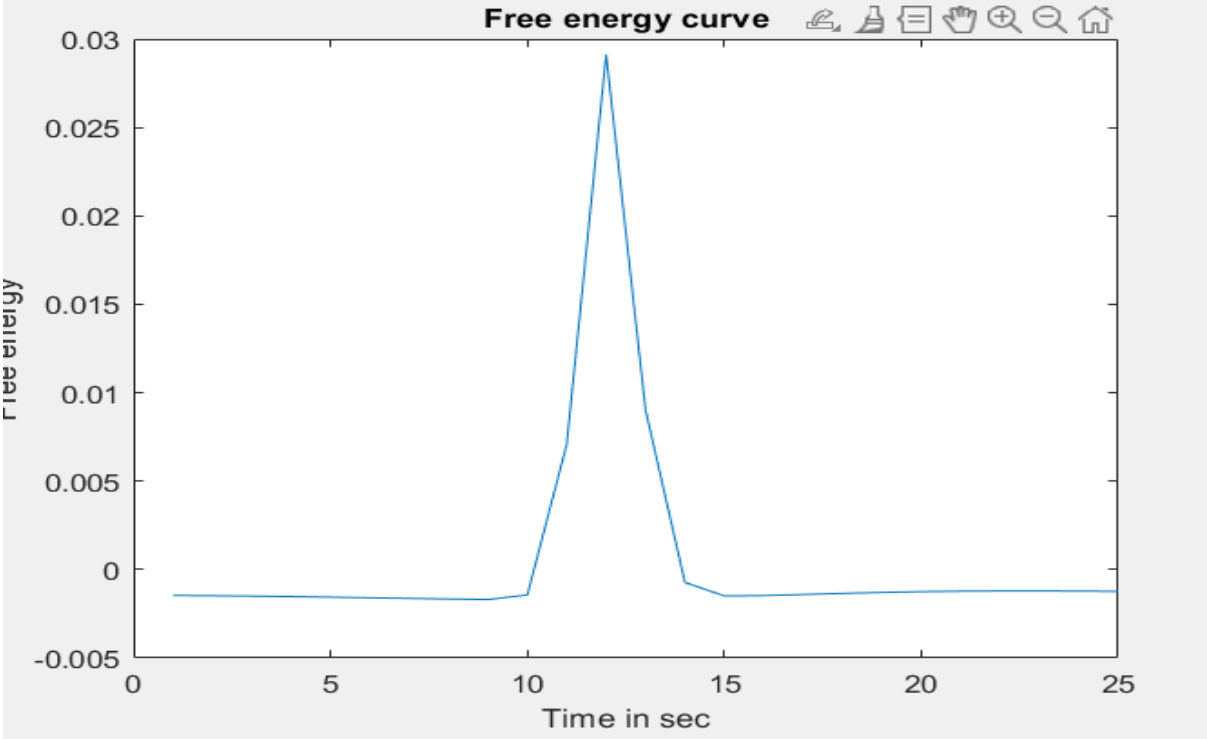
Graph1:-

Concenteration VS Length



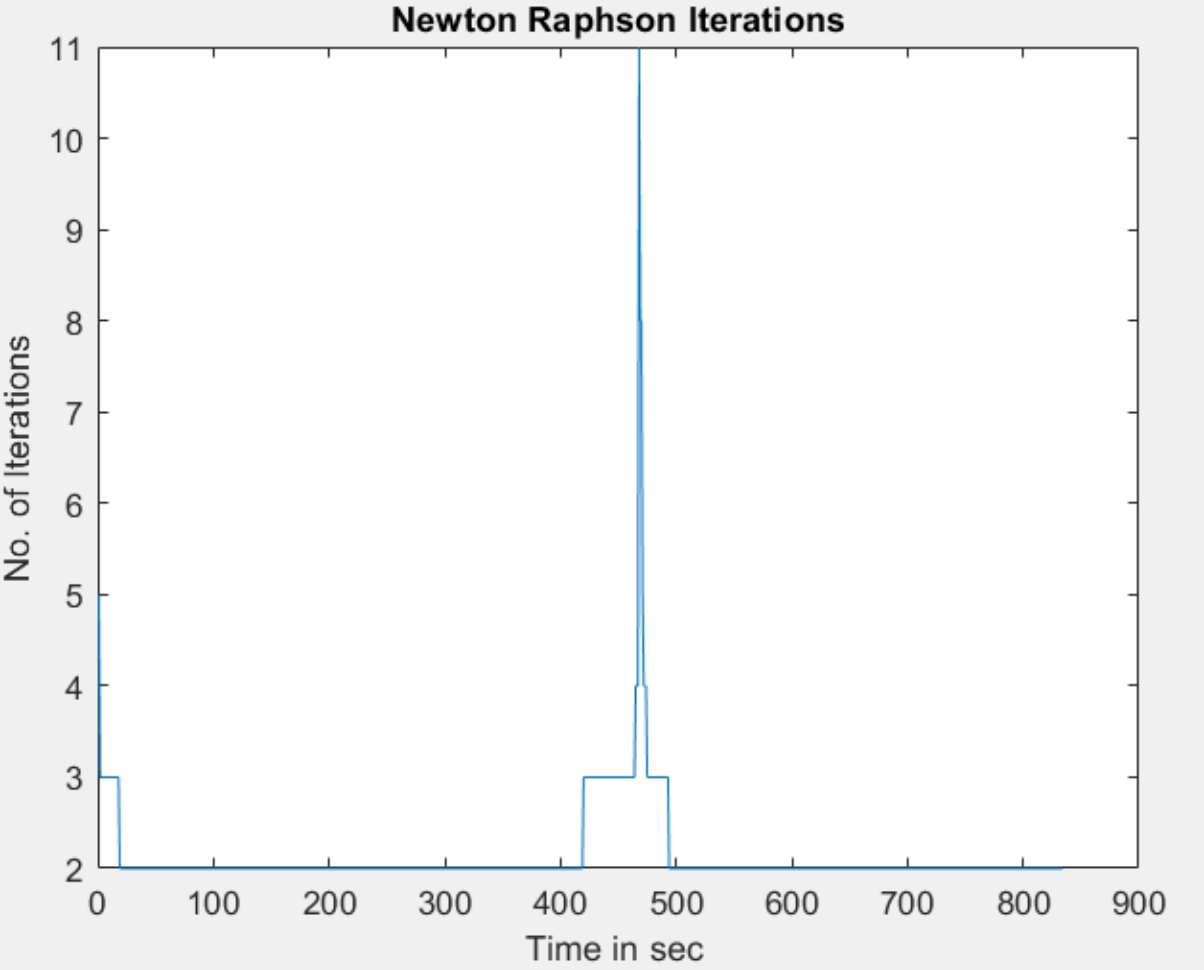

Graph2:-

Free energy curve vs Time



Graph3:-

Newton Raphson Iterations Vs Time




- As per my matriculation number,t is greater than 6000 and the size of the element is h<0.11.
- The above graphs are plotted for the t=2000 sec and the size of the element (h) =0.4mm since the given desired time steps and size of the element are unable to plot the desired graph.This occurs because how the $\Delta c$ is calculated.I have tried the iterative solvers for solving sparse matrices like PCG,CGS but still it was unable to reach the convergence.Only in the mldivide method,the while loop converges and thus giving the result.