EE2703 : Applied Programming Lab
End Semester Examination

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1 Objective :

To analyse the magnetic field along the axis of a loop antenna using vector operations an fitting it to an exponential function.

2 Theory :

The loop antenna carries a current given by,

$$I = 4\pi/\mu_o * cos(\phi) * exp(j\omega t)$$

where ϕ is the angle in cylinderical polar coordinates. The wire is on the x-y plane and is centered at the origin. The radius r of the loop is equal to $1/k = c/\omega$. In order to calculate the magnetic field we first calcutate the magnetic vector potential which is given by,

$$\vec{A}(r,\phi,z) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}(\phi)\hat{\phi}exp(-jkR)ad\phi}{R}$$

where $k = 1/r$ and $R = |\vec{r} - \vec{r'}|$, where $\vec{r'} = r\hat{r'}$ is the point on the loop antenna. This integration can be reduced to a summation as,

$$\vec{A}ijk = \sum_{l=0}^{N-1} \frac{\cos(\phi_l')\exp(-jkR_{ijkl})\,\overrightarrow{dl'}}{R_{ijkl}}$$

where $\vec{r} = r_i,\ \phi_j,\ z_k$ and $\vec{r'} =$ rcos$(\phi_l')\hat{x}$ + rsin$(\phi_l')\hat{y}$. From the vector potential $\vec{A}$, we can obtain the magnetic field $\vec{B}$ by using the relation,

$$\vec{B} = \nabla \times \vec{A}$$

Along the z direction this becomes,

$$B_z(z) = \frac{A_y(\Delta x, 0, z) - A_x(0, \Delta y, z) - A_y(-\Delta x, 0, z) + A_x(0, -\Delta y, z)}{4\Delta x\Delta y}$$

2.1 Pseudo Code:

- Define a meshgrid of size $3 * 3 * 1000$ for the x, y, z coordinates .
- Divide the loop into 100 sections and compute $\vec{dl_l}$ and $\vec{r_l'}$ for the sections.
- Compute the currents in the sections and make a quiver plot.
- Define the function `calc(l)` to compute $R_{ijkl} = |\vec{r_{ijk}} - \vec{r_l'}|$ and extend it to further compute and return A_{ijkl}
- Using a for loop, compute $\vec{A_{ijk}}$ using `calc(l)` and $\vec{dl'}$.
- Compute $B_z(z)$ using $\vec{A_{ijk}}$.
- Fit $B_z(z)$ to the model $B_z(z) = c * z^b$ using the least square function.
- Plot the magnetic fielda and the fitted function.

2.2 Code:

2.2.1 Importing modules and defining constants:

```
# importing necessary modules
import numpy as np
import matplotlib.pyplot as plt

# defining constants
PI = np.pi
mu0 = 4*PI*1e-7
```

2.2.2 Defining the meshgrid:

```
# Question 2:

    # Breaking the volume into a 3x3x1000 mesh, with mesh points separated
    # by 1cm


x = np.linspace(-0.01 , 0.01 ,3)
y = np.linspace(-0.01, 0.01,3)
z = np.linspace(0.01, 10,1000)
```

```
X,Y,Z = np.meshgrid(x,y,z,indexing = 'ij')
```

We have shifted the x axis by a small value, because the field along the z-axis is ideally zero. If we try to compute the field strength exactly on the z-axis we get only errors which is unreliable.

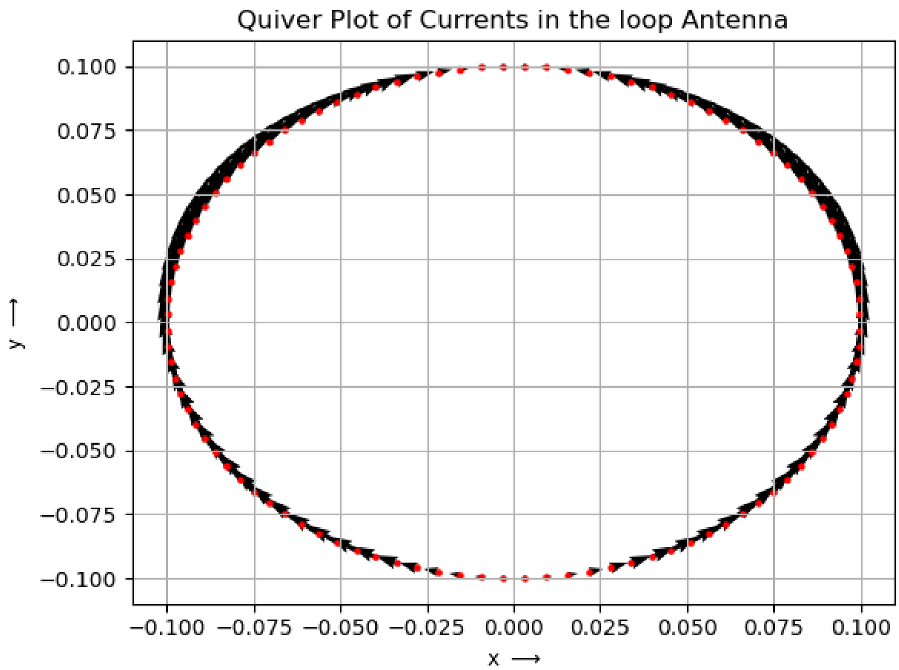
2.2.3 Computing $\vec{dl_l}$ and $\vec{r_l'}$ vectors for the sections:

```
# Question 4:

    # Obtaining the vectors rd_l and dl_l, where l indexes the   segments of the loop.

# number of sections
sec = 100
# radius of the loop
a = 0.1
k = 1/a
# array to store the indices of the loop segments
l = np.array(list(range(sec)))
# angular locations of the segments
phi = (l+0.5)*2*PI/sec
# dl vector
dl = 2*PI*a*(1/sec)*np.array([-np.sin(phi), np.cos(phi)]).T
# rd (r') vector
rd = a*np.array([np.cos(phi), np.sin(phi)]).T
```

2.2.4 Computing and plotting the currents in the sectiosn:

```
 # Question 3:

    # Plotting the currents in the x-y plane as a quiver plot


I = 4*PI*(1/mu0)*np.array([-np.cos(phi)*np.sin(phi), np.cos(phi)*np.cos(phi)]).T
plt.figure(1)
# quiver plot of currents
plt.quiver(rd[:,0],rd[:,1],I[:,0],I[:,1])
plt.scatter(rd[:,0], rd[:,1], s = 5, c = 'r')
plt.xlabel("x $\longrightarrow$")
plt.ylabel("y $\longrightarrow$")
```

```python
plt.title("Quiver Plot of Currents in the loop Antenna")
plt.grid()
plt.show()
```



2.2.5 Defining `calc(l)` function:

```python
# Question 5,6:

    # Defining function calc(l) to calculate Rijk_l and then extending it to find Aijk_l


def calc(l):
    rl = rd[l]
    # Matix of distances from segment l to r
    Rijk_l = np.sqrt((X-rl[0])**2 + (Y - rl[1])**2 + (Z )**2 )
    # Aijk_l matrix
    Aijk_l = np.cos(l*2*PI/sec)*np.exp(-1j*k*Rijk_l)/Rijk_l
    return Aijk_l
```

2.2.6 Computing the vector potential $\vec{A_{ijk}}$:

```
# Question 7:

    # Using the calc(l) function to compute the vector potential
    # Aijk( A_x and A_y)

# We have used a for loop to reduce space complexity. Vectorized code would
# require us to compute multiplication of large matrices.

Aijk_x = np.zeros(X.shape)
Aijk_y = np.zeros(Y.shape)

for l in range(sec):
  Aijk_l = calc(l)
  # potential in x direction
  Aijk_x = Aijk_x + Aijk_l*dl[l,0]
  # potential in y direction
  Aijk_y = Aijk_y + Aijk_l*dl[l,1]
```

Here, we have used a for loop instead of vectorized code because though the time complexity in the vectorized code is better, the space complexity would be far worse due the large size of the matrices we have to multiply.

2.2.7 Computing the magnetic field along the z-axis:

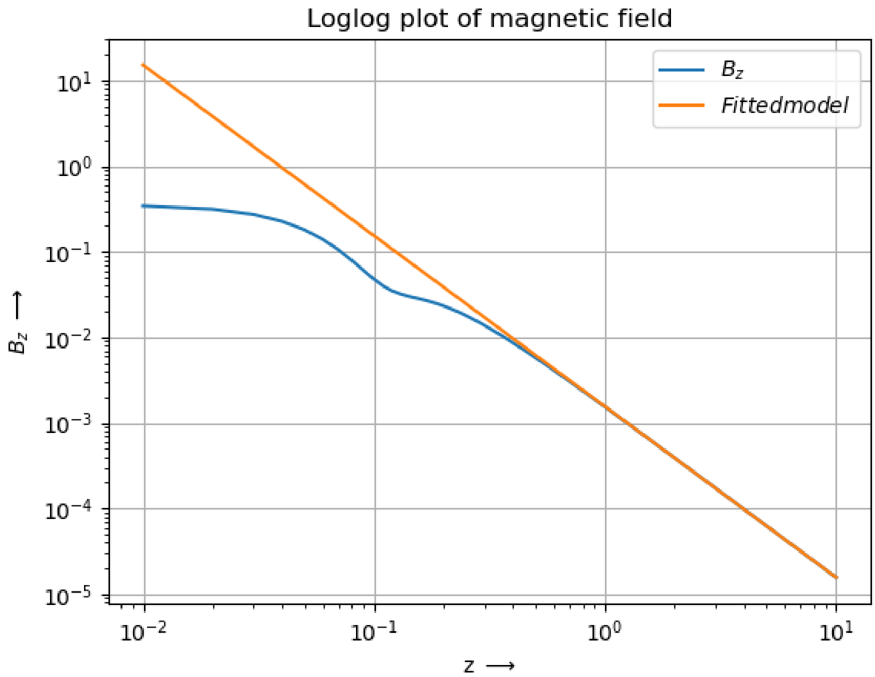
```
# Question 8:

    # Compution B along the z axis (B_z)

# we divide by 1e-4 to get the field magnitude in SI units
B_z = (Aijk_y[-1,1,:] - Aijk_x[1,-1,:] - Aijk_y[0,1,:] + Aijk_x[1,0,:])/(4*1e-4)
```

Here, we have divided further by $1e-4$ to get the magnetic field maginitude in SI units

2.2.8 Fitting the field and plotting the field with the fit:

```
# Question 9:

    # Plotting the magnetic field
```

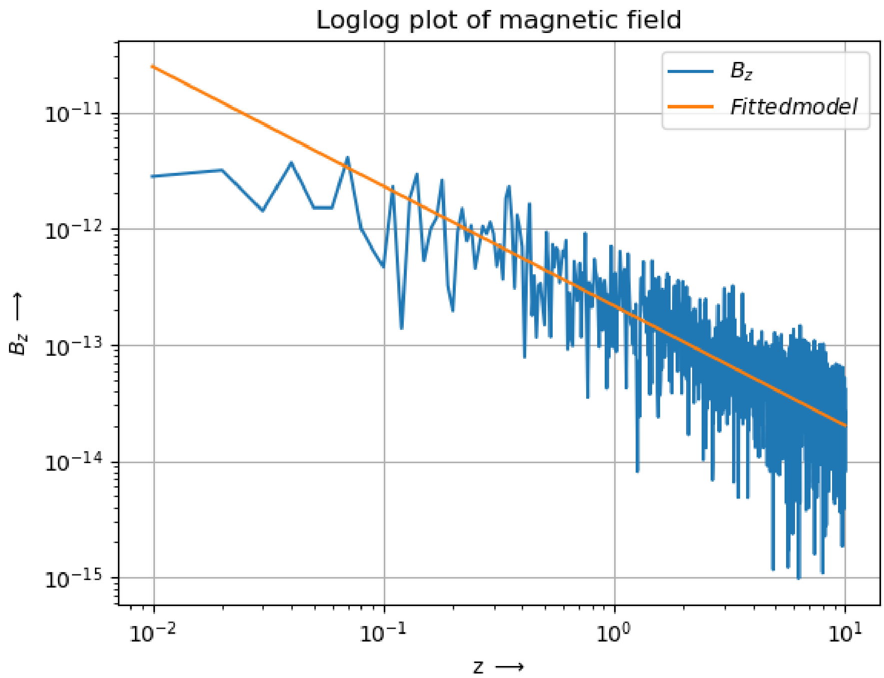
```python
plt.figure(2)
plt.loglog(z,np.abs(B_z),label = '$B_z$')
plt.title('Loglog plot of magnetic field')
plt.xlabel("z $\longrightarrow$")
plt.ylabel("$B_{z}$ $\longrightarrow$")
plt.grid()

# Question 10:

    # Fitting the field B_z to c*z^b


M = np.zeros((len(z[100:]),2))
M[:,0] = np.log(z[100:])
M[:,1] = 1
b,logc = np.linalg.lstsq(M, np.log(np.abs(B_z[100:])).T, rcond=None)[0]
print("The values of the parameters fitted to c*b^z are b = ", "{0:.5f}".format(b), " and c
↪  = ","{0:.5E}".format(np.exp(logc)),".")

# Plotting the fitted model upon the magnetic field plot
plt.loglog(z, np.exp(logc)*np.power(z,b), label = "$Fitted model$")
plt.legend()
plt.show()
```



The fitting parameters for the fitting function $c * z^b$ obtained by least square method are b = -1.99527 and c = 1.55536E-03 .

Observations and Inference:

- We get a fitting function given by $(1.55536E - 03) * z^{-1.99527}$. The obtained value of b is very close to the ideal value of -2 which can be calculated using some magnetic field analysis.
- We had calculated the magnetic field slightly shifted from the axis of the current loop. This is because the field exactly along the axis is *zero*.
- When we make our computations exactly at the axis we get the plot for the fields and the corresponding fitting function as below.



- The plot is very noisy and we can also notice that the magnitudes are many orders smaller than the previous case. This is because all we have now are errors which arise due to numerical inaccuracy.
- The corresponding fitting function is $(2.16015E-13)*z^{-1.02937}$. The value of c is very small as expected and the value of b now obtained is far from ideal. These parameters are unreliable, and are now rather a fit of the numerical errors.

3 Conclusion:

We have analyzed the magnetic field 'almost' along the axis of a loop antenna and found a corresponding exponential fit.