MACHINE LEARNING (CSE-242)
ASSIGNMENT-3

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(1.1) Perceptron can be used when:
* Data has only two classes (Binary classification)
* If data contains linearly seperable sets.
* The data is shuffled properly.

(1.2)

x_1	x_2	x_3	y
1	0	1	+1
0	-1	1	-1
1	1	1	+1
-1	2	0	-1

$b=0$: Initially considering $w=(0,0,0)$

Step ①: $a \leftarrow \sum_{d=1}^{D} w_d x_d + b$

$a = (0)(1) + (0)(0) + (0)(1)$

$a = 0. \rightarrow ya = 0$

Since $ya \leq 0$, so

$w_d \leftarrow w_d + y x_d$

$w = (0+(+1)(1), 0+(+1)(0), 0+(+1)(1))$

$\boxed{w=(1,0,1)}$

After first step (or) example:

$w = (1,0,1)$

$w = (1, 0, 1)$

Step ②: $a = (1)(0) + (0)(-1) + (1)(1)$

$a = 1$

$ya = (-1)(1) = -1$

Since $ya \le 0$, so

$$w = \left(1 + (-1)(0),\ 0 + (-1)(-1),\ 1 + (-1)(1)\right)$$

$$w = (1, 1, 0)$$

After second step (or) example:

$$\boxed{w = (1, 1, 0)}$$

Step ③: $w = (1, 1, 0)$

$a = (1)(1) + (1)(1) + (0)(1)$

$a = 2$

$ya = (+1)(2) = 2$

Since $ya > 0$, so

$$w = (1, 1, 0)$$

After third step (or) example:

$$\boxed{w = (1, 1, 0)}$$

Step ④: $w = (1, 1, 0)$

$a = (-1)(1) + (1)(2) + (0)(0)$

$a = 1$

$ya = (-1)(+1) = -1$

Since $ya \leq 0$, so

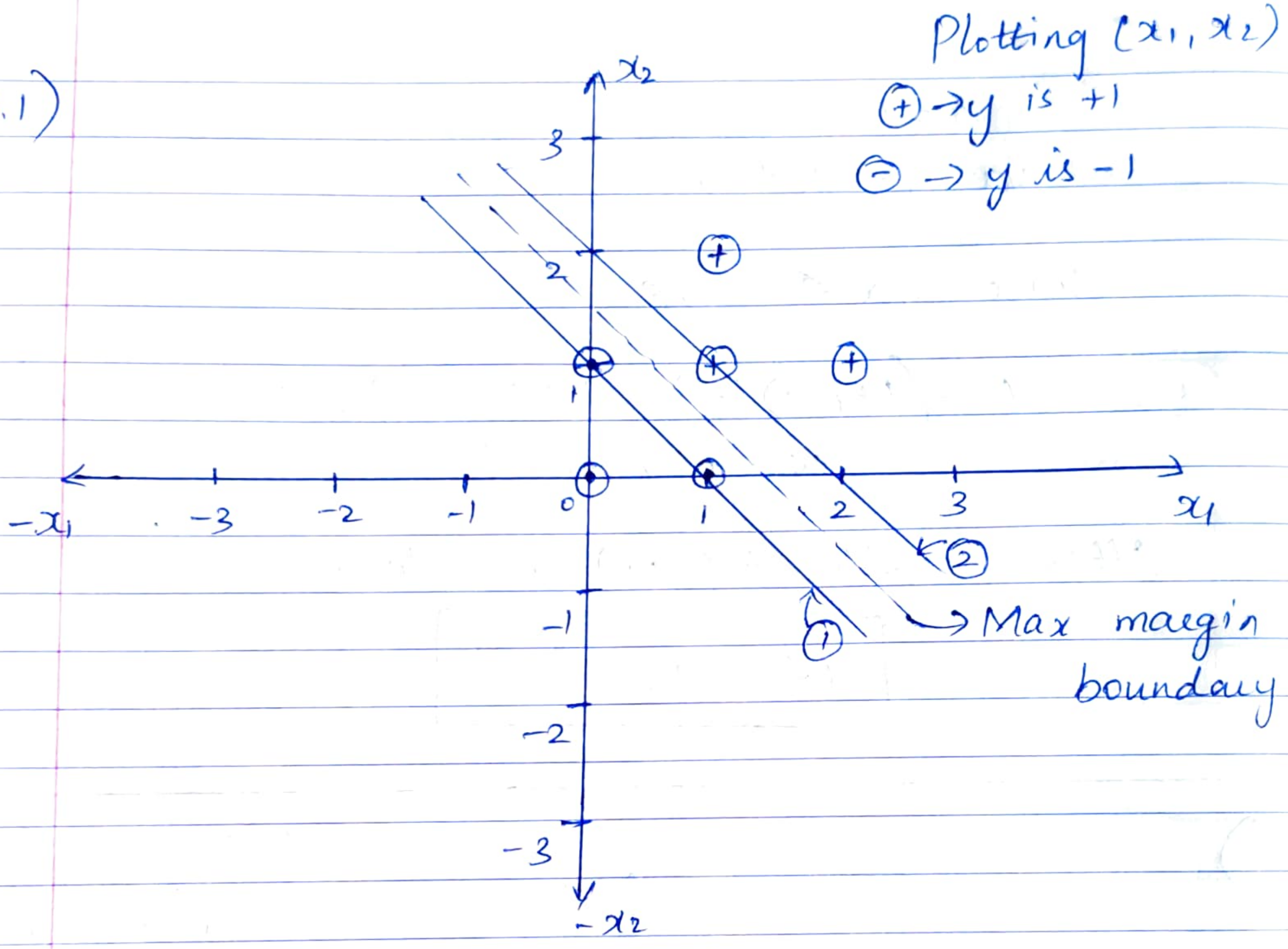
$w = (1 + (-1)(-1),\ 1 + (-1)(2),\ 0 + (-1)(0))$

$w = (2, -1, 0)$

After fourth step (or) example :

$$\boxed{w = (2, -1, 0)}$$

(2.1)



The support vectors are

x_1	x_2	y
0	1	-1
1	1	$+1$
1	0	-1

(2.2) The equation of the line ① passing through the support vectors is given by

support vectors $(0,1)$ & $(1,0)$:

$$x_2' - x_2 = m(x_1' - x_1)$$

$$m \rightarrow \text{slope} = \frac{x_2' - x_2}{x_1' - x_1} = \frac{1-0}{0-1} = -1$$

$\therefore$ The y intercept of this line is '1'.

$$\therefore \quad y = mx + c$$

$$x_2 = (-1)x_1 + 1$$

$$\boxed{x_1 + x_2 = 1} \rightarrow \text{line ① equation}.$$

Similarly for line ②

Slope $= -1$ since it is parallel to ①

$\therefore$

$$(x_2 - 1) = (-1)(x_1 - 1)$$

$$x_2 - 1 = -x_1 + 1$$

$$\boxed{x_1 + x_2 = 2} \rightarrow \text{line ② equation}.$$

So, they max margin hyperplane should lie between line ① & line ② with the same slope as it is parallel to both of them.

Points $(1,1)$ and $(0,0)$ — the line passing through it intersects line ①. So midpoint of these points gives a point on line ① perpendicular to the hyperplane (max margin).

(Since the whole shape being square)

midpoint (point on ①) $= \left(\frac{1+0}{2}, \frac{1+0}{2}\right)$

$= \left(1/2, 1/2\right)$ — ③

So, the point ③ on line ① & (1,1) on line ②, its midpoint gives a point on the maximum margin hyperplane which is perpendicular to both line ① & line ②

$= \left(\frac{1+1/2}{2}, \frac{1+1/2}{2}\right)$

$= \left(3/4, 3/4\right)$

$\therefore$ (3/4, 3/4) is a point on the hyperplane.

Equation of max-margin hyperplane:

Slope → similar to ① & ② since parallel

$x_2 - 3/4 = (-1)(x_1 - 3/4)$

$x_2 - 3/4 = -x_1 + 3/4$

$\therefore \boxed{x_1 + x_2 = 1.5}$

Geometric margin: $= 1/\|w\|$ → ⊥ distance between hyperplane & support vector plane.

we have two points: (1,1) & (3/4, 3/4)

$\therefore$ Geometric margin $= \sqrt{(1-3/4)^2 + (1-3/4)^2} = \frac{1}{2\sqrt{2}}$ (or) $\frac{\sqrt{2}}{4}$

(or)

Geometric maergin $= \frac{1}{||w||}$

'w' for the hypeeplane $= (1, 1)$

based on the equation of hypeeplane,

$= \frac{1}{\sqrt{1^2+1^2}}$

Geometric maergin $= \frac{1}{\sqrt{2}} \Big/ 2$ ($\because$ it is the distance between hypeeplane & the point)

$= \frac{1}{2\sqrt{2}}$ //