Mailam Engineering College

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Department of Mechanical Engineering

Sub Code/Name : **ME3591/DESIGN OF MACHINE ELEMENTS**

Year/Sem : **III/V**

UNIT – I FUNDAMENTAL CONCEPTS IN DESIGN

Introduction to the design process - factors influencing machine design, selection of materials based on mechanical properties - Preferred numbers- Direct, Bending and torsional loading- Modes of failure - Factor of safety – Combined loads – Principal stresses – Eccentric loading – curved beams – crane hook and 'C' frame- theories of failure – Design based on strength and stiffness – stress concentration – Fluctuating stresses – Endurance limit –Design for finite and infinite life under variable loading - Exposure to standards.

UNIT – II DESIGN OF SHAFTS AND COUPLINGS

Shafts and Axles - Design of solid and hollow shafts based on strength, rigidity and critical speed – Keys and splines – Rigid and flexible couplings.

UNIT – III DESIGN OF TEMPORARY AND PERMANENT JOINTS

Threaded fasteners - Bolted joints including eccentric loading, Knuckle joints, Cotter joints – Welded joints Butt, Fillet and parallel transverse fillet welds – welded joints subjected to bending, torsional and eccentric loads, riveted joints for structures - theory of bonded joints.

UNIT – IV DESIGN OF ENERGY STORING ELEMENTS AND ENGINE COMPONENTS

Types of springs, design of helical and concentric springs–surge in springs, Design of laminated springs - rubber springs - Flywheels considering stresses in rims and arms for engines and punching machines-- Solid and Rimmed flywheels- connecting rods and crank shafts

UNIT – V DESIGN OF BEARINGS AND MISCELLANEOUS ELEMENTS

Sliding contact and rolling contact bearings - Hydrodynamic journal bearings, Sommerfeld Number, Raimondi & Boyd graphs, -- Selection of Rolling Contact bearings –Design of Seals and Gaskets

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Principal

DEPARTMENT OF MECHANICAL ENGINEERING

Subject Title : DESIGN OF MACHINE ELEMENTS
Subject Code : ME 3591
Year/ SEM : III / V
UNIT I - STEADY STRESSES AND VARIABLE STRESSES IN MACHINE MEMBERS

SYLLABUS

Introduction to the design process - factors influencing machine design, selection of materials based on mechanical properties - Preferred numbers, fits and tolerances – Direct, Bending and torsional stress equations – Impact and shock loading – calculation of principle stresses for various load combinations, eccentric loading – curved beams – crane hook and 'C' frame- Factor of safety - theories of failure – Design based on strength and stiffness – stress concentration – Design for variable loading.

SUMMARY

Introduction to the design process.

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings.

Classifications of Machine Design:

The machine design may be classified as follows:

1. ***Adaptive design.*** In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training

2. ***Development design.*** This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture.

3. ***New design.*** This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design.

Design process and factors considered for designing, materials, and classification are all discussed.

Factors like (load, stress, motion, and kinematics, material selection, for, size, lubrication, and safety) are all explained as the factor have to be considered while designing a part or components.

Mechanical properties of metals like (strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, machinability, resilience, creep, fatigue) are all explained as the important consideration while designing the parts.

Simple stresses its introduction, reaction to force acting on it, varying load conditions, stress – strain relation, tensile stress and strain, compressive stress and strain, young's modulus of elasticity and the influenced factors of the above stated are explained.

Shear stress and strain, shear modulus, bearing stress, stress – strain diagram, working stress, factor of safety, selection of factor of safety, stresses in composite bars, thermal stresses, linear strain, lateral strain, poisson's ratio, bulk modulus, its relations, impact stress, are all explained.

Twisting force, torsional shear stress, shafts arrangements, bending stress in straight beam, curved beam, principle stresses and principle planes, determining principle stress and plane, its applications, failure theories under static load condition and variable load conditions, eccentric loading, direct and bending stresses combined, shear stresses in beams, are discussed.

Variable stresses in machine parts, reversed cyclic stresses, fatigue stresses, endurance limit, endurance limit factor, loading conditions, surface finish factors, size factor, factor of safety for fatigue loading, stress concentrations, theoretical or form stress concentration factors, stress concentration due to holes and notches, methods of reducing stress concentration, factors considered while designing machine parts, fatigue failure, notch sensitivity, combined steady and variable stress, Geber method, Goodman method, Soderberg methods are discussed with examples and applications.

PART – A (2 MARKS)

Introduction to the design process:

1. Define machine design.

Machine design is defined as the use of scientific principles, technical information and imagination in the description of a machine or a mechanical system to perform specific functions with maximum economy and efficiency.

Machine Design is the creation of new and better machines which improves the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of pictorial representations.

2. Write down the basic procedure of machine design.

Step 1: Product specification
Step 2: selection of materials
Step 3: Layout of configuration
Step 4: Design of individual component
Step 5: Preparation of drawing.

3. What do you understand by machine elements?

A machine consists of machine elements. Each part of machine which has motion with respect to some other parts are called machine element, may consists of several parts which are manufactured separately.

These machines elements can be classified into two groups are general - purpose and special-purpose machine elements.

a. **General** - purpose machine elements: include shafts, couplings clutches, bearing, springs, gears and machine frames.

b. **Special**- purpose of machine elements: include piston, valves, spindles, etc. The special purpose of machine elements is used only in certain types of applications. On the contrary, general purpose machine elements are used in a large number of machines.

4. What are the basic requirements of machine elements?

1) Strength
2) Rigidity
3) Wear resistance
4) Minimum dimensions and weight
5) Manufacturability
6) Safety
7) Conformance to standards
8) Reliability
9) Maintainability
10) Minimum - life cycle cost.

5. What are the steps involved in the design of machine elements? (Nov/Dec 2022)

Design of machine elements is the most important step in the complete procedure of machine design. In order to ensure the basic requirements of machine elements, calculations are carried out to find out the dimensions of the machine elements. These calculations form an integral part of the design of machine elements. The basic procedure of the design of machine elements as

Step 1: Specification of function
Step 2: Determination of forces
Step 3: Selection of materials
Step 4: Failure criterion
Step 5: Determination of dimensions
Step 6: Design modifications
Step 7: Working drawing

6. Define design synthesis and standardization.

Design synthesis is defined as the process of creating or selection configuration, materials, shapes and dimensions for a product. It is a decision making process with the main objective of optimization.

Standardization is defined as obligatory norms, to which various characteristics of a product should conform. The characteristics include materials dimensions and shape of the component, method of testing and method of making, packing and storing of the product.

7. Enumerate the standards used in mechanical engineering design.

1.Standard for materials their chemical compositions, mechanical properties and heat treatment.
2.Standards for shapes and dimensions of commonly used machine elements.
3.Standards for Fits, tolerance and surface finish of components
4.Standards for testing of products
5.Standards for engineering drawing of components.

8. How do you classify machine design?(Nov/Dec-16)

1. Adaptive design.

This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2. Development design.

This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. New design.

This type of design needs a lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design.

9. What are the factors should be considered in machine design?

1. Type of loading – static, dynamic, simple, complicated
2. Size of object – simple , complicated, small large
3. Environment conditions – pure atmosphere
4. Material properties – hard, soft, rigid, elastic, tough etc.
5. Place of employment – hazardous place, safe, place , on road, on water, in air etc,

6. Human safety – fool proof arrangement should be considered.
7. Cost - the design is in such way that the product should be manufactured in lower cost.

<u>Factors influencing machine design:</u>

10. List some factors that influence the process of machine design.[Nov/Dec-2018](April/may 2019)

a. strength and stiffness
b. surface finish and tolerances
c. manufacturability
d. ergonomics and aesthetics
e. working atmosphere
f. safety and reliability
g. cost

11. What are the methods of Optimization?

Optimization is the process of maximizing a desired quantity or minimizing an undesired one.

The various optimization methods available are

1. Optimization by evolution
2. Optimization by intuition
3. Optimization by trial and error
4. Optimization by numerical algorithm.

<u>Selection of materials based on mechanical properties</u>

12. Classify engineering materials. Give examples.

The engineering materials are mainly classified as :

1. Metals and their alloys, such as iron, steel, copper, aluminium, etc.

The metals may be further classified as:

a. Ferrous metals, and
b. Non-ferrous metals.

The **ferrous metals** are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

The **non-ferrous** metals are those which have a metal other than iron as their main constituent, such as copper, aluminium, brass, tin, zinc, etc.

2. Non-metals, such as glass, rubber, plastic, etc.

The Non - metals may be further classified as:

a. Organics – plastics, rubber, wood etc.,
b. Ceramics – brick, send cement etc.,

13. Which are mechanical properties of the metals? List any four mechanical properties.
(M/J – 2012)

The properties of machine elements which undergo any changes in shape and structure during the application of force on these elements are called as mechanical properties.

For example if rod is subjected to a tensile load, its length can be increased and so on.

Some of the mechanical properties are elasticity, plasticity, ductility, malleability, hardness, toughness, brittleness etc,

14. What are the factors that govern selection of materials while designing a machine component?

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials,
2. Suitability of the materials for the working conditions in service, and
3. The cost of the materials.

15. Define strength and stiffness of material.

1. Strength: It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.

2. Stiffness: It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

16. List the important factors that influence the magnitude of factor of safety .N/D2011

The reliability of the properties of the material and change of these properties during service ;

- The reliability of test results and accuracy of application of these results to actual machine parts.
- The reliability of applied load ;
- The certainty as to exact mode of failure ;
- The extent of simplifying assumptions ;
- The extent of localized stresses ;
- The extent of initial stresses set up during manufacture ;
- The extent of loss of life if failure occurs ; and
- The extent of loss of property if failure occurs.

Each of the above factors must be carefully considered and evaluated.

17. Write short notes on Elasticity and plasticity.

1. Elasticity: It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

2. Plasticity: It is the property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

18. What do you understand by the term Ductility and Brittleness?

1. Ductility. It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper,aluminium, nickel, zinc, tin and lead.

2. **Brittleness.** It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.

19. Write short notes on Malleability and Machinability.

1. **Malleability:** It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.
2. **Machinability:** It is the property of a material which refers to a relative case withwhich a material can be cut. The Machinability of a material can be measured in a number ofways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of theMaterial. It may be noted that brass can be easily machined than steel.

20. What do you understand about Resilience and creep?

1. Resilience: It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

2. Creep: When a part is subjected to constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called **creep.** This property is considered in designing internal combustion engines, boilers and turbines.

21. Define Toughness.

Toughness: It is the property of a material to resist fracture due to high impact loads like Hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed up to the point of fracture. This property is desirable in parts subjected to shock and impact loads.

22. Briefly explain about the cause of fatigue failure.

Fatigue: When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue**. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

23. Mention the significance of hardness as a mechanical property.

Hardness: It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation andMachinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

(a) Brinell hardness test,

(b) Rockwell hardness test,

(c) Vickers hardness (also called Diamond Pyramid) test, and

(d) Shore scleroscope.

24. Describe the material properties hardness, stiffness and resilience (April/ May -2009)

Hardness is the ability of material to resist scratching and indentation.

Stiffness is the ability of material to resist deformation under loading.

Resilience is the ability of material to resist absorb energy and to resist shock and impact load.

25. Mention some standard specification of steels (Nov/Dec – 2008)

ASTM's steel standards are instrumental in classifying, evaluating, and specifying the material, chemical, mechanical, and metallurgical properties of the different types of steels, which are primarily used in the production of mechanical components, industrial parts, and construction elements, as well as other accessories related to them. The steels can be of the carbon, structural, stainless, ferrite, austenitic, and alloy types.

ANSI B4.1, ANSI B4.2, ISO 286, ISO 1829, ISO 2768, EN 20286, JIS B 0401

Designation of plain carbon steels

These are denoted like: x.C.y

Where x – number 100 times the average percent of carbon

Y – Number 100 times the average percent of manganese

Designation of alloy steels:

Alloy steels are denoted by arranging the alloying elements in the descending order of their proportion and the average % of each element is shown with the chemical symbol before that number. The letter C is omitted here and just the number is written to denote carbon percentage.

25. (A) How are the plain carbon steels designated in B IS?(Nov/Dec 2021)

Designation of Plain carbon steels

These are denoted like x, C, y

Where,

x – Number 100 times the average percent of carbon

y – Number 100 times the average percent of Manganese

Example:

30C8

Where,

30 = 100 times average % of Carbon

8 = 100 times average % of Manganese

Average % of Carbon = 30/100 = 0.3%

Average % of Manganese = 8/100 = 0.08%

Actual means, the actual carbon % is not 0.3, but varies between 0.2 – 0.4%, so that the average becomes (0.2 + 0.4)/2 = 0.3%, and

Manganese from say 0.07 – 0.09%, so that the average (0.07+0.09)/2 = 0.08%

Preferred numbers, fits and tolerances

26. What do you understand by preferred numbers? Explain.

When a machine is to be made in several sizes with different powers or capacities, it is necessary to decide what capacities will cover a certain range efficiently with minimum number of sizes. It has been shown by experience that a certain range can be covered efficiently when it follows a geometrical progression with a constant ratio. The preferred numbers are the conventionally rounded off values derived from geometric series including the integral powers of 10 and having as common ratio of the following factors:

$$5\sqrt{10}, \qquad 10\sqrt{10}, \qquad 20\sqrt{10}, \quad and \qquad 40\sqrt{10}$$

These ratios are approximately equal to 1.58, 1.26, 1.12 and 1.06. The series of preferred Numbers are designated as R5, R10, R20 and R40 respectively.

These four series are called basic series. The other series called derived series whichmay be obtained by simply multiplying or dividing the basic sizes by 10, 100, etc.

27. Define the term interchangeability.

The term interchangeability is normally employed for the mass production of identical items Within the prescribed limits of sizes. A little consideration will show that in order to maintain the sizes of the part within a close degree of accuracy, a lot of time is required. But even then there will be small variations.

If the variations are within certain limits, all parts of equivalent size will be equally fit for operating in machines and mechanisms. Therefore, certain variations are recognized and allowed in the sizes of the mating parts to give the required fitting. This facilitates to select at random from a large number of parts for an assembly and results in a considerable saving in the cost of production.

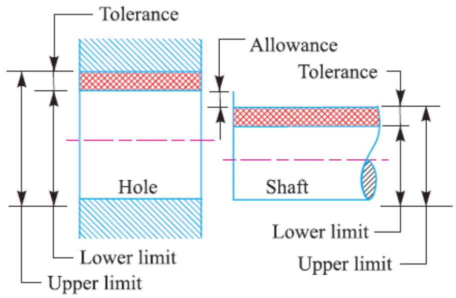
28. What do you understand by limits?

- In order to control the size of finished part, interchangeable parts is called limit.
- It may be noted that when an assembly is made of two parts, the part which enters into the other, is known as enveloped surface (or shaftfor cylindrical part).
- The term shaft refers not only to the diameter of a circular shaft, but it is also used to designate any external dimension of a part.
- The term holerefers not only to the diameter of a circular hole, but it is also used to designate any internal dimension of a part.

29. List out important terms in limit system.

1. Nominal size.2. Basic size.3.Actual size.4.Limits of sizes.5.Allowance.
6. Tolerance.7. Tolerance zone.8.Zero line.9. Upper deviation.10.Lower deviation.
11. Actual deviation.12. Mean deviation.13. Fundamental deviation.

30. Briefly explain limit of sizes.



1. Nominal size: It is the size of a part specified in the drawing as a matter of convenience.

2. Basic size: It is the size of a part to which all limits of variation (i.e. tolerances) are applied to arrive at final dimensioning of the mating parts. The nominal or basic size of a part is often thesame.

3. Actual size: It is the actual measured dimension of the part. The difference between the basic size and the actual size should not exceed a certain limit otherwise it will interfere with the interchangeability of the mating parts.

4. Limits of sizes: There are two extreme permissible sizes for a dimension of the part as Shown in Fig. The largest permissible size for a dimension of the part is called **upper** or **high** or **maximum limit,** whereas the smallest size of the part is known as **lower** or **minimum limit.**

31. Define tolerance.

(*a*) Unilateral tolerance. (*b*) Bilateral tolerance.

It is the difference between the upper limit and lower limit of a dimension. In other words, it is the maximum permissible variation in a dimension. The tolerance may be **unilateral orbilateral.** When all the tolerance is allowed on one side of the nominal size, e.g. $20^{+0.000}_{-0.004}$, then it is said to be **unilateral system of tolerance.** The unilateral system is mostly used in industries as it permits changing the tolerance value while still retaining the same allowance or type of fit. When the tolerance is allowed on both sides of the nominal size, e.g. $20^{+0.002}_{-0.002}$, then it is said tobe **bilateral system of tolerance.** In this case + 0.002 is the upper limit and – 0.002 is the lower limit.

32. Write short notes on allowance.

It is the difference between the basic dimensions of the mating parts.

The allowance may be

1. **Positive and**
2. **Negative**.

When the shaft size is less than the hole size, then the allowance is **positive** and

When the shaft size is greater than the hole size, then the allowance is **negative.**

33. Brief about the tolerance zone with neat sketch.

Tolerance zone: It is the zone between the maximum and minimum limit size, as shown in fig.1

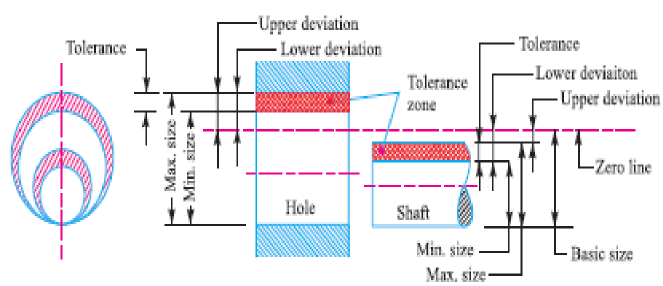


fig 1 Tolerance zone.

34. Write short notes on fundamental deviations.

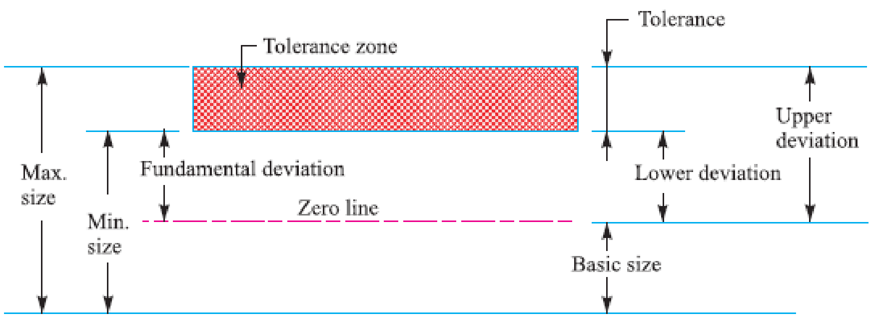


fig 2 Fundamental deviation.

a. Zero line: It is a straight line corresponding to the basic size. The deviations are measured from this line. The positive and negative deviations are shown above and below the zero line respectively.

b. Upper deviation: It is the algebraic difference between the maximum size and the basic size. The upper deviation of a hole is represented by a symbol ES (Ecart Superior) and of a shaft, it is represented by e_s.

c. Lower deviation: It is the algebraic difference between the minimum size and the basic size. The lower deviation of a hole is represented by a symbol EI (Ecart Inferior) and of a shaft, it is represented by e_i.

d. Actual deviation: It is the algebraic difference between an actual size and the corresponding basic size.

e. Mean deviation: It is the arithmetical mean between the upper and lower deviations.

f. Fundamental deviation: It is one of the two deviations which is conventionally chosen to define the position of the tolerance zone in relation to zero line, as shown in Figure. 2

35. Distinguish Fit, clearance and interference.

Fit: The degree of tightness or looseness between the two mating parts is known as a fit of **the parts. The nature of fit is characterized by the presence and size of clearance and** interference.

Clearance: The clearanceis the amount by which the actual size of the shaft is less than the actual size of the mating hole in an assembly. In other words, the clearance is the difference between the sizes of the hole and the shaft before assembly. The difference must be **positive.**

The interferenceis the amount by which the actual size of a shaft is larger than the actual

Finished size of the mating hole in an assembly. In other words, the interference is the arithmetical difference between the sizes of the hole and the shaft, before assembly. The difference must benegative.

36. What is adaptive design? Where it is used? Give example. (N/D 2012)(A/M 2015)

In most cases, the designer's work isconcerned with adaptation of existing designs. This type ofdesign needs no special knowledge or skill and can be attempted by designers of ordinary technical training. Thedesigner only makes minor alternation or modification in theexisting designs of the product.

Example: a press is to be designed for more capacity or a turret lathe is to be redesigned to accommodate large stock, etc.

37. Write about the clearance fit.

In this type of fit, the size limits for mating parts are so selected that clearance between them always occur, as in figure 3. So it may be noted that in a clearance fit, the tolerance zone of the hole is entirely above the tolerance zone of the shaft. In a clearance fit, the difference between the minimum size of the hole and the maximum size of the shaft is known as minimum clearance whereas the difference between the maximum size of the hole and minimum size of the shaft is called maximum clearance

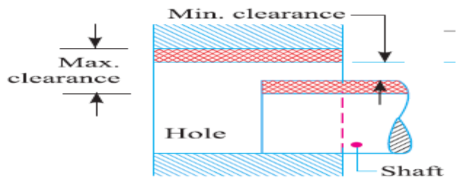


fig 3 (a) Clearance fit.

38. Write about the Interference fit.

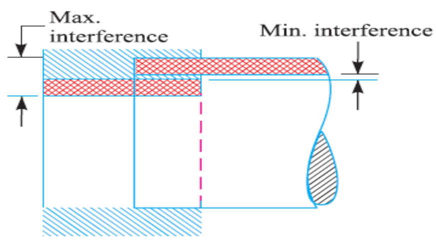


fig 4 (b) Interference fit.

Types of fits.

In this type of fit, the size limits for the mating parts are so selected thatinterference between them always occur, as shown in Fig. It may be noted that in an interference fit, the tolerance zone of the hole is entirely below the tolerance zone of the shaft.

In an interference fit, the difference between the maximum size of the hole and the minimum size of the shaft is known as **minimum interference,** whereas the difference between the minimum size of the hole and the maximum size of the shaft is called **maximum interference,** as shown in Fig.4The interference fits may be shrink fit, heavy drive fit and light drive fit.

39. Write about the Transition fit.

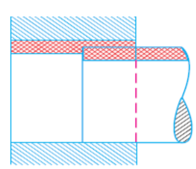
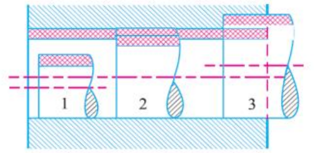


fig 5 (c) Transition fit.

In this type of fit, the size limits for the mating parts are so selected that either a clearance or interference may occur depending upon the actual size of the mating parts as shown in fig 5 may be noted that in a transition fit, the tolerance zones of hole and shaft overlap.

The transition fits may be force fit, tight fit and push fit.

40. What is meant by 'hole basis system'?



1. Clearance fit. 2. Trar

fig 6 (a) Hole basis system.

Ba

Hole basis system: When the hole is kept as a constant member (i.e. when the lower deviationof the hole is zero) and different fits are obtained by varying the shaft size, as shown in Fig.6(a), then the limit system is said to be on a hole basis.

41. Why Hole basis system it's preferred than Shaft basis system?

It may be noted that from the manufacturing point of view, a hole basis system is always preferred. This is because the holes are usually produced and finished by standard tooling like drill, reamers, etc., whose size is not adjustable easily. On the other hand, the size of the shaft (which is to go into the hole) can be easily adjusted and is obtained by turning or grinding operations.

<u>Direct, Bending and Torsional stress equations Impact and shock loading</u>

42. What are the different forces acting on the machine parts?

In engineering practice, the machine parts are subjected to various forces which may be due to either one or more of the following:

1. Energy transmitted,
2. Weight of machine,
3. Frictional resistances,
4. Inertia of reciprocating parts,
5. Change of temperature, and
6. Lack of balance of moving parts.

43. List out the types of loads.

It is defined as **any external force acting upon a machine part.**

The following four types of the load are important from the subject point of view:

1. Dead or steady load: A load is said to be a dead or steady load, when it does not change inmagnitude or direction.

2. Live or variable load: A load is said to be a live or variable load, when it changes continually.

3. Suddenly applied or shock loads: A load is said to be a suddenly applied or shock load, whenit is suddenly applied or removed.

4. Impact load: A load is said to be an impact load, when it is applied with some initial velocity.

44. Discuss about the bearing stress.

A localized compressive stress at the surface of contact between two members of a machinepart that are relatively at rest is known as **bearing stress** or **crushing stress.**

The bearing stress is taken into account in the design of riveted joints, cotter joints, knuckle joints, etc.

$$\sigma_b or\ \sigma_c = \frac{P}{d.t.n}$$

45. Briefly explain about on a). proportional limit b. elastic limit c. yield point d. ultimate Limit.

a. **Proportional limit:** it is the amount of stress up to which, the strain rate is linearly or directly proportional to the stress applied on the metal specimen.

b. **Elastic limit** is the amount of stress up to which if the applied load is removed the elongation produced due to load will be vanished and thus the specimen will regain its original length.

c. **Yield point** is the amount of stress, at which the specimen will enter into plastic nature from elastic nature that is after yield point even through the applied load is removed the elongation will remain and the specimen will not get its original length. For most of the materials, the elastic limit and yield point are one and the same.

d. **The ultimate limit** is the amount of stress which is the maximum value of stress that is load sustained by the metal specimen. Beyond that, the strained specimen will be broken in tow pieces.

46. What is meant by working stress?

When designing machine parts, it is desirable to keep the stress lower than the maximum orultimate stress at which failure of the material takes place. This stress is known as the **working stress** or **design stress.** It is also known as **safe** or **allowable stress.**

(By failure it is not meant actual breaking of the material. Some machine parts are said to fail when they have plastic deformation set in them, and they no more perform their function satisfactory.)

47. What do you mean by factor of safety? N/D2011

It is defined, in general, as the **ratio of the maximum stress to the working stress.** Mathematically,

$$\text{Factor of safety } = \frac{\text{Maximum stress}}{\text{Working or design stress}}$$

In case of ductile materials *e.g.* mild steel, where the yield point is clearly defined, the factor ofSafety is based upon the yield point stress. In such cases,

$$\text{Factor of safety } = \frac{\text{Yield point stress}}{\text{Working or design stress}}$$

In case of brittle materials *e.g.* cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

$$\text{Factor of safety } = \frac{\text{Ultimate stress}}{\text{Working or design stress}}$$

48. List out some mechanical properties of metals. M/J2012

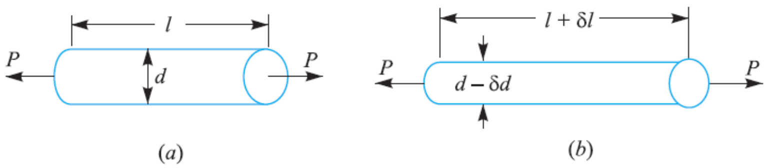
The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load.

These mechanical properties of the metal include

a. strength,
b. stiffness,
c. elasticity,
d. plasticity,
e. ductility,

f. brittleness,
g.malleability,
h.toughness,
i. resilience,
j. creep and
k.Hardness.

49. Distinguish between linear and lateral strain.



Linear and lateral strain.

A little consideration will show that due to tensile force, the length of the bar increases by anamount δland the diameter decreases by an amount δd, as shown in Fig. Similarly, if the bar is subjected to a compressive force, the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompanied by a strain in its own direction whichis known as linear strainand an opposite kind of strain in every direction, at right angles to it, isknown as lateral strain**.**

50. Write short notes on Poisson's ratio.

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, mathematically

$$\frac{\text{lateral strain}}{\text{linear strain}} = \text{constant}$$

This constant is known as **Poisson's ratio** and is denoted by $1/m$ or μ.

51. Write down the formula for relation between Bulk Modulus vs Young's Modulus and Young's Modulus vs Modulus of Rigidity

The bulk modulus (K) and Young's modulus (E) are related by the following relation,

$$K = \frac{m.E}{3(m-2)} = \frac{E}{3(1-2\mu)}$$

The Young's modulus (E) and modulus of rigidity (G) are related by the following relation,

$$G = \frac{m.E}{2(m+1)} = \frac{E}{2(1+\mu)}$$

Where

E= young's modulus N/mm^2 m – mass kg

G = modulus of rigidity N/mm^2 μ - Poisson's ratio

<u>Principle stresses for various load combinations, eccentric loading – curved beams – crane hook and 'C' frame- Factor of safety – theories of failure – Design based on strength and stiffness – stress concentration</u>

52. How do you determine the principle stresses in machine elements?

There are many cases in practice, in which machine members are subjected to combined stressesdue to simultaneous action of either tensile or compressive stresses combined with shear stresses.In many shafts such as

a. Propeller shafts, b. C-frames etc.,

There are direct tensile or compressive stresses due to the external force and shear stress due to torsion, which acts normal to direct tensile or compressive stresses.

The shafts like crank shafts are subjected simultaneously to torsion and bending. In such cases, the maximum principal stresses, due to the combination of tensile or compressive stresses with shear stresses may be obtained.

53. What are the various theories of failures?(M/J2013)

1. Maximum principal (or normal) stress theory (also known as Rankine's theory).
2. Maximum shear stress theory (also known as Guest's or Tresca's theory).
3. Maximum principal (or normal) strain theory (also known as Saint Venant theory).
4. Maximum strain energy theory (also known as Haigh's theory).
5. Maximum distortion energy theory (also known as Hencky and Von Mises theory).

54. Discuss about maximum principal stress theory.

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test.

$$\sigma_{t1} = \frac{\sigma_{yt}}{FOS} \ (for\ ductile)$$

$$\sigma_{t1} = \frac{\sigma_{u}}{FOS} \ (for\ brittle)$$

$$\sigma_{yt} - \ yield\ point\ stress\ in\ tension\ as\ determined\ from\ simple\ tension\ test$$

$$\sigma_{u} - \ ultimate\ stress$$

55. State the maximum shear stress theory.

According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test. Mathematically,

$$\tau_{max} = \frac{\tau_{yt}}{FOS}$$

$$\tau_{max} - Maximum\ shear\ stress\ in\ a\ bi-axal\ stress\ system$$

$$\tau_{yt} - shear\ stress\ at\ yield\ point\ as\ determined\ from\ simple\ tension\ test$$

FOS – Factor of safety $\tau_{max} = \frac{\sigma_{yt}}{2\times FOS}$

This theory is mostly used for designing members of ductile materials.

56. Write short notes on maximum principal strain theory.

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain (*i.e.* strain at yield point) as determined from a simple tensile test. The maximum principal (or normal) strain in a bi-axial stress system is given by

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t1}}{m.E}$$

According to the above theory,

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t1}}{m.E} = \frac{\sigma_{yt}}{E \times FS}$$

σt1 and σt2= Maximum and minimum principal stresses in a bi-axial stress system,
ε = Strain at yield point as determined from simple tension test,
1/*m* = Poisson's ratio,*E* = Young's modulus, and*F.S.* = Factor of safety.

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m.E} = \frac{\sigma_{yt}}{E \times FS}$$

$$\sigma_{t1} - \frac{\sigma_{t2}}{m.E} = \frac{\sigma_{yt}}{E \times FS}$$

This theory is not used, in general, because it only gives reliable results in particular cases.

57. Explain about maximum strain energy theory.

According to this theory, the failure or yielding occurs at a point in a member when the strainenergy per unit volume in a bi-axial stress system reaches the limiting strain energy (*i.e.* strain energy at the yield point) per unit volume as determined from simple tension test.

We know that strain energy per unit volume in a bi-axial stress system,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\sigma_{t1} \times \sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{FS}\right)^2$$

This theory may be used for ductile materials.

58. State Maximum Distortion Energy Theory (Hanky and Von Mises Theory). M/J 2009

According to this theory, the failure or yielding occurs at a point in a member when the distortionstrain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (*i.e.* distortion energy at yield point) per unit volume as determined from a simple tension test. Mathematically, the maximum distortion energy theory for yielding is expressed as

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = \left(\frac{\sigma_{yt}}{FS}\right)^2$$

This theory is mostly used for ductile materials in place of maximum strain energy theory.

59. Write short notes on endurance limit. What are the factors affecting endurance Strength?

Endurance limit is the maximum value of completely reversed stress that the standard specimen can sustain an infinite number (10^7) of cycles without failure

Factor affecting endurance strength are

1. Load
2. Size
3. Surface finish
4. Temperature
5. Impact 6. Reliability

60. Distinguishes variable load and give some applications.

If the magnitude of load at any point of a machine member varies time to time,then that kind of load is known as variable load. This variable load is also called as fluctuating load or fatigue load.

- Some of the machine members which are subjected to variable loading are axis, shafts, crankshafts, connecting rod, piston rods, spring, gear etc.

61. Briefly discuss about stress –concentration and stress concentration factor.(May/June -2012)(May/June 14))(May/June-16)

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called stress concentration.

It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc.

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically, theoretical or form stress concentration factor,

$$K_t = \frac{\text{Maximum stress}}{\text{Nominal stress}}$$

The value of K_t depends upon the material and geometry of the part.

62.State the difference between straight beams and curved beams.(N/D2012)

	Straight beams	Curved beams
1	Neutral axis of the cross –section passes through the centroid of the section	Neutral axis does not coincide with the cross – section, but is shifted towards the centre of curvature of the beam
2	The variation of bending stress is linear, magnitude being proportional to the distance of a fibre from the neutral axis	The distribution of the stress in the case of curved beam is non - linear because of the neutral axis is initially curved.
3	No stress concentration	Stress concentration is higher at the inner fibre
4	Neutral axis remain undisturbed along the CG	Neutral axis always shifts towards the centre of curvature

63. Differentiate between hardness and toughness.(M/J 2014) (Nov/Dec 2017) (Nov/Dec 2018)

Hardness is resistance to indentation while **toughness** is resistance to fracture or area under the stress-strain curve. Hardness can be measured on the Mohs scale or various other scales.

Toughness is the amount of energy the material absorbs per unit volume before it breaks. Resilience is the amount of energy absorbed by the material until its elastic limit.

Toughness, often expressed as the Modulus of Toughness, is measured in units of joules per cubic meter (J/m^3) in the SI system and pound-force per square inch.

64.What are the methods used to improve fatigue strength? N/D 2013

- High frequency mechanical impact (HFMI)
- Weld toe improvement.
- Fatigue strength improvement

The past decade has seen steady increase in thenumber of HFMI equipment manufacturers and service providers. Numerous power sources are employed, e.g., ultrasonic piezoelectric elements, ultrasonic magnetostrictive elements orcompressed air.

65. List at least two methods to improve the fatigue strength. (Nov/Dec 2014)

There are essentially four basic ways of improving fatigue resistance

1. Improve the actual welding procedure
2. Alter the material microstructure
3. Reduce geometrical discontinuities

Induce surface compressive residual stresses

66. Determine the force required to punch a hole of 20mm diameter in a 5mm thick plate with ultimate shear strength of 250 MPa (Nov/Dec 2014)

Given:

Force required to punch one hole= area sheared X ultimate shear strength

$$= \Pi * dt * S_s$$

Where **d= diameter of hole in mm**

t= thickness of plate in mm

S_s= ultimate shear strength in Mpa

$$\frac{\pi * 20 * 5 * 250}{1000}$$

= 78.5398 kN

67. Define limits and fits. Types of limits and fits (April/May 2015)

LIMITS: - These are two extreme permissible sizes of dimension between which actual size of dimension is contained. The greater of these two is called high limit and the smaller low limit.

Fits: - The degree of tightness or looseness between the two mating parts is known as a fit of the parts.

LIMITS: 1.High limit, 2. Low limit

FITS: 1.Clearance fit, 2. Interference fit & 3.Transition fit

68. What are the methods to reduce stress concentration? (Nov/Dec 2008) (A/M'2023)

Avoiding sharp corners, providing fillets, Use of multiple holes instead of single hole, undercutting the shoulder parts.

69. What are the types of variable stresses?

i. Completely reversed or cyclic stresses
ii. Fluctuating stresses
iii. Repeated stresses
iv. Alternating stresses

70. What are unilateral and bilateral tolerances? (May/June 2013)

It is the difference between the upper limit and lower limit of a dimension. In other words, it is the maximum permissible variation in a dimension. The tolerance may be unilateral or bilateral.

When all the tolerance is allowed on one side of the nominal size, e.g. $20_{-0.000}^{+0.004}$, then it is said to be unilateral system of tolerance.

When the tolerance is allowed on both sides of the nominal size, e.g.$20_{-0.002}^{+0.002}$, then it is said to be bilateral system of tolerance. In this case + 0.002 is the upper limit and – 0.002 is the lower limit.

71. Describe material properties hardness, stiffness and resilience. (A/ M- 2009 & N/D- 2009),(N/D- 2013)(M/J-16)

Hardness is the ability of material to resist scratching and indentation.
Stiffness is the ability to resist deformation under loading.
Resilience is the ability of material to resist absorbs energy and to resist shockand impact loads.

72. What is an S-N Curve? (Nov/Dec 2016)

- An S- N curve has fatigue stress on Y axis and number of loading cycles in X axis. It is used to find the fatigue stress value corresponding to a given number of cycles.

73. Define modulus of resilience and proof resilience (April/May 2017)

Proof resilience is defined as the maximum energy that can be absorbed up to the elastic limit, without creating a permanent distortion.

The **modulus of resilience** is defined as the maximum energy that can be absorbed per unit volume without creating a permanent distortion.

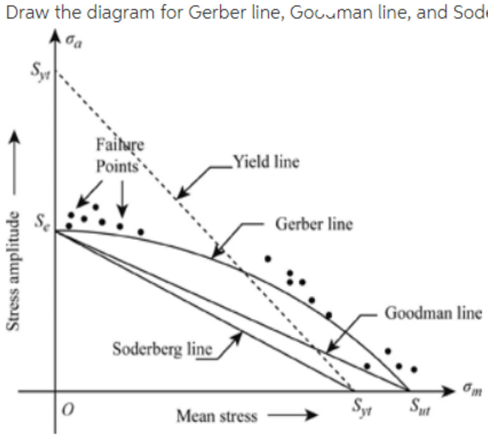
74. why non symmetrical I and T section are preferred in design of curved beams?(April/May 2017)

- If the section is symmetrical such as a circle, rectangle, I-beam with equal flanges, then the maximum bending stress will always occur at the inside fibre.
- If the section is unsymmetrical, then the maximum bending stress may occur at either the inside fibre or the outside fibre.

75. What is shock factor and what does it indicate ?(Nov/Dec 2017) (Nov/Dec 2018)
shock load is type of load which is applied suddenly or with some velocity examples for this type of loading include punching presses, hammers loads exerted on cams during the motion due to eccentricity.

76.Brief about Soderberg and Goodman lines (April/May 2018)

Draw the diagram for Gerber line, Goo..man line, and Soderberg line as below.



77.What are preferred numbers? (April/May 2018)

Preferred numbers (also called preferred values of preferred series) are standard guidelines for choosing exact product dimensions within a given set of constrains. Product developers must choose numerous lengths, distances, diameters, volumes and other characteristic quantities while all of these choices are constrained by considerations of functionality, usability, compatibility, safety or cost there usually remains considerable leeway in the exact choice for many dimensions. They are chosen such that when a product is manufactured in many different sizes, these will end up roughly equally spaced on a logarithmic scale. They therefore help to minimize the number of different sizes that need to be manufactured or kept in stock.

78. **Brief Saint venant's theory of failure. (April/May 2019)** (April/May 2023)

This **Theory** assumes that **failure** occurs when the maximum strain for a complex state of stress system becomes equals to the strain at yield point in the tensile test for the three dimensional complex state of stress system.

$$U_t = 1/2\sigma_1 \in_1 + 1/2\sigma_2 \in_2 + 1/2\sigma_3 \in_3$$

substituting the values of $\in_1, \in_2$ and $\in_3$

$$\in_1 = \frac{1}{E}[\sigma_1 - \gamma(\sigma_2 + \sigma_3)]$$

$$\in_2 = \frac{1}{E}[\sigma_2 - \gamma(\sigma_1 + \sigma_3)]$$

$$\in_3 = \frac{1}{E}[\sigma_3 - \gamma(\sigma_1 + \sigma_2)]$$

Thus, the failure criterion becomes

$$\left(\frac{\sigma_1}{E} - \gamma\frac{\sigma_2}{E} - \gamma\frac{\sigma_3}{E}\right) = \frac{\sigma_{yp}}{E}$$

or

$$\boxed{\sigma_1 - \gamma\sigma_2 - \gamma\sigma_3 = \sigma_{yp}}$$

79. A component is loaded with normal and shear stresses as σx= 15 MPa ; σy = 5 MPa ; and τxy = 10 MPa. Find the maximum shear stress developed in the component. Nov/Dec-2020, April/May-2021

Given: **σx= 15 MPa ; σy = 5 MPa ; and τxy = 10 MPa.**

Find the maximum shear stress τ_{max} **= 1/2** $\sqrt{\sigma x^2 + 4xy^2} = \sqrt{15^2 + 4(10)^2}$ **= 12.5 N/mm²**

80. Which theory of failure is suitable for the design of cast iron component subjected to steady state loading? Nov/Dec-2020, April/May-2021

Rankinis Theory is suitable for design of cast iron component subjected to steady state loading**.**

PART – B (16 Marks)

Introduction to the design process - factors influencing machine design, selection of materials based on mechanical properties - Preferred numbers, fits and tolerances

1. **(i) Explain various phases in design using a flow diagram and enumerate the factors influencing the machine design (12).**
(ii) What is meant by Hole basis system and shaft basis system? Which one is preferred and why? (4) (May /June– 2013)

Solution:

Classifications of Machine Design:

The machine design may be classified as follows:

1. **Adaptive design**. In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.
2. **Development design**. This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. **New design.** This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design.

The designs, depending upon the methods used, may be classified as follows:

(a) **Rational design.** This type of design depends upon mathematical formulae of principle of mechanics.

(b) **Empirical design.** This type of design depends upon empirical formulae based on the practice and past experience.

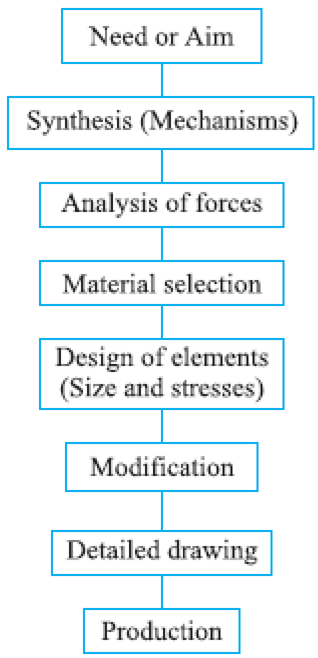
(c) **Industrial design.** This type of design depends upon the production aspects to manufacture any machine component in the industry.

(d) **Optimum design**. It is the best design for the given objective function under the specified constraints. It may be achieved by minimizing the undesirable effects.

(e) **System design.** It is the design of any complex mechanical system like a motor car.

(f) **Element design.** It is the design of any element of the mechanical system like piston, Crankshaft, connecting rod, etc.

(g) Computer aided design. This type of design depends upon the



General Procedure in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows :

1. **Recognition of need**. First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. **Synthesis (Mechanisms).** Select the possible mechanism or group of mechanisms which will give the desired motion.

3. **Analysis of forces**. Find the forces acting on each member of the machine and the energy transmitted by each member.

4. **Material selection.** Select the material best suited for each member of the machine.

5. **Design of elements** (Size and Stresses). Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. **Modification.** Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. **Detailed drawing**. Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. **Production.** The component, as per the drawing, is manufactured in the workshop.

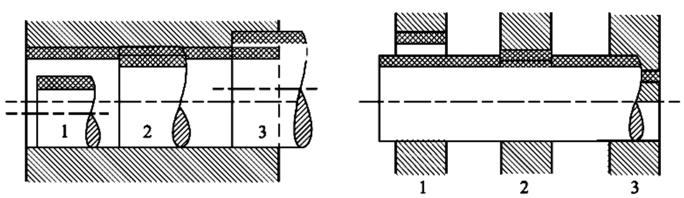
The flow chart for the general procedure in machine design is shown in Fig.

2. Factors influencing the machine design (or) Discuss in detail about the factor influencing machine design. (MAY/JUNE 2012) (May/June 2014)

Machine design is greatly influenced by the factors arising from the customer's requirement and factor concerned with the manufacture. Machine components design should meet the following important requirements.

1. Functional
2. Operational
3. Maintenance
4. Material used
5. Manufacturing methods.

(ii) Hole basis system:



1. Clearance fit. 2. Transition fit. 3. Interference fit.

fig 7 (a) Hole basis system. (b) Shaft basis system.

Bases of limit system.

The following are two bases of limit system:

1. **Hole basis system:** When the hole is kept as a constant member (i.e. when the lower deviationof the hole is zero) and different fits are obtained by varying the shaft size, as shown in Fig.7(a), then the limit system is said to be on a hole basis.

2. Shaft basis system: When the shaft is kept as a constant member (i.e. when the upper deviation of the shaft is zero) and different fits are obtained by varying the hole size, as shown in Fig.7then the limit system is said to be on a shaft basis.

3.Write short notes in preferred numbers fits and types of fits. (MAY/JUNE 2012)

PREFERRED NUMBERS(April/May 2017)

When a machine is to be made in several sizes with different powers or capacities, it is necessary to decide what capacities will cover a certain range efficiently with minimum number of sizes. It has been shown by experience that a certain range can be covered efficiently when it follows a geometrical progression with a constant ratio. The preferred numbers are the conventionally rounded off values derived from geometric series including the integral powers of 10 and having as common ratio of the following factors:

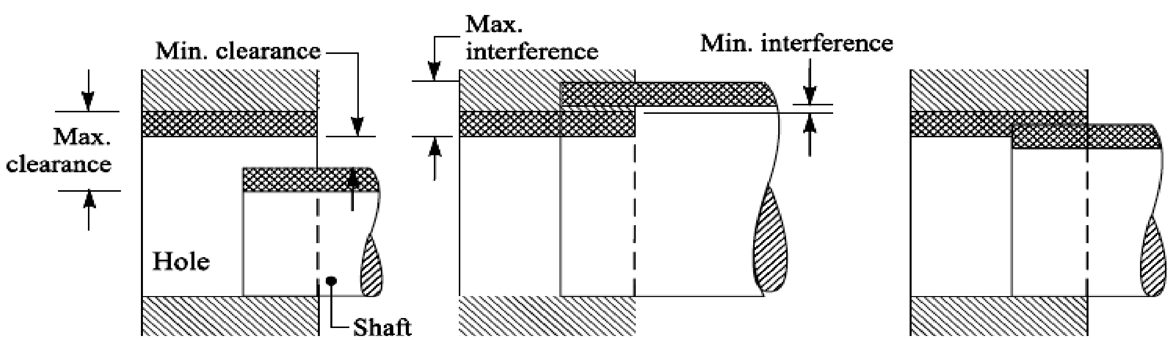
$$\sqrt[5]{10},\ \sqrt[10]{10},\ \sqrt[20]{10} \text{ and } \sqrt[40]{10}$$

These ratios are approximately equal to 1.58, 1.26, 1.12 and 1.06. The series of preferred numbers are designated as *R5, R10, R20 and R40 respectively. These four series are called ***basic series.*** The other series called ***derived series*** may be obtained by simply multiplying or dividing the basic sizes by 10, 100, etc. The preferred numbers in the series R5 are 1, 1.6, 2.5, 4.0 and 6.3. Table 3.12 shows basic series of preferred numbers according to IS : 1076 (Part I) – 1985 (Reaffirmed 1990).

FITS

The degree of tightness or looseness between the two mating parts is known as a *fit* of the parts. The nature of fit is characterised by the presence and size of clearance and interference.

The ***clearance*** is the amount by which the actual size of the shaft is less than the actual size of the mating hole in an assembly as shown in Fig.. 1.1 . In other words, the clearance is the difference between the sizes of the hole and the shaft before assembly. The difference must be ***positive.***



(*a*) Clearance fit. (*b*) Interference fit. (*c*) Transition fit.

fig1.1

The ***interference*** is the amount by which the actual size of a shaft is larger than the actual finished size of the mating hole in an assembly as shown in Fig. 1.1 (*b*). In other words, the interference is the arithmetical difference between the sizes of the hole and the shaft, before assembly. The difference must be ***negative.***

TYPES OF FITS

According to Indian standards, the fits are classified into the following three groups :

1. ***Clearance fit.*** In this type of fit, the size limits for mating parts are so selected that clearance between them always occur, as shown in Fig. 1.1 (*a*). It may be noted that in a clearance fit, the tolerance zone of the hole is entirely above the tolerance zone of the shaft.

In a clearance fit, the difference between the minimum size of the hole and the maximum size of the shaft is known as ***minimum clearance*** whereas the difference between the maximum size of the hole and minimum size of the shaft is called ***maximum clearance*** as shown in Fig. 1.1 (*a*).

The clearance fits may be slide fit, easy sliding fit, running fit, slack running fit and loose running fit.

2. ***Interference fit.*** In this type of fit, the size limits for the mating parts are so selected that interference between them always occur, as shown in Fig. 1.1 (*b*). It may be noted that in an interference fit, the tolerance zone of the hole is entirely below the tolerance zone of the shaft.

In an interference fit, the difference between the maximum size of the hole and the minimum size of the shaft is known as ***minimum interference,*** whereas the difference between the minimum size of the hole and the maximum size of the shaft is called ***maximum interference,*** as shown in Fig. 1.1(*b*).

The interference fits may be shrink fit, heavy drive fit and light drive fit.

3. ***Transition fit.*** In this type of fit, the size limits for the mating parts are so selected that either a clearance or interference may occur depending upon the actual size of the mating parts, as shown in Fig. 1.1 (*c*). It may be noted that in a transition fit, the tolerance zones of hole and shaft overlap.

The transition fits may be force fit, tight fit and push fit.

3. (a) A rod of length 100 mm and dia 20 mm is subjected to (i) pure tension (ii) pure bending (iii) combined bending and torsional load. Then draw the typical stress distribution on the critical section of each case. (Nov/Dec 2021)

Transmission shafts are subjected to axial tensile force, bending moment or torsional moment or their combinations. Most of the transmission shafts are subjected to combined bending and torsional moments. The design of transmission shaft consists of determining the correct shaft diameter from strength and rigidity considerations. When the shaft is subjected to axial tensile force, the tensile stress is given by,

$$\sigma_t = \frac{P}{\left(\frac{\pi d^2}{4}\right)}$$

$$\sigma_t = \frac{4P}{\pi d^2}$$

When the shaft is subjected to pure bending moment, the bending stresses are given by,

$$\sigma_b = \frac{M_b y}{I} = \frac{M_b\left(\frac{d}{2}\right)}{\left(\frac{\pi d^4}{64}\right)}$$

or, $$\sigma_b = \frac{32 M_b}{\pi d^3} \qquad (9.2)$$

When the shaft is subjected to pure torsional moment, the torsional shear stress is given by,

$$\tau = \frac{M_t r}{J} = \frac{M_t\left(\frac{d}{2}\right)}{\left(\frac{\pi d^4}{32}\right)}$$

or, $$\tau = \frac{16 M_t}{\pi d^3} \qquad (9.3)$$

When the shaft is subjected to combination of loads, the principal stress and principal shear stress are obtained by constructing Mohr's circle as shown in Fig. 1.1 The normal stress is denoted by σ_x while the shear stress, by τ. We will consider two cases for calculating the value of σ_x.

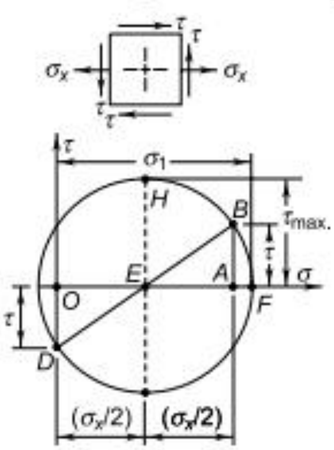


Fig. 1.1 *Mohr's Circle*

Case I In this case, the shaft is subjected to a combination of axial force, bending moment and torsional moment.

$$\sigma_x = \sigma_t + \sigma_b \qquad (9.4)$$

Case II In this case, the shaft is subjected to a combination of bending and torsional moments without any axial force.

$$\sigma_x = \sigma_b \tag{9.5}$$

The values of σ_t and σ_b in Eqs (9.4) and (9.5) are obtained from Eqs (9.1) and (9.2) respectively.

The Mohr's circle is constructed by the following steps:

(i) Select the origin O.

(ii) Plot the following points:
$\overline{OA} = \sigma_x \quad \overline{AB} = \tau \quad \overline{OD} = \tau$

(iii) Join $\overline{DB}$. The point of intersection of $\overline{DB}$ and $\overline{OA}$ is E.

(iv) Construct Mohr's circle with E as centre and $\overline{EB}$ as radius.

The principal stress σ_1 is given by,

$$\sigma_1 = \overline{OF} = \overline{OE} + \overline{EF} = \overline{OE} + \overline{EB}$$

or
$$\sigma_1 = \left(\frac{\sigma_x}{2}\right) + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \tag{9.6}$$

The principal shear stress τ_{max} is given by,

$$\tau_{max.} = \overline{EH} = \overline{EB}$$

or
$$\tau_{max.} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \tag{9.7}$$

Equations (9.1) to (9.7) are fundamental equations for design of shafts. However, every time, it is not necessary to use all these equations. For the design of shafts, simple expressions can be developed by combining the above equations. The shaft can be designed on the basis of maximum principal stress theory or maximum shear stress theory. We will apply these theories to transmission shaft subjected to combined bending and torsional moments.

The torsional shear stress and bending stresses in the shaft are given by,

$$\tau = \frac{M_t r}{J} \quad \sigma_b = \frac{M_b y}{I}$$

The distribution of bending stresses and torsional shear stress is shown in Fig. 1.2 It is observed that the torsional shear stress as well as bending stresses are zero at the shaft centre (r = 0 and y = 0) and negligibly small in the vicinity of the shaft centre, where the radius is small. As the radius increases, the resisting stresses due to external bending and torsional moments increase. Therefore, outer fi bres are more effective in resisting the applied moments.

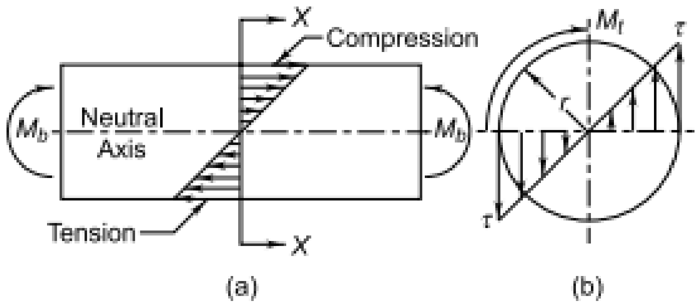


Fig. 1.2 *(a) Distribution of Bending Stresses*
(b) Distribution of Torsional Shear Stress

DIRECT, BENDING AND TORSIONAL

4.A cast iron pulley transmits 10 kW at 400 r.p.m. The diameter of the pulley is 1.2 Meters and it has four straight arms of elliptical cross-section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa. (Nov/Dec – 2011) (May/June 2011)(April/May 2008)

P = 10 kW = 10 × 103 W
N = 400 r.p.m
D = 1.2 m = 1200 mm or
R = 600 mm
σ_b = 15 MPa = 15 N/mm^2

To Find:

Dimensions of the arm a,b

Let T = Torque transmitted by the pulley.
We know that the power transmitted by the pulley (P),

$$10 \times 10^3 = \frac{2\pi NT}{60} = \frac{2\pi \times 400 \times T}{60} = 42\ T$$

$$T = \frac{10 \times 10^3}{42} = 238\ N-m = 238 \times 10^3 N-mm$$

Since the torque transmitted is the product of the tangential load and the radius of the pulley, therefore tangential load acting on the pulley

$$= \frac{T}{R} = \frac{238 \times 10^3}{600} = 396.7\ N$$

Since the pulley has four arms, therefore tangential load on each arm,

W = 396.7/4 = 99.2 N

and maximum bending moment on the arm,

$$M = W \times R = 99.2 \times 600 = 59\ 520 \text{ N-mm}$$

Let 2b = Minor axis in mm, and

2a = Major axis in mm = 2 × 2b = 4b ...(Given)

∴ Section modulus for an elliptical cross-section,

$$Z = \frac{\pi}{4} \times a^2 b = \frac{\pi}{4}(2b)^2 \times b = \pi b^3$$

We know that bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{59520}{\pi b^3} = \frac{18943}{b^3}$$

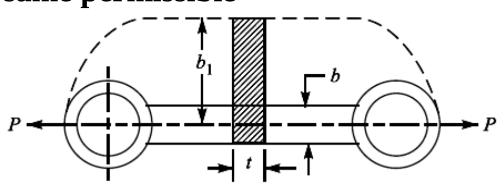
or b^3 = 18 943/15 = 1263 or b = 10.8 mm

∴ Minor axis, 2b = 2 × 10.8 = **21.6 mm Ans**.

And major axis, 2a = 2 × 2b = 4 × 10.8 = **43.2 mm Ans.**

5. A mild steel link, as shown in Fig. by full lines, transmits a pull of 80 kN. Find the dimensions b and t if b = 3t. Assume the permissible tensile stress as 70 MPa. If the original link is replaced by an unsymmetrical one, as shown by dotted lines in, having the same thickness t, find the depth b1, using the same permissible

Solution. Given : P = 80 kN = 80×10^3 N ; σ_t = 70 MPa = 70 N/mm^2



When the link is in the position shown by full lines in Fig. 5.24, the area of cross-section,

$$A = b \times t = 3\,t \times t = 3\,t^2 \qquad ...(\because\ b = 3\,t)$$

We know that tensile load (P),

$$80 \times 10^3 = \sigma_t \times A = 70 \times 3\,t^2 = 210\,t^2$$

$\therefore$ $t^2 = 80 \times 10^3 / 210 = 381$ or $t = 19.5$ say 20 mm **Ans.**

and $b = 3\,t = 3 \times 20 = 60$ mm **Ans.**

When the link is in the position shown by dotted lines, it will be subjected to direct stress as well as bending stress. We know that area of cross-section,

$$A_1 = b_1 \times t$$

$\therefore$ Direct tensile stress,

$$\sigma_o = \frac{P}{A} = \frac{P}{b_1 \times t}$$

and bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{P\,.\,e}{Z} = \frac{6\,P\,.\,e}{t\,(b_1)^2} \qquad ...\left(\because Z = \frac{t\,(b_1)^2}{6}\right)$$

$\therefore$ Total stress due to eccentric loading

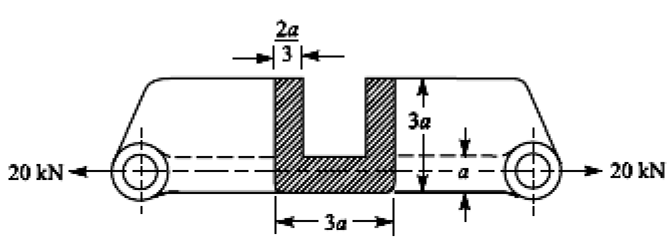
$$= \sigma_b + \sigma_o = \frac{6\,P\,.\,e}{t\,(b_1)^2} + \frac{P}{b_1 \times t} = \frac{P}{t\,.\,b_1}\left(\frac{6\,e}{b_1} + 1\right)$$

Since the permissible tensile stress is the same as 70 N/mm^2, therefore

$$70 = \frac{80 \times 10^3}{20\,b_1}\left(\frac{6 \times b_1}{b_1 \times 2} + 1\right) = \frac{16 \times 10^3}{b_1} \qquad ...\left(\because \text{Eccentricity, } e = \frac{b_1}{2}\right)$$

$\therefore$ $b_1 = 16 \times 10^3 / 70 = 228.6$ say 230 mm **Ans.**

6.A cast-iron link, as shown in Fig. 5.25, is to carry a load of 20 kN. If the tensile and compressive stresses in the link are not to exceed 25 MPa and 80 MPa respectively, obtain the dimensions of the cross-section of the link at the middle of its length. (Nov/Dec – 2013)



Given :

P = 20 kN = 20 × 103 N

$\sigma t_{(max)}$ = 25 MPa = 25 N/mm2

$\sigma c_{(max)}$ = 80 MPa= 80 N/mm2N/mm2

Since the link is subjected to eccentric loading, therefore therewill be direct tensile stress as well as bending stress. The bendingstress at the bottom of the link is tensile and in the upper portion iscompressive.

We know that cross-sectional area of the link,

$$A = 3a \times a + 2 \times \frac{2a}{3} \times 2a = 5.67\ a^2 mm^2$$

Now let us find the position of centre of gravity (or neutral axis) in order to find the bending stresses.

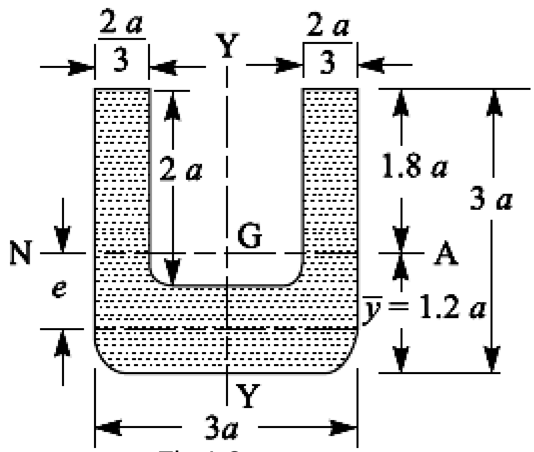


Fig 1.3

Let y = Distance of neutral axis (N.A.) from the bottom of the link as shownin Fig 1.3

$$.\ \bar{y} = \frac{3a^2 \times \frac{a}{2} + 2 \times 4\frac{a^2}{3} \times 2a}{5.67a^2} = 1.2\ a\ mm$$

Moment of inertia about N.A.,

$$I = \left[\frac{3a \times a^2}{12} + 3a^2(1.2a - 0.5\ a)^2\right] + 2\left[\frac{\frac{2}{3}a \times (2a)^2}{12} + \frac{4a^2}{3}(2a - 1.2\ a)^2\right]$$

$$= (0.25\ a^4 + 1.47\ a^4) + 2(0.44a^2 + 0.85\ a^4) = 4.3\ a^2 mm^4$$

Distance of N.A. from the bottom of the link,

$$y_t = y = 1.2\ a\ mm$$

Distance of N.A. from the top of the link,

$$y_c = 3\ a - 1.2\ a = 1.8\ a\ mm$$

Eccentricity of the load (i.e. distance of N.A. from the point of application of the load),

$$e = 1.2\ a - 0.5\ a = 0.7\ a\ mm$$

We know that bending moment exerted on the section,

$$M = P.e = 20 \times 10^3 \times 0.7\ a = 14 \times 10^3 a\ N\text{-}mm$$

∴Tensile stress in the bottom of the link,

$$\sigma_t = \frac{M}{Z_t} = \frac{M}{\frac{I}{y_t}} = \frac{My_t}{I} = \frac{14 \times 10^3 \times 1.2\ a}{4.3\ a^4} = \frac{3907}{a^2}$$

and compressive stress in the top of the link,

$$\sigma_c = \frac{M}{Z_c} = \frac{M}{\frac{I}{y_c}} = \frac{My_c}{I} = \frac{14 \times 10^3 \times 1.8\, a}{4.3\, a^4} = \frac{5860}{a^2}$$

We know that maximum tensile stress [σ_t(max)],

$$25 = \sigma_t + \sigma_c = \frac{3907}{a^2} + \frac{5860}{a^2} = \frac{9767}{a^2}$$

$$a^2 = \frac{9767}{25} = 390.7 \text{ or } a = 19.76 \text{ mm} \text{ -------------- } 1$$

and maximum compressive stress [σ_c(max)],

$$80 = \sigma_c - \sigma_o = \frac{5860}{a^2} - \frac{3530}{a^2} = \frac{2330}{a^2}$$

$\therefore a^2 = 2330 / 80 = 29.12$ or $a = 5.4$ mm ---------------------- 2

We shall take the larger of the two values, i.e.

a = 19.76 mm Ans.

7. A hollow shaft is required to transmit 600 KW at 110 r.p.m., the maximum torque being 20% greater than the mean. The shear stress is not to exceed 63 MPa and twist in a length of 3 meters not to exceed 1.4 degrees. Find the external diameter of the shaft, if the internal diameter to the external diameter is 3/8. Take modulus of rigidity as 84 GPa. [Aprl/May-2019]

Solution:

Given : P = 600 kW = 600 × 103 W ; N = 110 r.p.m. ; Tmax = 1.2 Tmean ; τ = 63 MPa = 63 N/mm2 ; l = 3 m = 3000 mm ; θ = 1.4 × π / 180 = 0.024 rad ; k = di / do = 3/8 ; C = 84 GPa = 84 × 109 N/m2 = 84 × 103 N/mm2

Let Tmean = Mean torque transmitted by the shaft,

do = External diameter of the shaft, and

di = Internal diameter of the shaft.

We know that power transmitted by the shaft (P),

$$600 \times 10^3 = \frac{2\,\pi\,N.T_{mean}}{60} = \frac{2\pi \times 110 \times T_{mean}}{60} = 11.52\,T_{mean}$$

$$\therefore \quad T_{mean} = 600 \times 10^3/11.52 = 52 \times 10^3 \text{ N-m} = 52 \times 10^6 \text{ N-mm}$$

and maximum torque transmitted by the shaft,

$$T_{max} = 1.2\,T_{mean} = 1.2 \times 52 \times 10^6 = 62.4 \times 10^6 \text{ N-mm}$$

Now let us find the diameter of the shaft considering strength and stiffness.

1. *Considering strength of the shaft*

We know that maximum torque transmitted by the shaft,

$$T_{max} = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1-k^4)$$

$$62.4 \times 10^6 = \frac{\pi}{16} \times 63 \times (d_o)^3 \left[1-\left(\frac{3}{8}\right)^4\right] = 12.12\,(d_o)^3$$

$$\therefore \quad (d_o)^3 = 62.4 \times 10^6/12.12 = 5.15 \times 10^6 \text{ or } d_o = 172.7 \text{ mm} \qquad ...(i)$$

2. *Considering stiffness of the shaft*

We know that polar moment of inertia of a hollow circular section,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{32}(d_o)^4\left[1-\left(\frac{d_i}{d_o}\right)^4\right]$$

$$= \frac{\pi}{32}(d_o)^4\,(1-k^4) = \frac{\pi}{32}(d_o)^4\left[1-\left(\frac{3}{8}\right)^4\right] = 0.0962\,(d_o)^4$$

We also know that

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{62.4 \times 10^6}{0.0962\,(d_o)^4} = \frac{84 \times 10^3 \times 0.024}{3000} \quad \text{or} \quad \frac{648.6 \times 10^6}{(d_o)^4} = 0.672$$

$$\therefore \quad (d_o)^4 = 648.6 \times 10^6/0.672 = 964 \times 10^6 \text{ or } d_o = 176.2 \text{ mm} \qquad ...(ii)$$

Taking larger of the two values, we shall provide

$$d_o = 176.2 \text{ say } 180 \text{ mm Ans.}$$

8.The stresses induced at a critical point in a machine components made of C45 steel are as follows :σx = 120 N/mm2, σy= 50 N/mm2. Calculate the factor of safety by a. maximum normal stress theory b. maximum shear stress theory c. distortion energy theory. (Nov/Dec -2009)

Given :

$\sigma_x = 120$ N/mm^2, σ_y= 50 N/mm^2

Material C45steel

To Find:

Factor of safety (FOS)

Solution:

From PSG Data book, ref page no 1.9 for C45 steel

$$\text{yield stress } \sigma_y = 36 \text{ kgf/mm}^2$$

$$\sigma_y = 360 \text{ N/mm}^2 \tau_{xy} = 0$$

Maximum principal stress = $\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{(\sigma_x-\sigma_y)}{2}\right)^2 + \tau_{xy}{}^2}$

$$= \frac{120+50}{2} + \sqrt{\left(\frac{120-50}{2}\right)^2 + 0} = 85 + 35 = 120\ \mathrm{N/mm^2}$$

Minimum principal stress = $\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{(\sigma_x-\sigma_y)}{2}\right)^2 + \tau_{xy}^2}$

$$= \frac{120+50}{2} - \sqrt{\left(\frac{120-50}{2}\right)^2 + 0} = 85 - 35 = 50\ \mathrm{N/mm^2}$$

Let ‘n’ is the factor of safety

Refer PSG data book, Page No 7.3 – Failure theories

a. <u>Maximum normal stress or Maximum stress theory</u>

$$\sigma_1 \text{ or } \sigma_2 \text{or } \sigma_3 (\text{whichever is maximum}) = \sigma_y$$

For design

$$\sigma_1 \text{ or } \sigma_2 \text{or } \sigma_3 (\text{whichever is maximum}) = \frac{\sigma_y}{n}$$

Since

$$\sigma_1 \text{ is maximum}$$

$$\sigma_1 = \frac{\sigma_y}{2}$$

$$120 = \frac{360}{2}$$

Factor of safety = n = 360/120 = 3

b. <u>Maximum shear stress theory</u>

$$(\sigma_1 - \sigma_2) \text{ or } (\sigma_2 - \sigma_3) \text{ or } (\sigma_3 - \sigma_1) \text{ whichever is maximum}$$

For design

$$(\sigma_1 - \sigma_2) \text{ or } (\sigma_2 - \sigma_3) \text{ or } (\sigma_3 - \sigma_1) \text{ whichever is maximum} = \frac{\sigma_y}{n}$$

Since

$$\sigma_1 \text{ and } \sigma_2 \text{ are positive}$$

$$(\sigma_1 - \sigma_3) = \frac{\sigma_y}{n}$$

$$\sigma_1 = \frac{\sigma_y}{n} \qquad 120 = \frac{360}{n}$$

Factor of safety = n = 360/120 = 3

c. <u>Distortion energy theory</u>

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1^2\sigma_3^2 - \sigma_2^2\sigma_3^2 = \sigma_y^2$$

For design$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1 = \left(\frac{\sigma_y}{n}\right)^2$

$$120^2 + 50^2 - (120 \times 50) = \left(\frac{360}{n}\right)^2$$

$$10900 = \left(\frac{360}{n}\right)^2$$

$$\frac{360}{n} = 10900^{1/2} = 104.403$$

n=Factor of safety=360/104.403=3.448

Therefore select factor of safety as 3.448, according to Distortion energy theory.

Factor of safety = n = 3.448 Ans

9.The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN Find the diameter of bolt required according to 1. Maximum principal stress theory; 2.Maximum shear stress theory; 3.Maximum principal strain theory; 4.Maximum strain energy theory; and 5. Maximum distortion energy theory.Take permissible tensile stress at elastic limit =100MPa and poisson's ratio=0.3. (Nov/Dec 15)

Given:

P_{t1}=10kN; +

P_s=5kN; $\sigma_{t(el)}$=100MPa=100 N/mm^2;

1/m=0.3

To Find:

Diameter of bolt, d

Solution:

Let d=diameter of the bolt in mm.

Cross-sectional area of the bolt,

$$A=\frac{\pi}{4}\text{xd}^2 = 0.7854\text{xd}^2 mm^2$$

We know that axial tensile stress,

$$\sigma_1 = \frac{P_{t1}}{A} = \frac{10}{0.7854d^2} = \frac{12.73}{d^2} kN/mm^2$$

Transverse shear stress,

$$\tau=\frac{P_s}{A}=\frac{5}{0.7854d^2}=\frac{6.365}{d^2}\text{kN/mm}^2$$

1. <u>**According to maximum principal stress theory**</u>

We know that maximum principal stress,

$$\sigma_{t1}=\frac{\sigma_1+\sigma_2}{2}+\frac{1}{2}\left[\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}\right](\because \sigma_2=0)$$

$$=\frac{\sigma_1}{2}+\frac{1}{2}\left[\sqrt{(\sigma_1)^2+4\tau^2}\right]=\frac{12.73}{2d^2}+\frac{1}{2}\left[\sqrt{\left(\frac{12.73}{2d^2}\right)^2+4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$=\frac{6.365}{d^2}+\frac{1}{2}\text{x}\frac{6.365}{d^2}\left[\sqrt{4+4}\right]=\frac{6.365}{d^2}\left[1+\frac{1}{2}\sqrt{4+4}\right]=\frac{15.365}{d^2}\text{kN/mm}^2=\frac{15365}{d^2}\text{N/mm}^2$$

According to maximum principal stress theory,

$$\sigma_{t1}=\sigma_{t(el)};\frac{15365}{d^2}=100$$

$$d^2=\frac{15365}{100}=153.65$$

$$d=12.4\text{mm}$$

2. <u>According to maximum shear stress theory</u>

We know that maximum shear stress,

$$\tau_{\max}=\frac{1}{2}\left[\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}\right](\because\sigma_2=0)$$

$$=\frac{1}{2}\left[\sqrt{(\sigma_1)^2+4\tau^2}\right]=\frac{1}{2}\left[\sqrt{\left(\frac{12.73}{2d^2}\right)^2+4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$=\frac{1}{2}\text{x}\frac{6.365}{d^2}\left[\sqrt{4+4}\right]=\frac{9}{d^2}\text{kN/mm}^2=\frac{9000}{d^2}\text{N/mm}^2$$

According to maximum shear stress theory,

$$\tau_{\max}=\frac{\sigma_{t(el)}}{2}$$

$$\frac{9000}{d^2}=\frac{100}{2}=50$$

$$d^2=\frac{9000}{50}=180$$

$$d=13.42mm$$

3. <u>According to maximum principal strain theory</u>

We know that the maximum principal stress,

$$\sigma_{t1}=\frac{\sigma_1}{2}+\frac{1}{2}\left[\sqrt{(\sigma_1)^2+4\tau^2}\right]=\frac{15365}{d^2}$$

Minimum principal stress,

$$\sigma_{t2}=\frac{\sigma_1}{2}-\frac{1}{2}\left[\sqrt{(\sigma_1)^2+4\tau^2}\right]$$

$$=\frac{12.73}{2d^2}-\frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2+4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$=\frac{6.365}{d^2}-\frac{1}{2}\text{x}\frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$=\frac{6.365}{d^2}\left[1-\sqrt{2}\right]=-\frac{2.635}{d^2}kN/mm^2=-\frac{2635}{d^2}N/mm^2$$

We know that according to maximum principal strain theory,

$$\frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{mE} = \frac{\sigma_{t(el)}}{E}$$

$$\sigma_{t1} - \frac{\sigma_{t1}}{m} = \sigma_{t(el)}$$

$$\frac{15365}{d^2} + \frac{2635x03}{d^2} = 100$$

$$d^2 = 16156/100 = 161.56$$

$$d = 12.7mm.$$

4. **<u>According to maximum strain energy theory</u>**

We know that according to maximum strain energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\sigma_{t1}x\sigma_{t2}}{m} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2x\frac{15365}{d^2}x\frac{-2635}{d^2}x0.3 = (100)^2$$

d= 12.78mm

5. **<u>According to maximum distortion energy theory</u>**

According to maximum distortion energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1}x\sigma_{t2} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2x\frac{15365}{d^2}x\frac{-2635}{d^2} = (100)^2$$

d=13.4mm

10.A bolt is subjected to a direct load of25 KN and shear load of 15 KN. Considering following theories of failure, determine a suitable size of the bolt if the material of the bolt is C15 having 200 N/mm2yield strength. Assume F.O.S. as 2 and also give your comments.

i) Maximum normal stress theory

ii) Maximum shear stress theory

iii) Von misses theory.(Nov/Dec 2017)

Solution $F = 25$ kN, $F_s = 15$ kN, and $\sigma_y = 200$ N/mm^2. Although the material properties may be well-known, there may be some change in the magnitude of the load. Thus to account for uncertainties we take the factor of safety, $n = 2.0$. Therefore,

$$\text{Allowable or design stress, } \sigma_d = \frac{\sigma_y}{n} = \frac{200}{2} = 100 \text{ N/mm}^2$$

Tensile stress due to direct load, if A is the resisting area of cross-section

$$\sigma_t = \frac{25}{A} \text{ kN/mm}^2$$

$$\text{Shear stress, } \tau = \frac{15}{A} \text{ kN/mm}^2$$

Since it is a case of biaxial stress condition, the principal stresses are:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

In this problem $\sigma_x = \sigma_t$ and $\sigma_y = 0$. Thus,

Since it is a case of biaxial stress condition, the principal stresses are:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

In this problem $\sigma_x = \sigma_t$ and $\sigma_y = 0$. Thus,

$$\sigma_{1,2} = \frac{\sigma_t}{2} \pm \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$$

$$= \frac{25}{2A} \pm \sqrt{\left(\frac{25}{2A}\right)^2 + \left(\frac{15}{A}\right)^2}$$

or

$$\text{Maximum principal stress, } \sigma_1 = \frac{32.02}{A} \text{ kN/mm}^2$$

$$\text{Minimum principal stress, } \sigma_2 = -\frac{7.02}{A} \text{ kN/mm}^2$$

(i) *Maximum normal stress theory.* According to the maximum normal stress theory, the maximum principal stress should be less than or equal to the design stress, i.e.

$$\sigma_1 \le \sigma_d$$

$$\left(\frac{32.02}{A} + \frac{7.02}{A}\right) \times 10^3 \leq 100$$

or

$$A = 390.4 \text{ mm}^2$$

Therefore, the root diameter of the bolt,

$$d_c = 22.3 \text{ mm}$$

(iii) *von-Mises theory.* According to the von-Mises criterion of failure, the equation of failure is

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \leq \left(\frac{\sigma_y}{n}\right)^2$$

or

$$10^6 \times \left[\left(\frac{32.02}{A}\right)^2 + \frac{32.02}{A} \times \frac{7.02}{A} + \left(\frac{7.02}{A}\right)^2\right] \leq 100^2$$

or

$$A = 360.4 \text{ mm}^2$$

Therefore,

$$\text{root diameter, } d_c = 21.4 \text{ mm}$$

A comparison of these three theories of failure reveals that maximum shear stress theory is more conservative which has predicted 22.3 mm root diameter whereas von-Mises has predicted 21.4 mm diameter.

or

$$\frac{32.02}{A} \times 1000 \leq 100$$

or

$$A = 320.2 \text{ mm}^2$$

if d_c is the root diameter of the bolt, then

$$\frac{\pi}{4} d_c^2 = 320.2 \qquad \text{or} \qquad d_c = 20.2 \text{ mm}$$

(ii) *Maximum shear stress theory.* According to the maximum shear stress theory, the failure criterion is

$$(\sigma_1 - \sigma_2) \leq \frac{\sigma_y}{n}$$

or

$$\left(\frac{32.02}{A} + \frac{7.02}{A}\right) \times 10^3 \leq 100$$

or

$$A = 390.4 \text{ mm}^2$$

Therefore, the root diameter of the bolt,

$$d_c = 22.3 \text{ mm}$$

11. A mass of 50 kg drops through 25 mm at the centre of a 250 mm long simply supported beam. The beam has a square cross section. It is made of steel 30C8 (Syt= 400 N/mm 2) and

the factor of safety is 2. The modulus of elasticity is 207000 N/mm2. Determine the dimension ofthe cross section of the beam.(Nov/Dec 2017)

Given:

m = 50 kg, h = 25 mm, l = 250 mm, S_{yt}= 400 N/mm^2 (f_s) = 2, E = 207 000 N/mm^2

Step I: Impact stress (σ_i)

From Eq. $$\sigma_i = \frac{W}{A}\left[1+\sqrt{1+\frac{2\,h\,A\,E}{W\,l}}\right]$$

In above equation, $\frac{W}{A}$ = static stress = σ_s **...(a)**

$$\frac{W\,l}{A\,E} = \text{static deflection} = \delta_s \quad \textbf{....... (b)}$$

Substituting (a) and (b) in Eq.

$$\sigma_i = \sigma_s\left[1+\sqrt{1+\frac{2\,h}{\delta_s}}\right] \quad \textbf{......... (c)}$$

Step II: Static stress (σ_s)

For simply supported beam, $W = m\,g = 50(9.81)$
= 490.5 N

$$M_b = \frac{W\,l}{4} = \frac{490.5(250)}{4} = 30\,656.25 \text{ N-mm}$$

$$I = \frac{b\,d^3}{12} = \frac{a\,(a)^3}{12} = \frac{a^4}{12}\text{ mm}^4 \qquad y = \frac{a}{2}$$

where a is the side of square cross-section. Therefore,

$$\sigma_s = \sigma_b = \frac{M_b\,y}{I} = \frac{(30\,656.25)\left(\frac{a}{2}\right)}{\left(\frac{a^4}{12}\right)}$$

$$= \frac{183\,973.5}{a^3}\text{ N/mm}^2 \qquad \text{(d)}$$

Step III: Static deflection

$$\delta_s = \frac{W\,l^3}{48\,E\,I} = \frac{(490.5)(250)^3(12)}{48(207\,000)\,a^4} = \frac{9256.11}{a^4}\text{ mm} \quad \text{(e)}$$

Step IV: Cross-section of beam

Equating impact stress to permissible stress,

$$\sigma_i = \frac{S_{yt}}{(fs)} = \frac{400}{2} = 200 \text{ N/mm}^2 \qquad \text{(f)}$$

Substituting (d), (e) and (f) in Eq. (c),

$$200 = \frac{183\,973.5}{a^3}\left[1 + \sqrt{1 + \frac{2(25)a^4}{9256.11}}\right]$$

$$\text{or} \quad \frac{a^3}{919.87} = \left[1 + \sqrt{1 + \frac{2(25)a^4}{9256.11}}\right]$$

$$\left(\frac{a^3}{919.87} - 1\right)^2 = 1 + \frac{a^4}{185.12}$$

$$\text{Simplifying,} \quad \frac{a^3}{846\,160.82} - \frac{1}{459.94} = \frac{a}{185.12}$$

The term (1/459.94) is very small and neglected. Therefore,

$$\frac{a^3}{846\,160.82} = \frac{a}{185.12}$$

$$a^2 = \frac{846160.82}{185.12} = 4570.88\,; \quad a = 67.6 \text{ or } 70 \text{ mm.}$$

The cross-section of the beam is 70 × 70 mm.

Step V: Check for impact stresses

$$\sigma_s = \frac{183\,973.5}{a^3} = \frac{183\,973.5}{(70)^3} = 0.5363 \text{ N/mm}^2$$

$$\delta_s = \frac{9256.11}{a^4} = \frac{9256.11}{(70)^4} = 3.855 \times 10^{-4} \text{ mm}$$

$$\sigma_i = \sigma_s\left[1 + \sqrt{1 + \frac{2h}{\delta_s}}\right]$$

$$= 0.5363\left[1 + \sqrt{1 + \frac{2\,(25)}{(3.855 \times 10^{-4})}}\right]$$

$$= 193.68 \text{ N/mm}^2$$

$$\sigma_i < 200 \text{ N/mm}^2$$

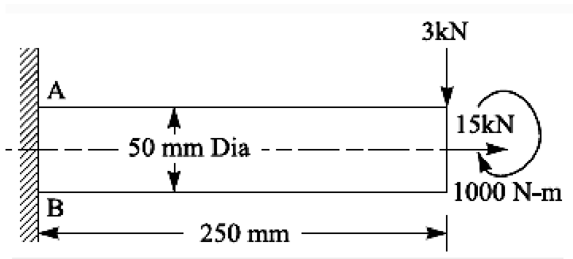
PRINCIPAL STRESS AND PRINCIPAL STRAIN

12. A shaft, as shown in Fig., is subjected to a bending load of 3 KN, pure torque of 1000 N-m and an axial pulling force of 15 kN. Calculate the stresses at A and B.

(or)

A hypothetical machine member by 50 mm in diameter and 250 mm long is supported in one end as cantilever is subjected to various types of loadings, as given below. Find the principal stresses and maximum shear stress in each case.

i. **Axial load 15 KN.**
ii. **Transverse load 3 KN at the free end.**
iii. **Twisting moment of 1 KN at the free end, clockwise while viewing form free end side.**
iv. **(i) and (ii) together.**
v. **(i) (ii) and (iii) together.** **(M/J 16) (Nov/Dec 2018)**



Given:

W=3kN=3000N;

T=1000 N-m=$1\text{x}10^6$ N-mm

P=15kN=$15\text{x}10^3$N

d=50mm

x=250mm

To Find:

Stresses at A,B

We know that cross-sectional area of the shaft,

$$A=\frac{\pi}{4}\text{x}d^2=\frac{\pi}{4}\text{x}(50)^2=1964\text{mm}^2$$

Tensile stress due to axial pulling at point A & B,

$$\sigma_o=\frac{P}{A}=\frac{15\text{x}10^3}{1964}=7.64\text{N/mm}^2=7.64\text{MPa}$$

Bending moment at point A & B,

$$M=W.x=3000\text{x}250=750\text{x}10^3\text{N-mm}$$

Section modulus for the shaft,

$$Z=\frac{\pi}{32}\text{x}d^3=\frac{\pi}{32}\text{x}(50)^3=12.27\text{x}10^3\text{mm}^3$$

Bending stress at point A & B,

$$\sigma_b=\frac{M}{Z}=\frac{750\text{x}10^3}{12.27\text{x}10^3}=61.1\text{N/mm}^2=61.1\text{MPa}$$

This bending stress is tensile at point A & compressive at B.

Resultant tensile stress at point A,

$$\sigma_A = \sigma_b + \sigma_o = 61.1 + 7.64 = 68.74\text{MPa}$$

Resultant compressive stress at point B,

$$\sigma_B = \sigma_b \text{-} \sigma_o = 61.1 - 7.64 = 53.46\text{MPa}$$

We know that shear stress at point A and B due to the torque transmitted,

$$\tau = \frac{16\text{T}}{\pi \text{d}^3} = \frac{16\text{x}1\text{x}10^6}{\pi(50)^3} = 40.74\text{N/mm}^2 = 40.74\text{MPa}$$

Stresses at point A

We know that maximum principal stress at point A,

$$\sigma_{A(\max)} = \frac{\sigma_A}{2} + \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\tau^2}\right] = \frac{68.74}{2} + \frac{1}{2}\left[\sqrt{(68.74)^2 + 4(40.74)^2}\right] = 34.37 + 53.3 = 87.67MPa(tensile)$$

Minimum principal stress at point A,

$$\sigma_{A(\min)} = \frac{\sigma_A}{2} - \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\tau^2}\right] = \frac{68.74}{2} - \frac{1}{2}\left[\sqrt{(68.74)^2 + 4(40.74)^2}\right] = 34.37 - 53.3 = -18.93MPa(compressive)$$

Maximum shear stress at point A,

$$\tau_{A(\max)} = \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(68.74)^2 + 4(40.74)^2}\right] = 53.3MPa.$$

Stresses at point B

We know that the maximum principal stress at point B,

$$\sigma_{\text{B(max)}} = \frac{\sigma_B}{2} + \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\tau^2}\right] = \frac{53.46}{2} + \frac{1}{2}\left[\sqrt{(53.46)^2 + 4(40.74)^2}\right] = 26.73 + 48.73 = 75.46MPa(compressive)$$

Minimum principal stress at point B,

$$\sigma_{\text{B(min)}} = \frac{\sigma_B}{2} - \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\tau^2}\right] = \frac{53.46}{2} - \frac{1}{2}\left[\sqrt{(53.46)^2 + 4(40.74)^2}\right] = 26.73 - 48.73 = -22MPa(\text{tensile})$$

Maximum shear stress at point B,

$$\tau_{\text{B(max)}} = \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(53.46)^2 + 4(40.74)^2}\right] = 48.73MPa.$$

13. A cantilever beam of rectangular cross-section is used to support a pulley as shown in fig. (a). the tension in the wire rope is 10 KN. The beam is made of cast iron FG 240 and the factor of safety is 3. The ratio of depth to width of the cross-section is 2. Determine the dimensions of the cross-section of the beam. Nov/Dec-2020,Aprl/May-2021

Solution:

Given: P = 5 KN: S_{ut} = 200 N/mm^2 : (fs) = 2.5: d/w = 2

Step 1:

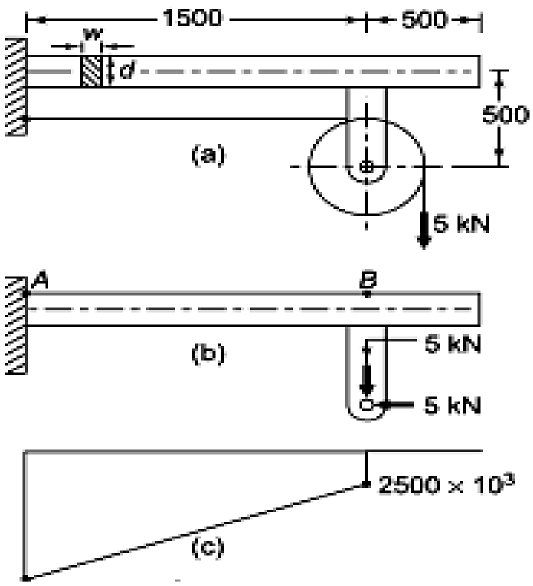
Calculation of permissible bending stress

$$\sigma_b = \frac{S_{ut}}{(fs)} = \frac{200}{2.5} = 80N/mm^2$$

Step 2:

Calculation of bending moments The force acting on the beam are shown in Fig. Referring to the figure,

$$(M_b)_{\text{at B}} = 5000 \times 500 = 2500 \times 10^3 \text{ N-mm}$$
$$(M_b)_{\text{at A}} = 5000 \times 500 + 5000 \times 1500$$
$$= 10000 \times 10^3 \text{ N-mm}$$



Step 3:

Calculation of dimensions of cross-section.

The bending moment diagram is shown in Fig. the cross-section at A is subjected to maximum bending stress. For this cross section,

$$y = \frac{d}{2} = w$$
$$I = \frac{1}{12}[(w)(2w)^3] = \frac{2}{3}w^4 mm^4$$
$$\sigma_b = \frac{M_b y}{I}(or)80 = \frac{(10000*10^3)(w)}{(\frac{2}{3}w^4)}$$

Therefore,

W= 57.24 mm or 60mm d= 2w= 120mm.

14.A simply supported beam has a concentrated load at the centre which fluctuates from a value of P to 4 P. The span of the beam is 500 mm and its cross-section is circular with a diameter of 60 mm. taking for the beam material an ultimate stress of 700 MPa, a yield stress of 500 MPa, endurance limit of 330 MPa for reversed bending, and a factor of safety of 1.3, calculate the maximum value of P. Take a size factor of 0.85 and a surface finish factor of 0.9.(Nov/Dec – 2010)

Given :

Wmin = P, Wmax = 4P

L = 500 mm

d = 60 mm

u = 700 MPa = 700 N/mm2 ;

σy = 500 MPa = 500 N/mm2 ; σe = 330 MPa = 330 N/mm2

F.S. = 1.3

Ksz = 0.85 ;Ksur = 0.9

To Find;

Maximum value of P

Solution:

We know that maximum bending moment,

$$\text{Mmax } = \frac{W_{max} \times L}{4} = \frac{4P \times 500}{4} = 500P\ N - mm$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times L}{4} = \frac{P \times 500}{4} = 125P\ N - mm$$

Mean or average bending moment,

$$\text{Mm } = \frac{M_{max} + M_{min}}{2} = \frac{500P \times 125P}{2} = 312.5\ P\ N - mm$$

and variable bending moment,

$$\text{Mv } = \frac{M_{max} - M_{min}}{2} = \frac{500P - 125P}{2} = 187.5\ P\ N - mm$$

Section modulus,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} 60^3 = 21.21 \times 10^3 mm^2$$

∴ Mean bending stress

$$\sigma_m = \frac{M_z}{Z} = \frac{312.5P}{21.21 \times 10^3} = 0.0147\ PN/mm^2$$

and variable bending stress,

$$\sigma_v = \frac{M_m}{Z} = \frac{187.5P}{21.21 \times 10^3} = 0.0088\ P \qquad \text{N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{FOS} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147\ P}{700} + \frac{0.0088\ P \times 1}{330 \times 0.9 \times 0.85}$$

$$\frac{1}{FOS} = \frac{21P}{10^6} + \frac{34.8\ P}{10^6} = \frac{55.8P}{10^6}$$

$$P = \frac{1}{1.3} \times \frac{10^6}{55.8} = 13785\ N = 13.785\ kN$$

and according to Soderberg's formula,

$$\frac{1}{FOS} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147\ P}{500} + \frac{0.0088\ P \times 1}{330 \times 0.9 \times 0.85}$$

$$\frac{1}{FOS} = \frac{29.4P}{10^6} + \frac{34.8\ P}{10^6} = \frac{64.2P}{10^6}$$

$$P = \frac{1}{1.3} \times \frac{10^6}{64.2} = 11982\ N = 11.982\ kN$$

Maximum value of P = 13.785 kN Ans.

15. A hot rolled steel shaft is subjected to a torsion moment that varies from 330 N-m clockwise to 110 N-m counterclockwise and an applied bending moment at a critical section varies from 440 N-m to – 220 N-m. The shaft is of uniform cross-section and no keyway is present at the critical section. Determine the required shaft diameter. The material has an ultimate strength of 550 MN/m2 and yield strength of 410 MN/m2. Take the endurance limit as half the ultimate strength, factor of safety of 2, size factor of 0.85 and a surface finish factor of 0.62. (Nov/Dec – 2013) (April/May'2023)

Given :

Tmax= 330 N-m (clockwise)
Tmin= 110 N-m (counterclockwise) = – 110 N-m(clockwise) ;
Mmax = 440 N-m ; Mmin = – 220 N-m
σu= 550 MN/m2 = 550 × 106 N/m2 ;
σy= 410 MN/m2 = 410 × 106 N/m2

σe= 1 2 σu = 275 × 106 N/m2 ; F.S. = 2 ; Ksz = 0.85 ; Ksur = 0.62

To Find:

Shaft diameter, d

Solution:

Let d = Required shaft diameter in metres.

We know that mean torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{330 + (-110)}{2} = 110\ N - m$$

And variable torque

$$T_v = \frac{T_{max} - T_{min}}{2} = \frac{330 - (-110)}{2} = 220\ N - m$$

Mean shear stress

$$\tau_m = \frac{16T_m}{\pi d^3} = \frac{16 \times 110}{\pi d^3} = \frac{560}{d^3}\ N/m^2$$

And variable shear stress,

$$\tau_v = \frac{16T_v}{\pi d^3} = \frac{16 \times 110}{\pi d^3} = \frac{1120}{d^3}\ N/m^2$$

Since the endurance limit in shear (τe) is 0.55 σe, and yield strength in shear (τy) is 0.5 σy, therefore

$$\tau_e = 0.55 \times 275 \times 10^6 = 151.25 \times 10^6\ N/m^2$$

$$\tau_y = 0.5 \times 410 \times 10^6 = 205 \times 10^6\ N/m^2$$

We know that equivalent shear stress,

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

$$= \frac{560}{d^3} + \frac{1120 \times 205 \times 10^6 \times 1}{d^3 \times 151.25 \times 10^6 \times 0.62 \times 0.85}$$

$$= \frac{560}{d^3} + \frac{2880}{d^3} = \frac{3440}{d^3}\ N/m^2$$

Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{440 + (-220)}{2} = 110\ N - m$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{440 - (-220)}{2} = 330\ N - m$$

Section modulus,

$$Z = \frac{\pi}{32} d^3 = 0.0982\ d^3 m^3$$

Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{110}{0.0982 d^3} = \frac{1120}{d^3}\ N/m^2$$

And variable bending stress

$$\sigma_v = \frac{M_v}{Z} = \frac{330}{0.0982d^3} = \frac{3360}{d^3}\ N/m^2$$

Since there is no reversed axial loading, therefore the equivalent normal stress due to reversed bending load,

$$\sigma_{nep} = \sigma_{ne} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{ep} \times K_{sur} \times K_{sz}}$$

$$= \frac{1120}{d^3} + \frac{9506}{d^3} = \frac{10626}{d^3}\ N/m^2$$

We know that the maximum equivalent shear stress,

$$\tau_{es(max)} = \frac{\tau_y}{FOS} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4(\tau_{es})^2}$$

$$\frac{205 \times 10^6}{2} = \frac{1}{2}\sqrt{\left(\frac{10625}{d^3}\right)^2 + 4\left(\frac{3440}{d^3}\right)^2}$$

$$205 \times 10^6 \times d^3 = \sqrt{113 \times 10^6 + 4 \times 11.84 \times 10^6} = 12.66 \times 10^3$$

$$d^3 = \frac{12.66 \times 10^3}{205 \times 10^6} = \frac{0.0617}{10^3}$$

$$\boldsymbol{d = \frac{0.935}{10} = 0.0.95m = 39.5\ say\ 40\ mm}$$

15.(a) A rod is subjected to axial tensile load of 20 KN and torsional load of10 KN. determine the diameter of the rod according to (i) Rankins theory,(2) St.venants theory, (3) Teresa theory. Take FOS is 2.5 poissions ratio 0.25, Sy=300 N/mm2.(Nov/Dec2021)

Given:

Tensile Load P = 20 kN = 20x10^3 N

Shear Load F = 10 kN = 10x10^3 N

FOS = 2.5

Yield stress σ_y = 300 N/mm^2

Poisson ratio v = 0.25

To find: Diameter of rod

Solution:

$$Allowable\,Stress = \frac{\sigma_y}{FOS} = \frac{300}{0.25} = 120 N/mm^2$$

$$Tensile\,Stress = \sigma_x = \frac{P}{A} = \frac{20\times10^3}{A} N/mm^2$$

$$Shear\,Stress = \tau_{xy} = \frac{F}{A} = \frac{10\times10^3}{A} N/mm^2$$

(i) *According to Rankine's theory*

$$\sigma_{all} = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$120 = \frac{1}{2}\left[\frac{20\times10^3}{A} + \sqrt{\left(\frac{20\times10^3}{A}\right)^2 + 4\left(\frac{10\times10^3}{A}\right)^2}\right]$$

$$120 = \frac{1}{2}\left[\frac{2\times10^4}{A} + \frac{6\times10^4}{A}\right]$$

$$120 = \frac{8\times10^4}{A}$$

$$A = \frac{8\times10^4}{120}$$

$$A = 666.66$$

$$\frac{\pi d^2}{4} = 666.66$$

$$d^2 = 848.83$$

$$d = 29.13\,mm$$

(2) According to St.Venan’s Theory:

$$\sigma_1 - \upsilon\sigma_2 = \frac{\sigma_y}{FOS}$$

$$\sigma_2 = \frac{1}{2}\left[\sigma_x - \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$\sigma_2 = \frac{1}{2}\left[\frac{2\times10^4}{A} - \frac{6\times10^4}{A}\right]$$

$$\sigma_2 = -\frac{2\times10^4}{A}$$

$$\frac{8\times10^4}{A} - 0.25\left(-\frac{2\times10^4}{A}\right) = 120$$

$$\frac{8\times10^4}{A} + \frac{0.5\times10^4}{A} = 120$$

$$\frac{8.5\times10^4}{A} = 120$$

$$A = \frac{8.5\times10^4}{120}$$

$$\frac{\pi d^2}{4} = 708.33$$

$$d^2 = 901.878$$

$$d = 30.03\,mm$$

(3) According to Tresa Theory:

$$\sigma_1 - \sigma_2 = \frac{\sigma_y}{n}$$

$$\frac{8\times10^4}{A} - \left(-\frac{2\times10^4}{A}\right) = 120$$

$$\frac{10\times10^4}{A} = 120$$

$$A = 833.33$$

$$\frac{\pi d^2}{4} = 833.33$$

$$d^2 = 1061.03$$

$$d = 32.57\,mm$$

16. **Determine the diameter of a circular rod made of ductile material with a fatigue strength (complete stress reversal), σe = 265 MPa and a tensile yield strength of 350 MPa. The member is subjected to a varying axial load from Wmin = – 300 × 103 N to Wmax = 700 × 103 N and has a stress concentration factor = 1.8. Use factor of safety as 2.0. (APRIL/MAY 2009)**

Solution. Given : $\sigma_e = 265$ MPa $= 265$ N/mm^2 ; $\sigma_y = 350$ MPa $= 350$ N/mm^2 ; $W_{min} = -300 \times 10^3$ N ; $W_{max} = 700 \times 10^3$ N ; $K_f = 1.8$; $F.S. = 2$

Let d = Diameter of the circular rod in mm.

$$\therefore \quad \text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that the mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{700 \times 10^3 + (-300 \times 10^3)}{2} = 200 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{200 \times 10^3}{0.7854\, d^2} = \frac{254.6 \times 10^3}{d^2} \text{ N/mm}^2$$

$$\text{Variable load,} \quad W_v = \frac{W_{max} - W_{min}}{2} = \frac{700 \times 10^3 - (-300 \times 10^3)}{2} = 500 \times 10^3 \text{ N}$$

$$\therefore \text{ Variable stress, } \sigma_v = \frac{W_v}{A} = \frac{500 \times 10^3}{0.7854\, d^2} = \frac{636.5 \times 10^3}{d^2} \text{ N/mm}^2$$

We know that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e}$$

$$\frac{1}{2} = \frac{254.6 \times 10^3}{d^2 \times 350} + \frac{636.5 \times 10^3 \times 1.8}{d^2 \times 265} = \frac{727}{d^2} + \frac{4323}{d^2} = \frac{5050}{d^2}$$

$$\therefore \quad d^2 = 5050 \times 2 = 10\,100 \text{ or } d = 100.5 \text{ mm } \textbf{Ans.}$$

17. **Determine the diameter of a circular rod made of ductile material with a fatigue strength (complete stress reversal), σe = 265MPa and a tensile yield strength of 350 MPa. The member is subjected to a varying axial load from Wmin = – 300 × 103 N to Wmax = 700 × 103 N and has a stress concentration factor = 1.8. Use factor of safety as 2.0.**

Given:

σ_e = 265MPa=265 N/mm^2
σ_y = 350MPa=350N/mm^2
W_{min}=-300x10^3 N
W_{max}=700x103 N
k_f=1.8; F.S.=2.

To Find:

d= diameter of the circular rod

Solution:

Let d= diameter of the circular rod in mm

$$\text{Area A}=\frac{\pi}{4}d^2 = 0.7854d^2 mm^2$$

We know that the mean load,

$$W_m = \frac{W_{\max}+W_{\min}}{2} = 200x10^3 N$$

Mean stress,

$$\sigma_m = \frac{W_m}{A} = \frac{200x10^3}{0.7854d^2} = \frac{254.6x10^3}{d^2} N/mm^2$$

Variable load,

$$W_v = \frac{W_{\max}-W_{\min}}{2} = 500x10^3 N$$

Variable stress,

$$\sigma_v = \frac{W_v}{A} = \frac{500x10^3}{0.7854d^2} = \frac{636.5x10^3}{d^2} N/mm^2$$

We know that according to Soderberg's formula,

$$\frac{1}{F.S} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v + k_f}{\sigma_e}$$

$$\frac{1}{2} = \frac{254.6x10^3}{d^2x350} + \frac{636.5x10^3}{d^2x265} = \frac{5050}{d^2}$$

$$d = 100.5mm$$

18. A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maximum value of 50 kN. Determine the diameter of bar by taking a factor of safety of 1.5, size effect of 0.85, surface finish factor of 0.9. The material properties of bar are given by : ultimate strength of 650 MPa, yield strength of 500 MPa and endurance strength of 350 MPa.(Nov/Dec – 2011)(Nov/Dec-2016)

Given :

l = 500 mm

W_{min} = 20 kN = 20 × 103 N

W_{max} = 50 kN = 50 × 103 N

F.S. = 1.5 ;Ksz = 0.85

K_{sur}= 0.9

σ_u = 650 MPa = 650 N/mm2

σ_y= 500 MPa = 500 N/mm2

σ_e = 350 MPa = 350 N/mm2

To Find:

Diameter of the bar

Solution:

Let d = Diameter of the bar in mm.

We know that the maximum bending moment,

We know that maximum bending moment,

$$\text{Mmax} = \frac{W_{max} \times L}{4} = \frac{50 \times 10^3 \times 500}{4} = 6250 \times 10^3 \text{ N} - \text{mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times L}{4} = \frac{20 \times 10^3 \times 500}{4} = 2500 \times 10^3 \text{ N} - \text{mm}$$

Mean or average bending moment,

$$\text{Mm} = \frac{M_{max} + M_{min}}{2} = \frac{6250 \times 10^3 + 2500 \times 10^3}{2} = 4375 \times 10^3 \text{ N} - \text{mm}$$

and variable bending moment,

$$\text{Mv} = \frac{M_{max} - M_{min}}{2} = \frac{6250 \times 10^3 - 2500 \times 10^3}{2} = 1875 \times 10^3 \text{ N} - \text{mm}$$

Section modulus, $Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} 0.0982\ d^3 mm^3$

∴ Mean bending stress,$\sigma_m = \frac{M_m}{Z} = \frac{44.5 \times 10^6}{d^3} N/mm^2$

and variable bending stress,$\sigma_v = \frac{M_v}{Z} = \frac{1875 \times 10^3}{0.0982\ d^3} = \frac{19.1 \times 10^6}{d^3} \quad N/mm^2$

and according to Soderberg's formula,

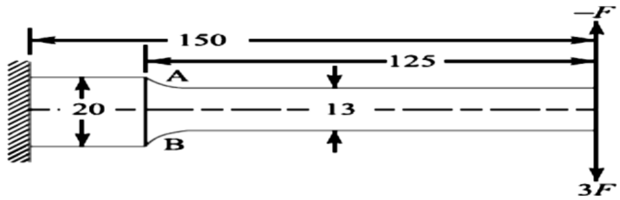
$$\frac{1}{FOS} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \textbf{taking } k_f = 1$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 650} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85}$$

$$\frac{1}{1.5} = \frac{68 \times 10^6}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{139 \times 10^3}{d^3}$$

$$d^3 = 139 \times 10^3 \times 1.5 = 209 \times 10^3 \text{or } d = 59.3\ mm$$

19. A cantilever beam made of cold drawn carbon steel of circular cross-section as shown in Fig. is subjected to a load which varies from – F to 3 F. Determine the maximum load that this member can withstand for an indefinite life using a factor of safety as 2. The theoretical stress concentration factor is 1.42 and the notch sensitivity is 0.9.



Assume the following values:Ultimate stress = 550 MPa, Yield stress = 470 MPa
Endurance limit = 275 MPa, Size factor = 0.85, Surface finish factor= 0.89(Nov/Dec-15)
Given :

W_{min}=-F
W_{max}=3F; F.S=2
k_t=1.42; q=0.9
σ_u=550MPa=550N/mm^2; σ_y=470MPa=470N/mm^2; σ_e=275MPa=275N/mm^2;
k_{sz}=0.85; k_{sur}=0.89

Solution:

We know that maximum bending moment at point A,

$$M_{max}=W_{max}x125=3Fx125=375\ F\ N\text{-}mm$$

Minimum bending moment at point A,

$$M_{min}=W_{min}x125=-Fx125=-125\ F\ N\text{-}mm$$

Mean bending moment,

$$M_m=\frac{M_{\max}+M_{\min}}{2}=\frac{375F+(-125F)}{2}=125FN-mm$$

Variable bending moment,

$$M_v=\frac{M_{\max}-M_{\min}}{2}=\frac{375F-(-125F)}{2}=250FN-mm$$

Section modulus,

$$Z=\frac{\pi}{32}d^3=\frac{\pi}{32}(13)^3=215.7mm^3$$

Mean bending stress,

$$\sigma_m=\frac{M_m}{Z}=\frac{125F}{215.7}=0.58FN/mm^2$$

Variable bending stress,

$$\sigma_v=\frac{M_v}{Z}=\frac{250F}{215.7}=1.16FN/mm^2$$

Fatigue stress concentration factor, k_f=1+q(k_t-1)=1+0.9(1.42-1)=1.378
We know that according to goodman’s formula

$$\frac{1}{F.S} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v + k_f}{\sigma_e x k_{sur} x k_{sz}}$$

$$\frac{1}{2} = \frac{0.58F}{550} + \frac{1.16Fx1.378}{275x0.89x0.85} = 0.00873F$$

$$F = 57.3N$$

According to Soderberg's formula,

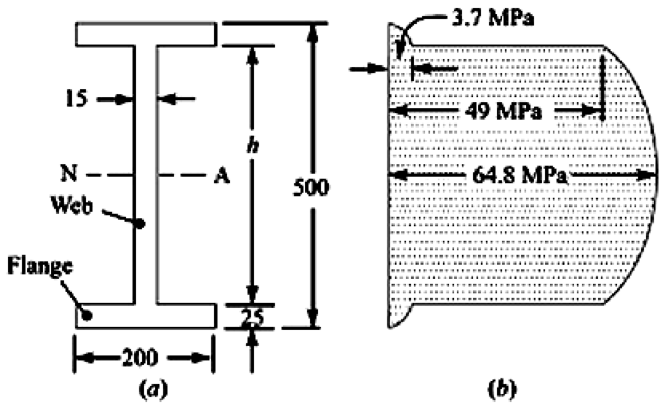
$$\frac{1}{F.S} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v + k_f}{\sigma_e x k_{sur} x k_{sz}}$$

$$\frac{1}{2} = \frac{0.58F}{470} + \frac{1.16Fx1.378}{275x0.89x0.85} = 0.00891F$$

$$F = 56N$$

Taking larger of the two, we have F=57.3 N

20. A beam of I- section 500mm deep and 200mm wide has flanges 25mm thick and web 15mm thick as shown in fig. (a). it carries a shearing force of 400kN. Find the maximum intensity of shear stress in the section, assuming the moment of inertia to be 645 x 106mm4. Also find the shear stress at the joint and at the junction of the top of the web and bottom of the flange.



All dimensions in mm.

Maximum intensity of shear stress

We know that maximum intensity of shear stress,

$$\tau_{max} = \frac{F}{I \cdot b}\left[\frac{B}{8}(H^2 - h^2) + \frac{b \cdot h^2}{8}\right]$$

$$= \frac{400 \times 10^3}{645 \times 10^6 \times 15}\left[\frac{200}{8}(500^2 - 450^2) + \frac{15 \times 450^2}{8}\right] \text{ N/mm}^2$$

$$= 64.8 \text{ N/mm}^2 = 64.8 \text{ MPa } \textbf{Ans.}$$

The maximum intensity of shear stress occurs at neutral axis.

Note : The maximum shear stress may also be obtained by using the following relation :

$$\tau_{max} = \frac{F \cdot A \cdot \bar{y}}{I \cdot b}$$

We know that area of the section above neutral axis,

$$A = 200 \times 25 + \frac{450}{2} \times 15 = 8375 \text{ mm}^2$$

Maximum shear stress at a point, according to Mohr's circle

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

where

$\sigma_x = \sigma_b$

$\sigma_y = 0$

$\tau_{xy} = \tau$

Thus,

$$\tau_{max} = \sqrt{\left(\frac{3055.77 \times 10^3}{2d^3}\right)^2 + \left(\frac{3055.77 \times 10^3}{d^3}\right)^2}$$

$$= \frac{3416.45 \times 10^3}{d^3} \text{ N/mm}^2$$

For safe design, τ_{max} should be less than or equal to the allowable shear strength. That is,

$$\frac{3416.45 \times 10^3}{d^3} \leq 80$$

or

$$d \geq 34.95 \text{ mm}$$

Let us adopt 35 mm as the shaft diameter.

21. A cylindrical shaft made of steel of yield strength 700 MPa is subjected to staticloads consisting of bending moment 10 kN-m and a torsional moment 30 kN-m. Determine the

diameter of the shaft using two different theories of failure, and assuming a factor of safety of 2. Take E = ***210GPa and Poisson's ratio = 0.25.***

Solution. Given : $\sigma_{yt} = 700$ MPa = 700 N/mm^2 ; $M = 10$ kN-m = 10×10^6 N-mm ; $T = 30$ kN-m = 30×10^6 N-mm ; $F.S. = 2$; $E = 210$ GPa = 210×10^3 N/mm^2 ; $1/m = 0.25$

Let d = Diameter of the shaft in mm.

First of all, let us find the maximum and minimum principal stresses.

We know that section modulus of the shaft

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

$\therefore$ Bending (tensile) stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{10 \times 10^6}{0.0982\, d^3} = \frac{101.8 \times 10^6}{d^3} \text{ N/mm}^2$$

and shear stress due to torsional moment,

$$\tau = \frac{16\, T}{\pi\, d^3} = \frac{16 \times 30 \times 10^6}{\pi\, d^3} = \frac{152.8 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \ldots(\because \sigma_2 = 0)$$

$$= \frac{101.8 \times 10^6}{2d^3} + \frac{1}{2}\left[\sqrt{\left(\frac{101.8 \times 10^6}{d^3}\right)^2 + 4\left(\frac{152.8 \times 10^6}{d^3}\right)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{1}{2} \times \frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{161 \times 10^6}{d^3} = \frac{211.9 \times 10^6}{d^3} \text{ N/mm}^2$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \ldots(\because \sigma_2 = 0)$$

$$= \frac{50.9 \times 10^6}{d^3} - \frac{161 \times 10^6}{d^3} = \frac{-110.1 \times 10^6}{d^3} \text{ N/mm}^2$$

Let us now find out the diameter of shaft (d) by considering the maximum shear stress theory and maximum strain energy theory.

1. *According to maximum shear stress theory*

We know that maximum shear stress,

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{1}{2}\left[\frac{211.9 \times 10^6}{d^3} + \frac{110.1 \times 10^6}{d^3}\right] = \frac{161 \times 10^6}{d^3}$$

We also know that according to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{yt}}{2\, F.S.} \quad \text{or} \quad \frac{161 \times 10^6}{d^3} = \frac{700}{2 \times 2} = 175$$

$$\therefore \qquad d^3 = 161 \times 10^6 / 175 = 920 \times 10^3 \quad \text{or} \quad d = 97.2 \text{ mm Ans.}$$

Note: The value of maximum shear stress (τ_{max}) may also be obtained by using the relation,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{101.8\times10^6}{d^3}\right)^2 + 4\left(\frac{152.8\times10^6}{d^3}\right)^2}\right]$$

$$= \frac{1}{2}\times\frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{1}{2}\times\frac{10^6}{d^3}\times 322 = \frac{161\times10^6}{d^3}\ \text{N/mm}^2 \qquad \text{...(Same as before)}$$

2. *According to maximum strain energy theory*

We know that according to maximum strain energy theory,

$$\frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1}\times\sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

or
$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1}\times\sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

$$\left[\frac{211.9\times10^6}{d^3}\right]^2 + \left[\frac{-110.1\times10^6}{d^3}\right]^2 - 2\times\frac{211.9\times10^6}{d^3}\times\frac{-110.1\times10^6}{d^3}\times 0.25 = \left(\frac{700}{2}\right)^2$$

or
$$\frac{44\,902\times10^{12}}{d^6} + \frac{12\,122\times10^{12}}{d^6} + \frac{11\,665\times10^{12}}{d^6} = 122\,500$$

$$\frac{68\,689\times10^{12}}{d^6} = 122\,500$$

$\therefore$ $d^6 = 68\,689\times10^{12}/122\,500 = 0.5607\times10^{12}$ or $d = 90.8$ mm **Ans.**

<u>IMPACT LOAD& ECCENTRIC LOAD</u>

22. An unknown weight falls through 10 mm on a collar rigidly attached to the lower end of a vertical bar 3 m long and 600 mm^2 in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight? Take E = 200 kN/mm^2. (Nov/Dec – 2014)

Given Data:

h = 10 mm;
l = 3 m = 3000 mm;
A = 600 mm^2; δl = 2 mm;
E = 200 kN/mm^2 = 200 × 10^3 N/mm^2

To Find: Unknown Weight, W

Let σ = stress in the bar
We know that Young's modulus,

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma \times l}{\delta l}$$

$$\sigma = \frac{E \times \delta ll}{l} = \frac{200 \times 10^3 \times 2}{3000} = \frac{400}{3} = 133.3\ \text{N/mm}^2$$

Value of the unknown weight

W = value of the unknown weight

We know that $\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right]$

$$\frac{400}{3} = \frac{W}{600}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right]$$

$$\frac{400}{3} = \frac{W}{600}\left[1 + \sqrt{1 + \frac{2 \times 10 \times 600 \times 200 \times 10^3}{W \times 3000}}\right]$$

$$\frac{400 \times 600}{3W} = 1 + \sqrt{1 + \frac{800000}{W}}$$

$$\frac{80000}{W} - 1 = \sqrt{1 + \frac{800000}{W}}$$

Squaring both sides,

$$\frac{6400 \times 10^6}{W^2} + 1 - \frac{160000}{W} = 1 + \frac{800000}{W}$$

$$\frac{6400 \times 10^2}{W} - 16 = 80$$

OR

$$\frac{6400 \times 10^2}{W} = 96$$

$$\mathbf{W} = \frac{\mathbf{6400 \times 10^2}}{\mathbf{96}} = \mathbf{6666.7\ N}$$

23. A wrought iron bar 50 mm in diameter and 2.5 m long transmits shock energy of 100 N-m. Find the maximum instantaneous stress and the elongation. Take E = 200 GN/m^2.

Given Data:

d=50mm; l=2.5m=2500mm;

U=100N-m=100x10^3N-mm;

$E=200GN/m^2=200x10^3N/mm^2$

To Find:

Maximum instantaneous stress

Let σ = Maximum instantaneous stress

We know that volume of the bar,

$$V = \frac{\pi}{4} x d^2 x l = \frac{\pi}{4} x (50)^2 x 2500 = 4.9 x 10^6 mm^3$$

We also know that shock or strain in the body (U),

$$100x10^3 = \frac{\sigma^2 x V}{2E} = \frac{\sigma^2 x 4.9 x 10^6}{2x200x10^3} = 12.25\sigma^2$$

$$\sigma^2 = \frac{100x10^3}{12.25} = 8163$$

$$\sigma = 90.3 N/mm^2$$

Elongation produced

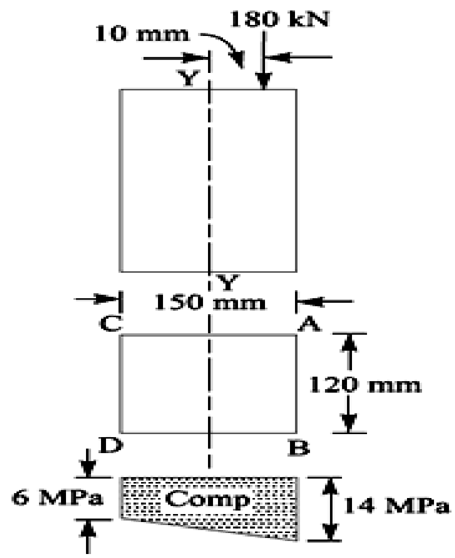
Let δl = Elongation produced

We know that Young's modulus,

$$E = \frac{Stress}{Strain} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\delta l / l}$$

$$\delta l = \frac{\sigma . l}{E} = \frac{90.3x2500}{200x10^3} = 1.13mm \textbf{ ANS}$$

24. A rectangular strut is 150 mm wide and 120 mm thick. It carries a load of 180 kN at an eccentricity of 10 mm in a plane bisecting the thickness as shown in Fig. Find the maximum and minimum intensities of stress in the section.



Given:

b=150 mm

d = 120 mm
P = 180 kN = 180 x 10^3 N
e = 10 mm.

To Find:

maximum and minimum intensities of stress in the section.

Solution:

We know that cross sectional area of the strut,

$$A = b.d = 18 \times 10^3 \text{ mm}^2$$

Direct compressive stress,

$$\sigma_o = P/A = 180 \times 10^3 / 18 \times 10^3$$
$$= 10 \text{N/mm}^2$$
$$= 10 \text{ MPa}$$

Section modulus for the strut,

$$Z = I_{YY}/y = (d.b^3/12)/(b/2) = d.b^2/6 = 450 \times 10^3 \text{ mm}^3$$

Bending moment,

$$M = P.e = 180 \times 10^3 \times 10$$
$$= 1.8 \times 10^6 \text{ N-mm}$$

Bending stress,

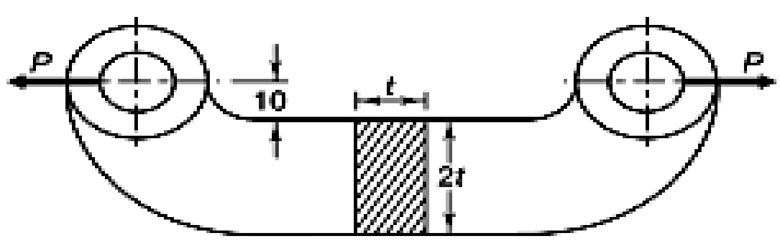
$$\sigma_b = M/Z = 1.8 \times 10^6 / 450 \times 10^3$$
$$= 4 \text{ N/mm}^2 = 4\text{MPa}$$

Since $\sigma_o$is greater than $\sigma_b$therefore the entire cross- section of the strut will be subjected to compressive stress. The maximum intensity of compressive stress will be at the edge AB and minimum at the edge CD.

Maximum intensity of compressive stress at the edge **AB = σ_o+ σ_b = 14 MPa.**

Minimum intensity of compressive stress at the edge **CD = σ_o- σ_b = 6MPa.**

25. An offset link subjected to a force of 25KN is shown in Fig. It is made of grey cast iron FG300 and the factor of Safety is 3. Determine the dimension of the cross-section of the link.



Offset Link

Solution:

Given: P = 25KN S_m = 300 N/mm^2 (fs) = 3

Step 1:

Calculation of permissible tensile stress for the link

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{300}{3} = 100\ \text{N/mm}^2$$

Step 2:

Calculation of direct tensile and bending stress

The cross section is subjected to direct tensile stress and bending stresses. The stresses are maximum at the top fiber. At the top fiber,

$$\sigma_t = \frac{P}{A} + \frac{M_b\, y}{I}$$

$$= \frac{25\times10^3}{t(2t)} + \frac{25\times10^3(10+t)(t)}{\left[\frac{1}{12}t(2t)^3\right]}$$

Step III Calculation of dimensions of cross-section

Equating (a) and (b),

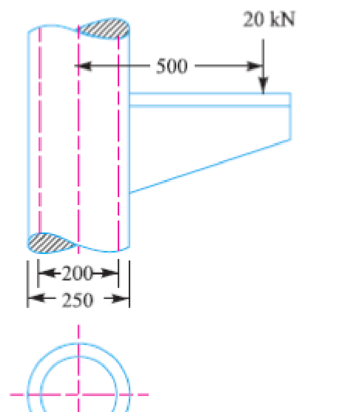
$$\frac{12500}{t^2} + \frac{37500\,(10+t)}{t^3} = 100$$

or, $t^3 - 500\,t - 3750 = 0$

Solving the above equation by trial and error method,

$$t \cong 25.5\ \text{mm}$$

27.A hollow circular column of external diameter250 mm and internal diameter 200 mm, carries a projecting bracketon which a load of 20 kN rests, as shown in Fig. 5.22. The centre ofthe load from the centre of the column is 500 mm. Find the stressesat the sides of the column.(Nov/Dec-2016)



Given:

D= 250mm, d=200mm

P=20KN

= 20X10³

e = 500mm

To find:

stresses

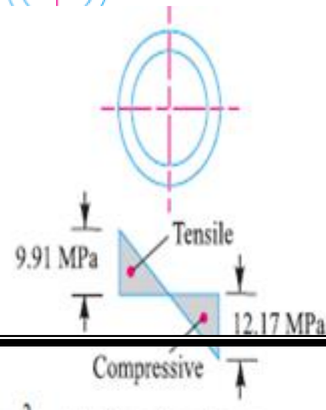
Solution:

We know that cross-sectional area of column,

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}[(250)^2 - (200)^2]$$

$$= 17\,674\ \text{mm}^2$$



∴ Direct compressive stress,

$$\frac{P}{} = \frac{20\times10^3}{}$$

Section modulus for the column,

$$Z = \frac{I}{y} = \frac{\frac{\pi}{64}\left[D^4 - d^4\right]}{D/2} = \frac{\frac{\pi}{64}\left[(250)^4 - (200)^4\right]}{250/2}$$

$$= 905.8 \times 10^3 \text{ mm}^3$$

Bending moment,

$$M = P.e$$

$$= 20 \times 10^3 \times 500$$

$$= 10 \times 10^6 \text{ N-mm}$$

∴ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{10 \times 10^6}{905.8 \times 10^3}$$

$$= 11.04 \text{ N/mm}^2$$

$$= 11.04 \text{ MPa}$$

Since σ_o is less than σ_b, therefore right hand side of the column will be subjected to compressive stress and the left hand side of the column will be subjected to tensile stress.

∴ Maximum compressive stress,

$$\sigma_c = \sigma_b + \sigma_o = 11.04 + 1.13$$

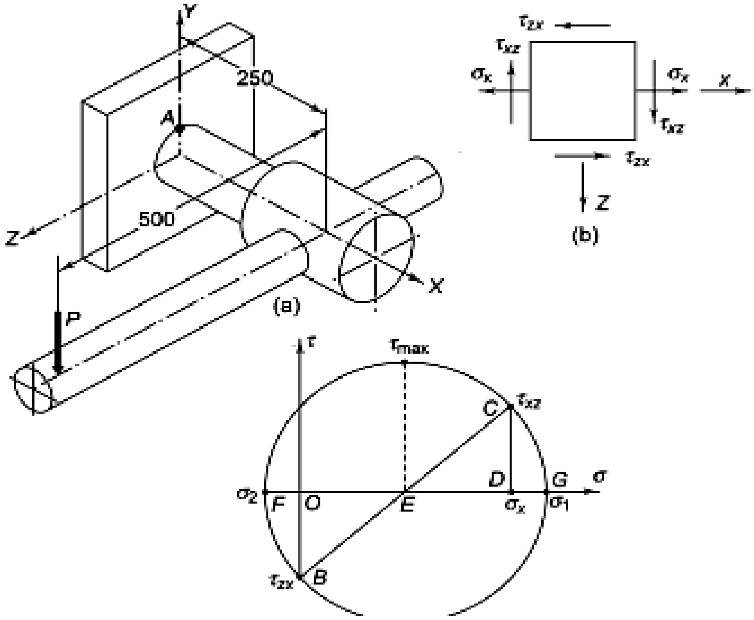
$$= 12.17 \text{ MPa}$$ **Ans.**

and maximum tensile stress,

$$\sigma_t = \sigma_b - \sigma_o = 11.04 - 1.13 = 9.91 \text{ MPa}$$ **Ans.**

ECCENTRIC AXIAL LOADING PROBLEM- [VBB]

28. The shaft of an overhang crank subjected to a force P of 1 KN is shown in fig. The shaft is made of plain carbon steel 45C8 and the tensile yield strength is 380 N/mm^2. The factor of safety is 2. Determine the diameter of the shaft using the maximum shear stress theory.



Solution:

Given: P= 1 KN: S_{yt} = 380 N/mm^2: (fs) = 2

Step 1:

Calculation of permissible shear stress, According to maximum shear stress theory,

S_{sy} = 0.5, S_{yt} = 0.5 (380) = 190 N/mm^2

The permissible shear stress is given by,

$$\tau_{\max} = \frac{S_{sy}}{(fs)} = \frac{190}{2} = 95N/mm^2$$

Step II :

Calculation of bending and torsion shear stresses.

The stresses are critical at the point A, which is subjected to combine bending and torsion moments, at the point A,

$$\tau_{max} = \frac{S_{sy}}{(fs)} = \frac{190}{2} = 95 N/mm^2$$

$$M_b = P \times (250) = (1000)(250) = 250 \times 10^3 N-mm$$

$$M_t = P \times (500) = (1000)(500) = 500 \times 10^3 N-mm$$

$$\sigma_b = \frac{M_b y}{I} = \frac{(250 \times 10^3)(d/2)}{(\pi d^4/64)}$$

$$= \left(\frac{2546.48 \times 10^3}{d^3}\right) N/mm^2$$

$$\tau = \frac{M_I r}{J} = \frac{(500 \times 10^3)(d/2)}{(\pi d^4/64)}$$

$$= \left(\frac{2546.48 \times 10^3}{d^3}\right) N/mm^2$$

Step III :

Calculation of maximum shear stress.

The stresses at point A and corresponding Mohr's circle are shown in Fig. (b) and (c) respectively.

In these figures,

$$\sigma_x = \sigma_b = \left(\frac{2546.48 \times 10^3}{d^3}\right) N/mm^2$$

$$\sigma_z = 0$$

$$\tau = \tau_{xy} = \tau_{zx} = \left(\frac{2546.48 \times 10^3}{d^3}\right) N/mm^2$$

From Mohr's circle,

$$\tau_{max} = \left(\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2}\right)$$

$$= \left(\sqrt{\left(\frac{2546.48}{2d^3}\right)^2 + \left(\frac{2546.48}{d^3}\right)^2}\right) \times 10^3$$

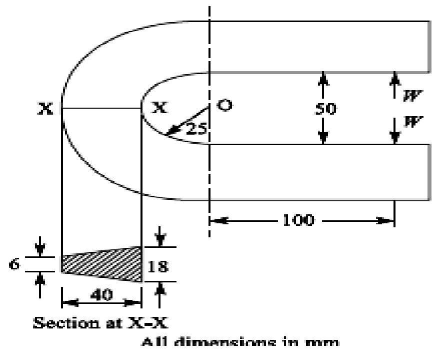
$$= \frac{2847.05 \times 10^3}{d^3}$$

Step IV: Calculation of shaft diameter equation (i) and (ii)

$$\frac{2847.05 \times 10^3}{d^3} = 95 \therefore d = 31.06 mm$$

<u>CRANE HOOK AND C- FRAME</u>

29.The frame of a punch press is shown in Fig. Find the stresses at the inner and outer surface at section X-X of the frame, if W = 5000 N. (April/ May 2014)



Given Data:

W=5000N; b_i=18mm

b_o=6mm; h=40mm

R_i=25mm

R_o=25+40=65mm

To Find:

Stress on the inner surface and outer surface.

Solution:

We know that area of section at X-X,

$$A = \frac{1}{2}(18+6)40 = 480 mm^2$$

We know that radius of curvature of the neutral axis,

$$R_n = \frac{\left(\frac{b_i + b_o}{2}\right)h}{\left(\frac{b_i R_o - b_o R_i}{h}\right)\log_e\left(\frac{R_o}{R_i}\right) - (b_i - b_o)}$$

$$= \frac{\left(\frac{18+6}{2}\right)40}{\left(\frac{18x65 - 6x25}{40}\right)\log_e\left(\frac{65}{25}\right) - (18-6)} = \frac{480}{(25.5x0.9555) - 12} = 38.83mm$$

Radius of curvature of the centroidal axis,

$$R = R_i + \frac{h(b_i + 2b_o)}{3(b_i + b_o)} = 25 + \frac{40(18+2x6)}{3(18+6)} \text{ mm}$$

$$= 25 + 16.67 = 41.67mm$$

Distance between the centroidal axis and neutral axis,

$$e=R-R_n=41.67-38.83=2.84mm$$

Distance between the load and centroidal axis

$$x=100+R=100+41.67=141.67mm$$

Bending moment about the centroidal axis,

$$M=W.x=5000x141.67=708350N-mm$$

The section at X-X is subjected to a direct tensile load of W=5000N and a bending moment of M=708350N-mm. We know that direct tensile stress at section X-X,

$$\sigma_t=\frac{W}{A}=\frac{5000}{480}=10.42N/mm^2=10.42MPa.$$

Distance from the neutral axis to the inner surface,

$$y_i=R_n-R_i=38.85-25=13.83mm$$

Distance from the neutral axis to the outer surface,

$$y_o=R_o-R_n=65-38.83=26.17mm$$

We know that maximum bending stress at the inner surface,

$$\sigma_{bi}=\frac{M.y_i}{A.e.R_i}=\frac{708350x13.83}{480x2.84x25}=287.4N/mm^2$$

$$\sigma_{bo}=\frac{M.y_i}{A_e.R_i}=\frac{708350\times 13.83}{480\times 2.84\times 25}=28704\ N/mm^2$$

$$\boldsymbol{\sigma_{bo}=287.4MPa}$$

Maximum stress at the outer surface,

$$\sigma_{bo}=\frac{M.y_o}{A_e.R_e}=\frac{708350\times 26.17}{480\times 2.84\times 65}=209.2$$

σ_{bo}= 209.2 MPa (compressive)

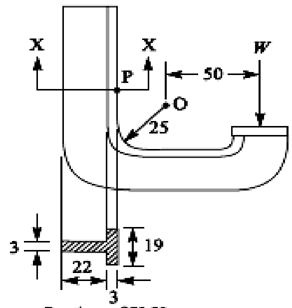
∴ Resultant stress on the inner surface

= σ_t+ σ_{bi}= 10.42 + 287.4 **= 297.82 MPa (tensile) Ans.**

And resultant stress on the outer surface,

= σ_t– σ_{bo}= 10.42 – 209.2 **= – 198.78 (compressive)**

30.A ‘C’ clamp is subjected to a maximum load of W, as shown in fig. if the maximum tensile stress in the clamp is limited to 140 N/mm^2, find the value of W.(Nov/Dec– 2012)



Section of X-X
All dimensions in mm.

Given:

σt(max) = 140 MPa = 140 N/mm2

R_i= 25 mm

R_o= 25 + 25 = 50 mm

b_i= 19 mm ;

t_i= 3 mm ; t = 3 mm

h = 25 mm

To Find:

Value of the Weight, W

Solution:

We know that area of section at X-X,

$$A = 3 \times 22 + 3 \times 19 = 123\ mm^2$$

The various distances are in fig 1.4 We know that radius of curvature of the neutral axis,

$$Rn = \frac{t_i(b_i - t) + t.h}{(b_i - t)\log_e\left(\frac{R_i + t_i}{R_i}\right) + t\log_e\left(\frac{R_o}{R_i}\right)}$$

$$= \frac{3(19-3) + 3 \times 25}{(19-3)\log_e\left(\frac{25+3}{25}\right) + 3\log_e\left(\frac{50}{25}\right)}$$

$$= \frac{123}{16 \times 0.113 + 3 \times 0.693} = \frac{123}{3.887} = 31.64\ \text{mm}$$

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{\frac{1}{2}h^2 \times t + \frac{1}{2}t_i^2(b_i - t)}{h \times t + t_i(b_i - t)}$$

$$R = 25 + \frac{\frac{1}{2}25^2 \times 3 + \frac{1}{2}3^2(19-3)}{25 \times 3 + 3(19-3)} = 25 + \frac{937.5 + 72}{75 + 48} = 25 + 8.2 = 33.2\ \text{mm}$$

Distance between the centroidal axis and neutral axis,

$$e = R - R_n = 33.2 - 31.64 = 1.56\ mm$$

and distance between the load W and the centroidal axis,

$$x = 50 + R = 50 + 33.2 = 83.2 \text{ mm}$$

∴ Bending moment about the centroidal axis,

$$M = W.x= W \times 83.2 = 83.2 \text{ W N-mm}$$

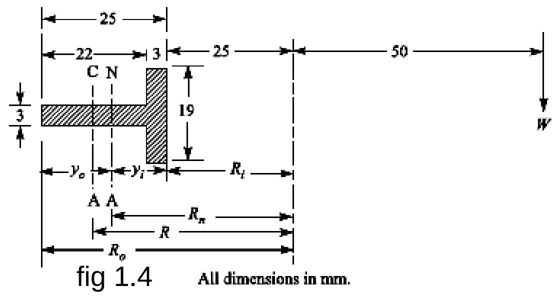


fig 1.4

The section at X-X is subjected to a direct tensile load of W and a bending moment of 83.2 W. The maximum tensile stress will occur at point P (i.e. at the inner fiber of the section). Distance from the neutral axis to the point P,

$$y_i= R_n– R_i= 31.64 – 25 = 6.64 \text{ mm}$$

Direct tensile stress at section X-X,

$$\sigma t = \frac{W}{A} = \frac{W}{123} = 0.008 \text{ N/mm2}$$

and maximum bending stress at point P,

$$\sigma bi = \frac{M \times y_i}{A \times e \times R_i} = \frac{83.2 \text{ W} \times 6.64}{123 \times 1.56 \times 25} = 0.115 \text{ W N/mm}^2$$

We know that the maximum tensile stress σt(max),

$$140 = \sigma_t + \sigma_{bi} = 0.008 \text{ W} + 0.115 \text{ W} = 0.123 \text{ W}$$

∴W = 140/0.123 = 1138 N Ans.

We know that distance from the neutral axis to the outer fibre,

$$y_o= R_o– R_n= 50 – 31.64 = 18.36 \text{ mm}$$

∴ Maximum bending stress at the outer fibre,

$$\sigma bo = \frac{M \times y_o}{A \times e \times R_o} = \frac{83.2 \text{ W} \times 18.36}{123 \times 1.56 \times 50} = 0.16 \text{ W}$$

and maximum stress at the outer fibre,

$$= \sigma_t – \sigma_{bo} = 0.008 \text{ W} – 0.16 \text{ W} = – 0.152 \text{ W N/mm2}$$

= 0.152 W N/mm2 (compressive)

31.A hollow shaft of 40 mm outer diameter and 25 mm inner diameter is subjected to a twisting moment of 120 N-m, simultaneously; it is subjected to an axial thrust of 10 kN and a bending moment of 80 N-m. Calculate the maximum compressive and shear stresses.

Given:

d_o=40mm; d_i=25mm

T=120N-m=120x10^{3}N-mm
P=10kN=10x10^{3}N
M=80 N-m=8x10^3 N-mm.

To Find:

maximum compressive and shear stresses

We know that cross-sectional area of the shaft,

$$A=\frac{\pi}{4}\left[\left(d_o\right)^2-(d_i)^2\right]=\frac{\pi}{4}\left[(40)^2-(25)^2\right]=766\text{mm}^2$$

Direct compressive stress due to axial thrust,

$$\sigma_o=\frac{P}{A}=\frac{10x10^3}{766}=13.05\text{N/mm}^2=13.05\text{MPa}$$

Section modulus of the shaft,

$$Z=\frac{\pi}{32}\left[\frac{\left(d_o\right)^4-(d_i)^4}{d_o}\right]=\frac{\pi}{32}\left[\frac{\left(40\right)^4-(25)^4}{40}\right]=5325\text{mm}^3$$

Bending stress due to bending moment,

$$\sigma_b=\frac{M}{Z}=\frac{80x10^3}{5325}=15.02\text{N/mm}^2=15.02\text{ MPa(compressive)}$$

Resultant compressive stress,

$$\sigma_c=\sigma_b+\sigma_o=15.02+13.05=28.07\text{N/mm}^2=28.07\text{MPa}$$

We know that twisting moment (T),

$$120x10^3=\frac{\pi}{16}x\tau\left[\frac{\left(d_o\right)^4-(d_i)^4}{d_o}\right]=\frac{\pi}{16}x\tau\left[\frac{\left(40\right)^4-(25)^4}{40}\right]=40650\tau$$

$$\tau=120x10^3/10650=11.27\text{ N/mm2}=11.27\text{ MPa}$$

Maximum compressive stress

We know that maximum compressive stress,

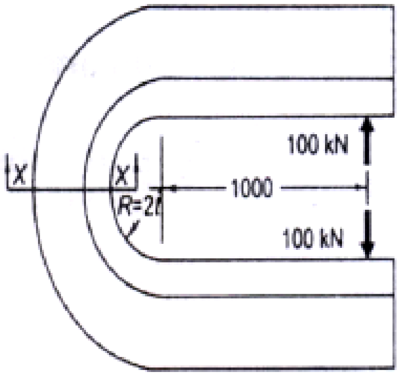
$$\sigma_{c(max)}=\frac{\sigma_c}{2}+\frac{1}{2}\left[\sqrt{(\sigma_c)^2+4\tau^2}\right]$$
$$=\frac{28.07}{2}+\frac{1}{2}\left[\sqrt{(28.07)^2+4(11.27)^2}\right]$$
$$=14.035+18=32.035\text{MPa.}$$

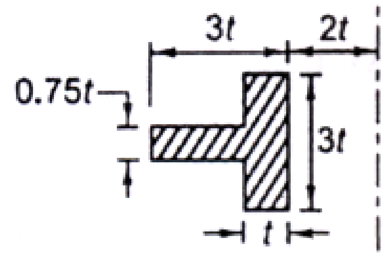
Maximum shear stress

We know that maximum shear stress,

$$\tau_{max}=\frac{1}{2}\left[\sqrt{(\sigma_c)^2+4\tau^2}\right]=\frac{1}{2}\left[\sqrt{(28.07)^2+4(11.27)^2}\right]=18\text{MPa}$$

32. The C-Frame of a 100KNrankins capacity press is shown in figure. The material of the frame is FG 200. Assuming the factor of safety as 3, determine the dimensions of the frame. (April/May 2018) (Nov/Dec 2018)





Solution **Given** $P = 100$ kN $S_{ut} = 200$ N/mm^2

$(fs) = 3$

Step I: Calculation of permissible tensile stress

$$\sigma_{max} = \frac{S_{ut}}{(fs)} = \frac{200}{3} = 66.67 \text{ N/mm}^2$$

Step II: Calculation of eccentricity (e)

Using notations of Eq. (4.79) and Fig. [4.65(e)],

$$b_i = 3t \qquad h = 3t \qquad R_i = 2t$$

$$R_o = 5t \qquad t_i = t \qquad t = 0.75t$$

From Eq. (4.79),

$$R_N = \frac{t_i (b_i - t) + t\,h}{(b_i - t)\log_e\left(\frac{R_i + t_i}{R_i}\right) + t\log_e\left(\frac{R_o}{R_i}\right)}$$

$$= \frac{t(3t - 0.75t) + 0.75t(3t)}{(3t - 0.75t)\log_e\left(\frac{2t + t}{2t}\right) + 0.75\,t\log_e\left(\frac{5t}{2t}\right)}$$

$$= 2.8134\,t$$

$$R = R_i + \frac{\frac{1}{2}t\,h^2 + \frac{1}{2}t_i^2 (b_i - t)}{t\,h + t_i (b_i - t)}$$

$$= 2t + \frac{\frac{1}{2}(0.75t)(3t)^2 + \frac{1}{2}t^2(3t - 0.75t)}{(0.75t)(3t) + t(3t - 0.75t)} = 3t$$

$$e = R - R_N = 3\,t - 2.8134\,t = 0.1866\,t$$

Step III: Calculation of bending stress

$$h_i = R_N - R_i = 2.8134\,t - 2\,t = 0.8134\,t$$

$$A = (3t)(t) + (0.75\,t)(2\,t) = (4.5\,t^2)\ \text{mm}^2$$

$$M_b = 100 \times 10^3(1000 + R)$$

$$= 100 \times 10^3\,(1000 + 3\,t)\ \text{N-mm}$$

From Eq. (4.69), the bending stress at the inner fibre is given by,

$$\sigma_{bi} = \frac{M_b\,h_i}{A\,e\,R_i} = \frac{100 \times 10^3\,(1000 + 3t)(0.8134t)}{(4.5t^2)(0.1866t)(2t)}$$

$$= \frac{100 \times 10^3\,(1000 + 3t)(2.1795)}{(4.5t^2)}\ \text{N/mm}^2$$

Step IV: Calculation of direct tensile stress

$$\sigma_t = \frac{P}{A} = \frac{100 \times 10^3}{(4.5\,t^2)}\ \text{N/mm}^2$$

Step V: Calculation of dimensions of cross-section

Adding the two stresses and equating the resultant stress to permissible stress,

$$\sigma_{bi} + \sigma_t = \sigma_{max}$$

$$\frac{100 \times 10^3\,(1000 + 3t)(2.1795)}{(4.5t^2)} + \frac{100 \times 10^3}{(4.5t^2)} = 66.67$$

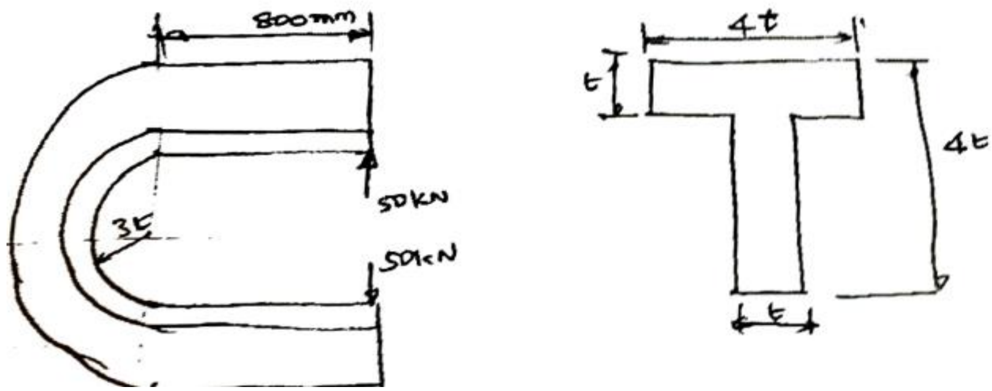
$$t^3 - 2512.83\,t = 726\,500$$

The above equation is cubic and solved by trial and error method. The trial values of 't' and the left hand side expression are tabulated here,

$\therefore \quad t \approx 99.5$ mm $\quad$ or $\quad t = 100$ mm

32.(a) **(Nov/Dec 2021)**

A punch press of capacity 50 KN has a c-frame of "T" cross section as shown in the fig. The Tensile strength of material is 350 MPa. Take F 0.5 as 3.5. Determine the dimensions of C-frame (13)



Given:

Load F = 50 kN = 50×10^3 N

σ_u = 350 Mpa = 350 N/mm^2

FOS = 3.5

σ_{all} = 350/3.5 = 100 N/mm^2

To find: Dimension of "**C"** frame:

Solution:

From DDB: 6.1

$$R = r_i + \frac{\frac{1}{2} h^2 t + \frac{1}{2} t_1^2 (b_i - t)}{ht + (b_i - t) t_i}$$

Where r_i=3t, h=4t, t=t, b_i=4t, r_o=7t

Substitute all the value R=4.357t

r_n=4.09t

e=(R- r_n) =0.267t

h_i=(r_n-r_i) =1.09t

Moment due to 50 kN force (about centroidal axis)

$$m_b = 50\times10^3\times(800+4.357t)$$

$$area\,of\,'T'\,Section = (4t\times t)+(3t\times t) = 7t^2$$

$$Bending\ stress\ \sigma_b = \frac{M_b h_i}{aer_i}$$

$$= \frac{50\times10^3\times(800+4.357t)\times1.09t}{7t^2\times0.267t\times3t}$$

$$Direct\ tensile\ stress\ \sigma_d = \frac{50\times10^3}{7t^2}$$

$$Total\ stress\ \sigma = \sigma_b+\sigma_d$$

$$Equating\ this\ to\ allowable\ stress$$

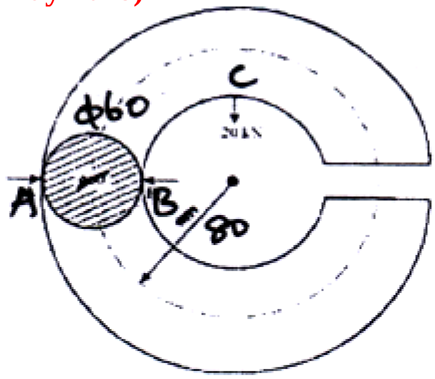
$$\sigma_{all} = \sigma_b+\sigma_d$$

$$100 = \frac{50\times10^3\times(800+4.357t)\times1.09t}{7t^2\times0.267t\times3t}+\frac{50\times10^3}{7t^2}$$

Solving this equation we get

Thickness of the frame **t= 55 mm**

33.Determine the stress at point A and B split ring shown in fig. if a compressive force = 20 KN is applied point 'C'.(April/May 2018)



Solution:

Redraw the critical section as shown in the figure.

Radius or cmroidal axis $r_c = 80mm$

lnner radius or curved beam

$$r_i = 80-\frac{60}{2} = 50mm$$

outer radius or curved beam

$$r_o = 80+\frac{60}{2} = 110mm$$

Radius or neutral axis

$$r_n = \frac{\left(\sqrt{r_0} + \sqrt{r_i}\right)^2}{4}$$

$$= \frac{\left(\sqrt{110} + \sqrt{50}\right)^2}{4} = 77.081mm$$

Applied force F = 20kN = 20.000N (compressive)

Area of cross section

$$A = \frac{\Pi}{4}d^2 = \frac{\Pi}{4} \times 60^2 = 2827.433mm^2$$

Distance from centroidal axis to force: $l = r_c = 80$mm
Bending moment about centroidal axis $M_b = Fl = 20000x80$
$= 16x\ 10^5$N -mm
Distance ofneutral axis to centroidal axis $c = r_e - r_n = 80 - 77.081 = 2.919$mm

Distance of neutral axis to inner radius
$C_i = r_n - r_i = 77.081 - 50 = 27.081$mm
Distance of neutral axis to outer radius
$C_o = r_o - r_n = 110 - 77.081 = 32.919$mm

Direct stress

$$\sigma_d = -\frac{F}{A} = \frac{20000}{2827.433} = -7.0736\ N/mm^2\ (compressive)$$

$$bending\ stress\ at\ the\ inner\ fiber\ \sigma_{bi} = \frac{M_b c_i}{Aer_i}$$

$$= \frac{-16 \times 10^5 \times 27.081}{2827.433 \times 2.919 \times 50} = -105\ N/mm^2 (compressive)$$

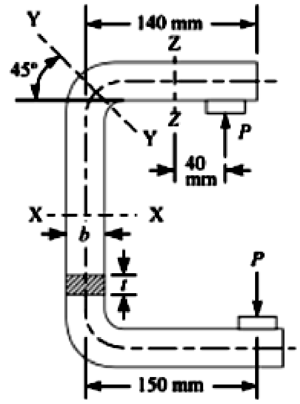
$$bending\ stress\ at\ the\ outer\ fiber\ \sigma_{bo} = \frac{M_b c_o}{Aer_o}$$

$$= \frac{16 \times 10^5 \times 27.081}{2827.433 \times 2.919 \times 110} = 58.016 N/mm^2 (tensile)$$

Combine stress at the inner fiber $\sigma_{ri} = \sigma_d + \sigma_{bi} = -7.0736 - 105.00$
$= -112.0736$N/mm^2 (Compressive)
Combine stress at the outer fiber $\sigma_{ro} = \sigma_d + \sigma_b = -7.0736 + 58.016$
$= 50.9424$N/mm^2 (tensile)
Maximum shear stress

$$\tau_{max} = 0.5 \times \sigma_{max} = 0.5 \times 112.0736 = 56.0368 N/mm^2 at\ B$$

34. **A C-clamp as shown in fig. carries a load P=25kN. The cross section of the clamp at X-X is rectangular having width equal to twice thickness. Assuming that the clamp is made of steel casting with an allowable stress of 100 Mpa, find its dimensions. Also determine the stresses at sections Y-Y and Z-Z.**



Solution. Given : $P = 25$ kN $= 25 \times 10^3$ N ; $\sigma_{t(max)} = 100$ MPa $= 100$ N/mm^2

Dimensions at X-X

Let $\quad t$ = Thickness of the section at X-X in mm, and

$\quad b$ = Width of the section at X-X in mm = $2t$

We know that cross-sectional area at *X-X*,

$$A = b \times t = 2\,t \times t = 2\,t^2 \text{ mm}^2$$

∴ Direct tensile stress at *X-X*,

$$\sigma_o = \frac{P}{A} = \frac{25 \times 10^3}{2\,t^2}$$

$$= \frac{12.5 \times 10^3}{t^3} \text{ N/mm}^2$$

Bending moment at *X-X* due to the load *P*,

$$M = P \times e = 25 \times 10^3 \times 140$$

$$= 3.5 \times 10^6 \text{ N-mm}$$

Section modulus, $$Z = \frac{t\,.\,b^2}{6} = \frac{t\,(2t)^2}{6} = \frac{4\,t^3}{6} \text{ mm}^3$$

...($\because b = 2t$)

∴ Bending stress at X-X,

$$\sigma_b = \frac{M}{Z} = \frac{3.5 \times 10^6 \times 6}{4\,t^3} = \frac{5.25 \times 10^6}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

We know that the maximum tensile stress $[\sigma_{t(max)}]$,

$$100 = \sigma_o + \sigma_b = \frac{12.5 \times 10^3}{t^2} + \frac{5.25 \times 10^6}{t^3}$$

or $$\frac{125}{t^2} + \frac{52.5 \times 10^3}{t^3} - 1 = 0$$

$\therefore$ $t = 38.5$ mm **Ans.** ...(By hit and trial)

and $b = 2\,t = 2 \times 38.5 = 77$ mm **Ans.**

Stresses at section Y-Y

Since the cross-section of frame is uniform throughout, therefore cross-sectional area of the frame at section *Y-Y*,

$$A = b \sec 45° \times t = 77 \times 1.414 \times 38.5 = 4192 \text{ mm}^2$$

Component of the load perpendicular to the section

$$= P \cos 45° = 25 \times 10^3 \times 0.707 = 17\,675 \text{ N}$$

This component of the load produces uniform tensile stress over the section.

$\therefore$ Uniform tensile stress over the section,

$$\sigma = 17\,675 / 4192 = 4.2 \text{ N/mm}^2 = 4.2 \text{ MPa}$$

Component of the load parallel to the section

$$= P \sin 45° = 25 \times 10^3 \times 0.707 = 17\,675 \text{ N}$$

This component of the load produces uniform shear stress over the section.

$\therefore$ Uniform shear stress over the section,

$$\tau = 17\,675 / 4192 = 4.2 \text{ N/mm}^2 = 4.2 \text{ MPa}$$

We know that section modulus,

$$Z = \frac{t\,(b \sec 45°)^2}{6} = \frac{38.5\,(77 \times 1.414)^2}{6} = 76 \times 10^3 \text{ mm}^3$$

Bending moment due to load (P) over the section *Y-Y*,

$$M = 25 \times 10^3 \times 140 = 3.5 \times 10^6 \text{ N-mm}$$

$\therefore$ Bending stress over the section,

$$\sigma_b = \frac{M}{Z} = \frac{3.5 \times 10^6}{76 \times 10^3} = 46 \text{ N/mm}^2 = 46 \text{ MPa}$$

Due to bending, maximum tensile stress at the inner corner and the maximum compressive stress at the outer corner is produced.

$\therefore$ Maximum tensile stress at the inner corner,

$$\sigma_t = \sigma_b + \sigma_o = 46 + 4.2 = 50.2 \text{ MPa}$$

and maximum compressive stress at the outer corner,

$$\sigma_c = \sigma_b - \sigma_o = 46 - 4.2 = 41.8 \text{ MPa}$$

Since the shear stress acts at right angles to the tensile and compressive stresses, therefore maximum principal stress (tensile) on the section *Y-Y* at the inner corner

$$= \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\tau^2}\right] = \frac{50.2}{2} + \frac{1}{2}\left[\sqrt{(50.2)^2 + 4 \times (4.2)^2}\right] \text{MPa}$$

$$= 25.1 + 25.4 = 50.5 \text{ MPa } \textbf{Ans.}$$

and maximum principal stress (compressive) on section *Y-Y* at outer corner

and maximum principal stress (compressive) on section *Y-Y* at outer corner

$$= \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\tau^2}\right] = \frac{41.8}{2} + \frac{1}{2}\left[\sqrt{(41.8)^2 + 4 \times (4.2)^2}\right] \text{MPa}$$

$$= 20.9 + 21.3 = 42.2 \text{ MPa } \textbf{Ans.}$$

Maximum shear stress $= \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(50.2)^2 + 4 \times (4.2)^2}\right] = 25.4$ MPa **Ans.**

Stresses at section Z-Z

We know that bending moment at section *Z-Z*,

$$= 25 \times 10^3 \times 40 = 1 \times 10^6 \text{ N-mm}$$

and section modulus, $Z = \dfrac{t \cdot b^2}{6} = \dfrac{38.5\,(77)^2}{6} = 38 \times 10^3 \text{ mm}^3$

∴ Bending stress at section *Z-Z*,

$$\sigma_b = \frac{M}{Z} = \frac{1 \times 10^6}{38 \times 10^3} = 26.3 \text{ N/mm}^2 = 26.3 \text{ MPa } \textbf{Ans.}$$

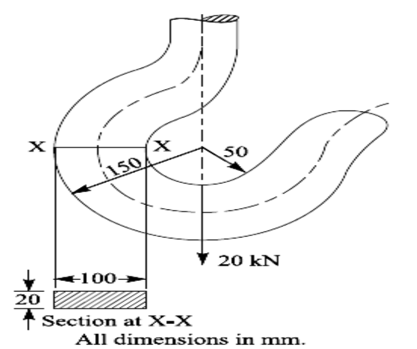
The bending stress is tensile at the inner edge and compressive at the outer edge. The magnitude of both these stresses is 26.3 MPa. At the neutral axis, there is only transverse shear stress. The shear stress at the inner and outer edges will be zero.

We know that *maximum transverse shear stress,

$$\tau_{max} = 1.5 \times \text{Average shear stress} = 1.5 \times \frac{P}{b.t} = 1.5 \times \frac{25 \times 10^3}{77 \times 38.5}$$

$$= 12.65 \text{ N/mm}^2 = 12.65 \text{ MPa } \textbf{Ans.}$$

35.The crane hook carries a load of 20kN as shown in Fig. The section at X-X is rectangular whose horizontal side is 100 mm. Find the stresses in the inner and outer fibers at the given section.



Given Data:

$W = 20KN = 20X10^3$
$R_i = 50$ mm
$R_o = 150$ mm
h=100 mm
b=20mm

To Find:

Stresses in the inner and outer fibres (σ_i&σ_o)

Solution:

We know that area of section at X-X,

$$A=b.h = 20x100=2000mm^2$$

The various distances are shown in fig.

We know that radius of curvature of the neutral axis.

$$R_n = \frac{h}{\log_e\left(\frac{R_o}{R_i}\right)} = \frac{100}{\log_e\left(\frac{150}{50}\right)} = \frac{100}{1.098} = 91.07mm$$

Radius of curvature of the centroidal axis,

$$R = R_i + \frac{h}{2} = 50 + \frac{100}{2} = 100mm$$

Distance between the centroidal axis and neutral axis,

$$e=R-R_n = 100 - 91.07 = 8.93mm$$

Distance between the load and the centroidal axis,

$$x=R=100mm$$

Bending moment about the centroidal axis

$$M=Wxx = 20x10^3x100=2x10^6N\text{-}mm$$

The section X-X is subjected to a direct tensile load of $W=20x10^3N$ and a bending moment of $M=2x10^6N$-mm.

We know that direct tensile stress at section X-X,

$$\sigma_t = \frac{W}{A} = \frac{20x10^3}{2000} = 10N/mm^2 = 10MPa$$

We know that the distance from the neutral axis to the inside fibre,

$$y_i=R_n\text{-}R_i=91.07\text{-}50=41.07mm$$

Distance from the neutral axis outside fibre,

$$y_o=R_o\text{-}R_n=150\text{-}91.07=58.93mm$$

Maximum bending stress at the inside fibre,

$$\sigma_{bi} = \frac{M.y_i}{A.e.R_i} = \frac{2x10^6x41.07}{2000x8.93x50} = 92N/mm^2=92MPa(tensile)$$

Maximum bending stress at the outside fibre,

$$\sigma_{bo} = \frac{M.y_o}{A.e.R_o} = \frac{2x10^6x58.93}{2000x8.93x150} = 44N/mm^2 = 44MPa(compressive)$$

Resultant stress at the inside fibre

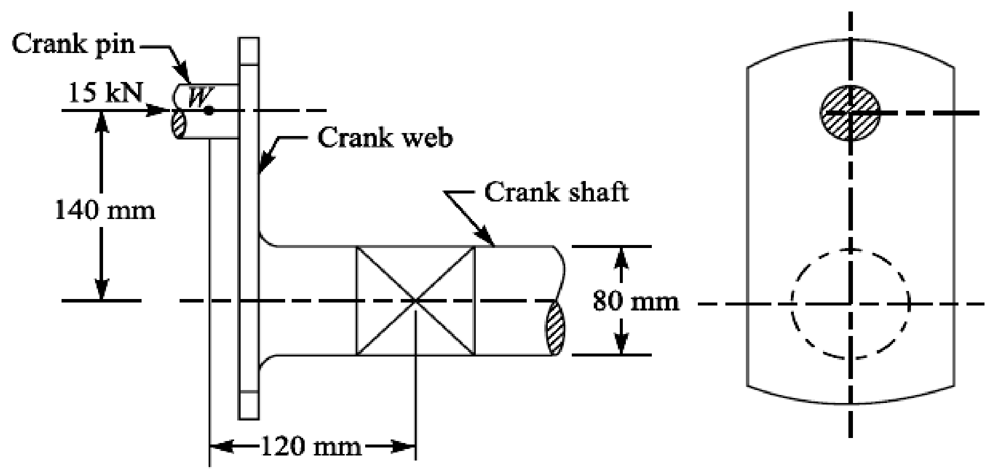
$$= \sigma_t + \sigma_{bi} = 10+92 = 102MPa(tensile)$$

Resultant stress at the outside fibre

$$= \sigma_t - \sigma_{bo} = 10-44 = -34MPa(compressive)$$

THEORIES OF FAILURE

36. An overhang crank with pin and shaft is shown in Figure. A tangential load of 15 kN acts on the crank pin. Determine the maximum principal stress and the maximum shear stress at the centre of the crankshaft bearing. (APRIL/MAY 2010)



Solution. Given : $W = 15$ kN $= 15 \times 10^3$ N ; $d = 80$ mm ; $y = 140$ mm ; $x = 120$ mm

Bending moment at the centre of the crankshaft bearing,

$$M = W \times x = 15 \times 10^3 \times 120 = 1.8 \times 10^6 \text{ N-mm}$$

and torque transmitted at the axis of the shaft,

$$T = W \times y = 15 \times 10^3 \times 140 = 2.1 \times 10^6 \text{ N-mm}$$

We know that bending stress due to the bending moment,

$$\sigma_b = \frac{M}{Z} = \frac{32\,M}{\pi\,d^3} \qquad \ldots\left(\because Z = \frac{\pi}{32} \times d^3\right)$$

$$= \frac{32 \times 1.8 \times 10^6}{\pi\,(80)^3} = 35.8 \text{ N/mm}^2 = 35.8 \text{ MPa}$$

and shear stress due to the torque transmitted,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16 \times 2.1 \times 10^6}{\pi\,(80)^3} = 20.9 \text{ N/mm}^2 = 20.9 \text{ MPa}$$

Maximum principal stress

We know that maximum principal stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

$$= \frac{35.8}{2} + \frac{1}{2}\left[\sqrt{(35.8)^2 + 4\,(20.9)^2}\right] \qquad \text{... (Substituting } \sigma_t = \sigma_b\text{)}$$

$$= 17.9 + 27.5 = 45.4 \text{ MPa } \textbf{Ans.}$$

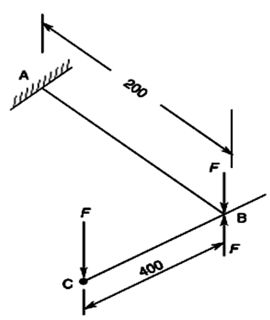
Maximum shear stress

We know that maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(35.8)^2 + 4\,(20.9)^2}\right]$$

$$= 27.5 \text{ MPa } \textbf{Ans.}$$

37.The shaft of an overhang crank is subjected to a force F of 2kN as shown in fig. below. The shaft is made of 30Mn2 steel having a allowable shear strength equal to 100N/mm2. Determine the diameter of the shaft. (April/May 2015)



Solution The stresses are critical at point A, the reason being that at this point combined bending and torsional moments are applied. Assume that two equal and opposite forces of magnitude *F* are applied at point B. The force *F* at a distance of 200 mm from the support is creating bending moment, i.e.

$$M = F \times 200$$
$$= 1.5 \times 200 = 300 \text{ N} \cdot \text{m}$$

The remaining two parallel but opposite forces produce torsion in the shaft. Therefore, torque

$$T = 1.5 \times 400 = 600 \text{ N} \cdot \text{m}$$

Let the diameter of the shaft be d mm.

$$\text{Bending stress, } \sigma_b = \frac{M}{Z} = \frac{300 \times 10^3}{\frac{\pi}{32} \times d^3} = \frac{3055.77 \times 10^3}{d^3} \text{ N/mm}^2$$

$$\text{Shear stress, } \tau = \frac{Tr}{J} = \frac{600 \times 10^3 \times d/2}{\frac{\pi}{32} d^4}$$

$$= \frac{3055.77 \times 10^3}{d^3} \text{ N/mm}^2$$

Maximum shear stress at a point, according to Mohr's circle

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

where

$\sigma_x = \sigma_b$
$\sigma_y = 0$
$\tau_{xy} = \tau$

Thus,

$$\tau_{max} = \sqrt{\left(\frac{3055.77 \times 10^3}{2d^3}\right)^2 + \left(\frac{3055.77 \times 10^3}{d^3}\right)^2}$$

$$= \frac{3416.45 \times 10^3}{d^3} \text{ N/mm}^2$$

For safe design, τ_{max} should be less than or equal to the allowable shear strength. That is,

$$\frac{3416.45 \times 10^3}{d^3} \leq 80$$

or

$$d \geq 34.95 \text{ mm}$$

Let us adopt 35 mm as the shaft diameter.

38.An axle 1 meter long supported in bearings at its ends carries a fly wheel weighing 30 kN at the centre. If the stress (bending) is not to exceed 60 MPa, find the diameter of the axle.

Given :

L=1m=1000mm;
W=30kN=3x10^3N;

σ_b=60MPa=60N/mm^2

To Find:

Diameter of axle, d

The axle with a flywheel is shown in fig. 1.5

Let d=diameter of the axle in mm.

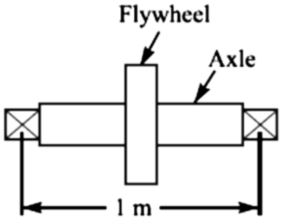


Fig 1.5

Section modulus,

$$Z = \frac{\pi}{32} x d^3 = 0.0982\,\mathrm{d}^3$$

Maximum bending moment at the centre of the axle

$$M = \frac{W.L}{4} = \frac{30x10^3 x1000}{4} = 7.5x10^6\,N - mm$$

We know that bending stress (σ_b),

$$60 = \frac{M}{Z} = \frac{7.5x10^6}{0.0982d^3} = \frac{76.4x10^6}{d^3}$$

$$d^3 = \frac{76.4xd^3}{60} = 1.27x10^6$$

$$d = 108.3mm$$

STRESS CONCENTRATION

39. Define Stress concentration. Give some methods of reducing stress concentration. (MAY/JUNE 2011)(Nov/Dec 2021) (April/May 2023)

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighbourhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration.***

It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc. In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Fig. 1.6

A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the crosssection is changing, a re-distribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

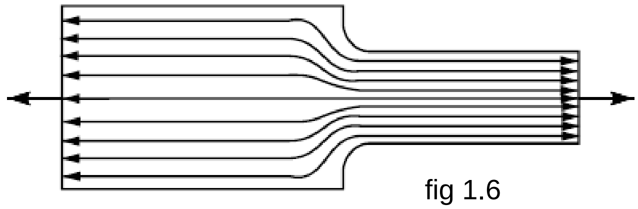


fig 1.6

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically, theoretical or form stress concentration factor,

$$K_t = \text{Maximum stress} / \text{Nominal stress}$$

The value of K_t depends upon the material and geometry of the part.

Methods of reducing stress concentration

Whenever there is a change in cross-section, such as shoulders, holes, notches or keyways and where there is an interference fit between a hub or bearing race and a shaft, then stress concentration results. The presence of stress concentration cannot be totally eliminated but it may be reduced to some extent. A device or concept that is useful in assisting a design engineer to visualize the presence of stress concentration and how it may be mitigated is that of stress flow lines, as shown in Fig 1.7 The mitigation of stress concentration means that the stress flow lines shall maintain their spacing as far as possible.

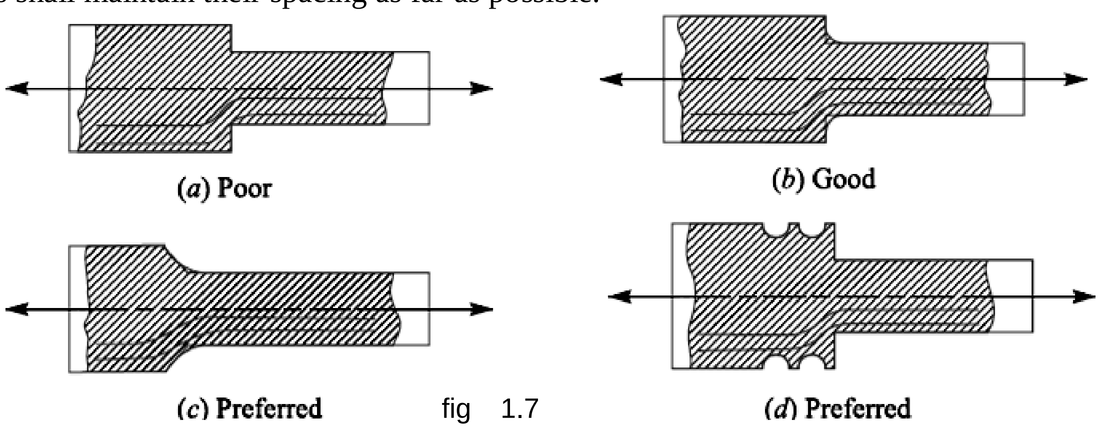


fig 1.7

In Fig. (*a*) we see that stress lines tend to bunch up and cut very close to the sharp re-entrant corner. In order to improve the situation, fillets may be provided, as shown in Fig. (*b*) and (*c*) to give more equally spaced flow lines. Figs show the several ways of reducing the stress concentration in shafts and other cylindrical members with shoulders, holes and threads respectively. It may be noted that it is not practicable to use large radius fillets as in case of ball and roller bearing mountings. In such cases, notches may be cut as shown in Fig. (*d*) and Fig. (*b*) and (*c*).

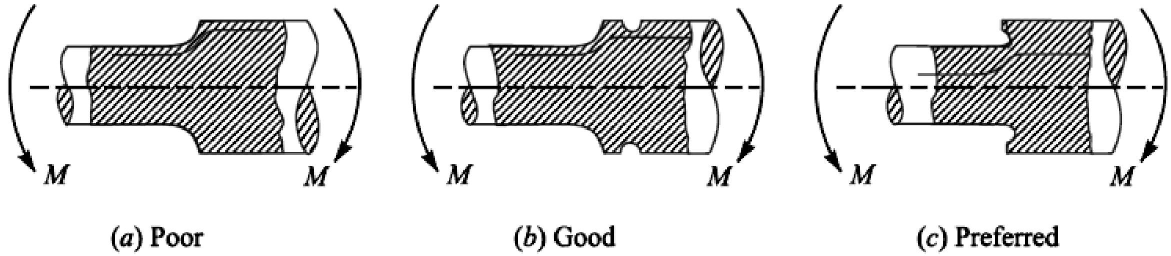
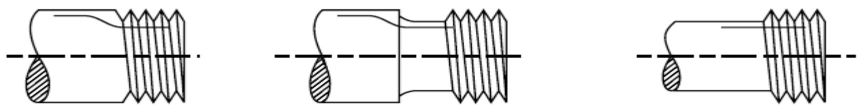


Fig 1.8

The stress concentration effects of a press fit may be reduced by making more gradual transition from the rigid to the more flexible shaft. The various ways of reducing stress concentration for such cases are in fig 1.8 . (*a*), (*b*) and (*c*).



40.Determine the diameter of a circular rod made of ductile material with a fatigue strength (complete stress reversal), σe = 265MPa and a tensile yield strength of 350 MPa. The member is subjected to a varying axial load from Wmin = – 300 × 103 N to Wmax = 700 × 103 N and has a stress concentration factor = 1.8. Use factor of safety as 2.0.

Given:

σ_e = 265MPa=265 N/mm^2

σ_y = 350MPa=350N/mm^2

W_{min}=-300x10^3 N

W_{max}=700x103 N

k_f=1.8; F.S.=2.

To Find:

d= diameter of the circular rod

Solution:

Let d= diameter of the circular rod in mm

$$\text{Area A}=\frac{\pi}{4}d^2 = 0.7854d^2 mm^2$$

We know that the mean load,

$$W_m = \frac{W_{\max} + W_{\min}}{2} = 200x10^3 N$$

Mean stress,

$$\sigma_m = \frac{W_m}{A} = \frac{200x10^3}{0.7854d^2} = \frac{254.6x10^3}{d^2} N/mm^2$$

Variable load,

$$W_v = \frac{W_{\max} - W_{\min}}{2} = 500x10^3 N$$

Variable stress,

$$\sigma_v = \frac{W_v}{A} = \frac{500x10^3}{0.7854d^2} = \frac{636.5x10^3}{d^2} N/mm^2$$

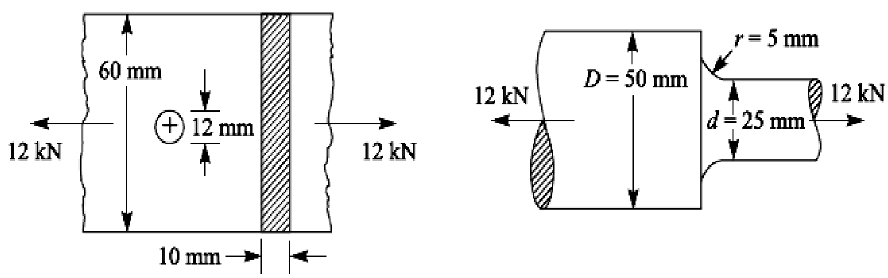
We know that according to Soderberg's formula,

$$\frac{1}{F.S} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v + k_f}{\sigma_e}$$

$$\frac{1}{2} = \frac{254.6x10^3}{d^2x350} + \frac{636.5x10^3}{d^2x265} = \frac{5050}{d^2}$$

$$d = 100.5mm$$

41.Find the maximum stress induced in the following cases taking stress concentration into account: 1. A rectangular plate 60 mm × 10 mm with a hole 12 diameter as shown in Fig. (a) and subjected to a tensile load of 12 kN. 2. A stepped shaft as shown in Fig. (b) and carrying a tensile load of 12 kN.



(a) (b)

Case 1:

Given: b=60mm; t=10mm; d=12mm; W=12kN = 12 x 10^3N

Solution:

W.K.T cross sectional area of the plate,

A = (b-d)t = (60-12)10 = 480 mm^2

Nominal stress

= W/A = 12 x 10^3 / 480 = 25 N/mm^2 = 25 MPa

Ratio of diameter of hole to width of plate,

d/b = 12/60 = 0.2

we find that for d/b = 0.2, theoretical stress concentration factor, k_t = 2.5

Maximum stress = k_t x Nominal stress

= 2.5 x 25 = 62.5 MPa

Case 2:

Given: D=50mm; d = 25 mm; r = 5mm; W = 12kN = 12 x 10^3N

Solution:

W.K.T cross sectional area of the stepped shaft,

$$A = \Pi/4\ d^2 = \Pi/4\ (25)^2 = 491\ mm^2$$

Nominal stress

$$= W/A = 12 \times 10^3 / 491 = 24.4\ N/mm^2 = 24.4\ MPa$$

Ratio of maximum diameter to minimum diameter,

$$D/d = 50/25 = 2$$

Ratio of radius of fillet to minimum diameter,

$$r/d = 5/25 = 0.2$$

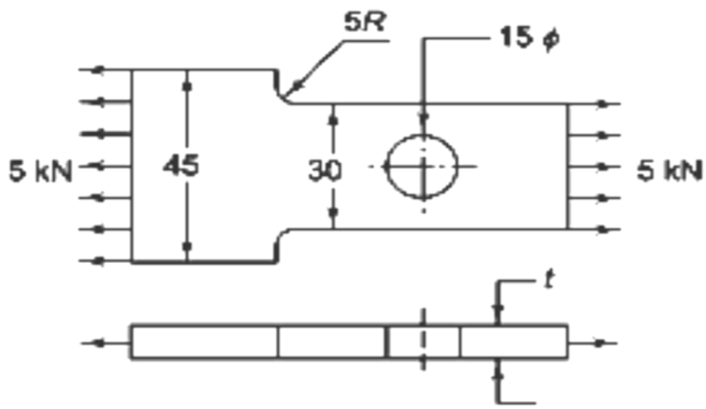
From table 6.3, we find that for

$$D/d = 2 \text{ and } r/d = 0.2,$$

Theoretical stress concentration factor $k_t = 1.64$.

Maximum stress = k_t x Nominal stress = 1.64 x 24.4 = 40 MPa.

42. A flat plate subjected to a tensile force of 5 KN is shown in fig. The plate material is grey cast iron FG 200 and the factor of safety is 2.5. Determine the thickness of the plate.



Solution:

Given:- P = 5 KN; S_{ut} = 200 N/mm²; (fs) = 2.5

Step I

Calculation of permissible tensile stress

$$\sigma_{max} = \frac{S_{ut}}{(fs)} = \frac{200}{2.5} = 80 N/mm^2$$

Step II

Tensile stress at fillet section.

The stress are critical at two sections- the fillet section and hole section. At the fillet section,

$$\sigma_0 = \frac{P}{dt} = \left(\frac{5000}{30t}\right) N/mm^2$$

$$\frac{D}{d} = \frac{45}{30} = 1.5$$

$$\frac{r}{d} = \frac{5}{30} = 0.167$$

$$K_t = 1.8$$

$$\therefore \sigma_{\max} = K_t \sigma_o = 1.8\left(\frac{5000}{30t}\right) = \left(\frac{300}{t}\right) N/mm^2$$

Step III

Tensile stress at hole section

$$\sigma_0 = \frac{P}{(W-d)t} = \frac{5000}{(30-15)t} N/mm^2$$

$$\frac{d}{w} = \frac{15}{30} = 0.5$$

Form fig.,

$$K_t = 2.16$$

$$\sigma_{\max} = K_t \sigma_o = 2.16\left[\frac{5000}{(30-15)t}\right] = \left(\frac{720}{t}\right) N/mm^2$$

Step IV

From (i) and (ii), it is seen that the maximum stress is induced at the hole section.

Equation it with permissible stress, we get

$$\left(\frac{720}{t}\right) = 80$$

Or $t = 9mm$

FACTOR OF SAFETY

43. What is the factor of safety? List the factor considered while deciding the factor of safety. (MAY/JUNE 2014)

It is defined, in general, as the **ratio of the maximum stress to the working stress.** Mathematically,

$$\text{Factor of safety} = \frac{\text{Maximum stress}}{\text{Working or design stress}}$$

In case of ductile materials *e.g.* mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

$$\text{Factor of safety} = \frac{\text{Yield point stress}}{\text{Working or design stress}}$$

In case of brittle materials *e.g.* cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

$$\therefore \quad \text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working or design stress}}$$

This relation may also be used for ductile materials.

Note: The above relations for factor of safety are for static loading.

Selection of Factor of Safety

The selection of a proper factor of safety to be used in designing any machine component depends upon a number of considerations, such as the material, mode of manufacture, type of stress, general service conditions and shape of the parts. Before selecting a proper factor of safety, a design engineer should consider the following points :

1. The reliability of the properties of the material and change of these properties during service ;
2. The reliability of test results and accuracy of application of these results to actual machine parts ;
3. The reliability of applied load ;
4. The certainty as to exact mode of failure ;
5. The extent of simplifying assumptions ;
6. The extent of localised stresses ;
7. The extent of initial stresses set up during manufacture ;
8. The extent of loss of life if failure occurs ; and
9. The extent of loss of property if failure occurs.

Each of the above factors must be carefully considered and evaluated. The high factor of safety results in unnecessary risk of failure. The values of factor of safety based on ultimate strength for different materials and type of load are given in the following table:

44.A machine component is subjected to a flexural stress which fluctuates between + 300 MN/m2 and – 150 MN/m^2. Determine the value of minimum ultimate strength according to 1. Gerber relation; 2.Modified Goodman relation; and 3.Soderberg relation. Take yield strength = 0.55 Ultimate strength; Endurance strength = 0.5 Ultimate strength; and factor of safety = 2. (Nov/Dec 2017)

Given:

σ_1=300 MN/m^2

σ_2 = -150MN/m^2

σ_y=0.55 σ_u

σ_e=0.5 σ_u;

F.S. = 2.

To Find:

Minimum ultimate strength

Solution:

Let σ_u = Minimum ultimate strength in MN/m^2

We know that the mean stress,

$$\sigma_m = \frac{\sigma_1 + \sigma_2}{2} = 75 MN/m^2$$

Variable stress, $\sigma_v = \frac{\sigma_1 - \sigma_2}{2} = 225 MN/m^2$

1. <u>According to Geber relation,</u>

We know that according to Geber relation,

$$\frac{1}{F.S} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \left(\frac{75}{\sigma_u}\right)^2 2 + \frac{225}{0.5\sigma_u}$$

$$\sigma_u = 924.35 MN/m^2$$

2. <u>According to modified Gooman's relation,</u>

We know that according to modified Goodman relation,

$$\frac{1}{F.S} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \frac{75}{\sigma_u} + \frac{225}{0.5\sigma_u}$$

$$\sigma_u = 1050 MN/m^2$$

3. According to Soderberg relation

We know that according to Soderberg relation,

$$\frac{1}{F.S} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \frac{75}{0.55\sigma_u} + \frac{255}{0.5\sigma_u}$$

$$\sigma_u = 1172.72 MN/m^2$$

45.Determine the thickness of a 120 mm wide uniform plate for safe continuous operation if the plate is to be subjected to a tensile load that has a maximum value of 250 kN and a minimum value of 100kN. The properties of the plate material are as follows: Endurance limit stress = 225MPa and Yield point stress = 300MPa. The factor of safety based on yield point may be taken as 1.5.

Given :

b=120mm

W_{max} = 250kN

W_{min}=100kN

σ_e = 225 MPa=225 N/mm^2

σ_y=300MPa=300N/mm^2

F.S.=1.5

To Find:

Thickness t

Solution:

Let t= thickness of the plate in mm.

Area A=bxt=120 t mm^2

We know that mean load,

$$W_m = \frac{W_{\max} + W_{\min}}{2} = 175x10^3 N$$

Mean stress

$$\sigma_m = \frac{W_m}{A} = \frac{175x10^3}{120t} N/mm^2$$

Variable load,

$$W_v = \frac{W_{\max} - W_{\min}}{2} = 75x10^3 N$$

Variable stress,

$$\sigma_v = \frac{W_v}{A} = \frac{75x10^3}{120t} N/mm^2$$

According to Sodderberg's formula

$$\frac{1}{F.S} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{1.5} = \frac{175x10^3}{120tx300} + \frac{75x10^3}{120tx225} = \frac{7.64}{t}$$

$$t = 7.64x1.5 = 11.46mm$$

46.(i) A bar of circular cross-section is subjected to alternating tensile forces varying from a minimum of 200 kN to a maximum of 500 kN. It is to be manufactured of a material with an ultimate tensile strength of 900 MPa and an endurance limit of 700 MPa. Determine the diameter of bar using safety factors of 3.5 related to ultimate tensile strength and 4 related to endurance limit and a stress concentration factor of 1.65 for fatigue load. Use Goodman straight line as basis for design.

Given:

W_{min}=200kN

W_{max} = 500kN

σ_u=900 MPa = 900N/mm^2

σ_e=700MPa=700N/mm^2

$(F.S)_u$=3.5; $(F.S)_e$ = 4

k_f = 1.65.

To Find:

d = diameter of bar in mm

Solution:

Let d = diameter of bar in mm.

$$\text{Area, A}=\frac{\pi}{4}d^2 = 0.7854d^2 mm^2$$

We know that the average force,

$$W_m = \frac{W_{\max} + W_{\min}}{2} = 350x10^3 N$$

Mean stress,

$$\sigma_m = \frac{W_m}{A} = \frac{446x10^3}{d^2} N/mm^2$$

Variable force,

$$W_v = \frac{W_{\max} - W_{\min}}{2} = 150x10^3 N$$

Variable stress,

$$\sigma_v = \frac{W_v}{A} = \frac{191x10^3}{d^2} N/mm^2$$

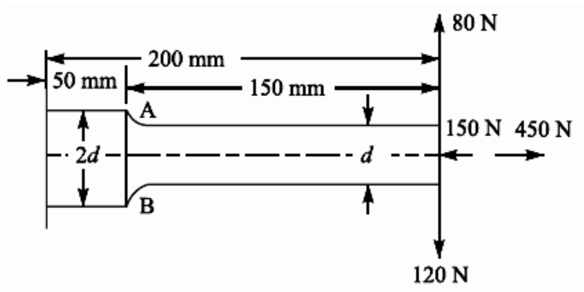
We know that according to Goodman's formula,

$$\frac{\sigma_v}{\sigma_e/(F.S)_e} = 1 - \frac{\sigma_m . k_f}{\sigma_u/(F.S)_u}$$

$$\frac{1100}{d^2} = 1 - \frac{2860}{d^2}$$

d=62.9 mm

47.A steel cantilever is 200 mm long. It is subjected to an axial load which varies from 150 N (compression) to 450 N (tension) and also a transverse load at its free end which varies from 80 N up to 120 N down. The cantilever is of circular cross-section. It is of diameter 2d for the first 50 mm and of diameter d for the remaining length. Determine its diameter taking a factor of safety of 2.Assume the following values: Yield stress = 330MPa; Endurance limit in reversed loading = 300MPa; Correction factors = 0.7 in reversed axial loading = 1.0 in reversed bending; Stress concentration factor = 1.44 for bending = 1.64 for axial loading; Size effect factor = 0.85; Surface effect factor = 0.90; Notch sensitivity index = 0.90 (May/June 16)



Given : l=200 mm;

$W_{a(max)}$ = 450 N; $W_{a(min)}$ = - 150 N; $W_{t(max)}$ = 120 N;

$W_{t(min)}$ = -80 N; F.S = 2;

σ_y=330 MPa = 330 N/mm^2;σ_e = 300MPa = 300 N/mm^2; k_a = 0.7; k_b=1;

k_{tb}=1.44; k_{ta}=1.64; k_{sz}=0.85; k_{sur}=0.90; q=0.90.

Solution:

Let us first consider the reverse axial load.

We know that mean axial load,

$$W_m = \frac{W_{a(max)} + W_{a(min)}}{2} = 150N$$

Variable axial load,

$$W_m = \frac{W_{a(max)} - W_{a(min)}}{2} = 300N$$

Mean axial stress,

$$\sigma_m = \frac{W_m}{A} = \frac{191}{d^3} N/mm^2$$

Variable axial stress,

$$\sigma_v = \frac{W_v}{A} = \frac{382}{d^3} N/mm^2$$

We know that fatigue stress concentration factor for reversed axial loading,

$$k_{fa}=1+q(k_{ta}-1) = 1+0.9(1.64-1)=1.576$$

Endurance limit stress for reversed axial loading,

$$\sigma_{ea}= \sigma_e \text{ x } k_a= 300\text{x}0.7 = 210 \text{ N/mm}^2$$

We know that equivalent normal stress at point A due to axial loading,

$$\sigma_{nea} = \sigma_m + \frac{\sigma_v x \sigma_y x k_{fb}}{\sigma_{eb} x k_{sur} x k_{sz}} = \frac{1428}{d^2} N/mm^2$$

Let us consider the reversed bending due to transverse load.

We know that mean bending load,

$$W_m = \frac{W_{t(max)} + W_{t(min)}}{2} = 20N$$

Variable bending load,

$$W_v = \frac{W_{t(max)} - W_{t(min)}}{2} = 100N$$

Bending moment at point A,

$$M_m = W_m(l-50) = 20(200-50) = 3000 \text{ N-mm}$$

Variable bending moment at point A,

$$M_v = W_v(l-50) = 100(200-50) = 15000 \text{ N-mm}$$

We know that section modulus, $Z = \frac{\pi}{32} x d^3 = 0.0982 d^3 mm^3$

Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{30550}{d^3} N/mm^2$$

Variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{152750}{d^3} N/mm^2$$

We know that fatigue stress concentration factor for reversed bending,

$$K_{fb} = 1 + q(k_{tb} - 1) = 1 + 0.9(1.44\ 3 - 1) = 1.396$$

Since the correction factor for reversed bending load is 1 (i.e, k_b=1), therefore the endurance limit for bending load,

$$\sigma_{eb} = \sigma_e x k_b = \sigma_e = 300 N/mm^2$$

We know that equivalent normal stress at point A due to bending,

$$\sigma_{neb} = \sigma_m + \frac{\sigma_v x \sigma_y x k_{fb}}{\sigma_{eb} x k_{sur} x k_{sz}} = \frac{337168}{d^3} N/mm^2$$

Total equivalent normal stress at point A,

$$\sigma_{ne} = \sigma_{neb} + \sigma_{nea} = \frac{337168}{d^2} + \frac{1428}{d^2} \text{ N}/mm^2 \qquad \text{(i)}$$

We know that equivalent normal stress at point A,

$$\sigma_{ne} = \frac{\sigma_y}{F.S} = \frac{330}{2} \text{ N}/mm^2 \qquad \text{(ii)}$$

Equating equations (i) & (ii), we have

$$165 = \frac{337168}{d^2} + \frac{1428}{d^2} \text{ N}/mm^2$$

d= 12.9 mm.

48.A pulley is keyed to a shaft midway between two bearings. The shaft is made of cold drawn steel for which the ultimate strength is 550 MPa and the yield strength is 400 MPa. The bending moment at the pulley varies from – 150 N-m to + 400 N-m as the torque on the shaft varies from – 50 N-m to + 150 N-m. Obtain the diameter of the shaft for an indefinite life. The stress concentration factors for the keyway at the pulley in bending and in torsion are 1.6 and 1.3 respectively. Take the following values: Factor of safety = 1.5 Load correction factors = 1.0 in bending, and 0.6 in torsion Size effect factor = 0.85 Surface effect factors = 0.88 (N/D – 2012) (N/D'22)

Given:

M_{min} = – 150 N-m; M_{max} = 400 N-m ;
T_{min} = – 50 N-m ; T_{max} = 150 N-m ;
K_{fb} = 1.6 ; K_{fs} = 1.3 ; F.S. = 1.5 ; Kb = 1 ; Ks = 0.6 ; Ksz = 0.85 ; Ksur = 0.88

To Find:

Shaftdiameter, d

Solution:

Let d = Diameter of the shaft in mm.
First of all, let us find the equivalent normal stress due to bending.
We know that the mean or average bending moment,
Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{400 + (-150)}{2} = 125\ N - m = 125 \times 10^3 N/mm$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{400 - (-150)}{2} = 275\ N - m = 275 \times 10^3 N/mm$$

Section modulus,

$$Z = \frac{\pi}{32} d^3 = 0.0982\ d^3 m^3$$

Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{1125 \times 10^3}{0.0982 d^3} = \frac{1273 \times 10^3}{d^3}\ N/m^2$$

And variable bending stress

$$\sigma_v = \frac{M_v}{Z} = \frac{275 \times 10^3}{0.0982 d^3} = \frac{2800 \times 10^3}{d^3}\ N/m^2$$

Assuming the endurance limit in reversed bending as one-half the ultimate strength and since the load correction factor for reversed bending is 1 (i.e. Kb = 1), therefore endurance limit in reversed bending,

$$\sigma_{ep} = \sigma_e = \frac{\sigma_u}{2} = \frac{550}{2} = 275\ N/mm^2$$

Since there is no reversed axial loading, therefore equivalent normal stress due to bending,

$$\sigma_{nep} = \sigma_{ne} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{ep} \times K_{sur} \times K_{sz}}$$

$$= \frac{1273 \times 10^3}{d^3} + \frac{2800 \times 10^3 \times 400 \times 1.6}{d^3 \times 275 \times 0.88 \times 0.85}$$

$$= \frac{1273 \times 10^3}{d^3} + \frac{8712 \times 10^3}{d^3} = \frac{9985 \times 10^3}{d^3} N/mm^2$$

Now let us find the equivalent shear stress due to torsional moment. We know that the mean torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{150 + (-50)}{2} = 50\ N - m = 50 \times 10^3\ N - mm$$

And variable torque

$$T_v = \frac{T_{max} - T_{min}}{2} = \frac{150 - (-50)}{2} = 100\ N - m = 100 \times 10^3\ N - mm$$

Mean shear stress

$$\tau_m = \frac{16T_m}{\pi d^3} = \frac{16 \times 50 \times 10^3}{\pi d^3} = \frac{225 \times 10^3}{d^3}\ N/m^2$$

And variable shear stress,

$$\tau_v = \frac{16T_v}{\pi d^3} = \frac{16 \times 110 \times 10^3}{\pi d^3} = \frac{510 \times 10^3}{d^3}\ N/m^2$$

Endurance limits stress for reversed torsional or shear loading,

$$\tau e = \sigma e \times Ks = 275 \times 0.6 = 165\ N/mm^2$$

Assuming yield strength in shear,

$$\tau y = 0.5\ \sigma y = 0.5 \times 400 = 200\ N/mm^2$$

We know that equivalent shear stress,

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

$$= \frac{225 \times 10^3}{d^3} + \frac{510 \times 10^3 \times 200 \times 1.3}{d^3 \times 165 \times 0.88 \times 0.85}$$

$$= \frac{225 \times 10^3}{d^3} + \frac{1074 \times 10^3}{d^3} = \frac{1329 \times 10^3}{d^3}\ N/mm^2$$

$$\tau_{es(max)} = \frac{\tau_y}{FOS} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4(\tau_{es})^2}$$

$$\frac{200}{1.5} = \frac{1}{2}\sqrt{\left(\frac{9985 \times 10^3}{d^3}\right)^2 + 4}\left(\frac{1329 \times 10^3}{d^3}\right)^2 = \frac{5165 \times 10^3}{d^3}$$

$$d^3 = \frac{15165 \times 10^3 \times 1.5}{200} = 38740\ or\ \boldsymbol{d = 33.84\ say\ 35\ mm}$$

49.A mild steel bracket as shown in Fig., is subjected to a pull of 6000 N acting at 45° to its horizontal axis. The bracket has a rectangular section whose depth is twice the thickness. Find the cross-sectional dimensions of the bracket, if the permissible stress in the material of the bracket is limited to 60 MPa. (NOV/DEC 2007) April/May'2023)

Solution. Given : $P = 6000$ N ; $\theta = 45°$; $\sigma = 60$ MPa $= 60$ N/mm^2

Let t = Thickness of the section in mm, and

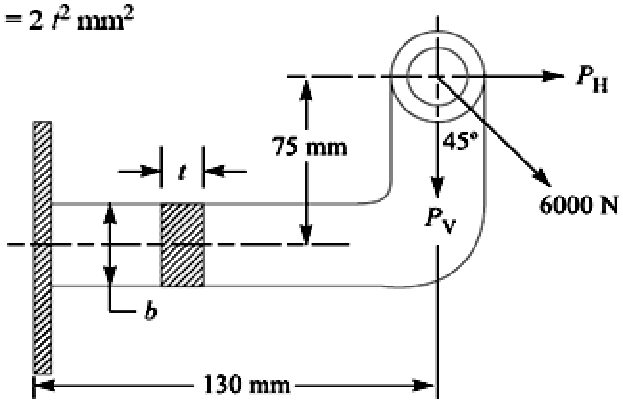
b = Depth or width of the section $= 2\,t$ ...(Given)

We know that area of cross-section,

$$A = b \times t = 2\,t \times t = 2\,t^2 \text{ mm}^2$$

and section modulus,

$$Z = \frac{t \times b^2}{6} = \frac{t\,(2t)^2}{6} = \frac{4\,t^3}{6} \text{ mm}^3$$



Horizontal component of the load,

$$P_H = 6000 \cos 45° = 6000 \times 0.707 = 4242 \text{ N}$$

$\therefore$ Bending moment due to horizontal component of the load,

$$M_H = P_H \times 75 = 4242 \times 75 = 318\ 150 \text{ N-mm}$$

A little consideration will show that the bending moment due to the horizontal component of the load induces tensile stress on the upper surface of the bracket and compressive stress on the lower surface of the bracket. ∴ Maximum bending stress on the upper surface due to horizontal component,

$$\sigma_{bH} = \frac{M_H}{Z} = \frac{318\ 150 \times 6}{4\,t^3} = \frac{477\ 225}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

Vertical component of the load,

$$P_V = 6000 \sin 45° = 6000 \times 0.707 = 4242 \text{ N}$$

$\therefore$ Direct stress due to vertical component,

$$\sigma_{oV} = \frac{P_V}{A} = \frac{4242}{2t^2} = \frac{2121}{t^2} \text{ N/mm}^2 \text{ (tensile)}$$

Bending moment due to vertical component of the load,

$$M_V = P_V \times 130 = 4242 \times 130 = 551\ 460 \text{ N-mm}$$

This bending moment induces tensile stress on the upper surface and compressive stress on the lower surface of the bracket.

∴ Maximum bending stress on the upper surface due to vertical component,

$$\sigma_{bV} = \frac{M_V}{Z} = \frac{551\,460 \times 6}{4\,t^3} = \frac{827\,190}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

and total tensile stress on the upper surface of the bracket,

$$\sigma = \frac{477\,225}{t^3} + \frac{2121}{t^2} + \frac{827\,190}{t^3} = \frac{1\,304\,415}{t^3} + \frac{2121}{t^2}$$

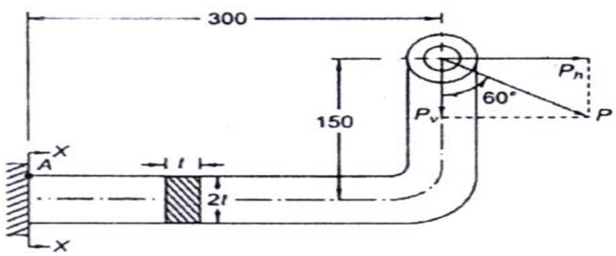
Since the permissible stress (σ) is 60 N/mm^2, therefore

$$\frac{1\,304\,415}{t^3} + \frac{2121}{t^2} = 60 \quad \text{or} \quad \frac{21\,740}{t^3} + \frac{35.4}{t^2} = 1$$

∴ t = 28.4 mm **Ans.** ... (By hit and trial)

and $b = 2t = 2 \times 28.4 = 56.8$ mm **Ans.**

50. A wall bracket with rectangular cross section is shown in figure . The depth of the cross section is twice that of the width. The force P acting on the bracket at 60°to the vertical is 5 KN. The material of the bracket is Grey Cast iron FG 200 and the factor of safety is 3.5. Determine the dimensions of the cross sections of the bracket. Assume maximum normal stress theory of failure. (Aprl/May 2018)



Solution

The stress is maximum at point A in section XX. The point is subjected to combined bending and direct tensile stresses. The force P is resolved into two components—horizontal component P_h and vertical component P_v.

$$P_h = P \sin 60° = 10(10)^3 \sin 60°$$

$$P_h = 8660.25 \text{ N}$$

$$P_v = P \cos 60° = 10(10)^3 \cos 60°$$

$$P_v = 5000 \text{ N}$$

The bending moment at section XX is given by

$$M_b = P_h(150) + P_v(300) \text{ N.mm} = 8660.25(150) + 5000(300) = 10(10)^3 \cos 60°$$

$$M_b = 2799(10)^3 \text{ N.mm}$$

$$\sigma_b = \frac{M_b y}{I} = \frac{2799(10)^3(t)}{\left[\left(\frac{1}{12}\right)t(2t)^3\right]}$$

$$= \frac{4198.5(10)^3}{t^3} \text{ N/mm}^2$$

The direct tenstile stress due to component P_h is given by

$$\sigma_t = \frac{P_h}{A} = \frac{8660.5}{t^2} \text{ N/mm}^2$$

The vertical component P_v induces shear stress at section XX. It is however small and neglected. The resultant tenstile stress σ_{max} at point A is given by

$$\sigma_{max} = \sigma_b + \sigma_t = \frac{4198.5(10)^3}{t^3} + \frac{8660.25(10)^3}{t^2} \text{ N/mm}^2$$

According to maximum normal stress theory, the maximum permissible tenstile stress is given by

$$[\sigma_{max}] = \frac{\sigma_{ut}}{(f_s)}$$

$$= \frac{200}{3.5}$$

$$= 57.14 \text{ N/mm}^2$$

$$\sigma_{max} = \frac{4198.5(10)^3}{t^3} + \frac{8660.25(10)^3}{t^2} = 57.14$$

$$t^3 - 151.5t - 73477.4 = 0$$

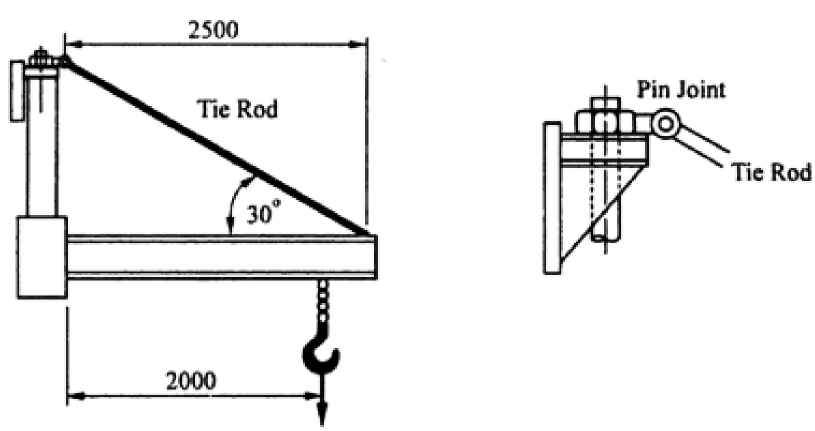
$$[\because \sigma_{max} = [\sigma_{max}]]$$

By trial and error method

$$t = 43.25 \text{ mm} \approx 44 \text{ mm}.$$

The dimension of the cross-section are 45 × 90 mm.

51.A wall crane with a pin- joint tie rod is as shown in fig. The crane hook is to take a maximum load 35 KN, when the load is at a distance of 2m from the wall. The tie rod and pin are made of steel FeG 250 (S_{yt} = 250 N/mm^2) and the factor of safety is 5. Calculate the diameter of the tie rod and the pin.(April/May-17)



Givendata:

Load F= 35KN, Distance from the wall = 2m
S_{yt} = 250 N/mm^2 ,Factor Of Safety = 5

To find:

i) Diameter of the tie rod, ii) Diameter of the pin

Solution:

$$\sigma_{perm} = \frac{S_{yt}}{fos} = \frac{250}{5} = 50 \text{ N/mm}^2$$

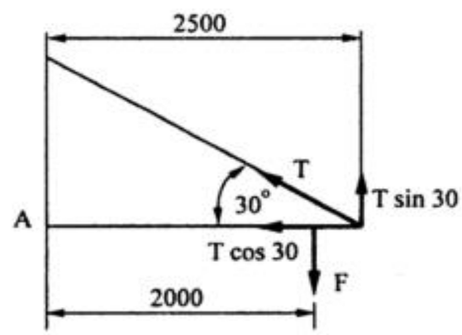


Fig.

Taking moment about A,

$$F \times 2000 = T \sin 30 \times 2500$$
$$35 \times 10^3 \times 2000 = T \sin 30 \times 2500$$
$$T = 56 \times 10^3 \text{ N}$$

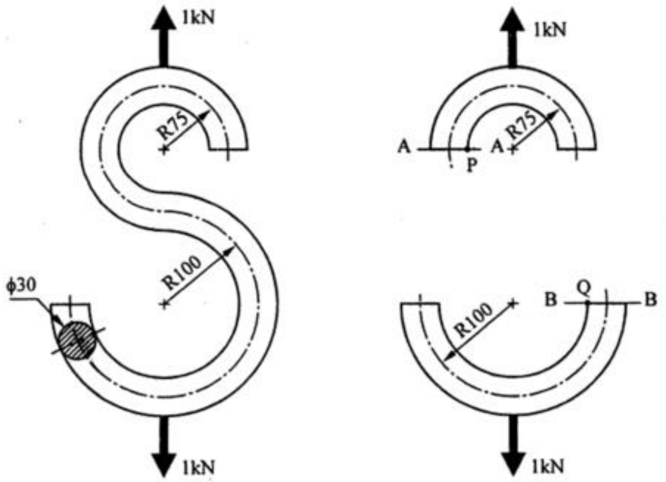
The tie rod is subjected to a tensile pull, T. Let the tie rod be of diameter 'd'.

$$\therefore \quad \sigma_{perm} = \frac{T}{A} = \frac{4T}{\pi d^2}$$
$$50 = \frac{4 \times 56 \times 10^3}{\pi d^2}$$
$$d = 37.763 \text{ mm} \approx 38 \text{ mm} \quad \ldots \text{Ans.}$$

Design of Pin : The pin is considered to be subjected to a double shear. Let the pin diameter be d_P.

$$\tau_{perm} = \frac{\sigma_{perm}}{2} = \frac{50}{2} = 25 \text{ N/mm}^2$$
$$\therefore \quad \tau_{perm} = \frac{4T}{2 \times \pi d_P^2} = \frac{2T}{\pi d_P^2}$$
$$25 = \frac{2 \times 56 \times 10^3}{\pi d_P^2}$$
$$d_P = 37.763 \text{ mm} \approx 38 \text{ mm} \quad \ldots \text{Ans.}$$

52.A link is shaped in the form of a letter S is made up of 30mm diameter bar, as shown in fig. Determine the maximum tensile stress and maximum shear stress in the link.(April/May-17)



Given:

Force F = 1KN, Diameter d = 30mm,

Radius of smaller bend r_i = 75mm, Radius of bigger bend r_o = 100mm

To find:

Maximum tensile stress and Maximum shear stress

Solution:

$$F = 1000\text{ N}$$
$$d = 30\text{ mm}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 30^2 = 706.858\text{ mm}^2$$

$$\sigma_D = \frac{F}{A} = \frac{1000}{706.858} = 1.415\text{ N/mm}^2$$

From Fig. , it is seen that the critical sections of the curved beam are Section A-A and Section B-B. From the previous examples on curved beams, it is observed that the bending stress increases with reduction in curvature. In this case, the smaller curvature at Section A-A, will have smaller bending moment, and the larger curvature at Section B-B will have larger bending moment. Hence both the sections need to be analysed.

Section A-A : It is observed that point P is subjected to eccentric loading, which causes a direct tensile stress and bending stress, (also tensile at point P).

$$M_B = F \times 75 = 75 \times 10^3\text{ Nmm}$$
$$r_G = 75\text{ mm}$$
$$r_i = 75 - 15 = 60\text{ mm}$$
$$r_o = 75 + 15 = 90\text{ mm}$$

$$r_N = \frac{\left(\sqrt{r_o}+\sqrt{r_i}\right)^2}{4} = \frac{\left(\sqrt{90}+\sqrt{60}\right)^2}{4}$$

$$= 74.242 \text{ mm}$$

$$e = r_G - r_N = 75 - 74.242$$

$$= 0.758 \text{ mm}$$

$$y_i = r_N - r_i = 74.242 - 60$$

$$= 14.242 \text{ mm}$$

$$\sigma_y = \sigma_D + \frac{M_B y_i}{A e r_i}$$

$$= 1.415 + \frac{75\times10^3\times14.242}{\quad}$$

Section B-B : It is observed that point Q is subjected to eccentric loading, which causes a direct tensile stress and bending stress, (also tensile at point Q).

$$M_B = F \times 100 = 100 \times 10^3 \text{ Nmm}$$

$$r_G = 100 \text{ mm}$$

$$r_i = 100 - 15 = 85 \text{ mm}$$

$$r_o = 100 + 15 = 115 \text{ mm}$$

$$r_N = \frac{\left(\sqrt{r_o}+\sqrt{r_i}\right)^2}{4} = \frac{\left(\sqrt{115}+\sqrt{85}\right)^2}{4}$$

$$= 99.434 \text{ mm}$$

$$e = r_G - r_N = 100 - 99.434$$

$$= 0.566 \text{ mm}$$

$$y_i = r_N - r_i = 99.434 - 85$$

$$= 14.434 \text{ mm}$$

$$\therefore \quad \sigma_y = \sigma_D + \frac{M_B y_i}{A e r_i}$$

$$= 1.415 + \frac{100\times10^3\times14.434}{706.858\times0.566\times85}$$

$$= 43.859 \text{ N/mm}^2 \qquad \ldots \text{(b)}$$

Therefore comparing results (a) and (b), it is observed that the maximum normal stress will be at point Q and its value will be 43.859 N/mm² ... Ans.

$$\therefore \quad \tau_{max} = \frac{\sigma_y}{2} = \frac{43.859}{2}$$

$$= 21.930 \text{ N/mm}^2 \qquad \text{(at point Q)} \ldots \text{Ans.}$$

53. A solid circular shaft of diameter 45 mm is loaded by bending moment 650 Nm torque 900 Nm and an axial tensile force of 30 KN the shaft metal is ductile with yield strength of 280 MPa. Determine the factor of safety according to Maximum principle stress, Tresca and Von misses theories of failure. (April/May 17)

Given:

Shaft Dia = 45mm, Bending moment M = 650 Nm,

Tordue = 900Nm,tensile load = 30KN, yield strength= 280 Mpa

To find:

Factor of safety according to the following,

Max.principal stress, von misses theories of failure, Tresca theories of failure

SOLUTION:

Stress due to axial compressive load F = 30 kN

$$\sigma_a = -\frac{4F}{\pi d^2} = -\frac{4 \times 30{,}000}{\pi \times 45^2} = 18.863 \text{ N/mm}^2 \text{ (compressive)}$$

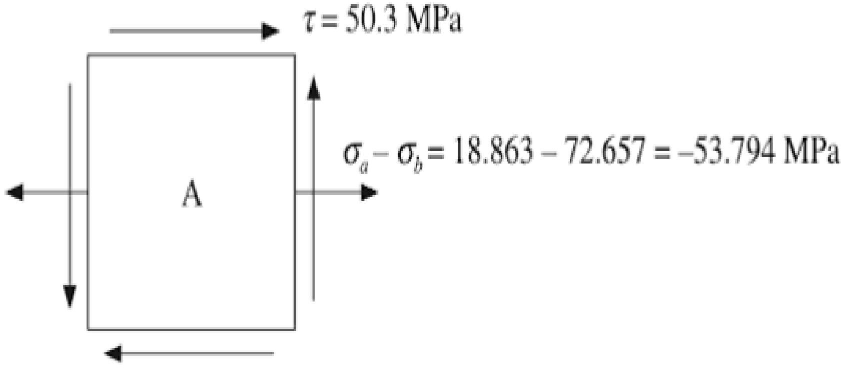
Stress due to bending moment M = 650 N·m

$$\sigma_b = \frac{32M}{\pi d^3} = \frac{32 \times 650{,}000}{\pi \times 45^3} = 72.657 \text{ N/mm}^2$$

Due to torque T = 900 N·m

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 900{,}000}{\pi \times 45^3} = 50.30 \text{ N/mm}^2$$

Combined stress on an element A is



There is a plane stress case

$$\sigma_{xx} = -53.794 \text{ MPa} \quad \sigma_{yy} = \sigma_{zz} = 0$$
$$\sigma_{xy} = 50.3 \text{ MPa} \quad \sigma_{yz} = \sigma_{zx} = 0$$

The principal stresses are

$$\sigma_{1,3} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau^2}$$

$$\sigma_{1,3} = \frac{-53.794 + 0}{2} \pm \sqrt{\left(\frac{-53.794 - 0}{2}\right)^2 + (50.3)^2}$$

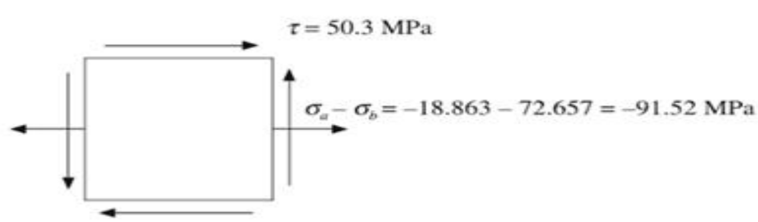
$$\sigma_{1,3} = -26.897 \pm 57.03$$
$$\sigma_1 = 30.133 \text{ MPa}$$
$$\sigma_3 = -83.927 \text{ MPa}$$

and $\sigma_2 = 0$

Maximum shear stress theory

$$N = \frac{\sigma_y}{\sigma_1 - \sigma_3} = \frac{280}{30.133 - (-83.927)} = 2.45$$

Combined stress on an element B just opposite to A is



This is a plane stress case.

$$\sigma_{xx} = -91.52 \text{ MPa} \quad \sigma_{yy} = \sigma_{zz} = 0$$
$$\sigma_{xy} = 50.3 \text{ MPa} \quad \sigma_{yz} = \sigma_{zx} = 0$$

The principal stresses are

$$\sigma_{1,3} = \frac{-91.52 + 0}{2} \pm \sqrt{\left(\frac{-91.52 - 0}{2}\right)^2 + (50.3)^2}$$

$$\sigma_{1,3} = -45.76 \pm 68.0$$
$$\sigma_1 = 22.24 \text{ MPa}$$
$$\sigma_3 = -113.76 \text{ MPa}$$

and $\sigma_2 = 0$

Maximum shear stress theory

$$N = \frac{\sigma_y}{\sigma_1 - \sigma_3} = \frac{280}{22.24 - (-113.76)} = 2.06$$

54. A 50 mm diameter shaft is made from carbon steel having ultimate tensile strength of 630 MPa. It is subjected to a torque which fluctuates between 2000 N-m to – 800 N-m. Using Soderberg method, calculate the factor of safety. Assume suitable values for any other data needed. (April/May 2019)

Solution. Given : $d = 50$ mm ; $\sigma_u = 630$ MPa $= 630$ N/mm^2 ; $T_{max} = 2000$ N-m ; $T_{min} = -800$ N-m

We know that the mean or average torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{2000 + (-800)}{2} = 600 \text{ N-m} = 600 \times 10^3 \text{ N-mm}$$

$\therefore$ Mean or average shear stress,

$$\tau_m = \frac{16\,T_m}{\pi d^3} = \frac{16 \times 600 \times 10^3}{\pi (50)^3} = 24.4 \text{ N/mm}^2 \qquad \ldots\left(\because\ T = \frac{\pi}{16} \times \tau \times d^3\right)$$

Variable torque,

$$T_a = \frac{T_{max} - T_{min}}{2} = \frac{2000 - (-800)}{2} = 1400 \text{ N-m} = 1400 \times 10^3 \text{ N-mm}$$

$\therefore$ Variable shear stress, $\tau_a = \dfrac{16\,T_a}{\pi d^3} = \dfrac{16 \times 1400 \times 10^3}{\pi (50)^3} = 57 \text{ N/mm}^2$

Since the endurance limit in reversed bending (σ_e) is taken as one-half the ultimate tensile strength (*i.e.* $\sigma_e = 0.5\,\sigma_u$) and the endurance limit in shear (τ_e) is taken as $0.55\,\sigma_e$, therefore

$$\tau_e = 0.55\,\sigma_e = 0.55 \times 0.5\,\sigma_u = 0.275\,\sigma_u$$

$$= 0.275 \times 630 = 173.25 \text{ N/mm}^2$$

Assume the yield stress (σ_y) for carbon steel in reversed bending as 510 N/mm^2, surface finish factor (K_{sur}) as 0.87, size factor (K_{sz}) as 0.85 and fatigue stress concentration factor (K_{fs}) as 1.

Since the yield stress in shear (τ_y) for shear loading is taken as one-half the yield stress in reversed bending (σ_y), therefore

$$\tau_y = 0.5\,\sigma_y = 0.5 \times 510 = 255 \text{ N/mm}^2$$

Let $\quad F.S.$ = Factor of safety.

We know that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} = \frac{24.4}{255} + \frac{57 \times 1}{173.25 \times 0.87 \times 0.85}$$

$$= 0.096 + 0.445 = 0.541$$

$\therefore \quad F.S. = 1 / 0.541 = 1.85$ **Ans.**

55. A 50mm wide , 5mm high rectangular plate has a 5mm diameter central hole. The allowable stress due to applying a tensile force is 700MPa. Find (a) the maximum tensile force that can be applied; (b) the maximum bending moment that can be applied; (c) the maximum tensile force and bending moment if the hole is not present. Express the results as a ratio when compared to parts (i) and (ii).

Solution $\frac{d}{b}=\frac{5}{50}=0.1$ $\quad Area = A = (b-d)h = 0.225\times10^{-3}m^2$

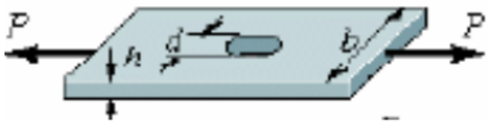


fig 1.9

From the Figure 1.9

$K_t = 2.70 \qquad P_{Max} = \frac{\sigma_{Allowable}A}{K_t} = 58.33kN$

Without a hole

$$Area = bh = 0.25\times10^{-3}m^2$$

$$P_{Max} = \sigma_{Allowable}\times A = 175kN$$

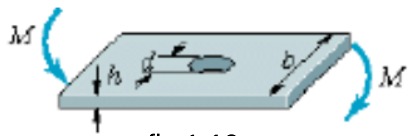
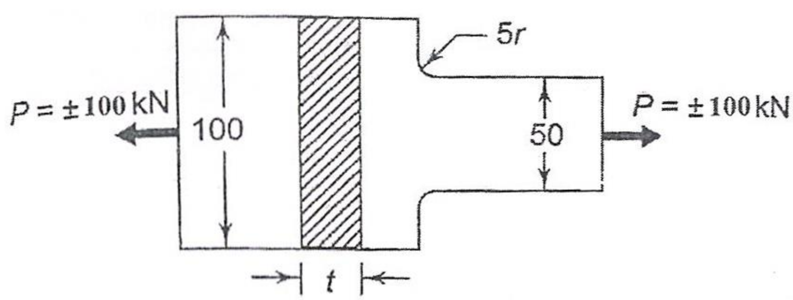


fig 1.10

$\frac{d}{b}=0.1 \qquad \frac{d}{h}=1 \qquad K_t = 2.04 \qquad M_{Max} = \frac{Ah\sigma_{Allow}}{6K_t} = 64.34N.m$

Without a hole $\quad M_{Max} = \frac{\sigma_{Allow}bh^2}{6} = 145.8N.m$

56. A component machined from a plate made of 45C8 (σu= 650 MPa) as shown in Fig. It is subjected to a complete reversed axial force of 100 kN. The reliability factor, kc = 0.897; factor of safety = 2. The size factor, kb = 0.8, surface finish factor, ka = 0.76. Determine the thickness of the plate, for infinite life, if the notch sensitivity factor, q = 0.8. Nov/Dec-2020,Aprl/May-2021



All dimension are in “mm”

Give Data:

σ_u = 650 Mpa, P_{max} = 100 KN, P_{min} = -100 KN, n = 2, Q = 0.8, Reliability = 0.897

Solution:

The thickness fillet section is critical the thickness (+) at that section is to be calculate

Mean Load **p_m** $= \dfrac{P_{max} + P_{min}}{2} = \dfrac{100x10^3 - 100x10^3}{2} = 0\,N$

Variable Load

p_a $= \dfrac{P_{max} - P_{min}}{2} = \dfrac{100x10^3 - (-100x10^3)}{2} = 100000\,N$

Mean stress σ_m= 0 N/mm^2

Variable Stress

σ_a= P_a / A = 100000/50x t = 2000/t N/mm^2

Flat bar with a should fillet in tension (From PSG Data 7.9)

D/d = 100/2 =2 and r/d = 5/50 =0.1

kf = 1+ q (kf-1) = 1+ 0.8 (2.275 -1)

kf = 2.02

from solving equation:

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + kf\,\frac{\sigma_u}{\sigma - 1kR}$$

$\sigma - 1$= 0.36 X 650 (PSG data 1.42)

$\sigma - 1$= 234 N/mm^2

$$\frac{1}{2} = \frac{0}{\sigma_y} + k\,\frac{\frac{2000}{t}}{234\,x\,0.897} = \frac{1}{2} = \frac{9.528}{t}\;\; t = 19.056mm$$

57.A manufacturer in starting a business with five difference models of tractors ranging from 7.5 to 75 KW capacities. Specify power capacities of the models. There is an expansion plan to further increase the number of models from five to nine fulfil the requirements of farmers. Specify the power capacities of the additional models(NOV/DEC'2022)

Solution

P_{min}=7.5KW

P_{max}=75kw

$$Ø = \left(\frac{Pmax}{Pmin}\right)^{(\frac{1}{n}-1)}$$

N=7

$Ø_1=\frac{75}{7.5}^{(1/7-1)}=10^{(1/6)}=1.4677$

$Ø_1=\frac{75}{7.5}^{(1/9-1)}=10^{(1/8)}=1.333$

For n=7	**For n=9**
P_1=7.5KW	P_1=7.5KW
P_2=11 KW	P_2=10KW
P_3=16.156KW	P_3=13.33KW
P_4=23.71KW	P_4=17.77KW
P_5=34.80KW	P_5=23.87KW
P_6=51.07KW	P_6=31.60KW
P_7=75KW	P_7=75KW
	P_8=56.17KW
	P_9=75KW

58.Determine the Numbers of R20/4(100…,1000) derived series(NOV/DEC'22)

Solution : Given data :

R 20/4 series, First number = 100, Last number = 1000

To find : Numbers of series

Step - 1 : Calculate the step ratio factor for given series

The step ratio factor is,

$$\phi = \sqrt[20/4]{10} = 1.5848$$

Step - 2 : Calculate the numbers of derived series

The numbers of derived series are ;

1^{st} number = **100**

2^{nd} number = $100\times\phi = 100\times1.5848 = 158.48 =$ **160**

3^{rd} number = $100\times\phi^2 = 100\times1.5848^2 = 251.1591 =$ **250**

4^{th} number = $100\times\phi^3 = 100\times1.5848^3 = 398.0369 =$ **400**

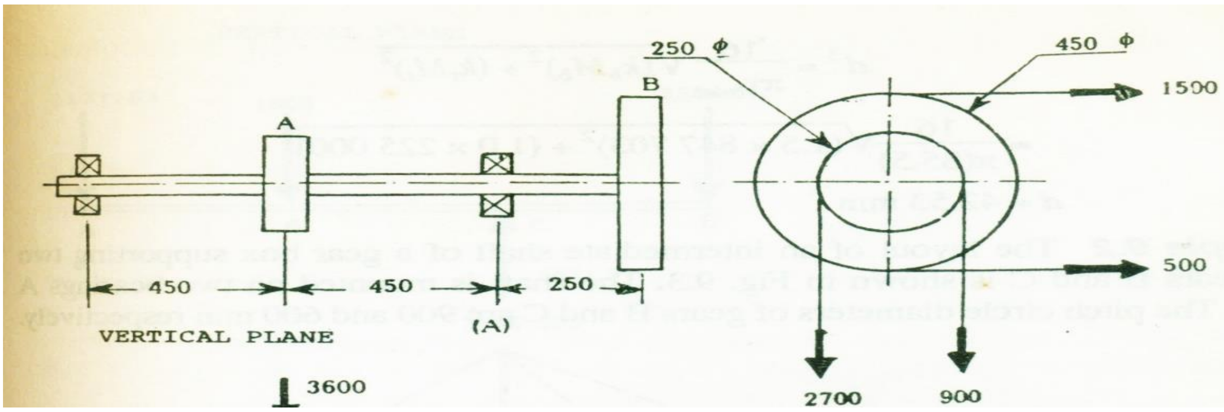
5^{th} number = $100\times\phi^4 = 100\times1.5848^4 = 630.8089 =$ **630**

6^{th} number = $100\times\phi^5 = 100\times1.5848^5 = 999.7060 =$ **1000** **... Ans.**

$\therefore$ The series consists of following numbers :

100, 160, 250, 400, 630, 1000.

59.A line shaft supporting to pulleys A and B is shown in fig. power is supplied to the shaft by means of a vertical belt on pulley SA, which is then transmitted to the pulley B carrying a horizontal belt. The ratio of belt tension on tight and loose sides is 3: 1.the limiting value of tension in the belts is 2.7 KN. The shaft is made of plain carbon steel 40c8 with (ultimate tensile strength=650Mpa and yield strength in tension=380 Mpa). The pulleys are keyed to the shafts. Determine the diameter of the shafts according to the ASME code if Kb=1.5 and Kt=1.0(N/D'22)



Solution $0.30S_{yt} = (0.30)(380) = 114 \text{ N/mm}^2$

$$0.18S_{ut} = (0.18)(650) = 117 \text{ N/mm}^2$$

The lower of the two values is 114 N/mm² and there are keyways on sh
Therefore

$$\tau_d = (0.75)(114) = 85.5 \text{ N/mm}^2$$

The maximum belt tension will occur in belt on pulley A due to smaller diam
The tensions on the tight and loose sides of this belt are 2700 N and 900
respectively.

The torque supplied to the shaft is given by

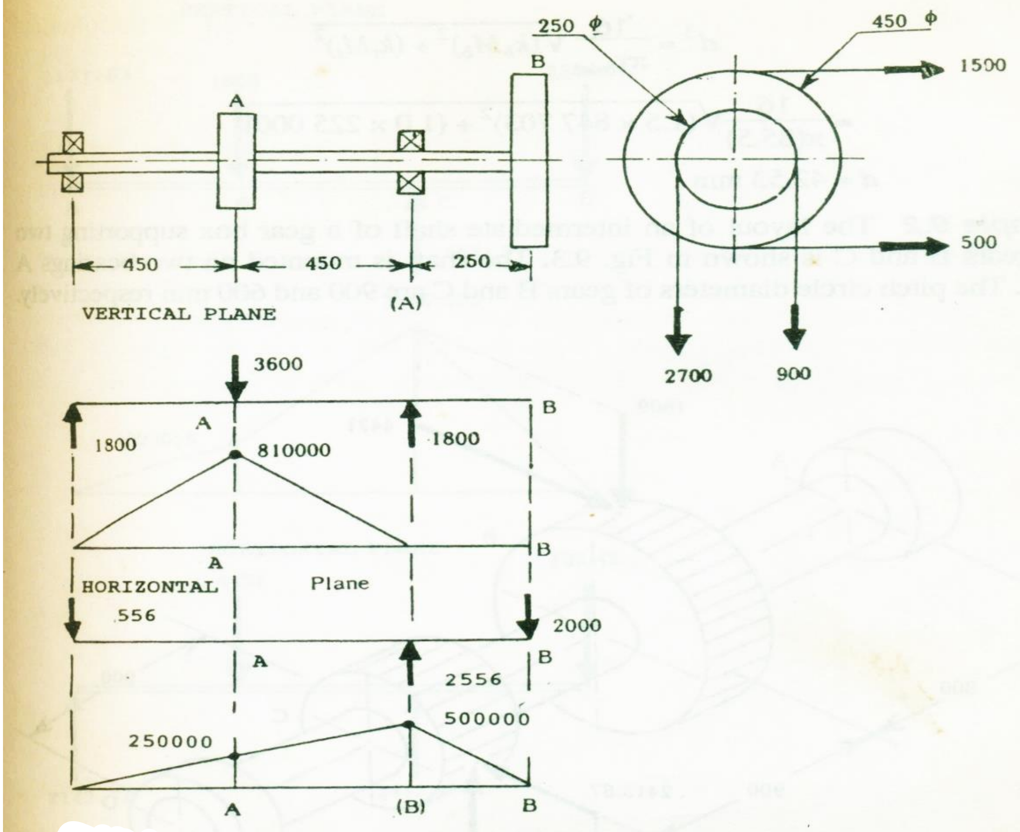


fig 1.11

$$M_t = (2700 - 900)\left(\frac{250}{2}\right) = 225\,000 \text{ N-mm}$$

The belt tensions for pulley B are P_{B1} and P_{B2}. Therefore,

$$\frac{P_{B1}}{P_{B2}} = 3 \qquad \text{(a)}$$

and $$(P_{B1} - P_{B2})\left(\frac{450}{2}\right) = 225\,000 \qquad \text{(b)}$$

Solving Eqs (a) and (b),

$$P_{B1} = 1500 \text{ N} \qquad \text{and} \qquad P_{B2} = 500 \text{ N}$$

The forces and bending moments in vertical and horizontal planes are shown in Fig.1.11 The maximum bending moment is at A. The resultant bending moment is given by

$$M_b = \sqrt{(810\,000)^2 + (250\,000)^2} = 847\,703 \text{ N-mm}$$

From Eq. (9.4),

$$d^3 = \frac{16}{\pi\tau_{max}}\sqrt{(k_b M_b)^2 + (k_t M_t)^2}$$

$$= \frac{16}{\pi(85.5)}\sqrt{(1.5 \times 847\,703)^2 + (1.0 \times 225\,000)^2}$$

$$\therefore \qquad d = 42.53 \text{ mm}$$

UNIT – II: SHAFTS AND COUPLINGS

SYLLABUS
Design of solid and hollow shafts based on strength, rigidity and critical speed – Keys, keyways and splines - Rigid and flexible couplings.

SUMMARY

Shaft:A shaft is a rotating machine element, which transits power from one point to another point.

Types of Shafts:
1. Transmission shafts: These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, overhead shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.
2. Machine shafts: These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Stresses in Shafts:
1. Shear stresses due to the transmission of torque (i.e. due to torsional load).
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
3. Stresses due to combined torsional and bending loads.

Design of Shafts
The shafts may be designed on the basis of
1. Strength and **2.** Rigidity and stiffness.
In designing shafts on the basis of strength, the following cases may be considered:
(a) Shafts subjected to twisting moment or torque only,
(b) Shafts subjected to bending moment only,
(c) Shafts subjected to combined twisting and bending moments, and
(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.
Critical Speed: The speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical speed

Key: A key is a device which is used for connecting two machine parts for preventing relative motion of rotation with respect to each other. Keys areused as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot orrecess in a shaft and hub of the pulley to accommodate key.
Types of Keys: 1. Sunk keys, **2.** Saddle keys, **3.** Tangent keys,**4.** Round keys and **5.**Splines.
Stresses in key: The following stresses are induced in the key: Shear stress and crushing stress.

Crankshaft: A crankshaft (i.e. a shaft with a crank) is used to convert reciprocating motion of the piston interrogatory motion or vice versa.
The crankshaft consists of the shaft parts which revolve in the mainbearings, the crankpins to which the big ends of the connecting rod are connected, the crank arms orwebs (also called cheeks) which connect the crankpins and the shaft parts.

Classification of crankshaft:

The crankshaft, **depending upon the position of crank**, may be divided into the following two types:

1. Side crankshaft or overhung crankshaft and 2. Centre crankshaft.

The crankshaft, **depending upon the number of cranks** in the shaft, may also be classified as

1. Single throw crankshafts and 2. Multi-throw crankshafts

Coupling: The elements which join two shafts are coupling. It is used to connect sections of long transmissions shaft to the shaft of a driving machine. Couplings are used to connect sections of long transmission shafts and to connect the shaft of a driving machine to the shaft of a driven machine.

Uses: Shaft couplings are used in machinery for several purposes, the most common of which are the following:

1. To provide for the connection of shafts of units those are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.

2. To provide for misalignment of the shafts or to introduce mechanical flexibility.

3. To reduce the transmission of shock loads from one shaft to another.

4. To introduce protection against overloads.

5. It should have no projecting parts.

Requirements of a Good Shaft Coupling

1. It should be easy to connect or disconnect.

2. It should transmit the full power from one shaft to the other shaft without losses.

3. It should hold the shafts in perfect alignment.

4. It should reduce the transmission of shock loads from one shaft to another shaft.

5. If should have no projecting parts

Types of Couplings

1. Rigid coupling: It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view:**(a)** Sleeve or muff coupling.

(b) Clamp or split-muff or compression coupling, and**(c)** Flange coupling.

2. Flexible coupling: It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point ofview:**(a)** Bushed pin type coupling, **(b)** Universal coupling, and**(c)** Oldham coupling.

PART – A (2MARKS)

Design of solid and hollow shafts based on strength, rigidity and critical speed

1. **What is meant by a shaft?**

   Shaft is a rotating machine element which is used to transmit power from one place to another. It is used in engines, machines and equipment for transmitting power form one point to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. The two shafts are connected by gear drive, belt or rope drive or chain drive.

2. **Discuss about the axle and spindle.(APR/MAY-15)**

   The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

   a. Axle: Anaxle, though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.

   b. Spindle : Aspindleis a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (e.g. lathe spindles).

3. **List out the various materials used for shafts.**

   When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.The material used for ordinary shafts:

   Carbon steel of **grades**

   a) 40 C 8,
   b) 45 C 8,
   c) 50 C 4 and
   d) 50 C 12.

4. **List Four required properties of good shaft material.(Nov/Dec2021)**

   (or) Mention the material properties of shafts.

   The material used for shafts should have the following properties:

   1. It should have high strength.
   2. It should have good machinability.
   3. It should have low notch sensitivity factor.
   4. It should have good heat treatment properties.
   5. It should have high wear resistant properties.

5. **Discuss the various types of shafts and mention its application.**

   The following two types of shafts are important from the subject point of view :

   i. **Transmission Shaft**: these are also called as line shaft, counter shaft or head shaft. These are used to transmit power between the power sources. That is the electric motor or IC engine and the machine to be operated.

   ii. **Machine Shaft:** A machine shaft is an integral part of machine, the examples being the crankshaft of an IC engine, lathe spindle, milling machine arbor etc.

6. A shaft of 750 mm long is subjected to shear stress of 40 MPa and has an angle of twist equal to 0.017 radian. Determine the diameter of the shaft. Take G=80 GPa. (Nov/Dec - 2013)

Given:

Length of the shaft l = 750 mm,

Shear stress , τ=40 N/mm^2,

Angel of twist, θ=radian,

Modulus of rigidity G= 0.8 x 10^5N/mm^2

To Find: Diameter of the shaft, d

Solution:

We know that torsional moment of the shaft

$$T = \frac{\pi}{16} \times \tau \times d^3$$

$$angle\ of\ twist \qquad \theta = \frac{T \times l}{GJ} = \frac{\frac{\pi}{16} \times \tau \times d^3 \times l}{G \times \left(\frac{\pi d^4}{32}\right)} = \frac{2\tau l}{Gd}$$

$$0.017 = \frac{2 \times 40 \times 750}{0.8 \times 10^5 \times d}$$

d=44.11 mm

The standard diameter is d= 45 mm

7. How areshafts manufactured?.

1) Shafts are generally manufactured by hot rolling and finished to size bycold drawing or turning and grinding.
2) The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses.
3) The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

8. List out standard size of transmission shaft.

The standard sizes of transmission shafts are:

a. 25 mm to 60 mm with 5 mm steps
b. 60 mm to 110 mm with 10 mm steps
c. 110 mm to 140 mm with 15 mm steps
d. 140 mm to 500 mm with 20 mm steps.

The standard lengths of the shafts are 5 m, 6 m and 7 m.

9.What are the various stresses induced in shafts? (May/June – 2014)

Based on the type of loading different kinds of stresses are induced in shaft, such as

(b) Torsional shear stress,
(c) Bending shear stress,
(d) Tensile and compressive stress (separately) or
(e) Tensile and compressive stress (in combined form)

The following stresses are induced in the shafts:
1. Shear stresses due to the transmission of torque (i.e. due to torsional load).
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
3. Stresses due to combined torsional and bending loads.

10.Discuss the steps involved in designing of shafts on the basis of strength

In designing shafts on the basis of strength, the following cases may be considered :

a. Shafts subjected to twisting moment or torque only,
b. Shafts subjected to bending moment only,
c. Shafts subjected to combined twisting and bending moments, and
d. Shafts subjected to axial loads in addition to combined torsional and bending loads.

10.(a) write short notes design of shafts on the basis of critical speed.(Nov/Dec 2021)

- The critical speed is the theoretical angular velocity which excites the natural frequency of a rotating object, such as a shaft.
- As the speed of rotation approaches the objects natural frequency, the object begins to resonate which dramatically increases systemic vibration.
- The unbalanced mass of the rotating object causes deflection that will create resonant vibration at certain speeds, known as the critical speeds.
- The magnitude of deflection depends upon the Stiffness of the shaft and its support,total mass of shaft and attached parts.

11.What do you mean by stiffness and rigidity with reference to shafts? Nov/Dec-2010.

Stiffness: It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.
Stiffness is the resistance of the shaft to the deformation by an applied force along given degree of freedom.

$$\frac{T}{\theta} = \frac{GJ}{L}$$

Where,

T – Torque in N-m
θ- Twisting angle
J – Polar moment of inertia in mm^4

Rigidity of the shaft is the property to resist deformation

12. Distinguish clearly spindle, axle and shaft with appropriate examples.

A spindle is a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (e.g. lathe spindles).

An axle, though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.

The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

13. Write an expression for the shaft when it is subjected to twisting moment and bending moment

Shafts Subjected to Twisting Moment Only

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\mathrm{T} = \frac{\pi}{16} \times \frac{d0^4}{d0}\left[1 - \left(\frac{di}{do}\right)^4\right] = \frac{\pi}{16} \times \tau (do)^3 (1 - k^4)$$

Where

T = Twisting moment (or torque) acting upon the shaft.

τ= Torsional shear stress.

k = Ratio of inside diameter and outside diameter of the shaft .

` $= \frac{d_i}{d_o}$, andd$_o$ and d$_i$ = Outside and inside diameter of the shaft resp.

Shafts Subjected to Bending Moment Only

$$\mathrm{M} = \frac{\pi}{32} \times \sigma_b (d_0)^3 (1 - k^4)$$

Where M = Bending moment,

k = Ratio of inside diameter and outside diameter of the shaft = d_i / d_o, and

d_o and d_i = Outside and inside diameter of the shaft MM.

σ_b= Bending stress N/MM^2

14. Define equivalent twisting moment. (or) equivalent torsional moment (April/May 2017)

The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. The expression $\sqrt{M^2 + T^2}$is known as equivalent twisting moment and is denoted by T_e.

$$T_e = \frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

Where,

M = Bending moment,

T = Twisting moment (or torque) acting upon the shaft.

T_{eq} = equivalent twisting moment

d = diameter of the shaft MM.

τ_{max}= Maximum Torsional shear stress.

15. What is meant by equivalent bending moment? April/May – 2012, Dec-20, Aprl/May-21

The equivalent bending moment may be defined as that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment. By limiting the maximum normal stress [σ_b (max)] equal to the allowable bending stress (σ_b).

The expression $\frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$ is known as equivalent bending moment and is denotedbyM_e.

$$M_e = \frac{\pi}{16} \times \sigma_{b\,(\max)} \times d^3$$

$$M_e = \frac{1}{2}[M + \sqrt{M^2 + T^2}]$$

Where,

M = Bending moment,

T = Twisting moment (or torque) acting upon the shaft.

d = diameter of the shaft MM.

σ_{bmax}= Maximum allowable stress in N/mm^2

16. When the shaft is subjected to fluctuating loads, what will be the equivalent twisting moment and the equivalent bending moment?

We have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment (T) and bending moment (M). Thus for a shaft subjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

and equivalent bending moment,

$$M_e = \frac{1}{2}[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}]$$

Where

K_m = Combined shock and fatigue factor for bendingand

K_t = Combined shock and fatigue factor for torsion.

17. What do you understand by the term torsional rigidity?

Torsional rigidity: The torsional rigidity is important in the case of camshaft of an I.C.engine where the timing of the valves would be effected. The permissible amount of twist should notexceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to3 degree per metre length may be used as limiting value. The widely used deflection for the shafts islimited to 1 degree in a length equal to twenty times the diameter of the shaft.

The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G.\theta}{L} \quad or \quad \theta = \frac{T.L}{J.G}$$

Where θ= Torsional deflection or angle of twist in radians, T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$J = \frac{\pi}{32} \times d^4$ (For solid shaft)

$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4]$ (For hollow shaft)

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

18. Define the term of lateral rigidity.

It is important in case of transmission shafting and shafts running at highspeed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity isalsoimportant for maintaining proper bearing clearances and for correct gear teeth alignment. If theshaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using thedeflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of abeam, i.e. $\frac{d^2y}{d^2x} = \frac{M}{EI}$

19. State the significance of Critical speed. Or what is meant by critical speed (April/May 2019)

All rotating shaft, even in the absence of external load, deflect during rotation.The combined weight of a shaft andwheel can cause deflection that willcreate resonant vibration at certainspeeds, known as **Critical Speed**.

Critical speed, $N_{s=} \frac{30}{\pi}\sqrt{\frac{g}{\delta_{st}}}$

Where,

g = gravity acceleration (9.81 m/s2)

δ_{st} = total maximum static deflection

20. Write Rayleigh-Ritz equation to determine the critical speed of shaft subjected to point loads. (April/May 2018)

Rayleigh Ritz method critical speed formula tofind the critical speed N_c of a rotating shaft. $N_c = [(30/\pi) \times \sqrt{(g/\Delta st)}]$. Standard gravity g = 9.81 m/s^2 & the shaft total deflection Δst in meter are the key terms of this calculation. This method recommends that the rotating shaft or object speed should not be exceeded more than 75% of it's critical speed

21. A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.

Given:

N= 200 r.p.m.;*P*= 20 kW = 20 × 103 W; τ□= 42 MPa = 42 N/mm2

Let *d* = Diameter of the shaft.

Solution:

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 N-m = 955 \times 10^3 \text{ N-mm}$$

We also know **that** torque transmitted by the shaft (*T*),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25 d^3$$

$$d^3 = 115733\, d = 48.7 mm \approx 50 mm$$

22. A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.

Given:

P = 1 MW = 1 × 10^6W; N = 240 r.p.m.; T_{max} = 1.2 T_{mean}; τ = 60 MPa = 60 N/mm^2

Let d = Diameter of the shaft.

Solution:

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\,784\, N-m = 39\,784 \times 10^3\, N-mm$$

Maximum torque transmitted,

$$T_{max} = 1.2\, T_{mean} = 1.2 \times 39784 \times 10^3 = 47741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, \mathrm{d}^3$$

$$d = 159.4\ mm \approx 160 mm$$

23. Why a hollow shaft has greater strength and stiffness than a solid shaft of equal weight?Nov/Dec - 2012

By shaft one means a rod which has to handle torsional loads. For any shaft, the stresses are proportional to the radius. This means that the central part is having very less stress but it is adding weight.

Its advantage is to remove the central material and get a hollow shaft.So an obvious advantage of hollow shaft is the weight being less for the same strength. Another advantage can be the space you get inside the shaft to play around.

The disadvantages come into picture when this shaft has other kinds of loads. These may be bending loads (perpendicular to axis). For these cases the strength is reduced.

24. Discussthe functions of counter shaft and jacks shaft

Counter shaft: It is a secondary shaft, which is driven by the main shaft and from which power is supplied to a machine component. Often the counter shaft is driven from the main shaft by means of pair of spur or helical gears and thus rotates counter to the direction of the main shaft. Counter shafts are used in multi-stage gear boxes.

Jackshaft: It is an auxiliary or intermediate shaft between two shafts that are used in transmission of power. Its function is same as that of counter shaft.

25. List out the advantages and disadvantagesof solid shaft over hollow shaft.

Compare with solid shaft, hollow shaft offers following advantages:

a. The stiffness of the hollow shaft is more than that of solid shaft with same weight.
b. The strength of hollow shaft is more than that of solid shaft with same weight.
c. The natural frequency of hollow shaft is higher than that of solid shaft with same weight

Compare with solid shaft, hollow shaft offers following disadvantages:

1. Hollow shaft is costlier than solid shaft
2. The diameter of hollow shaft is more than that of solid shaft and requires more space

26. Narrate the uses of Flexible shaft.

The flexible shafts are used to transmit torque between machine units, which may change their relative position during operation.

Flexible shafts have two important properties

a. They have low rigidity in bending, making them flexible.
b. They have high rigidity in torsion, making them capable to transmit torque.

27. What is the significance of 'slenderness ratio' in shaft design? (Nov/Dec – 2008)

Slenderness ratio is the ratio between the length of shaft and its least radius of gyration.

$$\text{Slanders ratio } \frac{L}{K} = \frac{Length\ of shaft}{Leastradius\ of gyration}$$

This ratio value should be limited based on the type and amount of load. For example if the nature of loading is tensile, the slenderness ratio will not have much effect. Due to this tensile load there will not be any bending or buckling. At the same time if the loading is compressive and the slenderness ratio is more, the shaft will buckle, which is also a functional failure and hence this buckling should be avoided.

28. List out the various causes of failures in machine component.

a. Tensile failure – The material exceeds its yield limit.
b. Compressive failure - The material exceeds its compressive limit.
c. Shear failure -The material exceeds its ultimate limit.
d. Edge fracture- Due to stress concentration.
e. Failure due to corrosion – Due to chemical reaction with environment.
f. Failure due to sudden loading – Due to high velocity loading.

29. List out the applications of Bushed pin type coupling.

a. Portable power driven tools
b. Speedometer drives
c. Positioning devices
d. Portable grinders
e. Concrete vibrators
f. Remote control devices
g. Servo drives

30. What are the various stresses induced in the shaft? (MAY/JUNE 2014)

Torsional Shear Stress
Compression stress
Bending Stresses

31. How the length and diameter of a shaft affects its critical speed?[APR/MAY-15]

The critical speed Nc depends on the magnitude and location of the load or loads carried by the shafts, the length of the shaft, moment of inertia I of the shaft, the modulus of the shaft material and the type of end supports.

32. What is meant by design of shaft based on rigidity?(Nov/Dec-15)

The shaft are designed on the basis of either torsional rigidity or lateral rigidity. A transmission shaft is said to be rigid on the basis of torsional rigidity, if it does not twist too much under the action of an external torque.

Similarly, the transmission shaft is said to be rigid on the basis of lateral rigidity, if it does not deflect too much under the action of external forces and bending moment.

33.Write the advantages that hollow shafts offer as compared to solid shafts(April/May 2018)

- The stiffness of the hollow shaft is more than that of solid shaft with same weight.
- The strength of hollow shaft is more than that of solid shaft with same weight.
- The natural frequency of hollow shaft is higher than that of solid shaft with same weight.
- Write formulas for axial, torsion and bending stiffness for both solid and hollow shaft.

Keys, keyways and splines

34. What is meant by a key? State its function.

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate key.

35. Brief about the types of keys.

The following types of keys are important from the subject point of view:

1. Sunk keys,
   a. Rectangular sunk key,
   b. Square sunk key,
   c. Parallel sunk key,
   d. Gib-head key,
   e. Feather key and
   f. Woodruff key.
2. Saddle keys,
   a. Flat saddle key
   b. Hollow saddle key
3. Tangent keys,
4. Round keys, and
5. Splines.

36. Discuss about various forcesacting on the keys. (Nov/Dec -2014)

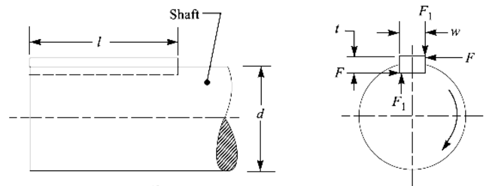


fig 1 Forces acting on a sunk key.

When a key is used in transmitting torque from a shaft to a rotoror hub, the following two types of forces act on the key :

1. Forces (F_1) due to fit of the key in its keyway, as in a tight fitting straight key or in a taperedkey driven in place. These forces produce compressive stresses in the key which are difficultto determine in magnitude. as shown in fig 1

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing andcompressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces areconcentrated near the torque-input end. The non-uniformity of distribution is caused by the twistingof the shaft within the hub.

37. Define the term Sunk key and list out the types of sunk key and draw anyone.(Nov/Dec 2017)(Nov/Dec-2018)

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub orboss of the pulley. The sunk keys are of the following types:

1. Rectangular sunk key,
2. Square sunk key,
3. Parallel sunk key,
4. Gib-head key,
5. Feather key and
6. Woodruff key.

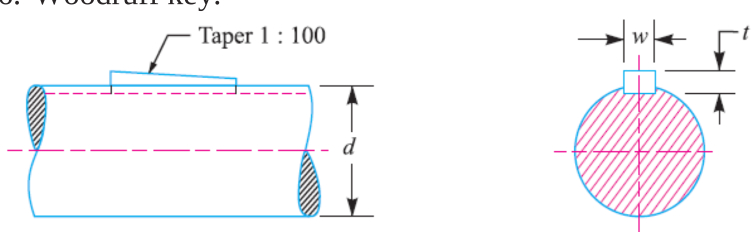


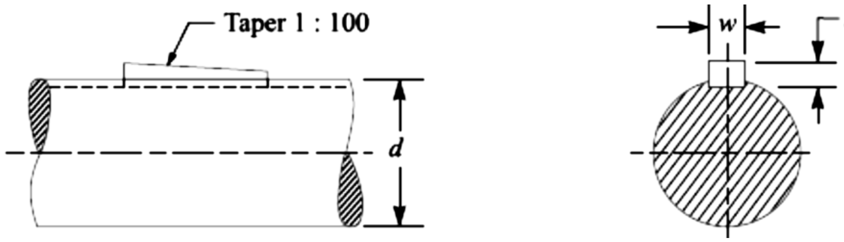
fig 2 Rectangular sunk key

38. List the various failures occurred in sunk keys (April/May 2008)

Shear failure

Crushing failure.

39. Differentiate the rectangular and square sunk keys.



Rectangular sunk key: A rectangular sunk key is shown figure 2 The usual proportionsofthiskey are:

Width of key, $w = \frac{d}{4}$ and

Thickness of key, $t = \frac{2w}{3} = \frac{d}{6}$.

Where,

d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.

Square sunk key: The only differencebetween a rectangular sunk key and a squaresunk key is that its width and thickness areequal, i.e.

$$w = t = \frac{d}{4}.$$

40. Write short notes on feather key.

A key attached to one member of a pair and which permits relative axial movement is known as feather key.

It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being sliding fit in the key way of the moving piece. The various proportions of a feather key are same as that of rectangular sunk key and gib head key.

41. What is meant by woodruff keys? (MAY/JUNE 2013)

A woodruff key is used to transmit small value of Torque in automotive and machine tool industries. The keyway in the shaft is milled in a curves shape whereas the key way in the hub is usually straight.

42. List out the advantages and disadvantages of Woodruff key.

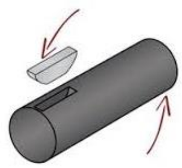
The main advantages of a woodruff key are as follows:

1. The woodruff key is an easily adjustable key
2. It accommodates itself to any taper in the hub or boss of the mating piece.
3. It is useful on tapering shaft ends. Its extra depth in the shaft prevents any tendency to turnover in its keyway.

The disadvantages are:

1. The depth of the keyway weakens the shaft.
2. It cannot be used as a feather.

43. What is the main use of woodruff keys? Nov/Dec 2013 &May/June 2013



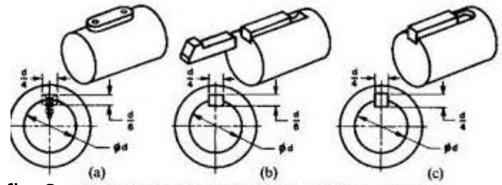


fig 3 (a) Feather key, (b) rectangular key, and (c) square key

A woodruff key is used to transmit small value of torque in automotive and machine tool industries. The keyway in the shaft is milled in a curved shape whereas the key way in the hub is usually straight.Common application include shown in fig 3

- Machine tools
- Automotive applications
- Snow blowers
- Marine propellers

44. Differentiate between keys and splines. (Nov/Dec – 2011)&(April -2008)

Sl. No	**KEYS**	**SPLINES**
1	A shaft which is having single key way	A shaft which is having multiple key way
2	Keys are used in coupling	Splines are used in automobiles and machine tools.
3	The key width is nominally ¼ of the shaftdiameter.	4 or more splines are used, as comparedwith 1 or 2 keys, a more uniform transfer of torqueand a lower loading on a givenpart of the shaft/hubinterface result.
4	Tapered keys are installed after mating the hub and shaft. The taperextends over the length of the hub.	Splines can either be straight-sided or involute.
5	Pin keys reduce stress concentration, but requires a tight fit.	The splines are integral with the shaft, so no relative motion can occur as between a key and the shaft.

45.What are the types of saddle keys? Explain.

The saddle keys are of the following two types:

1. **Flat saddle key:**Itis a taper key which fits in a keyway in the hub and is flat on the shaft. It is likely to slip round the shaft under load. Therefore, it is used for comparatively light loads

2. **Hollow saddle key:** It is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

46.What is the effect of keyway cut into the shaft ?(May/June -16)

A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. It other words, the torsional strength of the shaft isreduced. The following relation for the weakening effect of the keyway is based on the experimentalresults by H.F. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

Where

e = Shaft strength factor. It is the ratio of the strength of the shaft withkeyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

h = Depth of key way $= \left(\frac{Thickness\ of\ key(t)}{2}\right)$

47.Sketch the cross section of a splined shaft. April/May – 2012

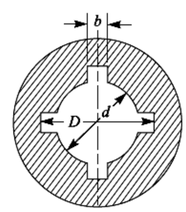


fig 5

Where

D= Outer diameter of spline D = 1.25d as shown in fig 5

Width of key , b = 0.25 D

The splined shafts are used when the force to be transmitted islarge in proportion to the size of the shaft as in automobile transmissionand sliding gear transmissions. By using splined shafts, we obtainaxial movement as well as positive drive is obtained.

48. Write short notes on Gib-head key.

Gib-head key is a rectangular sunk key with a head at one end known as gib head. It is usually provided to facilitate the removal of key. A gib head key is shown in Fig6(a) and its use in shown in Fig6(b).

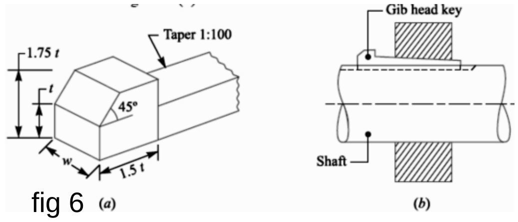


fig 6

The usual proportions of the gib head key are:

Width, w = d / 4;and thickness at large end, t = 2w / 3 = d / 6

49. What is spline shaft? Give example for its application.

Spline shaft is a shaft in which one or more rectangular projection, similar to sunk keys are made integral with the shaft, partly or throughout its length. The main aim of using spline shaft is to make its mating parts sliding on do it.

Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as splined shafts

This kind of spline shaft are mostly employed inGear box of automobiles, Automobile parts.

Rigid and flexible couplings.

50. Discuss the function of a coupling. Give at least three practical applications.

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

a. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alterations.
b. To provide for misalignment of the shafts or to introduce mechanical flexibility.
c. To reduce the transmission of shock loads from one shaft to another.
d. To introduce protection against overloads.
e. To alter the vibration characteristics of rotating units.

51. List out the uses of couplings in machinery.

Shaft couplings are used in machinery for several purposes, the most common of which are the following:

1. To provide for the connection ofshafts of units those are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
3. To reduce the transmission of shock loads from one shaft to another.
4. To introduce protection against overloads.
5. It should have no projecting parts.

52. What are the Requirements of a Good Shaft Coupling?

A good shaft coupling should have the following requirements:

1. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shafts in perfect alignment.
4. It should reduce the transmission of shock loads fromone shaft to another shaft.
5. If should have no projecting parts.

53. Name any two of the rigid and flexible couplings. (May/June -2013)(May/June-14) (or) Differentiate between rigid coupling and flexible coupling(May/June-16)(Nov/Dec 2017)(Nov/Dec-2018)(April/May 2019) (April/May 2021)

Shaft couplings are divided into two main groups as follows:

1. Rigid coupling: It is used to connect two shafts whichare perfectly aligned. Following types of rigid coupling are important from the subject point of view:

(a) Sleeve or muff coupling.
(b) Clamp or split-muff or compression coupling, and
(c) Flange coupling.

2. Flexible coupling: It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view:

(a) Bushed pin type coupling,
(b) Universal coupling, and
(c) Oldham coupling.

54. What are the types of Flange coupling?

The flange couplings are of thefollowing three types:

Unprotected type flange coupling: In an unprotected type flange coupling, each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts.

Protected type flange coupling: In a protected type flange coupling, the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman.

Marine type flange coupling: In a marine type flange coupling, the flanges are forged integral with the shafts. The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending upon the diameter of shaft.

55. Give the proportions of the Sleeve or Muff coupling with neat sketch.

Sleeve or Muff coupling:

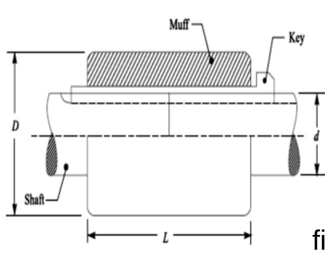


fig 7

The usual proportions of a cast iron sleeve coupling are as follows:

Outer diameter of the sleeve, D = 2d + 13 mmand   as shown in fig 7
length of the sleeve, L = 3.5 d
Where,
d is the diameter of the shaft.

56. What are the purposes in machinery for which couplings are used?

- To provide the connection of shafts of units those are manufactured separately such as motor and generator and to provide for disconnection for repairs or alteration.
- To provide misalignment of the shafts or to introduce mechanical flexibility.
- To reduce the transmission of shock from one shaft to another.
- To introduce protection against over load.

57. Under what circumstances and where are flexible couplings are used?Nov/Dec – 2012

They are used to join the abutting ends of shafts when they are not in exact alignment.
They are used to permit an axial misalignment of the shafts without under absorption of the power, which the shafts are transmitting.

1) When there is misalignment between the two shafts
2) When the shafts are aligned at angular position.

Uses: Vehicle, Stationery machinery, automotive drives   and Machine tools.

58. what are the advantages of flexile coupling over rigid coupling?(Nov/Dec2021)

- Flexible coupling can accommodate a slight misalignment between shafts but a rigid coupling cannot accommodate any misalignment.
- Flexible coupling can absorb shocks and vibrations but a rigid coupling cannot absorb shocks and vibrations.

59. Suggest suitable couplings for a. Shaft with parallel misalignment. b. shafts with angular misalignment of 10o c. Shaft in perfect alignment. (Nov/Dec – 2010 & May/june -2012)

1. **Oldhams coupling:**

An Oldham coupling has three discs, one coupled to the input, one coupled to the output, and a middle disc that is joined to the first two by tongue and groove.

2. **Universal coupling:**

Gear couplings and universal joints are used in similar applications. Gear couplings have higher torque densities than universal joints designed to fit a given space while universal joints induce lower vibrations. The limit on torque density in universal joints is due to the limited cross sections of the cross and yoke.

3. **Flange coupling**

This coupling has two separate cast iron flanges. Each flange is mounted on the shaft end and keyed to it.

60. What is meant by Flange coupling?

A flange coupling usually applies to a coupling having two separate cast iron flanges. Each flange is mounted on the shaft end and keyed to it. The faces are turned up at right angle to the axis of the shaft. One of the flange has a projected portion and the other flange has a corresponding recess. This helps to bring the shafts into line and to maintain alignment. The two flanges are coupled together by means of bolts and nuts. The flange coupling is adopted to heavy loads and hence it is used on large shafting.

61. Enumerate various types of Flange coupling.

The flange couplings are of thefollowing three types:

1. **Unprotected type flange coupling:**In an unprotected type flange coupling, each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts.
2. **Protected type flange coupling:**In a protected type flange coupling, the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman.
3. **Marine type flange coupling:**In a marine type flange coupling, the flanges are forged integral with the shafts. The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending on the diameter of shaft.

62. Explain with neat sketch on universal coupling.

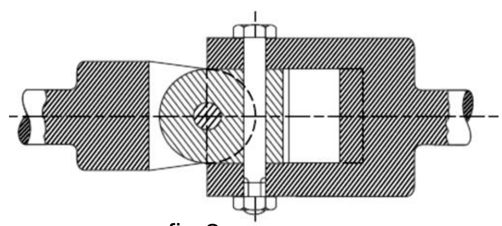


fig 8

A universal or Hooke's coupling is used to connect two shafts whose axes intersect at a small angle. The inclination of the two shafts may be constant, but in actual practice, it varies when the motion is transmitted from one shaft to another.as shown in fig 8

63. What is the difference between the clutch and coupling?

Clutch: It is a machine member used to connect a driving shaft to a driven shaft so that the driven shaft may be started or stopped at will, without stopping the driving shaft. The use of a clutch is mostly found in automobiles.

Coupling : To provide for the connection ofshafts of units that are manufacturedseparately such as a motor and generatorand to provide for disconnectionfor repairs or alternations.

A coupling is a device used to make permanent or semi permanent connection where as a clutch permits rapid connection or disconnection at will of the operator.

64. List out the applications of universal coupling.

The main application of the universal or Hooke'scoupling is found in the transmission from the gear box to the differential or back axle of theautomobiles.

In such a case, we use two Hooke's coupling, one at each end of the propeller shaft,connecting the gear box at one end and the differential on the other end.

A Hooke's coupling is alsoused for transmission of power to different spindles of multiple drilling machines.

It is used as a kneejoint in milling machines.

65. At what angle of the crank the twisting moment is maximum in the crank shaft? Nov/Dec – 2011

Crank angle is 25º to 40º

The twisting moment on the crankshaft will be maximum when the tangential force on the crank(FT) is maximum. The maximum value of tangential force lies when the crank is at angle of 25° to 30ºfrom the dead centre for a constant volume combustion (i.e. petrol engines) and 30º to 40º for constant pressure combustion engines (i.e. diesel engines).

66.In relation with the mechanics material approach,how a statically ndeterminate structural problems can be solved?(N/D'2022)

Statically indeterminate structures are solved by the displacement method as if unknown displacements and rotations were chosen. From a system of equilibrium equations we calculate deformations from which internal forces and reactions are calculated.

Indicate the type of coupling used under following condition.

1.Shafs having colliner axis

2.Shafts having intersecting axis

3.Shafts having parallel axis with a small distance apart.

Ans:

a. Rigid or flexible coupling
b. Universal coupling
c. Double slider crank principle mechanism.

66.a)What are the specifications used to define a coupling?

a. Diameter of shaft
b. Diameter of sleeve or muff
c. Length of sleeve or muff
d. Outer diameter of hub
e. PCD of bold circle

67.What are the advantages of gear coupling?

b. Gear coupling is a rigid coupling with some flexibility because of using curved external teeth.
c. Strength of gear coupling is very high
d. Most compact coupling for high power transmission
e. It is used to connect two shafts which are perfectly aligned

68.What are the possible modes of failure of the pin(bolt) in a flexible coupling?(Nov/Dec-15)

There are two types possible modes of failure of the pin in a flexible coupling is

a. Torsional

b. Shear

69. Define the term critical speed of a shaft. (Nov/Dec 16)

- ✓ The speed, at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical or whirling speed**.**

70. What are the types of flexible coupling and rigid coupling? (Nov/Dec 16)

1. Rigid coupling.

- ✓ It is used to connect two shafts which are perfectly aligned.
  1. Sleeve or muff coupling.
  2. Clamp or split-muff or compression coupling, and
  3. Flange coupling.

2. Flexible coupling.

- ✓ It is used to connect two shafts having both lateral and angular misalignment.
  1. Bushed pin type coupling,
  2. Universal coupling, and
  3. Oldham coupling.

71. State the reasons for which the couplings are located near the bearings (April/May 2017)

Rigid couplings are used when precise shaft alignment is required; shaft misalignment will affect the coupling's performance as well as its life.

72.List the various failures that occurs in keys(A/M'2023)

There Are Two Modes Of Failure In Key: Shear Failure Occurs When Key Is Sheared Across Its Width At The Interface Between Shaft And Hub Bearing Failure Bearing Failure Occurs By Crushing Either Side In Compression.

73.A Hollow Circular Shaft of inner Radius 15mm,Outer Radius 30mm And Length 1m is To Be Used As A Torsional Spring,If The Shear Modulus Of The Material Of The Shaft Is 250 Gpa,The Torsional Stiffness Of The Shaft(In KN-M/Rad)is?(A/M'2023)

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

∴ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32}[(d_o)^4 - (d_i)^4] \text{ ---}$$

$$S = \frac{T}{\theta} = \frac{250 \times \pi}{1 * 32}(0.0.3^4 - 0.015^4)$$

$$= 1.86 \text{KN-m/rad}$$

74.A Shaft Requires To Be designed and manufactured with ductile material.list out the commonly used theories of failure applied of ductile material.justify your choice for conservative approach towards shaft design(N/D'2022)

The maximum shear stress theory is more accurate in predicting the failure of ductile materials because it considers the direction of the principal stresses. In conclusion, the suitable theory of failure for ductile materials is the maximum shear stress theory**.**

PART – B (16 MARKS)

Design of solid and hollow shafts based on strength, rigidity and critical speed

1. A hoisting drum 0.5 m in diameter is keyed to a shaft which is supported in two bearings and driven through a 12 : 1 reduction ratio by an electric motor. Determine the power of the driving motor, if the maximum load of 8 kN is hoisted at a speed of 50 m/min and the efficiency of the drive is 80%. Also determine the torque on the drum shaft and the speed of the motor in r.p.m. Determine also the diameter of the shaft made of machinery steel, the working stresses of which are 115 MPa in tension and 50 MPa in shear. The drive gear whose diameter is 450 mm is mounted at the end of the shaft such that it overhangs the nearest bearing by 150 mm. The combined shock and fatigue factors for bending and torsion may be taken as 2 and 1.5 respectively. (Nov/Dec-2013)

Given :

D = 0.5 m or R = 0.25 m ;
Reduction ratio = 12 : 1 ; W = 8 kN = 8000 N ;
v= 50 m/min ;
η = 80% = 0.8 ;
σt= 115 MPa = 115 N/mm2 ; τ = 50 MPa = 50 N/mm2 ;
D1 = 450 mm or R1 = 225 mm = 0.225 m ; Overhang = 150 mm = 0.15 m ;
Km = 2 ;Kt= 1.5

Solution:

Power of the driving motor

We know that the energy supplied to the hoisting drum per minute

= W × v = 8000 × 50 = 400 × 103 N-m/min

∴ Power supplied to the hoisting drum

$=400\times 10^3/60 = 6670$ W = 6.67 kW ...(∵ 1 N-m/s = 1 W)

Since the efficiency of the drive is 0.8, therefore power of the driving motor

= 6.67/ 0.8 = 8.33 kW **Ans.**

Torque on the drum shaft

We know that the torque on the drum shaft,

T = W.R = 8000 × 0.25 = 2000 N-m **Ans.**

Speed of the motor

Let N = Speed of the motor in r.p.m.

We know that angular speed of the hoisting drum

= Linear speed / Radius of the drum

= v/R = 50/0.25= 200 rad / min

Since the reduction ratio is 12 : 1, therefore the angular speed of the electric motor,

ω = 200 × 12 = 2400 rad/min

and speed of the motor in r.p.m.,

$N = \omega/2\pi = 2400/2\pi = 382$ r.p.m. **Ans**

Diameter of the shaft

Let d = Diameter of the shaft.

Since the torque on the drum shaft is 2000 N-m, therefore the tangential tooth load on the drive gear,

$$F_t = \frac{T}{R_t} = \frac{2000}{0.225} = 8900 \text{ N}$$

Assuming that the pressure angle of the drive gear in 20°, therefore the maximum bending load on the shaft due to tooth load

$$= \frac{F_t}{\cos 20^o} = \frac{8900}{0.9397} = 9470 \text{ N}$$

Since the overhang of the shaft is 150 mm = 0.15 m, therefore bending moment at the bearing,

$$M = 9470 \times 0.15 = 1420 \text{ N-m}$$

We know that the equivalent twisting moment,

$$T_{eq} = \sqrt{(K_b M)^2 + (K_t M)^2}$$

$$T_{eq} = \sqrt{(2 \times 1420)^2 + (1.5 \times 2000)^2} = 4130 \text{ N} - \text{m} = 4130 \times 10^3 \text{N} - \text{mm}$$

We also know that equivalent twisting moment (Te),

$$4130 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 50 \times d^3 = 9.82\ d^3$$

$\therefore d^3 = 4130 \times 103 / 9.82 = 420.6 \times 103$ or d = 75 mm

Again we know that the equivalent bending moment,

$$M_e = \frac{1}{2}[K_m \times M\sqrt{(K_m \times M)^2 + (K_t \times T)^2} = \frac{1}{2}[K_m \times M \times T_e]$$

$$M_e = \frac{1}{2}[2 \times 1420 \times 4130] = 3485 \text{ N} - \text{m} = 3485 \times 10^3 \text{ N} - \text{mm}$$

We also know that equivalent bending moment (M_e),

$$3485 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 115 \times d^3 = 11.3 d^3$$

Taking the larger of the two values, we have

d = 75 mm Ans.

2. (i) In an axial flow rotary compressor, the shaft is subjected to maximum twisting moment and maximum bending moment of 1500 Nm and 3000 Nm respectively. Neglecting the axial load, determine the diameter, if the permissible shear stress is 50 N/mm². Assume minor shocks. If the shaft is hollow one with K=di/do=0.4 . What will be material saving in hollow shat which is subjected to same loading and material as a solid shaft.?**(Nov/Dec -2014)**

Given:

Maximum twisting moment = 1500 N-m
Maximum bending moment = 3000 N-m
Permissible shear stress = 50 N/mm²
K=di/do=0.4

Solution:

For a solid shaft without axial load the diameter of the shaft required is given by

$$d > \left\{\frac{16 \times \text{Equivalent torque}}{\pi \times \text{Allowableshear stress}}\right\}^2$$

From the Design data book page no: 7.21for gradually applied load Kb = 1.5 and Kt = 1.0

Equivalent torque $T_{eq} = \sqrt{(K_b M)^2 + (K_t M)^2}$

$$T_{eq} = \sqrt{(1.5 \times 3000)^2 + (1 \times 1500)^2} = 4743.5\ \mathrm{N-m}$$

$$d > \left\{\frac{16 \times T_{eq}}{\pi \times \tau}\right\}^{1/3} = \left\{\frac{16 \times 4743.5 \times 10^3}{\pi \times 50}\right\}^{1/3}$$

$$d > 78.5\ \mathrm{mm}$$

Take the nearest standard size = 80mm

For hollow shaft

$$D > \left\{\frac{16 \times T_{eq}}{\pi \times \tau(\,1 - k^4)}\right\}^{1/3}$$

Where k=d/D=0.4 D and d are outer and inner diameter respectively

$$D > \left\{\frac{16 \times 4743.5 \times 10^3}{\pi \times 50(\,1 - 0.4^4)}\right\}^{1/3}$$

>76 mm

The nearest standard diameter of shaft D= 80 mm

Percentage saving in material

$$= \frac{\text{Area of solid shaft} - \text{area of hollow shaft}}{\text{Area of solid shaft}}$$

$$= \frac{80^2 - (80^2 - 32^2)}{80^2} \times 100$$

$$= 0.4^2 \times 100 = 16\%$$

Stiffness of the shaft can be expressed in torque required produce unit angular deflection per unit length,In other words

$$\frac{T}{\frac{\theta}{l}} = \text{is a measure of the stiffness}$$

$$\frac{T}{\frac{\theta}{l}} = GJ$$

The stiffness of the solid and hollow shafts are proportional to their polar moments of inertia.
Effect of axial load: Neglecting the column effect produced by the axial load and taking α=1
Shear stress in the shaft,

$$\tau = \frac{16}{\pi \times D^3(\,1 - k^4)}\left\{(K_t T)^2 + \left(\left(K_b M + \frac{FD}{8}\right)^2 (1 + k^2)\right)\right\}^{1/2}$$

Where F is the axial load = 10000N

$$\tau = \frac{16 \times 1000^3}{\pi \times 80^3(\,1 - 0.0256^4)}\left\{(1500)^2 + \left(\left(1.5 \times 3000 + \frac{10000 \times 80}{8 \times 1000}\right)^2 (1 + 0.4^2)\right)\right\}^{1/2}$$

τ=49.3 N/mm²

2(ii) Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Given:

Power, P = 20 kW = 20 × 103 W ;
Speed, N = 200 r.p.m. ;
Ultimate shear stress, τ_u = 360 MPa = 360 N/mm2 ;
Factor of safety F.S. = 8 ;
k = di / do = 0.5

To Find:

Indie and outside diameter of the shafts.

Solution:

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S} = \frac{360}{8} = 45\ N/mm^2$$

Diameter of the solid shaft
Let d = Diameter of the solid shaft.
We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955\ \text{N} - \text{m} = 955 \times 10^3 \text{N} - \text{mm}$$

We also know that torque transmitted by the solid shaft (*T*),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = 955 \times 10^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

$$d^3 = \frac{(955 \times 10^{3)}}{8.84} = 108032$$

$$\boldsymbol{d = 47.6\ say\ 50\ mm}$$

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3(1 - k^4)$$

$$= \frac{\pi}{16} \times 45\ (d_o)^3(1 - 0.5^4) = 8.3(d_o)^3$$

$$(d_o)^3 = \frac{955 \times 10^3}{8.3} = 115060$$

$$\boldsymbol{d_o = 48.6\ say\ 50mm}$$

$$\boldsymbol{d_i = 0.5\ d_o = 0.5\ \times 50 = 25\ mm}$$

3. A steel solid shaft transmitting 15 kW at 200 r.p.m. is supported on two bearings 750 mm apart and has two gears keyed to it. The pinion having 30 teeth of 5 mm module is located 100 mm to the left of the right hand bearing and delivers power horizontally to the right. The gear having 100 teeth of 5 mm module is located 150 mm to the right of the left hand bearing and receives power in a vertical direction from below. Using an allowable stress of 54 MPa in shear, determine the diameter of the shaft.(M/J -2014) & M/J 2013 (April/May 2019)

Given:

P = 15 kW = 15 × 103 W

N = 200 r.p.m. ;AB= 750 mm ; TD = 30 ;

mD = 5 mm ; BD = 100 mm ; TC = 100 ;

mC = 5 mm ; AC = 150 mm ;

τ = 54 MPa = 54 N/mm2

Solution:

The space diagram of the shaft is shown in Fig.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = 716\ N - m = 716 \times 10^3 N - mm$$

The torque diagram is shown in Fig.

We know that diameter of gear

= No. of teeth on the gear × module

∴ Radius of gear C,

$$R_c = \frac{T_c m_c}{2} = \frac{100 \times 5}{2} = 250 \text{ mm}$$

and radius of pinion D,

$$R_D = \frac{T_D m_D}{2} = \frac{30 \times 5}{2} = 75 \text{ mm}$$

Assuming that the torque at C and D is same (i.e. 716 × 103 N-mm), therefore tangential force on the gear C, acting downward,

$$F_{tC} = \frac{T}{R_C} = \frac{716 \times 10^3}{250} = 2870 \text{ N}$$

and tangential force on the pinion D, acting horizontally,

$$F_{tD} = \frac{T}{R_D} = \frac{716 \times 10^3}{75} = 9550 \text{ N}$$

The vertical and horizontal load diagram is shown in Fig. (c) and (d) respectively.Now let us find the maximum bending moment for vertical and horizontal loading.First of all, considering the vertical loading at C.

Let R_{AV} and R_{BV} be the reactions at the bearingsAandB respectively.

We know that

$R_{AV} + R_{BV} = 2870$ N

Taking moments about A, we get

$R_{BV} \times 750 = 2870 \times 150$

$R_{BV} = 2870 \times 150 / 750 = 574$ N

And $R_{AV} = 2870 - 574 = 2296$ N

We know that B.M. at A and B,

$M_{AV} = M_{BV} = 0$

B.M. at C, $M_{CV} = R_{AV} \times 150 = 2296 \times 150 = 344\,400$ N-mm

B.M. at D, $M_{DV} = R_{BV} \times 100 = 574 \times 100 = 57\,400$ N-mm

The B.M. diagram for vertical loading is shown in Fig. (e).Now considering horizontal loading at D. Let R_{AH} and R^{BH} be the reactions at the bearings A and B respectively. We know that

$R_{AH} + R_{BH} = 9550$ N

Taking moments about A, we get

$R_{BH} \times 750 = 9550\,(750 - 100) = 9550 \times 650$

$\therefore R_{BH} = 9550 \times 650 / 750 = 8277$ N

and $R_{AH} = 9550 - 8277 = 1273$ N

We know that B.M. at A and B,

$M_{AH} = M_{BH} = 0$

B.M. at C, $M_{CH} = R_{AH} \times 150 = 1273 \times 150 = 190\,950$ N-mm

B.M. at D, $M_{DH} = R_{BH} \times 100 = 8277 \times 100 = 827\,700$ N-mm

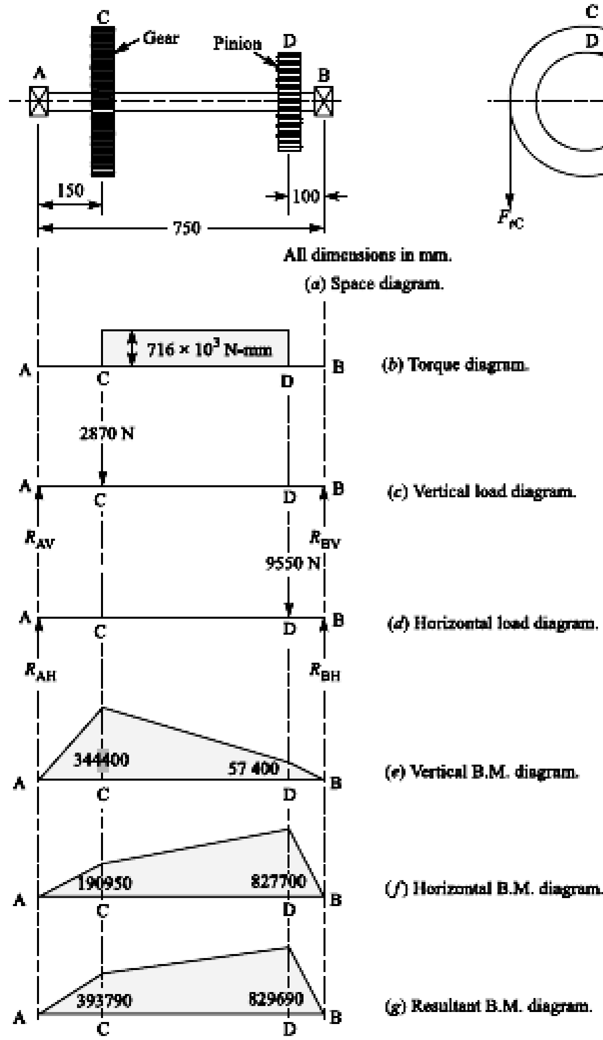


fig 2.1

The B.M. diagram for horizontal loading is shown in Fig. d

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(344400)^2 + (190950)^2} = 393790\ \mathrm{N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(57400)^2 + (827700)^2} = 829690\ \mathrm{N-mm}$$

The resultant B.M. diagram is shown in Fig. (g). We see that the bending moment is maximum at D.

∴ Maximum bending moment,M= M_D = 829 690 N-mm

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{M829690^2 + (716 \times 10^3)^2} = 1096 \times 10^3\mathrm{N-mm}$$

We also know that equivalent twisting moment (Te),

$$1096 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 54 \times d^3 = 10.6\ d^3$$

$$\therefore d^3 = 1096 \times 103/10.6 = 103.4 \times 10^3$$

or

d = 47 say 50 mm Ans.

4. Design a plain carbon steel centre crankshaft for a single acting four stroke, single cylinder engine for the following data:Piston diameter = 200 mm; stoke = 400 mm, Maximum combustion pressure = 2 N/mm², Weight of the flywheel = 15 KN, Total belt pull = 3N ,Length of connecting rod = 900 mm ,When the crank has turned through 30° from top dead centre, the pressure on the pistion is 1 N/mm² and the torque on the crank is maximum. Any other data required for the design may be assumed. (Apr/May – 2012)

Given:

D= 200mm,
L=400mm r =200mm,
$p=2N/mm^2$,
$W=15kN=15x10^3N$,
$T1+T2= 3kN=3x10^3N$,
$\theta=30^o$,l/r=900/200=4.5,
Plain carbon steel, single acting 4 stroke, single cylinder engine.

Solution:

I. Selection of materials:

Select C40 steel for crankpin, web and crankshaft from PSG. Data book

$\sigma_y= 330N/mm^2$, Factor of safety = 5

Permissible bending stress $[\sigma_b]=\sigma_y/FOS = 330/5=66\ N/mm^2$

Permissible shear stress $[\tau] = \frac{\sigma_y}{2} \times \frac{1}{FOS} = \frac{330}{2\times5} = 33\ N/mm^2$

Design of crank shaft when crank is at top dead centre

A. Find maximum gas force (F)

$$F = \frac{\pi D^2}{4} \times p = \frac{\pi}{4} \times 200^2 \times 2 = 62840\ N$$

B. Design of crankpin

Horizontal reaction due to gas force at each bearing

$$H_g = \frac{F}{2} = \frac{62840}{2} = 31420\ N$$

Horizontal reaction due to belt tension on each bearing

$$H_6 = \frac{3}{2} = 1.5\ kN = 1500\ N$$

Vertical reaction due to height of flywheel

$$V_f = \frac{W}{2} = \frac{15}{2} = 7.5 \text{ kN} = 7500 \text{ N}$$

Let l_c' ; length of crank d_c: diameter of crank pin.

From PSG. DB: 7.31 for gas and oil engine 4stroke for crankpin l_c'/d_c = 0.5 -15

p_b=10.8 12.6 N/mm^2

Let us select l_c'/d_c = 0.8 and p_b = 11 n/mm^2

Bearing pressure $p_b = \frac{F}{l_c' d_c} = \frac{62840}{0.8 d_c}$

$$d_c^2 = \frac{62840}{0.8 \times 11} = 7141$$

$$d_c = 84.51 \text{ say } 85 \text{ mm}$$

$$l_c' = 0.8 \times 85 = 68 \text{ mm say } 70 \text{ mm}$$

From PSG. DB: 7.123 other dimensions are

Thickness   t = 0.7 d= 0.7x85 = 59.5 say 60 mm

Width   w = 1.14 d = 1.14x85 = 96.5 say 97 mm

lc = 1.1 d = 1.1x85 = 93.5 mm say 94mm

C. Calculate length of crankshat, L and diameter of crankshaft D

From PSG. DB: 7.31 for maid L/D = 0.6-2 and pb = 4.9 to 10.4 n/mm^2

Let us take L/D = 1 and ,pb = 10 N/mm^2

Bearing pressure$p_b = \frac{F}{L \times D}$

$$10 = \frac{62840}{2D \times D}$$

D= 79.27 mm say 80mm, L = 80mm

D. Check for bending stress in crank pin

Max. B.M $M = \frac{F \times S}{4}$

F: Maximum gas force

$$S = \left(\frac{L}{2} + 2t + lc + \frac{L}{2}\right) = \left(\frac{80}{2} + 2 \times 60 + 94 + \frac{80}{2}\right) = 294 \text{ mm}$$

Bending stress $\sigma_b = \frac{M}{Z} = \frac{M}{\frac{\pi d^3}{32}} = \frac{62840 \times 294}{\frac{\pi \times 85^3 \times 4}{32}} = 76.6 \text{ N/mm}^2$

Since $(\sigma_b) > [\sigma_b]$design is not satisfactory increase the value of d_c

Let us take d_c=100 mm say l_c=0.8x100 = 80mm

Calculate other parameter

Thickness t= 0.7 d_c = 0.7x100 = 70mm

Width w = 1.14 d = 1.14x100 = 114 mm

lc = 1.1 d = 1.1x100 = 110 mm

$$S = \left(\frac{L}{2} + 2t + lc + \frac{L}{2}\right) = \left(\frac{80}{2} + 2 \times 70 + 100 + \frac{80}{2}\right) = 320 \text{ mm}$$

Check

$$\sigma_b = \frac{M}{Z} = \frac{62840 \times 400}{\frac{\pi \times 100^3 \times 4}{32}} = 64 \text{ N/mm}^2$$

$$(\sigma_b) > [\sigma_b] = 64 \frac{N}{mm^2} \text{ design is safe}$$

Check of crank web stress

$$\emptyset = \sin^{-1}\left(\frac{r}{l} \sin\theta\right)$$

$$\emptyset = \sin^{-1}\left(\frac{1}{4.5} \sin 45^o\right)$$

ϕ=6.38º

Radial force $F_r = P.\cos(\theta + \emptyset)$

$$F_r = 62840 \cos\theta(30 + 6.38\,) = 50593 \text{ N}$$

Tangential force

Radial force $F_t = P.\sin\,(\theta + \emptyset)$

$$F_t = 62840 \sin\,(30 + 6.38\,) = 37273 \text{ N}$$

a. Direct compressive stress due to radial force (Fr)

$$\sigma_c = \frac{\frac{F_r}{2}}{w \times t} = \frac{50593}{2 \times 114 \times 70} = 3.17 \text{ N/mm}^2$$

b. Bending stress due to radial force(F_r)

$$\sigma_{b1} = \frac{\frac{1}{2} F_r.x}{\frac{w \times t^2}{6}}$$

$$x = \frac{L}{2} + \frac{t}{2} = \frac{160}{2} + \frac{70}{2} = 115$$

$$\sigma_{b1} = \frac{6 \times 50593 \times 115}{2 \times 114 \times 70^2} = 31.25 \text{ N/mm}^2$$

Maximum principal stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2 \sigma_y} = 0$$

$$\sigma_1 = \frac{59.87}{2} + \sqrt{\left(\frac{59.87}{2}\right)^2 + 15.44^2}$$

=30+33.68 = 63.68 N/mm²

$$(\boldsymbol{\sigma_1}) > [\boldsymbol{\sigma_1}] = \mathbf{66} \frac{\mathbf{N}}{\mathbf{mm^2}} \mathbf{designissafe}$$

E. Design of right hand crank web

For balancing right hand crank web, dimensions are same as that of left web.

F. Design of shaft under flywheel

Length of main bearing are same at all places

Let width of flysheel = 100 mm

Total distance (c) = 100+80=180 say 200 mm

Bending moment due to weight of flywheel

$$M_p = V_f \times \frac{c}{2} = 7500 \times \frac{200}{2} = 75000 \text{ N} - \text{mm}$$

Bending moment due to belt pull

$$M_B = H_b \times \frac{c}{2} = 1500 \times \frac{200}{2} = 1.5 \times 10^5 \text{ N} - \text{mm}$$

Resultant bending moment

$$M_s = \sqrt{M_f^2 + M_B^2} = \sqrt{(7.5 \times 10^5)^2 + (1.5 \times 10^5)^2} = 7.65 \times 10^5 \text{ N} - \text{mm}$$

Diameter of flywheel shaft MsM$_s = \frac{\pi}{32} d_s^3 \sigma_b$

$$d_s^3 = \frac{7.65 \times 10^5 \times 32}{\pi \times 66} = 49.2 \text{ mm}$$

d_s = 49.2 say 50 mm

5. A horizontal nickel steel shaft rests on two bearings, A at the left and B at the right end and carries two gears C and D located at distances of 250 mm and 400 mm respectively from the centre line of the left and right bearings. The pitch diameter of the gear C is 600 mm and that of gear D is 200 mm. The distance between the centreline of the bearings is 2400 mm. The shaft transmits 20 kW at 120 r.p.m. The power is delivered to the shaft at gear C and is taken out at gear D in such a manner that the tooth pressure FtC of the gear C and FtD of the gear D act vertically downwards. Find the diameter of the shaft, if the working stress is 100 MPa in tension and 56 MPa in shear. The gears C and D weighs 950 N and 350 N respectively. The combined shock and fatigue factors for bending and torsion may be taken as 1.5 and 1.2 respectively. (Nov/Dec – 2012)

Given :

AC = 250 mm ;BD= 400 mm ; DC = 600 mm or RC = 300 mm ;

DD = 200 mm or RD = 100 mm ;AB= 2400 mm ;

P = 20 kW = 20 × 103 W ;

N = 120 r.p.m ;σt= 100 MPa = 100 N/mm2 ;

τ = 56 MPa = 56 N/mm2 ;

WC = 950 N ;WD = 350 N ;

Km = 1.5 ;Kt= 1.2

The shaft supported in bearings and carrying gears is shown in Fig. 2.2

Solution:

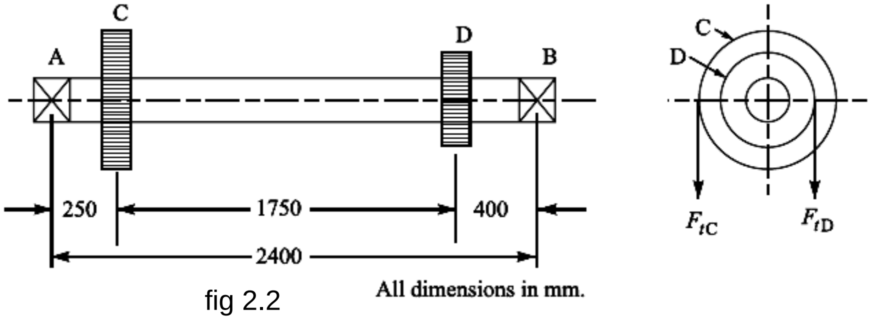


fig 2.2

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20000 \times 60}{2\pi \times 120} = 1590\ \mathrm{N-m} = 1590 \times 10^3 \mathrm{N-mm}$$

Since the torque acting at gears C and D is same as that of the shaft, therefore the tangential force acting at gear C,.

$$F_{tc} = \frac{T}{R_c} = \frac{1590 \times 10^3}{300} = 5300\mathrm{N}$$

and total load acting downwards on the shaft at C

$$= F_{tC} + W_C = 5300 + 950 = 6250\ \mathrm{N}$$

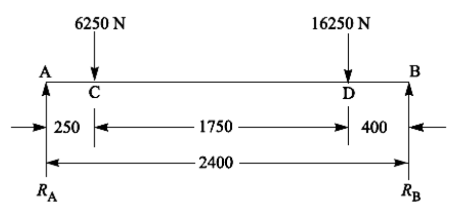
Similarly tangential force acting at gear D,

$$F_{tD} = \frac{T}{R_D} = \frac{1590 \times 10^3}{100} = 15900\mathrm{N}$$

and total load acting downwards on the shaft at D

$$= F_{tD} + W_D = 15\,900 + 350 = 16\,250\ \mathrm{N}$$

Now assuming the shaft as a simply supported beam as shown in Fig. the maximumbending moment may be obtained as discussed below :



Let R_A and R_B = Reactions at A and B respectively.

$\therefore R_A + R_B$ = Total load acting downwards at C and D

$= 6250 + 16\,250 = 22\,500$ N

Now taking moments about A,

$$R_B \times 2400 = 16\,250 \times 2000 + 6250 \times 250 = 34\,062.5 \times 10^3$$

$$\therefore R_B = 34\,062.5 \times 10^3 / 2400 = 14\,190\ \mathrm{N}$$

And$R_A = 22\,500 - 14\,190 = 8310$ N

A little consideration will show that the maximum bending moment will be either at C or D.

We know that bending moment at C,

$$M_C = R_A \times 250 = 8310 \times 250 = 2077.5 \times 10^3\ \mathrm{N\text{-}mm}$$

Bending moment at D,

$$*M_D = R_B \times 400 = 14\ 190 \times 400 = 5676 \times 10^3 \text{ N-mm}$$

∴ Maximum bending moment transmitted by the shaft,

$$M = {}_{MD} = 5676 \times 10^3 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_{eq} = \sqrt{(K_b M)^2 + (K_t M)^2}$$

$$T_{eq} = \sqrt{(1.5 \times 5676000)^2 + (1.2 \times 1590000)^2} = 8725 \times 10^3 N - mm$$

We also know that equivalent twisting moment (Te),

$$8725 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 56 \times d^3 = 11\ d^3$$

$$\therefore d^3 = 8725 \times 10^3 / 11 = 793 \times 10^3 \text{ or } d = 92.5 \text{ mm}$$

Again we know that the equivalent bending moment,

$$M_e = \frac{1}{2}[K_m \times M\sqrt{(K_m \times M)^2 + (K_t \times T)^2} = \frac{1}{2}[K_m \times M \times T_e]$$

$$M_e = \frac{1}{2}[2 \times 5676 \times 10^3 \ + 8725 \times 10^3\ 130] = 8620 \times 10^3\ N - mm$$

We also know that equivalent bending moment (M_e),

$$8620 \times 10^3 \ = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82 d^3$$

Taking the larger of the two values, we have

d = 95.7 say 100 mmAns.

6. Design a shaft to transmit power from an electric motor to a lathe head stock through a pulley by means of a belt drive. The pulley weighs 200 N and is located at 300 mm from the centre of the bearing. The diameter of the pulley is 200 mm and the maximum power transmitted is 1 kW at 120 rpm. The angle of lap of the belt is 180° and coefficient of friction between the belt and the pulley is 0.3. The shock arid fatigue factors for bending and twisting are 1.5 and 2 respectively. The allowable shear stress in the shaft may be taken as 35 MPa. N/D – 2011

Given :

W = 200 N ;L= 300 mm ; D = 200 mm or R = 100 mm ;
P = 1 kW = 1000 W ;N= 120 r.p.m. ;
$\theta = 180° = \pi$ rad ; $\mu = 0.3$;
Km = 1.5 ;Kt= 2 ;
$\tau = 35$ MPa = 35 N/mm2
The shaft with pulley is shown in Fig. 2.3

Solution:

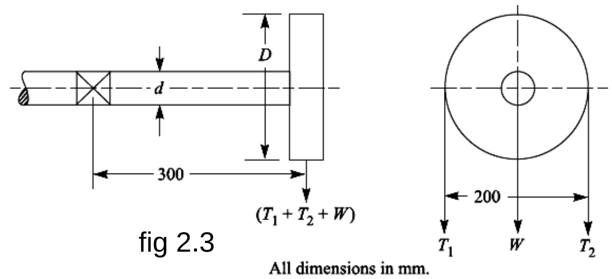


fig 2.3

We know that torque transmitted by the shaft,

$$T = \frac{P\times 60}{2\pi N} = \frac{1000\times 60}{2\pi\times 120} = 79.6\ N - m = 79.6\ \times 10^3 N - m\text{m}$$

Let T1 and T2 = Tensions in the tight side and slack side of the belt respectively in newtons.

∴ Torque transmitted (T),

79.6 × 103 = (T_1 – T_2) R = (T_1 – T_2) 100

∴T1 – T2 = 79.6 × 103 / 100 = 796 N ……………..**(i)**

We know that

$$2.3\, log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3\,\pi = 0.9426$$

$$log\left(\frac{T_1}{T_2}\right) = \frac{0.9426}{2.3} = 0.4098\ or \left(\frac{T_1}{T_2}\right) = 2.57$$

(Taking antilog of 0.4098)

From equations **(i)** and **(ii)**, we get,

T_1 = 1303 N, and T_2 = 507 N

We know that the total vertical load acting on the pulley,

W_T = T_1 + T_2 + W = 1303 + 507 + 200 = 2010 N

∴Bending moment acting on the shaft,

M = W_T × L = 2010 × 300 = 603 × 10^3 N-mm

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_b + M)^2 + (K_t + T)^2}$$

$$T_{eq} = \sqrt{(1.5 \times 603 \times 10^3)^2 + (2 \times 79.6 \times 10^3)^2} = 918 \times 10^3 N - mm$$

We also know that equivalent twisting moment (Te),

$$918 \times 10^3 = \frac{\pi}{16}\tau \times d^3 = \frac{\pi}{16} \times 35 \times d^3 = 6.87\, d^3$$

∴d ^{3} = 918 × 103 / 6.87 = 133.6 × 10^3 or d = 51.1 say 55 mm **Ans.**

7. A transmission shaft is supported on tow bearings which are 1m apart power is supplied to shaft by means of a flexible coupling which is located to the left of left hand bearing. Power is transmitted from the shaft by means of a belt pulley, 250 mm diameter. Which is located at a distance of 300 mm from the left hand bearing? The mass of the pulley is 20 kg and the ratio of belt tension on tight and slack sides is2:1. The belt tension act vertically downward. The shaft is made of steel with yield stress 300 N/mm^2 and the factor of safety is 3. Determine the shaft diameter, if it transmits 10kW power at 360 rpm from the coupling to the pulley. (Nov/Dec – 2010)

Given Data:

D= 250mm,

l=300mm;

T1/T2 = 2

σy=300 N/mm^2

$$[\tau] = \frac{\sigma_y}{2} \times \frac{1}{FS} = \frac{300}{2} \times \frac{1}{3} = 50N/mm^2$$

FOS = 3

P= 10kW

N=360 rpm

m=20 kg

W= 20x9.81 N

Solution:

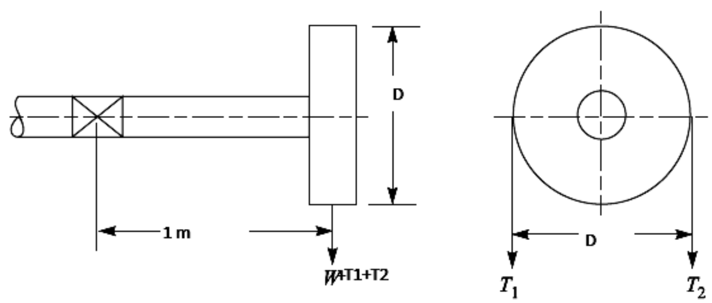


fig 2.4

We know that

$$P = \frac{T \times 2\pi N}{60}$$

$$T = \frac{P \times 60}{2\pi N}$$

$$= \frac{10 \times 10^3 \times 60}{2\pi \times 360}$$

T = 265.26 N-m

T= 265.26 x10^3 N/mm

Torque transmitted

$$T = (T_1 - T_2)R$$

$$265.26 \times 10^3 = (T_1 - T_2) \times \frac{250}{2}$$

$$T_1 - T_2 = 2.122 \times 10^3 N \text{ ------------1}$$

$$\frac{T_1}{T_2} = 2 \; given \text{ -------------2}$$

$$T_1 = 2T_2 \text{ ---------------3}$$

Substitute (3) in (1)

$$T_1 - T_2 = 2.122 \times 10^3 N$$

$$2T_2 - T_2 = 2.122 \times 10^3 N$$

$$T_2 = 2122 \, N$$

Substitute T_2 in 3

$$T_1 = 2 \times 2122 = 4244$$

Total vertical load acting on the pulley in fig 2.4

$$W_T = T_1 + T_2 + W = 4244 + 2122 + (20 \times 9.81) = 6562.2$$

Bending moment acting on the shaft

M=WT x L = 6562.2 x1000=6562.2 x 10^3 N-mm

Equivalent twisting moment

$$T_e = \sqrt{(K_b + M)^2 + (K_t + T)^2}$$

$$T_{eq} = \sqrt{(1.5 \times 6562.2 \times 10^3)^2 + (2 \times 265.26 \times 10^3)^2}$$

$$T_e == 9857.6 \times 10^3 N - mm$$

$$T_e = \frac{\pi}{16} \times \tau_s \times d^3$$

$$9857.6 \times 10^3 = \frac{\pi}{16} \times 50 \times d^3$$

$$d^3 = 1004086.9$$

$$\boldsymbol{d = 100.13 \, mm}$$

8. Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.

Given :

do= d ; di = do / 2 or

k = di / do = 1 / 2 = 0.5

Solution:

We know that weight of a hollow shaft,

W_H = Cross-sectional area × Length × Density

$$= \frac{\pi}{4}[(d_o)^2 - (d_i)^2] \times Length \times Density \text{-----------}1$$

and weight of the solid shaft,

$$W_S = 2 \text{ Length} \times \text{Density}$$

Since both the shafts have the same material and length, therefore by dividing equation **(i)** by equation **(ii)**, we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2}$$

$$= 1 - \frac{(d_i)^2}{(d_o)^2} = (1 - k^2) = 1 - 0.5^2$$

$$\boldsymbol{\frac{W_H}{W_S} = 0.75 \;\; mmAns}$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau \times (d_o)^2 \times (1 - k^4) \text{--------------------}3$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \text{--------------------}4$$

Dividing equation 3 by equation 4 we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3 \quad (1 - k^4)^2}{d^3} = \frac{(d_o)^3 \quad (1 - k^4)^2}{(d_o)^3} = 1 - k^4$$

$$\boldsymbol{\frac{T_H}{T_S} = 1 - 0.5^4 = 0.9375}$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

∴ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32}[(d_o)^4 - (d_i)^4] \text{ -------------------}5$$

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4 \text{ ------------------}5$$

Dividing equation (v) by equation (vi), we get

$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - 4}{(d_o)^4} = 1 - k^4$$

$$1 - k^4 = 1 - 0.5^4 = 0.9375 \; ans$$

9.A solid shaft of diameter d is used in power transmission. Due to modification of the existing transmission system, it is required to replace the solid shaft by hollow shaft of the same material and equally strong in torsion. Further the weight of the hollow shaft per meter length should be half of the solid shaft. Determine the outer diameter of the hollow shaft in terms of d.

Solution:(N/D'2022)

Step-I:

Torque transmitted by solid shaft

$$M_t = \left(\frac{\pi d^3}{16}\right)\tau \text{ ---------------------1}$$

Step – II :

Torque transmitted by hollow shaft

$$M_t = \left(\frac{J}{r}\right)\tau$$

Where $J = \frac{\pi(d_o^4 - d_i^4)}{32}$

$$M_t = \left(\frac{\frac{\pi(d_o^4 - d_i^4)}{32}}{16 d_o}\right)\tau \text{ ------------------2}$$

Step –III :

Outer diameter of hollow shaft

The solid and hollow shaft are equally strong in torsion. Equating (1) and (2)

$$d^3 = \frac{(d_o^4 - d_i^4)}{d_o}$$

$$d_i^4 = d_o^4 - d^3 d_o \text{ -----------------3}$$

Since the weight of hollow shat per metre length is half of the solid shaft.

$$\frac{\pi(d_o^2 - d_i^2)}{4} = \frac{1}{2} \times \frac{\pi d^2}{4}$$

$$d_i^2 = d_o^2 - \frac{d^2}{2}$$

$$d_i^4 = \left(d_o^2 - \frac{d^2}{2}\right)^2 \text{ ---------------------4}$$

Eliminating the term d_i^4 from equation (3) and (4)

$$d_o^2 - d^3 d_o = \left(d_o^2 - \frac{d^2}{2}\right)^2$$

$$d_o^4 - d^3 d_o = d_o^4 - d_o^2 d^2 + \frac{d^4}{4}$$

$$d_o^2 d^2 - d_o d^3 - \frac{d^4}{4} = 0 \text{ -------------------------5}$$

Equation (5) is a quadratic equation and the roots are given by

$$d_o = \frac{d^3 \pm \sqrt{d^6 - 4(d^2)\left(-\frac{d^4}{4}\right)}}{2d^2} = \frac{d \pm \sqrt{2}d}{2}$$

Taking positive root

$$d_o = \frac{d \pm \sqrt{2}d}{2}$$

10. Design an overhung crankshaft for an ic engine for the following data: stroke(L) = 350 mm, cylinder bore d_1= 250 mm, length of the connecting rod = 5times the crank radius. Maximum gas pressure = 2N/mm^2

Given Data:

Stroke L = 350 mm , Radius of crank r= L/2 = 350/2 =175 mm

Cylinder bore d_1 = 250 mm, l/r=5

Length of the connecting rod = 5(r) = 5x175 = 875 mm

p=2 N/mm^2

(Torque is maximum when θ varies from 25^{o}to 40^o)

Select θ=30^o

Solution:

Step – I: Selection of materials:

Select C40 steel for crankpin, web and crankshaft from PSG. Data book

σ_y= 330N/mm^2 , Factor of safety = 5

Permissible bending stress $[\sigma_b]=\sigma_y$/FOS = 330/5=66 N/mm^2 =$[\sigma_c]$

Permissible shear stress $[\tau] = \frac{\sigma_y}{2} \times \frac{1}{FOS} = \frac{330}{2\times 5} = 33\ N/mm^2$

Step – II : Find gas force

$$Gas force F = \frac{\pi d_1^2}{4} \times p = \frac{\pi}{4} \times 250^2 \times 2 = 98.17 \times 10^3 N$$

$$F = 98.17 \times 10^3 N$$

This force 'F' induces bending stress in the crankpin

Step – III : To find l_c and d_c (crank pin)

Select $\frac{l_c}{d_c} and [p_b] from the data book page no: 7.31$

Consider 4-stroke engine

Crank pin $\frac{l_c}{d_c} = 0.6\ to\ 1.5\ p_b = 108 - 126 \frac{kgf}{cm^2} = 10.8 - 12.6\ n/mm^2$

Select $\frac{l_c}{d_c} = 0.6$

$$F = l_c d_c [p_b]$$

$$98.17 \times 10^3 = (0.6 d_c) d_c \times 1$$

Diameter of crankshaft = d_c = 121.96 mm

Length of crank pin l_c = 73.17 mm

Step – IV: check for the stress in the crankpin

The load F induces bending moment
The maximum BM occurs at 0.75 l_c from the web
BM= M = Fx0.75l_c=98.17x10^3x0.75x73.17 = 5.387 x10^6 N-mm
Bending stress $\sigma_b = \frac{M}{Z} = \frac{M}{\frac{\pi d^3}{32}} = \frac{5.387\times10^6}{\frac{\pi\times121.96^3}{32}} = 30.24\ N/mm^2$

Since $(\boldsymbol{\sigma_b}) > [\boldsymbol{\sigma_b}]\boldsymbol{thenchange}\,\frac{\boldsymbol{l_c}}{\boldsymbol{d_c}}\boldsymbol{ratioand}[\boldsymbol{pb}]\boldsymbol{value}.$

Step – V : to find L and d crank shaft (main bearing)

From PSG. DB: 7.31 for maid L/D = 0.6-2 and pb = 4.9 to 10.4 n/mm^2
Let us take L/D = 0.6 and pb = 5 N/mm^2

$$F = L.D[p_b]$$
$$98.17 \times 10^3 = 0.6d(d)5$$

Diameter of crankshaft d=233.53 mm
Length of bearing L = 140.12 mm

Step –VI: calculation of (t) and (w)

From PSG. DB: 7.123
Proportions $t = Thicknessofwebvariesfromt_p tot_b$

$$= 1.4d_c to(1\ to\ 1.25\ d)$$

Select t=180 mm $w = widthofthewebvariesfromatob$

$$= 1.5d_c to(1\ to\ 1.35\ d)$$

Select w=250 mm (average approximately)

Step – VII: Calculate F_r and F_t (components of force F)

$$sin\,\emptyset = \frac{sin\,\theta}{\frac{l}{r}} = \frac{sin\,30^o}{5}$$
$$\emptyset = 5.73^o$$
$$F_r = Radialcomponent = F.cos(\theta + \emptyset) = 98.17\ \times 10^3 cos(30^o + 5.73^o)$$
$$F_r = 79.68\ \times 10^3 N$$
$$F_t = Tangentailcomponent = F.sin(\theta + \emptyset) = 98.17\ \times 10^3 sin(30^o + 5.73^o)$$
$$F_t = 57.32\ \times 10^3 N$$

Step – VIII : check the stress in the web

In the web portion consider point A
At A consider two equal and opposite forces F_{r1} and F_{r2}
Such that $F_{r1} = F_{r2} = F_r$
F_{r1} and $F_r$produce couple, the effect of coupleproduces bending moment, which induces bending stress (compressive).

$$C = \left(0.75\ lc + \frac{t}{2}\right) F_r = M = BM$$

M= 11.54x10^6 N-mm

1. **Bending stress due to couple**

$$\sigma_b = \frac{11.54 \times 10^6}{1.35 \times 10^6} = 8.548\ N/mm^2$$

2. **Force induced direct compressive stress.**

$$\sigma_c = \frac{79.68 \times 10^3}{250 \times 180} = 1.1770\ N/mm^2$$

Total stress in the web = $\sigma_u = \sigma_b + \sigma_c = 10.318\ N/mm^2 (compressive)$

$$= \boldsymbol{\sigma_w < \sigma_b = 66\ \ N/mm^2}$$

The design of web is safe.

Step – IX : checking the stress in the crank shaft (d)

The crank shaft is subjected to bending stress due to F_{r2} and shear stress due to F_t.

Bending moment = $= F_{r2}\left(0.75\ lc + t + \frac{L}{2}\right)$

$$M_b = 24.29 \times 10^6 N - mm$$

Twisting moment

$$M_t = F_t \times crankradius$$

$$M_t = 8.598 \times 10^6 N - mm$$

$$\sqrt{M_b^2 + M_t^2} = \frac{\pi}{16} \times \tau \times d^3$$

$$\tau = 10.3\ N/mm^2$$

$$\boldsymbol{(\tau) < [\tau] = 33\ N/mm^2}$$

The design of crankshaft satisfactory.

11. A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Solution. Given : M = 3000 N-m = 3×10^6 N-mm ; T = 10 000 N-m = 10×10^6 N-mm ; σ_{tu} = 700 MPa = 700 N/mm^2 ; τ_u = 500 MPa = 500 N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$\therefore$ $d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6$ or $d = 83.7$ mm

Taking the larger of the two values, we have

d = 86 say 90 mm **Ans.**

12. A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulleydirectly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mmdiameter is placed 200 mm to the left of right hand bearing and is driven with the help of electricmotor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on onepulley is equal to that on the other pulley.(April/May 17)(A/M'2023)

Solution. Given : $AB = 1$ m ; $D_C = 600$ mm or $R_C = 300$ mm $= 0.3$ m ; $AC = 300$ mm $= 0.3$ m ; $T_1 = 2.25$ kN $= 2250$ N ; $D_D = 400$ mm or $R_D = 200$ mm $= 0.2$ m ; $BD = 200$ mm $= 0.2$ m ; $\theta = 180° = \pi$ rad ; $\mu = 0.24$; $\sigma_b = 63$ MPa $= 63$ N/mm^2 ; $\tau = 42$ MPa $= 42$ N/mm^2

The space diagram of the shaft is shown in Fig. 14.5 (*a*).

Let T_1 = Tension in the tight side of the belt on pulley $C = 2250$ N ...(Given)

T_2 = Tension in the slack side of the belt on pulley C.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.24 \times \pi = 0.754$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.754}{2.3} = 0.3278 \quad \text{or} \quad \frac{T_1}{T_2} = 2.127 \qquad \text{...(Taking antilog of 0.3278)}$$

and

$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058 \text{ N}$$

$\therefore$ Vertical load acting on the shaft at C,

$$W_C = T_1 + T_2 = 2250 + 1058 = 3308 \text{ N}$$

and vertical load on the shaft at D

$$= 0$$

The vertical load diagram is shown in Fig.

We know that torque acting on the pulley C,

$$T = (T_1 - T_2)\, R_C = (2250 - 1058)\, 0.3 = 357.6 \text{ N-m}$$

The torque diagram is shown in Fig.

Let T_3 = Tension in the tight side of the belt on pulley D, and

T_4 = Tension in the slack side of the belt on pulley D.

Since the torque on both the pulleys (*i.e.* C and D) is same, therefore

$$(T_3 - T_4)\, R_D = T = 357.6 \text{ N-m} \quad \text{or} \quad T_3 - T_4 = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788 \text{ N} \qquad \text{...}(i)$$

We know that

$$= \frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127 \quad \text{or} \quad T_3 = 2.127\, T_4 \qquad \text{...}(ii)$$

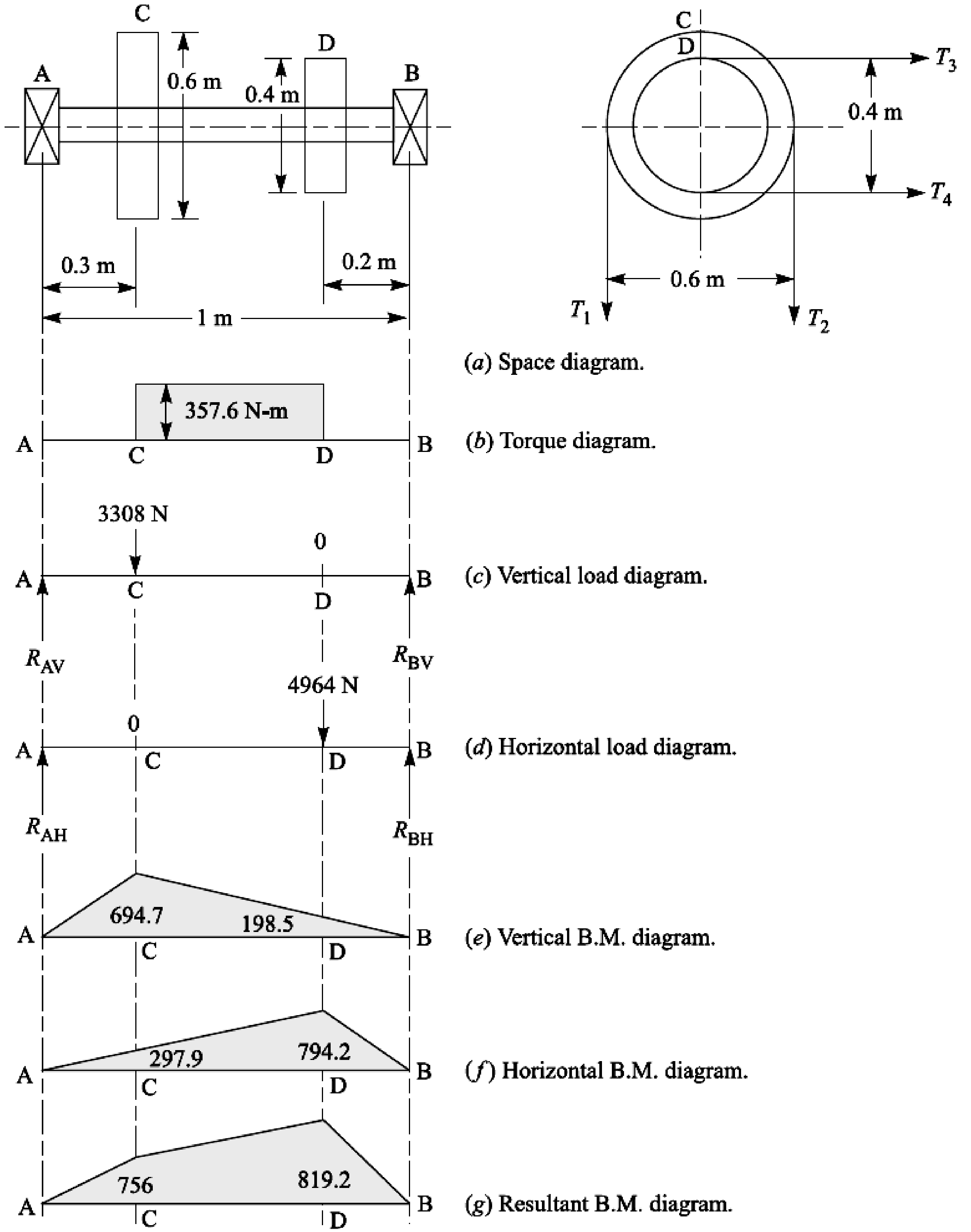

C
D
A
B
0.6 m
0.4 m
0.3 m
0.2 m
1 m
T3
T4
T1
T2
0.6 m
0.4 m
(a) Space diagram.
357.6 N-m
(b) Torque diagram.
3308 N
0
(c) Vertical load diagram.
RAV
RBV
4964 N
0
(d) Horizontal load diagram.
RAH
RBH
694.7
198.5
(e) Vertical B.M. diagram.
297.9
794.2
(f) Horizontal B.M. diagram.
756
819.2
(g) Resultant B.M. diagram.

fig 2.5

From equations **(i)** and **(ii),** we find that

$$T_3 = 3376 \text{ N, and } T_4 = 1588 \text{ N}$$

∴ Horizontal load acting on the shaft at D,

$$W_D = T_3 + T_4 = 3376 + 1588 = 4964 \text{ N}$$

and horizontal load on the shaft at C = 0

The horizontal load diagram is shown in Fig. (d). 2.5

Now let us find the maximum bending moment for vertical and horizontal loading

First of all, considering the vertical loading at *C*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AV} + R_{BV} = 3308 \text{ N}$$

Taking moments about *A*,

$$R_{BV} \times 1 = 3308 \times 0.3 \text{ or } R_{BV} = 992.4 \text{ N}$$

and $R_{AV} = 3308 – 992.4 = 2315.6$ N

We know that B.M. at *A* and *B*,

$$M_{AV} = M_{BV} = 0$$

B.M. at *C*, $M_{CV} = R_{AV} \times 0.3 = 2315.6 \times 0.3 = 694.7$ N-m

B.M. at *D*, $M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2 = 198.5$ N-m

The bending moment diagram for vertical loading in shown in Fig.(*e*). 2.5

Now considering horizontal loading at *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AH} + R_{BH} = 4964 \text{ N}$$

Taking moments about *A*,

$$R_{BH} \times 1 = 4964 \times 0.8 \text{ or } R_{BH} = 3971 \text{ N}$$

and $R_{AH} = 4964 – 3971 = 993$ N

We know that B.M. at *A* and *B*,

$$M_{AH} = M_{BH} = 0$$

B.M. at *C*, $M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9$ N-m

B.M. at *D*, $M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2$ N-m

The bending moment diagram for horizontal loading is shown in Fig.(*f*). 2.5

Resultant B.M. at *C*,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \text{ N-m}$$

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \text{ N-m}$$

The resultant bending moment diagram is shown in Fig. 2.5 (*g*).

We see that bending moment is maximum at *D*.

$\therefore$ Maximum bending moment,

$$M = M_D = 819.2 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$

$$= 894 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$\therefore$ $d^3 = 894 \times 10^3 / 8.25 = 108 \times 10^3$ or $d = 47.6$ mm

Again we know that equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(819.2 + 894) = 856.6 \text{ N-m} = 856.6 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

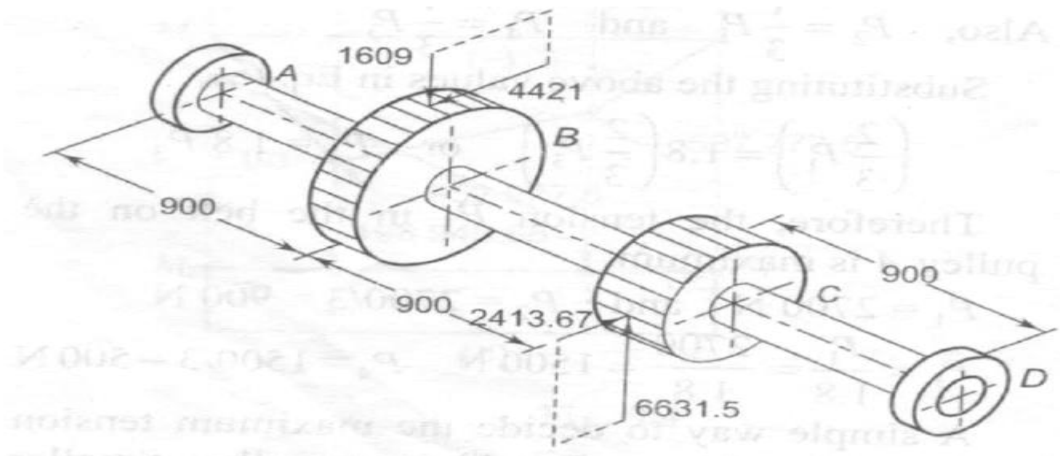
$$856.6 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 63 \times d^3 = 6.2\, d^3$$

$\therefore$ $d^3 = 856.6 \times 10^3 / 6.2 = 138.2 \times 10^3$ or $d = 51.7$ mm

Taking larger of the two values, we have

$$d = 51.7 \text{ say } 55 \text{ mm } \textbf{Ans.}$$

13. The layout of an intermediate shaft of a gear box supporting two spur gears B and C is shown in Fig. The shaft is mounted on two bearings A and D. The pitch circle diameters of gears Band Care 900 and 600 mm respectively. The material of the shaft is steel FeE 580 (S_{ut}= 770 and S_{yt} = 580 N/mm^2), The factors are $K_b$and K_t, ofASME code are 1.5 and 2.0 respectively. Determinethe shaft diameter using the ASME code.Assume that the gears are connected to the shaftby means of keys.



Solution

Given $S_{ut} = 770$ N/mm^2 $S_{yt} = 580$ N/mm^2

$k_b = 1.5$ $k_t = 2.0$

For gears, $(d'_p)_B = 900$ mm $(d'_p)_C = 600$ mm

Step I *Permissible shear stress*

$$0.30\ S_{yt} = 0.30(580) = 174 \text{ N/mm}^2$$

$$0.18\ S_{ut} = 0.18(770) = 138.6 \text{ N/mm}^2$$

The lower of the two values is 138.6 N/mm^2 and there are keyways on the shaft.

$$\therefore \qquad \tau_{max.} = 0.75(138.6) = 103.95 \text{ N/mm}^2$$

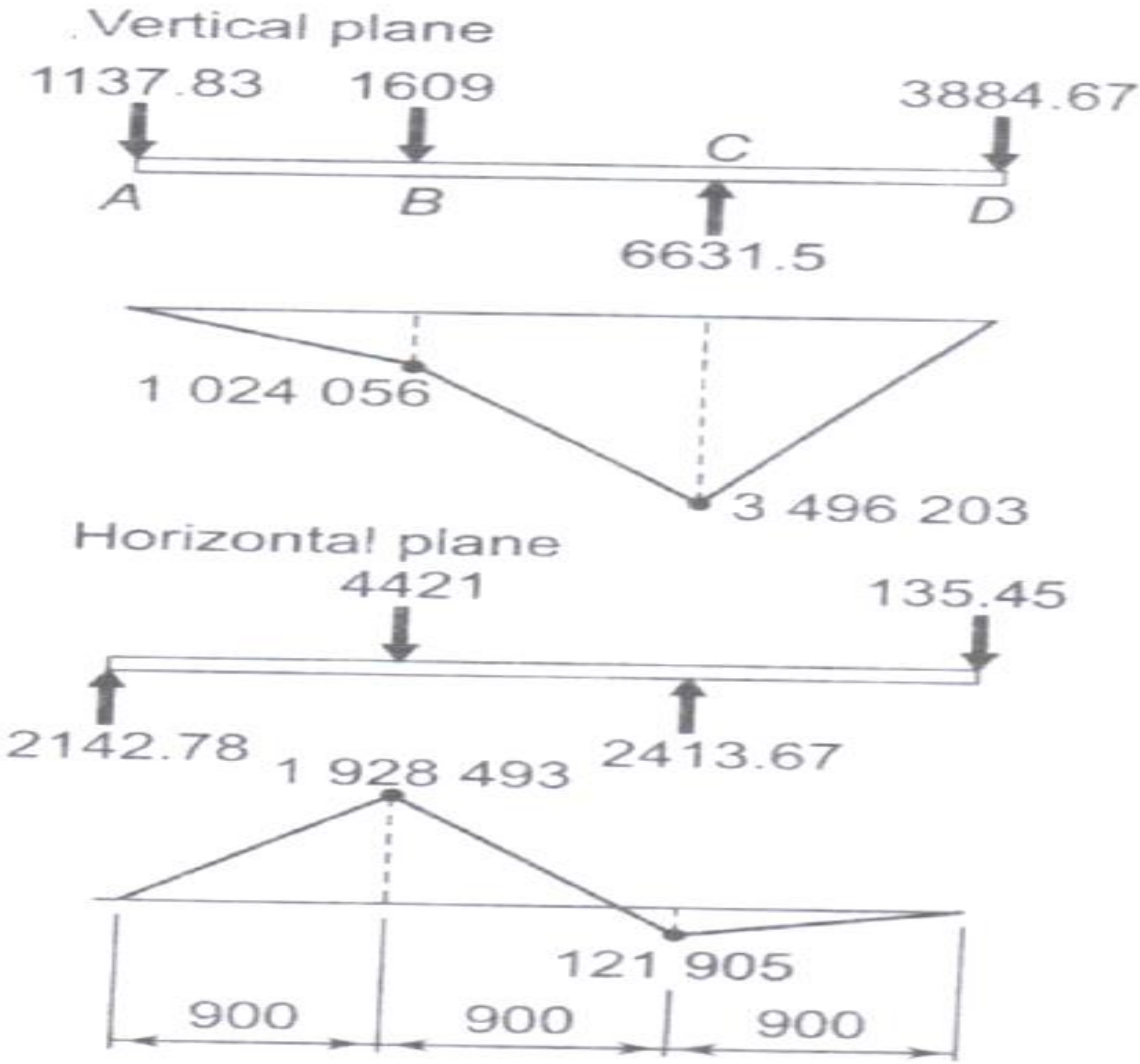
Step II *Bending and torsional moment*

The forces and bending moments in vertical and horizontal planes are shown in Fig. The maximum bending moment is at C. The resultant bending moment at C is given by,

$$M_b = \sqrt{(3\,496\,203)^2 + (121\,905)^2}$$

$$= 3\,498\,327.4 \text{ N-mm}$$

$$M_t = 4421(450) = 1\,989\,450 \text{ N-mm}$$



Step III Shaft diameter:

$$d^3 = \frac{16}{\pi\tau_{max.}}\sqrt{(k_b M_b)^2 + (k_t M_t)^2}$$

$$= \frac{16}{\pi(103.95)}\sqrt{(1.5 \times 3\,498\,327.4)^2 + (2.0 \times 1989\,450)^2}$$

or $\quad d = 68.59 \text{ mm}$

14. A shaft is supported by a two bearings placed 1100 mm apart. A pulley of diameter 620 mm is keyed at 400 mm to the right of the left hand bearing and this drives a pulley directly below it with a maximum tension of 2.75 KN. Another pulley of diameter 400 mm is placed 200 mm to the left of the right hand bearing and is driven with a motor placed horizontaly to the right. The angle of contact of the pulley is 180^0 and the coefficient of the friction between the belt and pulleys is 0.3. find the diameter of the shaft assume K_b = 3, K_t = 2.5, S_{yt} =190 MPa and S_{ut} = 300 Mpa. (April/May 17)

Solution:

We just find out the value of permissible shear stress

$$\tau_{\max} = 0.30 S_{yt} (or) 0.18 S_{ut}$$

Which ever is minimum, the above value are to be reduced by 25 Percent

$$\tau_{\max} = 0.30 * 190 = 57 N/mm^2$$

$$\tau_{\max} = 0.18 * 300 = 54 N/mm^2$$

The lowest of the two value is 54 N/mm^2 and they are the keyway on the shaft

$$\tau_{\max} = 0.75 * 54 = 40.5 N/mm^2$$

Similar procedure followed above problem

15. A shaft is supported on bearings A and B, 800 mm between centres. A 20°straight tooth spur gear having 600 mm pitch diameter, is located 200 mm to the right of the left handbearing A, and a 700 mm diameter pulley is mounted 250 mm towards the left of bearing B. The gearis driven by a pinion with a downward tangential force while the pulley drives a horizontal belthaving 180° angle of wrap. The pulley also serves as a flywheel and weighs 2000 N. The maximumbelt tension is 3000 N and the tension ratio is 3 : 1. Determine the maximum bending moment and thenecessary shaft diameter if the allowable shear stress of the material is 40 MPa.

Solution. Given : $AB = 800$ mm ; $\alpha_C = 20°$; $D_C = 600$ mm or $R_C = 300$ mm ; $AC = 200$ mm ; $D_D = 700$ mm or $R_D = 350$ mm ; $DB = 250$ mm ; $\theta = 180° = \pi$ rad ; $W = 2000$ N ; $T_1 = 3000$ N ; $T_1/T_2 = 3$; $\tau = 40$ MPa $= 40$ N/mm²

The space diagram of the shaft is shown in Fig. 14.6 (*a*).

We know that the torque acting on the shaft at *D*,

$$T = (T_1 - T_2)\,R_D = T_1\left(1 - \frac{T_2}{T_1}\right)R_D$$

$$= 3000\left(1 - \frac{1}{3}\right)350 = 700 \times 10^3 \text{ N-mm} \qquad ...(\because T_1/T_2 = 3)$$

The torque diagram is shown in Fig. 14.6 (*b*).

Assuming that the torque at *D* is equal to the torque at *C*, therefore the tangential force acting on the gear *C*,

$$F_{tc} = \frac{T}{R_C} = \frac{700 \times 10^3}{300} = 2333 \text{ N}$$

and the normal load acting on the tooth of gear *C*,

$$W_C = \frac{F_{tc}}{\cos\alpha_C} = \frac{2333}{\cos 20°} = \frac{2333}{0.9397} = 2483 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. Resolving the normal load vertically and horizontally, we get

Vertical component of W_C *i.e.* the vertical load acting on the shaft at *C*,

$$W_{CV} = W_C \cos 20°$$
$$= 2483 \times 0.9397 = 2333 \text{ N}$$

and horizontal component of W_C *i.e.* the horizontal load acting on the shaft at *C*,

$$W_{CH} = W_C \sin 20°$$
$$= 2483 \times 0.342 = 849 \text{ N}$$

Since $T_1 / T_2 = 3$ and $T_1 = 3000$ N, therefore

$$T_2 = T_1 / 3 = 3000 / 3 = 1000 \text{ N}$$

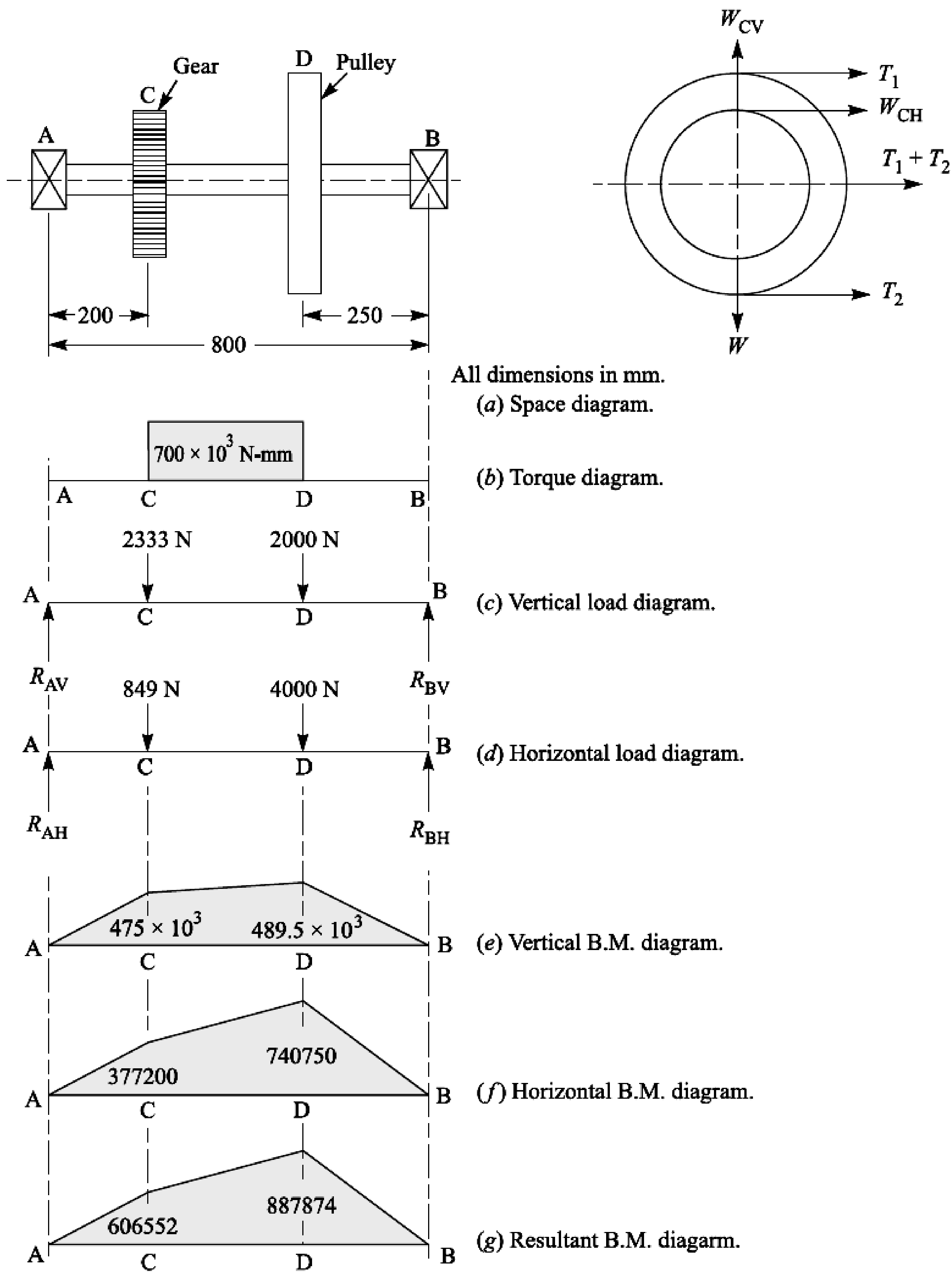

Gear
Pulley
A
C
D
B
200
250
800
W_CV
T_1
W_CH
T_1 + T_2
T_2
W
All dimensions in mm.
(a) Space diagram.
700 × 10^3 N-mm
(b) Torque diagram.
2333 N
2000 N
(c) Vertical load diagram.
R_AV
R_BV
849 N
4000 N
(d) Horizontal load diagram.
R_AH
R_BH
475 × 10^3
489.5 × 10^3
(e) Vertical B.M. diagram.
377200
740750
(f) Horizontal B.M. diagram.
606552
887874
(g) Resultant B.M. diagarm.

fig 2.6

∴ Horizontal load acting on the shaft at *D*,

$$W_{DH} = T_1 + T_2 = 3000 + 1000 = 4000 \text{ N}$$

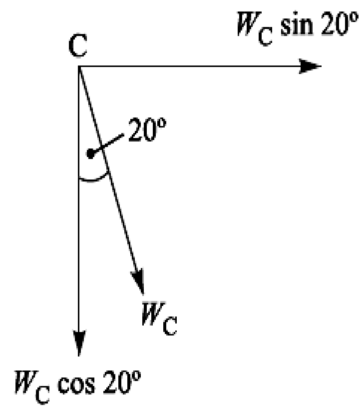
and vertical load acting on the shaft at *D*,

$$W_{DV} = W = 2000 \text{ N}$$

The vertical and horizontal load diagram at *C* and *D* is shown in Fig. 2.6 (*c*) and (*d*) respectively.

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all considering the vertical loading at *C* and *D*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that



$$R_{AV} + R_{BV} = 2333 + 2000 = 4333 \text{ N}$$

Taking moments about *A*, we get

$$R_{BV} \times 800 = 2000\,(800 - 250) + 2333 \times 200$$
$$= 1\,566\,600$$

$$\therefore \quad R_{BV} = 1\,566\,600 / 800 = 1958 \text{ N}$$

and $$R_{AV} = 4333 - 1958 = 2375 \text{ N}$$

We know that B.M. at *A* and *B*,

$$M_{AV} = M_{BV} = 0$$

B.M. at *C*, $$M_{CV} = R_{AV} \times 200 = 2375 \times 200$$
$$= 475 \times 10^3 \text{ N-mm}$$

B.M. at D, $$M_{DV} = R_{BV} \times 250 = 1958 \times 250 = 489.5 \times 10^3 \text{ N-mm}$$

The bending moment diagram for vertical loading is shown in Fig. 2.6 (*e*).

Now consider the horizontal loading at *C* and *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AH} + R_{BH} = 849 + 4000 = 4849 \text{ N}$$

Taking moments about *A*, we get

$$R_{BH} \times 800 = 4000\,(800 - 250) + 849 \times 200 = 2\,369\,800$$

$$\therefore \quad R_{BH} = 2\,369\,800 / 800 = 2963 \text{ N}$$

and $$R_{AH} = 4849 - 2963 = 1886 \text{ N}$$

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $M_{CH} = R_{AH} \times 200 = 1886 \times 200 = 377\ 200$ N-mm

B.M. at D, $M_{DH} = R_{BH} \times 250 = 2963 \times 250 = 740\ 750$ N-mm

The bending moment diagram for horizontal loading is shown in Fig. 2.6 (f).

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(475 \times 10^3)^2 + (377\,200)^2}$$

$$= 606\ 552 \text{ N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(489.5 \times 10^3)^2 + (740\ 750)^2}$$

$$= 887\ 874 \text{ N-mm}$$

Maximum bending moment

The resultant B.M. diagram is shown in Fig. 2.6 (g). We see that the bending moment is maximum at D, therefore

Maximum B.M., $M = M_D = 887\ 874$ N-mm **Ans.**

Diameter of the shaft

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(887\ 874)^2 + (700 \times 10^3)^2} = 1131 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1131 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 1131 \times 10^3 / 7.86 = 144 \times 10^3$ or $d = 52.4$ say 55 mm **Ans.**

16. A solid steel shaft is supported on two bearings 1.8 m apart and rotates at 250r.p.m. A 20° involute gear D, 300 mm diameter is keyed to the shaft at a distance of 150 mm to the lefton the right hand bearing. Two pulleys B and C are located on the shaft at distances of 600 mm and1350 mm respectively to the right of the left hand bearing. The diameters of the pulleys B and C are750 mm and 600 mm respectively. 30 kW is supplied to the gear, out of which 18.75 kW is taken off atthe pulley C and 11.25 kW from pulley B. The drive from B is vertically downward while from C thedrive is downward at an angle of 60° to the horizontal. In both cases the belt tension ratio is 2 andthe angle of lap is 180°. The combined fatigue and shock factors for torsion and bending maybetaken as 1.5 and 2 respectively.
Design a suitable shaft taking working stress to be 42 MPa in shear and 84 MPa in tension.(May/June-16)

Solution.

Given :PQ= 1.8 m ; N = 250 r.p.m ; α_D = 20° ; D_D = 300 mm or R_D = 150 mm = 0.15 m
Q_D= 150 mm = 0.15 m ; P_B= 600 mm = 0.6 m ; P_C= 1350 mm = 1.35 m ; D_B = 750 mm or R_B = 375mm = 0.375 m ; D_C = 600 mm or R_C = 300 mm = 0.3 m ; P_D = 30 kW = 30 × 103 W ; P_C = 18.75 kW= 18.75 × 103 W ; P_B = 11.25 kW = 11.25 × 103 W ; T_{B1}/T_{B2} = T_{C1}/T_{C2} = 2 ; θ = 180° = π rad ;K_t= 1.5 ; K_m= 2 ; τ = 42 MPa = 42 N/mm^2 ; σt= 84 MPa = 84 N/mm^2
First of all, let us find the total loads acting on the gear D and pulleys C and B respectively

For gear D

We know that torque transmitted by the gear D,

$$T_D = \frac{P_D \times 60}{2\pi N} = \frac{30 \times 10^3 \times 60}{2\pi \times 250} = 1146 \text{ N-m}$$

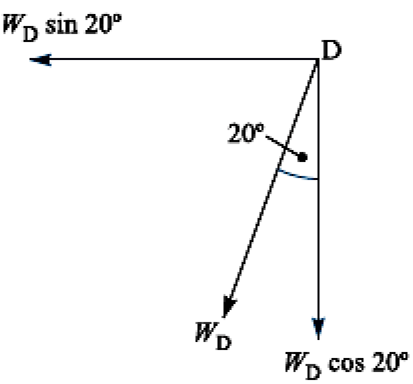


fig 2.7

∴ Tangential force acting on the gear D,

$$F_{tD} = \frac{T_D}{R_D} = \frac{1146}{0.15} = 7640 \text{ N}$$

and the normal load acting on the gear tooth,

$$W_D = \frac{F_{tD}}{\cos 20°} = \frac{7640}{0.9397} = 8130 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. 2.7 Resolving the normal load vertically and horizontally, we have

Vertical component of W_D

$$= W_D \cos 20° = 8130 \times 0.9397 = 7640 \text{ N}$$

Horizontal component of W_D

$$= W_D \sin 20° = 8130 \times 0.342 = 2780 \text{ N}$$

For pulley C

We know that torque transmitted by pulley C,

$$T_C = \frac{P_C \times 60}{2\pi N} = \frac{18.75 \times 10^3 \times 60}{2\pi \times 250} = 716 \text{ N-m}$$

Let T_{C1} and T_{C2} = Tensions in the tight side and slack side of the belt for pulley C.

We know that torque transmitted by pulley C (T_C),

$$716 = (T_{C1} - T_{C2}) R_C = (T_{C1} - T_{C2})\, 0.3$$

∴ $T_{C1} - T_{C2} = 716 / 0.3 = 2387$ N ...(*i*)

Since $T_{C1} / T_{C2} = 2$ or $T_{C1} = 2\,T_{C2}$, therefore from equation (*i*), we have

$$T_{C2} = 2387 \text{ N} ; \text{ and } T_{C1} = 4774 \text{ N}$$

∴ Total load acting on pulley C,

$$W_C = T_{C1} + T_{C2} = 4774 + 2387 = 7161 \text{ N}$$

...(Neglecting weight of pulley C)

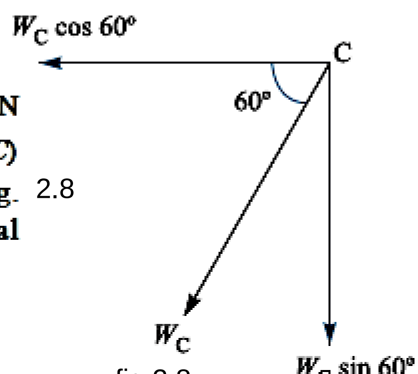


fig 2.8

This load acts at 60° to the horizontal as shown in Fig. 2.8

Resolving the load W_C into vertical and horizontal components, we have

Vertical component of W_C

$$= W_C \sin 60° = 7161 \times 0.866$$
$$= 6200 \text{ N}$$

and horizontal component of W_C

$$= W_C \cos 60° = 7161 \times 0.5$$
$$= 3580 \text{ N}$$

For pulley B

We know that torque transmitted by pulley *B*,

$$T_B = \frac{P_B \times 60}{2\pi N} = \frac{11.25 \times 10^3 \times 60}{2\pi \times 250} = 430 \text{ N-m}$$

Let T_{B1} and T_{B2} = Tensions in the tight side and slack side of the belt for pulley *B*.

We know that torque transmitted by pulley *B* (T_B),

$$430 = (T_{B1} - T_{B2})\, R_B = (T_{B1} - T_{B2})\, 0.375$$

$$\therefore T_{B1} - T_{B2} = 430 / 0.375 = 1147 \text{ N} \quad \ldots\textbf{(ii)}$$

Since $T_{B1} / T_{B2} = 2$ or $T_{B1} = 2T_{B2}$, therefore from equation **(*ii*)**, we have

$$T_{B2} = 1147 \text{ N, and } T_{B1} = 2294 \text{ N}$$

$\therefore$ Total load acting on pulley *B*,

$$W_B = T_{B1} + T_{B2} = 2294 + 1147 = 3441 \text{ N}$$

This load acts vertically downwards.

From above, we may say that the shaft is subjected to the vertical and horizontal loads as follows :

Type of loading	*Load in N*		
	At D	*At C*	*At B*
Vertical	**7640**	**6200**	**3441**
Horizontal	**2780**	**3580**	**0**

The vertical and horizontal load diagrams are shown in Fig. (*c*) and (*d*).

First of all considering vertical loading on the shaft. Let R_{PV} and R_{QV} be the reactions at bearings *P* and *Q* respectively for vertical loading. We know that

$$R_{PV} + R_{QV} = 7640 + 6200 + 3441 = 17\,281 \text{ N}$$

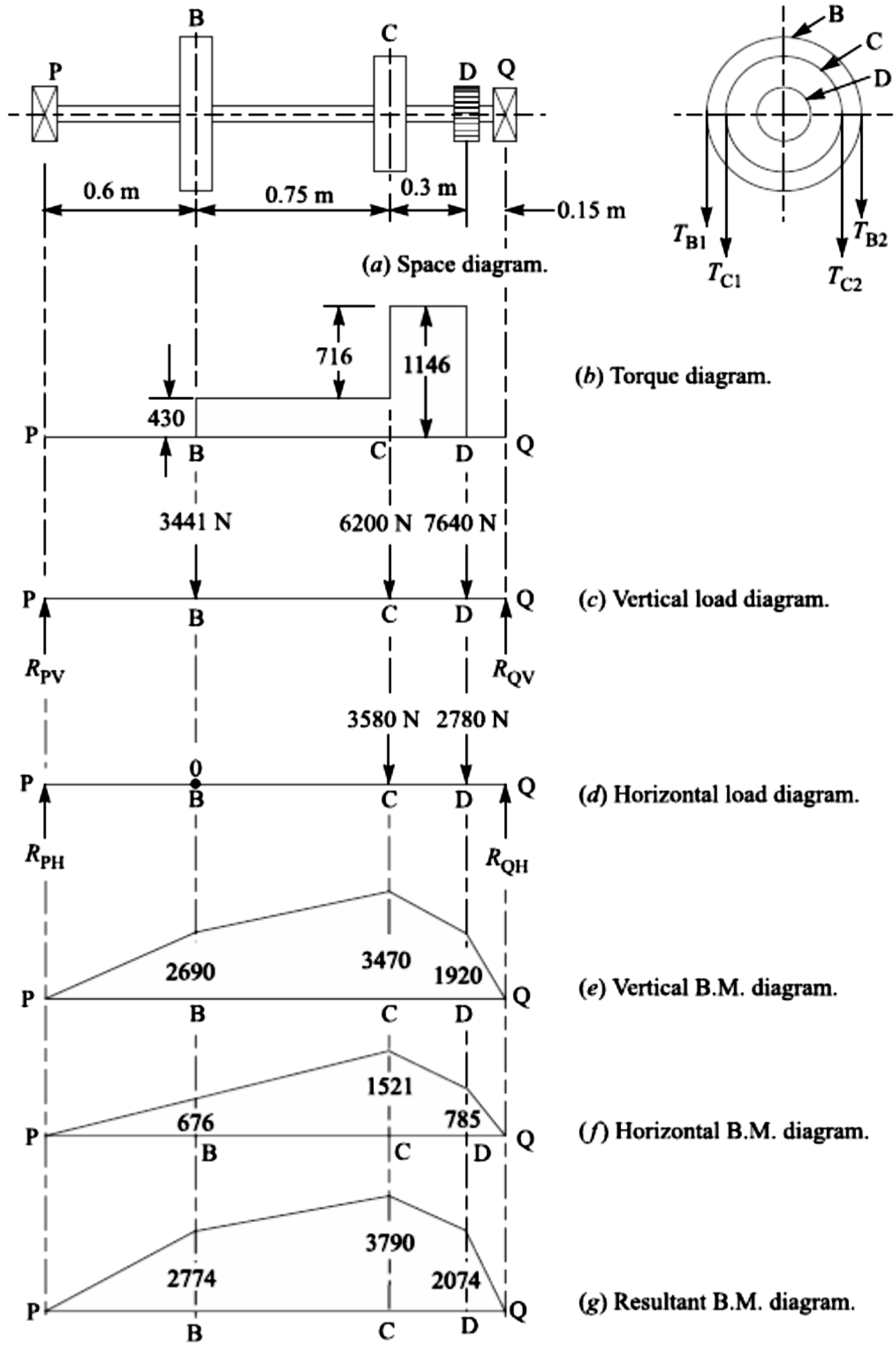

B
C
P
D
Q
0.6 m
0.75 m
0.3 m
0.15 m
T_{B1}
T_{C1}
T_{C2}
T_{B2}
(a) Space diagram.
716
1146
430
(b) Torque diagram.
3441 N
6200 N
7640 N
R_{PV}
R_{QV}
(c) Vertical load diagram.
3580 N
2780 N
0
R_{PH}
R_{QH}
(d) Horizontal load diagram.
2690
3470
1920
(e) Vertical B.M. diagram.
1521
676
785
(f) Horizontal B.M. diagram.
3790
2774
2074
(g) Resultant B.M. diagram.

fig 2.9

Taking moments about P, we get

$$R_{QV} \times 1.8 = 7640 \times 1.65 + 6200 \times 1.35 + 3441 \times 0.6 = 23\ 041$$

$$\therefore R_{QV} = 23\ 041 / 1.8 = 12\ 800 \text{ N}$$

and $R_{PV} = 17\ 281 - 12\ 800 = 4481$ N

We know that B.M. at P and Q,

$$M_{PV} = M_{QV} = 0$$

B.M. at B, $M_{BV} = 4481 \times 0.6 = 2690$ N-m

B.M. at C, $M_{CV} = 4481 \times 1.35 - 3441 \times 0.75 = 3470$ N-m

and B.M. at D, $M_{DV} = 12\ 800 \times 0.15 = 1920$ N-m

The bending moment diagram for vertical loading is shown in Fig. (*e*). 2.9

Now considering horizontal loading. Let R_{PH} and R_{QH} be the reactions at the bearings P and Q respectively for horizontal loading.

We know that

$$R_{PH} + R_{QH} = 2780 + 3580 = 6360 \text{ N}$$

Taking moments about P, we get

$$R_{QH} \times 1.8 = 2780 \times 1.65 + 3580 \times 1.35 = 9420 \text{ N}$$

$$\therefore R_{QH} = 9420 / 1.8 = 5233 \text{ N}$$

And $R_{PH} = 6360 - 5233 = 1127$ N

We know that B.M. at P and Q,

$$M_{PH} = M_{QH} = 0$$

B.M. at B, $M_{BH} = 1127 \times 0.6 = 676$ N-m

B.M. at C, $M_{CH} = 1127 \times 1.35 = 1521$ N-m

And B.M. at D, $M_{DH} = 5233 \times 0.15 = 785$ N-m

The bending moment diagram for horizontal loading is shown in Fig.(*f*). 2.9

The resultant bending moments for the points B, C and D are as follows :

Resultant.B.M.at B= $\sqrt{(M_{BV})^2 + (M_{BH})^2} = \sqrt{(2690)^2 + (676)^2}$ =2774 N-m

Resultant.B.M.at C= $\sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(3470)^2 + (1521)^2}$ =3790 N-m

Resultant.B.M.at D= $\sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(1920)^2 + (785)^2}$ =2074 N-m

From above we see that the resultant bending moment is maximum at *C*.

$$\therefore M = M_C = 3790 \text{ N-m}$$

And maximum torque at *C*,

T = Torque corresponding to 30 kW = T_D = 1146 N-m

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2} = \sqrt{(2 \times 3790)^2 + (1.5 \times 1146)^2}$$

$$= 7772 \text{ N-m} = 7772 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment (Te),

$$= \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$$d^3 = 7772 \times 10^3/8.25$$

$$d^3 = 942 \times 10^3 \text{ or}$$

$$d = 98 \text{ mm}$$

Again, we know that equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \frac{1}{2}\left[K_m \times M + T_e\right]$$

$$= \frac{1}{2}\left(2 \times 3790 + 7772\right) = 7676 \text{ N-m} = 7676 \times 10^3 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$7676 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 84 \times d^3 = 8.25 d^3$$

$$d^3 = 7676 \times 10^3 / 8.25 = 930 \times 10^3 \text{ or } d = 97.6 \text{ mm}$$

Taking the larger of the two values, we have d = 98 say 100 mm **Ans.**

17. A hollow shaft is subjected to a maximum torque of 1.5 kN-m and a maximumbending moment of 3 kN-m. It is subjected, at the same time, to an axial load of 10 kN. Assume thatthe load is applied gradually and the ratio of the inner diameter to the outer diameter is 0.5. If theouter diameter of the shaft is 80 mm, find the shear stress induced in the shaft.

Solution. Given :T= 1.5 kN-m = 1.5 × 10^3 N-m ; M = 3 kN-m = 3 × 10^3 N-m ;
F = 10 kN = 10 × 10^3N ;$k= d_i/ d_o$= 0.5 ; d_o= 80 mm = 0.08 m
Let τ = Shear stress induced in the shaft.
Since the load is applied gradually, therefore, we find that
K_m=1.5 ; and K_t= 1.0

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o\ (1+k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 3 \times 10^3 + \frac{1 \times 10 \times 10^3 \times 0.08\ (1+0.5^2)}{8}\right]^2 + (1 \times 1.5 \times 10^3)^2}$$

... ($\because$ α = 1, for axial tensile loading)

$$= \sqrt{(4500+125)^2 + (1500)^2} = 4862 \text{ N-m} = 4862 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$4862 \times 10^3 = \frac{\pi}{16} \times \tau\ (d_o)^3\ (1-k^4) = \frac{\pi}{16} \times \tau\ (80)^3\ (1-0.5^4) = 94\ 260\ \tau$$

$\therefore$ $\tau = 4862 \times 10^3 / 94\ 260 = 51.6 \text{ N/mm}^2 = 51.6$ MPa **Ans.**

16. A hollow shaft of 0.5 m outside diameter and 0.3 m inside diameter is used todrive a propeller of a marine vessel. The shaft is mounted on bearings 6 metre apart and it transmits5600 kW at 150 r.p.m. The maximum axial propeller thrust is 500 kN and the shaft weighs 70 kN.Determine :

1. The maximum shear stress developed in the shaft, and

2. The angular twist between the bearings. **(Nov/Dec 16)**

Given :d_o= 0.5 m ; d_i= 0.3 m ; P = 5600 kW = 5600 × 10^3 W ; L = 6 m ;
N = 150 r.p.m. ;F= 500 kN = 500 × 10^3 N ; W = 70 kN = 70 × 10^3 N

To Find:

1. The maximum shear stress developed in the shaft,

2. The angular twist between the bearings.

Solution.

1. ***Maximum shear stress developed in the shaft***

Let τ = Maximum shear stress developed in the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{5600 \times 10^3 \times 60}{2\pi \times 150} = 356460 \text{ N-m}$$

and the maximum bending moment,

$$M = \frac{W \times L}{8} = \frac{70 \times 10^3 \times 6}{8} = 52500 \text{ N-m}$$

Now let us find out the column factor α. We know that least radius of gyration,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2]\,[(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}}$$

$$= \frac{1}{4}\sqrt{(d_o)^2 + (d_i)^2} = \frac{1}{4}\sqrt{(0.5)^2 + (0.3)^2} = 0.1458 \text{ m}$$

$\therefore$ Slenderness ratio,

$$L/K = 6/0.1458 = 41.15$$

and column factor,

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)} \qquad \ldots\left(\because \frac{L}{K} < 115\right)$$

$$= \frac{1}{1 - 0.0044 \times 41.15} = \frac{1}{1 - 0.18} = 1.22$$

Assuming that the load is applied gradually, therefore from Table , we find that

$$K_m = 1.5 \text{ and } K_t = 1.0$$

Also $\quad k = d_i / d_o = 0.3 / 0.5 = 0.6$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o\,(1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 52\,500 + \frac{1.22 \times 500 \times 10^3 \times 0.5\,(1 + 0.6^2)}{8}\right]^2 + (1 \times 356\,460)^2}$$

$$= \sqrt{(78\,750 + 51\,850)^2 + (356\,460)^2} = 380 \times 10^3 \text{ N-m}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$380 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) = \frac{\pi}{16} \times \tau\,(0.5)^3\,[1 - (0.6)^4] = 0.02\,\tau$$

$\therefore \quad \tau = 380 \times 10^3 / 0.02 = 19 \times 10^6 \text{ N/m}^2 = 19 \text{ MPa}$ **Ans.**

2. ***Angular twist between the bearings***

Let θ = Angular twist between the bearings in radians.

We know that the polar moment of inertia for a hollow shaft,

$$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}[(0.5)^4 - (0.3)^4] = 0.005\,34 \text{ m}^4$$

From the torsion equation,

$$\frac{T}{J} = \frac{G \times \theta}{L}\text{, we have}$$

$$\theta = \frac{T \times L}{G \times J} = \frac{356\,460 \times 6}{84 \times 10^9 \times 0.00\,534} = 0.0048 \text{ rad}$$

... (Taking G = 84 GPa = 84×10^9 N/m^2)

$$= 0.0048 \times \frac{180}{\pi} = 0.275° \textbf{ Ans.}$$

18. A mild steel shaft transmits 20 kW at 200 r.p.m. It carries a central load of 900N and is simply supported between the bearings 2.5 metres apart. Determine the size of the shaft, ifthe allowable shear stress is 42 MPa and the maximum tensile or compressive stress is not to exceed56 MPa. What size of the shaft will be required, if it is subjected to gradually applied loads?

Solution. Given :P= 20 kW = 20×10^3 W ; N = 200 r.p.m. ; W = 900 N ; L = 2.5 m ;

τ = 42 MPa = 42 N/mm^2 ;σ_b= 56 MPa = 56 N/mm^2

Size of the shaft

Let d = Diameter of the shaft, in mm.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

and maximum bending moment of a simply supported shaft carrying a central load,

$$M = \frac{W \times L}{4} = \frac{900 \times 2.5}{4} = 562.5 \text{ N-m} = 562.5 \times 10^3 \text{ N-mm}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(562.5 \times 10^3)^2 + (955 \times 10^3)^2}$$
$$= 1108 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1108 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$$\therefore \quad d^3 = 1108 \times 10^3 / 8.25 = 134.3 \times 10^3 \text{ or } d = 51.2 \text{ mm}$$

We know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \tfrac{1}{2}(M + T_e)$$

$$= \tfrac{1}{2}(562.5 \times 10^3 + 1108 \times 10^3) = 835.25 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$835.25 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$$\therefore \quad d^3 = 835.25 \times 10^3 / 5.5 = 152 \times 10^3 \text{ or } d = 53.4 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 53.4 \text{ say } 55 \text{ mm } \textbf{Ans.}$$

Size of the shaft when subjected to gradually applied load

Let d = Diameter of the shaft.

From Table 14.2, for rotating shafts with gradually applied loads,

$$K_m = 1.5 \text{ and } K_t = 1$$

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 562.5 \times 10^3)^2 + (1 \times 955 \times 10^3)^2} = 1274 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1274 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$$\therefore \quad d^3 = 1274 \times 10^3 / 8.25 = 154.6 \times 10^3 \text{ or } d = 53.6 \text{ mm}$$

We know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \tfrac{1}{2}\left[K_m \times M + T_e\right]$$

$$= \tfrac{1}{2}\left[1.5 \times 562.5 \times 10^3 + 1274 \times 10^3\right] = 1059 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

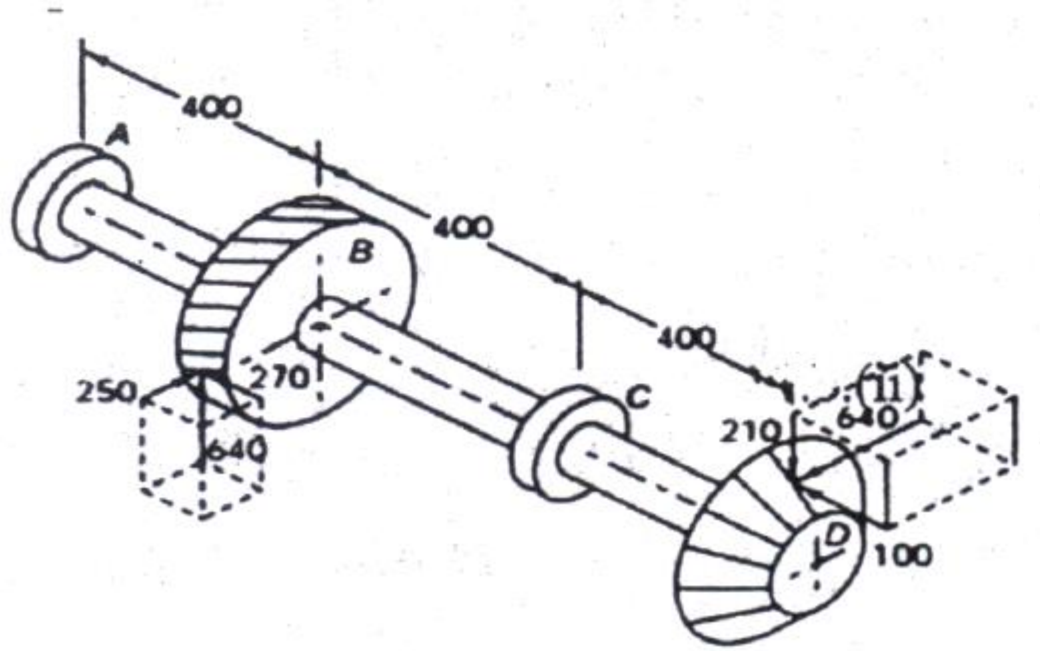
$$1059 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$$\therefore \quad d^3 = 1059 \times 10^3 / 5.5 = 192.5 \times 10^3 = 57.7 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 57.7 \text{ say } 60 \text{ mm } \textbf{Ans.}$$

19.A transmission shaft supporting a helical gear B and an overhang bevel gear D is shown in figure. The shaft is mounted-on two bearings A and C. The pitch circle diameter of the helical gear is 450 mm and the diameter of the bevel gear at the forces is 450 mm. Power is transmitted from the helical gear to the bevel gear. The gears are keyed to the shaft. The material of the shaft is steel 45C8 (Sut = 600 and Syt = 380 N/mm2). The factors kb and ~ of ASME code are 2.0 and 1.5 respectively. Determine the shaft diameter usingthe ASME code.(April/May 2018)



Solution **Given** $S_{ut} = 600\text{ N/mm}^2$ $S_{yt} = 380\text{ N/mm}^2$

$k_b = 2.0$ $k_t = 1.5$

For gears, $(d'_p)_B = 450\text{ mm}$ $(d'_p)_D = 450\text{ mm}$

Step I: Permissible shear stress

$$0.30\, S_{yt} = 0.30(380) = 114\text{ N/mm}^2$$

$$0.18\, S_{ut} = 0.18(600) = 108\text{ N/mm}^2$$

The lower of the two values is 108 N/mm^2 and there are keyways on the shaft.

$$\therefore \quad \tau_{max} = 0.75(108) = 81\text{ N/mm}^2$$

Step II: Bending moment

The forces and bending moments in vertical and horizontal planes are shown in Fig. 2.10 The resultant bending moments at B and C are as follows:

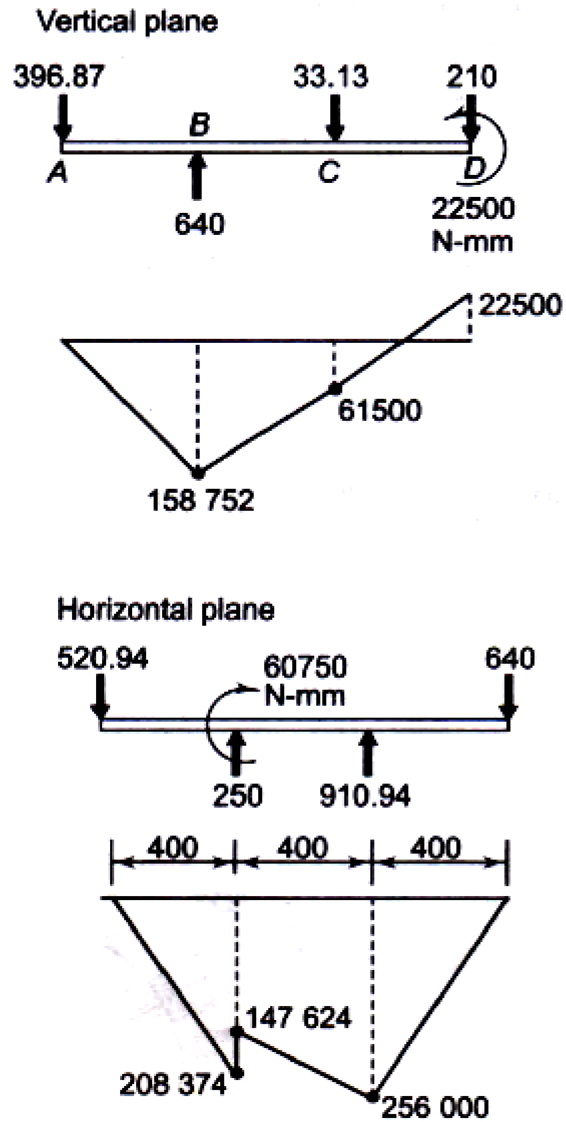


Fig. 2.10

At B, $M_b = \sqrt{(158\,752)^2 + (208\,374)^2}$

$= 261\,957.85$ N-mm

At C, $M_b = \sqrt{(61\,500)^2 + (256\,000)^2}$

$= 263\,283.59$ N-mm

Step III: Torsional moment

$M_t = 640 \times 225 = 144\,000$ N-mm

Step IV: Shaft diameter

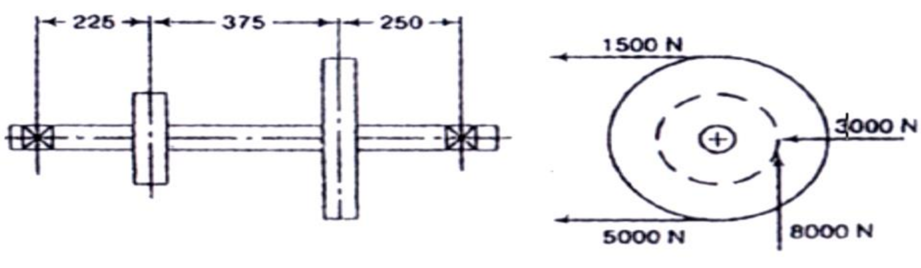
From Eq. (9.15),

$$d^3 = \frac{16}{\pi \tau_{max.}} \sqrt{(k_b M_b)^2 + (k_t M_t)^2}$$

$$= \frac{16}{\pi (81)} \sqrt{(2.0 \times 263\,283.59)^2 + (1.5 \times 144000)^2}$$

or $d = 32.95$ mm

20. A 600 mm diameter pulley driven by a horizontal belt transmits power through a solid shaft to a 262 mm diameter pinion which drives a matting gear. The pulley weighs 1200 N to provide some flywheel effect. The arrangement of elements, the belt tensions and components of the gear reactions on the pinion are as indicated in Figure 13(a). Determine the necessary shaft diameter using a suitable value for commercial shafting and shock fatigue factors of K, = 2 and K,= 1.5. (Nov/Dec 2017)



Given:

D = 600mm, P=1200N, K= 2, K=1.5

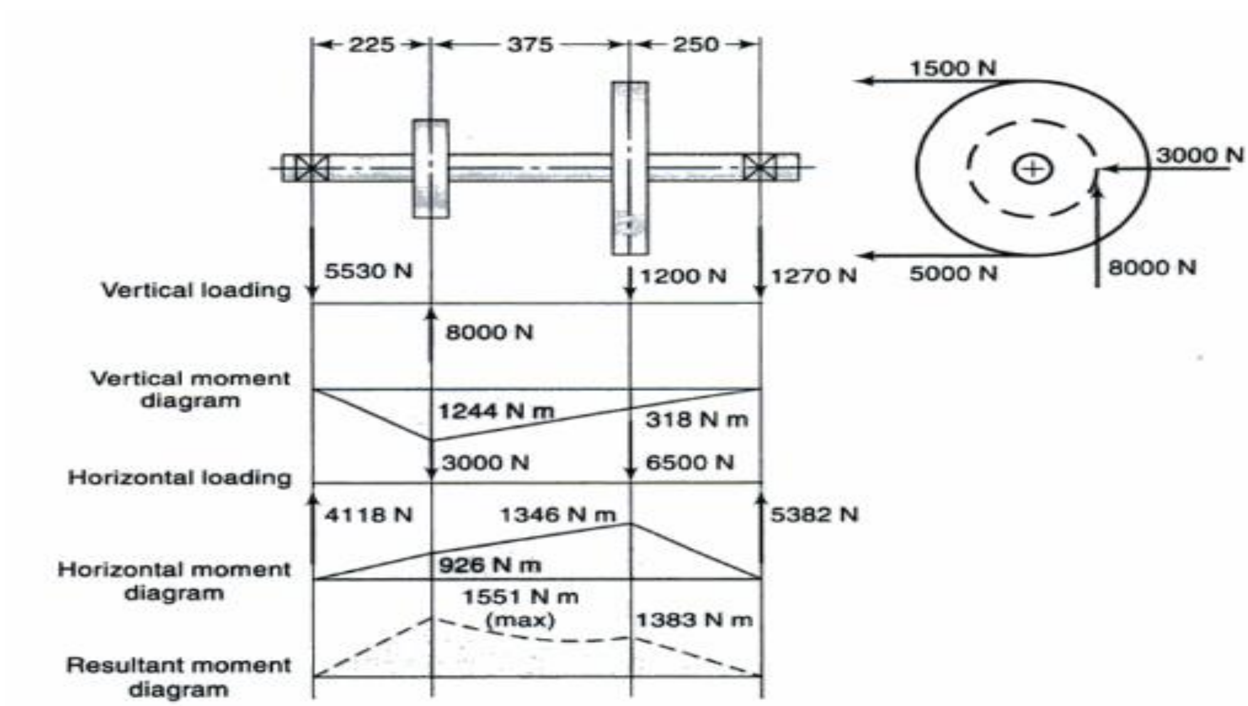


fig 2.11

from fig 2.11 $M_t(\max) = (T_1 - T_2)(0.3)$

$$= (5000 - 1500)(0.3) = 1050 \text{ N m}$$

$$M_b(\max) = \sqrt{(1244)^2 + (926)^2} = 1551 \text{ N m}$$

$s_s(\text{allowable}) = 40 \text{ MN/m}^2$

$$d^3 = \frac{16}{\pi \times 40 \times 10^6}\sqrt{(2\times 1551)^2 + (1.5\times 1050)^2}$$

$$= 443 \times 10^{-6} \text{ m}^3$$

$d = 76.2$ mm. Use 76 mm diameter shaft.

21. A section of commercial shafting 2 m long between bearing carries a 1000 n pulley at its mid point, as shown in fig. the pulley is keyed to the shaft and receives 30 KW at 150 rev/min which is transmitted to a flexible coupling just outside the right bearing. The belt drive is horizontal and some of the belt tensions is 8000 N. Assume K_t = K_b =1.5. Calculate the necessary shaft diameter and determine the angle of twist between bearings. G=80 GN/m^2.(Nov/Dec-2018)

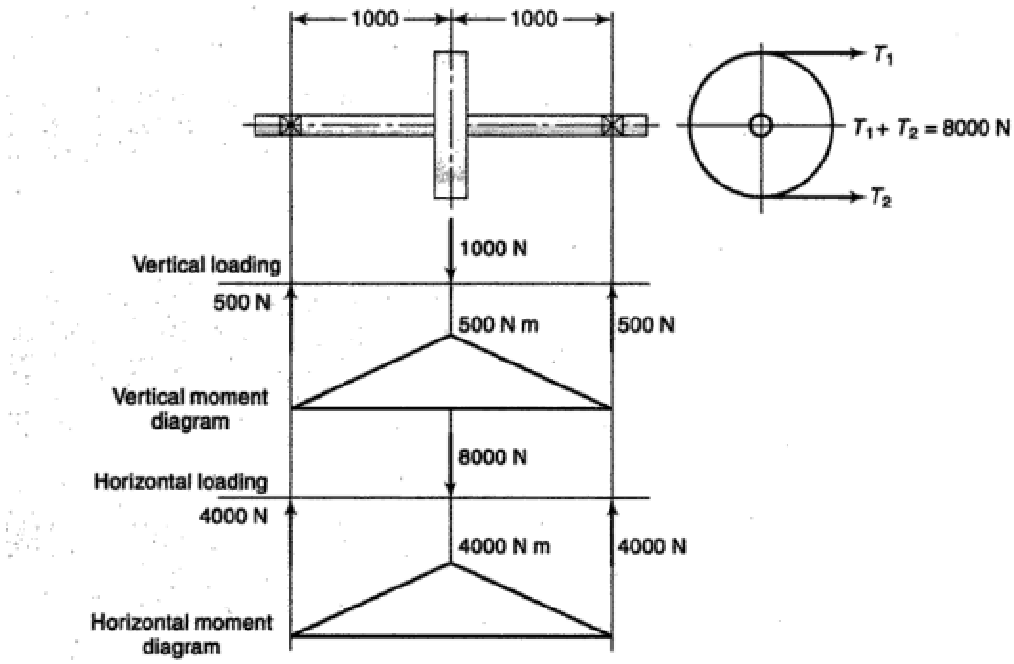


fig 2.12

It is first necessary to determine the maximum bending and torsional moments acting on the shaft.

from fig 2.12 $M_b(\max) = \sqrt{500^2 + 4000^2} = 4031$ N m,

$M_t(\max) = 30(9550)/150 = 1910$ N m

$s_s(\text{allowable}) = 40 \text{ MN/m}^2$ for shaft with keyway. Then

$$d^3 = \frac{16}{\pi s_s}\sqrt{(K_b M_b)^2 + (K_t M_t)^2}$$

$$= \frac{16}{40\pi \times 10^6}\sqrt{\begin{aligned}&(1.5 \times 4030)^2 \\ &+ (1.5 \times 1910)^2\end{aligned}}$$

from which d = 94.8 mm. Use 96 mm shaft, nearest standard size.

$$\theta = \frac{584\,M_t\,L}{Gd^4} = \frac{584 \times 1910 \times 1}{80 \times 10^9 \times 0.096^4}$$

$= 0.164°$ twist

Keys, keyways and splines – Rigid and flexible couplings

21. Two 35 mm shafts are connected by a flanged coupling. The flanges are fitted with 6 bolts on 25 mm bolt circle. The shafts transmit a torque of 800 N-m at 350 rpm. Forthe safe stresses mentioned below, calculate (i) diameter of bolts. (ii) thickness of flanges, (iii) key dimensions (iv) hub length and (v) power transmitted. Safe stress for shaft material 63 MPa, Safe stress for bolt material 56 MPa, Safe stress for cast iron coupling10 MPa and Safe stress for key material 46 MPa. (Nov /Dec – 2011)

Given :

d = 35 mm ;n= 6 ;

D1 = 125 mm ;T= 800 N-m = 800 × 103 N-mm ;

N = 350 r.p.m.;

τs= 63 MPa = 63 N/mm2 ; τb= 56 MPa = 56 N/mm2 ;

τc= 10 MPa = 10 N/mm2 ; τk= 46 MPa = 46 N/mm2

Solution:

1. Diameter of bolts

Let d1 = Nominal or outside diameter of bolt.

We know that the torque transmitted (T),

$800 \times 10^3 = \frac{\pi}{4} \times {d_1}^2 \times t_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} \times {d_1}^2 \times 56 \times 6 \times \frac{125}{2} = 16495{d_1}^2$

$${d_1}^2 = \frac{800 \times 10^3}{16495} = 48.5\ or d_1 = 6.96\ say\ 8\ mm$$

2. Thickness of flanges

Let t_f= Thickness of flanges.

We know that the torque transmitted (T),

$$800 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi\ (2 \times 35)^2}{2} \times 10 \times t_f = 76980\ t_f$$

∴t_f= 800 × 103 / 76 980 = 10.4 say 12 mm Ans.

3. Key dimensions

We find that the proportions of key for a 35 mm diameter shaft are :

Width of key, w = 12 mm **Ans.**

and thickness of key, t = 8 mm**Ans.**

The length of key (l) is taken equal to the length of hub (L).

∴l = L = 1.5 d = 1.5 × 35 = 52.5 mm

Let us now check the induced shear stress in the key. We know that the torque transmitted (T),

$$800 \times 10^3 = L \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11025\ \tau_k$$

∴τ_k= 800 × 103 / 11 025 = 72.5 N/mm2

Since the induced shear stress in the key is more than the given safe stress (46 MPa), therefore let us find the length of key by substituting the value of τk= 46 MPa in the above equation, i.e.

$$800 \times 10^3 = L \times 12 \times 46 \times \frac{35}{2} = 9660\ l$$

l = 800 × 103 / 9660 = 82.8 say 85 mm **Ans.**

4. Hub length

Since the length of key is taken equal to the length of hub, therefore we shall take hub length,

L = l = 85 mm **Ans.**

5. Power transmitted

We know that the power transmitted,

$$P = \frac{T \times 2\pi N}{60} = \frac{800 \times 2\pi \times 350}{60}$$

P = 29325W = 29.325 kW

22. A shaft made of A1SI 1030 cold drawn steel (σ_u = 520 MPa and σ_y = 440 MPa) transmits 50 kW at 900 rpm through a gear. Select an appropriate square key for the gear.(Nov/Dec 2017)(Nov/Dec-2018)

SOLUTION:

Given P = 50 kW, speed = 300 rpm, material = AISI 1030 CD steel

σ_{ult} = 520 MPa, σ_y = 440 MPa

Assumption: Equal strength in tension and compression

Allowable strength for shaft design

Permissible shear stress is

$$30\% \text{ of } \sigma_y = 0.3 \times 440 = 132.0 \text{ MPa},$$

$$18\% \text{ of } \sigma_{ult} = 0.18 \times 520 = 93.6 \text{ MPa}$$

Hence the allowable strength is 93.6 MPa.

Because of keyways, the allowable strength is further reduced by 25%

$$\tau_{all} = 0.75 \times 93.6 = 70.2 \text{ MPa}$$

Torque due to power transmission is

$$T = \frac{9550 \times \text{kW}}{\text{rpm}} = \frac{9550 \times 50}{900} = 530.56 \text{ N·m}$$

Shear stress induced due to T is

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 530.56 \times 10^3}{\pi \times d^3}$$

There is no other stress induced in the shaft as weight of the gear and shaft are not considered in this example.

Now equating Eq. (7.85) to allowable strength, we get

$$\frac{16 \times 530.56 \times 10^3}{\pi \times d^3} = 70.2$$

Solving, we get

$$d = 33.76 \text{ mm}$$

Design of the square key

Width of the key $b = d/4 = \frac{35}{4} = 8.75 \text{ mm}$

Height or thickness of the key $h = 8.75$ mm

Length of the key is obtained from the shear stress equation,

$$\tau = \frac{8T}{d^2 l} = \frac{8 \times 530.56 \times 10^3}{(35)^2 \times l}$$

Equating (7.86) to allowable strength

$$\frac{8 \times 530.56 \times 10^3}{(35)^2 \times l} = 70.2$$

Solving, we get

$$l = 49.35 \text{ mm}$$
$$= 50.0 \text{ mm}$$

From crushing failure consideration,

$$\sigma_c = \frac{16T}{d^2 l} = \frac{16 \times 530.56 \times 10^3}{(35)^2 \times 50} = 138.59 \text{ MPa}$$

Now the factor of safety is

$$\text{fos} = \frac{440}{138.59} = 3.17$$

Therefore, the design is safe. Hence, the dimensions of the key are 10 × 10 × 50 mm

23. Design a rigid flange coupling to transmit a torque of 250 N-m between two coaxial shafts. The shaft is made of alloy steel, flanges out of cast iron and bolts out of steel. Four bolts are used to couple the flanges. The shafts are keyed to the flange hub. The permissible stresses are given below:
Shear stress on shaft =100 MPa, Bearing or crushing stress on shaft =250 MPa
Shear stress on keys =100 MPa, Bearing stress on keys =250 MPa
Shearing stress on cast iron =200 MPa, Shear stress on bolts =100 MPa
After designing the various elements, make a neat sketch of the assembly indicating the important dimensions. The stresses developed in the various members may be checked if thumb rules are used for fixing the dimensions. (Nov /Dec – 2013)

Given :

T = 250 N-m = 250 × 103 N-mm ;n= 4;
τs= 100 MPa = 100 N/mm2 ;σcs= 250 MPa = 250 N/mm2 ;
τk= 100 MPa = 100 N/mm2 ; σck= 250 MPa = 250 N/mm2 ;
τc= 200 MPa = 200 N/mm2 ;τb= 100 MPa = 100 N/mm2
The cast iron flange coupling of the protective type is designed as discussed below :

1. Design for hub

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft (T),

$$250 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 100 \times d^3 = 19.64 d^3$$

∴d ^{3} = 250 × 10^3 / 19.64 = 12 729 or d = 23.35 say 25 mm Ans.

We know that the outer diameter of the hub,

D = 2 d = 2 × 25 = 50 mm

and length of hub, L = 1.5 d = 1.5 × 25 = 37.5 mm

Let us now check the induced shear stress in the hub by considering it as a hollow shaft. We know that the torque transmitted (T),

$$250 \times 10^3 = \frac{\pi}{16} \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \tau_c \left[\frac{50^4 - 25^4}{50}\right] = 23013 \tau_c$$

∴τ_c= 250 × 103/23 013 = 10.86 N/mm2 = 10.86 MPa

Since the induced shear stress for the hub material (i.e. cast iron) is less than 200 MPa, therefore the design for hub is safe.

2. Design for key

We find that the proportions of key for a 25 mm diameter shaft are :

Width of key, w = 10 mm **Ans.**
and thickness of key, t = 8 mm **Ans.**

The length of key (l) is taken equal to the length of hub,

∴l = L = 37.5 mm Ans.

Let us now check the induced shear and crushing stresses in the key. Considering the key in

$$250 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 37.5 \times 10^3 \tau_k \frac{25}{2} = 4688\tau_k$$

∴τ_k= 250 × 103 / 4688 = 53.3 N/mm2 = 53.3 MPa

Considering the key in crushing. We know that the torque transmitted (T),

$$250 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 37.5 \times \frac{8}{2} \times \sigma_{ck} \times \frac{25}{2} = 1875\ \sigma_{ck}$$

∴σ_{ck}= 250 × 103 / 1875 = 133.3 N/mm2 = 133.3 MPa

Since the induced shear and crushing stresses in the key are less than the given stresses,therefore the design of key is safe

3. Design for flange

The thickness of the flange (tf) is taken as 0.5 d.

∴t_f= 0.5 d = 0.5 × 25 = 12.5 mm Ans.

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear. We know that the torque transmitted (T),

$$250 \times 10^3 = \frac{\pi D^2}{2} \tau_c \times \tau_f = \frac{\pi (50)^2}{2} \times 12.5 \times \tau_c = 49094\ \tau_c$$

∴τ_c= 250 × 103 / 49 094 = 5.1 N/mm2 = 5.1 MPa

Since the induced shear stress in the flange of cast iron is less than 200 MPa, therefore design of flange is safe.

4. Design for bolts

Let d1 = Nominal diameter of bolts.

We know that the pitch circle diameter of bolts,

∴D1 = 3 d = 3 × 25 = 75 mm Ans.

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$250 \times 10^3 = \frac{\pi}{4} d_1^2 \times n \times \tau_b \times \frac{D_1}{2} = \frac{\pi}{4} d_1^2 \times 4 \times 100 \times \frac{75}{2} = 11780\ d_1^2$$

∴ $(d_1)^2$ = 250 × 103 / 11 780 = 21.22 or d1 = 4.6 mm

Assuming coarse threads, the nearest standard size of the bolt is M 6. **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

D_2 = 4 d = 4 × 25 = 100 mm **Ans.**

Thickness of the protective circumferential flange,

tp= 0.25 d = 0.25 × 25 = 6.25 mm **Ans.**

24. A V grooved pulley 200 mm pitch circle diameter is receiving 5 kW from a motor and rotates a shaft at 300 rpm. A crowned pulley 500 mm in diameter supplies power to a machine in a workshop. The angle of wrap for both pulley is x and the coefficient of friction between belt and pulley is 0.3.the semi groove angle for smaller pulley is 20^o. For the material of the shaft E=205 kN/mm^2 is G 84 kN/mm^2.Allowable shar stress = 60 N/mm^2, k_b=1.5, K_t = 2.

Neglect centrifugal tension in the belt. Check that θ = 0.5°, slope i_c<0.5 and δ at any point is <0.1 mm. (April/May -2011)

Given:

Power, P= 5 kW
Speed , N= 300 rpm
Young's Modulus E=205 kN/mm^2
G 84 kN/mm^2.
Allowable shar stress = 60 N/mm^2,
k_b=1.5, K_t = 2. Neglect centrifugal tension in the belt.
θ = 0.5°, slope i_c<0.5 and δ at any point is <0.1 mm

Solution:

Step – I

$$\omega = \frac{2\pi \times 300}{60} = 31.416 \quad rad/s$$

$$r = 100 \ mm$$

Smaller pulley V = 31.416 x0.1 = 3.1416 m/s
Bigger pulley R = 250 mm
V' = 31.416 x 0.25 = 7.854 m/s

Step – II Belt tension in smaller pulley

$$T_1 - T_2 = \frac{Power}{V} = \frac{5000}{3.1416} = 1591.5 \ N$$

$$\frac{T_1}{T_2} = e^{\mu\theta cosec\beta} = e^{0.3\times\pi\times cosec 20^o}$$

$$\frac{T_1}{T_2} = 15.73\text{-----------------}1$$

$$T_1 - T_2 = 1591.5 \text{ ----------------------}2$$

Solving 1 and 2

T_1 = 1699.5 kN; T_2 = 108.04 kN

Bigger Pulley

$$T_1{}' - T_2{}' = \frac{Power}{V'} = 78.54 = 636.6 \ N$$

$$\frac{T_1{}^1}{T_2{}'} = e^{\mu\theta} = e^{0.3\ x} = 2.566$$

$$(2.566 - 1)T_2' = 636.6 \ N$$

$$T_2' = 406.5 \ N$$

$$T_1' = 1043.1 \ N$$

Torque on the shaft $T = \frac{power}{\omega} = \frac{5000}{31.416} = 159.16 \ N - m$

Loading diagram for the shaft

Vertical load acting on shafts due to belt tension

$$T_1 + T_2 = 1699 + 108 = 1807.5\ N$$

$$T_1' + T_2' = 1043.1 + 406.5 = 1449.6\ \ N$$

At point B:

$$T_1' - T_2' = 1449.6\ \ N$$

D: $T_1 + T_2 = 1807.2\ N$

Reaction at C: $R_C = \frac{1449.6 \times 0.15 + 1807.5 \times 0.53}{0.45} = 2612\ N$

$$R_A = 1449 + 1807.5 - 2612 = 6415.1\ N$$

$$R_C = 2612\ N R_A = 6415.1\ N$$

Bending moment diagram

$$M_A = 0$$
$$M_B = 645.1 \times 0.15 = 96.76\ N - m$$
$$M_D = 0$$
$$M_C = -2162 \times 0.8 = 208.96\ N - m$$

Step – III

$$d^3 = \frac{16}{\pi \times Z_o}\sqrt{(K_b M)^2 + (KL_t T)^2} atC$$

$$d^3 = \frac{16}{\pi \times 60}\sqrt{(1.5 \times 208.96M)^2 + (2 \times 159.16)^2} \times 10^3 atC$$

$$\boldsymbol{d = 33.6\ mm}$$

Step – IV: Checking for rectangular fault between B and D

$$T = 159.16 \times 10^3 N - mm$$
$$l = 0.3 + 0.08 = 0.38\ m = 380\ mm$$

$$J = \frac{\pi d^4}{32} = \frac{\pi \times 33.6^4}{32} = 12.513 \times 10^3 mm^2$$

$$\theta = \frac{Tl}{T.I} = \frac{159.16 \times 10^3 \times 380}{84 \times 1000 \times 12.513 \times 10^4}$$
$$\theta = 57.4 \times 10^{-4} rad = \ 0.33^o < 0.5$$

Step – V: Deflection at loads and slope in bearing

Taking a reaction at a distance of x from the end A. In the portion CD of shaft

$$E.I\frac{d^2y}{dx^2} = 0.64x - 1.45(x-0.15) + 2.61(x-0.45)$$

$$E.I\frac{d^2y}{dx^2} = 0.32x^2 - 0.715(x-0.15)^2 + 1.305\,(x-0.45)^2 + C_1$$

$$E.Iy = 0.107x^3 - 0.242(x-0.15)^2 + 0.435\,(x-0.45)^3 + C_1x + C_2$$

When s=0,y=0, C_2=0

X=0.45y =0 (in the bearing)

C_1=7.146x10^{-3}

Slope at A

$$i_A = \frac{C_1}{E.I} = \frac{-7.146\times10^3}{E.I}$$

$$I = \frac{\pi d^4}{64} = \frac{\pi\times33.6^4}{64}\times10^{-12}m^4$$

$$I = 62.594\times10^{-9}m^4$$

$$E.I = 12825.7\ Nm^2$$

$$i_A = \frac{-7.146\times10^{-3}}{12825.7} = 5.57\times10^{-7}radian$$

$$i_A = 0.33\times10^{-4}\ degree < 0.05$$

Deflection at B

$$S_{B.x} = x = 0.15\ m$$

$$E.IS_B = 0.0107\times0.15^3 + C_1\times0.15$$

Substituting the values

$$S_B = -0.55\times10^{-7}$$

$$S_B = -0.55\times10^{-4}mm \ll 0.1\ mm$$

Slope in the bearing at C will also be much less than thepermissible limit

Deflection at D

$$E.IS_D = 0.107\times0.53^3 - 0.242(0.38)^3 + 0.435(0.08)^3 - 7.146\times10^{-3}\times0.53$$

Substituting the values

$$S_D = -0.96\times10^{-7}m = 0.96\times10^{-4}mm \ll 0.1\ mm$$

This increases the flexural and torsional rigidity of the shaft.

25. Design and draw a cast iron flange coupling for a mild steel shaft transmitting 90 kW at 250 r.p.m. The allowable shear stress in the shaft is 40 MPa and the angle of twist is not to exceed 1° in a length of 20 diameters. The allowable shear stress in the coupling bolts is 30 MPa. Take G = 84kN/mm^2(Nov/Dec -2014)

Given :

Power, P = 90 kW = 90 × 103 W

Speed, N = 250 × r.p.m.

Shear stress ,τ_s= 40 MPa = 40 N/mm^2
Twist angle θ = 1° = π / 180 = 0.0175 rad
Allowable shear stress τ_b= 30 MPa = 30 N/mm^2

Solution:
First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{90 \times 10^3 \times 60}{2\pi \times 250} = 3440N - m = 3440 \times 10^3 N - mm$$

Considering strength of the shaft, we know that

$$\frac{T}{J} = \frac{\tau_s}{\frac{d}{2}}$$

$$\frac{3440 \times 10^3}{\frac{\pi}{32} \times d^4} = \frac{40}{d/2} or \frac{3440 \times 10^3}{d^4} = \frac{80}{d}$$

$$d^3 = 35 \times \frac{10^6}{80} = 0.438 \times 10^6 ord = 76mm$$

Considering rigidity of the shaft, we know that

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$\frac{3440 \times 10^3}{\frac{\pi}{32} \times d^4} = \frac{84 \times 10^3 \times 0.0175}{20d} or \frac{35 \times 10^3}{d^4} = \frac{73.5}{d}$$

(taking C = 84Kn/mm^2)

$$d^3 = 35 \times \frac{10^6}{80} = 0.476 \times 10^6 ord = 78mm$$

d=78 say 80 mm

Let us now design the cast iron flange coupling of the protective type as discussed below

I. Design for hub

We know that

The outer diameter of hub, D = 2d = 2 × 80 = 160 mm **Ans.**

And length of hub, L = 1.5 d = 1.5 × 80 = 120 mm **Ans.**

Let us now check the induced shear stress in the hub by considering it as a hollow shaft. The Shear stress for the hub material (which is cast iron) is usually 14 MPa. We know that the torque transmitted (T),

$$3440 \times 10^3 = \frac{\pi}{16}\tau_s\left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16}\tau_s\left[\frac{160^4 - 80^4}{160}\right] = 754 \times 10^3 \tau_c$$

∴τ_c= 3440 × 103 / 754 × 103 = 4.56 N/mm2 = 4.56 MPa

Since the induced shear stress for the hub material is less than 14 MPa, therefore the design for hub is safe.

II. Design for key

We find that the proportions of key for a 80 mm diameter shaft are :

Width of key, w = 25 mm **Ans.**

and thickness of key, t = 14 mm **Ans.**

The length of key (l) is taken equal to the length of hub (L).

L = 120 mm **Ans.**

Assuming that the shaft and key are of the same material. Let us now check the induced shear stress in key. We know that the torque transmitted (T),

$$3440 \times 10^3 = l \times w \times \tau_k \times \frac{80}{2} = 120 \times 10^3 \tau_k$$

τk= 3440 × 103/120 × 103 = 28.7 N/mm2 = 28.7 MPa

Since the induced shear stress in the key is less than 40 MPa, therefore the design for key is safe.

III. Design for flange

The thickness of the flange (tf) is taken as 0.5 d.

∴tf= 0.5 d = 0.5 × 80 = 40 mm Ans.

Let us now check the induced shear stress in the cast iron flange by considering the flange at the junction of the hub under shear. We know that the torque transmitted (T),

$$3440 \times 10^3 = \frac{\pi D^2}{2} \tau_f \times \tau_c = \frac{\pi (160)^2}{2} \times 40 \times \tau_c = 1608 \times 10^3 \tau_c$$

∴τc= 3440 × 103/1608 × 103 = 2.14 N/mm2 = 2.14 MPa

Since the induced shear stress in the flange is less than 14 MPa, therefore the design for flange is safe.

IV. Design for bolts

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 80 mm, therefore let us take

Number of bolts, n = 4 and

Pitch circle diameter of bolts, D1 = 3 d = 3 × 80 = 240 mm

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$3440 \times 10^3 = \frac{\pi}{4} d_1^2 \times n \times \tau_b \times \frac{D_1}{2} = \frac{\pi}{4} d_1^2 \times 4 \times 30 \times \frac{240}{2} = 11311\ (d_1)^2$$

$(d1)^2$ = 3440 × 103/11 311 = 304 or d1 = 17.4 mm

Assuming coarse threads, the standard nominal diameter of bolt is 18 mm. **Ans.**

The other proportions are taken as follows:

Outer diameter of the flange, D2 = 4 d = 4 × 80 = 320 mm **Ans.**

Thickness of protective circumferential flange, tp= 0.25 d = 0.25 × 80 = 20 mm **Ans.**

26. Determine the dimensions of flange coupling that connects a motor and a pump shaft. The power to be transmitted a 2kW at a shaft speed of 960 rpm. Select suitable materials for the parts of the coupling and list the dimensions. (May/June -2014)

Given:

Power P=2kW=2 x10^3 W

Speed N=960 rpm

Tofind

Dimensions of flange coupling

Solution:

1. Diameter of shaft, d

$$T = \frac{P\times 60}{2\pi N} = \frac{2\times 10^3\times 60}{2\pi\times 960} = 19.8934\ N-m = 19894.36 N-mm \text{ -----------1}$$

Torque transmitted by the shaft, T

$$T = \frac{\pi}{16}\times \tau_s \times d^3 \text{ -------------2}$$

Assume that the shaft key and bolts are made of mild steel which is having

Allowable shear strength = 50N/mm^2

Allowable crushing strength = 90 N/mm^2

From equation 1 and 2

$$19894.36 = \frac{\pi}{16}\times 50 \times d^3$$

d=12.65 mm

say diameter of the shaft d= 13 mm

2. Outside diameter of hub, D

D=2d = 2 x 13 = 26mm

3. Pitch circle diameter of bolts, D_1

D_1 = 3d = 3x13 = 39 mm

4. Outer diameter of flange D2

D2 = 4d = 4x13 = 52 mm

5. Length of the hub, L

L = 1.5 d = 1.5x13 = 19.5 mm

6. Thickness of flange, t_f

t_f=0.5d = 0.5 x 13 = 6.5 mm

Design of hub:

Assume that the hub is made of cast iron. The allowable shear strength of cast iron is14N/mm2. We know that

$$T = \frac{\pi}{16}\tau_h\left[\frac{D^4 - d^4}{D}\right]$$

$$19894.36 = \frac{\pi}{16}\tau_s\left[\frac{26^4 - 13^4}{26}\right]$$

$$\tau_h = 6.15\ N/mm^2$$

Which is less than the allowable shear stress 14N/mm2. Hence the design is safe.

Design of key:

From PSGDB 5.16 corresponding to d = 13 mm

Width of the key, b = 6mm

Height of the key h = 6mm

Length of the key, l=19.5 mm

Check for shear

$$T = l \times b \times \tau_k \times \frac{d}{2}$$

$$19894.36 = 19.5 \times 6 \times \tau_k \times \frac{13}{2}$$

$$\boldsymbol{\tau_k = 26.16\ N/mm^2}$$

Which is less than the allowable value 50 N/mm^2. Hence the design is safe.

Check for crushing

$$T = l \times \frac{h}{2} \times \sigma_{ck} \times \frac{d}{2}$$

$$19894.36 = 19.5 \times \frac{6}{2} \times \sigma_{ck} \times \frac{13}{2}$$

$$\boldsymbol{\sigma_{ck} = 52.32\ N/mm^2}$$

Which is less than the allowable value 90 N/mm^2. Hence the design is safe.

Design of flange

$$T = \frac{\pi D^2}{2} \tau_h \times \tau_c$$

$$19894.36 = \frac{\pi (26)^2}{2} \times 6.5 \times \tau_h$$

$$\boldsymbol{\tau_h = 2.88\ N/mm^2}$$

Which is less than the allowable value 14 N/mm^2. Hence the design is safe.

Diameter of bolts, d_b

$$T = \frac{\pi}{4} d_b^2 \times n \times \tau_b \times \frac{D_1}{2}$$

Assume number of bolt, n = 3

$$19894.36 = \frac{\pi}{4} d_1^2 \times 50 \times 3 \times \frac{39}{2}$$

$$\boldsymbol{d_b = 2.94\ mm}$$

Say diameter of bolt db = 3mm

Check for crushing

$$T = d_b \times n \times t_f \times \sigma_{cb} \times \frac{D_1}{2}$$

$$19894.36 = 3 \times 6 \times 6.5 \times \sigma_{cb} \times \frac{39}{2}$$

$$\boldsymbol{\sigma_{cb} = 8.72\ N/mm^2}$$

27. A rigid type of coupling is used to connect tow shafts transmitting 15 kW at 200 rpm. The shafts, keys and bolts are made of C45 steel and the coupling is of cast iron. Design the coupling.(Nov/Dec -2006)&(May /June 2013)(Nov/Dec 2021)

Given:

Power P=15 kW=15 x10^3 W

Speed N=200 rpm

To find

Design of coupling

Solution:

The given coupling is a rigid type of coupling. So we can take the coupling as the clamp or split muff coupling.

1. **Diameter of shaft, d**

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 200} = 716197.2 N - mm$$

Torque transmitted by the shaft, T

$$T = \frac{\pi}{16} \times \tau_s \times d^3$$

Assume that the shaft key and bolts are made of C45 steel having stress = 65N/mm^2

$$716197.2 = \frac{\pi}{16} \times 65 \times d^3$$

d=38.28 mm

Say diameter of the shaft d= 40 mm

2. **Dimensions of the coupling**
   a. Outside diameter of sleeve or muff , D=2.5 d = 2.5 x 40 = 100 mm
   b. Length of the sleeve L = 3.5d = 3.5x40 = 140 mm
3. **Design of sleeve:**

Assume that the sleeve is made of cast iron having allowable shear stress of 14N/mm^2

Sleeve is considered as hollow shaft

$$T = \frac{\pi}{16} \tau_s \left[\frac{D^4 - d^4}{D}\right]$$

$$716197.2 = \frac{\pi}{16} \tau_s \left[\frac{100^4 - 40^4}{100}\right]$$

$$\tau_s = 3.74\ N/mm^2$$

Induced shear stress for sleeve is less that the permissible stress. Therefor the design is safe.

4. **Design of key:**

Length of the key, l=L/2 = 140/2=70 mm

a. **Check for shear strength,**

$$T = l \times b \times \tau_k \times \frac{d}{2}$$
$$716197.2 = 70 \times 12 \times \tau_k \times \frac{40}{2}$$
$$\tau_k = 42.63\ N/mm^2$$

Induced shear stress for key is less that the permissible stress. There for the design is safe.

b. Check for crushing:

$$T = l \times \frac{h}{2} \times \sigma_{ck} \times \frac{d}{2}$$
$$716197.2 == 70 \times \frac{8}{2} \times \sigma_{ck} \times \frac{40}{2}$$

$$\boldsymbol{\sigma_{ck} = 127.9\ N/mm^2}$$

Induced crushing stress is less than the permissible stress. There for the design is safe.

5. Design in bolts:

$$T = \frac{\pi^2}{16} \times \mu \times d_b^2 \times \sigma_t \times n \times d$$

Assume number of bolt, n = 4

Assume allowable tensile stress for bolt, $\sigma_t = 70N/mm^2$

Co-efficient of friction, μ = 0.3

$$716197.2 = \frac{\pi^2}{16} \times 0.3 \times d_b^2 \times 70 \times 4 \times 40$$

$$\boldsymbol{d_b = 18.594\ mm}$$

Say diameter of bolt d_b = 20 mm

28.Design a muff coupling to connect two shafts transmitting 40 kW at 120 r.p.m. The permissible shear and crushing stress for the shaft and key material (mild steel) are 30 MPa and 80 MPa respectively. The material of muff is cast iron with permissible shear stress of 15 MPa. Assume that the maximum torque transmitted is 25 per cent greater than the mean torque.(Apr/May – 2012)

Given Data:

Muff coupling

Power P = 40x10^3 Watts

N= 120 rpm

$\tau_s = \tau_{key} = 30\ MPa = 30N/mm^2$

$[\sigma_c]_{shaft} = [\sigma_c]_{key} = 80\ MPa = 80N/mm^2$

$[\tau]_{muff} = 15\ N/mm^2$

$[M_t]_{design} = 1.25\ T$

Solution:

1. To find T and diameter of shaft. D

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{40 \times 10^3 \times 60}{2\pi \times 120} = 3183\ Nm$$

$$M_t = 1.25\,T = 1.25 \times 3813 = 3979\ Nm$$

$$M_t = 3979\ \times 10^3 N - mm$$

$$T = \ \frac{\pi}{16} \times \tau_s \times d^3$$

$$or M_t = \frac{\pi\tau d^3}{16}$$

$$16 = \frac{3979 \times 10^3 \times 16}{\pi \times 30} = 675496$$

d=87.4mm

R20 standard shaft size d = 90mm

2. Design of sleeve

Outer diameter of sleeve D= 2d+13=103 mm

Inner diameter of sleeve d= diameter of shaft = 90mm

Length of sleeve L = 3.5d = 3.5x90 = 315 mm

Check for shear stress in sleeve

$$T = \frac{\pi}{16}\tau_s D^3(1 - K^4)K = \frac{d}{D} = \frac{90}{193} = 0.47$$

$$3979\ \times 10^3 = \frac{\pi}{16}\tau_s 193^3(1 - 0.47^4)$$

Induced shear stress in sleeve

$$\tau_s = 2.963\ \ N/mm^2$$

$$\tau sleeve < [\tau]_{sleeve<} = 15\ N/mm^2$$

Design is safe

3. Design of key

From PSG DB: 5.16 and 5.17 for shaft d=90mm we have

Width of key b= 25mm

Height of key h = 14 mm

Assume length of key on each side = l = sleeve length / 2

=315/2 = 157.5 mm key say 158 mm

Check for shear stress of key

We know for twisting moment

$$T = l \times b \times \tau_k \times \frac{d}{2}$$

$$3979000 = 25 \times 158 \times \tau_k \times \frac{90}{2}$$

$$\boldsymbol{\tau_k = 30\ N/mm^2}$$

Design is satisfactory

Check for crushing stress

$$T = l \times \frac{h}{2} \times \sigma_{ck} \times \frac{d}{2}$$

$$3979000 = 158 \times \frac{14}{2} \times \sigma_{ck} \times \frac{90}{2}$$

$$\sigma_{ck} = 80\ N/mm^2$$

So select the length from the PG DB: 517 select t=180 mm

Re checking

$$3979000 = 180 \times \frac{14}{2} \times \tau \times \frac{90}{2}$$

$$\tau = 19.65\ \ N/mm^2$$

$$.\tau = 19.65\ \frac{N}{mm^2} < \sigma_{key} = 30\ N/mm^2$$

Design is safe.

Rechecking

$$T = l \times \frac{h}{2} \times \sigma_c \times \frac{d}{2}$$

$$\sigma_c = \frac{3979000 \times 2 \times 2}{14 \times 180 \times 90} = 70\frac{N}{mm^2} < [\sigma_c]\ = 8070\frac{N}{mm^2}$$

Selection is satisfactory

B=25 mm, h=14mm, l=180 mm

29.Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 KW at 350 r.p.m.The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40MPa and 80MPa respectively. The material for the muff is cat iron for which the allowable shear stress may be assumed as 15MPa. (Nov/Dec 16)

Given : P = 40 kW = 40 × 10^3 W;

N = 350 r.p.m.;

τs= 40 MPa = 40 N/mm^2;

σcs= 80 MPa =80 N/mm^2;

τc= 15 MPa = 15 N/mm^2

To find:

Design a muff coupling

Solution:

1. *Design for shaft*

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft,key and muff,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} = 1100\text{ N-m}$$

$$= 1100 \times 10^3\text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3$$

$$1100 \times 10^3 = \frac{\pi}{16} \times 40 \times d^3$$

$$d^3 = \frac{1100 \times 10^3}{7.86}$$

$$d^3 = 140 \times 10^3$$

d=52mm say 55mm

2. *Design for sleeve*

We know that outer diameter of the muff,

$D = 2d + 13\text{ mm} = 2 \times 55 + 13 = 123$ say 125 mm **Ans.**

and length of the muff,

$L = 3.5\,d = 3.5 \times 55 = 192.5$ say 195 mm **Ans.**

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$

$$= 370 \times 10^3\ \tau_c$$

$\therefore \qquad \tau_c = 1100 \times 10^3/370 \times 10^3 = 2.97\text{ N/mm}^2$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm^2, therefore the design of muff is safe.

3. *Design for key*

we find that for a shaft of 55 mm diameter, Width of key, w = 18 mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square keymay be used.

$\therefore$ Thickness of key, $t = w = 18$ mm **Ans.**

We know that length of key in each shaft,

$$l = L / 2 = 195 / 2 = 97.5 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \tau_s$$

$$\therefore \quad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \sigma_{cs}$$

$$\therefore \quad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stressesare less than the permissible stresses, therefore the design of key is safe.

30.Design a bushed-pin type of flexible coupling to connect a pump shaft to amotor shaft transmitting 32 kW at 960 r.p.m. The overall torque is 20 percent more than mean torque. The material properties are as follows :(a) The allowable shear and crushing stress for shaft and key material is 40 MPa and80 MPa respectively.(b) The allowable shear stress for cast iron is 15 MPa.(c) The allowable bearing pressure for rubber bush is 0.8 N/mm^2.(d) The material of the pin is same as that of shaft and key.Draw neat sketch of the coupling. (Nov /Dec – 2012) (Nov/Dec-15)

Given:

$P = 32$ kW = 32 × 103 W ;

$N = 960$ r.p.m. ;

T_{max}= 1.2 Tmean;

$\tau_s = \tau_k$= 40 MPa= 40 N/mm2 ;

$\sigma_{cs} = \sigma_{ck}$= 80 MPa = 80 N/mm2 ;

τ_c= 15 MPa = 15 N/mm2 ;

p_b= 0.8 N/mm2

Solution:

The bushed-pin flexible coupling is designed as discussed below :

1. Design for pins and rubber bush

First of all, let us find the diameter of the shaft (d). We know that the mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{32 \times 10^3 \times 60}{2\pi \times 960} = 318.3 \ N - m$$

and the maximum or overall torque transmitted,

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 318.3 = 382 \text{ N-m} = 382 \times 10^3 \text{ N-mm}$$

We also know that the maximum torque transmitted by the shaft (Tmax),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86 d^3$$

d^3 = 382 × 103 / 7.86 = 48.6 × 10^3 or d = 36.5 say 40 mm

We have discussed in rigid type of flange coupling that the number of bolts for 40 mm diameter shaft are 3. In the flexible coupling, we shall use the number of pins (n) as 6.

$$Diameter of pins d1 = \frac{0.5\ d}{\sqrt{n}} = \frac{0.5 \times 40}{\sqrt{6}} = 8.2\ m$$

In order to allow for the bending stress induced due to the compressibility of the rubber bush ,the diameter of the pin (d1) may be taken as 20 mm. **Ans.**

The length of the pin of least diameter i.e. d1 = 20 mm is threaded and secured in the right hand coupling half by a standard nut and washer. The enlarged portion of the pin which is in the left hand coupling half is made of 24 mm diameter. On the enlarged portion, a brass bush of thickness 2 mm is pressed. A brass bush carries a rubber bush. Assume the thickness of rubber bush as 6 mm.

∴ Overall diameter of rubber bush,

d_2 = 24 + 2 × 2 + 2 × 6 = 40 mm Ans.

and diameter of the pitch circle of the pins,

$D_1 = 2\ d + d_2 + 2 \times 6 = 2 \times 40 + 40 + 12 = 132$ mm Ans.

Let l = Length of the bush in the flange.

We know that the bearing load acting on each pin,

$W = p_b \times d_2 \times l = 0.8 \times 40 \times l = 32\ l$ N

and the maximum torque transmitted by the coupling (Tmax),

$$382 \times 10^3 = W \times n \times \frac{D_1}{2} = 32\ l \times 6 \times \frac{132}{2} = 12672\ l$$

∴l = 382 × 103/12 672 = 30.1 say 32 mm

andW= 32 l = 32 × 32 = 1024 N

∴Direct stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4} {d_1}^2} = \frac{W}{\frac{\pi}{4} 20^2} = 3.26\ N/mm^2$$

Since the pin and the rubber bush are not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin. Assuming a uniform distribution of load (W) along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5\right) = 1024\left(\frac{32}{2} + 5\right) = 21504\ N - mm$$

And section modulus $Z = \frac{\pi}{32} (d_1)^3 = \frac{\pi}{32} (20)^3 = 785.5\ mm^3$

We know that bending stress

$$\sigma = \frac{M}{Z} = \frac{21504}{785.5} = 27.4\ N/mm^2$$

Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[27.4 + \sqrt{27.4^2 + 4(3.26)^2}\right]$$

=13.7+14.1 = 27.8 N/mm$^{2.}$

and maximum shear stress

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{27.4^2 + 4(3.26)^2}\right]$$

=14.1 N/mm$^{2.}$

Since the maximum principal stress and maximum shear stress are within limits, therefore the design is safe.

2. Design for hub

We know that the outer diameter of the hub,

D = 2 d = 2 × 40 = 80 mm

and length of hub, L = 1.5 d = 1.5 × 40 = 60 mm

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16}\tau_c\left[\frac{80 - 40^4}{100}\right] = 94.26 \times 10^3\tau_c$$

∴τ_c= 382 × 10^3 / 94.26 × 10^3 = 4.05 N/mm2 = 4.05 MPa

Since the induced shear stress for the hub material (i.e. cast iron) is less than the permissible value of 15 MPa, therefore the design of hub is safe.

3. Design for key

Since the crushing stress for the key material is twice its shear stress (i.e. σck= 2 τk), therefore a square key may be used. From Table 13.1, we find that for a shaft of 40 mm diameter,

Width of key, w = 14 mm **Ans.**

and thickness of key, t = w = 14 mm **Ans.**

The length of key (L) is taken equal to the length of hub, i.e.

L = 1.5 d = 1.5 × 40 = 60 mm

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted (Tmax),

$$382 \times 10^3 = L \times w \times \tau_k \times \frac{d}{2} = 60 \times 14 \times \tau_k \times \frac{40}{2} = 16800\tau_k$$

∴τ_k= 382 × 103/16 800 = 22.74 N/mm2 = 22.74 MPa

Considering the key in crushing. We know that the maximum torque transmitted (Tmax),

$$382 \times 10^3 = L \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 60 \times \frac{14}{2} \times \sigma_{ck} \times \frac{40}{2} = 8400\ \sigma_{ck}$$

∴σ_{ck}= 382 × 103/8400 = 45.48 N/mm2 = 45.48 MPa

Since the induced shear and crushing stress in the key are less than the permissible stresses of 40 MPa and 80 MPa respectively, therefore the design for key is safe.

4. Design for flange

The thickness of flange (tf) is taken as 0.5 d.

$$\therefore t_f = 0.5\ d = 0.5 \times 40 = 20\ mm$$

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (Tmax),

$$382 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi 80^2}{2} \times \tau_c \times 20 = 201 \times 10^3 \tau_c$$

$$\therefore \tau c = 382 \times 103 / 201 \times 103 = 1.9\ N/mm2 = 1.9\ MPa$$

Since the induced shear stress in the flange of cast iron is less than 15 MPa, therefore the design of flange is safe.

31.Design a rigid type of flange couplingto connect two shafts. The input shaft transmits 37.5 kW powerat 180 rpm to the output shaft through the coupling. The service factor for the applicationis 1.5. Select suitable material for various parts of the coupling.

(Nov/D ec– 2010)

Given:

P = 37.5 kW

N=180 rpm

σ_c = 80 MPa

FOS = 1.5

τ_B=40 MPaτ_{Cl}= 8 MPa

Solution:

1. Design for hub

$$P = \frac{2\pi NT}{60} =$$

$$37.5 = \frac{2\pi \times 180 \times T}{60}$$

T = 1.99 N-m

FOS is 1.5 the maximum torque transmitted by the shaft

T_{max}= 1.5 x 1.99 = 2.98 N-m

T_{max} = 2.98 x 10^3 N-mm

Torque transmitted by the shaft

$$T = \frac{\pi}{16} \times \tau_s \times d^3$$

$$2.98 \times 10^3 = \frac{\pi}{16} \times 40 \times d^3$$

d=7.23 mm

R20 series standard dis is d= 8mm

Outer diameter of the hub D= 2d = 2x8 =16 mm

Length of the hub L =1.5d = 1.5x8 = 12 mm

To check

$$T = \frac{\pi}{16}\tau_c\left[\frac{D^4 - d^4}{D}\right]$$

$$2.98 \times 10^3 = \frac{\pi}{16}\tau_c\left[\frac{16^4 - 8^4}{16}\right]$$

$$2.98 \times 10^3 = \frac{\pi}{16}\tau_c \times 3840$$

$$\tau = 3.95\ MPa < [\tau]$$

Since it is less than permissible stress, design is safe.

2. Design for key

From PGDB: 5.16

Shaft diameter = 8 mm

Width of key, w = 3 mm **Ans.**

and thickness of key, t = 3 mm **Ans.**

The length of key (l) is taken equal to the length of hub,

∴l = L = 1.5d=12mm **Ans.**

To check

$$T = l \times w \times \tau_k \times \frac{d}{2}$$

$$2.98 \times 10^3 = 12 \times 3 \times \tau_k \times \frac{8}{2}$$

$$\therefore \boldsymbol{\tau k\ =\ 13.73\ MPa < [\tau] = 40\,\frac{N}{mm^2} henceitissafe}$$

For crushing stress (T),

$$T = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2}$$

$$2.98 \times 10^3 = 10.855 \times \frac{3}{2} \times \sigma_c \times \frac{8}{2}$$

$$\therefore \boldsymbol{\sigma ck\ =\ 45.78\ MPa < [\sigma_c] = 80\ N/mm^2}$$

Since it is less than the permissible stress, design is safe.

3. Design for flange

The thickness of the flange (t_f) is taken as 0.5 d.

∴tf= 0.5 d = 0.5 × 8 = 4 mm **Ans.**

To check shear stress

$$T = \frac{\pi D^2}{2}\tau_c \times \tau_f$$

$$2.98 \times 10^3 = \frac{\pi(16)^2}{2} \times 4 \times \tau_c$$

$$\tau = 1.85\ MPa < [\tau] = 8\ MPa$$

It is less than 8 MPa the design is safe

4. Design for bolts

Take n = 2

Pitch circle diameter of bolts D1 = 3d = 24 mm

$$T = \frac{\pi}{4} d_1^2 \times n \times \tau_b \times \frac{D_1}{2}$$

$$2.98 \times 10^3 = \frac{\pi}{4} d_1^2 \times 40 \times 2 \times \frac{24}{2}$$

$$d_1 = 1.98\, mm$$

Standard bolt size (PGDB: 5.48) d= M3

Outer diameter of flange D2 = 4d = 4x8 = 32 mm

S.R – 2.130

32.A power of 5kW at 12 rps is transmitted through a flange coupling. Materials for bolts, shaft and key and flange are C60, C40, and CI grade 30 respectively. Design the coupling.

Given:

Power, P = 5kW

Speed , N= 12 rps

Solution:

Tensile stress for C60 = 420 N/mm^2

Tensile stress for C 40 = 330 N/mm^2

Tensile stress for CI grade 30 = 300 N/mm^2

Design shear stress for bolt $= \frac{420}{2} \times 0.6 = 126\ N/mm^2$

Design shear stress for shaft and key $= \frac{330}{2} \times 0.6 = 99\ N/mm^2$

Design shear stress for flange $= \frac{300}{2} \times 0.6 = 165\ N/mm^2$

Design crushing stress for key $= \frac{330}{2} \times 1.1 = 181\ N/mm^2$

Design crushing stress for bolt $= \frac{420}{2} \times 1.1 = 231\ N/mm^2$

Torque transmitted$T = \frac{5\times1000}{2\pi\times12} = 66.31\ N - m$

Shaft diameter $d = \sqrt[3]{\frac{16T}{\pi\times\tau}} =$

$$= \sqrt[3]{\frac{16 \times 66.31}{\pi \times 99} \times 1000} = 15mm$$

Take d=16 mm

Boss diameter d$_1$ = 2d = 32 mm

Shear stress in boss $= \frac{16\times T_{d1}}{\pi(d_1^4 - d^4)}$

$$= \frac{16 \times 66.31 \times 1000}{\pi(32^4 - 16^4)} = 0.29\ N/mm^2$$

This is within the allowable value.

Cross section of key for 16 mm shaft is 5mm x 5 mm
Length of key from shearing consideration

$$l > \frac{2T}{db\tau}$$

$$= \frac{2 \times 66310}{16 \times 5 \times 99} = 16.75\ mm$$

Length of key for crushing consideration

$$l > \frac{4T}{dh\sigma_c}$$

$$= \frac{4 \times 66310}{16 \times 5 \times 181} = 18.32\ mm$$

Take l = 20 mm

Number of bolts n=0.02 d+3 = 3.32

Take n=4

Bolt circle diameter d_2 = 2d+50 = 82 mm

Bolt diameter d_b

$$\text{Shear stress in each bolt} = \frac{\mathrm{T}}{\text{bolt circel radius} \times \mathrm{n} \times \text{area of bolt}}$$

$$= \frac{66310 \times 4}{41 \times 4 \times \pi \times \mathrm{d_b^2}}$$

$$\mathrm{d_b} = \sqrt{\frac{66310 \times 4}{41 \times 4 \times \pi \times 126}} = 2.02\ \mathrm{mm}$$

Take d_b= 5 mm

Shear stress at junction of flange and boss

$$\text{Thickness of flange} = \frac{66310 \times 4}{16 \times \pi \times 32 \times \tau}$$

$$\mathrm{t} = \frac{66310}{16 \times \pi \times 32 \times 8} = 5.15\ \mathrm{mm}$$

For case of casting thickness of flange = 10 mm
Check:
Crushing stress between bolt and flange

$$= \frac{\mathrm{T}}{\text{bolt of radius } \times \text{projected aread of bolt} \times \text{No. of holes}}$$

$$\frac{66310}{41 \times 7.3 \times 4} = 55\ \mathrm{N/mm^2}$$

This is within the allowable value.

Taking webs thickness 5mm

Outside diameter of flange

$$= \mathrm{B.C.d} + 2 \times \mathrm{d_b} + 2 \times 5 \text{ clearance } + 2 \times \text{ web thickness}$$

$$= 82 + 2 \times 5 + 10 + 2 \times 5$$

$$\mathbf{= 82 + 10 + 20 = 112\ mm}$$

33. Design and draw a protective type of cast iron flange coupling for a steel shaft transmitting 15 kW at 200 r.p.m. and having an allowable shear stress of 40 MPa. The working stress in the bolts should not exceed 30 MPa. Assume that the same material is used for shaft and key and that the crushing stress is twice the value of its shear stress. The maximum torque is 25% greater than the full load torque. The shear stress for cast iron is 14 MPa.(May/June-16)(A/M'2023)

Solution. Given : P = 15 kW = 15 × 10^3 W ; N = 200 r.p.m. ; τ_s = 40 MPa = 40 N/mm^2 ; τ_b = 30 MPa = 30 N/mm^2 ; $\sigma_{ck} = 2\tau_k$; $T_{max} = 1.25\ T_{mean}$; τ_c = 14 MPa = 14 N/mm^2

The protective type of cast iron flange coupling is designed as discussed below :

1. *Design for hub*

First of all, let us find the diameter of shaft (d). We know that the full load or mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 200} = 716 \text{ N-m} = 716 \times 10^3 \text{ N-mm}$$

and maximum torque transmitted,

$$T_{max} = 1.25\ T_{mean} = 1.25 \times 716 \times 10^3 = 895 \times 10^3 \text{ N-mm}$$

We also know that maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\ d^3$$

$\therefore$ $d^3 = 895 \times 10^3 / 7.86 = 113\ 868$ or d = 48.4 say 50 mm **Ans.**

We know that the outer diameter of the hub,

$$D = 2\,d = 2 \times 50 = 100 \text{ mm}$$ **Ans.**

and length of the hub, $L = 1.5\,d = 1.5 \times 50 = 75$ mm **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron, by considering it as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left(\frac{(100)^4 - (50)^2}{100}\right) = 184\ 100\ \tau_c$$

$\therefore$ $\tau_c = 895 \times 10^3 / 184\ 100 = 4.86$ N/mm^2 = 4.86 MPa

Since the induced shear stress in the hub is less than the permissible value of 14 MPa, therefore the design for hub is safe.

2. ***Design for key***

Since the crushing stress for the key material is twice its shear stress, therefore a square key may be used.

From Table 13.1, we find that for a 50 mm diameter shaft,

Width of key, $w = 16$ mm **Ans.**

and thickness of key, $t = w = 16$ mm **Ans.**

The length of key (l) is taken equal to the length of hub.

$\therefore \quad l = L = 75$ mm **Ans.**

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 75 \times 16 \times \tau_k \times \frac{50}{2} = 30 \times 10^3\ \tau_k$$

$$\therefore \quad \tau_k = 895 \times 10^3 / 30 \times 10^3 = 29.8 \text{ N/mm}^2 = 29.8 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 75 \times \frac{16}{2} \times \sigma_{ck} \times \frac{50}{2} = 15 \times 10^3\ \sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 895 \times 10^3 / 15 \times 10^3 = 59.6 \text{ N/mm}^2 = 59.6 \text{ MPa}$$

Since the induced shear and crushing stresses in key are less than the permissible stresses, therefore the design for key is safe.

3. ***Design for flange***

The thickness of the flange (*tf*) is taken as 0.5 *d*

$\therefore \quad t_f = 0.5 \times 50 = 25$ mm **Ans.**

Let us now check the induced shear stress in the flange, by considering the flange at the junction of the hub in shear. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (100)^2}{2} \times \tau_c \times 25 = 392\ 750\ \tau_c$$

$$\therefore \quad \tau_c = 895 \times 10^3 / 392\ 750 = 2.5 \text{ N/mm}^2 = 2.5 \text{ MPa}$$

Since the induced shear stress in the flange is less than the permissible value of 14 MPa, therefore the design for flange is safe.

4. ***Design for bolts***

Let d_1 = Nominal diameter of bolts.

Since the diameter of shaft is 50 mm, therefore let us take the number of bolts,

$$n = 4$$

and pitch circle diameter of bolts,

$$D_1 = 3\,d = 3 \times 50 = 150 \text{ mm}$$

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi}{4}(d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2\, 30 \times 4 \times \frac{150}{4} = 7070\,(d_1)^2$$

$$\therefore \quad (d_1)^2 = 895 \times 10^3 / 7070 = 126.6 \quad \text{or} \quad d_1 = 11.25 \text{ mm}$$

Assuming coarse threads, the nearest standard diameter of the bolt is 12 mm (M 12). **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 50 = 200 \text{ mm} \textbf{ Ans.}$$

Thickness of the protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 50 = 12.5 \text{ mm} \textbf{ Ans.}$$

34. Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shearstress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. Thepermissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and theshaft surface may be taken as 0.3. (April/May 2019)

Solution. Given : $P = 30$ kW $= 30 \times 10^3$ W ; $N = 100$ r.p.m. ; $\tau = 40$ MPa $= 40$ N/mm^2 ; $n = 6$; $\sigma_t = 70$ MPa $= 70$ N/mm^2 ; $\mu = 0.3$

1. ***Design for shaft***

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{30 \times 10^3 \times 60}{2\,\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\,d^3$$

$$\therefore \quad d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3 \text{ or } d = 71.4 \text{ say } 75 \text{ mm} \textbf{ Ans.}$$

2. ***Design for muff***

We know that diameter of muff,

$$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163 \text{ say } 165 \text{ mm} \textbf{ Ans.}$$

and total length of the muff,

$$L = 3.5\,d = 3.5 \times 75 = 262.5 \text{ mm} \textbf{ Ans.}$$

3. ***Design for key***

The width and thickness of the key for a shaft diameter of 75 mm (are as follows :

Width of key, w = 22 mm **Ans.**

Thickness of key, t = 14 mm **Ans.**

and length of key = Total length of muff = 262.5 mm **Ans.**

4. ***Design for bolts***

Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\ (d_b)^2\ 70 \times 6 \times 75 = 5830 (d_b)^2$$

$$\therefore \qquad (d_b)^2 = 2865 \times 10^3 / 5830 = 492 \quad \text{or} \quad d_b = 22.2 \text{ mm}$$

From Table 11.1, we find that the standard core diameter of the bolt for coarse series is 23.32 mm and the nominal diameter of the bolt is 27 mm (M 27). **Ans.**

35. The shaft and the flange of a marine engine are to be designed for flangecoupling, in which the flange is forged on the end of the shaft. The following particulars are to beconsidered in the design :

Power of the engine = 3 MW

Speed of the engine = 100 r.p.m.

Permissible shear stress in bolts and shaft = 60 MPa

Number of bolts used = 8

Pitch circle diameter of bolts = 1.6 × Diameter of shaft

Find : 1. diameter of shaft ; 2. diameter of bolts ; 3. thickness of flange ; and 4. diameter offlange.

Solution. Given : P = 3 MW = 3×10^6 W ; N = 100 r.p.m. ; $\tau_b = \tau_s$ = 60 MPa = 60 N/mm^2 ; n = 8 ; $D_1 = 1.6\, d$

1. ***Diameter of shaft***

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2 \pi N} = \frac{3 \times 10^6 \times 60}{2 \pi \times 100} = 286 \times 10^3 \text{ N-m} = 286 \times 10^6 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$286 \times 10^6 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$$\therefore \qquad d^3 = 286 \times 10^6 / 11.78 = 24.3 \times 10^6$$

or $\qquad d = 2.89 \times 10^2 = 289$ say 300 mm **Ans.**

2. ***Diameter of bolts***

Let d_1 = Nominal diameter of bolts.

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$286 \times 10^6 = \frac{\pi}{4}(d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} \times (d_1)^2\, 60 \times 8 \times \frac{1.6 \times 300}{2}$$

$$= 90\,490\,(d_1)^2 \qquad \text{... } (\because D_1 = 1.6\, d)$$

$\therefore$ $(d_1)^2 = 286 \times 10^6 / 90\,490 = 3160$ or $d_1 = 56.2$ mm

Assuming coarse threads, the standard diameter of the bolt is 60 mm (M 60). The taper on the bolt may be taken from 1 in 20 to 1 in 40. **Ans.**

3. ***Thickness of flange***

The thickness of flange (t_f) is taken as d / 3.

$\therefore$ $t_f = d / 3 = 300/3 = 100$ mm **Ans.**

Let us now check the induced shear stress in the flange by considering the flange at the junction of the shaft in shear. We know that the torque transmitted (T),

$$286 \times 10^6 = \frac{\pi d^2}{2} \times \tau_s \times t_f = \frac{\pi (300)^2}{2} \times \tau_s \times 100 = 14.14 \times 10^6\, \tau_s$$

$\therefore$ $\tau_s = 286 \times 10^6 / 14.14 \times 10^6 = 20.2$ N/mm^2 = 20.2 MPa

Since the induced shear stress in the *flange is less than the permissible shear stress of 60 MPa, therefore the thickness of flange (t_f = 100 mm) is safe.

4. ***Diameter of flange***

The diameter of flange (D_2) is taken as 2.2 d.

$\therefore$ $D_2 = 2.2\, d = 2.2 \times 300 = 660$ mm **Ans.**

36. A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa.A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPais to be used. Find the required length of key, if the shaft is loaded to transmit the maximumpermissible torque. Use maximum shear stress theory and assume a factor of safety of 2.

Solution. Given :d= 45 mm ; $\sigma_{yt}$for shaft = 400 MPa = 400 N/mm^2 ; w = 14 mm ;

t = 9 mm ;$\sigma_{yt}$for key = 340 MPa = 340 N/mm^2; F.S. = 2

Let l = Length of key.

According to maximum shear stress theory the maximum shear stress for the shaft,

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{400}{2 \times 2} = 100 \text{ N/mm}^2$$

and maximum shear stress for the key,

$$\tau_k = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{340}{2 \times 2} = 85 \text{ N/mm}^2$$

We know that the maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100\ (45)^3 = 1.8 \times 10^6 \text{ N-mm}$$

First of all, let us consider the failure of key due to shearing. We know that the maximum torque transmitted (T),

$$1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26\ 775\ l$$

$\therefore$ $l = 1.8 \times 10^6 / 26\ 775 = 67.2$ mm

Now considering the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key (T),

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17\ 213\ l$$

$$\ldots \left(\text{Taking } \sigma_{ck} = \frac{\sigma_{yt}}{F.S.}\right)$$

$\therefore$ $l = 1.8 \times 10^6 / 17\ 213 = 104.6$ mm

Taking the larger of the two values, we have

$l = 104.6$ say 105 mm **Ans.**

37. A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and theextension being 75 mm. The permissible shear and crushing stresses for the mild steel key are56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength ofthe key against the normal strength of the shaft.

Solution.Given :P= 15 kW = 15 × 10^3 W ; N = 960 r.p.m. ; d = 40 mm ; l = 75 mm ;

τ = 56 MPa = 56 N/mm^2 ;σ_c= 112 MPa = 112 N/mm^2

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

Let w = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted (T),

$$149 \times 10^3 = l \times w \times \tau \times \frac{d}{2} = 75 \times w \times 56 \times \frac{40}{2} = 84 \times 10^3\ w$$

$\therefore$ $w = 149 \times 10^3 / 84 \times 10^3 = 1.8$ mm

This width of keyway is too small. The width of keyway should be at least d / 4.

$$\therefore \qquad w = \frac{d}{4} = \frac{40}{4} = 10 \text{ mm Ans.}$$

Since $\sigma_c = 2\tau$, therefore a square key of w = 10 mm and t = 10 mm is adopted.

According to H.F. Moore, the shaft strength factor,

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right) \qquad \text{... } (\because h = t/2)$$

$$= 1 - 0.2\left(\frac{10}{20}\right) - \left(\frac{10}{2 \times 40}\right) = 0.8125$$

$\therefore$ Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e = \frac{\pi}{16} \times 56\ (40)^3\ 0.8125 = 571\ 844 \text{ N}$$

and shear strength of the key

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\ 000 \text{ N}$$

$$\therefore \quad \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840\ 000}{571\ 844} = 1.47 \text{ Ans.}$$

38. Design a muff coupling to connect two steel shafts transmitting 25kw power at 360rpm. The shafts and key are made of plain carbon steel 30C8 (Syt=Sye=400N/mm2). The sleeve is made of grey cast iron FG 200 (Sut = 200 N/mm2). The factor of safety for the shafts and key is 4. For the sleeve, the factor of safety is 6 based on ultimate strength. [Apr/May-15](April/May-17)

Given:

Power = 25 KW, N=360 rpm

For Shafts and key, Syt=Sye=400N/mm2 (fs)=4

For sleeve, Sut= 200 N/mm2 (fs)=6

Tofind:

Design a muff coupling

Solution:

Step I Permissible stresses

For the material of shafts and key,

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{4} = 100 N/mm^2$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{400}{4} = 100 N/mm^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} = \frac{0.5(400)}{4} = 50 N/mm^2$$

For sleeve material,

$$\tau = \frac{S_{su}}{(fs)} = \frac{0.5S_{ut}}{(fs)} = \frac{0.5(200)}{6} = 16.67 N/mm^2$$

Step II Diameter of the Each shaft

$$M_t = \frac{60\times10^6(KW)}{2\pi n} = \frac{60\times10^6(25)}{2\pi(360)} = 663145.60 N-mm$$

$$\tau = \frac{16M_t}{\pi d^3}$$

$$50 = \frac{16(663145.60)}{\pi d^3}$$

d=40.73 OR 45mm

Step III Dimensions of the Sleeve

D=(2d+13)= 2×45+13= 103 or 105mm

L=3.5d= 3.5(45)= 157.5 or 160 mm

The torsional shear stress in the sleeve is calculated by treating it as a hollow cylinder.

$$J = \frac{\pi(D^4 - d^4)}{32} = \frac{\pi(105^4 - 45^4)}{32}$$

= 11530626.79 mm^4

$$r = \frac{D}{2} = \frac{105}{32} = 52.5 \text{ mm}$$

$$\tau = \frac{Mtr}{J} = \frac{(663145.60)(52.5)}{(11530626.79)} = 3.02 \text{ N/mm}^2$$

τ <16.67 N/mm^2

Step IV Dimensions of Key

The standard cross-section of flat sunk key for a 45 mm diameter shaft is 14×9mm. The length of key in each shaft is one half of the length of sleeve.

Therefore,

$$l = \frac{L}{2} = \frac{160}{2} = 80\text{mm}$$

The dimensions of the key are 14×9×80 mm.

Step IV Check for stresses in Key

$$\tau = \frac{2M_t}{dbl} = \frac{2(663145.60)}{(45)(14)(80)} = 26.32 \text{ N/mm}^2$$

τ <50N/mm2

$$\sigma_c = \frac{4M_t}{dhl} = \frac{4(663145.60)}{(45)(9)(80)} = 81.87 \text{ N/mm}^2$$

σ_c <100 N/mm^2

The design of the key is safe from shear and compression considerations.

39. It is required to design a square key for fixing a gear on a shaft of 25mm diameter. The shaft is transmitted 15kw power at 720rpm to the gear. The key is made of steel 50C4 (Syt =460 N/mm2) and the factor of safety is 3. For key material, the yield strength in compression can be assumed to be equal to the yield strength in tension. Determine the dimenstions of the key.[Apr/May-15]

Given:

Power = 15 KW, N = 720 rpm, S_{yt} = 460 N/mm^2

(fs) = 3, d = 25mm

Solution:

Step I Permissible compressive and shear stresses

$S_{yc} = S_{yt} = 460$ N/mm^2

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{460}{3} = 153.33 \text{ N/mm}^2$$

According to the Maximum shear stress theory of failure,

$$S_{yc} = 0.5S_{yt} = 0.5(460) = 230 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{230}{3} = 76.67 \text{ N/mm}^2$$

Step II Torque transmitted by the shaft

$$M_t = \frac{(60\times10^6)(KW)}{2\pi N} = \frac{(60\times10^6)(15)}{2\pi(720)}$$

$$= 198943.68 \text{ N-mm.}$$

Step III Key Dimensions

The industrial practice is to use a square key with sides equal to one quarter of the shaft diameter.

Therefore,

$$b = h = \frac{d}{4} = \frac{25}{4} = 6.25 \text{ or } 6 \text{ mm}$$

$$l = \frac{2Mt}{\tau db} = \frac{2(198943.68)}{(76.67)(25)(6)} = 34.60\text{mm} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(a)}$$

$$l = \frac{4M_t}{\sigma_c dh} = \frac{4(198943.68)}{(153.33)(25)(6)} = 34.60mm \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(b)}$$

From (a) and (b), the length of the key should be 35mm. The dimensions of the key are 6×6×35 mm

40.Design the rectangular key for a shaft of 50 mm diameter. The shearing andcrushing stresses for the key material are 42 MPa and 70 MPa(April/May-17)

Given:

$$d = 50 \text{ mm} ; \tau = 42 \text{ MPa} = 42 \text{ N/mm}^2 ; \sigma_c = 70 \text{ MPa} = 70 \text{ N/mm}^2$$

Tofind: Design a rectangular key.

Solution:

The rectangular key is designed as discussed below:

From Table , we find that for a shaft of 50 mm diameter,

Width of key, $w = 16$ mm **Ans.**

and thickness of key, $t = 10$ mm **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\ 800\ l \text{ N-mm} \quad \ldots(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\ (50)^3 = 1.03 \times 10^6 \text{ N-mm} \quad \ldots(ii)$$

From equations *(i)* and *(ii)*, we have

$$l = 1.03 \times 10^6 / 16\ 800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\ l \text{ N-mm} \quad \ldots(iii)$$

From equations *(ii)* and *(iii)*, we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

l = 117.7 say 120 mm **Ans.**

41. A rigid coupling is used to transmit 50 kW power at 300 rpm. There are "six bolts the outer diameter of the flanges is 220 mm, while the recess diameter is 150mm. The coefficient of friction between the flanges is 0.15 mm. The bolts are made of steel 45C8 (Syt = 380 N/mm2) and the factor of safety is 3. Determine the diameter of the bolts. Assume that the bolts are fitted in large clearance holes.(April/May 2018)(N/D'2022)

Solution

Given kW = 50 $n = 300$ rpm $\mu = 0.15$

For bolts, $S_{yt} = 380$ N/mm^2 $(fs) = 3$ $N = 6$

For flanges, $D_o = 200$ mm $D_i = 150$ mm

Step I *Permissible tensile stress*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{380}{3} = 126.67 \text{ N/mm}^2$$

Step II *Preload in bolts*

The torque transmitted by the shaft is given by,

$$M_t = \frac{60\times10^6(\text{kW})}{2\pi n} = \frac{60\times10^6(50)}{2\pi(300)}$$

$$= 1\,591\,549.4 \text{ N-mm}$$

$$R_f = \frac{2}{3}\frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)} = \frac{2}{3}\frac{(100^3 - 75^3)}{(100^2 - 75^2)} = 88.1 \text{ mm}$$

From Eq. (9.43),

$$P_i = \frac{M_t}{\mu N R_f} = \frac{1\,591\,549.4}{0.15(6)(88.1)} = 20\,072.51 \text{ N}$$

Step III *Diameter of bolts*

Due to pre-load of 20 072.51 N, the bolts are subjected to tensile stresses.

$$P_i = \left(\frac{\pi}{4}\right) d_1^2 \sigma_t$$

or $$d_1^2 = \frac{4P_i}{\pi\sigma_t} = \frac{4(20\,072.51)}{\pi(126.67)}$$

$\therefore$ $$d_1 = 14.2 \text{ mm}$$

42.The intermediate shaft in a multi-stage gear box carries a pinion and a gear as show in fig. The dimensions and the tooth loads are given in figure. The material of the shaft is plain carbon steel whose yield strength is 380 MPa. The factor of safety is specified as 3. The power flowing through the shaft is approximately 38 kW at a speed of approximately 200 rpm. Determine the size of the shaft on the basis of strength.nov/dec 2020,April/may 2021)

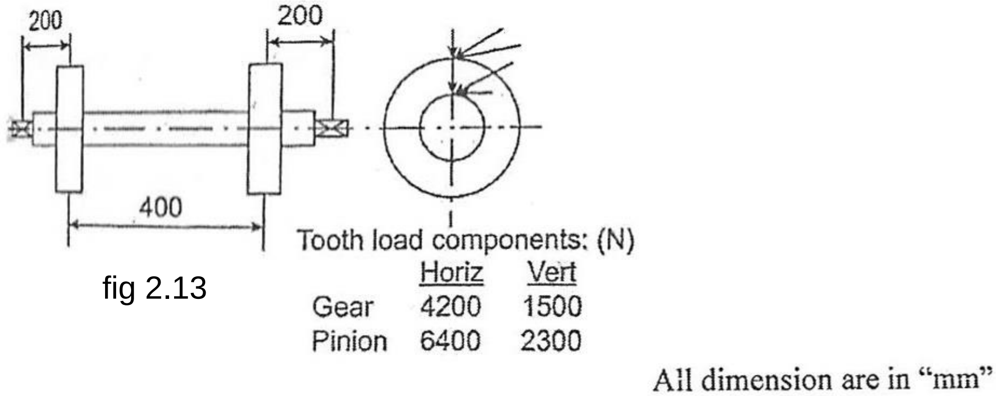


fig 2.13

Given :

AC = 250 mm ;BD= 400 mm ; DC = 600 mm or RC = 300 mm ;

DD = 200 mm or RD = 100 mm ;AB= 2400 mm ;

P = 20 kW = 20 × 103 W ;

N = 120 r.p.m ;σ_t= 100 MPa = 100 N/mm2 ;

τ = 380MPa = 56 N/mm2 ;

W_C = 950 N ;W_D = 350 N ;

K_m = 1.5 ;K_t= 1.2

The shaft supported in bearings and carrying gears is shown in Fig. 2.13

Solution:

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20000 \times 60}{2\pi \times 120} = 1590\ \text{N} - \text{m} = 1590 \times 10^3 \text{N} - \text{mm}$$

Since the torque acting at gears C and D is same as that of the shaft, therefore the tangential force acting at gear C,.

$$F_{tc} = \frac{T}{R_c} = \frac{1590 \times 10^3}{300} = 5300\text{N}$$

and total load acting downwards on the shaft at C

$$= F_{tC} + W_C = 5300 + 950 = 6250\ \text{N}$$

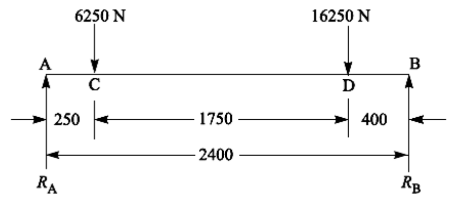
Similarly tangential force acting at gear D,

$$F_{tD} = \frac{T}{R_D} = \frac{1590 \times 10^3}{100} = 15900\text{N}$$

and total load acting downwards on the shaft at D

$$= F_{tD} + W_D = 15\,900 + 350 = 16\,250\ \text{N}$$

Now assuming the shaft as a simply supported beam as shown in Fig. the maximumbending moment may be obtained as discussed below :



Let R_A and R_B = Reactions at A and B respectively.

$$\therefore R_A + R_B = \text{Total load acting downwards at C and D}$$

$$= 6250 + 16\,250 = 22\,500 \text{ N}$$

Now taking moments about A,

$$R_B \times 2400 = 16\,250 \times 2000 + 6250 \times 250 = 34\,062.5 \times 10^3$$

$$\therefore R_B = 34\,062.5 \times 10^3 / 2400 = 14\,190 \text{ N}$$

AndR_A = 22 500 – 14 190 = 8310 N

A little consideration will show that the maximum bending moment will be either at C or D.

We know that bending moment at C,

$$M_C = R_A \times 250 = 8310 \times 250 = 2077.5 \times 10^3 \text{ N-mm}$$

Bending moment at D,

$$*M_D = R_B \times 400 = 14\,190 \times 400 = 5676 \times 10^3 \text{ N-mm}$$

∴ Maximum bending moment transmitted by the shaft,

$$M = {}_{MD} = 5676 \times 10^3 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_{eq} = \sqrt{(K_b M)^2 + (K_t M)^2}$$

$$T_{eq} = \sqrt{(1.5 \times 5676000)^2 + (1.2 \times 1590000)^2} = 8725 \times 10^3 N - mm$$

We also know that equivalent twisting moment (Te),

$$8725 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 56 \times d^3 = 11\, d^3$$

$$\therefore d^3 = 8725 \times 10^3 / 11 = 793 \times 10^3 \text{ or } d = 92.5 \text{ mm}$$

Again we know that the equivalent bending moment,

$$M_e = \frac{1}{2}[K_m \times M \sqrt{(K_m \times M)^2 + (K_t \times T)^2} = \frac{1}{2}[K_m \times M \times T_e]$$

$$M_e = \frac{1}{2}[2 \times 5676 \times 10^3 + 8725 \times 10^3\ 130] = 8620 \times 10^3\ N - mm$$

We also know that equivalent bending moment (M_e),

$$8620 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82 d^3$$

Taking the larger of the two values, we have

d = 95.7 say 100 mmAns.

43.A split muff coupling is used to connect two shafts for transmitting 40 kW at 200 rpm. Plain carbon steel is used as material for the shafts whose yield strength is 380 MPa. The number of clamping bolts is 8 and the factor of safety for shafts, bolts and key is 4. The coefficient of friction between the coupling halves is given as 0.3. Calculate (a) diameter of the shafts (b) draw a line sketch of the coupling halves and mark the dimensions, bore diameter, OD, and hub length. (c) Assuming that power is transmitted by friction between the two halves of the coupling, determine the diameter of the clamping bolt.nov/dec 2020,april/May 2021

Given:

Power P=40 kW=40 x10^3 W

Speed N=200 rpm

To find

Design of coupling

Solution:

The given coupling is a rigid type of coupling. So we can take the coupling as the clamp or split muff coupling.

6. **Diameter of shaft, d**

$$T = \frac{P \times 60}{2\pi N} = \frac{40 \times 10^3 \times 60}{2\pi \times 200} = 716197.2N - mm$$

Torque transmitted by the shaft, T

$$T = \frac{\pi}{16} \times \tau_s \times d^3$$

Assume that the shaft key and bolts are made of C45 steel having stress = 65N/mm^2

$$716197.2 = \frac{\pi}{16} \times 65 \times d^3$$

d=38.28 mm

Say diameter of the shaft d= 40 mm

7. **Dimensions of the coupling**
   c. Outside diameter of sleeve or muff , D=2.5 d = 2.5 x 40 = 100 mm
   d. Length of the sleeve L = 3.5d = 3.5x40 = 140 mm
8. **Design of sleeve:**

Assume that the sleeve is made of cast iron having allowable shear stress of 14N/mm^2

Sleeve is considered as hollow shaft

$$T = \frac{\pi}{16} \tau_s \left[\frac{D^4 - d^4}{D}\right]$$

$$716197.2 = \frac{\pi}{16} \tau_s \left[\frac{100^4 - 40^4}{100}\right]$$

$$\tau_s = 3.74\ N/mm^2$$

Induced shear stress for sleeve is less that the permissible stress. Therefor the design is safe.

9. Design of key:

Length of the key, l=L/2 = 140/2=70 mm

c. Check for shear strength,

$$T = l \times b \times \tau_k \times \frac{d}{2}$$

$$716197.2 = 70 \times 12 \times \tau_k \times \frac{40}{2}$$

$$\tau_k = 42.63\ N/mm^2$$

Induced shear stress for key is less that the permissible stress. There for the design is safe.

d. Check for crushing:

$$T = l \times \frac{h}{2} \times \sigma_{ck} \times \frac{d}{2}$$

$$716197.2 == 70 \times \frac{8}{2} \times \sigma_{ck} \times \frac{40}{2}$$

$$\boldsymbol{\sigma_{ck} = 127.9\ N/mm^2}$$

Induced crushing stress is less than the permissible stress. There for the design is safe.

10. Design in bolts:

$$T = \frac{\pi^2}{16} \times \mu \times d_b^2 \times \sigma_t \times n \times d$$

Assume number of bolt, n = 4

Assume allowable tensile stress for bolt, $\sigma_t = 70N/mm^2$

Co-efficient of friction, μ = 0.3

$$716197.2 = \frac{\pi^2}{16} \times 0.3 \times d_b^2 \times 70 \times 4 \times 40$$

$$\boldsymbol{d_b = 18.594\ mm}$$

Say diameter of bolt d_b = 20 mm

44.Design a wood ruff key to transmit 4 KW power at 400 rpm.the key is made up of C 45 steel and take FOS=2.(Nov/Dec 2021)

Given:

Power P = 4 kW
Speed N = 400 rpm
Material C45
FOS = 2

To find:

Dimensions of key

Solution:

Assume

$$Yield\ strength\ of\ C45 = 360\,N/mm^2$$

$$Allowable\ tensile\ stress\ \sigma_u = \frac{360}{2} = 180\,N/mm^2$$

$$Maximum\ shear\ stress\ \tau = 0.55\sigma_u = 0.55 \times 180 = 99\,N/mm^2$$

Diameter of shaft :

$$P = \frac{2\pi NT}{60}$$

$$4 \times 10^3 = \frac{2\pi \times 400 \times T}{60}$$

$$T = 95.49\,Nm$$

$$\frac{\pi d^3}{16} \times 99 = 95.49 \times 10^3$$

$$d^3 = 4912.54\,mm^2$$

$$d = 16.95\,mm,\ std\ dia = d = 25\,mm$$

From DDB: 5.23

For d = 25 mm,

Result:-

Width of key (b) = 8 mm

Height of the key (h)= 11 mm

Length of the key (l)= 27.35 mm

DEPARTMENT OF MECHANICAL ENGINEERING

Subject Title : DESIGN OF MACHINE ELEMENTS

Subject Code : ME 3591

Year/ SEM : III / V

UNIT – III: TEMPORARY AND PERMANENT JOINTS

QUESTION BANK & SOLVED

SYLLABUS:

Threaded fasteners - Bolted joints including eccentric loading, Knuckle joints, Cotter joints – Welded joints,Butt, Fillet and parallel transverse fillet welds – welded joints subjected to bending, torsional and eccentric loads, riveted joints for structures - theory of bonded joints

SUMMARY

Threaded fasteners:

A screw thread is formed by cutting a continuous helical groove on a cylindrical surface. A screw made by cutting a single helical groove on the cylinder is known as single threaded (or single-start) screw and if a second thread is cut in the space between the grooves of the first, a double threaded (or double-start) screw is formed. Similarly, triple and quadruple (i.e. multiple-start) threads may be formed.

The helical grooves may be cut either right hand or left hand.

A screwed joint is mainly composed of two elements i.e. a bolt and nut. The screwed joints are widely used where the machine parts are required to be readily connected or disconnected without damage to the machine or the fastening.This may be for the purpose of holding or adjustment in assembly or service inspection, repair, or replacement or it may be for the manufacturing or assembly reasons.

Bolted joints :

In addition to sizing a bolt on the basis of axial tensile stress, the threads must be checked to ensure that they will not be stripped off by shearing. The variables involved in the shear strength of the threads are the materials of the bolt, the nut, or the internal threads of a tapped hole, the length of engagement, L, and the size of the threads. The details of analysis depend on the relative strength of the materials.

Most bolts and screws have enlarged heads that bear down on the part to be clamped and thus exert the clamping force. *Set screws are* headless, are inserted into tapped holes, and are designed to bear directly on the mating part, locking it into place. Caution must be used with set screws, as with any threaded fastener, so that vibration does not loosen the screw.

When a bolt and nut is made of mild steel, then the effective height of nut is made equal to thenominal diameter of the bolt. If the nut is made of weaker material than the bolt, then the height of nutshould be larger, such as 1.5 *d* for gun metal, 2 *d*for cast iron and 2.5 *d* for aluminium alloys (where*d* is the nominal diameter of the bolt).

In case cast iron or aluminium nut is used, then *V*-threads arepermissible only for permanent fastenings, because threads in these materials are damaged due torepeated screwing and unscrewing.

When these materials are to be used for parts frequently removedand fastened, a screw in steel bushing for cast iron and cast-in-bronze or model metal insert should beused for aluminium and should be drilled and tapped in place.

Bolted Joints under Eccentric Loading:

There are many applications of the bolted joints which are subjected to eccentric loading suchas a wall bracket, pillar crane, etc. The eccentric load may be

1. Parallel to the axis of the bolts,

2. Perpendicular to the axis of the bolts, and

3. In the plane containing the bolts.

Riveted joints for structures:

A rivet is a short cylindrical bar with a head integralto it. The cylindrical portion of the rivet is called ***shank*** or ***body***and lower portion of shank isknown as ***tail,*** The rivets are used to makepermanent fastening between theplates such as in structural work,ship building, bridges, tanks andboiler shells. The riveted jointsare widely used for joining lightmetals.
The fastenings (*i.e.* joints)may be classified into the following two groups :

1. Permanent fastenings, and

2. Temporary or detachable fastenings.

The ***permanent fastenings*** are those fastenings which cannot be disassembled withoutdestroying the connecting components. The examples of permanent fastenings in order of strengthare soldered, brazed, welded and riveted joints.

The ***temporary*** or ***detachable fastenings*** are those fastenings which can be disassembled without destroying the connecting components. The examples of temporary fastenings are screwed,keys, cotters, pins and splined joints.

Rivets are non threaded fasteners, usually made of steel or aluminum. They are originally made with one head, and the opposite end is formed after the rivet is inserted through holes in the parts to be joined. Steel rivets are formed hot, whereas aluminum can be formed at room temperatures. Of course, riveted joints are not designed to be assembled more than once.

A large variety *of quick-operating fasteners* is available. Many are of the quarter-tum type, requiring just a 90° rotation to connect or disconnect the fastener. Access panels, hatches, covers, and brackets for removable equipment are attached with such fasteners. Similarly, many varieties of *latches* are available to provide quick action with, perhaps, added holding power.

Welded joints:

Welding involves the metallurgical bonding of metals, usually by the application of heat with an electric arc, a gas flame, or electrical resistance heating under heavy pressure.
A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material. The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding). The latter method is extensively used because of greater speed of welding.

Welding is extensively used in fabrication as an alternative method for casting or forging and as a replacement for bolted and riveted joints. It is also used as a repair medium e.g. to reunite metal at a crack, to build up a small part that has broken off such as gear tooth or to repair a worn surface such as a bearing surface.

Knuckle joints, Cotter joints:

A knuckle joint is used to connect two rods which are under the action of tensile loads. However, if the joint is guided, the rods may support a compressive load. A knuckle joint may be readily disconnected for adjustments or repairs.

Its use may be found in the link of a cycle chain, tierod joint for roof truss, valve rod joint with eccentric rod, pump rod joint, tension link in bridgestructure and lever and rod connections of various types.

A cotter is a flat wedge shaped piece of rectangularcross-section and its width is tapered (either on one side orboth sides) from one end to another for an easy adjustment.
The taper varies from 1 in 48 to 1 in 24 and it may beincreased up to 1 in 8, if a locking device is provided. Thelocking device may be a taper pin or a set screw used on thelower end of the cotter. The cotter is usually made of mildsteel or wrought iron. A cotter joint is a temporary fasteningand is used to connect rigidly two co-axial rods or barswhich are subjected to axial tensile or compressive forces.

It is usually used in connecting a piston rod to the crossheadof a reciprocating steam engine, a piston rod and itsextension as a tail or pump rod, strap end of connecting rodetc.

Theory of bonded joints:

If the load is not very large adhesive joints become very useful in joining metallic or non–metallic dissimilar materials. No special device is needed. But the disadvantage of this joint is that the joint gets weakened by moisture or heat and some adhesive needs meticulous surface preparation. In an adhesive joint, adhesive are applied between two plates known as adhered. The strength of the bond between the adhesive and adhered arise become of various reasons given below.The adhesive materials may penetrate into the adherend material and locks the two bodies.

a. Long polymeric chain from the adhesive diffuse into the adhered body to form a strong bond.
b. Electrostatic force may cause bonding of two surfaces.

Common types of adhesives are epoxies, polyester resins, nitric rubber phenolics. Epoxies are extensively used for mechanical purposes because of their high internal strength in cohesion, low shrinkage stresses, low temperature cure and creep, insensitivity to moisture etc. Often fillers like aluminum oxides, boron fibers are used to improve mechanical strength. Polyester resins are widely used in commercial fields for various structural applications involving plastics operating at moderate temperature.

PART – A (2MARKS)

THREADED FASTENERS:

1. **What do you understand by single start and twin start threads?Nov/Dec 2011**

   Single-start threads:

   A single-start thread screw has one continuous thread running along the body of the screw.Normally single start threads are used where heavy loads are to be applied. Generally, the more starts, the steeper the helix and the faster the transit of nut. High loads need a flat helix, given a better mechanical advantage, and are usually a single start thread.

   Twin start threads:

   A screw with a twin or double start thread has two threads running along the body of the screw instead of just one. Screws with twin-start threads often have a larger pitch, which means they can be inserted or removed twice as fast as a screw with a single-start thread. They will also hold material more securely. However, screws with twin/double start threads are usually more expensive

2. **Screw joint is composed with two elements justify**

   A screwed joint is mainly composed of two elements*i.e*a bolt and nut. The screwed joints are widely used where the machine parts are required to be readily connected or disconnected without damage to the machine or the fasten-ing. This may be for the purpose of holding or adjustment in assembly or service inspection, repair, or replacement or it may be for the manufacturing or assembly reasons.The parts may be rigidly connected or provision may be made for predetermined relative motion.

3. **Write the advantages and disadvantages of screwed joints**

   Following are the major advantages and disadvantages of the screwed joints.

   Advantages:

   a. Screwed joints are highly reliable in operation.
   b. Screwed joints are convection to assemble and disassemble.
   c. A wide range of screwed joints may be adopted to various operating conditions.
   d. Screws are relatively cheap to produce due to standardizations and highly efficient manufacturing processes.

   Disadvantages:

   a. The maindisadvantages of the screwed joints are the stress concentration in the threaded portions which are vulnerable points under variable load conditions.
   b. The strength of the screwed joints is not comparable with welded or riveted joints.
   c. Screwed joints become loose due to machine vibrations

4. **Define the terms. a) Outside diameter b) Core Diameter c) Pitch diameter**

   a. Outside diameter

   It is the largest diameter of an external or internal screw thread. The screw is specified by this diameter. It is also known as outside or nominal diameter.

   b. Core diameter

   It is the smallest diameter of an external or interior screw thread. It is also known as core or root diameter.

c. Pitch diameter

It is the diameter of an imaginary cylinder, on a cylindrical screw thread, the surface of which would pass through the thread at such points as to make equal the width of the thread and the width of the spaces between the threads. It is also called an effective diameter. In a nutand bolt assembly, it is the diameter at which the ridges on the bolt are in complete touch with the ridges of the corresponding nut.

5. How the screw threads are designated?

The screw threads are designated by indian standard,

Size designation:

The size of the screw thread is designated by the letter 'M' followed by the diameter and pitch, the two beigns separated by the sign×. When there is no indication of the pitch, it shall mean that coarse pitch is implied.

Tolerance designation:

This shall include

(a) A figure designating tolerance grade as indicated below:
'7' for fine grade, '8' for normal (medium) grade, and '9' for coarse grade.

(b) A letter designating the tolerance position as indicated below :
'*H*' for unit thread, '*d*' for bolt thread with allowance, and '*h*' for bolt thread withoutallowan ce.

6. What types of stresses are induced in a bolt?

The following stresses are induced in a bolt when it is subjected to an external load.

Tensile stress

The bolt, suds and screws usually carry a load in the direction of the bolt axis which induces a tensile stress in the bolt.

Shear stress

Sometimes, the bolts are used to prevent the relative movement of two or more parts, as in case of flange coupling, then shear stress is induced in the bolt. The shear stresses should be avoided as far as possible.

Combined tension and shear stress

When a bolt is subjected to both tension and shear loads, as in case of coupling bolts are bearing, then the diameter of the shank of the bolt is obtained from the shear load and that of threaded part from the tensile load.

7. How do you prevent the fasteners which become loose under the action of loads?

In order to prevent this loosening of fasteners in action of vibration, using one of the locking devices is locking with pin.as shown in fig 1

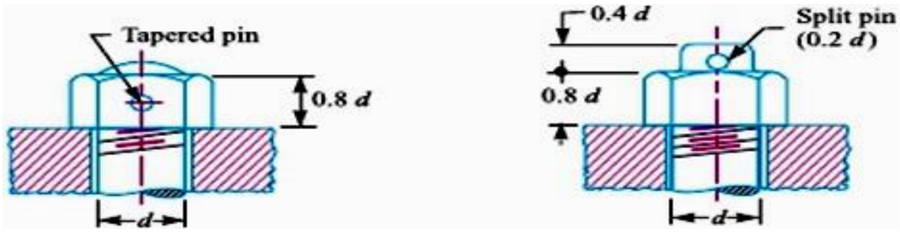


fig 1 Locking with pin

Locking with pin:

The nuts may be locked by means of atapper pin or cotter pin passing through the middle of the nuts as shown in figure (a). But a split pin is often driven through the bolt above the nut, as shown in figure (b).

8. State the difference between differential and compound screws

1) A differential screw is a device consisting of two screws of same hand of helix in series that will give a resultant motion equal to the differences of the two pitches.

2) If p_1 and $p_2$are the two pitches, then the resultant motion is $p_1 - p_2$.

3) A compound screw is a device consisting of two screws of different hands of helix in series that will give a resultant motion equal to the sum of the two pitches.

4) For p_1 and p_2 pitches, the resultant motion is p_1+p_2.

9. Give the merits and demerits of threads

The merits and demerits of threads are following

a. Merits of square threads:

1. Efficiency is more.
2. It can transmit motion and force in both directions

b. Demerits of square threads and Buttress thread:

1. Difficult to manufacture
2. Weaker than ACME and trapezoidal threads
3. Wear cannot be compensated.
4. Can transmit

c. Merits of ACME, Trapezoidal and Buttress threads:

1. Manufacturing and finishing easy.

2. Stronger because of more thread thickness.

3. Wear compensation is possible

10. Define the term "self-locking of power screw". Nov/Dec 2012&May/June 2013

The term self - lockingiss defined by the angle of their threads. The threads of self-locking screws are precisely angled so that, once the screw is placed, they will not slip or move unless some additional force is applied. After you have screwed a self-locking screw into position, it will not move again unless you use a screwdriver or similar tool to remove it from position.

If the friction angle (φ) is greater than helix angle (α) of the power screw, the torque required to lower the load will be positive, indicating that an effort is applied to lower the load. This type is screw is known as self-locking screws. The efficiency of the self-locking screw is less than 50%.

11. Why are ACME threads preferred over square thread for power transmission? (Nov/Dec-2014)

In power square, the square thread is more efficient than the Acme thread because in the Acme thread the effective coefficient of friction increases, yet for power screws it is the Acme thread which is used more predominantly. The Acme thread can be machined more easily than the square thread and more importantly the clearance in the Acme thread can be adjusted to take care of the wear or machining inaccuracy.

12. Define the terms: a) Machine Screws b) Studs.

Machine screws:

In this screw very small cap screws are known as machine screws. The heads of these screws are slotted so that they can be tightened by means of a screwdriver. They are used in assembly of small machines such as typewriter, jigs, carburetors, etc.

Studs:

A stud is a bolt in which the head is replaced by a threaded end. It passes through one of the parts to be connected and is screwed into the other part. Thus the stud always remains in position when thetwo parts are disconnected. Clearance between the threads and hole facilltates the removal of the part without injury to the free end of the stud. With this construction, the wear and crumbling of the threads in a weak material are avoided. Studs are employed for connecting heads of cylinders in engine and pumps.

13. What are the different applications of screwed fasteners?

The different applications of screwed fasteners are

a. For readily connecting & disconnecting machine parts with out damage

b. The parts can be rigidly connected.

c. Used for transmitting power.

14. What are the different types of metric thread?

1. BSW (British standard whit worth)
2. BSE (British standard End)

15.What is known as proof strength of the bolts?(APRL/MAY-15)(Nov/Dec 2017)

The bolt is tightened by applying torque on nut which after free turns stretches the bolt and induces tensile load in the bolt called preload. The torque applied to tighten the bolt should be sufficient enough to produce a preload that induces stress up to 90% of the proof strength for static loading and 75% of proof strength for dynamic loading. Proof strength is the stress at which the bolt begins to take permanent deformation. It is very close but less than the yield strength.

16. Determine the safe tensile load for a bolt of M20, assuming a safe tensile stress of 40 MPa (May/June 2012)

For M 20 bolt the nominal diameter d p = 18.37 6mm & the effective diameter is 16.933 mm.

Safe tensile load = Permissible stress × Cross-sectional area at bottom of the thread

$= 40 \text{ x } \pi/4(dp+dc/2)^2 = 40 \text{ x } \pi/4(18.376 + 16.933 /2)^2 = 9786.759$ N

17.List out the advantages of the V-threads (April/May 2018)

Fastening threads are usually V-threads. They offer the following advantages.

- V-threads result in higher friction, which lessen the possibility of loosening.
- V-threads have higher strength due to increased thread thickness at the core diameter.
- V-threads are more convenient to manufacture.

18. List out the advantages of threaded joints (April/May 2019)

Advantages of threaded joints

Threaded joints are widely used in mechanical assemblies its been like 60% of the parts have made by threads. The popularity of threaded joints is due to certain advantages.
Now we can see the advantages of threaded joint below :

- Threaded joints are reliable joints. there is no loosening of the parts that are held together by means of large clamping force.
- The parts are assembled by means of a spanner and the length of spanner is large as compared with the radius of the thread therefore, the mechanical advantages is more and force required to tighten the joint is small.
- It has small overall dimensions resulting in compact construction.
- The threads are self-locking so it can be placed in any position (vertical, horizontal or inclined also).
- High accuracy can be maintained for the threaded components.
- Manufacturing of thread is very simple.
- In threaded joints the thread can be detached easily when required.

Bolted joints including eccentric loading, knuckle joints, cotter joints

18.In which conditions eccentric loading occurs?

Following conditions are the eccentric load may occur,
There are many applications of the bolted joints which are subjected to eccentric loading such as a wall bracket, pillar crane, etc.
The eccentric load may be

1) Parallel to the axis of the bolts
2) Perpendicular to the axis of the bolts, and
3) Plane of loading is same as the plane of the bolts

19. Discuss the details of socket and spigot cotter joint.

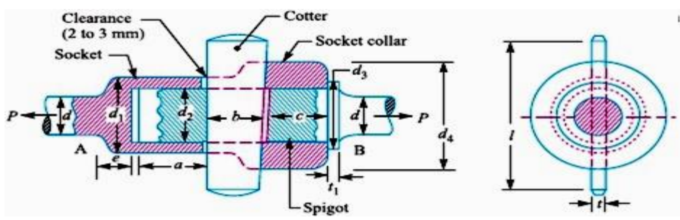


fig 2

In a socket and spigot cotter joint, one end of the rods (say A) is provided with a socket type of end as shown in figure and the other end of the rod (say B) is inserted into a

socket. The end of the rod which goes into a socket is also called spigot. A rectangular hole is made in the socket and spigot. A cotter is then driven tightly through a hole in order to make the temporary connection between the two rods. The load is usually acting axial, but it changes its direction and hence then the cotter joint must be designed to carry both the tensile and compressive loads. The compressive load is taken up by the collar on the spigot. as shown in fig 2

20. State the three conditions when tap bolts are used. Nov/Dec 2010

The tap bolts are used under the following condition:

i. The parts that are fastened have medium thickness, e.g. plates, flanges or beams and space is available to accommodate to bolt head and the nut. Space should also be available to accommodate the spanner to tighten the nut.

ii. The parts that are fastened are made of materials, which are too weak to make durable threads.

iii. The parts that are fastened require frequent dismantling and reassembly.

21. What is a gib? Why is it provided in a cotter joint? Nov/Dec 2013

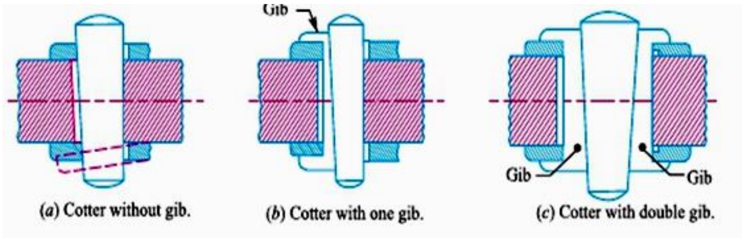


fig 3

A gib is a sacrificial piece that can be replaced once worn, allowing a joint to function correctly. A gib and cotter joint is usually used in a strap end of a connecting rod. Gibs are used which hold together the end of the strap in cotter joint. Moreover, gibs provide a larger bearing surface for the cotter to slide on, due to the increased holding power. Thus, the tendency of cotter to slacken back owing to friction it considerably decreased. The gib, also, enables parallel holes to be used.

1. When one gib is used, the cotter with one side tapered is provided and the gib is always on the outside.
2. When two gibs are used, the cotter with the both sides tapered is provided as shown in Figure. 3

22.Determine the safe tensile load for a bolt of M20, assuming a safe tensile stress of 40Mpa. (May/June 2012)

Given: Safe tensile stress (σ_t) = 40 MPa = 40 N/mm^2.

To Find: Bolt of M20

Solution:

The safe tensile load for a bolt of M 20,

We find that the core diameter of the bolt corresponding to M 20 is d_c = 16.93 mm.

We know that safe tensile stress,

$$\sigma t = 40MPa = 40N/mm^2$$

$$P = \frac{\pi}{4} dc^2 \times \sigma t$$

Where,

P = tensile load, dc = core diameter, σ_{t} = tensile stress

$$P = \frac{\pi}{4}(16.93)^2 \times 40$$

$$P = 225.114669 \times 40 = 90 \times 10^3 \text{N/mm}^2.$$

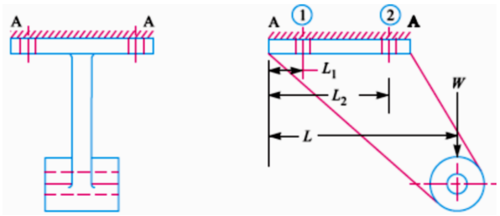
23. How arenuts for fastenings designed?

The nuts for fastening are designed by the when a bolt and nut is made of mild steel, then the effective height of nut is made equal to thenominal diameter of the bolt. If the nut is made of weaker material than the bolt, then the height of nut should be larger, such as 1.5 *d* for gun metal, 2 *d* for cast iron and 2.5 *d* for aluminium alloys (where *d* is the nominal diameter of the bolt).

In case cast iron or aluminium nut is used, then *V*-threads are permissible only for permanent fastenings, because threads in these materials are damaged due to repeated screwing and unscrewing. When these materials are to be used for parts frequently removed and fastened, a screw in steel bushing for cast iron and cast-in-bronze or monel metal insert should be used for aluminium and should be drilled and tapped in place.

24. A bracket, as shown in Fig., supports a load of 30 kN. Determine the sizeof bolts, if the maximum allowable tensile stress in the bolt material is 60 MPa. The distances are:

L1 = 80 mm, L2 = 250 mm, and L = 500 mm



Given: fig 4

W= 30 kN. ; σ_t= 60 MPa = 60 N/mm^2;L_1 = 80 mm ; L_2 = 250 mm; L = 500 mm

Solution:

We know that the direct tensile load carried by each bolt,

$$W_{t1} = \frac{W}{n} = \frac{30}{4} = 7.5 \text{ kN.}$$

and load in a bolt per unit distance,

$$W = \frac{W\ L}{2[L1^2]+[L2^2]} = \frac{30 \times 500}{2[80^2]+[250^2]} = 0.109\ kN/mm$$

Since the heavily loaded bolt is at a distance of *L*2 mm from the tilting edge, therefore load on the heavily loaded bolt,

$$Wt2 = w.L2 = 0.109 \times 250 = 27.25 \text{ kN.}$$

Maximum tensile load on the heavily loaded bolt,

$$W_t = W_{t1} + W_{t2} = 7.5 + 27.25 = 34.75 \text{ kN} = 34\ 750 \text{ N}$$

Let d_c= Core diameter of the bolts.

We know that the maximum tensile load on the bolt (W_t), by fig 4

$$34750 = \frac{\pi}{4} dc^2 \times \sigma_t = \frac{\pi}{4} dc^2 \times 60 = 47\ (d_c)^2$$

$$(d_c)^2 = 34750 / 47 = 740$$

$$d_c = 27.2mm$$

Or

We find that the standard core diameter of the bolt is 28.706 mm and the corresponding size of the **bolt is M 33.**

25. Discuss about the modes of failure in socket and spigot cotter joints.

The following failures for socket and spigot cotter joints

a) Failure of the rod in tension.
b) Failure of the spigot in tension across the weakest section (or slot).
c) Failure of the rod or cotter in crushing.
d) Failure of the socket in tension across the slot.
e) Failure of cotter in shear.
f) Failure of socket collar in crushing.
g) Failure of socket end in shearing
h) Failure of rod end in shear.
i) Failure of spigot collar in crushing.
j) Failure of spigot collar in shearing.
k) Failure of cotter in bending.

26.What are the different types of cotter joints? May/June 2014

There are three commonly used cotter joints to connect two rods by a cotter is

Socket and spigot cotter joint:

In a socket and spigot cotter joint, one end of the rods (say A) is provided with a socket type of end as shown in figure and the other end of the rod (say B) is inserted into a socket. The end of the rod which goes into a socket is also called spigot. A rectangular hole is made in the socket and spigot. A cotter is then driven tightly through a hole in order to make the temporary connection between the two rods. The load is usually acting axial, but it changes its direction and hence then the cotter joint must be designed to carry both the tensile and compressive loads. The compressive load is taken up by the collar on the spigot.

Sleeve and cotter joint :

A sleeve and cotter joint is used to connect two round rods or bars. In this type of joint, a sleeve or muff is used over the two rods and then two cotters (one on each rod end) are inserted in the holes provided for them in the sleeve and rods. The taper of cotter is usually 1 in 24.

Gib and cotter joint:

A gib and cotter joint is usually used in a strap end of a connecting rod. Gibs are used which hold together the end of the strap in cotter joint. Moreover, gibs provide a larger bearing surface for the cotter to slide on, due to the increased holding power. Thus, the tendency of cotter to slacken back owing to friction it considerably decreased. The gib, also, enables parallel holes to be used.

27.Write short notes on turnbuckle

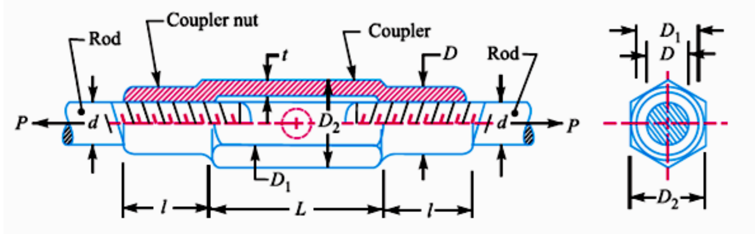
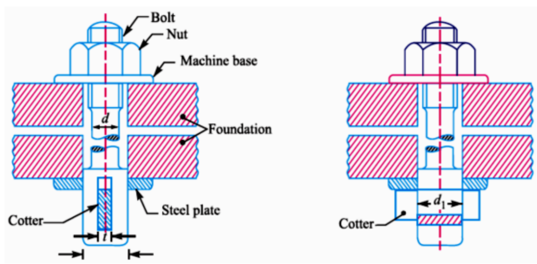


fig 5

In this two round tie rods, are connected by means of a coupling known as turnbuckle. A turnbuckle commonly used in engineering practice (mostly in airplanes). This type of turnbuckle is made hollow in the middle to reduce its weight. In this case two ends of the rods may also be

seen. But it is not necessary that the material of the rods and the turnbuckle may be same or different. It depends on the pull acting on the joint.

28.Under various application why cotter foundation bolt is preferred over ordinary bolt in heavy machinery foundation?



The cotter foundation bolt is mostly used in conjunctionwith foundation and holding down bolts to fasten heavymachinery to foundations. It is generally used where anordinary bolt or stud cannot be conveniently used. Figure shows the two views of the application of such a cotterfoundation bolt. In this case, the bolt is dropped down fromabove and the cotter is driven in from the side. Now thisassembly is tightened by screwing down the nut. It may benoted that two base plates (one under the nut and the otherunder the cotter) are used to provide more bearing area inorder to take up the tightening load on the bolt as well as todistribute the same uniformly over the large surface.

Let,

d = Diameter of bolt, d_1 = Diameter of the enlarged end of bolt, t = Thickness of cotter, and b= Width of cotter.

29.What are the failures which occurs in cotter foundation bolt?

The various modes of failure of the cotter foundation bolt are,

- **a)** Failure of bolt in tension
- **b)** Failure of the enlarged end of the bolt in tension at the cotter
- **c)** Failure of cotter in shear
- **d)** Failure of cotter in crushing.

30.Under the action of tensile load which type of joint isprefered? Why?

A knuckle joint is used to connect two rods which are under the action of tensile loads.However, if the joint is guided, the rods may support a compressive load. A knuckle joint may bereadily disconnected for adjustments or repairs. Its use may be found in the link of a cycle chain, tierod joint for roof truss, valve rod joint with eccentric rod, pump rod joint, tension link in bridgestructure and lever and rod connections of various types.In knuckle joint, one end of one of therods is made into an eye and the end of theother rod is formed into a fork with an eye ineach of the fork leg. The knuckle pin passesthrough both the eye hole and the fork holesand may be secured by means of a collar andtaper pin or spilt pin.

30.(a) state any two conditions under which the use of knuckle joint is recommended(Nov/Dec 2021)

A knuckle joint is a mechanical joint used to connect two rods which are under a tensile load, when there is a requirement of small amount of flexibility, or angular moment is necessary

31.Draw the neat sketch of knuckle jointand give its dimensional parts

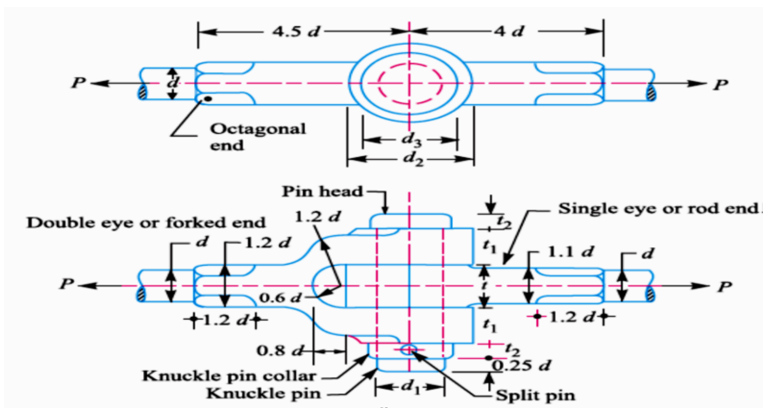


fig 7

The dimensions of various parts of theknuckle joint are fixed by empirical relations asgiven below. It may be noted that all the partsshould be made of the same material *i.e.* mildsteel or wrought iron.

If *d* is the diameter of rod, then diameter ofpin, $d_1 = d$

Outer diameter of eye, $d_2 = 2\,d$

Diameter of knuckle pin head and collar, $d_3 = 1.5\,d$

Thickness of single eye or rod end, $t = 1.25\,d$

Thickness of fork, $t_1 = 0.75\,d$

Thickness of pin head, $t_2 = 0.5d$

Other dimensions of the joint are shown in Figure. 7

32.State the eight methods of failure of knuckle joints

In determining the strength of the joint for the various methods of failure, it is assumed that

1. There is no stress concentration.
2. The load is uniformly distributed over each part of the joint.

Due to these assumptions, the strengths are approximate; however they serve to indicate a well-proportioned joint. Following are the various methods of failure of the joint:

a. Failure of the solid rod in tension

b. Failure of the knuckle pin in shear
c. Failure of the single eye or rod end in tension
d. Failure of the single eye or rod end in shearing
e. Failure of the single eye or rod end in crushing
f. Failure of the forked end in tension
g. Failure of the forked end in shear
h. Failure of the forked end in crushing

33.Write the empirical relations used in knuckle joint.

The emprical relation used in kunckle joint is a designer should consider the empirical relations in designing a knuckle joint. The following procedure may be adopted:

First of all, find the diameter of the rod by considering the failure of the rod in tension. We know that tensile load acting on the rod,

$$P = \frac{\pi}{4} dc^2 \times \sigma_t$$

Where, d = Diameter of the rod, and

σ_t= Permissible tensile stress for the material of the rod.

After determining the diameter of the rod, the diameter of pin (*d*1) may be determined by considering the failure of the pin in shear. We know that load,

$$P = 2\times\frac{\pi}{4} d1^2 \times \tau$$

A little consideration will show that the value of *d*1 as obtained by the above relation is less thanthe specified value (*i.e.* the diameter of rod). So fix the diameter of the pin equal to the diameter of therod.

<u>WELDED JOINTS, RIVETED JOINTS FOR STRUCTURE</u>

34.Write short notes on permanent joints.

A welded joint is a permanent joint which is obtainedby the fusion of the edges of the two parts to be joinedtogether, with or without the application of pressure and afiller material. The heat required for the fusion of thematerial may be obtained by burning of gas (in case of gaswelding) or by an electric arc (in case of electric arcwelding). The latter method is extensively used because ofgreater speed of welding.Welding is extensively used in fabrication as analternative method for casting or forging and as areplacement for bolted and riveted joints.

35.Write any two advantages and disadvantages of welded joints over riveted joints.(May/June 2013)

Following are the advantages and disadvantages of welded joints over riveted joints are,

Advantages:

a) The welded structures are usually lighter than riveted structures. This is due to the reason,that in welding, gussets or other connecting components are not used.
b) The welded joints provide maximum efficiency (may be 100%) which is not possible incase of riveted joints.

c) Alterations and additions can be easily made in the existing structures.
d) As the welded structure is smooth in appearance, therefore it looks pleasing.
e) In welded connections, the tension members are not weakened as in the case of riveted joints.
f) A welded joint has a great strength. Often a welded joint has the strength of the parentmetal itself.
g) Sometimes, the members are of such a shape (*i.e*. circular steel pipes) that they afforddifficulty for riveting. But they can be easily welded.
h) The welding provides very rigid joints. This is in line with the modern trend of providingrigid frames.
i) It is possible to weld any part of a structure at any point. But riveting requires enoughclearance.
j) The process of welding takes less time than the riveting.

Disadvantages

a) Since there is an uneven heating and cooling during fabrication, therefore the members may get distorted or additional stresses may develop.
b) It requires a highly skilled labour and supervision.
c) Since no provision is kept for expansion and contraction in the frame, therefore there is a possibility of cracks developing in it.
d) The inspection of welding work is more difficult than riveting work.

36. Properly made butt welds are equal or better strength than the plate. Justify.

Butt welded joint, when properly made, has equal and better strength than the plates and there is no need for determining the stresses in the weld or the size and the length of the weld. All that is required is to match the strength of the weld material to the strength of the plates.

$$\mathbf{P = \sigma_t\, t / \eta}$$

Where, η = efficiency of the welded joints

There are certain codes, like code for unfired pressure vessel, which suggest reduction in strength of butt welded joint by a factor called efficiency of the joint.

37. Define welding.

Welding can be defined as a process of joining two similar or dissimilar metals with or without application of pressure along with or without addition of filler material.

37.(a) Write the significance of weld specifications.(Nov/Dec 2021)

- A Welding Procedure Specification (WPS), is a document that serves as a guide for the effective creation of a weld that meets all applicable code requirements and production standards.
- Think of a WPS as a recipe for welders.
- A Welding Procedure Specification provides direction to the welder for making sound and quality production welds as per the code requirements.

38. Explain about the types of welding process.

The welding processes may be classified in the following types such as,

a) Forge welding

b) Electric resistance welding and
c) Fusion welding

(a) Forge welding

In forge welding, the parts are heated to reach the plastic stage and the joint is prepared by applying impact force.

(b) Electric resistance welding

Inelectric resistance welding, the parts are pressed together and an electric current is passed through them. So, the metal is heated to the fusion temperature.

(c)Fusion welding

In fusion welding, the two pieces to be joined are heated to the fusion temperature by an oxy-acetylene flame or by an electric arc and then joined together by an additional filler material from a welding rod. It is the mostly used welding.

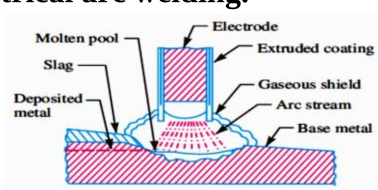
The fusion welding, according to the method of heat generated, may be classified as:

a. Thermit welding.
b. Gas welding and
c. Electric arc welding.

39.Write short notes on gas welding.

A gas welding is made by applying the flame of an oxy-acetylene or hydrogen gas from a welding torch upon the surfaces of the prepared joint. The intense heat at the white cone of the flame heats up the local surfaces to fusion point while the operator manipulates a welding rod to supply the metal for the weld. A flux is being used to remove the slag. Since the heating rate in gas welding is slow, therefore it can be used on thinner materials.

40.Discuss in detail about the electrical arc welding.



In electric arc welding, the work is prepared in the same manner as for gas welding. In this case the filler metal is supplied by metal welding electrode. The operator, with his eyes and face protected, strikes an arc by touching the work of base metal with the electrode. The base metal in the path of the arc stream is melted, forming a pool of molten metal, which seems to be forced out of the pool by the blast from the arc, as shown in Figure. A small depression is formed in the base metal and the molten metal is deposited around the edge of this depression, which is called the ***arc crater.*** The slag is brushed off after the joint has cooled.

41.Narrate the various type of welded joints.

Following two types of welded joints are

1. Lap joint of Fillet joint
2. Butt joint

Lap or fillet joint:

The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet joints may be 1. Single transverse fillet,2. Double transverse fillet, and 3. Parallel fillet joints.

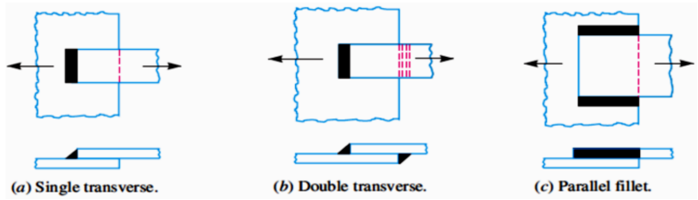


fig 8

Butt joint:

The butt joint is obtained by placing the plates edge to edge as shown in Fig.9 . In butt welds,the plate edges do not require bevelling if the thickness of plate is less than 5 mm. On the other hand, ifthe plate thickness is 5 mm to 12.5 mm, the edges should be bevelled to V or U groove on both sides.The butt joints may be

- Square butt joint,
- Single V-butt joint
- Single U-butt joint,
- Double V-butt joint, and
- Double U-butt joint.

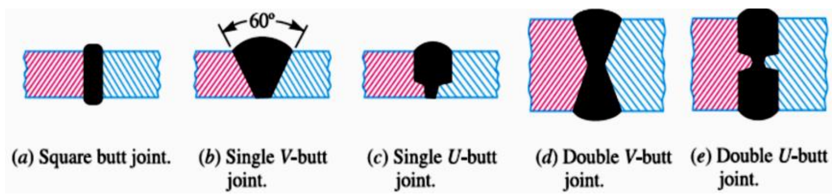


fig 9

42.What are the disadvantages of welding Joints? (Nov/Dec 2014),(Nov/Dec 2017) (Nov/Dec 2018)

The following are the disadvantages of welding

a) Welded joints cannot be used for collision and vibration.
b) Welded joints cannot be used for assembled and reassembled.
c) Welded joints are more brittle and therefore their fatigue strength is less than the members joined.
d) Due to uneven heating & cooling of the members during the welding, the members may distort resulting in additional stresses.
e) Skilled labor and electricity are required for welding.
f) No provision for expansion and contraction is kept in welded connection & therefore, there is possibility of racks.
g) The inspection of welding work is more difficult and costlier than the riveting work.
h) Defects like internal air pocket, slag inclusion and incomplete penetration are difficult to detect.

43.What are the methods to remove a seized or rusted bolt?

There are two possible methods to remove a seized or rusted bolt which is followed

1. If the assembly the bolt is in wall tolerates heat without damage, use an oxyfuel torch to heat the bolt head and its assembly to just below red heat. A rapid water quench will further help screw removal. Two or more heating and cooling cycles may beneeded.

2. If the slot, head, or allen cap screw socket is damaged or missing placed nut over the bolt head (or remaining stub end), hold this nut in this place, and fill inside of the nut with weld metal using any welding method. This weld will join the nut to the bolt stub.

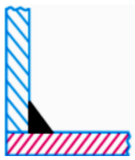
44.Define the terms: Tee- joint, corner joint and edge joint.

1. Tee- joint

The two plates are arranged in 'T' shape i.e. located at right angle to each other and the overlapping edges are welded by fillet weld as shown in figure.

2. Corner joint

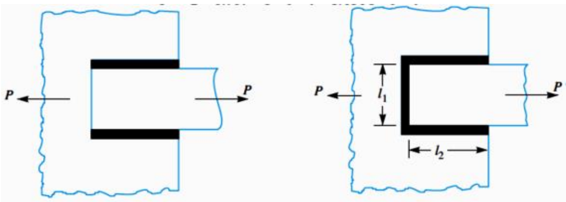
In this type of joint, two plates are arranged at right angles such that it forms an angle i.e. L-shape. The adjacent edges are joined by a fillet weld as shown in figure.



3. Edge joint

For plates of thickness less than 6mm, the ends of the overlapping plates can be directly welded at the edges are joined by a fillet weld.

45.Enumerate the designfor Strength of parallel fillet welded joints.



(a) Double parallel fillet weld.

(b) Combination of transverse and parallel fillet weld.

fig 10

The strength of parallel fillet welded joints are designed for shear strength. Consider a double parallel filletwelded joint as in figure 10 We have already discussed in the previous article, that theminimum area of weld or the throat area,

$$A = 0.707\ s \times l$$

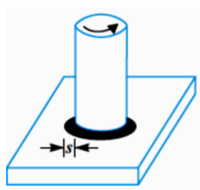
If τ is the allowable shear stress for the weld metal, then the shear strength of the joint for singleparallel fillet weld,

P = Throat area × Allowable shear stress = $0.707\ s \times l \times \tau$

and shear strength of the joint for double parallel fillet weld,

$$P = 2 \times 0.707 \times s \times l \times \tau = 1.414\ s \times l \times \tau$$

46.A 50 mm diameter solid shaft is welded to a flat plate by 10 mm fillet weld as shown in Fig. 10.12. Find the maximum torque that the welded joint can sustain if the maximum shear stress intensity in the weld material is not to exceed 80 MPa.



Given:

d = 50 mm; s = 10 mm; τ_{max}= 80 MPa = 80 N/mm^2

To find: T = Maximum torque that the welded joint can sustain.

Solution:

Let, We know that the maximum shear stress (τ_{max}),

$$80 = \frac{2.83\ T}{\pi s \times d^2} = \frac{2.83\ T}{\pi(10)\times(50)^2} = \frac{2.83\ T}{78550}$$

T = 80 × 78 550/2.83

= 2.22 × 106 N-mm **= 2.22 kN-m**

47. State the two types of eccentric welded connections.(Nov/Dec 2013)

1. Welded connection subjected to moment in a plane of the weld.
2. Welded connection subjected to moment in a plane normal to the plane ofthe weld.

48. Why are welded joints preferred over riveted joints? (AU. Apr 2009, 08, 03)(A/M'2023)

Material is saved in welding and hence the machine element will be light if welded joints are used instead of riveted joints. Leak proof joints can be easily obtained by welded joints compared riveted joints.

49. How butt joint is designed based on strength?

The butt joints are designed for tension or compression. Consider a single V-butt joint as shownin Fig.(a)

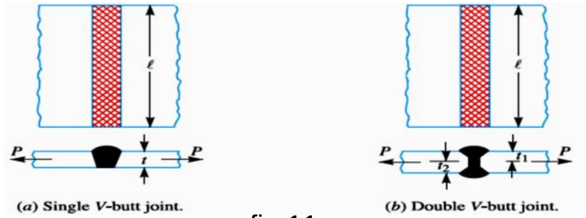


fig 11

In case of butt joint, the length of leg or size of weld is equal to the throat thickness s which isequal to thickness of plates.

Tensile strength of the butt joint (single-*V* or square butt joint),

$$P = t \times l \times \sigma_t$$

Where,

l= Length of weld. It is generally equal to the width of plate.

and tensile strength for double-*V* butt joint as in figure 11

$$P = (t1 + t2)\ l \times \sigma_t$$

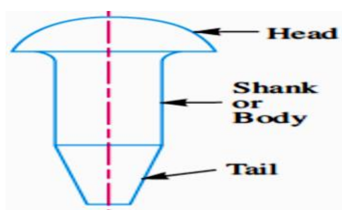
Where,

t_1 = Throat thickness at the top, andt_2 = Throat thickness at the bottom.

It may be noted that size of the weld should be greater than the thickness of the plate, but it maybe less.

50. What do you understand by the rivet joint?

The term riveted joint is understood by, a rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called ***shank*** or ***body*** and lower portion of shank is known as ***tail,*** as shown in Fig. The rivets are used to make permanent fastening between the plates such as in structural work, ship building, bridges, tanks and boiler shells. The riveted joints are widely used for joining light metals.



51. Discussabout the methods of riveting.

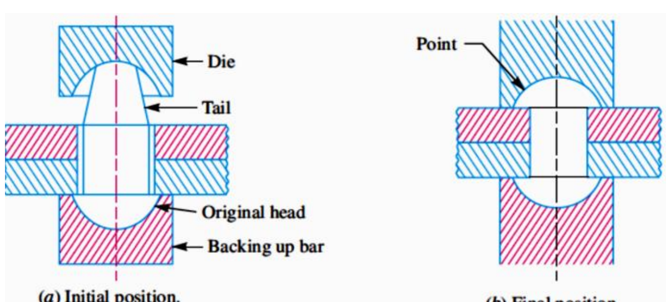


fig 12

The method of riveting is followed by

The function of rivets in a joint is to make a connection that has strength and tightness. Thestrength is necessary to prevent failure of the joint.

When two plates are to be fastened together by a rivet as in fig 12.a the holes in the plates are punched and reamed or drilled. Punching is the cheapest method and is used for relatively thin plates and in structural work. Since punching injures the material around the hole, therefore drilling is used in most pressure-vessel work. In structural and pressure vessel riveting, the diameter of the rivet hole is usually 1.5 mm larger than the nominal diameter of the rivet.

The plates are drilled together and then separated to remove any burrs or chips so as to have a tight flush joint between the plates. A cold rivet or a red hot rivet is introduced into the plates and the ***point*** (*i.e.* second head) is then formed. When a cold rivet is used, the process is known as ***coldriveting***and when a hot rivet is used, the process is known as ***hot riveting*.** The cold riveting process is used for structural joints while hot riveting is used to make leak proof joints.

The riveting may be done by hand or by a riveting machine. In hand riveting, the original rivet head is backed up by a hammer or heavy bar and then the die or set, as shown in

Fig.12 Is placed against the end to be headed and the blows are applied by a hammer. This causes the shank to expand thus filling the hole and the tail is converted into a ***point*** as shown in Fig. 12 (*b*)As the rivet cools, it tends to contract. The lateral contraction will be slight, but there will be a longitudinal tension introduced in the rivet which holds the plates firmly together.

52. List the different ways by which a riveted joint may fail May/June 2012

Or

What are the possible failure modes of riveted joint? Nov/Dec 2012

A riveted joint may fail in the following ways:

1. **Tearing of the plate at an edge:**

A joint may fail due to tearing of the plate at an edge as shown in Fig. 13

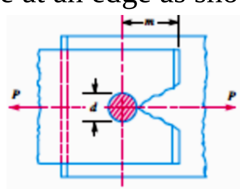
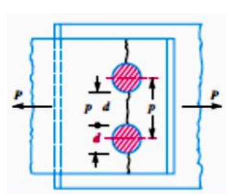


fig 13

2. **Tearing of the plate across a row of rivets:**

Due to the tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets as shown in Fig. 13



3. **Shearing of the rivets:**

The plates which are connected by the rivets exert tensile stress onthe rivets, and if the rivets are unable to resist the stress, they are sheared off as shown in Fig 14

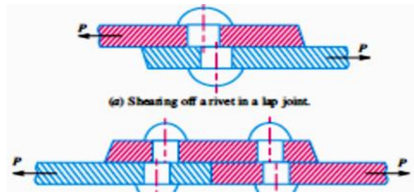


fig 14

Crushing of the plate or rivets:

Sometimes, the rivets do not actually shear off under the tensile stress, but are crushed as shown in Fig. 15

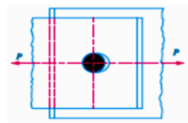


fig 15

53. What are the materials to be selected for rivets under fluid tight conditions?

The material of the rivets must be tough and ductile. They are usually made of steel (low carbonsteel or nickel steel), brass, aluminium or copper, but when strength and a fluid tight joint is the mainconsideration, then the steel rivets are used.

The rivets for general purposes shall be manufactured from steel conforming to the followingIndian Standards:

(*a*) IS: 1148–1982 (Reaffirmed 1992) – Specification for hot rolled rivet bars (up to 40 mmdiameter) for structural purposes; or

(*b*) IS: 1149–1982 (Reaffirmed 1992) – Specification for high tensile steel rivet bars forstructural purposes.

The rivets for boiler work shall be manufactured from material conforming to IS: 1990 – 1973(Reaffirmed 1992) – Specification for steel rivets and stay bars for boilers.

54. What are the types of riveted joints based on connected plates?

Following are the two types of riveted joints, depending upon the way in which the plates are connected.

i. Lap joint and
ii. Butt joint

i.Lap joint

A lap joint is that in which one plate overlaps the other and the two plates are then riveted together.

ii.Butt joint

A butt joint is that in which the main plates are kept inalignment butting (*i.e.* touching) each other and a cover plate(*i.e.* strap) is placed either on one side or on both sides of the main plates. The cover plate is then riveted together with themain plates. Butt joints are of the following two types:

a. Single strap butt joint, and
b. Double strap buttjoint.

a. Single strap butt joint, and

In a single strap butt joint, the edges of the main plates butt against each other and only one cover plate is placed on one side of the main plates and then riveted together.

b. Double strap butt joint.

In a double strap butt joint, the edges of the main plates butt against each other and two cover plates are placed on both sides of the main plates and then riveted together.

54.(a) Explain the method adopted for designing of economic riveted joints (Nov/Dec 2021)

- For the design of a lap joint or butt joint, the thickness of plates to be joined is known and the joints are designed for the full strength of the plate.
- For the design of a structural steel work, force (pull or push) to be transmitted by the joint is known and riveted joints can be designed.

DESIGN PROCEDURE FOR RIVETED JOINT

Step 1:

The size of the rivet is determined by the Unwin's formula

$$d = 6.04\sqrt{t}$$

Where d= nominal diameter of rivet in mm and t= thickness of plate in mm.

The diameter of the rivet computed is rounded off to available size of rivets. Rivets are manufactured in nominal diameters of 12, 14, 16, 18, 20, 22, 24, 27, 30, 33, 36, 39, 42 and 48 mm

Step 2:

The strength of rivets in shearing and bearing are computed. Working stresses in rivets and plates are adopted as per ISI. Rivet value R is found. For designing lap joint or butt joint tearing strength of plate is determined as follows

Pt=(p-D).t.pt

Where p=pitch of rivets adopted, t=thickness of plate and pt = working stress in direct tension for plate. Tearing strength of plate should not exceed the rivet value R (Ps or Pb whichever is less) or

$$(p - D).t.p_t \leq \text{R}$$

From this relation pitch of the rivets is determined.

Step 3:

In structural steel work, force to be transmitted by the riveted joint and the rivet value are known. Hence number of rivets required can be computed as follows

$$Number\ of\ rivets\ required\ in\ the\ joint = \frac{Force}{Rivet\ value}$$

The number of rivets thus obtained is provided on one side of the joint and an equal number of rivets is provided on the other side of joint also.

Step 4:

For the design of joint in a tie member consisting of a flat, width/thickness of the flat is known. The section is assumed to be reduced by rivet holes depending upon the arrangements of the rivets to be provided, strength of flat at the weakest section is equated to the pull transmitted by the joint. For example, assuming the section to be weakened by one rivet and also assuming that the thickness of the flat is known we have

$$(b - D).t.p_t = \text{P}$$

Where b= width of flat, t=thickness of flat, pt=working stress in tension in plate and P=pull to be transmitted by the joint. From this equation, width of the flat can be determined.

55. Classify the rivet heads according to IS specification Nov/Dec 2011

The rivet heads are classifiedIndian standard specifications into the following types:

b. Rivet heads for general purposes (below 12 mm diameter) as in figure 16 according toIS: 2155 – 1982 (Reaffirmed 1996).

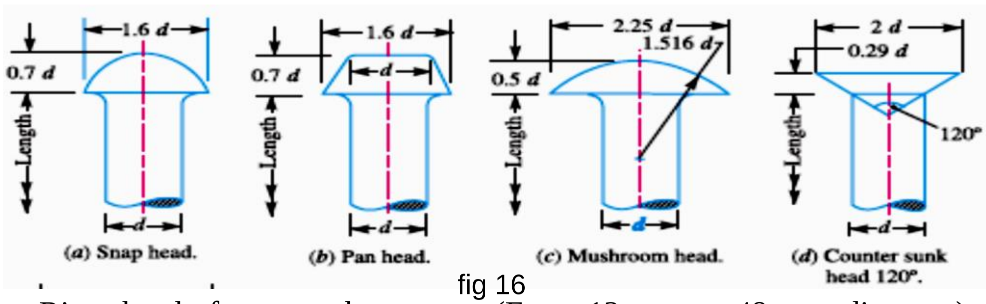


fig 16

c. Rivet heads for general purposes (From 12 mm to 48 mm diameter) as shown in fig 17 ,according to IS: 1929 – 1982 (Reaffirmed 1996).

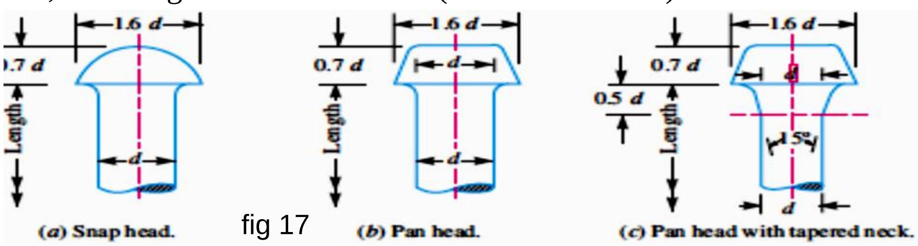


fig 17

d. Rivet heads for boiler work (from 12 mm to 48 mm diameter, as shown in Fig, 18 according toIS: 1928 – 1961 (Reaffirmed 1996).

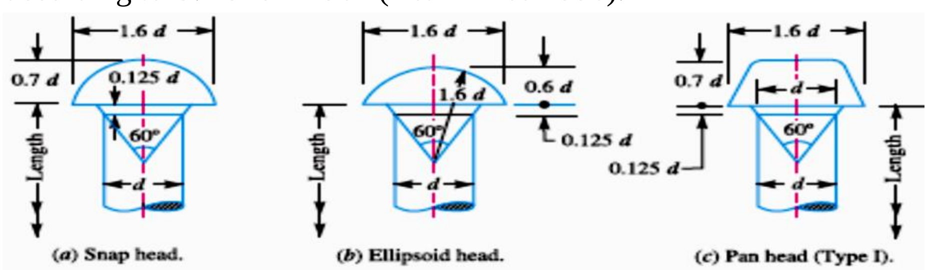


fig 18

56. What are the important terms used in riveted joints?

The following terms in connection with the riveted joints are important from the subject pointof view:

1. Pitch: It is the distance from the centre of one rivet to the centre of the next rivet measured parallel to the seam as shown in Fig. 19 It is usually denoted by p.

2. Back pitch: It is the perpendicular distance between the centre lines of the successive rows as shown in Fig. 19 It is usually denoted by p_b.

3. Diagonal pitch: It is the distance between the centres of the rivets in adjacent rows of zig-zagriveted joint as shown in Fig 19 It is usually denoted by p_d.

4. Margin or marginal pitch: It is the distance between the centre of rivet hole to the nearest edge of the plate as shown in fig 19 It is usually denoted by *m*.

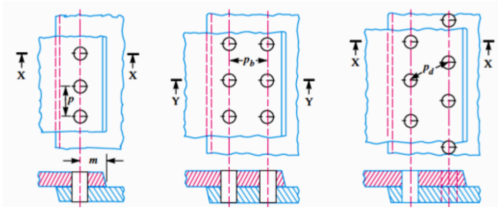
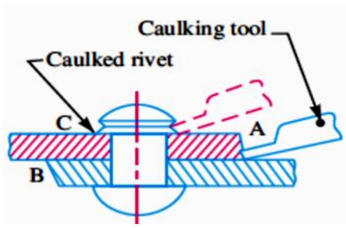


fig 19

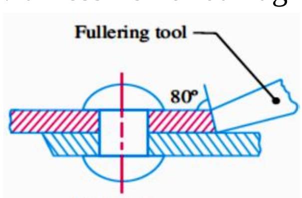
57. Which Process is employed to make the joints leak proofs or fluid tight in steam boiler.?

In order to make the joints leak proof or fluid tight in pressure vessels like steam boilers, air receivers and tanks etc. a process known as caulking is employed. In this process, a narrow blunt tool called caulking tool, about 5 mm thick and 38 mm in breadth, is used. The edge of the tool is ground to an angle of 80°. The tool is moved after each blow along the edge of the plate, which is planed to a bevel of 75° to 80° to facilitate the forcing down of edge. It is seenthat the tool burrs down the plate at *A*inFig. (*a*) Forming a metal to metal joint. In actual practice, both the edges at *A* and *B* are caulked. The head of the rivets as shown at *C* are also turned down with a caulking tool to make a joint steam tight. A great care is taken to prevent injury to the plate below the tool.



58. Define the term fullering with neat sketch.

The term fullering is defined as a more satisfactory way of making the joints staunch is known as fulleringwhich has largelysuperseded caulking. In this case, a fullering tool with a thickness at the end equal to that of the plateis used in such a way that the greatest pressure due to the blows occur near the joint, giving a cleanfinish, with less risk of damaging the plate.



(*b*) Fullering.

59. Write short notes on strength of ariveted joint

The strength of riveted joint is a joint which may be defined as the maximum force, which it can transmit, withoutcausing it to fail. That P_t, P_s and $P_c$are the pulls required to tear off the plate,shearing off the rivet and crushing off the rivet.

A little consideration will show that if we go onincreasing the pull on a riveted joint, it will fail when the least of these three pulls is reached, becausea higher value of the other pulls will never reach since the joint has failed, either by tearing off theplate, shearing off the rivet or crushing off the rivet.If the joint is continuous as in case of boilers, the strength is calculated per pitch length. But ifthe joint is small,the strength is calculated for the whole length of the plate.

60. What is meant by efficiency of riveted joint ?

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to thestrength of the un-riveted or solid plate.
We have already discussed that strength of the riveted joint

= Least of Pt, Ps and Pc

Strength of the un-riveted or solid plate per pitch length,

$$P = p \times t \times \sigma t$$

Efficiency of the riveted joint,

$$\eta = \frac{least\ of\ Pt, Ps\ and\ Pc}{P \times t \times \sigma}$$

Where p = Pitch of the rivets,
t = Thickness of the plate, and
σ_t= Permissible tensile stress of the plate material.

61. What is Caulking and Fullering process in riveted joints? Why is it used? (April/May 2018)

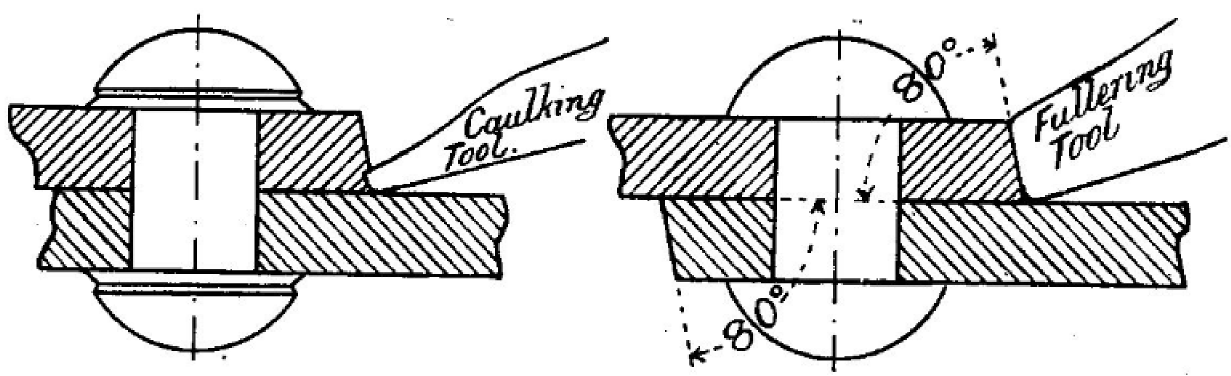
Caulking:

In order to make the joints leaks proof or fluid tight in pressure vessels like steam boiler, air receives and tanks etc. A process known as caulking is employed. In this process narrow blunt tool called caulking tool, about 5 mm thick and 38 mm in breadth is used.

Fullering:

A more satisfactory way of making the joints staunch is known as fullering which has largely superseded caulking. In this process a fullering tool with a thickness at the end equal to the plate is used in such a way that the greater pressure due to the blow occur near the joint, giving a clean finish, with less risk of damaging the plate.

To provide leak proof and fluid tight joints



62. What are the assumptions are made while designing a joint for boilers?

The following assumptions are made while designing a joint for boilers:

1. The load on the joint is equally shared by all the rivets. The assumption implies that theshell and plate are rigid and that all the deformation of the joint takes place in the rivetsthemselves.

2. The tensile stress is equally distributed over the section of metal between the rivets.

3. The shearing stress in all the rivets is uniform.
4. The crushing stress is uniform.
5. There is no bending stress in the rivets.
6. The holes into which the rivets are driven do not weaken the member.
7. The rivet fills the hole after it is driven.
8. The friction between the surfaces of the plate is neglected.

63. Differentiate with neat sketch the fillet welds subjected to parallel loading and transverse loading May/June 2014

Fillet welds subjected to parallel loading:

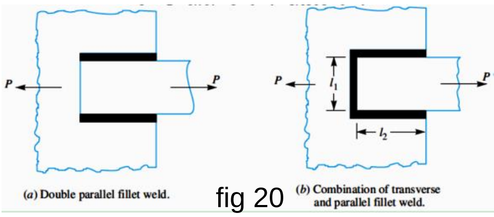


fig 20

The parallel fillet welded joints are designed for shear strength. Consider a double parallel filletwelded joint as shown in Fig.thattheminimum area of weld or the throat area.If there is a combination of single transverse and double parallel fillet welds as in figure 20.b ,then the strength of the joint is given by the sum of strengths of single transverse and double parallel fillet welds.

Mathematically,$P= 0.707s \times l1 \times \sigma t + 1.414\ s \times l2 \times \tau$

Where, $l1$ is normally the width of the plate.

In order to allow for starting and stopping of thebead, 12.5 mm should be added to the length of each weld obtained by the above expression.

Fillet welds subjected to transverse loading:

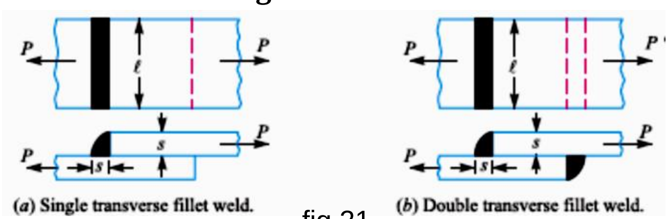


fig 21

The fillet or lap joint is obtained by overlapping the platesandthen welding the edges of the plates. The transverse fillet welds are designed for tensile strength. Letus consider a single and double transverse fillet welds as in figure 21 (*a*) and (*b*) respectively.

64.What are the reasons for replacing riveted joint by a welded joint in modern equipment? Nov/Dec 2010

The ultimate replacement of riveted by welded sections in large buildings is inevitable, because of the tremendous saving which will result from the fact that there is no loss of strength at the joints welded, while the loss of strength on riveted sections runs from 30 to 50 per cent.

Where welding is applied to the supporting members of a structure, this, of course, means that the necessity of having great, heavy steel pieces in the lower stories of a building in order to carry the load higher up, as required on riveted jobs, is eliminated.

65.Write down the design procedure of longitudinal Butt joint for a boiler.

The following design procedures are longitudinal butt joint for a boiler according to Indian Boiler Regulations (I.B.R),

a. Thickness of boiler shell.
b. Diameter of rivets.
c. Pitch of rivets.
d. Distance between the rows of rivets.
e. Thickness of butt strap.
f. Margin.

66. Under what circumstances riveted joints are preferred over welded joints? (Apri/May 2019)

- It is used where we have to avoid after thermal effects, as in case of welding
- Used for metals which have poor weldability
- Used for heterogeneous materials like asbestos friction lining and steel
- As welded joints have poor vibration damping capabilities so where required rivets are used in place of it
- Used for aircraft structure where Aluminium is used

THEORY OF BONDED JOINTS

66.List out the types of adhesive?(or) Classiy adhesives used in adhesive joint.Nov/Dec 2021)

Adhesives can be classified as follows.

1. According to their function.

a. Structural adhesives
b. Holding adhesives
c. Sealing adhesives

2. According to chemical structure

a. Natural adhesives
b. Inorganic adhesives
c. Synthetic organic adhesives

3. According to the parts being joined

a. Metal – metal adhesives
b. Metal – plastic adhesives
c. Plastic – glass adhesives

67. State the selection of an adhesion for various parameters.

The selection of an adhesive is to be made taking into account various parameters. Some of them are as follows:

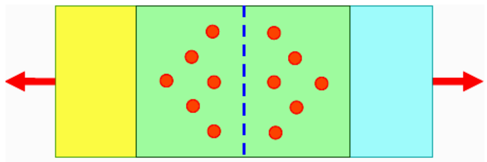
One – part adhesives are better than two parts. Two parts may result in improper metering and mixing due to human ignorance or negligence.

Tape and film adhesives are better than liquid and paste systems because of ease of handling. Also shrinkage problems are eliminated, resulting in strong bond.

Better to go for adhesives which are less critical about the cleanliness of the surfaces being joined

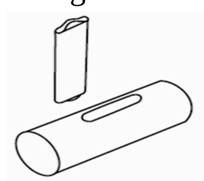
68. Diamond riveting used for structural joints, exhibits uniform strength at all sections. Justify.

Diamond riveting used for structural joints,the procedure by which uniform strength in a riveted joint is obtained is known as diamond riveting, whereby the number of rivets is increasedprogressively from the outermost row to the innermost row. A common joint, where this type of riveting is done, is Lozenge joint used for roof, bridge work etc.



69. Why bearing edges of the cotter and bearing slots in the rods of a cotter joint are made semi-circular?

A cotter is a flat wedge shaped piece, made of steel. It is uniform in thickness but tapering inwidth, generally on one side; the usual taper being 1:30. The lateral (bearing) edges of the cotter and the bearing slots are generally made semi-circular instead of straight. This increases the bearing area, and permits drilling while making the slots.


Cotter and the bearing slot

70. Narrate the advantages and disadvantages of bonded joints.

The following are shows the advantages and disadvantages of bonded joints:

Advantages :

1) The bond prevents electrochemical corrosion between dissimilar metals

2) Vibration and noise are reduced because of internal damping provided by the adhesive.

3) Thin and fragile components can be joined without increasing the weight.

4) Stress concentration is minimized because the entire bond area is utilized.

5) Easy assembling of the parts.

Disadvantages:

1) Not suitable for high temperature services.

2) Bonding and curing is a lengthy process

3) Surface preparation is essential.

4) Some adhesives may be toxic or inflammable. Thus ventilation and fire extinguishingsystems may be necessary

5) Limited reliability.

71. What are the different applications of screwed fasteners? (Nov/Dec 16)

1. for readily connecting & disconnecting machine parts without damage
2. The parts can be rigidly connected
3. Used for transmitting power

72. State the two types of eccentric welded connections. (Nov/Dec 16)

1. Welded connections subjected to moment in a plane of the weld.
2. Welded connections subjected to moment in a plane normal to the plane of the Weld

73. list the advantages of cotter joint over threaded joints (April/May 17)

- It is simple to design and manufacture,
- It is easy to assemble and disassemble.
- The wedge of the cotter produce high tightening force which prevents loosening of the parts.

74. Why throat is considered while calculating stress in fillet welds? (april/May 17)

The weld thickness is very small compared to the diameter of the shaft, maximum shear stress occurs in the throat area.

75. What is known as bolt of uniform strength?(Nov/Dec-2018)

The **bolt**, in this way, becomes stronger and lighter and it increases the shock absorbing capacity of the**bolt** because of an increased modulus of resilience. This gives us **bolts of uniform strength**

76.What is meant by set screw ? Nov/Dec-20.Aprl/May-21

A set screw is a type of screw generally used to secure an object within or against another object. The most common examples are securing a pulley or gear to a shaft. Set screws are usually headless, meaning that the screw is fully threaded and has no head projecting past the major diameter of the screw thread. A blind set screw is almost always driven with an internal-wrenching drive, such as a hex socket, star, square socket, or slot.

77. Differentiate between butt and fillet welded joints. Nov/Dec-20.Aprl/May-21

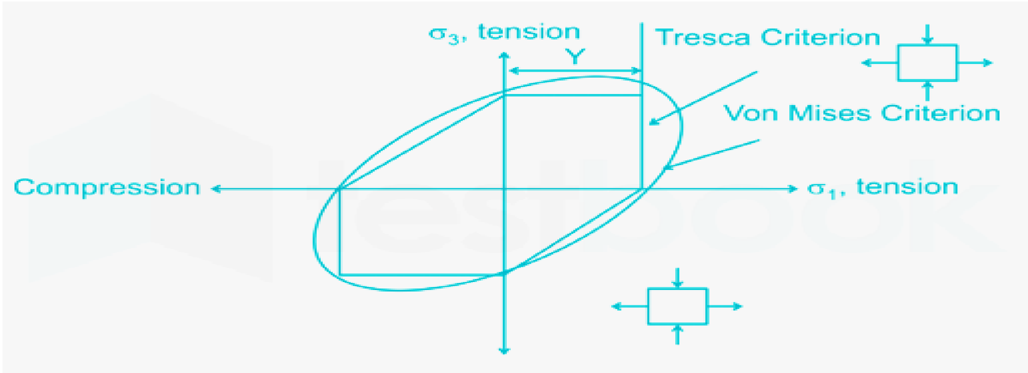
The main difference between a butt and a fillet weld is the angle between the joining workpieces. If the surfaces to be joined are on the same plane, then it is a butt weld. If the surfaces are perpendicular (with an angle of 90°), then they are usually joined with a fillet weld.

78.Define the term stress concentration also state whether stress concentration a material property or not(N/D'2022)

A stress concentration, also known as a stress riser/raiser, is a point in a part where the stress is significantly greater than its surrounding area. Stress concentrations occur as a result of irregularities in the geometry or within the material of a component structure that cause an interruption of the stress flow

It is not a propert of a material

79.What will be the shape of yield locus for two dimensional and three dimensional state of stress of?(N/D'2022)



1.Tresca(maximum shear stress theory)
2.von mises theory

80. How is bolt designated?mention with example?(A/M'2023)

Metric nuts and bolts are commonly referenced using 'M' sizes, for example: M3, M8, M12. But the size of a metric fastener is more accurately specified using diameter, pitch and length dimensions, in millimeters. For nuts the size dimensions used are simply diameter and pitch

PART – B (16 MARKS)

Threaded fasteners - Bolted joints including eccentric loading:

1. **A mild steel cover plate is to be designed for an inspection hole in the shell of a pressure vessel. The hole is 120 mm in diameter and the pressure inside the vessel is 6 N/mm². Design the cover plate along with the bolts. Assume allowable tensile stress for mild steel as 60 MPa and for bolt material as 40 MPa. (16)**

Given:

$D = 120$ mm;
$p = 6$ N/mm^2;
$\sigma_t = 60$ MPa $= 60$N/mm^2;
$\sigma_{tb} = 40$ MPa $= 40$ N/mm^2

Solution:

Let us find the thickness of the pressure vessel. According to Lame's equation, thickness of the pressure vessel,

$$t = r\left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1\right] = 6\text{mm}$$

Let us adopt t = 10 mm

Design of bolts

Let d = nominal diameter of studs
d_c = core diameter of studs
n = number of studs

W.K.T upward force acting on the cylinder cover

$$P = \frac{\pi}{4}(D)^2\, p = 67860 \text{ N} \qquad \text{(i)}$$

Let the nominal diameter of the bolt is 24 mm. we find that the corresponding core diameter (d_c) of the bolt is 20.32 mm.

Resisting force offered by n number of bolts,

$$P = \frac{\pi}{4}(d_c)^2\ \sigma_{tb} \text{ x } n = 12973 \text{ n N} \qquad \text{(ii)}$$

From equation (i) and (ii), we get

$$n = 67860 / 12973 = 5.23$$

Taking the diameter of the bolt hole (d_1) as 25 mm, we have pitch circle diameter of bolts,

$$D_p = D + 2t + 3d_1 = 215 \text{ mm.}$$

Circumferential pitch of the bolts

$$= \frac{\pi}{n}(D_p)^2 = 112.6 \text{ mm}$$

W.K.T for a leak proof joint, the circumferential pitch of the bolts should lie between

$20\sqrt{d_1}$ to $30\sqrt{d_1}$, where d1 is the diameter of the bolt hole in mm.

minimum circumferential pitch of the bolts

$$= 20\sqrt{d_1} = 100 \text{ mm}$$

Maximum circumferential pitch of the bolts

$$= 30\sqrt{d_1} = 150 mm$$

since the circumferential pitch of the bolts obtained is within 100 mm and 150 mm, therefore size of the bolt chosen is satisfactory.

size of the bolt = M 24

Design of cover plate

Let t_1 = thickness of the cover plate

W.K.T the bending moment at A-A

$$M = 0.053 \text{ P x } D_p = 773265 \text{ N-mm}$$

Outside diameter of the cover plate,

$$D_0 = D_p + 3d_1 = 290 \text{ mm}$$

Width of the plate

$$W = D_0 - 2d_1 = 240 \text{ mm}$$

Section modulus

$$Z = 1/6 \text{ x } w (t_1)^2 = 40 (t^1)^2 \text{ mm}^3$$

W.K.T bending (tensile) stress,

$$\sigma_t = M/Z$$

$$(t_1)^2 = 322;$$

$$\mathbf{t_1 = 18 \text{ mm}}$$

2. **The cylinder head of a steam engine is subjected to a steam pressure of 0.7 N/mm^2. It is held in position by means of 12 bolts. A soft copper gasket is used to make the joint leak-proof. The effective diameter of cylinder is 300 mm. Find the size of the bolts so that the stress in the bolts is not to exceed 100 MPa.(8)**

Given:

p = 0.7 N/mm2;

n = 12; D = 300 mm;

σ_t = 100MPa = 100 N/mm2

Solution:

W.K.T the total force acting on the cylinder head (on 12 bolts),

$$= \frac{\pi}{4}(D)^2 \, p = 49490 \text{ N}$$

External load on the cylinder head per bolt,

P_2 = 4124 N

d = nominal diameter of the bolt,

d_c = core diameter of the bolt.

W.K.T initial tension due to tightening of bolt,

P_1 = 2840 d N

From the table, we find that for soft copper gasket with long through bolts, the minimum value of K = 0.5.

Resultant axial load on the bolt,

P = P1 + K. P2 = (2840 d + 2062) N

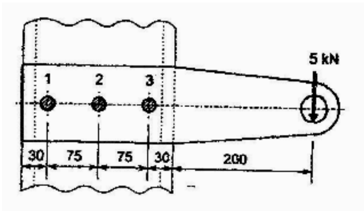
W.K.T load on bolt (P),

$$2840 \text{ d} + 2062 = \frac{\pi}{4}(d_c)^2 \ \sigma_t \ = 55.4 \text{ d}^2$$

D = 52 mm

Thus we shall use a bolt of size M 52.

3.A steel plate subjected to force of 5 kN and fixed to a channel by means of three identical bolts is shown in fig. The bolts are made from plain carbon steel 45 C8 and factor of safety is 3. Specify the size of bolts. (8) (Nov/ Dec – 2010) .(April/May 17) (Nov/Dec 2018)



Given Data:

P= 5 kN = 5000 N

FOS = 3

N= 180 r.p.m

e= 200 mm

r_1 = 75 mm

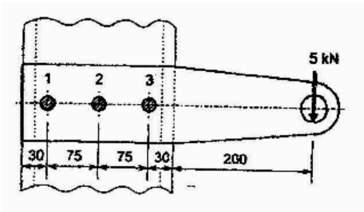
r_2 = 75 mm

To Find :

Bolt size

Solution:

CG of the bolt group is at (105,0) with the left end as reference i.e the CG is same the location of bolt 2.



So for bolt 1 distance from CG

$$x = 75\ , y = 0$$

$$r_1 = \sqrt{x^2 + y^2} = 75$$

For bolt 2, x=y=0, r_2=0

For bolt 3, x=75, y = 0

$$r_3 = \sqrt{x^2 + y^2} = 75$$

$$F_1 = \frac{P}{FOS} = \frac{5000}{3} = 16666.67N$$

$$F_{21} = \frac{P.e.r_1}{r_1^2 + r_2^2 + r_3^2} = \frac{5000 \times (75 + 30 + 200) \times 75}{75^2 + 0 + 75^2} = 10166.67N$$

$$F_{23} = \frac{P.e.r_3}{r_1^2 + r_2^2 + r_3^2} = F_{21} = 10166.67N$$

That is F_2 = 10166.67 N

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{1667.67^2 + 10166.67^2 + 2 \times 1667.67 \times 10166.67 \times \cos\theta}$$

$$F_R = 11833.34\ N$$

We know that

$$\frac{F_R}{A_c} = \frac{\sigma_y}{FOS}$$

Assuming σy = 300 N/mm^2 and FOS, n = 3

$$\frac{11833.34}{A_c} = \frac{300}{3}$$

$$A_c = 118.33\ mm^2$$

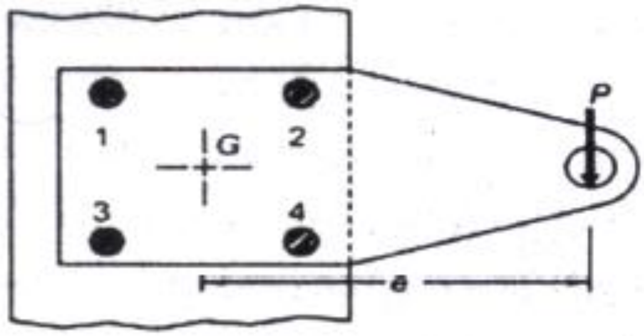
From PSGDB 5.42 for 118.33 mm^2, We choose

M16 x 2

Result:

Bolt chosen is M16 x 2

4.The structural connection shown in figure is subjected to an eccentric force P of 10 kN with an eccentricity of 500 mm from the CG of the bolts. The centre distance between bolts 1 and 2 is 200 mm, and the centre distance between bolts 1 and 3 is 150 mm. All the bolts are identical. The bolts are made from plain carbon steel 30C8 (Syt = 400 N/mm2) and the factor of safety is 2.5. Determine the size of the bolts. (April/May 2018)



Solution ***Given*** P = 10 kN $S_{yt} = 400\ N/mm^2$ (fs) = 2.5 e = 500 mm

Step I: Permissible shear stress

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} = \frac{0.5(400)}{2.5} = 80\ N/mm^2$$

Step II: Primary and secondary shear forces

By symmetry, the centre of gravity G is located at a distance of 100 mm to the right of bolts 1 and 3 and 75 mm below bolts 1 and 2. Thus,

$$r_1 = r_2 = r_3 = r_4 = r$$

and

$$r = \sqrt{(100)^2 + (75)^2} = 125\ mm$$

The primary and secondary shear forces are Shown in Fig.

$$P_1' = P_2' = P_3' = P_4' = \frac{10\,000}{4} = 2\,500\ \mathbf{N}$$

$$P_1'' = \frac{(Pe)r_1}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)} = \frac{(Pe)r}{4r^2}$$

$$= \frac{Pe}{4r} = \frac{(10\,000)(500)}{4(125)} = 10\,000\ \mathbf{N}$$

Similarly it can be proved that

$$P_1'' = P_2'' = P_3'' = P_4'' = 10\ 000\ N$$

Step III: Resultant shear force

Referring to Fig. 3.1

$$\tan\theta = \frac{75}{100} = 0.75 \quad \text{or} \quad \theta = 36.87^\circ$$

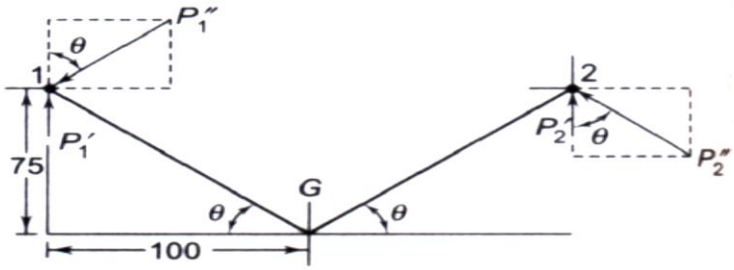


fig 3.1 Vector Addition of Shear Forces

The vertical component of P_1'' is ($P_1''\cos\Theta$), which acts in downward direction, while the primary force P_1' acts in upward direction at bolt 1. Therefore, thenet downward component is ($P_1''\cos\Theta - P'_1$) The horizontal component is ($P_1''\sin\Theta$). The resultant force acting on bolt 1 is given by,

$$P_1 = \sqrt{(P_1''\cos\theta - P_1')^2 + (P_1''\sin\theta)^2}$$

$$= \sqrt{[10000\cos(36.87) - 2500]^2 + [10000\sin(36.87)]^2}$$

$$= 8139.41 \text{ N}$$

Similarly the resultant force acting on bolt 2 **is given** by,

$$P_2 = \sqrt{(P_2''\cos\theta + P_2')^2 + (P_2''\sin\theta)^2}$$

$$= \sqrt{[10000\cos(36.87) + 2500]^2 + [10000\sin(36.87)]^2}$$

$$= 12093.38 \text{ N}$$

Size of bolts

$$\tau = \frac{P_2}{A} \qquad 80 = \frac{12093.38}{\frac{\pi}{4}d_c^2}$$

$$d_c = 13.87 \text{ mm}$$

$$d = \frac{d_c}{0.8} = \frac{13.87}{0.8} = 17.34 \text{ or } 18 \text{ mm}$$

the standard size of bolts is M 20.

5.A steel bolt of M16x2 is 300 mm long carries an impact load of 5000 Nm. If the threads stop adjaced to the Nut and E=2.1 x 10^6 MPa

1. **Find the stress in the root area**
2. **Find the stress if the shank area is reduced to root area. (8) (Nov/Dec – 2014)**

Given Data:

d=16 mm

l=300 mm

U = 5000 N-mm

$E = 2x10^5 N/mm^2$

To find:

stresses

Solution:

$$\text{Shank area } A_b = \frac{\pi}{4} \times 16^2 = 201.06 \text{ mm}^2$$

a. Bolt stiffness

$$q_b = \frac{A_b E}{l} = \frac{201.06 \times 2.1 \times 10^5}{300} = 140742 \text{ N/mm}$$

Load acting on the bolt is found from the energy applied on to it

Energy $U = \frac{P^2}{2q_b}$

$$P = \sqrt{2Uq_b} = \sqrt{2 \times 5000 \times 140742} = 37515.6 \text{ N}$$

From PSGDB 5.42 for M16, stress area $A_c = 157 \text{ mm}^2$

b. When shank area = root area

For this case,

$$A_c = 157 \text{ mm}^2$$

Bolt stiffness

$$k_b = \frac{AE}{l} = \frac{157 \times 2.1 \times 10^5}{300} = 109933 \text{ N/mm}$$

Stress

$$\sigma = \frac{P}{A_c} = \frac{33151.168}{157} = 211.153 \text{ MPa}$$

Inference

If the shank diameter is reduced to root diameter, the stress developed is reduced.

6. A cast iron cylinder head is fastened to cylinder of 500 mm bore with 8 stud bolts. The maximum pressure inside the cylinder is 2 MPa. The stiffness of part is thrice the stiffness of the bolt. What should be the initial tightening load so that the point is leak proof at maximum pressure? Also choose a suitable bolt for the above application. (08)(May/June – 2014)

Given:

Diameter, D = 500 mm

Number of stud bolts, n = 8

Pressure, p = 2 MPa = 2 N/mm^2

$$q_p = 2K_b \text{or} \frac{q_b}{q_p} = \frac{1}{3}$$

To Find:

Maximum pressure, P1

Solution:

Total external load,

$$P_t = p \times \frac{\pi}{4} \times D^2 = 2 \times \frac{\pi}{4} \times 500^2 = 392699.08 \text{ N}$$

This load is shared by 8 bolts

$$\frac{\text{Load}}{\text{Bolt}} = \frac{P}{P_{max}} = \frac{392699.08}{8} = 49087.38 \text{ N}$$

This equation is

$$P_{max} = P_i\left(\frac{q_b + q_c}{q_p}\right) = P_i\left(\frac{q_b}{q_p} + 1\right) = P_i\left(\frac{1}{3} + 1\right)$$

$$49087.38 = P_i\left(\frac{1}{3} + 1\right)$$

$$P_i = 36824.45 \text{ N}$$

To ensure safety, let us have a higher value

$$\text{Say } P_i = 37000 \text{ N}$$

Total load on the bolt,

$$P_b = P_i + P\left(\frac{q_b}{q_b + q_p}\right) = P_i + P\left(\frac{1}{1 + (q_b/q_{p)}}\right)$$

$$= 37000 + 49087\left(\frac{1}{1+3}\right) = 49271.75 \text{ N}$$

Assuming σy=300 N/mm^2

$$\text{Using } A_C = \left(\frac{60 \times P_b}{\sigma_y}\right)^{2/3} = \left(\frac{60 \times 49271.75}{300}\right)^{2/3} = 459.64 \text{ mm}^2$$

From PSGDB 5.42 for 561 mm^2 that is nearest to the A_c value of 459.64 the bolt chosen is M30

Result:

Bolt chosen is M30

7. A bolt M20x2.5 ISO metric thread is subjected to a fluctuating load of 0 to 12000N. Endurance strength = 210MPa. Bolt and the past are of same material and length. Yield stress = 490MPa stress concentration factor 3.85, component area 362mm^2. Calculate

i) Factor of safety without preload

ii) Minimum initial load to prevent joint opening.

iii) Factor of safety with 10KN preload. Comment on it.

iv)Minimum force in the part for a given loading and a preloading of 10KN.

(Apr/May – 2006)

Given Data:

M20 (i.e.) d = 20mm

σ_{-1} = 210MPa

σ_y = 490MPa

K_f = 3.85

A_p = 362mm^2

P_{min} = 0

P_{max} = 12000N

To find:

i) FOS without preload

ii) F_i (minimum)

iii) FOS with 10KN preload

iv) Minimum 10KN preload force

Solution:

i) FOS without preload [P_i =0]

Mean load, $P_m = \dfrac{P_{\min} + P_{\max}}{2} = \dfrac{1200}{2}$ = 6000N

Unable load, $P_v = \dfrac{P_{\min} - P_{\max}}{2} = \dfrac{1200}{2}$ = 6000N

If P_i =0, P_m=P_v= 6000N

From PSGDB page 5.42, for M20,

Stress area A_c =245mm^2

$$\sigma_m = \sigma_v = P_m \text{ (or) } P_v = \frac{P_m (or) P_v}{2} = \frac{6000}{245} = 24.49 \text{ N/mm}^2$$

Using Soderberg equation,

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + \frac{K_t \sigma_v}{\sigma_{-1}} = \frac{24.49}{490} + \frac{(3.85 \times 24.49)}{210}$$

n=2 (i.e.) Factor of safety = 2

(ii) F_i (minimum): Limiting condition for joint opening is,

$$P_{\max} = P_i \left(\frac{q_b + q_p}{q_p} \right) = P_i \left(\frac{q_b}{q_p} + 1 \right)$$

$$P_i = \frac{P_{\max}}{\left[\dfrac{q_b}{q_p} + 1 \right]}$$

$$q_b = \frac{A_b . E_b}{lb}; \; q_p = \frac{A_p . E_p}{lp}$$

$$\frac{q_b}{q_p} = \frac{A_b . E_b}{lb} \times \frac{lp}{A_p . E_p}$$

But, E_b = E_p and l_b=l_p (Given)

$$\frac{q_b}{q_p} = \frac{A_b}{Ap} \qquad \text{……….(a)}$$

$$A_b = \frac{\pi}{4} d^2 = \frac{\pi}{4} 20^2 = 314 \text{mm}^2$$

$$A_p = 362 \text{ mm}^2$$

$$\frac{q_b}{q_p} = \frac{A_b}{Ap} = \frac{314}{362} = 0.8674$$

P_{max} = 12000N (Given)

$$P_i = \frac{P_{\max}}{\left[\dfrac{q_b}{q_p} + 1 \right]} = \frac{12000}{0.8674 + 1} = 6426.05\text{N}$$

Minimum preload to avoid joint opening **= 6426.05N**

(iii)FOS when P_i = 10000N:

When P_i = 10000N, Minimum load bolt P_{min}= P_i=10000N

In addition to P_i external load P also acts

Maximum external load, P = 12000N

$$\text{Total load on bolt, } P_b = P_i + P\frac{q_p}{q_b + q_p} = P\frac{q_b / q_p}{1 + q_b / q_p}$$

$$\text{From equation (a), } \frac{q_b}{q_p} = 0.8674$$

$$P_b = P_i + P\left(\frac{0.8674}{1+0.8674}\right)$$

$$= 10000+12000\left(\frac{0.8674}{1.8674}\right)$$

$$P_b = 15574\text{N}$$

So, for this case, load varies from 10000N to 15574N

(i.e.) P_{min} = 10000N

P_{max} = 15574N

$$\text{Mean load, } P_m = \frac{P_{\min} + P_{\max}}{2}$$

$$= \frac{10000+15574}{2} = 12787\text{N}$$

$$\text{Variable load, } P_v = \frac{P_{\min} - P_{\max}}{2}$$

$$= \frac{10000-15574}{2} = 2787\text{N}$$

$$\text{Mean stress, } \sigma_m = \frac{P_m}{A_c}$$

$$= \frac{12787}{245} = 52.19\text{N/mm}^2$$

$$\text{Variable stress, } \sigma_v = \frac{P_v}{A_c}$$

$$= \frac{2787}{245} = 11.37\text{N/mm}^2$$

From Soderberg equation,

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + \frac{K_t \sigma_v}{\sigma_{-1}}$$

$$= \frac{52.19}{490} + \frac{(3.85\times 11.37)}{210} = 3.378$$

(i.e.) Factor of safety is 3.378

Comment:

Application of preload improves the factor of safety [comparing cases (i) and (iii)]

(iv) Minimum 10KN preload force

Minimum force on the part,

$P_p = P_b - P = 15574 – 12000 =$ **3574N**

Results:

FOS = 2, P_i = 6426.05N ,FOS = 3.378, P_P = 3574N

8. A bolted assembly is subjected to an external force, which varies from 0 to 15KN. The combined stiffness of the parts, held together by the bolt is three times the bolt stiffness. The bolt is initially so tightened that at 50% over load condition, the partsheld together by the bolt are adjusted by the bolt are just about to separate. The bolt material has yield strength of 450N/mm² and the ultimate strength of 650N/mm². Fatigue stress concentration factor is 2.5 and reliability expected is 95%. Assuming a FOS of 2, determine the bolt size.

Given data:

$P_{min} = 0$

P_{max} = 15000N

$K_f = 2.5$

Reliability = 0.95

q_p =3qb

σ_y = 450 N/mm²

σ_u = 650N/mm²

Factor of safety, n = 2

To find:

The Bolt size.

Solution:

For repeated loading, assuming σ^1_{-1} = 0.5 σ_u = 0.5 (650) = 325

For 95% reliability, factor K_c =0.868

Factor considering K_f, K_d = $\frac{1}{K_f} = \frac{1}{2.5}$ 0.4

Endurance strength, $\sigma^1_{-1} = K_c . K_d . \sigma^1_{-1}$ (assuming other factor is unity)

= 0.868 x 0.4 x 325 = 112.84N/mm²

At 50% over load the joint is to separate.

So, $P_{max} = P_i \left(\frac{q_b + q_p}{q_p} \right)$

But at 50% over load, $P_{max}{}^1$ = 1.5 x 15000 = 22500N

$P_{\max}{}^{1}$ = 22500 = $P_i\left(\frac{q_b+3q_b}{3q_b}\right)$

Pi = 16875N

Maximum load on bolt, $(P_b)_{\max}{}^{1}$ = Pi + Pmax $\left(\frac{q_b}{q_b+q_p}\right)$

= Pi + Pmax $\left(\frac{q_b}{q_b+3q_p}\right)$

= Pi + Pmax $\left(\frac{1}{4}\right)$

= 16875 + 15000 $\left(\frac{1}{4}\right)$

$$\frac{1875}{18750}=0.1\frac{P_{b(\max)}-P_{b(\min)}}{2}\frac{\sigma_{bm}}{\sigma_u}+\frac{\sigma_{ba}}{\sigma_{-i}}=1\frac{\sigma_{bm}}{650}+\frac{0.1\sigma_{bm}}{112.84}=1$$

$$\tan\theta=\frac{\sigma_{bm}}{\sigma_{ba}}=\frac{P_a/A_c}{P_m/A_c}=\frac{P_a}{P_m}\Rightarrow\sigma_a=\frac{\sigma_{ba}}{n}$$

$$\frac{1875}{A_c}=\frac{41.243}{n}$$

Minimum load on bolt,

$$P_{b(\min)}=P_i+P_{\min}\left(\frac{1}{4}\right)=16875+0$$

$$P_{b(\min)}=16875N$$

Mean load on bolt, Pm = $\frac{P_{b(\max)}+P_{b(\min)}}{2}$

Pm = $\frac{20625+16875}{2}$

Pm = 18750 N

Variable load on bolt,

Pa = $\frac{P_{b(\max)}-P_{b(\min)}}{2}$

= $\frac{20625-16875}{2}$

Pa = 1875 N

Let

Mean stress on bolt = σ_{bm}

Variable stress on bolt = σ_{ba}

Using Goodman equation,

$$\frac{\sigma_{bm}}{\sigma_u} + \frac{\sigma_{ba}}{\sigma_{-i}} = 1$$

$$\frac{\sigma_{bm}}{650} + \frac{\sigma_{ba}}{112.84} = 1 \qquad \text{........ (1)}$$

Slope of Goodman line,

$$\tan\theta = \frac{\sigma_{bm}}{\sigma_{ba}} = \frac{P_a / A_c}{P_m / A_c} = \frac{P_a}{P_m}$$

$$= \frac{1875}{18750} = 0.1$$

(i.e.) $\frac{\sigma_{bm}}{\sigma_u} = 0.1$ ...... (2)

(i.e.) $\sigma_{ba} = 0.1\,\sigma_{bm}$

Sub.in (1) and solving

$$\frac{\sigma_{bm}}{650} + \frac{0.1\sigma_{bm}}{112.84} = 1$$

$$\Rightarrow (0.00242)\sigma_{bm} = 1$$

$$\Rightarrow \sigma_{bm} = 412.43 N / mm^2$$

$$\sigma_{ba} = 0.1\,\sigma_{bm} = 41.243 \; N / mm^2$$

$$\sigma_a = \frac{\sigma_{ba}}{n}$$

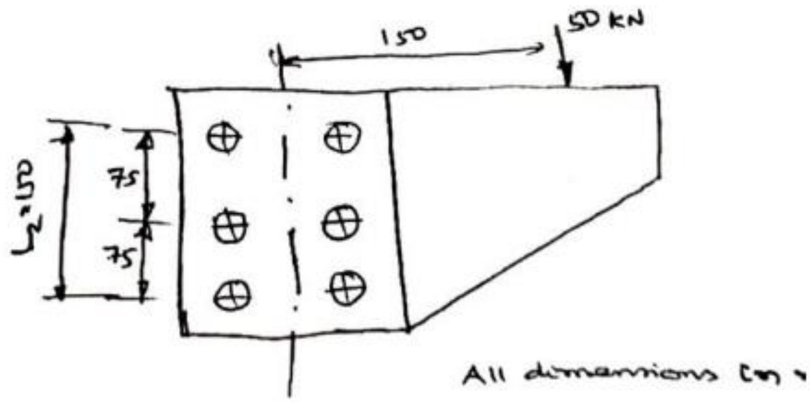
$$\frac{P_a}{A_c} = \frac{\sigma_{ba}}{n}$$

$$\frac{1875}{A_c} = \frac{41.243}{n}$$

A_c = 90.92mm^2, From PSGDB table, we choose **M16 bolt**

Result: The bolt chosen is M16

8.(a) A Bracket is bolted to a column by six bolts as shown fig,.It carries a load of 50 KN at a distance of 150 mm,from the center of coloumn.if the maximum stress in the bolts is to be limited to 150 MPa. Determine the diameter of the bolt.(NoV/Dec 2021)



Given Data:-

load (P) = 50 KN

distance of load,from the center of coloumn. e = 150 mm

maximum stress in the bolts = 150 MPa

to Find :-

The diameter of the bolt

Solution:-

When the load applied eccentrically both primary and secondary effects will be present

Primary effect:-

Po=P/ N

P=50/6 = 8.33kN (in the direction of applied load.

Secondary effect:-

Ps=K x Lj

Where, Li = distance of i th rivet rom C.G

$$K=\frac{P\times e}{I^2 Li}$$

Where, e=150 mm (distance from CG to load)

K =(50 × 150)/(2 (75) ^{2} 4(75√2)^2

K=0.133

Ps_{max} = 0.133 x 75 $\sqrt{2}$

= 14.14 KN

Ps – will be perpendicular to Li

Maximum force will be on rivet 2 & 6,

Resultant P=$\sqrt{Po^2 + Ps^2 + 2PoPs\cos\theta}$

$$= \sqrt{(8.33)^2 + (14.14)^2 + 2(8.33)(14.14)(\frac{1}{r})}$$

$$= \sqrt{435.91}$$

= 20.88 kN

Wkt,

Maximum stress = $\dfrac{20.88}{\frac{\pi d^2}{4}}$

$\dfrac{20.88}{\frac{\pi d^2}{4}}$ = 150 x 10^{-3}

d^2 = $\dfrac{20.88 \times 4}{\pi \times 150 \times 10^{-3}}$

d = 13.313 mm

Standard diameter of the bolt d=14 mm

9.A cylindrical steam pressure vessel of 1 m inside diameter is subjected to an internal pressure of 2.5 MPa. Design a double-riveted, double-strap longitudinal butt joint for the vessel. The straps are of equal width. The pitch of the rivets in the outer row should be twice of the pitch in the inner row. A zig-zag pattern is used for rivets in inner and outer rows. The efficiency of the riveted joint should be at least 70%. The permissible tensile strength for the steel plate of pressure vessel is 80 N/mm2. The permissible shear stress for the rivet material is 60 N/mm2. Assume that the rivets in double shear are 1.875 times stronger than in single shear and the joint do not fail by crushing. Calculate i) thickness of the plate ii) diameter of the rivets iii) pitch of the rivets iv) distance between inner and outer rows of the rivets v) margin vi) thickness of the straps vii) efficiency of the joint. Make neat sketch showing all the calculated dimensions .(April/May 2018)

Solution

Given For vessel, $D_i = 1$ m $P_i = 2.5$ MPa

$\eta = 70\%$ $\sigma_t = 80$ N/mm^2 $\tau = 60$ N/mm^2

A double-riveted double-strap butt joint, with equal straps,

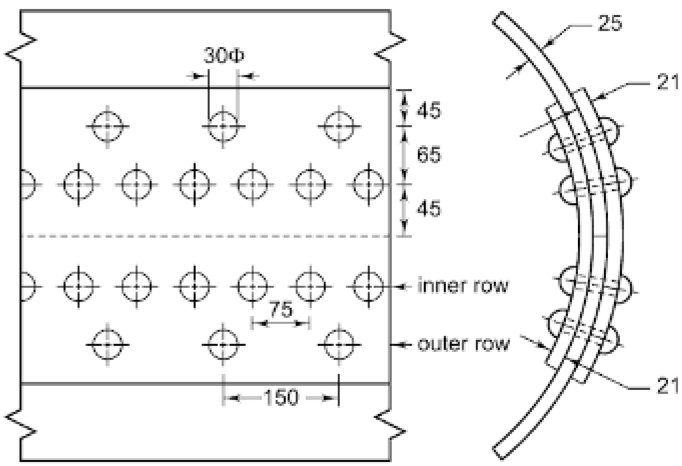


fig 3.2

Step I *Thickness of plate*

From Eq. (8.43),

$$t = \frac{P_i D_i}{2\sigma_t \eta} + \text{CA} = \frac{2.5(1000)}{2(80)(0.7)} + 2$$

$$= 24.32 \quad \text{or} \quad 25 \text{ mm} \qquad \text{(i)}$$

Step II *Diameter of rivets*

$t > 8$ mm

From Eq. (8.45),

$$d = 6\sqrt{t} = 6\sqrt{25} = 30 \text{ mm} \qquad \text{(ii)}$$

Step III: Pitch of rivets

The pitch of rivets is obtained by equating the shear strength of the rivets to the tensile strength of the plate

$$p = \frac{(n_1 + 1.875 n_2)\pi d^2 \tau}{4t\sigma_t} + d$$

As shown in Fig. the number of rivets per Pitch length in two rows are as follows:

(i) Outer row: There is one-half rivet on left side and one-half rivet on right side. These make one rivet per pitch length.

(ii) Inner row: There are two rivets per pitch length. as shown in fig 3.2

Adding these values, the total number of rivets per pitch length is 3. Since the straps are of equal width, all rivets connect the inner strap, the shell plate, and the outer strap. This results in double shear in all the rivets.Therefore,

$n_l = 0$ $n_2 = 3$

Substituting values in Eqn,

$$p = \frac{(0 + 1.875 \times 3)\pi (30)^2 (60)}{4(25)(80)} + 30$$

$$= 149.28 \text{ or } 150 \text{ mm}$$

From Eqs. , $P_{min} = 2d = 2\ (30) = 60$ mm

From Table , (3 rivets per pitch length) and

(double strap butt joint)

C= 4.63

$$P_{max} = Ct + 41.28 = 4.63\ (25) + 41.28$$

$$= 157.03 \text{ mm}$$

The pitch of 150 mm is within the limits from 60 mm to 157.03 mm. Therefore,

$$P = 150 \text{ mm}$$

The pitch of rivets in outer row is 150 mm. The pitch of rivets in inner row is (150/2) or 75 mm.

Step IV: Distance between inner and outer rows

The number of rivets in outer row is one-half of the number of rivets in inner row. Also, the rivets are arranged in zigzag pattern. From Eq., the distance between inner and outer rows is given by,

$$P_t = 0.2p + 1.15d = 0.2\ (150) + 1.15\ (30)$$

= 64.5 or 65 mm

Step V: Margin

From Eq. , m = l.5d = 1.5 (30) = 45 mm

Step VI: Thickness of straps

The straps are of equal width and every alternate rivet in outer row is omitted. From Eq,

$$t_1 = 0.625\, t\left[\frac{p-d}{p-2d}\right] = 0.625\ (25)\ \left[\frac{150-30}{150-2\times 30}\right]$$
$$= 20.83 \text{ or } 21 \text{ mm}$$

Step VII: Efficiency of joint

The tensile strength of the plate per pitch length, in outer row of rivets is given by,

P_t = (p - d) t σ_t = (150 - 30)(25)(80)

= 240 000 N

The shear strength of the rivets per pitch length

is given by,

$$P_s = (n_1 + 1.875\ n_2)\left[\frac{\pi}{4}d^2\tau\right]$$
$$= (0 + 1.875 \times 3)\left[\frac{\pi}{4}(30)^2(60)\right]$$
$$= 238\ 564.\ 69\ \text{N}$$

It is assumed that the joint does not fail in crushing. Also, the tensile strength of the solid plateper pitch length is given by,

P = p t σ_t = 150(25)(80) = 300 000 N

$$\eta = \frac{238\ 564.69}{300\ 000} = 0.7952 \quad \text{or} \quad 79.52\%$$

10. A steam engine of effective diameter 300 mm is subjected to a steam pressure of 1.5 N/mm2. The cylinder head is connected by 8 bolts having yield point 330 MPa and endurance limit at 240 MPa. The bolts are tightened with an initial preload of 1.5 times the steam load. A soft copper gasket is used to make the joint leak-proof. Assuming a factor of safety 2, find the size of bolt required. The stiffness factor for copper gasket may be taken as 0.5. (NOV/DEC 2007/2011)

Solution. Given : D = 300 mm ; p = 1.5 N/mm^2 ; n = 8 ; σ_y = 330 MPa = 330 N/mm^2; σ_e = 240 MPa = 240 N/mm^2 ; $P_1 = 1.5\,P_2$; $F.S.$ = 2 ; K = 0.5

We know that steam load acting on the cylinder head,

$$P_2 = \frac{\pi}{4}(D)^2\ p = \frac{\pi}{4}(300)^2\ 1.5 = 106\ 040\ \text{N}$$

$\therefore$ Initial pre-load,

$$P_1 = 1.5\,P_2 = 1.5 \times 106\ 040 = 159\ 060\ \text{N}$$

We know that the resultant load (or the maximum load) on the cylinder head,

$$P_{max} = P_1 + K.P_2 = 159\ 060 + 0.5 \times 106\ 040 = 212\ 080\ \text{N}$$

This load is shared by 8 bolts, therefore maximum load on each bolt,

$$P_{max} = 212\ 080 / 8 = 26\ 510\ \text{N}$$

and minimum load on each bolt,

$$P_{min} = P_1 / n = 159\ 060/8 = 19\ 882\ \text{N}$$

We know that mean or average load on the bolt,

$$P_m = \frac{P_{max} + P_{min}}{2} = \frac{26\ 510 + 19\ 882}{2} = 23\ 196\ \text{N}$$

and the variable load on the bolt,

$$P_v = \frac{P_{max} - P_{min}}{2} = \frac{26\ 510 - 19\ 882}{2} = 3314\ \text{N}$$

Let d_c = Core diameter of the bolt in mm.

Let d_c = Core diameter of the bolt in mm.

$\therefore$ Stress area of the bolt,

$$A_s = \frac{\pi}{4}(d_c)^2 = 0.7854\,(d_c)^2\ \text{mm}^2$$

We know that mean or average stress on the bolt,

$$\sigma_m = \frac{P_m}{A_s} = \frac{23\ 196}{0.7854\,(d_c)^2} = \frac{29\ 534}{(d_c)^2}\ \text{N/mm}^2$$

and variable stress on the bolt,

$$\sigma_v = \frac{P_v}{A_s} = \frac{3314}{0.7854\,(d_c)^2} = \frac{4220}{(d_c)^2}\ \text{N/mm}^2$$

According to *Soderberg's formula, the variable stress,

$$\sigma_v = \sigma_e\left(\frac{1}{F.S} - \frac{\sigma_m}{\sigma_y}\right)$$

$$\frac{4220}{(d_c)^2} = 240\left(\frac{1}{2} - \frac{29\ 534}{(d_c)^2\ 330}\right) = 120 - \frac{21\ 480}{(d_c)^2}$$

or $\frac{4220}{(d_c)^2} + \frac{21\,480}{(d_c)^2} = 120$ or $\frac{25\,700}{(d_c)^2} = 120$

$\therefore$ $(d_c)^2 = 25\,700 / 120 = 214$ or $d_c = 14.6$ mm

From Table 11.1 (coarse series), the standard core diameter is d_c = 14.933 mm and the corresponding size of the bolt is M18. **Ans.**

11. Determine the size of the bolts and the thickness of the arm for the bracket as shown in Fig. if it carries a load of 40 kN at an angle of 60° to the vertical. The material of the bracket and the bolts is same for which the safe stresses can be assumed as 70, 50 and 105 MPa in tension, shear and compression respectively. (NOV/DEC 2008)

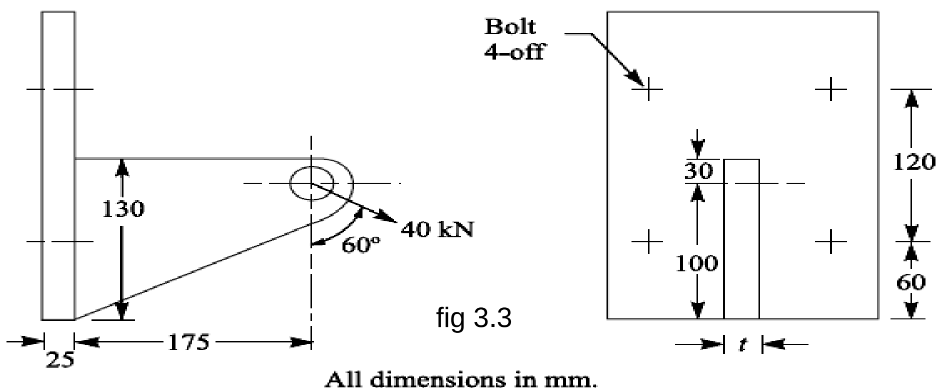


fig 3.3

Solution. Given : W = 40 kN = 40 × 10^3 N ; σ_t = 70 MPa = 70N/mm^2 ; τ = 50 MPa = 50 N/mm^2; σ_c = 105 MPa = 105 N/mm^2

Since the load W = 40 kN is inclined at an angle of 60° to the vertical, therefore resolving it into horizontal and vertical components. We know that horizontal component of 40 kN,

$$W_H = 40 \times \sin 60° = 40 \times 0.866 = 34.64 \text{ kN} = 34\,640 \text{ N}$$

and vertical component of 40 kN,

$$W_V = 40 \times \cos 60° = 40 \times 0.5 = 20 \text{ kN} = 20\,000 \text{ N}$$

Due to the horizontal component (W_H), which acts parallel to the axis of the bolts as shown in Fig. 3.3 , the following two effects are produced :

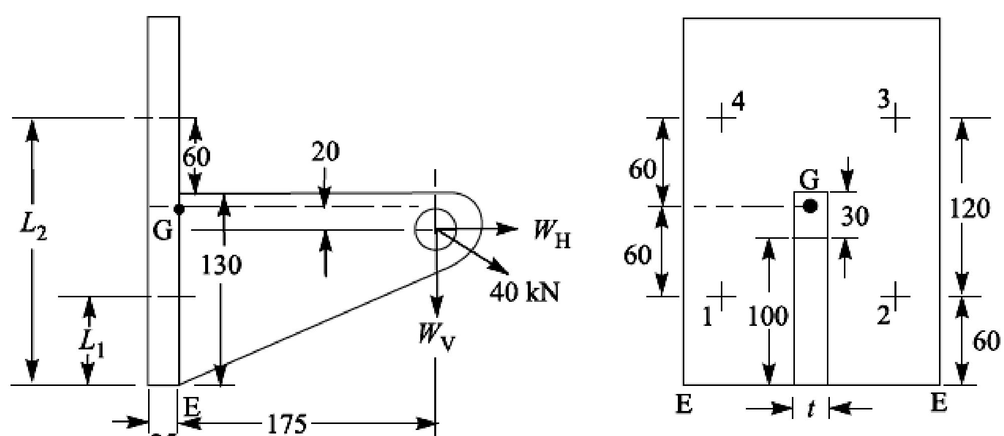


fig 3.4

1. A direct tensile load equally shared by all the four bolts, and
2. A turning moment about the centre of gravity of the bolts, in the anticlockwise direction.

$\therefore$ Direct tensile load on each bolt,

$$W_{t1} = \frac{W_H}{4} = \frac{34\ 640}{4} = 8660 \text{ N}$$

Since the centre of gravity of all the four bolts lies in the centre at G (because of symmetrical bolts), therefore the turning moment is in the anticlockwise direction. From the geometry of the Fig. 3.3 we find that the distance of horizontal component from the centre of gravity (G) of the bolts

$$= 60 + 60 - 100 = 20 \text{ mm}$$

$\therefore$ Turning moment due to W_H about G,

$$T_H = W_H \times 20 = 34\ 640 \times 20 = 692.8 \times 10^3 \text{ N-mm} \quad \text{...(Anticlockwise)}$$

Due to the vertical component W_V, which acts perpendicular to the axis of the bolts as shown in Fig. 3.4 the following two effects are produced:

1. A direct shear load equally shared by all the four bolts, and
2. A turning moment about the edge of the bracket in the clockwise direction.

$\therefore$ Direct shear load on each bolt,

$$W_s = \frac{W_V}{4} = \frac{20\ 000}{4} = 5000 \text{ N}$$

Distance of vertical component from the edge E of the bracket,

$$= 175 \text{ mm}$$

$\therefore$ Turning moment due to W_V about the edge of the bracket,

$$T_V = W_V \times 175 = 20\ 000 \times 175 = 3500 \times 10^3 \text{ N-mm} \quad \text{(Clockwise)}$$

From above, we see that the clockwise moment is greater than the anticlockwise moment, therefore,

Net turning moment $= 3500 \times 10^3 - 692.8 \times 10^3 = 2807.2 \times 10^3$ N-mm (Clockwise) ...(*i*)

Due to this clockwise moment, the bracket tends to tilt about the lower edge E.

Let w = Load on each bolt per mm distance from the edge E due to the turning effect of the bracket,

L_1 = Distance of bolts 1 and 2 from the tilting edge E = 60 mm, and

L_2 = Distance of bolts 3 and 4 from the tilting edge E

$= 60 + 120 = 180$ mm

$\therefore$ Total moment of the load on the bolts about the tilting edge E

$$= 2\,(w.L_1)\,L_1 + 2\,(w.L_2)\,L_2$$

... ($\because$ There are two bolts each at distance L_1 and L_2.)

$$= 2w\,(L_1)^2 + 2w(L_2)^2 = 2w\,(60)^2 + 2w(180)^2$$

$$= 72\ 000\ w \text{ N-mm} \quad \text{...(ii)}$$

From equations (*i*) and (*ii*),

$$w = 2807.2 \times 10^3 / 72\ 000 = 39 \text{ N/mm}$$

Since the heavily loaded bolts are those which lie at a greater distance from the tilting edge, therefore the upper bolts 3 and 4 will be heavily loaded. Thus the diameter of the bolt should be based on the load on the upper bolts. We know that the maximum tensile load on each upper bolt,

$$W_{t2} = w.L_2 = 39 \times 180 = 7020 \text{ N}$$

$\therefore$ Total tensile load on each of the upper bolt,

$$W_t = W_{t1} + W_{t2} = 8660 + 7020 = 15\,680 \text{ N}$$

Since each upper bolt is subjected to a tensile load (W_t = 15 680 N) and a shear load (W_s = 5000 N), therefore equivalent tensile load,

$$W_{te} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4(W_s)^2}\right]$$

$$= \frac{1}{2}\left[15\,680 + \sqrt{(15\,680)^2 + 4(5000)^2}\right] \text{ N}$$

$$= \frac{1}{2}[15\,680 + 18\,600] = 17\,140 \text{ N} \qquad \text{...}(\textit{iii})$$

Size of the bolts

Let d_c = Core diameter of the bolts.

We know that tensile load on each bolt

$$= \frac{\pi}{2}(d_c)^2\,\sigma_t = \frac{\pi}{4}(d_c)^2\,70 = 55\,(d_c)^2 \text{ N} \qquad \text{...}(\textit{iv})$$

From equations ***(iii)*** and ***(iv)***, we get

$$(d_c)^2 = 17\,140 / 55 = 311.64 \quad \text{or} \quad d_c = 17.65 \text{ mm}$$

From Table 11.1 (coarse series), we find that the standard core diameter is 18.933 mm and corresponding size of the bolt is M 22. **Ans.**

Thickness of the arm of the bracket

Let t = Thickness of the arm of the bracket in mm, and

b = Depth of the arm of the bracket = 130 mm ...(Given)

We know that cross-sectional area of the arm,

$$A = b \times t = 130\,t \text{ mm}^2$$

and section modulus of the arm,

$$Z = \frac{1}{6}\,t\,(b)^2 = \frac{1}{6} \times t\,(130)^2 = 2817\,t \text{ mm}^3$$

Due to the horizontal component W_H, the following two stresses are induced in the arm :

1. Direct tensile stress,

$$\sigma_{t1} = \frac{W_H}{A} = \frac{34\,640}{130\,t} = \frac{266.5}{t} \text{ N/mm}^2$$

2. Bending stress causing tensile in the upper most fibres of the arm and compressive in the lower most fibres of the arm. We know that the bending moment of W_H about the centre of gravity of the arm,

$$M_H = W_H\left(100 - \frac{130}{2}\right) = 34\,640 \times 35 = 1212.4 \times 10^3 \text{ N-mm}$$

$$\therefore \quad \text{Bending stress,} \quad \sigma_{t2} = \frac{M_H}{Z} = \frac{1212.4 \times 10^3}{2817\,t} = \frac{430{\cdot}4}{t} \text{ N/mm}^2$$

Due to the vertical component W_V, the following two stresses are induced in the arm :

1. Direct shear stress,

$$\tau = \frac{W_V}{A} = \frac{20\,000}{130\,t} = \frac{154}{t} \text{ N/mm}^2$$

2. Bending stress causing tensile stress in the upper most fibres of the arm and compressive in the lower most fibres of the arm.

Assuming that the arm extends upto the plate used for fixing the bracket to the structure. This assumption gives stronger section for the arm of the bracket.

$\therefore$ Bending moment due to W_V,

$$M_V = W_V (175 + 25) = 20\,000 \times 200 = 4 \times 10^6 \text{ N-mm}$$

and bending stress,
$$\sigma_{t3} = \frac{M_V}{Z} = \frac{4 \times 10^6}{2817\, t} = \frac{1420}{t} \text{ N/mm}^2$$

Net tensile stress induced in the upper most fibres of the arm of the bracket,

$$\sigma_t = \sigma_{t1} + \sigma_{t2} + \sigma_{t3} = \frac{266.5}{t} + \frac{430.4}{t} + \frac{1420}{t} = \frac{2116.9}{t} \text{ N/mm}^2 \quad ...(v)$$

We know that maximum tensile stress $[\sigma_{t(max)}]$,

$$70 = \frac{1}{2}\sigma_t + \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2}$$

$$= \frac{1}{2} \times \frac{2116.9}{t} + \frac{1}{2}\sqrt{\left(\frac{2116.9}{t}\right)^2 + 4\left(\frac{154}{t}\right)^2}$$

$$= \frac{1058.45}{t} + \frac{1069.6}{t} = \frac{2128.05}{t}$$

$\therefore$ $t = 2128.05 / 70 = 30.4$ say 31 mm **Ans.**

Let us now check the shear stress induced in the arm. We know that maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2} = \frac{1}{2}\sqrt{\left(\frac{2116.9}{t}\right)^2 + 4\left(\frac{154}{t}\right)^2}$$

$$= \frac{1069.6}{t} = \frac{1069.6}{31} = 34.5 \text{ N/mm}^2 = 34.5 \text{ MPa}$$

Since the induced shear stress is less than the permissible stress (50 MPa), therefore the design is safe.

Notes : 1. The value of 't' may be obtained as discussed below :

Since the shear stress at the upper most fibres of the arm of the bracket is zero, therefore equating equation (*v*) to the given safe tensile stress (*i.e.* 70 MPa), we have

$$\frac{2116.9}{t} = 70 \quad \text{or} \quad t = 2116.9 / 70 = 30.2 \text{ say } 31 \text{ mm } \textbf{Ans.}$$

2. If the compressive stress in the lower most fibres of the arm is taken into consideration, then the net compressive stress induced in the lower most fibres of the arm,

$$\sigma_c = \sigma_{c1} + \sigma_{c2} + \sigma_{c3}$$

$$= -\sigma_{t1} + \sigma_{t2} + \sigma_{t3}$$

... ($\because$ The magnitude of tensile and compressive stresses is same.)

$$= -\frac{266.5}{t} + \frac{430.4}{t} + \frac{1420}{t} = \frac{1583.9}{t} \text{ N/mm}^2$$

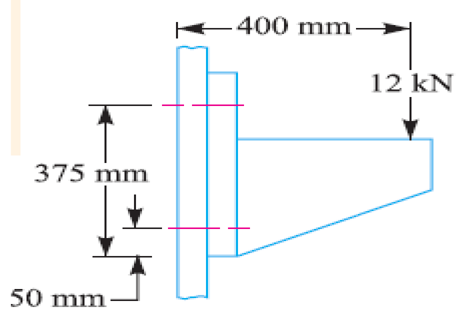
Since the safe compressive stress is 105 N/mm², therefore

$$105 = \frac{1583.9}{t} \quad \text{or} \quad t = 1583.9 / 105 = 15.1 \text{ mm}$$

This value of thickness is low as compared to 31 mm as calculated above. Since the higher value is taken, therefore

$$t = 31 \text{ mm } \textbf{Ans.}$$

12. For supporting the travelling crane in a workshop, the brackets are fixed on steel columns as shown in Fig. 11.35. The maximum load that comes on the bracket is 12 kN acting vertically at a distance of 400 mm from the face of the column. The vertical face of the bracket is secured to a column by four bolts, in two rows (two in each row) at a distance of 50 mm from the lower edge of the bracket. Determine the size of the bolts if the permissible value of the tensile stress for the bolt material is 84 MPa. Also find the cross-section of the arm of the bracket which is rectangular (NOV/DEC 2013)(NOV/DEC'2022)



Solution. Given : $W = 12$ kN $= 12 \times 10^3$ N ; $L = 400$ mm ; $L_1 = 50$ mm ; $L_2 = 375$ mm ; $\sigma_t = 84$ MPa $= 84$ N/mm² ; $n = 4$

We know that direct shear load on each bolt,

$$W_s = \frac{W}{n} = \frac{12}{4} = 3 \text{ kN}$$

Since the load W will try to tilt the bracket in the clockwise direction about the lower edge, therefore the bolts will be subjected to tensile load due to turning moment. The maximum loaded bolts are 3 and 4 (See Fig. 11.34), because they lie at the greatest distance from the tilting edge A–A (*i.e.* lower edge).

We know that maximum tensile load carried by bolts 3 and 4,

$$W_t = \frac{W.L.L_2}{2\,[(L_1)^2 + (L_2)^2]} = \frac{12 \times 400 \times 375}{2\,[(50)^2 + (375)^2]} = 6.29 \text{ kN}$$

Since the bolts are subjected to shear load as well as tensile load, therefore equivalent tensile load,

$$W_{te} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4(W_s)^2}\right] = \frac{1}{2}\left[6.29 + \sqrt{(6.29)^2 + 4 \times 3^2}\right] \text{ kN}$$

$$= \frac{1}{2}(6.29 + 8.69) = 7.49 \text{ kN} = 7490 \text{ N}$$

Size of the bolt

Let d_c = Core diameter of the bolt.

We know that the equivalent tensile load (W_{te}),

$$7490 = \frac{\pi}{4}(d_c)^2 \sigma_t = \frac{\pi}{4}(d_c)^2\, 84 = 66\,(d_c)^2$$

$\therefore$ $(d_c)^2 = 7490 / 66 = 113.5$ or $d_c = 10.65$ mm

From Table 11.1 (coarse series), the standard core diameter is 11.546 mm and the corresponding size of the bolt is M 14. **Ans.**

Cross-section of the arm of the bracket

Let t and b = Thickness and depth of arm of the bracket respectively.

$\therefore$ Section modulus,

$$Z = \frac{1}{6} t.b^2$$

Assume that the arm of the bracket extends upto the face of the steel column. This assumption gives stronger section for the arm of the bracket.

$\therefore$ Maximum bending moment on the bracket,

$$M = 12 \times 10^3 \times 400 = 4.8 \times 10^6 \text{ N-mm}$$

We know that the bending (tensile) stress (σ_t),

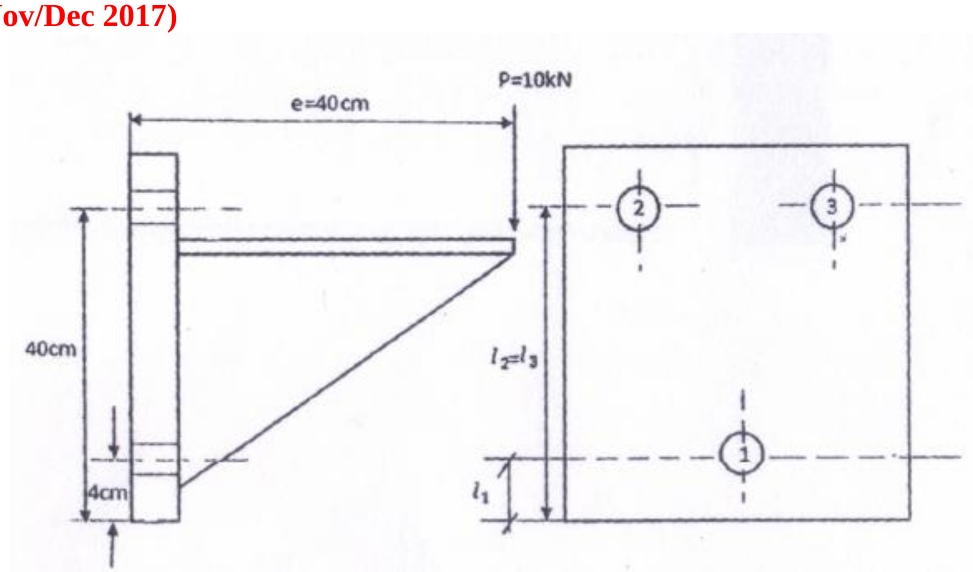
$$84 = \frac{M}{Z} = \frac{4.8 \times 10^6 \times 6}{t.b^2} = \frac{28.8 \times 10^6}{t.b^2}$$

$\therefore$ $t.b^2 = 28.8 \times 10^6 / 84 = 343 \times 10^3$ or $t = 343 \times 10^3 / b^2$

Assuming depth of arm of the bracket, $b = 250$ mm, we have

$$t = 343 \times 10^3 / (250)^2 = 5.5 \text{ mm } \textbf{Ans.}$$

13. shows a bracket fixed on a steel column by means of 3 bolts of same size. If the permissible tensile and shear stress are limited to 75 N/mm2 and 55 N/mm2 respectively. Find the size of bolts. (Nov/Dec 2017)



Given:

W=10kN, n=3, σ_t=75N/mm^2

Tofind:

The Size of bolt

Solution:

We know that direct shear load on each bolt

$$w_t = \frac{w}{n} = \frac{10}{3} = 3.33kN$$

We know that maximum tensile load carried by bolts 2 and 3

$$W_t = \frac{w.L.L_2}{2[(L_1)^2 + (L_2)^2]}$$

$$W_t = \frac{10 \times 400 \times 360}{2[(40)^2 + (360)^2]} = 5.48kN$$

Since the bolts are subjected to shear load as well as tensile load, therefore equivalent tensile load,

$$W_{te} = \frac{1}{2}[w_t + \sqrt{(w_t)^2 + 3(w_s)^2}$$

$$W_{te} = \frac{1}{2}[5.48 + \sqrt{(5.48)^2 + 3(3.33)^2}$$
$$= 6.71kN$$

Size of the bolt

Let dc = Core diameter of the bolt.

We know that the equivalent tensile load (*Wte*),

$$6710 = \frac{\pi}{4}(d_c)^2 \sigma_t$$

$$6710 = \frac{\pi}{4}(d_c)^2 \times 75$$
$$d_c = 10.67$$

the standard core diameter is 11.546 mm and the corresponding size of the bolt is M 14.

14. A steel bolt of M16x2 is 300mm long carries an impact load of 5000 Nmm. If the threads stop adjacent to the Nut and E=2.1 x 105 MPa

(i) Find the stress in the root area

(ii) Find the stress if the shank area is reduced to root area. (Nov/Dec-14)

Given:

d = 16mm

l = 300mm

U = 5000N-mm

E=2.1 x 10^5 MPa

To find:

Stresses

Solution:

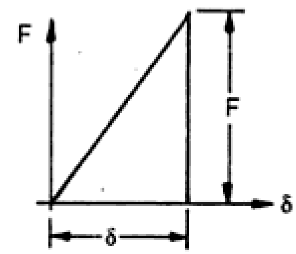


fig 3.5

(a) From Fig. 3.5 energy stored $= \frac{1}{2} F \cdot \delta$

$$\delta = F/k \text{ where } k \text{ is the stiffness}$$

$$\therefore \quad U = \frac{F^2}{2k} \times \text{ for tensile load } \delta = \frac{P}{AE}$$

$\therefore$ Stiffness of bolt $k = \frac{AE}{l}$

For 16 × 2 threads shank area $A = \frac{\pi}{4} \times 16^2 = 201.06 \text{ mm}^2$

$$\therefore \quad k = \frac{201.06 \times 2.1 \times 10^5}{300} = 140742 \text{ N/mm}$$

Again, $U = \frac{F^2}{2k}$ $\quad \therefore F = \sqrt{2Uk}$

$$= \sqrt{2 \times 5000 \times 140742} = 37515.6 \text{ N}$$

From Table 7 root area for M 16 × 2 bolt is 157 mm²

$$\therefore \quad \sigma = \frac{37515.6}{157} = 238.94 \text{ MPa}$$

(b) Now, $A_C = 157 \text{ mm}^2 = A$

$$\therefore \quad k = \frac{157 \times 2.1 \times 10^5}{300} = 109900.00 \text{ N/mm}$$

$$F = \sqrt{2 \times 5000 \times 109900.00} = 33151.168 \text{ N}$$

$$\sigma = \frac{33151.168}{157} = 211.153 \text{ MPa.}$$

15. A cylindrical beam of size 60mm is attached to support by a complete circumferential fillet weld of 6mm. find (i) torque and (ii) bending moment that can be applied if limiting shear stress is 140 Mpa.[Nov/Dec-14]

Given:

D = 60mm

h = 6mm

Shear stress = 140Mpa

To find: mt & mb

Solution:

$$\tau = \frac{2.83 \times m_t}{hD^2\pi}$$

$$140 \times 10^6 = \frac{2.83 \times m_t}{0.006 \times .06^2 \times \pi}$$

$m_t = 3356.95 Nm.$

$\sigma = 2 \times \tau$

$\sigma = 2 \times 140 \times 10^6 = 280 \times 10^6$

$$\sigma = \frac{5.66 \times m_b}{hD^2\pi}$$

$$280 \times 10^6 = \frac{5.66 \times m_b}{0.006 \times .06^2 \times \pi}$$

$m_b = 3356.95 Nm.$

16.Fig. shows a solid forged bracket to carry a vertical load of 13.5 KNapplied through the centre of hole. The square flange is secured to the flat side of a vertical stanchionthrough four bolts. Calculate suitable diameter D and d for the arms of the bracket, if the permissiblestresses are 110 MPa in tension and 65 MPa in shear.Estimate also the tensile load on each top bolt and the maximum shearing force on each bolt.(Nov/Dec-16)

Given:

$W = 13.5$ kN = 13 500 N ; $\sigma_t = 110$ MPa = 110 N/mm^2 ; $\tau = 65$ MPa = 65 N/mm^2

Tofind:

i) Diameter D and d for the arms of the bracket

ii)Tensile load on each top bolt and the maximum shearing force on each bolt

Solution:

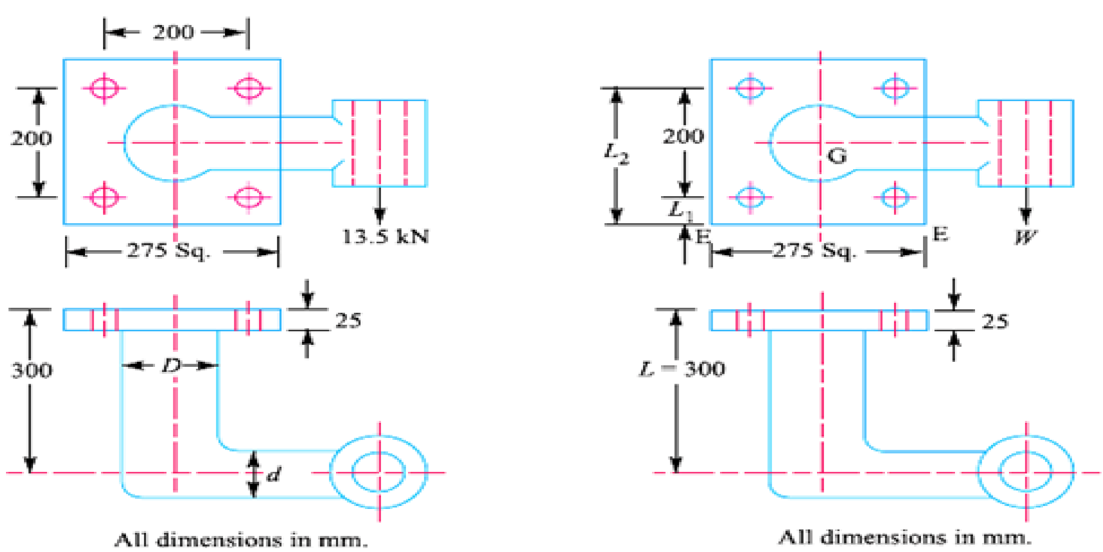


fig 3.6

Diameter D for the arm of the bracket

The section of the arm having D as the diameter is subjected to bending moment as well as twisting moment. We know that bending moment,

$$M = 13\,500 \times (300 - 25) = 3712.5 \times 10^3 \text{ N-mm}$$

Diameter (d) for the arm of the bracket

The section of the arm having d as the diameter is subjected to bending moment only. We know that bending moment,

$$M = 13\,500 \left(250 - \frac{75}{2}\right) = 2868.8 \times 10^3 \text{ N-mm}$$

and section modulus, $Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$

We know that bending (tensile) stress (σ_t),

$$110 = \frac{M}{Z} = \frac{2868.8 \times 10^3}{0.0982\, d^3} = \frac{29.2 \times 10^6}{d^3}$$

$\therefore$ $d^3 = 29.2 \times 10^6 / 110 = 265.5 \times 10^3$ or $d = 64.3$ say 65 mm **Ans.**

Tensile load on each top bolt

Due to the eccentric load W, the bracket has a tendency to tilt about the edge $E–E$, as shown in Fig. 3.6

Let w = Load on each bolt per mm distance from the tilting edge due to the tilting effect of the bracket.

Since there are two bolts each at distance L_1 and L_2 as shown in Fig. 3.6 , therefore total moment of the load on the bolts about the tilting edge $E–E$

$$= 2\,(w.L_1)\,L_1 + 2(w.L_2)\,L_2 = 2w\,[(L_1)^2 + (L_2)^2]$$

$$= 2w\,[(37.5)^2 + (237.5)^2] = 115\,625\; w \text{ N-mm} \quad ...(i)$$

...($\because L_1 = 37.5$ mm and $L_2 = 237.5$ mm)

and turning moment of the load about the tilting edge

$$= W.L = 13\,500 \times 300 = 4050 \times 10^3 \text{ N-mm} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$w = 4050 \times 10^3 / 115\,625 = 35.03 \text{ N/mm}$$

$\therefore$ Tensile load on each top bolt

$$= w.L_2 = 35.03 \times 237.5 = 8320 \text{ N}$$ **Ans.**

Maximum shearing force on each bolt

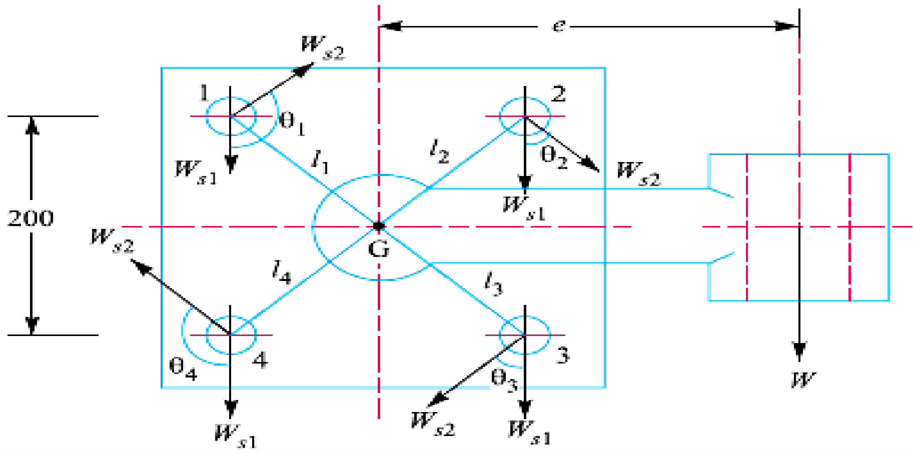
We know that primary shear load on each bolt acting vertically downwards,

$$W_{s1} = \frac{W}{n} = \frac{13\,500}{4} = 3375 \text{ N} \qquad ...(\because \text{No. of bolts, } n = 4)$$

Since all the bolts are at equal distances from the centre of gravity of the four bolts (G), therefore the secondary shear load on each bolt is same.

Distance of each bolt from the centre of gravity (G) of the bolts,

$$l_1 = l_2 = l_3 = l_4 = \sqrt{(100)^2 + (100)^2} = 141.4 \text{ mm}$$



$\therefore$ Secondary shear load on each bolt,

$$W_{s2} = \frac{W.e.l_1}{(l_1)^2 + (l_2)^2 + (l_3)^3 + (l_4)^2} = \frac{13\,500 \times 250 \times 141.4}{4\,(141.4)^2} = 5967 \text{ N}$$

Since the secondary shear load acts at right angles to the line joining the centre of gravity of the bolt group to the centre of the bolt as shown in Fig. , therefore the resultant of the primary and secondary shear load on each bolt gives the maximum shearing force on each bolt.

From the geometry of the Fig. , we find that

$$\theta_1 = \theta_4 = 135°, \text{ and } \theta_2 = \theta_3 = 45°$$

$\therefore$ Maximum shearing force on the bolts 1 and 4

$$= \sqrt{(W_{s1})^2 + (W_{s2})^2 + 2\,W_{s1} \times W_{s2} \times \cos 135°}$$

$$= \sqrt{(3375)^2 + (5967)^2 - 2 \times 3375 \times 5967 \times 0.7071} = 4303 \text{ N } \textbf{Ans.}$$

and maximum shearing force on the bolts 2 and 3

$$= \sqrt{(W_{s1})^2 + (W_{s2})^2 + 2\,W_{s1} \times W_{s2} \times \cos 45°}$$

$$= \sqrt{(3375)^2 + (5967)^2 + 2 \times 3375 \times 5967 \times 0.7071} = 8687 \text{ N } \textbf{Ans.}$$

Knuckle joints, Cotter joints:

17.Design a knuckele joint to transmit 150 kN. The design stress may be taken as 75 MPa in tension, 60 MPa in shear and 150 MPa in compression. (16) (Nov/ Dec – 2011) & (Nov/Dec – 2012) (April/May 2019)

Given :

P = 150 kN = 150 × 103 N ;

σt= 75 MPa = 75 N/mm2 ;

τ = 60 MPa = 60 N/mm2 ;

σc= 150 MPa = 150 N/mm2

Solution:

The knuckle joint is shown in The joint is designed by considering the variousmethods of failure as discussed below :

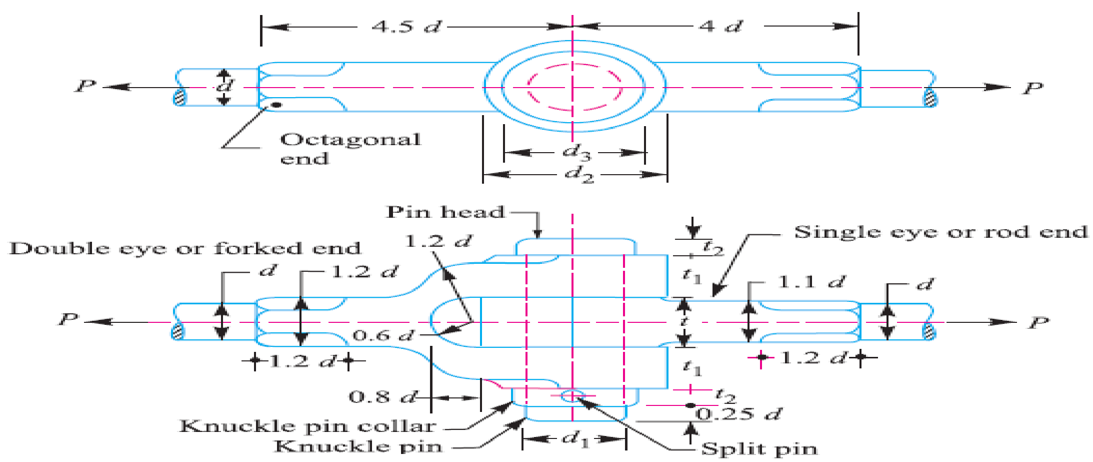


fig 3.7 Kunckle joint.

1. **Failure of the solid rod in tension**

Let d = Diameter of the rod.

We know that the load transmitted (P),

$$150 \times 10^3 = \frac{\pi}{4} d^2 \times \sigma_t = \frac{\pi}{4} d^2 \times 75 = 59 d^2$$

$$d^2 = \frac{150 \times 10^3}{59} = 2540 \quad \text{or } 50.4 \text{ say } 52 \text{ mm}$$

Now the various dimensions are fixed as follows :

from fig 3.7 Diameter of knuckle pin,

$d_1 = d = 52$ mm

Outer diameter of eye, $d_2 = 2\,d = 2 \times 52 = 104$ mm

Diameter of knuckle pin head and collar,

$d_3 = 1.5\,d = 1.5 \times 52 = 78$ mm

Thickness of single eye or rod end,

$t = 1.25\,d = 1.25 \times 52 = 65$ mm

Thickness of fork, $t_1 = 0.75\,d = 0.75 \times 52 = 39$ say 40 mm

Thickness of pin head, $t_2 = 0.5\,d = 0.5 \times 52 = 26$ mm

2. Failure of the knuckle pin in shear

Since the knuckle pin is in double shear, therefore load (P),

$$150 \times 10^3 = 2 \times \frac{\pi}{4} \times d_1^2 \times \tau = 2 \times \frac{\pi}{4} \times 52^2 \times \tau = 4248\tau$$

$$\tau = 150 \times 10^3 / 4248 = 35.3 \text{ N/mm}^2 = 35.3 \text{ MPa}$$

3. Failure of the single eye or rod end in tension

The single eye or rod end may fail in tension due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\ t \times \sigma_t = (104 - 52)\ 65 \times \sigma_t = 3380\ \sigma_t$$

$$\therefore \sigma_t = 150 \times 10^3 / 3380 = 44.4 \text{ N / mm}^2 = \mathbf{44.4\ MPa}$$

3. **Failure of the single eye or rod end in shearing**

The single eye or rod end may fail in shearing due to the load. We know that load (P),

$150 \times 10^3 = (d2 - d1)\ t \times \tau$□$= (104 - 52)\ 65 \times \tau = 3380\ \tau$

$\tau = 150 \times 10^3 / 3380 = 44.4 \text{ N/mm}^2 = 44.4 \text{ MPa}$

5**. Failure of the single eye or rod end in crushing**

The single eye or rod end may fail in crushing due to the load. We know that load (P),

$$150 \times 10^3 = d_1 \times t \times \sigma_c = 52 \times 65 \times \sigma_c = 3380\ \sigma_c$$

$$\therefore \sigma_c = 150 \times 10^3 / 3380 = 44.4 \text{ N/mm}^2 = 44.4 \text{ MPa}$$

6**. Failure of the forked end in tension**

The forked end may fail in tension due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\ 2\ t_1 \times \sigma_t = (104 - 52)\ 2 \times 40 \times \sigma_t = 4160\ \sigma_t$$

$$\therefore \sigma_t = 150 \times 10^3 / 4160 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

7**. Failure of the forked end in shear**

The forked end may fail in shearing due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\ 2\ t_1 \times \tau = (104 - 52)\ 2 \times 40 \times \tau = 4160\ \tau$$

$$\therefore \tau = 150 \times 10^3 / 4160 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

8. Failure of the forked end in crushing

The forked end may fail in crushing due to the load. We know that load (P),

$$150 \times 10^3 = d_1 \times 2\ t_1 \times \sigma_c = 52 \times 2 \times 40 \times \sigma_c = 4160\ \sigma_c$$

$$\therefore \sigma_c = 150 \times 10^3 / 4180 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

From above, we see that the induced stresses are less than the given design stresses, therefore **the joint is safe.**

18. It is required to design a knuckle joint to connect two circular rods subjected to an axial tensile force of 50 kN. The rods are co-axial and a small amount of angular movement between their axes is permissible. Design the joint and specify the dimensions of its components. Select suitable materials for the parts. Assume rod materials as 30CB and FOS = 5. (Nov/Dec 2017)

Given;

$P = (50 \text{ X } 10^3)$ N

solution:

Part I: Selection of material

The rods are subjected to tensile force. Therefore, yield strength is the criterion for the selection of material for the rods. The pin is subjected to shear stress and bending stresses. Therefore, strength is also the criterion for material selection of the pin. On strength basis, the material for two rods and pin is selected as plain carbon steel of Grade 30C8 (S_{yt}= 400 N/mm^2). It is further assumed that the yield strength in compression is equal to yield strength in tension. In practice, the compressive strength of steel is much higher than its tensile strength.

Part II: Selection of factor of safety

In stress analysis of knuckle joint, the effect of stress concentration is neglected. To account for this effect, a higher factor of safety of 5 is assumed in present design.

Part III: Calculation of permissible stresses

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{5} = 80 \text{ N/mm}^2$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{S_{yt}}{(fs)} = \frac{400}{5} = 80 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\,S_{yt}}{(fs)} = \frac{0.5(400)}{5} = 40 \text{ N/mm}^2$$

Part IV: Calculation of dimensions

The dimensions of the knuckle joint are calculated by the procedure

Step I: Diameter of rods

$$D = \sqrt{\frac{4P}{\pi\sigma_t}} = \sqrt{\frac{4(50\times10^3)}{\pi(80)}} = 28.21 \quad \text{or} \quad 30 \text{ mm}$$

Step II: Enlarged diameter of rods (D_1)

$$D_1 = 1.1\,D = 1.1(30) = 33 \text{ or } 35 \text{ mm}$$

Step III: Dimensions a and b

$$a = 0.75\,D = 0.75(30) = 22.5 \text{ or } 25 \text{ mm}$$
$$b = 1.25\,D = 1.25(30) = 37.5 \text{ or } 40 \text{ mm}$$

Step IV: Diameter of pin

$$d = \sqrt{\frac{2P}{\pi\tau}} = \sqrt{\frac{2(50\times10^3)}{\pi(40)}} = 28.21 \quad \text{or} \quad 30 \text{ mm}$$

$$d = \sqrt[3]{\frac{32}{\pi\sigma_b}\times\frac{P}{2}\left[\frac{b}{4}+\frac{a}{3}\right]}$$
$$= \sqrt[3]{\frac{32}{\pi(80)}\times\frac{(50\times10^3)}{2}\left[\frac{40}{4}+\frac{25}{3}\right]}$$
$$= 38.79 \quad \text{or} \quad 40 \text{ mm}$$
$$d = 40 \text{ mm}$$

Step V: Dimensions d_o and d_1

$$d_o = 2\,d = 2(40) = 80 \text{ mm}$$
$$d_1 = 1.5\,d = 1.5(40) = 60 \text{ mm}$$

Step VI: Check for stresses in eye

$$\sigma_t = \frac{P}{b(d_0 - d)} = \frac{(50 \times 10^3)}{40(80 - 40)} = 31.25 \text{ N/mm}^2$$

$$\sigma_t < 80 \text{ N/mm}^2$$

$$\sigma_c = \frac{P}{bd} = \frac{(50 \times 10^3)}{40(40)} = 31.25 \text{ N/mm}^2$$

$$\sigma_c < 80 \text{ N/mm}^2$$

$$\tau = \frac{P}{b(d_0 - d)} = \frac{(50 \times 10^3)}{40(80 - 40)} = 31.25 \text{ N/mm}^2$$

$$\tau < 40 \text{ N/mm}^2$$

Step VII: Check for stresses in fork

$$\sigma_t = \frac{P}{2a(d_0 - d)} = \frac{(50 \times 10^3)}{2(25)(80 - 40)} = 25 \text{ N/mm}^2$$

$$\sigma_t < 80 \text{ N/mm}^2$$

$$\sigma_c = \frac{P}{2ad} = \frac{(50 \times 10^3)}{2(25)(40)} = 25 \text{ N/mm}^2$$

$$\sigma_c < 80 \text{ N/mm}^2$$

$$\tau = \frac{P}{2a(d_0 - d)} = \frac{(50 \times 10^3)}{2(25)(80 - 40)} = 25 \text{ N/mm}^2$$

$$\tau < 40 \text{ N/mm}^2$$

It is observed that stresses are within limits.

19.Design a gib and cottor joint as shown in Fig. 12.13, to carry a maximum load of 35 kN. Assuming that the gib, cotter and rod are of same material and have the following allowable stresses :σt = 20 MPa ; τ = 15 MPa ; and σc= 50 MPa. (16)

Given :

P = 35 kN = 35 000 N ;
σt= 20 MPa = 20 N/mm2 ;
τ = 15 MPa = 15 N/mm2 ;
σc= 50 MPa = 50 N/mm2

Solution:

1. Side of the square rod

Let x = Each side of the square rod.Considering the failure of the rod in tension. We know that load (P),

$$35\ 000 = x^2 \times \sigma_t = x^2 \times 20 = 20\, x^2$$

$$\therefore x^2 = 35\ 000 / 20 = 1750 \text{ or } x = 41.8 \text{ say } \mathbf{42\ mm\ Ans.}$$

Other dimensions are fixed as follows :

Width of strap, B_1 = x = 42 mm Ans.

Thickness of cotter, $t = B_1/4 = 42/4 = 10.5$ say 12 mm Ans.

Thickness of gib = Thickness of cotter = 12 mm Ans.

Height (t_2) and length of gib head (l_4)= Thickness of cotter = 12 mm Ans.

2. **Width of gib and cotter**

Let B = Width of gib and cotter.

Considering the failure of the gib and cotter in double shear. We know that load (P),

$$35\,000 = 2\,B \times t \times \tau = 2\,B \times 12 \times 15 = 360\,B$$

$$\therefore B = 35\,000 / 360 = 97.2 \text{ say } 100 \text{ mm Ans.}$$

Since one gib is used, therefore

Width of gib, $b_1 = 0.55\,B = 0.55 \times 100 = 55$ mm Ans.

and width of cotter, $b = 0.45\,B = 0.45 \times 100 = 45$ mm Ans.

3. **Thickness of strap**

Let t1 = Thickness of strap.

Considering the failure of the strap end in tension at the location of the gib and cotter. We know that load (P),

$$35\,000 = 2\,(x \times t_1 - t_1 \times t)\,\sigma_t = 2\,(42 \times t_1 - t_1 \times 12)\,20 = 1200\,t_1$$

$$\therefore t_1 = 35\,000 / 1200 = 29.1 \text{ say } 30 \text{ mm Ans.}$$

Now the induced crushing stress may be checked by considering the failure of the strap or gib in crushing. We know that load (P),

$$35\,000 = 2\,l_1 \times t \times \sigma c = 2 \times 30 \times 12 \times \sigma_c = 720\,\sigma_c$$

$$\boldsymbol{\therefore \sigma_c = 35\,000 / 720 = 48.6 \text{ N/mm}^2}$$

Since the induced crushing stress is less than the given crushing stress, therefore the joint is safe.

4. **Length *(l_1)* of the rod**

Considering the failure of the rod end in shearing. Since the rod is in double shear, therefore load (P),

$$35\,000 = 2l_1 \times x \times \tau = 2l_1 \times 42 \times 15 = 1260l_1$$

$$\therefore l_1 = 35\,000 / 1260 = 27.7 \text{ say } 28 \text{ mm Ans.}$$

5. **Length (l_2) of the rod**

Considering the failure of the strap end in shearing. Since the length of the rod (l2) is in double shear, therefore load (P),

$35\,000 = 2 \times 2\,l_2 \times t_1 \times \tau = 2 \times 2\,l_2 \times 30 \times 15 = 1800\,l_2$

$\therefore l_2 = 35\,000 / 1800 = 19.4$ say 20 mm Ans.

Length (l_3) of the strap end

$$= \frac{2}{3} \times x = \frac{2}{3} \times 42 = \boldsymbol{28\ mm\ Ans}$$

and length of cotter = 4 x= 4 × 42 = **168 mm Ans.**

20.Design a cotter joint to connect piston rod to the crosshead of a double actingsteam engine. The diameter of the cylinder is 300 mm and the steam pressure is 1 N/mm2. Theallowable stresses for the material of cotter and piston rod are as follows :σt= 50 MPa ; τ = 40 MPa ; and σc= 84 MPa (16)

Given :

D = 300 mm ;

p = 1 N/mm2 ;

σt= 50 MPa = 50 N/mm2 ;

τ = 40 MPa = 40 N/mm2 ;

σc= 84 MPa = 84 N/mm2

Solution:

We know that maximum load on the piston rod,

$$P = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} 300^2 \times 1 = 70695\ N$$

The various dimensions for the cotter joint are obtained by considering the different modes of failure as discussed below :

1. Diameter of piston rod at cotter

Let d_2 = Diameter of piston rod at cotter, and

t = Thickness of cotter. It may be taken as 0.3 d_2.

Considering the failure of piston rod in tension at cotter. We know that load (P),

$$70695 = \left[\frac{\pi}{4} \times d_2^2 - d_2 \times t\right] \sigma_t = \left[\frac{\pi}{4} \times d_2^2 - 0.3 \times d_2^2\right] 50 = 24.27\ (d_2)^2$$

$$(d_2)^2 = \frac{70695}{24.27} = 2913 \text{ or } d_2 = 53.97 \text{ say } 55 \text{ mm}$$

andt= 0.3 d_2 = 0.3 × 55 **= 16.5 mm Ans.**

2. Width of cotter

Let b = Width of cotter.

Considering the failure of cotter in shear. Since the cotter is in double shear, therefore load (P),

70 695 = 2 b × t × τ = 2 b × 16.5 × 40 = 1320 b

∴b = 70 695 / 1320 = 53.5 say **54 mm Ans.**

3. Diameter of socket

Let d3 = Diameter of socket.

Considering the failure of socket in tension at cotter. We know that load (P),

$$70695 = \left[\frac{\pi}{4} \times \{(d_3)^2 - (d_2)^2\} - (d_3 - d_2)t\right] \sigma_1$$

$$= \left[\frac{\pi}{4} \times \{(d_3)^2 - (55)^2\} - (d_3 - 55)16.5\right] 50$$

$$= 39.27\ d_3^2 - 118792 - 825 d_3 + 45375$$

Or$d_3^2 - 21 d_3 - 3670 = 0$

$$d_3 = \frac{21 \pm \sqrt{(21)^2 + 4 \times 3670}}{2} = \frac{21 \pm 123}{2} = 72\ mm \qquad \text{..(Taking +ve sign)}$$

Let us now check the induced crushing stress in the socket. We know that load (P),

$$70\ 695 = (d_3 - d_2)\ t \times \sigma_c = (72 - 55)\ 16.5 \times \sigma_c = 280.5\ \sigma_c$$

$$\therefore \sigma_c = 70\ 695 / 280.5 = \mathbf{252\ N/mm^2}$$

Since the induced crushing is greater than the permissible value of 84 N/mm^2, therefore let us find the value of d3 by substituting σc= 84 N/mm^2 in theabove expression, i.e.

$$70\ 695 = (d_3 - 55)\ 16.5 \times 84 = (d_3 - 55)\ 1386$$

$$\therefore d_3 - 55 = 70\ 695 / 1386 = 51$$

$$\text{or} d_3 = 55 + 51 = \mathbf{106\ mm\ Ans.}$$

We know the tapered length of the piston rod,

$$L = 2.2\ d_2 = 2.2 \times 55 = \mathbf{121\ mm\ Ans.}$$

Assuming the taper of the piston rod as 1 in 20, thereforethe diameter of the parallel part of the piston rod,

$$d = d_2 - \frac{L}{2} \times \frac{1}{20} = 55 - \frac{121}{2} \times \frac{1}{20} = \mathbf{58\ mm}$$

$$d_1 = d_2 - \frac{L}{2} \times \frac{1}{20} = 55 - \frac{121}{2} \times \frac{1}{20} = \mathbf{52\ mm}$$

21.Design and draw a cotter joint to support a load varying from 30 kN in compression to 30 kN in tension. The material used is carbon steel for which the following allowable stresses may be used. The load is applied statically. Tensile stress = Compressive stress = 50 MPa, Shear stress = 35 MPa and crushing stress = 90 MPa. (16) (May/June – 2013) (April/May 2019)(Nov/Dec 2021)

Given :

P = 30 kN = 30 × 103 N ;

σt= 50 MPa = 50 N / mm2 ;

τ= 35 MPa = 35 N / mm2 ;

σc= 90 MPa = 90 N/mm2

Solution:

The cotter joint is designed as discussed below :

1. Diameter of the rods

Letd = Diameter of the rods.

Considering the failure of the rod in tension.

We know that load (P),

$$30 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 50 = 39.3d^2$$

$\therefore d^2$ = 30 × 103 / 39.3 = 763 or d = 27.6 say **28 mm Ans**.

2.Diameter of spigot and thickness of cotter

Let d_2 = Diameter of spigot or inside diameter of socket, and

t = Thickness of cotter. It may be taken as d2 / 4.

Considering the failure of spigot in tension across the weakest section. We know that load (P),

$$30 \times 10^3 = \left[\frac{\pi}{4} \times d_2^2 - d_2 \times t\right] \sigma_t = \left[\frac{\pi}{4} \times d_2^2 - d_2 \times \frac{d_2}{4}\right] 50 = 26.8\ (d_2)^2$$

$$(d_2)^2 = \frac{30 \times 10^3}{26.8} = 1119.4 \text{ or} d_2 = 33.4 \text{ say } 34 \text{ mm}$$

and thickness of cotter, $t = \frac{d_2}{4} = \frac{34}{4} = 8.5$ mm

Let us now check the induced crushing stress. We know that load (P),

$$30 \times 10^3 = d_2 \times t \times \sigma_c = 34 \times 8.5 \times \sigma_c = 289\ \sigma_c$$

$$\therefore \sigma_c = 30 \times 10^3 / 289 = 103.8 \text{ N/mm}^2$$

Since this value of σcis more than the given value of σ_c= 90 N/mm2, therefore the dimensions d_2 = 34 mm and t = 8.5 mm are not safe. Now let us find the values of d_2 and t by substituting the value of σ_c= 90 N/mm2 in the above expression, i.e.

$$30 \times 10^3 = \left[d_2 \times \frac{d_2}{4}\right] 90 = 22.5\ (d_2)^2$$

$$(d_2)^2 = \frac{30 \times 10^3}{22.5} = 1333 \text{ or} d_2 = 36.5 \text{ say } 40 \text{ mm}$$

and $\quad t = d_2 / 4 = 40 / 4 =$ **10 mm Ans.**

3. Outside diameter of socket

Let $\quad d_1$ = Outside diameter of socket.

Considering the failure of the socket in tension across the slot. We know that load (P),

$$30 \times 10^3 = \left[\frac{\pi}{4} \times \{(d_1)^2 - (d_2)^2\} - (d_1 - d_2)t\right] \sigma_t$$

$$= \left[\frac{\pi}{4} \times \{(d_1)^2 - (40)^2\} - (d_1 - 40)10\right] 50$$

$$\frac{30 \times 10^3}{50} = 0.7854 d_1^2 - 1256.6 - 10 d_1 + 400$$

Or $d_1^2 - 12.7\ d_1 - 1854.6 = 0$

$$d_1 = \frac{12.7 \pm \sqrt{(12.7)^2 + 4 \times 1854.6}}{2} = \frac{12.7 \pm 87.1}{2}$$

= 49.9 say 50 mm Ans. ..(Taking +ve sign)

4. Width of cotter

Let $\quad$ b = Width of cotter.

Considering the failure of the cotter in shear. Since the cotter is in double shear, therefore load (P),

$$30 \times 10^3 = 2\ b \times t \times \tau = 2\ b \times 10 \times 35 = 700\ b$$

$\therefore b = 30 \times 10^3 / 700 =$ **43 mm Ans.**

5. Diameter of socket collar

Let $\quad d_4$ = Diameter of socket collar.

Considering the failure of the socket collar and cotter in crushing. We know that load (P),

$$30 \times 10^3 = (d_4 - d_2)t \times \sigma_c = (d_4 - 40)10 \times 90 = (d_4 40)900$$

$$d_4 - 40 = \frac{30 \times 10^3}{900} = 33.3 \text{ or} d_4 = 33.3 + 40 = \mathbf{73.3 \text{ say } 75 \text{ mm}}$$

6. Thickness of socket collar

Let $\quad$ c = Thickness of socket collar.

Considering the failure of the socket end in shearing. Since the socket end is in double shear, therefore load (P),

$$30 \times 10^3 = 2(d_4 - d_2)\ c \times \tau = 2\ (75 - 40\ )c \times 35 = 2450\ c$$

$$\therefore c = 30 \times 10^3 / 2450 = \mathbf{12\ mm\ Ans.}$$

7. **Distance from the end of the slot to the end of the rod**

Let a = Distance from the end of slot to the end of the rod.

Considering the failure of the rod end in shear. Since the rod end is in double shear, thereforeload (P),

$$30 \times 10^3 = 2\ a \times d2 \times \tau = 2a \times 40 \times 35 = 2800\ a$$

$$\therefore a = 30 \times 10^3 / 2800 = 10.7\ \textbf{say 11 mm Ans.}$$

8. Diameter of spigot collar

Let d_3 = Diameter of spigot collar.

Considering the failure of spigot collar in crushing. We know that load (P),

$$30 \times 10^3 = \left[\frac{\pi}{4} \times (d_3)^2 - (d_2)^2\right] \sigma_c = \left[\frac{\pi}{4} \times (d_3)^2 - (40)^2\right] 90$$

$$(d_3)^2 - (40)^2 = \frac{30 \times 10^3 \times 4}{90\pi} = 424$$

$$(d_3)^2 = 424 + 40^2 = 2024 \quad \text{or}\ \mathbf{d_3 = 45\ mm}$$

9. Thickness of spigot collar

Let t_1 = Thickness of spigot collar.

Considering the failure of spigot collar in shearing. We know that load (P),

$$30 \times 10^3 = \pi\ d_2 \times t_1 \times \tau = \pi \times 40 \times t_1 \times 35 = 4400\ t_1$$

$$\therefore t_1 = 30 \times 10^3 / 4400 = 6.8 \text{ say } \mathbf{8\ mm\ Ans.}$$

10. The length of cotter (l) is taken as 4 d.

$$\therefore l = 4\ d = 4 \times 28 = \mathbf{112\ mm}\text{Ans.}$$

11. The dimension e is taken as 1.2 d.

$$\therefore e = 1.2 \times 28 = 33.6 \text{ say } \mathbf{34\ mm}\text{Ans.}$$

22. Design a sleeve and cotter joint to resist a tensile load of 60 kN. All parts of thejoint are made of the same material with the following allowable stresses :

σt = 60 MPa ; τ = 70 MPa ; and σc = 125 MPa.

Solution.Given : P = 60 kN = 60 × 103 N ; σt = 60 MPa = 60 N/mm^2 ; τ = 70 MPa = 70 N/mm^2 ; σc = 125 MPa = 125 N/mm^2

1. ***Diameter of the rods***

Let d = Diameter of the rods.

Considering the failure of the rods in tension. We know that load (P),

$$60 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 60 = 47.13\, d^2$$

$$\therefore \quad d^2 = 60 \times 10^3 / 47.13 = 1273 \quad \text{or} \quad d = 35.7 \text{ say } 36 \text{ mm Ans.}$$

2. ***Diameter of enlarged end of rod and thickness of cotter***

Let d_2 = Diameter of enlarged end of rod, and

t = Thickness of cotter. It may be taken as $d_2 / 4$.

Considering the failure of the rod in tension across the weakest section (*i.e.* slot). We know that load (P),

$$60 \times 10^3 = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times \frac{d_2}{4}\right]60 = 32.13\,(d_2)^2$$

$$\therefore \quad (d_2)^2 = 60 \times 10^3 / 32.13 = 1867 \quad \text{or} \quad d_2 = 43.2 \text{ say } 44 \text{ mm Ans.}$$

and thickness of cotter,

$$t = \frac{d_2}{4} = \frac{44}{4} = 11 \text{ mm Ans.}$$

Let us now check the induced crushing stress in the rod or cotter. We know that load (P),

$$60 \times 10^3 = d_2 \times t \times \sigma_c = 44 \times 11 \times \sigma_c = 484\,\sigma_c$$

$$\therefore \quad \sigma_c = 60 \times 10^3 / 484 = 124 \text{ N/mm}^2$$

Since the induced crushing stress is less than the given value of 125 N/mm^2, therefore the dimensions d_2 and t are within safe limits.

3. ***Outside diameter of sleeve***

Let d_1 = Outside diameter of sleeve.

Considering the failure of sleeve in tension across the slot. We know that load (P)

$$60 \times 10^3 = \left[\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\,t\right]\sigma_t$$

$$= \left[\frac{\pi}{4}[(d_1)^2 - (44)^2] - (d_1 - 44)\,11\right]60$$

$$\therefore \quad 60 \times 10^3 / 60 = 0.7854\,(d_1)^2 - 1520.7 - 11\,d_1 + 484$$

$$\text{or} \quad (d_1)^2 - 14\,d_1 - 2593 = 0$$

$$\therefore \quad d_1 = \frac{14 \pm \sqrt{(14)^2 + 4 \times 2593}}{2} = \frac{14 \pm 102.8}{2}$$

$$= 58.4 \text{ say } 60 \text{ mm Ans.} \qquad \text{...(Taking +ve sign)}$$

4. ***Width of cotter***

Let b = Width of cotter.

Considering the failure of cotter in shear. Since the cotter is in double shear, therefore load (P),

$$60 \times 10^3 = 2\,b \times t \times \tau = 2 \times b \times 11 \times 70 = 1540\,b$$

$$\therefore \quad b = 60 \times 10^3 / 1540 = 38.96 \text{ say } 40 \text{ mm Ans.}$$

5. *Distance of the rod from the beginning to the cotter hole (inside the sleeve end)*

Let a = Required distance.

Considering the failure of the rod end in shear. Since the rod end is in double shear, therefore load (P),

$$60 \times 10^3 = 2\,a \times d_2 \times \tau = 2\,a \times 44 \times 70 = 6160\,a$$

$\therefore$ $a = 60 \times 10^3 / 6160 = 9.74$ say 10 mm **Ans.**

6. *Distance of the rod end from its end to the cotter hole*

Let c = Required distance.

Considering the failure of the sleeve end in shear. Since the sleeve end is in double shear, therefore load (P),

$$60 \times 10^3 = 2\,(d_1 - d_2)\,c \times \tau = 2\,(60 - 44)\,c \times 70 = 2240\,c$$

$\therefore$ $c = 60 \times 10^3 / 2240 = 26.78$ say 28 mm **Ans.**

<u>Weldedjoints:</u>

23.A rectangular cross-section bar is welded to a support by means of fillet welds as shown in fig (i). Determine the size of the welds, if the permissible shear stress in the weld is limited to 75 MPa. (16M) (Nov/ Dec – 2011) Ref: 367

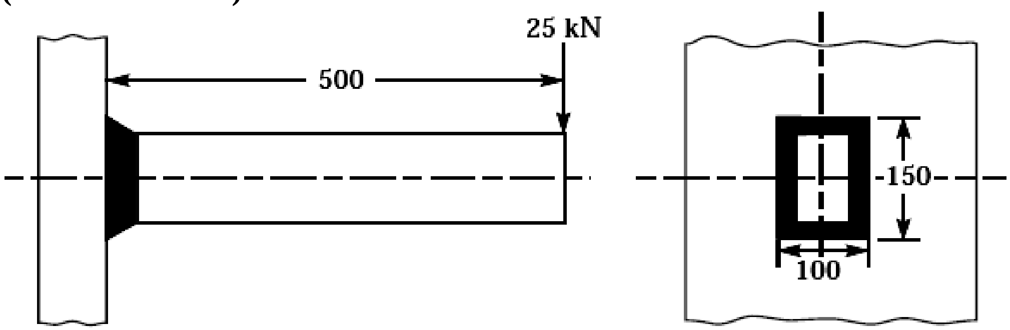


fig 3.8 All dimensions in mm

Given :

P = 25 kN = 25 × 103N ;

τ_{max}= 75 MPa = 75N/mm2 ;

l= 100 mm ;

b = 150 mm ;

e = 500 mm

Solution:

Let s = Size of the weld, and

t = Throat thickness.

The joint, as shown in Fig. 3.8 is subjected to direct shear stress and the bending stress. We know that the throat area for a rectangular fillet weld,

A = t (2b + 2*l*)

$= 0.707\ s\ (2b + 2l)$

$= 0.707s\ (2 \times 150 + 2 \times 100) = 353.5\ s\ mm^2$ ... ($\because t = 0.707s$)

Direct shear stress $= \frac{P}{A}$

$$= \frac{25 \times 10^3}{353.5\ s} = \frac{70.72}{s} N/mm^2$$

We know that bending moment,

M = P × e = 25 × 103 × 500 = 12.5 × 106 N-mm

we find that for a rectangular section, section modulus,

$$Z = t\left(b.l + \frac{b^2}{3}\right) = 0.707\ s$$

$$= \left[150 \times 100 + \frac{150^2}{3}\right] = 15907.5\ smm^3$$

$$\text{Bendingstress}, \sigma_b = \frac{M}{Z}$$

$$= \frac{12.5 \times 10^6}{15907.5\ s} = \frac{785.8}{s} N/mm^2$$

We know that maximum shear stress (τmax),

$$75 = \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{785.8}{s}\right)^2 + 4\left(\frac{70.72}{s}\right)^2} = \frac{399.2}{s}$$

$$s = \frac{399.2}{75} = \mathbf{5.32\ mm}$$

Result:

Size of the weld, s = 5.32 mm

24.What is an eccentric loads welded joint? Describe procedure for designing such a joint. (8) (May/June – 2013)

Eccentrically Loaded Welded Joints:

An eccentric load may be imposed on welded joints in many ways. The stresses induced on the joint may be of different nature or of the same nature. The induced stresses are combined depending upon the nature of stresses. When the shear and bending stresses are simultaneously present in a joint(see case 1), then maximum stresses are as follows:

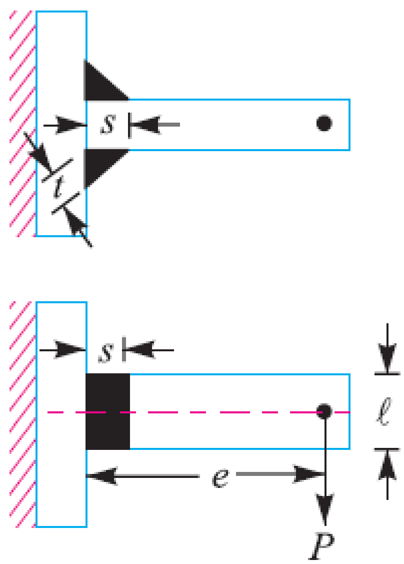


fig 3.9 Eccentricallyloaded welded joint.

Maximum normal stress,

$$\sigma_{t(\max)} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

where σ_b= Bending stress, andτ = Shear stress.

When the stresses are of the same nature, these may be combinedvectorially (see case 2).

Case 1

Consider a *T*-joint fixed at one end and subjected to an eccentric load *P* at a distance *e* as shown in Fig. 3.9

Let *s* = Size of weld,

l = Length of weld, and

t = Throat thickness.

The joint will be subjected to the following two types of stresses:

1. Direct shear stress due to the shear force *P* acting at the welds, and

2. Bending stress due to the bending moment $P \times e$.

We know that area at the throat,

A = Throat thickness × Length of weld

$= t \times l \times 2 = 2\, t \times l$ ... (For double fillet weld)

$= 2 \times 0.707\, s \times l = 1.414\, s \times l$ ... ($\because t = s \cos 45° = 0.707\, s$)

∴ Shear stress in the weld (assuming uniformly distributed),

$$\tau = \frac{P}{A} = \frac{P}{1.414\, s \times l}$$

Section modulus of the weld metal through the throat,

$$Z = \frac{t \times l^2}{2} \times 2$$

$$= \frac{0.707\, s \times l^2}{4.242} \times 2 = \frac{s \times l^2}{4.242} \; (\mathit{for\ both\ sides\ weld})$$

Bending moment, $M = P \times e$

$\therefore$ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{P \times e \times 4.242}{s \times l^2} = \frac{4.242\, P \times e}{s \times l^2}$

We know that the maximum normal stress,

$$\sigma_{t(\max)} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Case 2

When a welded joint is loaded eccentrically as shown in Fig. 3.9 ,the following two types of the stresses are induced:

1. Direct or primary shear stress, and

2. Shear stress due to turning moment.

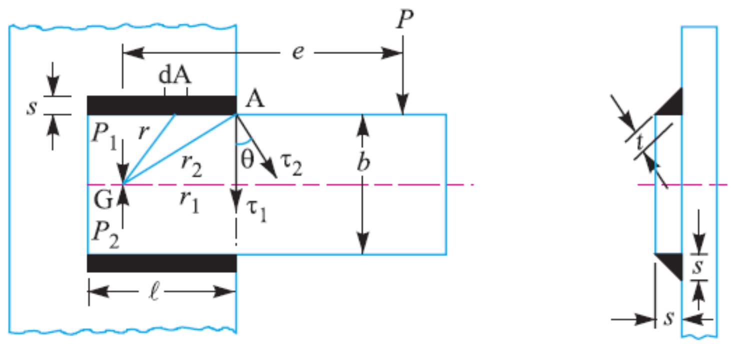


fig 3.10 Eccentrically loaded welded joint.

Let *P* = Eccentric load,

e = Eccentricity *i.e.* perpendicular distance between the line of action ofload and centre of gravity (*G*) of the throat sectionor fillets, from the fig 3.10

l = Length of single weld,

s = Size or leg of weld, and

t = Throat thickness.

Let two loads $P1$ and $P2$ (each equal to P) are introduced at the centre of gravity 'G' of the weld system. The effect of load $P_1 = P$ is to produce direct shear stress which is assumed to be uniform overthe entire weld length. The effect of load $P_2 = P$ is to produce a turning moment of magnitude $P \times e$which tends of rotate the joint about the centre of gravity 'G' of the weld system. Due to the turningmoment, secondary shear stress is induced.

We know that the direct or primary shear stress,

$$\tau_1 = \frac{Load}{Throut\ area} = \frac{P}{A} = \frac{P}{2t \times l}$$

$$\tau_1 = \frac{P}{2 \times 0.707\ s \times l} = \frac{P}{1.414\ s\ \times l}$$

(Throat area for single fillet weld = txl = 0.707 s x1)

Since the shear stress produced due to the turning moment ($T = P \times e$) at any section is proportional to its radial distance from G, therefore stress due to $P \times e$ at the point A is proportional to AG ($r2$) andis in a direction at right angles to AG. In other words,

$$\frac{\tau_2}{r_2} = \frac{\tau}{r} = constant$$

$$\tau = \frac{\tau_2}{r_2} \times r \quad -----------------1$$

where τ_2 is the shear stress at the maximum distance (r_2) and τ is the shear stress at any distance r.

Consider a small section of the weld having area dAat a distance r from G.

∴ Shear force on this small section

$$= \tau \times dA$$

and turning moment of this shear force about G,

$$dT = \tau \times dA \times r = \frac{\tau_2}{r_2} \times dA \times r^2$$

∴ Total turning moment over the whole weld area,

$$T = P \times e = \int \frac{\tau_2}{r_2} \times d\,A \times r^2 = \frac{\tau_2}{r_2} \int d\,A \times r^2$$

$$= \frac{\tau_2}{r_2} \times J J = \int d\,A \times r^2$$

where$J$= Polar moment of inertia of the throat area about G.

∴ Shear stress due to the turning moment *i.e*. secondary shear stress,

$$\tau_2 = \frac{T \times r_2}{J} = \frac{P \times e \times r_2}{J}$$

In order to find the resultant stress, the primary and secondary shear stresses are combined vectorially.

∴ Resultant shear stress at A,

$$\tau_A = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1 \times \tau_2 \times cos\theta}$$

where θ = Angle between τ1 and τ2, and

cos θ = *r*1 / *r*2

Note: The polar moment of inertia of the throat area (*A*) about the centre of gravity (*G*) is obtained by the parallelaxis theorem, *i.e.*

$J = 2\,[I_{xx} + A \times x^2]$ ... (∵ of double fillet weld)

$$= 2\left[\frac{A \times l^2}{12} + A \times x^2\right] = 2A\left(\frac{l_2}{12} + x^2\right) A = \pi r^2$$

where*A*= Throat area = *t* × *l* = 0.707 *s* × *l*,

l = Length of weld, and

x = Perpendicular distance between the two parallel axes.

25. A rectangular steel plate is welded as a cantilever to a vertical column and supports a single concentrated load P, as shown in fig. Determine the weld size if shear stress in the same is not to exceed 140 MPa. ((8)) (Nov/Dec – 2012) &(May/June – 2013) (Apr/May15)(APRIL/MAY'2023)

Given :

P = 60 kN = 60×10^3N ;

b = 100 mm ; l = 50 mm ;

τ = 140 MPa = 140 N/mm^2

Solution:

Let s = Weld size, and

t = Throat thickness.

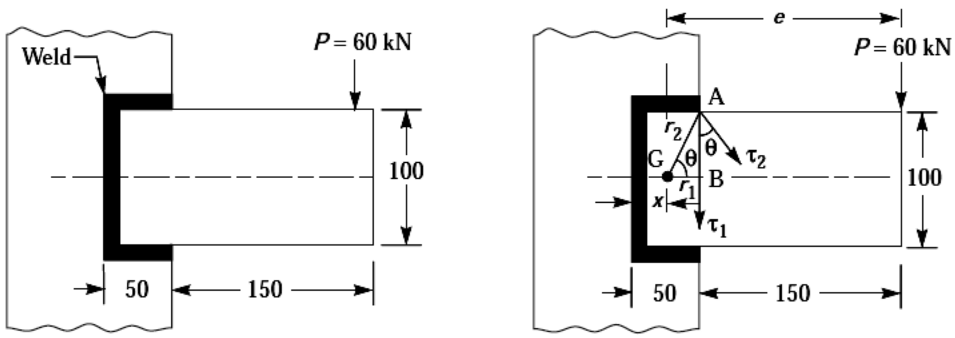


fig 3.11 All dimensions in mm.

First of all, let us find the centre of gravity (G) of the weld system, as shown in Fig.a. 3.11

Let x be the distance of centre of gravity (G) from the left hand edge of the weld system. From Table 10.7 we find that for a section as shown in Fig.b, 3.11

$$x = \frac{l^2}{2l + b} = \frac{50^2}{2 \times 50 + 100} = 12.5 \text{ mm}$$

and polar moment of inertia of the throat area of the weld system about G,

$$J = t\left(\frac{(b+2l)^3}{12} - \frac{l^2(b+l)^2}{b+2l}\right) = 0.707\ s\left[\frac{(100+2\times50)^3}{12} - \frac{50^2(100+50)^2}{100+2\times50}\right]\ (t=0.0707\ s)$$

$$= 0.707s[670 \times 10^3 - 281 \times 10^3] = 275 \times 10^3 smm^4$$

Distance of load from the centre of gravity (G) i.e. eccentricity,

$$e = 150 + 50 - 12.5 = 187.5 \text{ mm}$$

$$r_1 = BG = 50 - x = 50 - 12.5 = 37.5 \text{ mm}$$

$$AB = 100 / 2 = 50 \text{ mm}$$

We know that maximum radius of the weld,

$$r_2 = \sqrt{(AB)^2 + (BG)^2} = \sqrt{(50)^2 + (37.5)^2} = 62.5 \text{ mm}$$

$$\cos\theta = \frac{r_1}{r_2} = \frac{37.5}{62.5} = 0.6$$

We know that throat area of the weld system,

A = 2 × 0.707s × l + 0.707s × b = 0.707 s (2l + b)

= 0.707s (2 × 50 + 100) = 141.4 s mm2

Direct or primary shear stress,

$$\tau_1 = \frac{P}{A} + \frac{60 \times 10^3}{141.4\ s} = \frac{424}{s}\frac{N}{mm^2}$$

And shear stress due to the turning moment or secondary shear stress,

$$\tau_2 = \frac{P \times e \times r_2}{J} = \frac{60 \times 10^3 \times 187.5 \times 62.5}{275 \times 10^3} = \frac{2557}{s} N/mm^2$$

We know that the resultant shear stress

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1 \times \tau_2 \times \cos\theta}$$

$$140 = \sqrt{\left(\frac{424}{s}\right)^2 + \left(\frac{2557}{s}\right)^2 + 2 \times \frac{424}{s} \times \frac{2557}{s} \times 0.6} = \frac{2832}{s}$$

$$s = \frac{2832}{140} = 20.23 \text{ mm}$$

Result:

Size of the weld **s=20.23 mm**

26.Find the maximum shear stress induced in the weld of 6 mm size when a channel, as shown in fig, is welded to plate and loaded with 20 kN force at a distance of 200 mm. (16)(Nov/Dec – 2013)

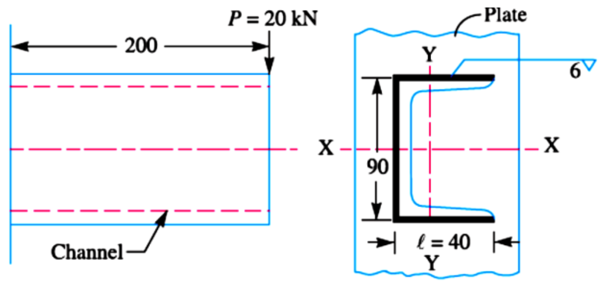


fig 3.12 All dimensions in mm.

Given:

s = 6 mm;

P = 20 kN = 20 × 103N ;

l= 40 mm ;

b = 90 mm

Solution:

Let t = Throat thickness.

First of all, let us find the centre of gravity (G) of the weld system

Let x be the distance of centre of gravity from the left hand edge of the weld system. we find that for a section as shown in Fig. 3.12

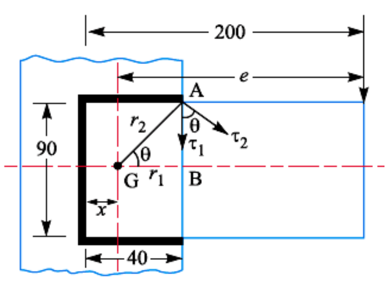
$$x = \frac{l^2}{2l + b} = \frac{40^2}{2 \times 40 + 90} = 9.4 \text{ mm}$$

and polar moment of inertia of the throat area of the weld system about G,

$$J = t\left(\frac{(b + 2l)^3}{12} - \frac{l^2(b + l)^2}{b + 2l}\right)$$

$$= 0.707\, s\left[\frac{(90 + 2 \times 40)^3}{12} - \frac{40^2(90 + 40)^2}{90 + 2 \times 40}\right]$$

$$= 0.707 \times 6\,[409.4 \times 10^3 - 159 \times 10^3] = 1062.2 \times 10^3 \text{ mm}^4$$



Distance of load from the centre of gravity (G), i.e. eccentricity,

$e = 200 - x = 200 - 9.4 = 190.6$ mm

r_1 = BG = 40 – x = 40 – 9.4 = 30.6 mm

AB = 90 / 2 = 45 mm

We know that maximum radius of the weld,

$$r_2 = \sqrt{(AB)^2 + (BG)^2} = \sqrt{(45)^2 + (30.6)^2} = 54.4 \text{ mm}$$

$$\cos\theta = \frac{r_1}{r_2} = \frac{30.6}{54.4} = 0.5625$$

We know that throat area of the weld system,

A = 2 × 0.707s × l + 0.707s × b = 0.707 s (2l+ b)

= 0.707 × 6 (2 × 40 + 90) = 721.14 mm2

Direct or primary shear stress,

$$\tau_1 = \frac{P}{A} + \frac{20 \times 10^3}{721.14} = 27.7 \text{ N/mm}^2$$

and shear stress due to the turning moment or secondary shear stress,

$$\tau_2 = \frac{P \times e \times r_2}{J} = \frac{20 \times 10^3 \times 190.6 \times 54.4}{1062.2 \times 10^3} = 195.2 \text{ N/mm}^2$$

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1 \times \tau_2 \times \cos\theta}$$

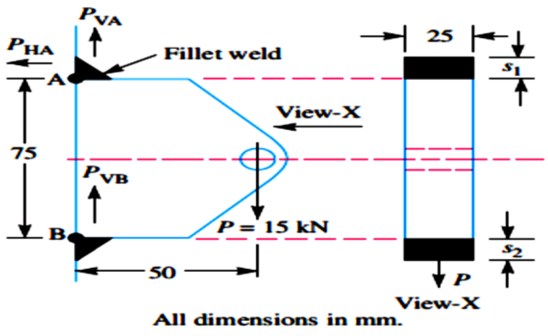
$$140 = \sqrt{(27.7)^2 + (195.2)^2 + 2 \times 27.7 \times 195.2 \times 0.5625}$$

$$= 212 \text{N/mm}^2 = \mathbf{212\ MPa}$$

Result:

Maximum shear stress=**212 MPa**

27.The bracket, as shown in Fig., is designed to carry a dead weight of P = 15 kN. What sizes of the fillet welds are required at the top and bottom of the bracket? Assume the forces act through the points A and B. The welds are produced by shielded arc welding process with a permissible strength of 150 MPa. (10)



Given:

P =15 kN

τ = 150 MPa = 150 N/mm2

l = 25 mm

To Find:

Size of the fillet weld

Solution:

In the joint, the weld at A is subjected to a vertical force P_{VA} and a horizontal force P_{HA}, whereas the weld at B is subjected only to a vertical force P_{VB}.

We know that

$$P_{VA} + P_{VB} = P \text{ and } P_{VA} = P_{VB}$$

Vertical force at A and B,

$$P_{VA} = P_{VB} = P/2 = 15/2 = 7.5 \text{ kN} = 7500 \text{ N}$$

The horizontal force at A may be obtained by taking moments about point B.

$$\therefore P_{HA} \times 75 = 15 \times 50 = 750$$

orP_{HA} =750 / 75 = 10 kN

Size of the fillet weld at the top of the bracket

Let s_1 =Size of the fillet weld at the top of the bracket in mm.

We know that the resultant force at A,

$$P_A = \sqrt{(P_{VA}^2) + (P_{HA}^2)} = \sqrt{7.5^2 + 10^2}$$
$$= 12.5 kN$$
$$= 12500\, N \text{ ....}(i)$$

We also know that the resultant force at A,

PA = Throat area × Permissible stress

$$= 0.707\, s_1 \times l \times \tau = 0.707\, s_1 \times 25 \times 150 = 2650\, s_1 \text{ ……….(ii)}$$

$A \quad \pi$

From equation (i) and (ii) we get

s_1 =12 500 / 2650 = 4.7 mm **Ans.** $= \quad r^2$

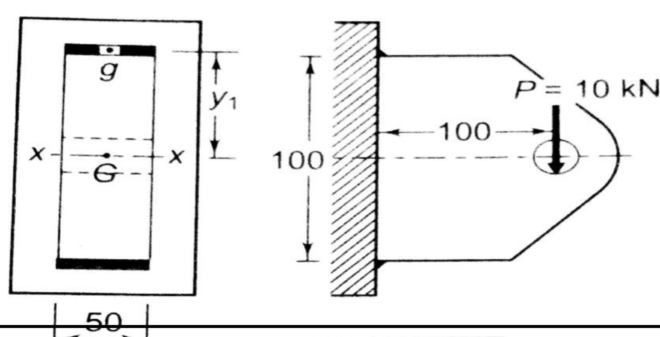
Size of fillet weld at the bottom of the bracket

Let s_2 =Size of the fillet weld at the bottom of the bracket.

The fillet weld at the bottom of the bracket is designed for the vertical force (P_{VB}) only. We know that

$$P_{VB} = 0.707\, s_2 \times l \times \tau$$
$$7500 = 0.707\, s_2 \times 25 \times 150 = 2650\, s_2$$
$$\therefore s_2 = 7500 / 2650 = 2.83 \text{ mm } \textbf{Ans.}$$

28.A bracket is welded to the vertical column by means of two fillet welds as shown in fig., Determine the size of the welds, if the permissible shear stress is limited to 70 N/mm²(April/May 17)



Given $P = 10$ kN $\tau = 70$ N/mm²

To find:

Size of the weld

Solution:

Step I *Primary shear stress*

The area of the two welds is given by,

$$A = 2\,(50\,t) = (100\,t)\ \text{mm}^2$$

The primary shear stress is given by,

$$\tau_1 = \frac{P}{A} = \frac{10 \times 10^3}{(100t)} = \frac{100}{t}\ \text{N/mm}^2 \qquad \text{(i)}$$

Step II *Bending stress*

The moment of inertia of the top weld about the X-axis passing through its centre of gravity g is $(50t^3/12)$. This moment of inertia is shifted to the centre of gravity of the two welds at G by the parallel axis theorem. It is given by,

$$I_{xx} = \frac{50t^3}{12} + Ay_1^2 = \frac{50t^3}{12} + (50t)(50)^2\ \text{mm}^4$$

The dimension t is very small compared with 50. The term containing t^3 is neglected. Therefore,

$$I_{xx} = (50\,t)\,(50)^2 = (50^3\,t)\ \text{mm}^4$$

Since there are two welds,

$$I = 2I_{xx} = 2(50^3 t) = (250\,000t)\ \text{mm}^4$$

The bending stress in the top weld is given by,

$$\sigma_b = \frac{M_b y}{I} = \frac{(10 \times 10^3 \times 100)(50)}{(250\,000t)}$$

$$= \left(\frac{200}{t}\right)\text{N/mm}^2 \qquad \text{(ii)}$$

$$\tau = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau_1)^2} = \sqrt{\left(\frac{200}{2t}\right)^2 + \left(\frac{100}{t}\right)^2}$$

$$= \frac{141.42}{t} \text{ N/mm}^2$$

Step IV *Size of weld*

The permissible shear stress in the weld is 70 N/mm². Therefore,

$$\frac{141.42}{t} = 70 \quad \text{or} \quad t = 2.02 \text{ mm}$$

$$h = \frac{t}{0.707} = \frac{2.02}{0.707} = 2.86 \quad \text{or} \quad 3 \text{ mm}$$

29.A plate 100 mm wide and 12.5 mm thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of 50 kN. Find the length of the weldso that the maximum stress does not exceed 56 MPa. Consider the joint first under static loading andthen under fatigue loading. (8)(Nov/Dec 2021)

Given:

Width = 100 mm ;

Thickness = 12.5 mm ;

P = 50 kN = 50 × 103N ;

τ = 56 MPa = 56 N/mm2

Solution:

Length of weld for static loading

Let l = Length of weld, and

s = Size of weld = Plate thickness

= 12.5 mm ... (Given)

We know that the maximum load which theplates can carry for double parallel fillet welds (P),

$$50 \times 10^3 = 1.414 \text{ s} \times l \times \tau$$

$$= 1.414 \times 12.5 \times l \times 56 = 990l$$

$$\therefore l = 50 \times 10^3 / 990 = 50.5 \text{ mm}$$

Adding 12.5 mm for starting and stopping ofweld run, we have

l= 50.5 + 12.5 = **63 mm Ans.**

Length of weld for fatigue loading ,we find that the stressconcentration factor for parallel fillet welding is 2.7.

∴ Permissible shear stress,

τ = 56 / 2.7 = 20.74 N/mm2

We know that the maximum load which the plates can carry for double parallel fillet welds (P),

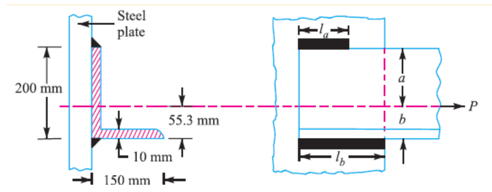
$$50 \times 10^3 = 1.414 \text{ s} \times l \times \tau = 1.414 \times 12.5 \times l \times 20.74 = 367 \text{ l}$$

$$\therefore l = 50 \times 10^3 / 367 = 136.2 \text{ mm}$$

Adding 12.5 for starting and stopping of weld run, we have

l= 136.2 + 12.5 = **148.7 mm Ans.**

30.A 200 × 150 × 10 mm angle is to be welded to a steel plate by fillet welds as shown in Fig. 10.21. If the angle is subjected to a static load of 200 kN, find the length of weld at the top and bottom. The allowable shear stress for static loading may be taken as 75 MPa.(Nov/Dec-2004)



Given :

(a + b)= 200 mm ;

P = 200 kN = 200 × 10^3N ;

τ = 75 MPa = 75 N/mm^2

Solution:

Let l_a= Length of weld at the top,

l_b= Length of weld at the bottom, and

l = Total length of the weld = $la + lb$

Since the thickness of the angle is 10 mm, therefore size of weld,

s = 10 mm

We know that for a single parallel fillet weld, the maximum load (P),

200 × 10^3 = 0.707 s × l× τ = 0.707 × 10 × l × 75 = 530.25 l

∴l= 200 × 10^3 / 530.25 = 377 mm

Or $la + lb$= 377 mm

Now let us find out the position of the centroidal axis.

Let b = Distance of centroidal axis from the bottom of the angle.

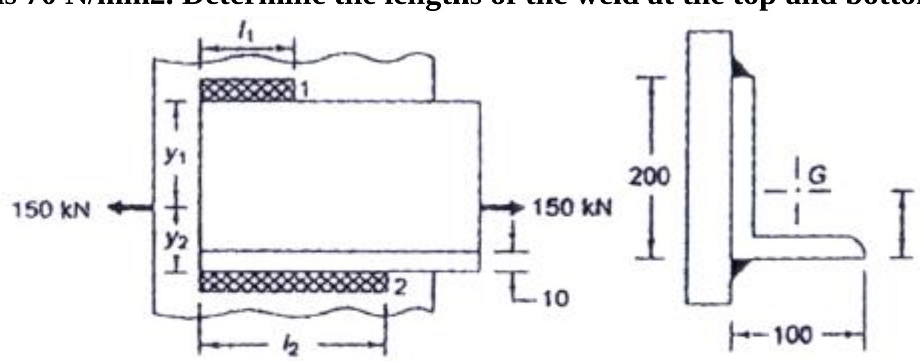
$$b = \frac{(200 - 10)10 \times 95 + 150 \times 10 \times 5}{190 \times 10 + 150 \times 10} = 60.88\, mm$$

anda= 200 – 55.3 = 144.7 mm

We know that$l_a = \frac{l \times b}{a+b} = \frac{377 \times 60.88}{200} = 114.76$

$$l_b = l - l_a = 377 - 114.76 = 262.24\, mm$$

31.An ISA 200 x 100 x 100 angle is welded to a steel plate by means of fillet welds as shown in Fig . The angle is subjected to a static force of 150 kN and permissible shear stress for the weld is 70 N/mm2. Determine the lengths of the weld at the top and bottom.(Nov/Dec 2017)



Given P = 150 kN, τ = 70 N/mm^2, h= 10mm
Solution:
Step1: **Total length of weld**

The total length (l) of the weld required to withstand
the load of 150 kN is given by Eq..

$P = 0.707\ hl\tau$

or 150x 103 = 0.707 (10) l(70)
1= 303.09 mm...................... (i)

StepII: Weld lengths l_1 and l_2

FromEq

$l_1 y_1 = l_2\ y_2$ or $l_1\ (200 - 71.8) = l_2\ (71.8)$
$128.2\ l_1 = 71.8\ l_2$............................ (ii)
Also, $l_1 + l_2 = l$ = 303.09 mm..... (iii)
From (ii) and (iii),
l_1 = 108.81 mm and l_2 = 194.28 mm

32.A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. 10.15. The maximum tensile and shear stresses are 70 MPa and 56 MPa respectively. Find the length of each parallel fillet weld, if the joint is subjected to both static and fatigue loading.(Aprl/May-2008)(NOV/DEC'2022)

Given :

Width = 75 mm ;
Thickness = 12.5 mm ;
σ = 70 MPa = 70 N/mm2 ;
τ = 56 MPa = 56 N/mm2.

Solution:

The effective length of weld (l1) for the transverse weld may beobtained by subtracting 12.5 mm from the width of the plate.

$\therefore l_1 = 75 - 12.5 = 62.5$ mm

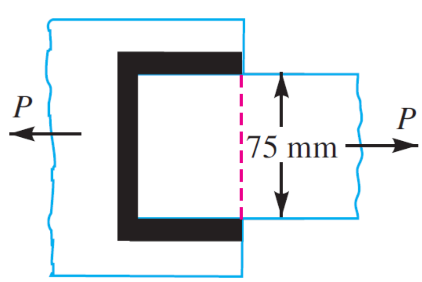


fig 3.13

Length of each parallel fillet for static loading

Let l_2= Length of each parallel fillet.

We know that the maximum load which the plate can carry is from fig 3.13

$$P = \text{Area} \times \text{Stress}$$

$$= 75 \times 12.5 \times 70 = 65\,625 \text{ N}$$

Load carried by single transverse weld,

$$P_1 = 0.707\, s \times l1 \times \sigma t$$

$$= 0.707 \times 12.5 \times 62.5 \times 70 = \mathbf{38\,664\ N}$$

and the load carried by double parallel fillet weld,

$$P_2 = 1.414\, s \times l2 \times \tau$$

$$= 1.414 \times 12.5 \times l2 \times 56 = \mathbf{990\ l2\ N}$$

Load carried by the joint (P),

$$65\,625 = P_1 + P_2$$

$$= 38\,664 + 990\, l_2 \text{ or}$$

$$l_2 = \mathbf{27.2\ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 27.2 + 12.5 = 39.7 \text{ say } \mathbf{40\ mm\ Ans.}$$

Length of each parallel fillet for fatigue loading

we find that the stress concentration factor for transverse welds is 1.5 and forparallel fillet welds is 2.7.

∴ Permissible tensile stress,

$$\sigma_t = 70 / 1.5 = 46.7 \text{ N/mm2}$$

and permissible shear stress,

$$\tau = 56 / 2.7 = 20.74 \text{ N/mm}^2$$

Load carried by single transverse weld,

$$P_1 = 0.707\, s \times l_1 \times \sigma_t$$

$$= 0.707 \times 12.5 \times 62.5 \times 46.7 = 25\,795 \text{ N}$$

and load carried by double parallel fillet weld,

$$P_2 = 1.414\, s \times l_2 \times \tau$$

$$= 1.414 \times 12.5\, l_2 \times 20.74 = 366\, l2 \text{ N}$$

∴ Load carried by the joint (P),

$$65\,625 = P_1 + P_2 = 25\,795 + 366\, l_2 \text{or } l_2 = 108.8 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 108.8 + 12.5 = 121.3 \text{ mm Ans.}$$

Result:

$$\mathbf{l_2 = 121.3\ mm}$$

33.Determine the length of the weld run for a plate of size 120 mm wide and 15 mm thick to be welded to another plate by means of 1. A single transverse weld; and 2. Double parallel fillet welds when the joint is subjected to variable loads. (16)

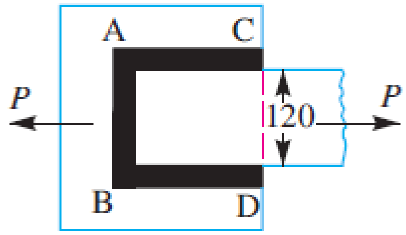


fig 3.14

Given :

Width = 120 mm ; Thickness = 15 mm

In Fig.3.14represents the single transverse weld and ACand BD represents double parallel fillet welds.

Solution:

1**. Length of the weld run for a single transverse weld**

The effective length of the weld run (l_1) for a single transverse weld may be obtained by subtracting12.5 mm from the width of the plate.

$$\therefore l_1 = 120 - 12.5 = 107.5 \text{ mm Ans.}$$

2. Length of the weld run for a double parallel fillet weld subjected to variable loads

Let l_2= Length of weld run for each parallel fillet,

ands= Size of weld = Thickness of plate = 15 mm

Assuming the tensile stress as 70 MPa or N/mm2 and shear stress as 56 MPa or N/mm2 for staticloading.

We know that the maximum load which the plate can carry is

$$P = \text{Area} \times \text{Stress} = 120 \times 15 \times 70 = 126 \times 10^3 \text{ N}$$

From Table ,we find that the stress concentration factor for transverse weld is 1.5 and for parallel fillet welds is 2.7.

∴ Permissible tensile stress,

$$\sigma_t = 70 / 1.5 = 46.7 \text{ N/mm}^2$$

and permissible shear stress,

$$\tau = 56 / 2.7 = 20.74 \text{ N/mm2}$$

∴ Load carried by single transverse weld,

$$P_1 = 0.707\, s \times l_1 \times \sigma_t$$

$$= 0.707 \times 15 \times 107.5 \times 46.7 = 53\,240 \text{ N}$$

and load carried by double parallel fillet weld,

$$P_2 = 1.414\, s \times l_2 \times \tau$$

$$= 1.414 \times 15 \times l2 \times 20.74 = 440\, l2 \text{ N}$$

∴ Load carried by the joint (P),

$$126 \times 10^3 = P_1 + P_2$$

$$= 53\,240 + 440\, l_2 \text{or } l2 = 165.4 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 165.4 + 12.5 = 177.9 \text{ say } 178 \text{ mm Ans.}$$

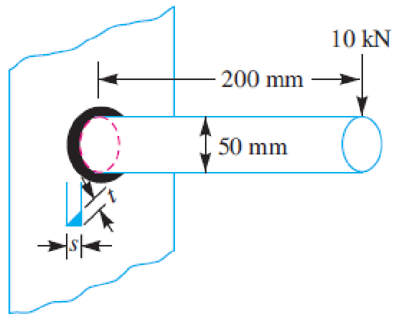
Result:

l_2 = 178 mm Ans.

34.The figure below shows a cylindrical rod of 50 mm diameter, welded to a flat plate. The cylindrical fillet weld is loaded eccentrically by a force of 10 KN acting at 200 mm from the welded end. If the size of the weld is 20 mm, determine the maximum normal stress in the weld.

(or)

A 50 mm diameter solid shaft is welded to a flat plate as shown in Fig. If the size of the weld is 15 mm, find the maximum normal and shear stress in the weld.(16)(May/June – 2014) (Nov/Dec 2018)



Given :

D = 50 mm ;

s = 15 mm ;

P = 10 kN= 10 000 N ;

e = 200 mm

Solution:

Let t = Throat thickness.

The joint, as shown in Fig., is subjected to directshear stress and the bending stress. We know that the throat areafor a circular fillet weld,

$$A = t \times \pi D$$
$$= 0.707\, s \times \pi D$$
$$= 0.707 \times 15 \times \pi \times 50 = 1666 \text{ mm}^2$$

∴Direct shear stress,

$$\tau = \frac{P}{A}$$
$$= \frac{100 \times 10^3}{1666} = 6 \frac{N}{mm^2} = 6 \text{ MPa}$$

We know that bending moment,

$$M = P \times e = 10\,000 \times 200 = 2 \times 10^6 \text{ N-mm}$$

From PGDB : we find that for a circular section, section modulus,

$$Z = \left(\frac{\pi . t . D^2}{4}\right)$$

$$= \frac{\pi \times 0.707\, s \times D^2}{4}$$

$$= \frac{\pi \times 0.707\ s \times 50^2}{4}$$

$$\mathbf{Z = 20825\ mm^3}$$

Bending stress

$$\sigma_b = \frac{M}{Z}$$

$$= \frac{2 \times 10^6}{20825} = 96 \frac{N}{mm^2} = 96\ MPa$$

Maximum normal stress

We know that the maximum normal stress,

$$\sigma_{t(max)} = \frac{1}{2}\sigma_b \sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2} \times 96 + \frac{1}{2}\sqrt{96^2 + 4 \times 6^2}$$

$$= 48 + 48.4 = \mathbf{96.4\ MPa}$$

Maximum shear stress

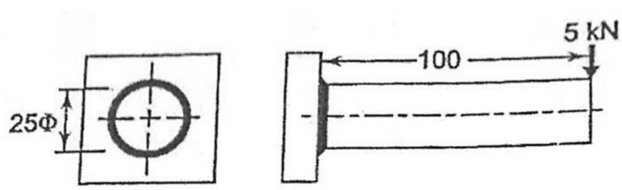
We know that the maximum normal stress,

$$\tau_{(max)} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{96^2 + 4 \times 6^2}$$

$$\boldsymbol{\tau_{(max)} = 48.4\ MPa}$$

34. A solid circular beam, 25 mm in diameter, is welded to a support by means of a fillet weld as shown in Fig. 13b. Determine the leg dimensions of the weld, if the permissible shear stress is 95 MPa Nov/Dec-20.Aprl/May-21



All dimension are in "mm"

Given :

D = 25 mm ;

P = 5 kN= 50 00 N ;

e = 100 mm

Solution:

Let t = Throat thickness.

The joint, as shown in Fig., is subjected to directshear stress and the bending stress. We know that the throat areafor a circular fillet weld,

$$A = t \times \pi D$$
$$= 0.707\ s \times \pi D$$
$$= 0.707\ s \times \pi \times 25 = 55.52\ s\ mm^2$$

∴Direct shear stress,

$$\tau = \frac{P}{A}$$
$$= \frac{5000}{55.52\ s} = 90/s \frac{N}{mm^2}$$

We know that bending moment,

$$M = P \times e = 5000 \times 100 = 500 \times 10^3\ N\text{-}mm$$

From PGDB : we find that for a circular section, section modulus,

$$Z = \left(\frac{\pi. t. D^2}{4}\right)$$

Bending stress

$$\sigma_b = \frac{M}{Z}$$

$$= \frac{5000\ X\ 100}{\Pi/4\ X\ 625\ X\ 0.707\ s} = 1440.7/s\ \frac{N}{mm^2}$$

Maximum shear stress

We know that the maximum normal stress,

$$\tau_{(max)} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$95 = \frac{1}{2}\sqrt{(720.35/s)^2 + (90/s)^{\quad 2}}$$

$$\mathbf{S = 7.64\ MPa}$$

35.A welded joint as shown in Fig. 10.24, is subjected to an eccentric load of 2 kN. Find the size of weld, if the maximum shear stress in the weld is 25 MPa. (NOV/DEC 2011)

Solution. Given: $P = 2kN = 2000$ N ; $e = 120$ mm ; $l = 40$ mm ; $\tau_{max} = 25$ MPa $= 25$ N/mm^2

Let s = Size of weld in mm, and

t = Throat thickness.

The joint, as shown in Fig. 3.15 will be subjected to direct shear stress due to the shear force, $P = 2000$ N and bending stress due to the bending moment of $P \times e$.

We know that area at the throat,

$$A = 2t \times l = 2 \times 0.707\ s \times l$$
$$= 1.414\ s \times l$$
$$= 1.414\ s \times 40 = 56.56 \times s\ mm^2$$

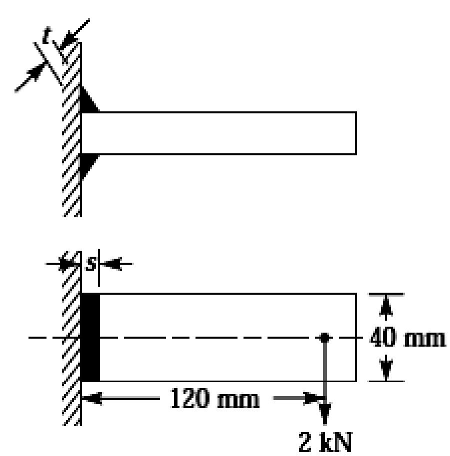


fig 3.15

$\therefore$ Shear stress, $\tau = \frac{P}{A} = \frac{2000}{56.56 \times s} = \frac{35.4}{s}$ N/mm^2

Bending moment, $M = P \times e = 2000 \times 120 = 240 \times 10^3$ N-mm

Section modulus of the weld through the throat,

$$Z = \frac{s \times l^2}{4.242} = \frac{s\,(40)^2}{4.242} = 377 \times s \text{ mm}^3$$

$\therefore$ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{240 \times 10^3}{377 \times s} = \frac{636.6}{s}$ N/mm^2

We know that maximum shear stress (τ_{max}),

$$25 = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{636.6}{s}\right)^2 + 4\left(\frac{35.4}{s}\right)^2} = \frac{320.3}{s}$$

$\therefore$ $s = 320.3 / 25 = 12.8$ mm **Ans.**

36. A butt welding joint with ground and flush surface is subjected to tensile load which varies from 25KN to 75KN. Plates are 10mm thick. Determine the length of weld required for over 2000000 cycles. (Aprl/May-15)

Solution.

Method 1 :

The load varies from 25 kN to 75 kN, hence the load variation factor K (defined as F_{min}/F_{max}) is $\frac{25}{75} = 0.333$.

For steady load $K = 1$, while for variation from O to F, $K = 0$. Thus the loading in the present case is less damaging than $K = 0$ and hence from Table 10.6, a design stress lying between 93 and 110, say 100, may be used. This gives

Required weld length $= \frac{F_{max}}{t \times \sigma} = \frac{75 \times 1000}{10 \times 100} = 75$ mm

Method 2 :

Mean load, $F_m = \frac{75 + 25}{2} = 50$ kN

Alternating load, $F_a = \frac{75 - 25}{2} = 25$ kN

Soderberg's equation gives $\frac{1}{A_w}\left[F_m + \frac{\sigma_y}{A\sigma_{en}} K_f F_a\right] = \frac{\sigma_y}{\text{F.O.S.}}$

where $\sigma_u = 410$ MPa for $E60$ weld electrode

$K_f = 1$ for ground and flush butt weld

$\sigma_y \simeq 0.7\,\sigma_u$

$\sigma_{en} \simeq 0.5\,\sigma_u$

$A = 0.7$ to 1 for axial load

F.O.S. = 3.75 on σ_u

Substitution gives $$A_w = \frac{3.75}{410}\left[50 + \left(\frac{0.7 \times 410}{0.8 \times 0.5 \times 410}\right)25\right] \times 1000 = 857 \text{ mm}^2$$

Length of weld $= \dfrac{A}{t} = \dfrac{857}{10} = 86$ mm.

37.The fig. below shows an angle welded to a column and carries a static load F as shown. Determine the ratio of the weld lengths La and Lb and Fa and Fb in terms of F [Apr/May-15]

Sometimes unsymmetrical sections such as angles, channels, T-sections etc, welded on the flange edges are loaded axially in fig 3.16 In such cases, the lengths of weld should be proportioned in such a way that the sum of resisting moments of the welds about the gravity axis is zero. Consider an angle section.

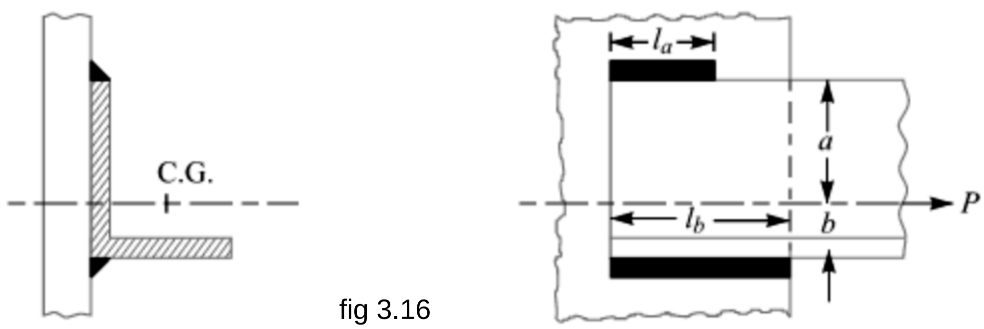


fig 3.16

Let l_a = Length of weld at the top,

l_b = Length of weld at the bottom,

l = Total length of weld $= l_a + l_b$

P = Axial load,

a = Distance of top weld from gravity axis,

b = Distance of bottom weld from gravity axis, and

f = Resistance offered by the weld per unit length.

$\therefore$ Moment of the top weld about gravity axis

$$= l_a \times f \times a$$

and moment of the bottom weld about gravity axis

$$= l_b \times f \times b$$

Since the sum of the moments of the weld about the gravity axis must be zero, therefore,

$$l_a \times f \times a - l_b \times f \times b = 0$$

or $$l_a \times a = l_b \times b \quad \ldots(i)$$

We know that $$l = l_a + l_b \quad \ldots(ii)$$

$\therefore$ From equations (*i*) and (*ii*), we have

$$l_a = \frac{l \times b}{a + b}, \quad \text{and} \quad l_b = \frac{l \times a}{a + b}$$

38. A bracket carrying a load of 15 kN is to be welded as shown in Fig.
Find the size of weld required if the allowable shear stress is not to exceed 80 MPa.

Given : P = 15 kN = 15 × 103 N ; τ = 80 MPa = 80 N/mm2 ; b = 80 mm ;
l = 50 mm; e = 125 mm
Let s = Size of weld in mm, and
t = Throat thickness.

We know that the throat area,

A = 2 × t × l = 2 × 0.707 s × l
= 1.414 s × l = 1.414 × s × 50 = 70.7 s mm2

∴ **Direct or primary shear stress,**

$$\tau_1 = \frac{P}{A} = \frac{15 \times 10^3}{70.7\, s} = \frac{212}{s}\ \text{N/mm}^2$$

we find that for such a section, the polar moment of inertia of the throat area of the weld about G is

$$J = \frac{t.l\,(3b^2 + l^2)}{6} = \frac{0.707\, s \times 50\,[3\,(80)^2 + (50)^2]}{6}\ \text{mm}^4$$

$$= 127\,850\, s\ \text{mm}^4 \qquad \text{... } (\because\ t = 0.707\, s)$$

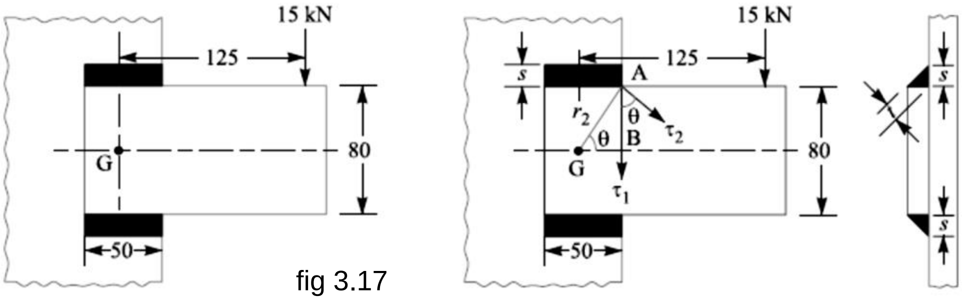


fig 3.17

from the fig 3.17 we find that AB = 40 mm and BG = r1 = 25 mm.∴ Maximum radius of the weld,

$$r_2 = \sqrt{(AB)^2 + (BG)^2} = \sqrt{(40)^2 + (25)^2} = 47\ \text{mm}$$

Shear stress due to the turning moment *i.e.* secondary shear stress,

$$\tau_2 = \frac{P \times e \times r_2}{J} = \frac{15 \times 10^3 \times 125 \times 47}{127\,850\, s} = \frac{689.3}{s}\ \text{N/mm}^2$$

and $$\cos\theta = \frac{r_1}{r_2} = \frac{25}{47} = 0.532$$

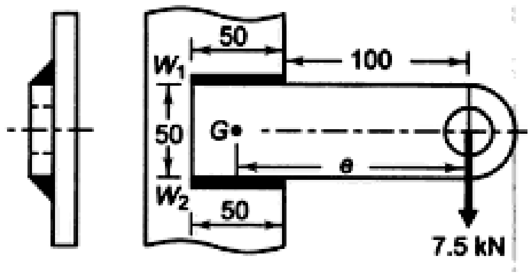
We know that resultant shear stress,

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\,\tau_1 \times \tau_2 \cos\theta}$$

$$80 = \sqrt{\left(\frac{212}{s}\right)^2 + \left(\frac{689.3}{s}\right)^2 + 2 \times \frac{212}{s} \times \frac{689.3}{s} \times 0.532} = \frac{822}{s}$$

$$\therefore \qquad s = 822 / 80 = 10.3\ \text{mm}\ \ \textbf{Ans.}$$

39. A welded connection as shown in fig. is subjected to an eccentric force of 7.5 KN. Determine the size of welds if the permissible shear stress for the weld is 100 N/mm^2. Assume static conditions.(NOV/DEC-2018)



Solution

Given $P = 7.5$ kN $\tau = 100$ N/mm^2

Step I Primary shear stress

Suppose t is the throat of each weld. There are two welds W_1 and W_2 and their throat area is given by,

$$A = 2(50t) = (100t) \text{ mm}^2$$

From Eq. the primary shear stress is given by,

$$\tau_1 = \frac{P}{A} = \frac{7500}{(100t)} = \left(\frac{75}{t}\right) \text{N/mm}^2 \quad \text{(a)}$$

Step II Secondary shear stress

The two welds are symmetrical and G is the centre of gravity of the two welds.

$$e = 25 + 100 = 125 \text{ mm}$$

$$M = P \times e = (7500)(125) = 937\,500 \text{ N-mm} \quad \text{(i)}$$

The distance r of the farthest point in the weld from the centre of gravity is given by

$$r = \sqrt{(25)^2 + (25)^2} = 35.36 \text{ mm} \quad \text{(ii)}$$

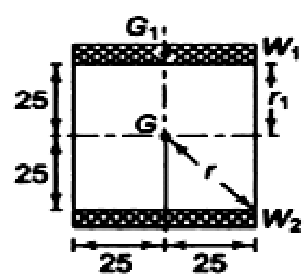


fig 3.18

From Eq. the polar moment of inertia J_1 of the weld W_1 about G is given by

$$J_1 = A\left[\frac{l^2}{12} + r_1^2\right] = (50t) \times \left[\frac{50^2}{12} + 25^2\right]$$
$$= (41\ 667t)\ \text{mm}^4$$

Due to symmetry, the polar moment of inertia of the two welds (J) is given by

$$J = J_1 + J_2 = 2J_1 = 2(41\ 667t) = (83\ 334t)\ \text{mm}^4$$

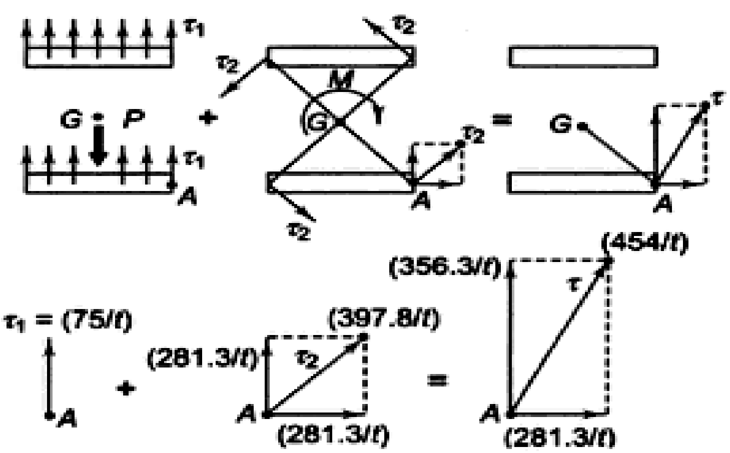
From Eq. the secondary shear stress is given by

$$\tau_2 = \frac{Mr}{J} = \frac{(937\ 500)(35.36)}{(83\ 334t)} = \left(\frac{397.8}{t}\right)\ \text{N/mm}^2$$

Step III Resultant shear stress

Figure 3.18 shows the primary and secondary shear stresses. The vertical and horizontal components of these shear stresses are added and the resultant shear stress is determined. Therefore,

$$\tau = \left(\frac{454}{t}\right)\ \text{N/mm}^2$$



Step IV Size of weld

Since the permissible shear stress for the weld material is 100 N/mm^2,

$$\left(\frac{454}{t}\right) = 100 \quad \text{or} \quad t = 4.54 \text{ or } 5 \text{ mm}$$

39.Design a sleeve and cotter joint to resist a tensile load of 60 kN. All parts of the joint are made of the same material with the following allowable stresses :σt= 60 MPa ; τ = 70 MPa ; and σc= 125 MPa.

Given :

P = 60 kN = 60 × 103 N ;

σt= 60 MPa = 60 N/mm^2 ;

τ = 70 MPa = 70 N/mm^2 ;

σc= 125 MPa = 125 N/mm^2

Solution:

1. Diameter of the rods

Let d = Diameter of the rods.

Considering the failure of the rods in tension. We know that load (P),

$$60 \times 10^3 = \frac{\pi}{4} \times d^2 \times 60 = 47.13\ d^2$$

$\therefore d^2 = 60 \times 10^3 / 47.13 = 1273$ or d = 35.7 say 36 mm **Ans.**

2. Diameter of enlarged end of rod and thickness of cotter

Let d_2 = Diameter of enlarged end of rod, and

t = Thickness of cotter. It may be taken as $d_2 / 4$.

Considering the failure of the rod in tension across the weakest section (i.e. slot). We know that load (P),

$$60 \times 10^3 = \left[\frac{\pi}{4} \times \{(d_2)^2 - d_2 \times t\}\right] \sigma_t$$

$$= \left[\frac{\pi}{4} \times \left\{(d_2)^2 - d_2 \times \frac{d_2}{4}\right\}\right] 60 = 32.13\ d_2^2$$

$\therefore (d_2)^2 = 60 \times 10^3 / 32.13 = 1867$ or d2 = 43.2 say 44 mm **Ans.**

and thickness of cotter,

$$t = d_2/4$$

d = 11mmAns.

Let us now check the induced crushing stress in the rod or cotter. We know that load (P),

$$60 \times 10^3 = d_2 \times t \times \sigma c = 44 \times 11 \times \sigma_c = 484\ \sigma_c$$

$$\therefore \sigma_c = 60 \times 10^3 / 484 = 124\ N/mm^2$$

Since the induced crushing stress is less than the given value of 125 N/mm^2, therefore the dimensionsd2 and t are within safe limits.

3. Outside diameter of sleeve

Let d_1 = Outside diameter of sleeve.

Considering the failure of sleeve in tension across the slot. We know that load (P)

$$60 \times 10^3 = \left[\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)t\right] \sigma_t$$

$$= \left[\frac{\pi}{4}[(d_1)^2 - (44)^2] - (d_1 - 44)11\right] 60$$

$$\frac{60 \times 10^3}{60} = 0.7854\ (d_1)^2 - 1520.7 - 11d_1 + 484$$

$$\therefore (d_1)^2 - 14\ d_1 - 2593 = 0$$

$$d_1 = \frac{14 \pm \sqrt{(14^2) + 4 \times 2593}}{2}$$

$$= \frac{14 \pm 102.8}{2}$$

$$d_1 = \mathbf{58.4 say 60 mm Ans}$$

4. Width of cotter

Let b = Width of cotter.

Considering the failure of cotter in shear. Since the cotter is in double shear, therefore load (P),

$$60 \times 10^3 = 2\ b \times t \times \tau = 2 \times b \times 11 \times 70 = 1540\ b$$

$\therefore b = 60 \times 10^3 / 1540 = 38.96$ say **40 mm Ans.**

5. Distance of the rod from the beginning to the cotter hole (inside the sleeve end)

Let a = Required distance.

Considering the failure of the rod end in shear. Since the rod end is in double shear, therefore load (P),

$$60 \times 10^3 = 2\ a \times d2 \times \tau = 2\ a \times 44 \times 70 = 6160\ a$$

$\therefore a = 60 \times 10^3 / 6160 = 9.74$ say 10 mm **Ans.**

6. Distance of the rod end from its end to the cotter hole

Let c = Required distance.

Considering the failure of the sleeve end in shear. Since the sleeve end is in double shear, therefore load (P),

$$60 \times 10^3 = 2\ (d1 - d2)\ c \times \tau = 2\ (60 - 44)\ c \times 70 = 2240\ c$$

$\therefore c = 60 \times 10^3 / 2240 = 26.78$ say 28 mm **Ans.**

RIVETED JOINTS FOR STRUCTURES:

40.A double riveted lap joint is made between 15 mm thick plates. The rivet diameter and pitch are 25 mm and 75 mm respectively. If the ultimate stresses are 400 MPa in tension,320 MPa in shear and 640 MPa in crushing, find the minimum force per pitch which will rupturethe joint.If the above joint is subjected to a load such that the factor of safety is 4, find out the actualstresses developed in the plates and the rivets.(16)

Given :

t = 15 mm ;

d = 25 mm ;

p = 75 mm ;

σ_{tu}= 400 MPa = 400 N/mm^2 ;

τ_u= 320MPa = 320 N/mm^2 ;

σ_{cu}= 640 MPa = 640 N/mm^2

Solution:

Minimum force per pitch which will rupture the joint

Since the ultimate stresses are given, therefore we shall find the ultimate values of the resistances of the joint. We know that ultimate tearing resistance of the plate per pitch,

$$P_{tu} = (p - d)\, t \times \sigma_{tu}$$
$$= (75 - 25)15 \times 400 = 300\,000\ N$$

Ultimate shearing resistance of the rivets per pitch,

$$P_{su} = n \times \frac{\pi}{4} d^2 \times \tau_u$$
$$= 2 \times \frac{\pi}{4} 25^2 \times 320 = 314200 N$$

and ultimate crushing resistance of the rivets per pitch,

$$P_{cu} = n \times d \times t \times \sigma_{cu}$$
$$= 2 \times 25 \times 640 = 480000\ N$$

From above we see that the minimum force per pitch which will rupture the joint is 300000 N or 300 kN.Ans.

Actual stresses produced in the plates and rivets

Since the factor of safety is 4, therefore safe load per pitch length of the joint

$$= 300\,000/4 = 75\,000 \text{ N}$$

Let σ_{ta}, $\tau_a$and σ_{ca}be the actual tearing, shearing and crushing stresses produced with a safeload of 75 000 N in tearing, shearing and crushing.

We know that actual tearing resistance of the plates (P_{ta}),

$$75\,000 = (p - d)\ t \times \sigma_{ta}$$
$$= (75 - 25)15 \times \sigma_{ta} = 750\ \sigma_{ta}$$
$$\therefore \sigma_{ta} = 75\,000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa Ans.}$$

Actual shearing resistance of the rivets (P_{sa}),

$$75000 = n \times \frac{\pi}{4} d^2 \times \tau_a$$
$$= 2 \times \frac{\pi}{4} 25^2 \times \tau_a = 982\ \tau_a$$

$$\tau_a = \frac{75000}{982} = 76.4 \frac{N}{mm^2} = 100\ MPa$$

and actual crushing resistance of the rivets *(P_{ca})*,

$$75\,000 = n \times d \times t \times \sigma_{ca}$$

$$= 2 \times 25 \times 15 \times \sigma_{ca} = 750\ \sigma_{ca}$$

$$\therefore \sigma_{ca} = 75000 / 750 = 100 \text{ N/mm}^2 = \mathbf{100\ MPa\ Ans.}$$

41.Find the efficiency of the following riveted joints :

1. Single riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 50 mm.

2. Double riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 65 mm. AssumePermissible tensile stress in plate = 120 MPa, Permissible shearing stress in rivets = 90 MPaPermissible crushing stress in rivets = 180 MPa (16) (April/May 2019)

Given :

t = 6 mm ; d = 20 mm ;

σt = 120 MPa = 120 N/mm^2 ;

τ = 90 MPa = 90 N/mm^2 ;

σc = 180 MPa = 180 N/mm^2

Solution:

1. Efficiency of the first joint

Pitch, p = 50 mm ...(Given)

First of all, let us find the tearing resistance of the plate, shearing and crushing resistances of the rivets.

(i) Tearing resistance of the plate

We know that the tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (50 - 20)\, 6 \times 120 = 21\,600\ N$$

(ii) Shearing resistance of the rivet

Since the joint is a single riveted lap joint, therefore the strength of one rivet in single shear is taken. We know that shearing resistance of one rivet,

$$P_s = n \times \frac{\pi}{4} d^2 \times \tau = 2 \times \frac{\pi}{4} 20^2 \times 90 = 28278\ N$$

(iii) Crushing resistance of the rivet

Since the joint is a single riveted, therefore strength of one rivet is taken. We know that crushing resistance of one rivet,

$$P_c = d \times t \times \sigma_c = 20 \times 6 \times 180 = 21\,600\ N$$

∴ Strength of the joint

$$= \text{Least of } P_t, P_s \text{ and } P_c = 21\,600\ N$$

We know that strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 50 \times 6 \times 120 = 36\,000\ N$$

∴ Efficiency of the joint, $\eta = \frac{Least\ of\ Pt, Ps\ and\ Pc}{P} = \frac{21600}{36000} = \mathbf{0.6\ or\ 60\%\ Ans}$

2. Efficiency of the second joint

Pitch, p = 65 mm ...(Given)

(i) Tearing resistance of the plate,

We know that the tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (65 - 20)\, 6 \times 120 = 32\,400\ N$$

(ii) Shearing resistance of the rivets

Since the joint is double riveted lap joint, therefore strength of two rivets in single shear is taken. We know that shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} d^2 \times \tau = 2 \times \frac{\pi}{4} 20^2 \times 90 = 56556 N$$

(iii) Crushing resistance of the rivet

Since the joint is double riveted, therefore strength of two rivets is taken. We know that crushing resistance of rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 20 \times 6 \times 180 = 43\ 200\ N$$

$$\therefore \text{Strength of the joint} = \textbf{Least of Pt, Ps and Pc = 32 400 N}$$

We know that the strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 65 \times 6 \times 120 = 46\ 800\ N$$

∴ Efficiency of the joint,

$$\eta = \frac{Least\ of\ Pt, Ps\ and\ Pc}{P} = \frac{32400}{46800} = \mathbf{0.692\ or\ 69.2\%\ Ans}$$

42.Two length of mild steel tie rod having width 200 mm are to be connected by means of Lozenge joint with two cover plates to withstand a tensile load of 180 kN. Completely design the joint, if the permissible stresses are 80 MPa in tension; 65 MPa in shear and 160 MPa crushing. Draw a neat sketch of the joint. (16) (Nov/Dec – 2011).

Given :

b = 200 mm ;
t = 12.5 mm ;
σt= 80 MPa = 80 N/mm^2 ;
τ = 65 MPa = 65 N/mm^2 ;
σc= 160 MPa = 160 N/mm^2

Solution:

1. Diameter of rivet

We know that the diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{12.5} = 21.2\ mm$$

we see that according to IS : 1929 – 1982 (Reaffirmed 1996), the standarddiameter of the rivet hole (d) is 21.5 mm and the corresponding diameter of rivet **is 20 mm. Ans.**

2. Number of rivets

Let n = Number of rivets.

We know that maximum pull acting on the joint,

$$Pt = (b - d)t \times \sigma t = (200 - 21.5)\ 12.5 \times 80 = 178\ 500\ N$$

Since the joint is a butt joint with double cover plates as shown in Fig. 3.19 therefore the rivets are in double shear. Assume that the resistance of the rivet in double shear is 1.75 times than in singleshear.

∴ Shearing resistance of one rivet,

$$P_s = 1.75 \times \frac{\pi}{4} d^2 \times \tau = 1.75 \times \frac{\pi}{4} 21.5^2 \times 65 = 471300\ N$$

and crushing resistance of one rivet,

$$Pc = d \times t \times \sigma c = 21.5 \times 12.5 \times 160 = 43\ 000\ N$$

Since the shearing resistance is less than the crushing resistance, therefore number of rivets required for the joint,

$$n = \frac{P_t}{P_s} = \frac{178500}{41300} = 4.32\ say\ \mathbf{5}\ \ \boldsymbol{Ans}$$

3. The arrangement of the rivets

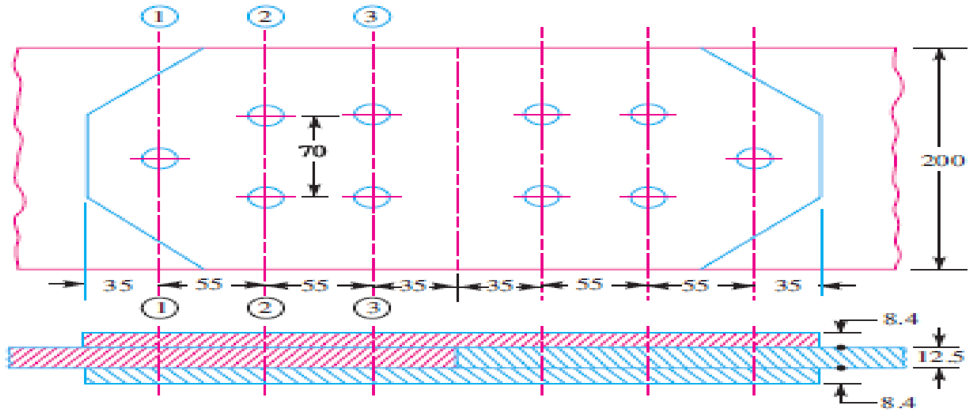


fig 3.19 All dimensions in mm.

4. Thickness of butt straps

We know that thickness of butt straps,

$$t_1 = 0.75\ t = 0.75 \times 12.5 = 9.375 \text{ say } \mathbf{9.4\ mm\ Ans}$$

***5.* Efficiency of the joint**

First of all, let us find the resistances along the sections 1-1, 2-2 and 3-3.

At section 1-1, there is only one rivet hole.

∴ Resistance of the joint in tearing along section 1-1,

$$P_{t1} = (b - d)\ t \times \sigma_t = (200 - 21.5)\ 12.5 \times 80 = 178\ 500\ N$$

At section 2-2, there are two rivet holes. In this case, the tearing of the plate will only take place if the rivet at section 1-1 (in front of section 2-2) gives way (i.e. shears).

∴ Resistance of the joint in tearing along section 2-2,

$$P_{t2} = (b - 2d)\ t \times \sigma_t + \text{Shearing resistance of one rivet}$$

$$= (200 - 2 \times 21.5)\ 12.5 \times 80 + 41\ 300 = \mathbf{198\ 300\ N}$$

At section 3-3, there are two rivet holes. The tearing of the plate will only take place if one rivet at section 1-1 and two rivets at section 2-2 gives way (i.e. shears).

∴ Resistance of the joint in tearing along section 3-3,

$$P_{t3} = (b - 2d)\ t \times \sigma_t + \text{Shearing resistance of 3 rivets}$$

$$= (200 - 2 \times 21.5)\ 12.5 \times 80 + 3 \times 41\ 300 = \mathbf{280\ 900\ N}$$

Shearing resistance of all the 5 rivets

$$Ps = 5 \times 41\ 300 = \mathbf{206\ 500\ N}$$

and crushing resistance of all the 5 rivets,

$$Pc = 5 \times 43\ 000 = \mathbf{215\ 000\ N}$$

Since the strength of the joint is the least value of Pt1, Pt2, Pt3, Ps and Pc, therefore strength of the joint

$$= 178\ 500 \text{ N along section 1-1}$$

We know that strength of the un-riveted plate,

$$= b \times t \times \sigma t = 20 \times 12.5 \times 80 = \mathbf{200\ 000\ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{strength\ of\ the\ joint}{strength\ of\ unriveted\ plate} = \frac{178500}{200000} = \mathbf{0.892\ or\ 89.2\%\ Ans}$$

6. Pitch of rivets,

$$p = 3\ d + 5\ mm = (3 \times 21.5) + 5 = 69.5 \text{ say } \mathbf{70\ mm\ Ans.}$$

7. **Marginal pitch**,

$$m = 1.5\ d = 1.5 \times 21.5 = 33.25 \text{ say } \mathbf{35\ mm\ Ans.}$$

8. Distance between the rows of rivets

$$= 2.5\ d = 2.5 \times 21.5 = 53.75 \text{ say } \mathbf{55\ mm\ Ans.}$$

42.(a) A tie-bar in a bridge consists of flat 350 mm wide and 20 mm thick. It is connected to a gusset plate of the same thickness by a double cover butt joint. Design an economical joint if the permissible stresses are : σt = 90 MPa, τ = 60 MPa and σc = 150 MPa(AP/MAY'2023)

Solution. Given : b = 350 mm ; t = 20 mm ; σt= 90 MPa = 90 N/mm2 ; τ = 60 MPa = 60 N/mm2 ;σc = 150 MPa = 150 N/mm2

1. ***Diameter of rivet***

We know that the diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{20} = 26.8 \text{ mm}$$

From Table 9.7, we see that according to IS : 1929–1982 (Reaffirmed 1996), the standard diameter of rivet hole (d) is 29 mm and the corresponding diameter of rivet is 27 mm. **Ans.**

2. ***Number of rivets***

Let n = Number of rivets.

We know that the maximum pull acting on the joint,

$$P_t = (b - d)\ t \times \sigma_t = (350 - 29)\ 20 \times 90 = 577\ 800 \text{ N}$$

Since the joint is double strap butt joint, therefore the rivets are in double shear. Assume that the resistance of the rivet in double shear is 1.75 times than in single shear.

∴ Shearing resistance of one rivet,

$$P_s = 1.75 \times \frac{\pi}{4} \times d^2 \times \tau = 1.75 \times \frac{\pi}{4}\ (29)^2\ 60 = 69\ 360 \text{ N}$$

and crushing resistance of one rivet,

$$P_c = d \times t \times \sigma_c = 29 \times 20 \times 150 = 87\ 000 \text{ N}$$

Since the shearing resistance is less than crushing resistance, therefore number of rivets required for the joint,

$$n = \frac{P_t}{P_s} = \frac{577\ 800}{69\ 360} = 8.33 \text{ say } 9 \quad \textbf{Ans.}$$

3. The arrangement of rivets

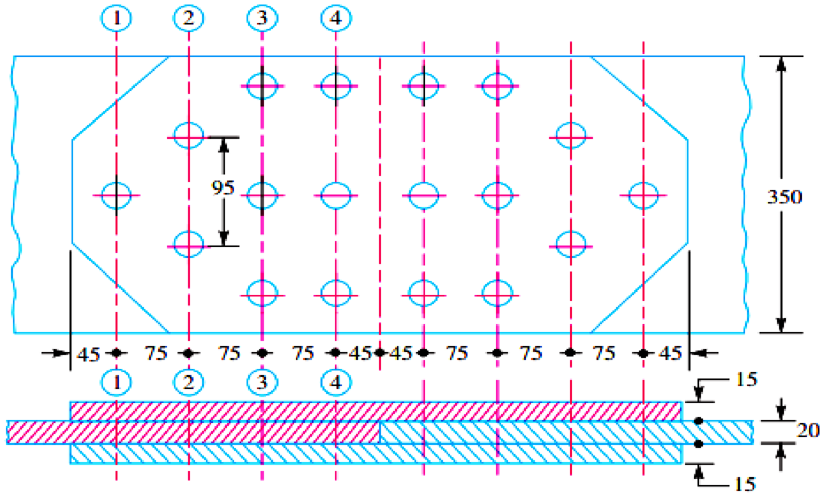


fig 3.20 All dimensions in mm.

4. ***Thickness of butt straps***

We know that the thickness of butt straps,

$$t_1 = 0.75\ t = 0.75 \times 20 = 15 \text{ mm } \textbf{Ans.}$$

5. *Efficiency of the johint*

First of all, let us find the resistances along the sections 1-1, 2-2, 3-3 and 4-4.

At section 1-1, there is only one rivet hole.

∴ Resistance of the joint in tearing along 1-1,

$$P_{t1} = (b - d)\, t \times \sigma_t = (350 - 29)\, 20 \times 90 = 577\,800 \text{ N}$$

from the fig 3.20 At section 2-2, there are two rivet holes. In this case the tearing of the plate will only take place if the rivet at section 1-1 (in front of section 2-2) gives way.

∴ Resistance of the joint in tearing along 2-2,

$$P_{t2} = (b - 2d)\, t \times \sigma_t + \text{Shearing strength of one rivet in front}$$
$$= (350 - 2 \times 29)\, 20 \times 90 + 69\,360 = 594\,960 \text{ N}$$

At section 3-3, there are three rivet holes. The tearing of the plate will only take place if one rivet at section 1-1 and two rivets at section 2-2 gives way.

∴ Resistance of the joint in tearing along 3-3,

$$P_{t3} = (b - 3d)\, t \times \sigma_t + \text{Shearing strength of 3 rivets in front.}$$
$$= (350 - 3 \times 29)\, 20 \times 90 + 3 \times 69\,360 = 681\,480 \text{ N}$$

Similarly, resistance of the joint in tearing along 4-4,

$$P_{t4} = (b - 3d)\, t \times \sigma_t + \text{Shearing strength of 6 rivets in front}$$
$$= (350 - 3 \times 29)\, 20 \times 90 + 6 \times 69\,360 = 889\,560 \text{ N}$$

Shearing resistance of all the 9 rivets,

$$P_s = 9 \times 69\,360 = 624\,240 \text{ N}$$

and crushing resistance of all the 9 rivets,

$$P_c = 9 \times 87\,000 = 783\,000 \text{ N}$$

The strength of the joint is the least of P_{t1}, P_{t2}, P_{t3}, P_{t4}, P_s and P_c.

∴ Strength of the joint

$$= 577\,800 \text{ N along section 1-1}$$

We know that the strength of the un-riveted plate,

$$P = b \times t \times \sigma_t = 350 \times 20 \times 90 = 630\,000 \text{ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{\text{Strength of the joint}}{\text{Strength of the un-riveted plate}} = \frac{577\,800}{630\,000}$$

$$= 0.917 \text{ or } 91.7\% \quad \textbf{Ans.}$$

6. Pitch of rivets, $p = 3d + 5 \text{ mm} = 3 \times 29 + 5 = 92$ say 95 mm **Ans.**

7. Marginal pitch, $m = 1.5\, d = 1.5 \times 29 = 43.5$ say 45 mm **Ans.**

8. Distance between the rows of rivets

$$= 2.5\, d = 2.5 \times 29 = 72.5 \text{ say } 75 \text{ mm} \quad \textbf{Ans.}$$

Note : If chain riveting with three rows of three rivets in each is used instead of diamond riveting, then

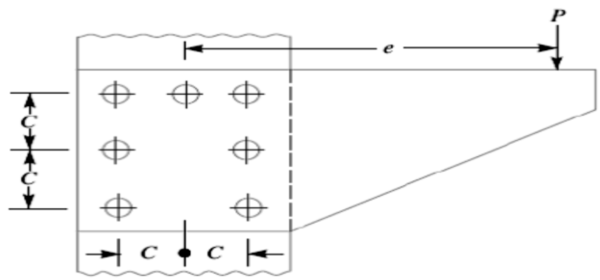
Least strength of the joint

$$= (b - 3\,d)\, t \times \sigma_t = (350 - 3 \times 29)\, 20 \times 90 = 473\,400 \text{ N}$$

∴ Efficiency of the joint $= \dfrac{473\,400}{630\,000} = 0.752$ or 75.2%

Thus we see that with the use of diamond riveting, efficiency of the joint is increased.

43. An eccentrically loaded lap riveted joint is to be designed for a steel bracket as shown in Fig. 2. The bracket plate is 25 mm thick. All rivets are to be of the same size. Load on the bracket, P = 50 kN ; rivet spacing, C =100 mm; load arm, e = 400 mm. shear stress is 65 MPa and crushing stress is 120 MPa. Determine the size of the rivets to be used for the joint.



Solution. Given: t = 25 mm ; P = 50 kN = 50 × 103 N ; e = 400 mm ; n = 7 ; τ= 65 MPa = 65 N/mm₂ ; □c = 120 MPa = 120 N/mm^2.

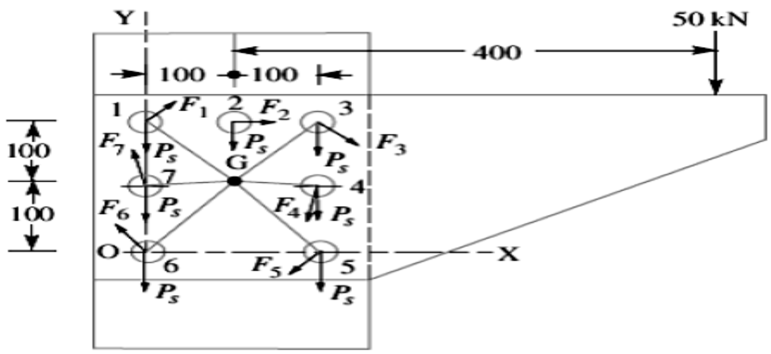


fig 3.21

First of all, let us find the centre of gravity (G) of the rivet system.
Let
x= Distance of centre of gravity from OY
y= Distance of centre of gravity fromOX
$x1$, $x2$, $x3$... = Distances of centre of gravity of each rivet from OY and
$y1$, $y2$, $y3$... = Distances of centre of gravity of each rivet from OX

We know that

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7}{n}$$

$$= \frac{100 + 200 + 200 + 200}{7} = 100 \text{ mm} \qquad ...(\because x_1 = x_6 = x_7 = 0)$$

$$\bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7}{n}$$

$$= \frac{200 + 200 + 200 + 100 + 100}{7} = 114.3 \text{ mm} \quad ...(\because y_5 = y_6 = 0)$$

The centre of gravity (G) of the rivet system lies at a distance of 100 mm from mm from OX, as shown in Fig. 3.21

We know that direct shear load on each rivet,

$$P_s = \frac{P}{n} = \frac{50 \times 10^3}{7} = 7143 \text{ N}$$

The direct shear load acts parallel to the direction of load P *i.e.* vertically downward

Turning moment produced by the load P due to eccentricity (e)

$$= P \times e = 50 \times 103 \times 400 = 20 \times 10^6 \text{ N-mm}$$

This turning moment is resisted by

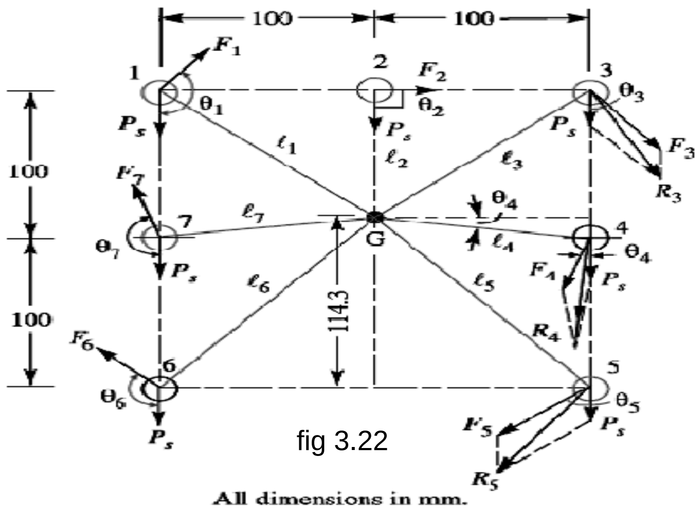


fig 3.22

All dimensions in mm.

Let $F1$, $F2$, $F3$, $F4$, $F5$, $F6$ and $F7$ be the secondary shear load on the rivets 1, 2, 3, 4, 5, 6 and 7 placed at distances l_1, l_2, l_3, l_4, l_5, l_6 and l_7 respectively from the centre of gravity of the rivet system

From the geometry of the fig3.22 we find that

$$l_1 = l_3 = \sqrt{(100)^2 + (200 - 114.3)^2} = 131.7 \text{ mm}$$

$$l_2 = 200 - 114.3 = 85.7 \text{ mm}$$

$$l_4 = l_7 = \sqrt{(100)^2 + (114.3 - 100)^2} = 101 \text{ mm}$$

$$l_5 = l_6 = \sqrt{(100)^2 + (114.3)^2} = 152 \text{ mm}$$

Now equating the turning moment due to eccentricity of the load to the resisting moment of the rivets, we have

$$P \times e = \frac{F_1}{l_1}\left[(l_1)^2 + (l_2)^2 + (l_3)^2 + (l_4)^2 + (l_5)^2 + (l_6)^2 + (l_7)^2\right]$$

$$= \frac{F_1}{l_1}\left[2(l_1)^2 + (l_2)^2 + 2(l_4)^2 + 2(l_5)^2\right] \quad \text{....}(\because l_1 = l_3;\ l_4 = l_7 \text{ and } l_5 = l_6)$$

$$50 \times 10^3 \times 400 = \frac{F_1}{131.7}\left[2(131.7)^2 + (85.7)^2 + 2(101)^2 + 2(152)^2\right]$$

$$20 \times 10^6 \times 131.7 = F_1(34\ 690 + 7345 + 20\ 402 + 46\ 208) = 108\ 645\ F_1$$

$$F_1 = 20 \times 10^6 \times 131.7 / 108\ 645 = 24\ 244 \text{ N}$$

Since the secondary shear loads are proportional to their radial distances from the centre of gravity, therefore

$$F_2 = F_1 \times \frac{l_2}{l_1} = 24\ 244 \times \frac{85.7}{131.7} = 15\ 776 \text{ N}$$

$$F_3 = F_1 \times \frac{l_3}{l_1} = F_1 = 24\ 244 \text{ N} \quad \text{...}(\because l_1 = l_3)$$

$$F_4 = F_1 \times \frac{l_4}{l_1} = 24\ 244 \times \frac{101}{131.7} = 18\ 593 \text{ N}$$

By drawing the direct and secondary shear loads on each rivet, we see that the rivets 3, 4 and 5 are heavily loaded. Let us now find the angles between the direct and secondary shear load for these three rivets. From the geometry of Fig we find that

$$\cos\theta_3 = \frac{100}{l_3} = \frac{100}{131.7} = 0.76$$

$$\cos\theta_4 = \frac{100}{l_4} = \frac{100}{101} = 0.99$$

$$\cos\theta_5 = \frac{100}{l_5} = \frac{100}{152} = 0.658$$

Now resultant shear load on rivet 3,

$$R_3 = \sqrt{(P_s)^2 + (F_3)^2 + 2P_s \times F_3 \times \cos\theta_3}$$

$$= \sqrt{(7143)^2 + (24\ 244)^2 + 2 \times 7143 \times 24\ 244 \times 0.76} = 30\ 033 \text{ N}$$

Resultant shear load on rivet 4,

$$R_4 = \sqrt{(P_s)^2 + (F_4)^2 + 2\ P_s \times F_4 \times \cos\theta_4}$$

$$= \sqrt{(7143)^2 + (18\ 593)^2 + 2 \times 7143 \times 18\ 593 \times 0.99} = 25\ 684 \text{ N}$$

And resultant shear load on rivet 5,

$$R_5 = \sqrt{(P_s)^2 + (F_5)^2 + 2\ P_s \times F_5 \times \cos\theta_5}$$

$$= \sqrt{(7143)^2 + (27\ 981)^2 + 2 \times 7143 \times 27\ 981 \times 0.658} = 33\ 121\ \text{N}$$

The resultant shear load may be determined graphically, as shown in Fig.3.

From above we see that the maximum resultant shear load is on rivet 5. If d is the diameter of rivet hole, then maximum resultant shear load (R_5),

$$33\ 121 = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times d^2 \times 65 = 51\ d^2$$

$$d^2 = 33\ 121 / 51 = 649.4 \quad \text{or} \quad d = 25.5\ \text{mm}$$

From DDB, we see that according the standard diameter of the rivet hole (*d*) is 25.5 mm and the corresponding diameter of rivet is 24 mm.

Let us now check the joint for crushing stress. We know that

$$\text{Crushing stress} = \frac{\text{Max. load}}{\text{Crushing area}} = \frac{R_5}{d \times t} = \frac{33\ 121}{25.5 \times 25}$$

$$= 51.95\ \text{N/mm}^2 = 51.95\ \text{MPa}$$

Since this stress is well below the given crushing stress of 120 MPa, therefore the design is satisfactory.

44.Design a double riveted butt joint with two cover plates for the longitudinalseam of a boiler shell 1.5 m in diameter subjected to a steam pressure of 0.95 N/mm2. Assume jointefficiency as 75%, allowable tensile stress in the plate 90 MPa ; compressive stress 140 MPa ; andshear stress in the rivet 56 MPa.(Nov/Dec-16)

Given:

$D = 1.5$ m $= 1500$ mm ; $P = 0.95$ N/mm^2 ; $\eta_l = 75\% = 0.75$; $\sigma_t = 90$ MPa $= 90$ N/mm^2 ; $\sigma_c = 140$ MPa $= 140$ N/mm^2 ; $\tau = 56$ MPa $= 56$ N/mm^2

To find:

Design a double riveted butt joint

Solution:

1. *Thickness of boiler shell plate*

We know that thickness of boiler shell plate,

$$t = \frac{P.D}{2\sigma_t \times \eta_l} + 1 \text{ mm} = \frac{0.95 \times 1500}{2 \times 90 \times 0.75} + 1 = 11.6 \text{ say } 12 \text{ mm} \quad \textbf{Ans.}$$

2. *Diameter of rivet*

Since the thickness of the plate is greater than 8 mm, therefore the diameter of the rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{12} = 20.8 \text{ mm}$$

From Table , we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of the rivet hole (d) is 21 mm and the corresponding diameter of the rivet is 20 mm. **Ans.**

3. *Pitch of rivets*

Let p = Pitch of rivets.

The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (p - 21)12 \times 90 = 1080\,(p - 21)\text{N} \qquad ...(i)$$

Since the joint is double riveted double strap butt joint, as shown in Fig. , therefore there are two rivets per pitch length (*i.e.* $n = 2$) and the rivets are in double shear. Assuming that the rivets in

$$p = p_{max} = 84 \text{ mm} \qquad \textbf{Ans.}$$

4. *Distance between rows of rivets*

Assuming zig-zag riveting, the distance between the rows of the rivets (according to I.B.R.),

$$p_b = 0.33\,p + 0.67\,d = 0.33 \times 84 + 0.67 \times 21 = 41.8 \text{ say } 42 \text{ mm} \quad \textbf{Ans.}$$

5. *Thickness of cover plates*

According to I.B.R., the thickness of each cover plate of equal width is

$$t_1 = 0.625\,t = 0.625 \times 12 = 7.5 \text{ mm} \qquad \textbf{Ans.}$$

6. *Margin*

We know that the margin,

$$m = 1.5\,d = 1.5 \times 21 = 31.5 \text{ say } 32 \text{ mm} \qquad \textbf{Ans.}$$

Let us now find the efficiency for the designed joint.

Tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (84 - 21)12 \times 90 = 68\,040 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 1.875 \times \frac{\pi}{4}(21)^2 \times 56 = 72\,745 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 21 \times 12 \times 140 = 70\,560 \text{ N}$$

Since the strength of riveted joint is the least value of P_t, P_s or P_c, therefore strength of the riveted joint,

$$P_t = 68\,040 \text{ N}$$

We know that strength of the un-riveted plate,

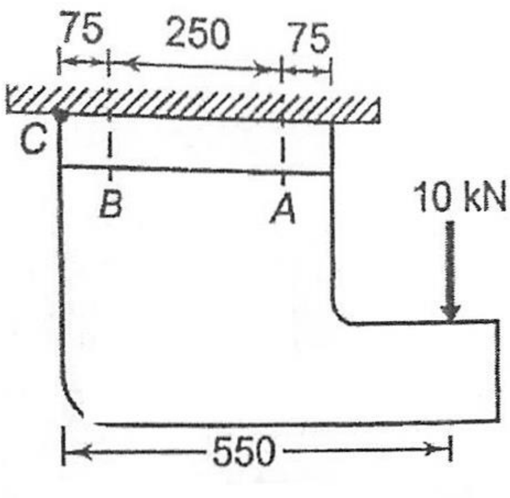
$$P = p \times t \times \sigma_t = 84 \times 12 \times 90 = 90\,720 \text{ N}$$

∴ Efficiency of the designed joint,

$$\eta = \frac{P_t}{P} = \frac{68\,040}{90\,720} = 0.75 \text{ or } 75\% \qquad \textbf{Ans.}$$

Since the efficiency of the designed joint is equal to the given efficiency of 75%, therefore the design is satisfactory.

45.A cast iron bracket, as shown in fig. 13a, supports a load of 10 kN. It is fixed to the horizontal channel by means of four identical bolts, two at A and two at B. The bolts are made of steel 30C8 whose yield strength is 400 MPa and the factor of safety is 6. Determine the major diameter of the bolts if dc = 0.8d.(nov/dec 2020,april/may 2021)



All dimension are in “mm”

Given:

W=10kN, n=4, σ_t=400N/mm²

Tofind:

The Size of bolt

Solution:

We know that direct shear load on each bolt

$$w_t = \frac{w}{n} = \frac{10}{3} = 3.33kN$$

We know that maximum tensile load carried by bolts 2 and 3

$$W_t = \frac{w.L.L_2}{2[(L_1)^2 + (L_2)^2]}$$

$$W_t = \frac{10 \times 400 \times 360}{2[(40)^2 + (360)^2]} = 5.48kN$$

Since the bolts are subjected to shear load as well as tensile load, therefore equivalent tensile load,

$$W_{te} = \frac{1}{2}[w_t + \sqrt{(w_t)^2 + 3(w_s)^2}$$

$$W_{te} = \frac{1}{2}[5.48 + \sqrt{(5.48)^2 + 3(3.33)^2}$$
$$= 6.71kN$$

Size of the bolt

Let dc = Core diameter of the bolt.

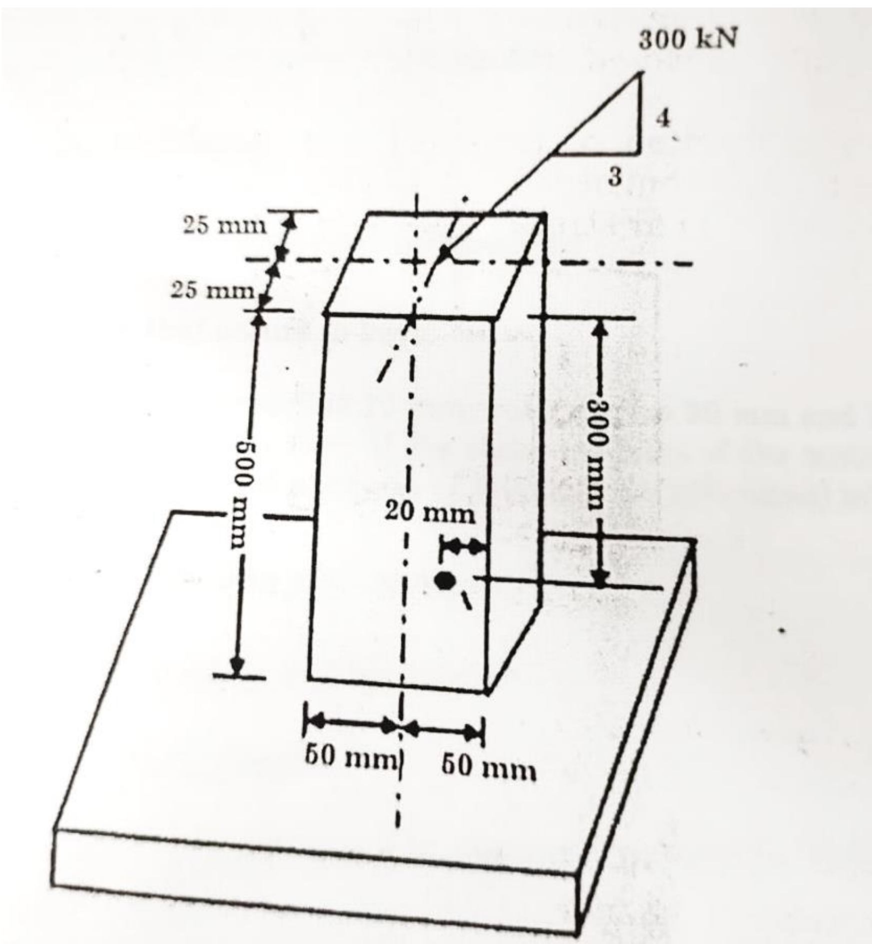
We know that the equivalent tensile load (Wte),

$$6710 = \frac{\pi}{4}(d_c)^2 \sigma_t$$

$$6710 = \frac{\pi}{4}(d_c)^2 \times 75$$
$$d_c = 10.67$$

the standard core diameter is 11.546 mm and the corresponding size of the bolt is M 14.

46.Calculate the stresses on the element at A as shown in fig(A/M'2023)



Solution. Given: b = 4 m ; d = 3 m ; P = 300 kN ; ex = 25 m ; e y = 15 m

We know that cross-sectional area of the pier,

A = b × d = 4 × 3 = 12 m2

Moment of inertia of the pier about X-axis,

$$I_{XX} = \frac{b \cdot d^3}{12} = \frac{4 \times 3^3}{12} = 9 \text{ m}^4$$

and moment of inertia of the pier about Y-axis,

$$I_{YY} = \frac{d \cdot b^3}{12} = \frac{3 \times 4^3}{12} = 16 \text{ m}^4$$

Distance between X-axis and the corners A and B,

$$x = 3/2 = 1.5 \text{ m}$$

D

Distance between Y-axis and the corners A and C,

$$y = 4/2 = 2 \text{ m}$$

We know that stress at corner A,

$$\sigma_A = \frac{P}{A} + \frac{P \cdot e_x \cdot x}{I_{XX}} + \frac{P \cdot e_y \cdot y}{I_{YY}}$$

$$= \frac{30}{12} + \frac{30 \times 0.5 \times 1.5}{9} + \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 + 2.5 + 3.75 = 8.75 \text{ kN/m}^2$$ **Ans.**

Similarly stress at corner B,

$$\sigma_B = \frac{P}{A} + \frac{P \cdot e_x \cdot x}{I_{XX}} - \frac{P \cdot e_y \cdot y}{I_{YY}}$$ ... [$\because$ At B, x is +ve and y is –ve]

$$= \frac{30}{12} + \frac{30 \times 0.5 \times 1.5}{9} - \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 + 2.5 - 3.75 = 1.25 \text{ kN/m}^2$$ **Ans.**

Stress at corner C,

$$\sigma_C = \frac{P}{A} - \frac{P \cdot e_x \cdot x}{I_{XX}} + \frac{P \cdot e_y \cdot y}{I_{YY}}$$ ... [At C, x is –ve and y is +ve]

$$= \frac{30}{12} - \frac{30 \times 0.5 \times 1.5}{9} + \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 - 2.5 + 3.75 = 3.75 \text{ kN/m}^2$$ **Ans.**

ınd stress at corner D,

$$\sigma_D = \frac{P}{A} - \frac{P \cdot e_x \cdot x}{I_{XX}} - \frac{P \cdot e_y \cdot y}{I_{YY}}$$ ... [At D, both x and y are – ve]

$$= \frac{30}{12} - \frac{30 \times 0.5 \times 1.5}{9} - \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 - 2.5 - 3.75 = -3.75 \text{ kN/m}^2 = 3.75 \text{ kN/m}^2 \text{ (tensile)}$$ **Ans.**

DEPARTMENT OF MECHANICAL ENGINEERING

Subject Title : DESIGN OF MACHINE ELEMENTS
Subject Code : ME 3591
Year/ SEM : III / V

UNIT IV - ENERGY STORING ELEMENTS AND ENGINE COMPONENTS

UNIT IV - ENERGY STORING ELEMENTS AND ENGINE COMPONENTS:

Types of springs, design of helical and concentric springs–surge in springs, Design of laminated springs - rubber springs - Flywheels considering stresses in rims and arms for engines and punching machines-- Solid and Rimmed flywheels- connecting rods and crank shafts

SUMMARY

Springs:

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows :

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.
2. To apply forces, as in brakes, clutches and spring loaded valves.
3. To control motion by maintaining contact between two elements as in cams and followers.
4. To measure forces, as in spring balances and engine indicators.
5. To store energy, as in watches, toys, etc.

The material of the spring should have high fatigue strength, high ductility, high resilience andit should be creep resistant. It largely depends upon the service for which they are used i.e. severe service, average service or light service.

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 percent carbon and 0.60 to 1.0 per cent manganese. Music wire is used for small springs. Non-ferrousmaterials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in specialcases to increase fatigue resistance, temperature resistance and corrosion resistance.

Leaf Springs:

Leaf springs (also known as **flat springs**) are made out of flat plates. The advantage of leafspring over helical spring is that the ends of the spring may be guided along a definite path as itdeflects to act as a structural member in addition to energy absorbing device. Thus the leaf springsmay carry lateral loads, brake torque, driving torque etc., in addition to shocks.

The material used for leaf springs is usually a plain carbon steel having 0.90 to 1.0% carbon.The leaves are heat treated after the forming process. The heat treatment of spring steel producesgreater strength and therefore greater load capacity, greater range of deflection and better fatigueproperties.

Flywheel:

A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than supply.

In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke. For example, in I.C. engines, the energy is developed only during power stroke which is much more than the engine load, and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed. A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.

In machines where the operation is intermittent like punching machines, shearing machines, riveting machines, crushers etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

Connecting rod:

The connecting rod is the intermediate member between the piston and the crankshaft. Its primaryfunction is to transmit the push and pull from the piston pin to the crankpin and thus convert thereciprocating motion of the piston into the rotary motion of the crank. The usual form of the connectingrod in internal combustion engines. It consists of a long shank, a small end and abig end. The cross-section of the shank may be rectangular, circular, tubular, I-section or H-section.Generally circular section is used for low speed engines while I-section is preferred for high speed engines.

Crank Shaft:

A crankshaft (i.e. a shaft with a crank) is used to convert reciprocating motion of the piston into rotatory motion or vice versa. The crankshaft consists of the shaft parts which revolve in the mainbearings, the crankpins to which the big ends of the connecting rod are connected, the crank arms orwebs (also called cheeks) which connect the crankpins and the shaft parts. The crankshaft, dependingupon the position of crank, may be divided into the following two types :

1. Side crankshaft or overhung crankshaft, **2.** Centre crankshaft

PART – A (Two marks)

VARIOUS TYPES OF SPRINGS, OPTIMIZATION OF HELICAL SPRINGS:

1. Define spring and state its various applications(A/M'2023)

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed.

The various important applications of springs are as follows

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.

2. To apply forces, as in brakes, clutches and springloadedvalves.

3. To control motion by maintaining contact betweentwo elements as in cams and followers.

4. To measure forces, as in spring balances andengine indicators.

5. To store energy, as in watches, toys, etc.

2. Mention any four types of springs (May/June-2012)

Following are the classification of the springs

1. Helical springs

   The helical springs are made up of a wire coiled in the form of a helix andis primarily intended for compressive or tensile loads.

2. Conical and volute springs

   The conical and volute springs are usedin special applications where a telescoping spring or a spring with a spring rate that increases with theload is desired.

3. Torsion springs

   Torsion spring is defined as these springs may be of helical or spiral type. The helical type may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms.

4. Laminated or leaf springs

   The laminated or leaf spring (also known as flat spring or carriage spring) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts

5. Disc or bellevile springs
6. Special purpose springs

3. What are the terms used in compression springs and explain any two of them in detail?

The following terms used in connection with compression springs are,

a. Solid length
b. Free length
c. Spring index
d. Spring rate
e. Pitch

a. Solid length:

In solid length, when the compression spring is compressed until the coils come in contact with each other, then the spring is said to be ***solid*.** The solid length of a spring is the product of total number of coils and the diameter of the wire.

Mathematically,

Solid length of the spring,$\boldsymbol{LS = n'.d}$

Where

n'= Total number of coils, and$d$= Diameter of the wire.

b. Pitch :

The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

The pitch of the coil may also be obtained by using the following relation, *i.e*.

$$p = \frac{Freelength}{n' - 1}$$

Pitch of the coil,

$$p = \frac{LF - LS}{n' - 1} + d$$

Where,

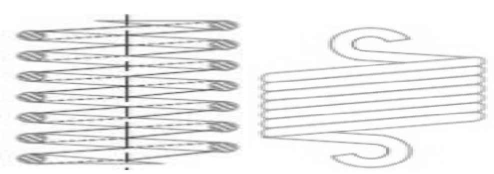
LF = Free length of the spring,

LS = Solid length of the spring,

n' = Total number of coils, and

d = Diameter of the wire.

4. **Write short notes on helical springs.**



The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. In these springs, the major stress is shear stress due to twisting. The two forms of helical springs are compression helical spring and tension helical spring.

5. **Distinguish between closed coiled and open coiled spring. Nov/ Dec 2014.**

Closely coiled:

The helical springs are said to be closely coiled when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small;

it is usually less than 10°. The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring.

Open coiled helical springs

In open coiled helical springs, the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open coiled helical springs are limited, therefore our discussion shall confine to closely coiled helical springs only.

6. What are the advantages of helical springs?

The helical springs have the following advantages are:

1. These are easy to manufacture.
2. These are available in wide range.
3. These are reliable.
4. These have constant spring rate.
5. Their performance can be predicted more accurately.
6. Their characteristics can be varied by changing dimensions.

7. While designing a helical compression spring what are the important considerations should be followed?

The design of a helical compression spring involves the following considerations

1. Modes of loading – i.e., whether the spring is subjected to static or infrequently varying load or alternating load.
2. The force deflection characteristic requirement for the given application.
3. Is there any space restriction.
4. Required life for springs subjected to alternating loads.
5. Environmental conditions such as corrosive atmosphere and temperature.
6. Economy desired.

8. Enumerate the design procedure for helical compression spring with circular cross section.

The following are the design procedure for helical compression spring

1. Diameter of wire

   LS = n'.d

   Wheren'= Total number of coils, and

   d = Diameter of the wire.

2. Mean diameter of coil
3. Number of coil or turns
4. Free length

   Free length of the spring,

   *L*F = Solid length + Maximum compression + *Clearance betweenadjacent coils (or clash allowance)

   $= n'.d + \delta max + 0.15\ \delta max$

5. Stiffness or rate of spring

   Spring rate, $k = W / \delta$

   Where, W = Load, and

   δ = Deflection of the spring.

6. Pitch

$$\text{Pitch of the coil} = \frac{\text{free length}}{n' - 1}$$

9. Differentiatebetween conical and volute springs.

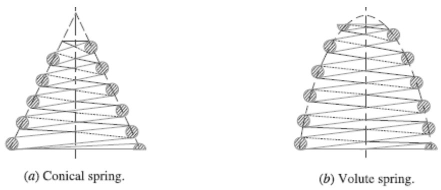


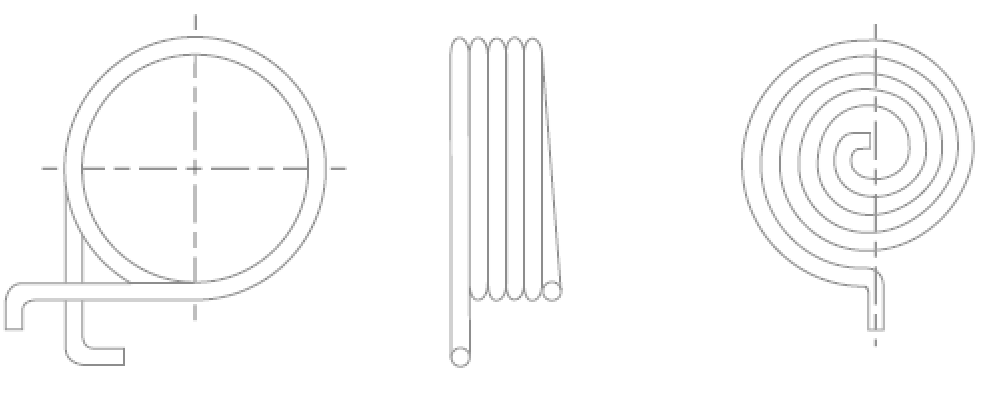
Fig 4.1

The conical and volute springs, are usedin special applications where a telescoping spring or a spring with a spring rate that increases with theload is desired. The conical spring, as shown in Fig. 4.1(*a*), is wound with a uniform pitch whereasthe volute springs,

A **volute spring** as shown in Fig. 4.1(*b*), are wound in the form of paraboloid with constant pitchand lead angles.The springs may be made either partially or completely telescoping. In either case,the number of active coils gradually decreases. The decreasing number of coils results in an increasingspring rate. This characteristic is sometimes utilized in vibration problems where springs are used tosupport a body that has a varying mass.The major stresses produced in conical and volute springs are also shear stresses due to twisting.

10. Which types of springs produce more tensile and compressive stresses due to bending?

The helical type may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms. The spiral type is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks. The major stresses produced in torsion springs are tensile and compressive due to bending.These springs may be of helical or spiral type as shown in Fig 4.2(a) and (b).



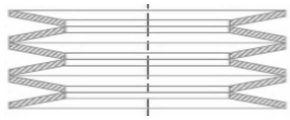
(*a*) Helical torsion spring. (*b*) Spiral torsion spring.

Fig 4.2

11. State the purpose of disc or bellevile springs and special purpose springs.

The following are the disc or bellevile springs and special purpose springs.

Disc or bellevile springs:



These springs consist of a number of conical discs held togetheragainst slipping by a central bolt or tube as shown in Fig. These springs are used in applicationswhere high spring rates and compact spring units are required.

Special purpose springs:

These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

12. Define (a) spring index (b) spring rate. Nov /Dec 2011& May/June 2016

Spring Index:

Spring index is the correlation between the mean diameter of a spring and the wire diameter of a spring. This proportion will determine the strength of the spring, the stress induced on the spring, and the manufacturability of the spring.

Formula for Calculating Spring Index:

Index = Mean Diameter (D) / Wire Diameter (d)

Mean Diameter (D) = Outer Diameter (OD) - Wire Diameter (d)

Spring Rate :

The spring rate is defined as the load required per unit deflection of the spring is known as the spring rate, defined as the load in pounds divided by the deflection of the spring in inches. A soft spring has a low rate and deflects a greater distance under a given load.

Spring rate, $k = W / \delta$

Where,

W = Load, and

δ = Deflection of the spring.

13. Discuss the objectives of spring used as machine member.

Following are the objectives of a spring when used as a machine member:

a. **Cushioning, absorbing, or controlling of energy due to shock and vibration:**
   1. Car springs or railway buffers
   2. To control energy, springs-supports and vibration dampers

b. **Control of motion:**

Maintaining contact between two elements (cam and its follower) in a cam and a follower arrangement, widely used in numerous applications, a spring maintains contact between the two elements. It primarily controls the motion.

c. Measuring forces:

Spring balances, gages

d. **Storing of energy**

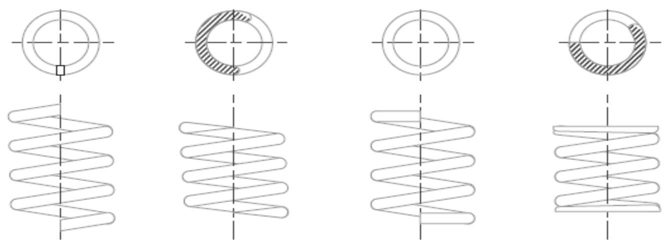
In clocks or starters, The clock has spiral type of spring which is wound to coil and then the stored energy helps gradual recoil of the spring when in operation. Nowadays we do not find much use of the winding clocks.

14. What are the important materials used in spring manufacturing?(N/D'2022)

One of the important considerations in spring design is the choice of the spring material. Some of the common spring materials are given below.

1. Hard-drawn wire
2. Oil-tempered wire
3. Chrome Vanadium steel
4. Chrome Silicon steel
5. Stainless steel
6. Phosphor Bronze / Spring BrassS

15. State the end connection for compression helical spring.



(a) Plain ends. (b) Ground ends. (c) Squared ends. (d) Squared and ground ends.

The end connections for compression helical springs are suitably formed in order to apply theload. In all springs, the end coils produce an eccentric application of the load, increasing the stress on one side of the spring. Under certain conditions, especially where the number of coils is small, thiseffect must be taken into account. The nearest approach to an axial load is secured by squared andground ends, where the end turns are squared and then ground perpendicular to the helix axis.

It maybe noted that part of the coil which is in contact with the seat does not contribute to spring action andhence are termed as inactive coils. The turns which impart spring action are known as active turns.As the load increases, the number of inactive coils also increases due to seating of the end coils andthe amount of increase varies from 0.5 to 1 turn at the usual working loads.

16. Enumerate the eccentric loading of springs.

The eccentric loading of springs is, the load on the springs does not coincide with the axis of the spring, and i.e. the spring is subjected to an eccentric load. In such cases, not only the safe load for the spring reduces, thestiffness of the spring is also affected. The eccentric load on the spring increases the stress on one sideof the spring and decreases on the other side. When the load is offset by a distance from the springaxis, then the safe load on the spring may be obtained by multiplying the axial load by the factor

$$D / 2e + D$$

Where,

D is the mean diameter of the spring.

17. Write the formula for calculating the natural frequency of spring Nov/Dec- 2012.

The natural frequency of spring should be at least twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies up to twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 n}\sqrt{\frac{6G.g}{\rho}}\, Cycles/s$$

Where,

d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity,

ρ= Density of the material of the spring

18. A helical spring of rate 12 N/mm is mounted on the top of another spring of rate 8 N/mm. Find the force required to give a deflection of 50mm?(Nov/Dec-2013)

Given:

Stiffness of first spring ,q_1 = 12 N/mm,

Stiffness of second spring ,q_2 = 8 N/mm,

Deflection, y=50 mm

To Find: Force, P

Solution:

When two springs are arranged in series

We know that

$$\frac{1}{q} = \frac{1}{q_1} = \frac{1}{q_2} = \frac{1}{12} + \frac{1}{8}$$

Total stiffness q=4.8 N/mm

Stiffness = Load/Deflection

48=P/50

Force, P = 240 N Ans

19. Define elastic strain energy.

The elastic strain energy is defined as when an elastic material is deformed, work is done. This work is stored as elastic strain energy in the material. The close coiled helical spring when subjected to axial load is subjected to torsion. This gives rise to strain energy stored in the spring. Strain energy stored in a close soiled helical spring under axial load

$$U = (4W^2 D^3 n) / Gd^4$$

$$\text{Deflection, } \delta = (8 W D^3 n) / Gd^4$$

$$\text{Then number of coils, } n = (\delta\, Gd^4) / (8 W D^3)$$

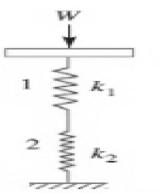
20. Using soderberg line method expalin how the helical springs are designed based on fatigue loading.

The helical springs subjected to fatigue loading are designed by using the Soderberglinemethod. The spring materials are usually tested for torsional endurance strength under a repeatedstress that varies from zero to a maximum. Since the springs are ordinarily loaded in one directiononly(the load in springs is never reversed in nature), therefore a modified Soderberg diagram is usedfor springs,

21. Write short notes on spring in series and spring in parallel.

Spring in series:

A little consideration will show that when the springs are connected in parallel, then the total deflection produced by the springs is same as the deflection of the individual springs. The two springs are connected in series as shown in fig.



Springs in series.

$$\frac{1}{\mathrm{k}} = \frac{1}{k_1} + \frac{1}{k_2}$$

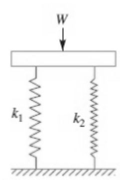
Where,

W = Load carried by the springs,δ1 = Deflection of spring 1,δ2 = Deflection of spring 2,

*k*1 = Stiffness of spring 1 = *W* / δ1, and*k*2 = Stiffness of spring 2 = *W* / δ2

Spring in parallel:

A little consideration will show that when the springs are connected in parallel, then the totaldeflection produced by the springs is same as the deflection of the individual springs



Springs in parallel.

Fig 4.3

We know that $W = W_1 + W_2$

Or

$$\delta.k = \delta.k_1 + \delta.k_2$$
$$k = k_1 + k_2$$

Where,

k = Combined stiffness of the springs, andδ = Deflection produced.

W = Load carried by the springs,*W*1 = Load shared by spring 1,

*W*2 = Load shared by spring 2,*k*1 = Stiffness of spring 1, and*k*2 = Stiffness of spring 2.

22. What are the uses of concentric or composite springs?

A concentric or composite spring is used for one of the following purposes:

1. To obtain greater spring forcewithin a given space.
2. To insure the operation of amechanism in the event of failure of one ofthe springs.
3. Sometimes concentric springs are used to obtain a spring force which does not increase in a direct relation to the deflection but increases faster. Such springs are made of different lengths

23. Define leaf spring or laminated spring.

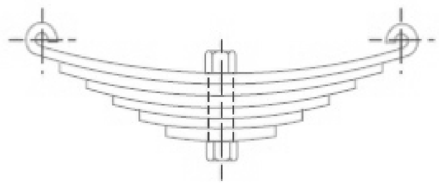
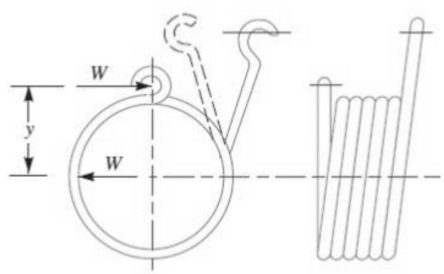


Fig 4.5

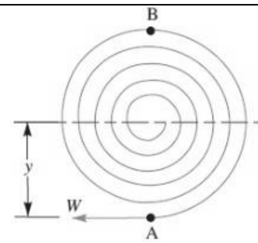
The laminated or leaf spring (also known as flat spring or carriage spring) consists of a number of flat plates of varying lengths held together by means of clamps and bolts, as shown in Fig. 4.5 These are mostly used in automobiles. A leaf spring may be of full elliptical, semi-elliptical or cantilever type.The advantage of leafspring over helical spring is that the ends of the spring may be guided along a definite path as itdeflects to act as a structural member in addition to energy absorbing device. Thus the leaf springsmay carry lateral loads, brake torque, driving torque etc., in addition to shocks.The major stresses produced in leaf springs are tensile and compressive stresses.

24. Discuss in detail about the helical torsion springs.



The helical torsion springs may be made from round, rectangular orsquare wire. These are wound in a similar manner as helical compression or tension springs but theends are shaped to transmit torque. The primarystress in helical torsion springs is bending stresswhereas in compression or tension springs, the stresses are torsional shear stresses. The helical torsion springs are widely used for transmitting small torques as in door hinges, brush holders in electric motors, automobile starters etc.

25. Write short notes on flat spiral spring



A flat spring is a long thin strip of elastic material wound like a spiral as shown in Fig.4.3 These springs are frequently used in watches and gramophones etc.When the outer or inner end of this type of spring is woundup in such a way that there is a tendency in the increase of numberof spirals of the spring, the strain energy is stored into its spirals.This energy is utilized in any useful way while the spirals open outslowly. Usually the inner end of spring is clamped to an arbor while the outer end may be pinned or clamped. Since the radius of curvature of every spiral decreases when the spring is wound up, therefore the material of the spring is in a state of pure bending

26. What is meant by semi-elliptical leaf spring? (April / May 2014)

This leaf spring commonly used in automobiles is of semi-elliptical form.It is built up of a number of plates (known as leaves). The leaves are usually given an initial curvatureor cambered so that they will tend to straighten under the load. The leaves are held together by means of a band shrunk around them at the centre or by a bolt passing through the centre. Since the band exerts stiffening and strengthening effect, therefore the effective length of the spring for bending will be overall length of the spring minus width of band.

27. What are the materials used in manufacturing of leaf spring?

The material used for leaf springs is usually a plain carbon steel having 0.90 to 1.0% carbon. The leaves are heat treated after the forming process. The heat treatment of spring steel produces greater strength and therefore greater load capacity, greater range of deflection and better fatigue properties.

The following applications are used,

1. **For automobiles:** 50 ruCr 1, 50 Cr 1 V 23, and 55 Si 2 Mn 90 all used in hardened and tempered state.

2. **For rail road springs:** C 55 (water-hardened), C 75 (oil-hardened), 40 Si 2 Mn 90 (water-hardened) and 55 Si 2 Mn 90 (oil-hardened).

28. What do you understand by full length and graduated leaves of a leaf spring?

Full length of leaf spring:

The longest leaf known as main leaf or master leafhas its ends formed in the shape of an eye through whichthe bolts are passed to secure the spring to its supports.Usually the eyes, through which the spring is attached tothe hanger or shackle, are provided with bushings of someantifriction material such as bronze or rubber.

Graduated leaves:

The other leaves of the spring are known as graduated leaves. In order to prevent digging in the adjacent leaves, the ends of the graduated leaves are trimmed in various forms. Since the master leaf has to withstand vertical bending loads as well as loads due to sideways of the vehicle and twisting, therefore due to the presence of stresses caused by these loads, it is usual to provide two full length leaves and the rest graduated leaves.

29. What are the disadvantages of Helical Springs of Non-circular Wire?

The helical springs may be made of non-circular wire such as rectangular or square wire, in order to provide greater resilience in a given space.

Thesesprings have the following main disadvantages:

1. The quality of material used for springs is not so good.
2. The shape of the wire does not remain square or rectangular while forming helix, resulting in trapezoidal cross-sections. It reduces the energy absorbing capacity of the spring.
3. The stress distribution is not as favorable as for circular wires.
4. But this effect is negligible where loading is of static nature.

30. What are the factors considered for designing a helical spring?

Design of a helical spring involves a trail-and-error method, and the result should be checked by actual testing of the spring. While designing a spring, the designers have to consider the following factors:

a. It should be able to carry the designed load
b. It should have the required load-deflection characteristics
c. It should not buckle under load
d. It should also satisfy the given set of constrain, such as space limitation, the minimum height, he desired life, the specific vibrational characteristics etc.
e. Material of the spring for the specific atmospheric condition such as temperature, humidity etc.

31. What do you understand by nipping in a leaf spring?(Nov/Dec 2018)(A/M'2023)

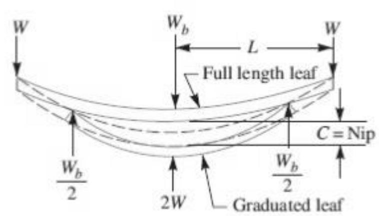


Fig 4.6

The nipping in a leaf spring is understood by giving a greater radius of curvature to the full length leaves than graduated leaves, before the leaves are assembled to form a spring. By doing so, a gap or clearance will be left between the leaves. This initial gap, as shown by C in Fig.,4.6 is called nip. Consider that under maximum load conditions, the stress in all the leaves is equal. Then atmaximum load, the total deflection of the graduated leaves will exceed the deflection of the full length leaves by an amount equal to the initial gap *C*.

32. Under what circumstances, disc springs are preferred?

Disc springs are preferred due to their predictability, high reliability and unparalleled fatigue life, Disc Springs are preferred over all other types of springs in critical applications such as safety valves, clutch and brake mechanisms for elevators and heavy equipment, and supports for industrial pipe systems. They can be used individually or assembled into stacks to achieve the desired deflection characteristic required for the application.

33. Why Wahl's factor is to be considered in the design of helical compression springs? (A/M 2015)

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A.M. Wahl may be used. When a wire is wound in the form of helix, the length of inner fiber of wire is reduced in comparison to the length of outer fiber. This result in stress concentration at the inner fiber. Wahl's factor takes into account the effect of curvature as well as shear stress correction factor.

$$\textbf{Wahl's stress factor, } K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

Where,

K=Stress factor, C= Spring Index

34. What do you know about the term "uniform strength" in the context of leaf spring mean?

The term uniform strength the leaf spring has a shape of uniformly varying width (say Lozenge shape) then the bending stress at all section remains uniform. The situation is also identical as before in case of varying thickness, the thickness should vary non-uniformly with length to make a beam of uniform strength (L/h2 = constant). These leaves require lesser material; have more resilience compared to a constant width leaf. These types of springs are called leaf springs of uniform strength.

35. State the purpose of using concentric springs.(or) Why concentric springs are used? (APRIL/MAY-2015)

a. To get greater spring force within a given space

b. To insure the operation of a mechanism in the event of failure of one of the spring

36. Explain about surge in springs (MAY/JUNE 2013)(April/May 2018)

When one end of the spring is resting on a rigid support and the other end is loaded suddenly, all the coils of spring does not deflect equally, because some time is required for the propagation of stress along the wire. Thus a wave of compression propagates to the fixed end from where it is reflected back to the deflected end this wave passes through the spring indefinitely. If the time interval between the load application and that of the wave to propagate are equal, then resonance will occur. This will result in very high stresses and cause failure. This phenomenon is called surge.

37. Define active turns.

Active turns of the spring are defined as the number of turns, which impart spring action while loaded. As load increases the no of active coils decreases.

38. Define inactive turns.

An inactive turn of the spring is defined as the number of turns which does not contribute to the spring action while loaded. As load increases number of inactive coils increases from 0.5 to 1 turn.

<u>RUBBER SPRING:</u>

39. Write notes on rubber spring.(Nov/Dec2021)

The rubber spring is a combined spring composed of rubber and a coil spring. Compared with general anti-vibration rubber mounts, it has a flexible spring constant. It has an intermediate property between an air spring and an anti-vibration rubber mount, making it is easy to combine with vibrating machines.

The term rubber is used in most suspension systems as bump and rebound stops. Increasing the load on a suspension causes the rubber cone to act like a spring being deformed. When the load is removed, the rubber's elastic properties tend to return it to its original state. Rubber has a number of advantages. It doesn’t need to be lubricated, it can be made into any shape, as required, and it’s silent during use. Rubber spring and cushioning devices are finding an increasing range of application in industry.

40. Discuss the variation between the predicted and actual behavior of rubber spring.

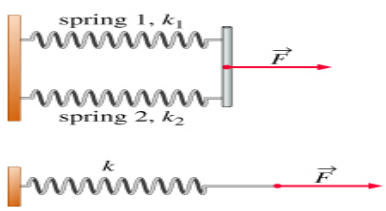
The reason for variation between the predicted and actual behavior of such a spring are following:

1. Variation in elastic or shear moduli may occur among different rubber compounds even though of the same hardness reading.
2. The static and dynamic moduli of elasticity will differ.
3. In the case of compression springs of rubber, friction between compression surfaces may vary through wide limits thus affecting the behavior of spring. Where rubber pads are bonded to steel plates, such variation will not occur, however.
4. In general, rubber springs are deflected by relatively large amount, and such deflections are more difficult to calculate accurately.

41. Determine the combined stiffness of two springs connected in parallel and series. (April/May 2019)

Parallel.

When two massless springs following Hooke's Law, are connected via a thin, vertical rod as shown in the figure below, these are said to be connected in parallel. Spring 1 and 2 have spring constants k1 and k2 respectively. A constant force →F is exerted on the rod so that remains perpendicular to the direction of the force. So that the springs are extended by the same amount. Alternatively, the direction of force could be reversed so that the springs are compressed.



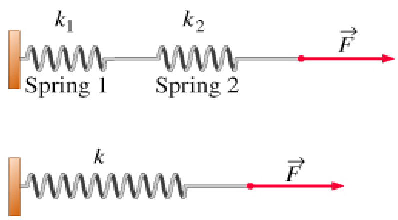
This system of two parallel springs is equivalent to a single Hookean spring, of spring constant k. The value of k can be found from the formula that applies to capacitors connected in parallel in an electrical circuit.

k=k1+k2

Series

.

When same springs are connected as shown in the figure below, these are said to be connected in series. A constant force $\vec{F}$ is applied on spring 2. So that the springs are extended and the total extension of the combination is the sum of elongation of each spring. Alternatively, the direction of force could be reversed so that the springs are compressed.



This system of two springs in series is equivalent to a single spring, of spring constant k. The value of k can be found from the formula that applies to capacitors connected in series in an electrical circuit.

For spring 1, from Hooke's Law

$$F = k_1 x_1$$

where x_1 is the deformation of spring.

Similarly if x_2 is the deformation of spring 2 we have

$$F = k_2 x_2$$

Total deformation of the system

$$x_1 + x_2 = \frac{F}{k_1} + \frac{F}{k_2}$$
$$\Rightarrow x_1 + x_2 = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

Rewriting and comparing with Hooke's law we get

$$k = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

FLYWHEELS CONSIDERING STRESSES IN RIMS AND ARMS FOR ENGINES AND PUNCHING MACHINES:

41. What do you understand by the term torsional rigidity?

Torsional rigidity:

The torsional rigidity is important in the case of camshaft of an I.C.engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to3 degree per metre length may be used as limiting value. The widely used deflection for the shafts islimited to 1 degree in a length equal to twenty times the diameter of the shaft.

The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G.\theta}{L} or \theta = \frac{T.L}{J.G}$$

Where

θ= Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$J = \frac{\pi}{32} \times d^4$ (For solid shaft)

$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4]$ (For hollow shaft)

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

42. Define the term lateral rigidity.

The lateral rigidity is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of abeam i.e.

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

43. Define Critical speed.

The critical speed is definedas the rotational speed of the rotor or rotating element at which resonance occurs in the system. The shaft speed at which at least one of the "critical" or natural frequencies of a shaft isexcitedknown as Critical Speed.

. Critical speed, $N_{s=} \frac{30}{\pi}\sqrt{\frac{g}{\delta_{st}}}$

Where,

g = gravity acceleration (9.81 m/s2

44. Define the term self-locking power screw. Nov/Dec 2012

Self-locking screws are defined by the angle of their threads. The threads of self-locking screws are precisely angled so that, once the screw is placed, they will not slip or move unless some additional force is applied. After you have screwed a self-locking screw into position, it will not move again unless you use a screwdriver or similar tool to remove it from position

If the friction angle (ϕ) is greater than helix angle (α) of the power screw, the torque required to lower the load will be positive, indicating that an effort is applied to lower the load. This type is screw is known as self-locking screws. The efficiency of the self-locking screw is less than 50%.

45. What is the use of flywheel or Function of flywheel? (April/May 2018)(N/D'2022) (or) write short notes on the working of flywheel as a speed regulator (nov/Dec 2021)

A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than supply.In machines where the operation is intermittent like punching machines, shearing machines,riveting machines, crushers etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

46. What is the main function of a flywheel in an engine? (Nov/Dec2011&Nov/Dec 2013)

The main function of flywheel in I.C. engine is, the energy is developedonly during power stroke which is much more than theengine load, and no energy is being developed duringsuction, compression and exhaust strokes in case of fourstroke engines and during compression in case of two strokeengines. The excess energy developed during power strokeis absorbed by the flywheel and releases it to the crankshaftduring other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed. A little consideration will show that when the flywheelabsorbs energy, its speed increases and when it releases, the speed decreases. Hence a flywheel does not maintain a constant speed; it simply reduces the fluctuation of speed.

46.(a) State the reasons why the size of multi cylinder engine flywheel size is smaller than that of single cylinder engine.(Nov/Dec 2021)

In a single cylinder engine, merely one power stroke for every two revolution of crankshaft, Whereas in multi cylinder engine the power stroke is given by number of cylinders. So that in Single cylinder engine there is more rotational energy required which is given by heavier flywheel. ... so, it has lighter flywheel.

47.Brief why fly wheels are used in punching machines.(Nov/Dec 2017)

The main function of flywheel in Punching press driven by electric motor the flywheel store energy during the idel portion of the work cycles by increasing its speed and delivers this energy during the peak load of punching.

48. Define co-efficient of fluctuation of a speed and energy in flywheel .(April/May 2013 & Nov/ Dec 2014).(May/June 2016)

The co-efficient of fluctuation of speed in flywheel is, the difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed. The ratio of the maximum fluctuation of speed to the mean speed is calledcoefficient of fluctuation of speed.Thecoefficient of fluctuation of speed is a limiting factor in the design of flywheel. It varies depending upon the nature of service to which the flywheel is employed.

Let

N1 = Maximum speed in r.p.m. during the cycle,

N2 = Minimum speed in r.p.m. during the cycle, and

N = Mean speed in r.p.m $= \dfrac{N_1 + N_2}{2}$

Coefficient of fluctuation of speed,

$$C_s = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$C_1 = \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$$

$$C_2 = \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2}$$

Where,

C_1 – angular speed, C_2 – linear speed

The ratio of fluctuation of energy to the mean energy is called co efficient of fluctuation of energy.

$K_E = \dfrac{E_{max} - E_{min}}{E}$ $= \Delta E/E$

49. How does the function of a flywheel differ from governor? (Nov/Dec 2012)

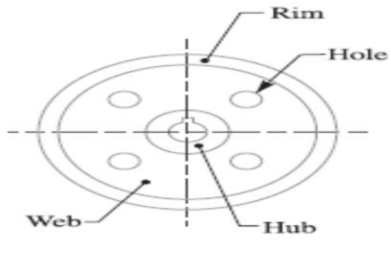
The function of a governor in engine is entirely different from that of a flywheel. It regulates the mean speed of an engine when there are variations in the load, e.g. when the load on the engine increases, it becomes necessary to increase the supply of working fluid. On the other hand, when the load decreases, less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load condition and keeps the mean speed within certain limits

50. Define whipping stress.

Whipping stress is defined as the general practice is to design a connecting rod by assuming the force in the connecting rod equal to the maximum forcesdue to pressure, neglecting the piston inertia effects and then checked for bending stress due to inertia force. The parallel components adds up algebracically to the force acting on the connecting rod and produces thrust on the pins. The perpendicular components produces bending action and the stress induced in the connecting rod is called whipping stress

51. How stresses are formed in flywheel rim and state its types?

A flywheel, as consists of a rim at which the major portion of the mass or weight of flywheel is concentrated, a boss or hub for fixing the flywheel on to the shaft and a number of arms for supporting the rim on the hub



(a) Flywheel with web.

The following types of stresses are induced in the rim of a flywheel:

1. Tensile stress due to centrifugal force,
2. Tensile bending stress caused by the restraint of the arms, and
3. The shrinkage stresses due to unequal rate of cooling of casting. These stresses may bevery high but there is no easy method of determining. This stress is taken care of by a factor of safety.

52. What are the various types of stresses induced in flywheel arms?

The following stresses are induced in the arms of a flywheel.

1. Tensile stress due to centrifugal force acting on the rim.
2. Bending stress due to the torque transmitted from the rim to the shaft or from the shaft to the rim.
3. Shrinkage stresses due to unequal rate of cooling of casting. These stresses are difficult to determine.

53. How the cross-sectional dimensions of arms of a flywheel are determined?

The cross-section of the arms is usually elliptical with major axis as twice the minor axis, as shown in Fig.4.7, and it is designed for the maximum bending stress.

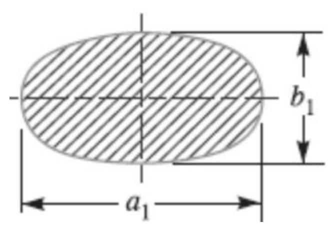


Fig 4.7

Let,

a_1 = Major axis, and

b_1 = Minor axis.

Section modulus, $Z = \frac{\pi}{32} \times b_1 (a_1)^2$ ……… (i)

We know that maximum bending moment, $M = \frac{T}{R.n}(R - r)^2$

Maximum bending stress, $\sigma_b = \frac{M}{Z} = \frac{T}{R.nZ}(R - r)$ ………… (ii)

Assuming $a_1 = 2\, b_1$, the dimensions of the arms may be obtained from equations (i) and (ii).

54. Briefly writeabout the design procedure of shaft, hub and key for flywheel.

Design of shaft:

The diameter of shaft for flywheel is obtained from the maximum torque transmitted. We know that the maximum torque transmitted,

$$T_{max} = \frac{\pi}{16} \times \tau\ (d_1)^3$$

Where,d1 = Diameter of the shaft, and , τ = Allowable shear stress for the material of the shaft

Design of hub:

The hub is designed as a hollow shaft, for the maximum torque transmitted. We know that the maximum torque transmitted,

$$Tmax = \frac{\pi}{16} \times \tau \frac{(d^4 - d1^4)}{d}$$

Where, d = Outer diameter of hub, and

d1 = Inner diameter of hub or diameter of shaft.

The diameter of hub is usually taken as twice the diameter of shaft and length from 2 to 2.5 times the shaft diameter. It is generally taken equal to width of the rim.

Design of hub:

A standard sunk key is used for the shaft and hub. The length of key is obtained by considering the failure of key in shearing. We know that torque transmitted by shaft,

$$Tmax = L \times W \times \tau \times \frac{d1}{2}$$

Where, L = Length of the key,

τ = Shear stress for the key material, and d1 = Diameter of shaft

55. Write short notes on importance of crankshaft in I.C engine.

The crankshaft is an important part of IC engine that converts the reciprocating motion of the piston into rotary motion through the connecting rod. The crankshaft consists of three portions – crank pin, crank web and shaft. The big end of the connecting rod is attached to the crank pin. The crank web connects the crank pin to the shaft portion. The shaft portion rotates in the main bearings and transmits power to the outside source through the belt drive, gear drive or chain drive.

56. Discuss the advantages and disadvantages of flywheel.

The following are the advantages and disadvantages of flywheel

Advantages:

1. Initial cost and ongoing maintenance.
2. They work out cheaper
3. It is environmentally friendly
4. It work in almost any climate
5. Very quick to get up to speed
6. It is extremely efficient
7. Take up less space than batteries or other forms of energy storage

Disadvantages

1. It is more weight they add
2. Heavy wheel spinning inside a moving car will tend to act like a gyroscope
3. Further difficulty is the huge stresses and strains that flywheels experience when they rotate at extremely high speeds.
4. Making higher speeds and energies possible without compromising on safety.

57. Define the term fluctuation of energy.

Fluctuation of energy

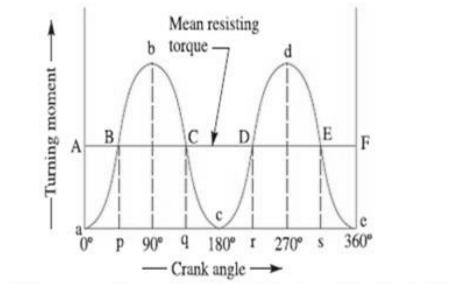


Fig 4.8

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider a turning moment diagram for a single cylinder double acting steam engine as shown in Fig.4.8 The vertical ordinate represents the turning moment and the horizontal ordinate (abscissa) represents the crank angle.

Similarly when the crank moves from q to r, more work is taken from the engine than is developed. This loss of work is represented by the area CcD. To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from q to r. As the crank moves from r to s, excess energy is again developed given by the area DdE and the speed again increases. As the piston moves from s to e, again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called fluctuation of energy. The areas BbC, CcD, DdE etc. represent fluctuations of energy.

58. In a flywheel, the major axis of the elliptical section of the arm is the plane of rotation Write down the reason for this arrangement.

The arms may have to carry the full torque load due to high inertia of the flywheel when the energy input to its shaft is cut off. The arm may be assumed as a cantilever fixed at the hub and carrying the load at the rim end. This bending moment lies in the plane of rotation of the flywheel. Therefore, the major axis of the arms must be parallel to the tangential force F acting on the flywheel.

Bending moment on the arm,

$$\mathbf{M = F(R\text{-}d_h/2)}$$

$$\mathbf{F = t/\ (nR)}$$

Where,

n = number of arms

Section modulus of arm, $Z = (\pi/32)\, b_1 a_1^2$

Bending stress $\sigma_b = M/Z$

59. What type of stresses are induced in disc flywheel? (Nov/Dec 2010)

The stresses produced in a flywheel being a rotating disc, centrifugal stresses acts upon its distributed mass and attempts to pull it apart. Its effect is similar to those caused by an internally pressurized cylinder. When a circular disc flywheel rotates at high speed, two types of stresses are set up. They are radial stress $\sigma_{rad}$and tangential stress σ_{Θ}. The numerical equationfor these stresses at any radius r given below:

$$\sigma rad = \frac{3+\mu}{8}\rho\omega^2(R^2 - r^2)$$

$$\sigma\Theta = \frac{\rho\omega^2}{8}[(3+\mu)R^2(1+3\mu)r^2$$

60. What are the applications of flywheel?

The application of flywheel is some cases the power is supplied at uniform rate. While the requirement of power from the driven machinery is variable. E.g.: punching press driven by the electric motor, rolling mill driven by an electric motor. In this case the flywheel store energy during the idle portion of the work cycle by increasing its speed and delivers this energy. During the peak load period of punching

60.(a) What do you mean by inertial bending in the design of connecting rod?(N0v/Dec 2021)

- Force due to inertia of the connecting rod and reciprocating mass is known as inertial bending
- Following are the forces acting on connecting rod (i) Force on the piston due to gas pressure. (ii) Force due to inertia of the connecting rod and reciprocating mass (iii) Force due to friction of the piston rings and of the piston.

61. What are the two planes of buckling of connecting rod?

The buckling of connecting rod in two different planes:

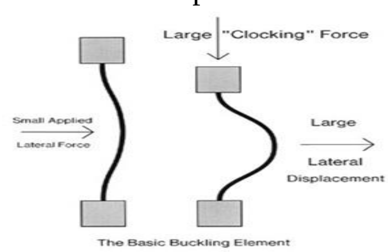


Fig 4.9

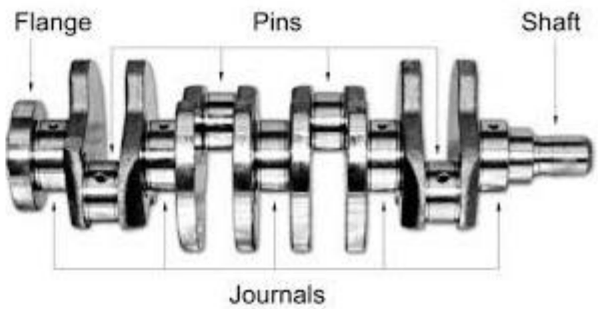
1. Plane of motion

The buckling of connecting rod in the plane of motion as shown in fig4.9. in this this plane, the end of connecting rod are hinged in the crank pin and piston pin. Therefore, for buckling about the XX-axis, the end fixity coefficient (n) is one.

2. Plane of perpendicular

The buckling of the connecting rod in a plane perpendicular to the plane of motion is shown in fig. in this plane; the ends of the connecting rod are fixed due to the constraining effect of bearing at the crank.

62. Define crankshaft.



Crankshaft is defined as to converts the reciprocating motion of the piston into rotary (circular) motion. It transmits engine torque to a pulley or gear, so that some object may be driven by the engine. The crankshaft also drives thecamshaft (on four-cycle engines), supports the fly- wheel, and, in many engines, operates the ignition system.Crankshafts can be made of cast or drop- forged steel.

63. At what angle of the crank, the twisting moment is maximum in the crankshaft?

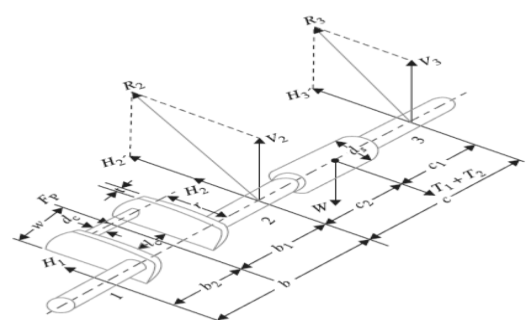


Fig 4.10

The twisting moment on the crankshaft will be maximum when the tangential force on the crank (F_T) is maximum.

The maximum value of tangential force lies when the crank is at angle of 25° to 30°from the dead centre for a constant volume combustion (i.e petrol engines) and 30° to 40° for constant pressure combustion engines (i.e. diesel engines).

Consider a position of the crank at angle of maximum twisting moment as shown in Fig.4.10 If p′ is the intensity of pressure on the piston at this instant, then the piston gas load at this position of crank.

$$F_T = \frac{\pi}{4} \times D^2 \times P'$$

Where,

F_T – tangential force, N

P - Intensity of pressure

64. Why is I-section prefered for connecting rod?

In the case of connecting rod, there are chances for buckling about x-axis and y-axis. Since the both ends of connecting rods are assumed as hnged about x-axis and fixed about y-axis, the area moment of inertia about x-axis and y-axis is having the relation as I_{XX} = $4I_{YY}$. Some time the slight buckling about x-axis is allowed whereas the buckling about y-axis will not be allowed. Since the required condition (i.e; $I_{xx} = 4I_{YY}$) is satisfied by I- section it is prefered.

65. Mention the various forces acting on the connecting rod. (Nov/Dec 2019)

The various forces acting on the connecting rod.

Force on the piston due to gas pressure and inertia of the reciprocating parts,

Force due to inertia of the connecting rod or inertia bending forces,

Force due to friction of the piston rings and of the piston,

Force due to friction of the piston pin bearing and the crankpin beraing.

The maximum compressive stress in the connecting rod will be

$$\sigma_{c(max)} = \text{direct compressive stress} + \text{maximum bending}$$

66. Define spring and state its various functions (Nov/Dec 16)

- A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed

Functions:

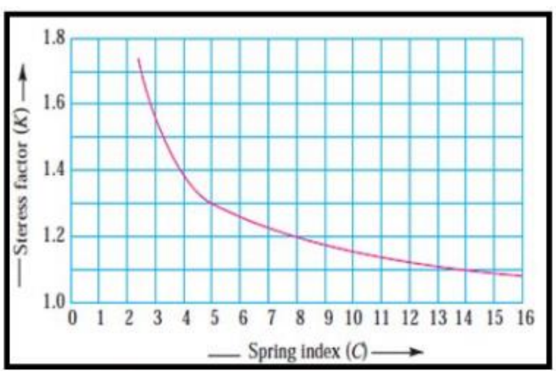
- To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.
- To apply forces, as in brakes, clutches and spring loaded valves cams and followers.

67. How does the function of flywheel differ from that of governor? (NOV/DEC 2012) (Nov/Dec 16)

- A governor regulates the mean speed of an engine when there are variations in the mean loads. It automatically controls the supply of working fluid to engine with the varying load condition and keeps the mean speed within the limits.
- A flywheel does not control the speed variation caused by the varying load A flywheel does not maintain constant speed.

68. While designing helical springs, K is introduced in the shear stress equation, why? (April/May 17)(Nov/Dec 2017)(Nov/Dec 2018)

The values of K for a given spring index(c) may be obtained from the graph as shown in fig 4.11



Note: The Wahl's stress factor (K) may be considered as composed of two sub-factors, KS and KC, such that

$$K = KS \times KC$$

Where,

KS = Stress factor due to shear, and

KC = Stress concentration factor due to curvature.

Fig 4.11

69. What are forces acting on connecting rod?(April/May 17)

- The force due to gas or steam pressure and inertia of the reciprocating part
- Inertia force
- Tensile force and compressive force

70. Name the common types of mechanical springs **Nov/Dec-20, Aprl/May-21**

Types of spring in mechanical

Based on the shape of the springs, it can be broadly classified into following types:

1. Helical Spring:
a.) Tension spring:
b.) Compression spring
c.) Torsion spring
d.) Spiral Springs
2. Leaf springs
3. Belleville spring
4. Volute and conical spring
5. Special purpose spring

71. A flywheel connected to a punching machine has to supply energy of 400 N-m while running at a mean angular speed of 20 rad/s. If the total fluctuation of speed is not to exceed ±2%, what is the mass moment of inertia of the flywheel ? **Nov/Dec-20, Aprl/May-21**

$\Delta E = 400 Nm$

$\omega_{mean} = 20$ rad/s

$C_S = \pm 2\%$

$= 2 \times \frac{2}{100} = 0.04$

$\Delta E = I\omega^2_{mean}C_S$

$400 = I(20)^2 \times 0.04$

$I = 25$ kg-m

PART- B(16 MARKS

HELICAL SPRINGS:

1. **A helical compression spring made of oil tempered carbon steel is subjected to a load which varies from 400 N to 1000 N. The spring index is 6 and the design factor of safety is 1.25. If the yield stress in shear is 110 MPa and endurance stress in shear is 350 MPa, find: 1. Size of the spring wire, 2. Diameters of the spring, 3. Number of turns of the spring, and 4. free length of the spring. The compression of the spring at the maximum load is 30 mm. The modulus of rigidity for the spring material may be taken as 80 kN/mm^2. (16) (Nov/Dec 2013)**

Given data:

Minimum load, P_{min} = 400 N
Maximum load, P_{max} = 1000 N
Spring index, C = 6
Factor of safety, n_s = 1.25
Compression (or) deflection of the spring, y = 30mm
Yield stress in shear, τ_y = 110 N/mm^2
Endurance stress in shear, τ_{-1} = 350 N/mm^2
Modulus of rigidity, G = 80kN/mm^2 = 80×10^3N/mm^2

To find:

1. Size of the spring wire
2. Diameters of the spring
3. Number of turns of the spring, and
4. Free length of the spring

Solution:

(i) Diameter of the spring wire, d:

Mean load of the spring,

$$\mathrm{Pm} = \frac{\mathrm{Pmax} + \mathrm{Pmin}}{2}$$

$$= \frac{1000 + 400}{2} = 700\mathrm{N}$$

Amplitude load on the spring,

$$\mathrm{Pa} = \frac{\mathrm{Pmax} - \mathrm{Pmin}}{2}$$

$$= \frac{1000 - 400}{2} = 300\mathrm{N}$$

Direct shear factor,

$$Ksh = 1 + \frac{0.615}{C}$$

$$Ksh = 1 + \frac{0.615}{6} = 1.1025$$

Mean shear stress,

$\tau_m = (8K_{sh}P_mC)/\pi d^2$

$= 8 \times 1.1025 \times 700 \times 6 / \pi d^2$

$= 11791.471 / d^2$

Wahl stress factor, $K_s = K_{sh} \times K_c$

$= 1.1025 \times 1.15$ (from table 4.4 for C = 6, $K_c = 1.15$)

$K_s = 1.267875$

Amplitude shear stress,

$\tau_m = (8K_s\ P_a\ C) / \pi d^2$

$= 8 \times 1.267875 \times 300 \times 6 / \pi d^2$

$= 5811.51 / d^2$

For repeated loading,

$$\frac{1}{n} = \frac{\tau_{m} - \tau_a}{\tau_y} + \frac{2\tau_a}{\tau_{-1}}$$

Substituting $\boldsymbol{\tau}_{m,\ a}$ value in the above equation,

$$\frac{1}{1.25} = \frac{\frac{11791.741}{d^2} - \frac{5811.51}{d^2}}{110} + \frac{\frac{2\times 5811.51}{d^2}}{350}$$

$$\frac{1}{1.25} = \frac{\frac{5879.96}{d^2}}{110} + \frac{\frac{11623.02}{d^2}}{350}$$

$$\frac{1}{1.25} = \frac{54.36327}{d^2} + \frac{33.208}{d^2}$$

$$\frac{1}{1.25} = \frac{87.57184}{d^2}$$

$$d^2 = 109.46$$

$$d^2 = 10.462\text{mm}$$

The next standard diameter from table 4.5, **d = 10.6mm**

(ii)Mean coil diameter, D:

Spring index, C = D / d

D = C × d = 6×10.6

D = 63.6mm

(iii) Number of active turns, n:

Deflection,

$$y = \frac{8\ P_{max\ C^3\ n}}{Gd}$$

$$30 = \frac{8 \times 1000 \times 6^3 \times n}{8 \times 10^3 \times 10.6}$$

$$= 14.72 \quad \mathbf{sayn = 15}$$

Assume that the end of the coil is squared and ground

Total number of coil, n_t = n+2 = 15 + 2 = **17**

(iv) Solid length of the spring, L_s:

Ls = dn + 2d = 10.6 × 15 + 2 × 10.6 = **180.2mm**

(v) Free length of the spring, L_f:

$L_f = L_S + y$ = 180.2 + 30 = **210.2mm**

(vi) Pitch of the coil $p = \frac{L_f - L_s}{n_t} + d$

$$p = \frac{210.2 - 180.2}{17} + 11 = \mathbf{12.76mm}$$

(vi) Helix angle of the coil, α:

$$\alpha = \tan^{-1}(\frac{p}{\pi\ D})$$

$$\alpha = \tan^{-1}(\frac{12.76}{\pi \times 63.6}) = \mathbf{3.65°}$$

(vii) Spring rate, q:

$$q = \frac{P_{max}}{y} = \frac{1000}{30} = \mathbf{33.33N/mmAns.}$$

2. A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn. (8)

Given data:

d = 6 mm;

D_o= 75 mm;

τ = 350 MPa = 350 N/mm²;

G = 84 kN/mm2 = 84 × 103 N/mm²

Solution:

We know that mean diameter of the spring,

D = Do – d = 75 – 6 = 69 mm

∴Spring index,

$$C = \frac{D}{d}$$

$$C = \frac{69}{6} = 11.5$$

Let,

W = Axial load, and

δ / *n* = Deflection per active turn.

1. Neglecting the effect of curvature

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C}$$

$$K_S = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S + \frac{8W.D}{\pi \times d^3}$$

$$350 = K_S + \frac{8W.D}{\pi \times d^3}$$

$$350 = 1.043 + \frac{8W \times 69}{\pi \times (6)^3} = 0.848W$$

W = 350 / 0.848 = **412.7 N**

∴We know that deflection of the spring,

$$\delta = \frac{8W.D^3\ .n}{G \times d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8W.D^3}{G \times d^4}$$

$$\frac{\delta}{n} = \frac{8 \times 412.7(69)^3}{84 \times 10^3 \times 6^4} = 9.96\text{mm}$$

2. Considering the effect of curvature

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.165}{C}$$

$$K = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.165}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi \times d^2}$$

$$350 = 1.123 \times \frac{8W \times 11.5}{\pi \times 6^2} = 0.913W$$

W = 350 / 0.913 = **383.4 N**

and deflection of the spring,

$$\delta = \frac{8W.D^3\ .n}{G \times d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\mathrm{W.D}^3}{\mathrm{G} \times \mathrm{d}^4} = \frac{8 \times 383.4(69)^3}{84 \times 10^3 \times 6^4} = 9.26\mathrm{mm}$$

3. Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, G = 84 kN/mm2. Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils. (8) (Nov/Dec 2010)

Given:

W_1 = 2250 N; W2 = 2750 N;

δ = 6 mm; C= D/d = 5;

τ = 420 MPa= 420 N/mm^2

G = 84 kN/mm^2 = 84 × 103 N/mm^2

Solution:

1. **Mean diameter of the spring coil**

Let, D = Mean diameter of the spring coil for a maximum load of

W_2 = 2750 N, and

d = Diameter of the spring wire.

We know that twisting moment on the spring,

$$T = W_2 \times \frac{D}{2}$$

$$T = 2750 \times \frac{5d}{2} = 6875\mathrm{d}$$

We also know that twisting moment (*T*),

$$6875d = \frac{\pi}{16} \times \tau \times d^3$$

$$6875d = \frac{\pi}{16} \times 420 \times d^3 = 42.84\mathrm{d}^3$$

$$\mathrm{d}^2 = 6875 / 82.48 = 83.35 \text{ or } \mathrm{d} = 9.13 \text{ mm}$$

We shall take a standard wire of size *SWG* 3/0 having diameter (*d*) = 9.49 mm.

∴ Mean diameter of the spring coil,

D = 5d = 5 × 9.49 = 47.45 mm

We know that outer diameter of the spring coil,

D_o= D + d = 47.45 + 9.49 = 56.94 mm

and inner diameter of the spring coil,

D_i= D – d = 47.45 – 9.49 = 37.96 mm

2. **Number of turns of the spring coil**

Let, n = Number of active turns.

It is given that the axial deflection (δ) for the load range from 2250 N to 2750 N (*i.e.* for *W* = 500 N) is 6 mm.

We know that the deflection of the spring (δ),

$$6 = \frac{8W.C^3.\text{n}}{G.d} = \frac{8 \times 500(5)^3 \text{ n}}{84 \times 10^3 \times 9.49} = 0.63\text{n}$$

$\therefore$ n = 6 / 0.63 = 9.5 say 10

For squared and ground ends, the total number of turns,n' = 10 + 2 = 12

3. **Free length of the spring**

Since the compression produced under 500 N is 6 mm, therefore maximum compression produced under the maximum load of 2750 N is

$$\delta_{\max} = \frac{6}{500} \times 2750 = 33mm$$

We know that free length of the spring,shown in fig 4.12

$$LF = n'.d + \delta_{max} + 0.15\ \delta_{max}$$
$$= 12 \times 9.49 + 33 + 0.15 \times 33$$
$$= 151.83 \text{ say } 152 \text{ mm.}$$

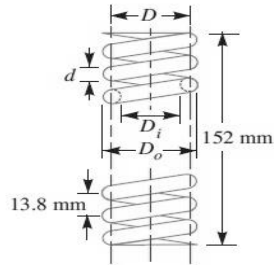


Fig 4.12

4. **Pitch of the coil**

We know that pitch of the coil

$$= \frac{Freelength}{n^{'} - 1} = \frac{152}{12 - 1} = 13.73 say 13.8mm$$

4. **(a)Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5. The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN/mm^2. Take whal'sfactor= $\frac{4C-1}{4C-4} + \frac{0.165}{C}$, where C = spring index. (8)**

Given data:

W = 1000 N; δ = 25 mm; C = D/d = 5;

τ = 420 MPa = 420 N/mm^2; G = 84 kN/mm^2 = 84 × 103 N/mm^2

Solution:

1. ***Mean diameter of the spring coil***

Let, *D* = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$\text{K} = \frac{4\text{C} - 1}{4\text{C} - 4} + \frac{0.165}{\text{C}}$$

$$\text{K} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.165}{11.5} = 1.31$$

and maximum shear stress (τ),

$$420 = K \times \frac{8W.C}{\pi \times d^2}$$

$$420 = 1.31 \times \frac{8 \times 1000 \times 5}{\pi \times d^2} = \frac{16677}{d^2}$$

$\therefore \quad d^2 = 16\ 677 / 420 = 39.7$ or **d = 6.3 mm**

we shall take a standard wire of size SWG 3 having diameter (d) = 6.401 mm.

∴ Mean diameter of the spring coil,

D = C.d= 5 d = 5 × 6.401 = 32.005 mm... (∵C = D/d = 5)

and outer diameter of the spring coil,

D_o= D + d = 32.005 + 6.401 = 38.406 mm**.**

2. Number of turns of the coils

Let, n = Number of active turns of the coils.

We know that compression of the spring (δ),

$$25 = \frac{8W.C^3.\text{n}}{G.d} = \frac{8 \times 500(5)^3\ \text{n}}{84 \times 10^3 \times 6.401} = 1.86n$$

∴n = 25 / 1.86 = 13.44 say 14

For squared and ground ends, the total number of turns,

n' = n + 2 = 14 + 2 = 16

3. Free length of the spring

We know that free length of the spring

= n'.d+ δ + 0.15 δ = 16 × 6.401 + 25 + 0.15 × 25

= 131.2 mm

4. Pitch of the coil

We know that pitch of the coil

$$\frac{Freelength}{n' - 1} = \frac{131.2}{16 - 1} = 8.75mm$$

5. Design a helical spring for a spring loaded safety valve (Ramsbottom safety valve) for the following conditions : Diameter of valve seat = 65 mm ; Operating pressure = 0.7 N/mm2; Maximum pressure when the valve blows off freely = 0.75 N/mm²; Maximum lift of the valve when the pressure rises from 0.7 to 0.75 N/mm² = 3.5 mm; Maximum allowable stress = 550 MPa; Modulus of rigidity = 84 kN/mm²; spring index = 6. Draw a neat sketch of the free spring showing the main dimensions.(8)

Given data:

D1 = 65 mm;

p1 = 0.7 N/mm^2; p2 = 0.75N/mm^2;

δ = 3.5 mm; τ = 550 MPa = 550 N/mm^2;

G = 84 kN/mm^2= 84 × 103 N/mm^2;

C = 6

Solution:

1. Mean diameter of the spring coil

Let, D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

Since the safety valve is a Ramsbottom safety valve, therefore the spring will be under tension. We know that initial tensile force acting on the spring (i.e. before the valve lifts),

$$W_1 = \frac{\pi}{4}(\mathrm{D}_1)^2 p_1 = \frac{\pi}{4}(65)^2 0.7 = 2323N$$

and maximum tensile force acting on the spring (*i.e.* when the valve blows off freely),

$$W_2 = \frac{\pi}{4}(\mathrm{D}_1)^2 p_1 = \frac{\pi}{4}(65)^2 0.75 = 2489N$$

∴ Force which produces the deflection of 3.5 mm,

$$W = W2 - W1 = 2489 - 2323 = 166 \text{ N}$$

Since the diameter of the spring wire is obtained for the maximum spring load (*W*2), therefore maximum twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2489 \times \frac{6d}{2} = 7467d$$

We know that maximum twisting moment (*T*),

$$7467d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 550 \times d^3 = 108d^3$$

∴ $d^2 = 7467 / 108 = 69.14$ or $d =$ **8.3 mm**

From Table, we shall take a standard wire of size *SWG* 2/0 having diameter (*d*)= 8.839 mm

∴ Mean diameter of the coil,

$$D = 6\,d = 6 \times 8.839 = 53.034 \text{ mm.}$$

Outside diameter of the coil,

$$D_o = D + d = 53.034 + 8.839 = 61.873 \text{ mm}$$

and inside diameter of the coil,

$$D_i = D - d = 53.034 - 8.839 = 44.195 \text{ mm}$$

2. Number of turns of the coil

Let, n = Number of active turns of the coil.

We know that the deflection of the spring (δ),

$$3.5 = \frac{8W.C^3.\mathrm{n}}{G.d} = \frac{8 \times 166 \times 6^3\,\mathrm{n}}{84 \times 10^3 \times 8.839} = 0.386n$$

∴ n = 3.5 / 0.386 = 9.06 say 10**.**

For a spring having loop on both ends, the total number of turns,

n' = n + 1 = 10 + 1 = 11

3. Free length of the spring

Taking the least gap between the adjacent coils as 1 mm when the spring is in free state, the freelength of the tension spring,

LF = n.d+ (n – 1) 1 = 10 × 8.839 + (10 – 1) 1 = 97.39 mm

4. Pitch of the coil

We know that pitch of the coil

$$\frac{Freelength}{n' - 1} = \frac{97.39}{10 - 1} = 10.82mm$$

6.A helical compression spring made of circular wire, is subjected to an axial force, which varies from 2.5 kN to 3.5 kN. Over this range of force, the deflection of the spring should be

approximately 5 mm. The spring index can be taken as 5. The spring has square and ground ends. The spring is made of patentedand cold-drawn steel wire and ultimate tensile strength of 1050 N/mm2 and modulus of rigidity of 81370 N/mm2. The permissible shear stress for the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate i) Wire diameter ii) mean coil diameter iii) numberof active coils iv) total number of coils v) solid length of spring vi) free lengthof spring vii) required spring rate viii) actual spring rate.(April/May 2018)(N/D'2022)

Given data:

P=2.5 to 3.5 KN, δ=5mm, C=5, S_{ut} = 1050N/mm^2, G= 81370N/mm^2, τ=0.05 S_{ut}

Solution:

(i) Wire diameter

The permissible shear stress is given by, = 0.5 Sut = 0.5(1090) = 545 N/mm^2

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(5)-1}{4(5)-4} + \frac{0.615}{5} = 1.3105$$

$$\tau = K\left(\frac{8PC}{\pi d^2}\right)$$

$$525 = 1.3105\left(\frac{8 \times 3500 \times 5}{\pi d^2}\right)$$

d= 10.55 or 11mm(i)

(ii) Mean Coil Diameter

D = cd =5x11 = 55mm (ii)

(iii) Number of Active Coils

$$\delta = \frac{8PD^3N}{Gd^4}$$

$$5 = \frac{8 \times (3500 - 2500) \times (55)^3 \times N}{81370 \times (11)^4}$$

N = 4.48 or 5 coils

Number active turns N=5

(iv) Total Number of Coils:

For square and ground ends.

The number of inactive coil is 2.

Therefore Nt = N+2 = 5+2 = 7 coils (iv)

(V) Solid Length of Spring

Solid length of spring = Nt d = 7x11 =77mm (v)

(vi) Free Length

The actual deflection of the spring under the maximum force of 3.5kN is given by.

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8\times 3500\times(55)^3\times 5}{81370\times(11)^4} = 19.55mm$$

It is assumed that there will be a gap of 0.5 mm between consecutive coils when the spring is subjectedto the maximum force3.5kN. The total number of coils is 7.
The total axial gap between the coilswill be (7-1) x 0.5 = 3mm.
Free length = solid length + total axial gap + δ
= 77+3+19.55 = 99.55 or 100mm

(vii) Required Spring Rate:

The required spring rate is given by,

$$K = \frac{P_1 - P_2}{\delta} = \frac{3500-2500}{5} = 200N/mm$$

(viii) Actual Spring Rate:

The actual spring rate is given by,

$$K = \frac{Gd^4}{8D^3N} = \frac{81370\times(11)^4}{8\times(55)^3\times 5} = 179.01N/mm$$

7.A spring loaded safety valve for a boiler is required to blow off at a pressure of0.8 MPa. The diamater of valve seat is 90 mm and maximum lift of valve is10 mm. Design a suitable spring for the valve assuming the spring index as 7.Provide an initial compression of 30 mm. Take allowable shear stress as420 MPa.(Nov/Dec 2017)

Solution Given data:

Blow off pressure $p_2 = 0.8$ MPa
Dia of valve seat $D_V = 90$ mm
Lift $y' = 10$ mm
Initial compression $y_1 = 30$ mm
Spring index $C = 7$
Final compression $y = y_2 = y_1 + y' = 30 + 10 = 40$ mm
Allowable shear stress $\tau = 420$ MPa
Select modulus of rigidity for steels $G = 80$ GPa $= 80 \times 10^3$ MPa
Force on the spring = Force on valve seat = Area of valve × pressure

$$F = \frac{\pi}{4} D_V^2 p$$

Maximum force (at the time of blow off) $= F_2 = \frac{\pi}{4} D_V^2 p_2$

$$F = F_2 = \frac{\pi}{4} \times 90^2 \times 0.8 = 5089.38 \text{ N}.$$

Step 1. Shear stress $\tau = \frac{8FCK}{\pi d^2}$

when $K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 7 - 1}{4 \times 7 - 4} + \frac{0.615}{7}$

$$K = 1.2129$$

$$420 = \frac{8 \times 5089.38 \times 7 \times 1.2129}{\pi d^2}$$

$\therefore$ dia of wire $d = 16.18$

Standard dia of wire $d = 17$ mm

Step 2. Mean coil dia $D = Cd = 7 \times 17 = 119$ mm

Inside dia of coil $D_i = D - d = 119 - 17 = 102$ mm

Outside dia of coil $D_o = D + d = 119 + 17 = 136$ mm

Step 3. No. of active coils $i = \frac{yGd^4}{8FD^3}$

$$= \frac{40 \times 80 \times 10^3 \times 17^4}{8 \times 5089.38 \times 119^3} = 3.895 \simeq 4 \text{ coils}$$

Step 4. Free length of spring

$$l_o \geq (i + n)d + y + a$$

Assuming squared and ground ends, $n = 2$

Assume clearance $a = 0.25y$

$0.25 \times 40 = 10$ mm

$\therefore$ $l_o \geq (4 + 2)17 + 40 + 10$

$l_o \geq 152 \simeq 152$ mm.

Step 5. Pitch $p = \frac{l_o - 2d}{i} = \frac{152 - 2 \times 17}{4} = 29.5$ mm

Step 6. Stiffness $F_o = \frac{F}{y} = \frac{5089.38}{40} = 127.23$ N/mm

Step 7. Length of wire $l_w = \pi D i'$

$= \pi D\,(i + n) = \pi \times 119 \times (4 + 2)$

$l_w = 2243.1$ mm.

8. A Bellevile spring is made of silicon steel. The spring is compressed completely flat when it is subjected to axial force of 4500N. The corresponding maximum stress is (1375×10⁶ N/m²). Assume $\frac{d_o}{d_i} = 1.75$ and $\frac{h}{t} = 1.5$ (16) (Apr/May - 2010)

Calculate:

1. **Thickness of washer;**
2. **Free height of washer minus thickness (h);**
3. **Outer diameter of washer;**
4. **Inner diameter of washer**

Solution:

When the spring is compressed completely flat.

$$\delta = \mathrm{h}$$

$$M = \frac{6}{\pi \log_e(d_o l d_i)}\left[\frac{(d_o l d_i) - 1^2}{(d_o l d_i))}\right]$$

$$\frac{6}{\pi \log_e(1.75)}\left[\frac{(1.75) - 1^2}{(1.75)}\right] = 0.6268$$

$$C1 = \frac{6}{\pi \log_e(d_o l d_i)}\left[\frac{(d_o l d_i) - 1}{(d_o l d_i))} - 1\right]$$

$$= \frac{6}{\pi \log_e(1.75)}\left[\frac{(1.75) - 1}{(1.75)} - 1\right] = 1.161$$

$$C2 = \frac{6}{\pi \log_e(d_o l d_i)}\left[\frac{(d_o l d_i) - 1}{2}\right]$$

$$= \frac{6}{\pi \log_e(1.75)}\left[\frac{(1.75) - 1}{2}\right] = 1.28$$

Dividing equation (10.42) by eq. (10.43) in v.b. bhandari 438

$$\frac{P}{Q} = \frac{\left(h - \frac{\delta}{2}\right)(h - \delta)t + t^3}{C1\left(h - \frac{\delta}{2}\right) + C2t}$$

$$= \frac{t^3}{C1\left(h - \frac{\delta}{2}\right) + C2t}$$

Because (h=δ)

Substituting values,

$$\frac{4500}{1375(10)^6} = \frac{t^3}{(1.161)\left(1.5t - \frac{1.5t}{2}\right) + (1.28)t} = \frac{t^2}{2.15}$$

t = 2.653 $(10)^{-3}$m = 2.653mm = 2.65mm …..(i)

h = 1.5t = 1.5(2.65) = 3.98mm = 4mm ………(ii)

From equation (10.42)

$$P = \frac{E\delta}{(1-\mu^2)M(d_o l2)^2}\left[\left(h - \frac{\delta}{2}\right)(h-\delta)t + t^3\right]$$

Or

$$P = \frac{E\delta}{(1-\mu^2)M(d_o l2)^2}[t^3]$$

Because (h=δ)

E = 207000 N/mm^2 = (207 000×10^6) N/m^2

$$\mu = 0.3$$

t = (2.65×10^{-3}) m

h = $\delta = 4 \times 10^{-3}$= 0.004 m

Substituting these values,

Or

$$4500 = \frac{(207\,000 \times 10^6)(0.004)}{(1-0.3^2)(0.6268)(d_o l2)^2}[(2.65 \times 10^{-3})^3]$$

d$_o$ = 154.96× 10^{-3} m = 155mm

9. A helical compression spring is used to absorb the shock. The initial compression of the spring is 30 mm and it is further compressed by 50 mm while absorbing the shock. The spring is to absorb 250 J of energy during the process. The spring index can be taken as 6. The made of patented and cold draw steel wire with ultimate tensile strength of 813750 N/mm^2. The permissible shear stress for the spring wire should be taken as 30% of the ultimate tensile strength. Design the spring and calculate (Apr/May 2012)

1. **Wire diameter**
2. **Mean coil diameter**
3. **Number of active turns**
4. **Free length**
5. **Pitch of the turns**

Given data:

Suppose P_1 and δ_1 denote initial spring force and deflection respectively before the shock.

δ_1= 30 mm

P_1 = kδ = (30 k) N

Where, k is the stiffness of the spring.

Solution:

Suppose P_2 and δ_2 denote initial spring force and deflection respectively after the shock.

$\delta_2 = 30+50 = 80$ mm

$P_2 = k\delta_2 = (80\ k)$ N

Average force during compression = $\frac{(30k+80k)}{2}$

= (55k) N

Energy absorbed during shock = average force × δ

$250\times10^3 = (55k)\times50$

k = 90.19 N/mm

The maximum spring force given by,

$P_2 = 80\ k = 80\ (90.19) = 7272.72$ N

The permissible shear stress for the spring wire is given by,

$\boldsymbol{\tau} = 0.3\ S_{ut} = 0.3\ (1500) = 450\ N/mm^2$

$$\boldsymbol{K} = \frac{4C-1}{4C+1} + \frac{0.615}{C}$$

$$= \frac{4(6)-1}{4(6)+1} + \frac{0.615}{6} = 1.2525$$

$$\tau = K\left(\frac{8PC}{\pi d^2}\right)$$

Or

$$450 = (1.2525)\left\{\frac{8(7272.72)(6)}{\pi d^2}\right\}$$

d = 17.59 or 18 mm (i)

D = Cd = 6(18) = 108 mm (ii)

We know that

$$k = \frac{Gd^4}{8D^3N}$$

Or

$$90.91 = \frac{(81370)(18)^4}{8(108)^3N}$$

N = 9.32 or 10 turns (iii)

It is assumed that the spring has square and ground ends. The number of inactive turns is 2. Therefore,

$N_t = N + 2 = 10 + 2 = 12$

Solid length = $N_t d = 12(18) = 216$ mm

It is assumed that there will be a gap of 2 mm between the adjacent turns when the spring is subjected to maximum force of 7272.72 N. the total number of turn is 12 therefore, the total axial gap will be (12-1)×2 = 22 mm. the maximum deflection is given by,

$$\delta = \frac{8PD^3N}{Gd^4}$$

$$= \frac{8(7272.72)(108)^3\ (10)}{(81370)(18)^4} = 85.80\ \text{mm}$$

Free length = solid length + total axial gap + δ

$$= 216+22 + 85.80 = 323.8 \text{ mm}$$

Or free length = 325 mm

Pitch of the coil = $\frac{\text{free length}}{(N_t-1)} = \frac{325}{(N_t-1)} = 29.54$ mm

10. A safety valve, 40 mm in diameter, is to blow off at a pressure of 1.2 MPa. It is held on its seat by means of a helical compression spring, with initial compression of 20 mm. The maximum lift of the valve is 12 mm. The spring index is 6. The spring is made of cold-drawn steel wire with ultimate tensile strength of 1400 MPa. The permissible shear stress can be taken as 50% of this strength. G= 81.37 GPa. Calculate (i) wire diameter, (ii) mean coil diameter and (iii) number of active coils. Nov/Dec-20, Aprl/May-21.

THE BELWO METHODS PROBLEM WILL BE FOLLOW

10. A safety valve 50 mm in diameter is to blow off at a pressure of 1.5MPa. it is held on its seat by means of helical compression of 25 mm. the maximum lift of the valve is 10 mm. the spring index can be taken as 6. The spring is made of patented and cold drawn steel wire with ultimate tensile strength of 1500 N/mm^2 and modulus of rigidity of 81370 N/mm^2. The permissible shear stress for the spring wire should be taken as 30% of the ultimate tensile strength. Design the spring and calculate.

1. **Wire diameter ,2. Mean coil diameter**
2. **Number of active turns**
3. **Total number of turns**
4. **Solid length**
5. **Free length**
6. **Pitch of the coil (16) (Apr/May- 2013)**

Given data:

Diameter, d = 50 mm

Pressure, p = 1.5 MPa

Solution:

Let P_1 and δ_1 denote initial spring force and deflection respectively, when the valve just begins to blow off.

$$P_1 = \frac{\pi}{4} d^2 p$$

$$= \frac{\pi}{4}(50)^2(1.5) = 2945.24 \text{ N}$$

Let P_2 and δ_2 denote spring force and deflection respectively, when the valve is open.

$$\delta_2 = \delta_1 + \text{valve lift} = 25 + 10 = 35 \text{ mm}$$

Also,

$$P \alpha\, \delta$$

Therefore

$$\frac{P_2}{P_1} = \frac{\delta_2}{\delta_1} \; or \; \frac{P_2}{(2945.24)} = \frac{35}{25}$$

P_2 = 4123.34 N (maximum force)

The permissible shear stress for the spring wire is given by,

τ = 0.3 S_{ut} = 0.3 (1500) = 450 N/mm^2

$$\boldsymbol{K} = \frac{4C - 1}{4C + 1} + \frac{0.615}{C}$$

$$= \frac{4(6) - 1}{4(6) + 1} + \frac{0.615}{6} = 1.2525$$

$$\tau = K\left(\frac{8PC}{\pi d^2}\right)$$

Or

$$450 = (1.2525)\left\{\frac{8(7272.72)(6)}{\pi d^2}\right\}$$

d= 13.24 or 14 mm (i)

D = Cd = 6 (14) = 84 mm (ii)

$$\delta = \frac{8PD^3N}{Gd^4}$$

Substituting values of P_1 and δ_1,

$$25 = \frac{8(2945.24)(84)^3N}{(81370)(14)^4}$$

N = 5.6 or 6 turns (iii)

It is assumed that spring has square and ground ends. The number of inactive coil is 2. Therefore

N_t = N + 2 = 6 + 2 = 8 turns (iv)

Solid length = N_t d = 8 (14) = 112 mm (v)

The maximum deflection of the spring under the force of 4123.34 N is given by,

$$\delta = \frac{8PD^3N}{Gd^4}$$

$$\delta = \frac{8(4123.34)(84)^3(\,6)}{(81370)(14)^4} = 37.53\text{mm}$$

It is assumed that there will a gap of 2mm between the adjacent turns, when the spring is subjected to the maximum compression. This gap is essential to avoid clashing of the coils. The total number of turns is 8. Therefore, the total axial gap will be (8-1) ×2 = 14 mm.

Free length = solid length + total axial gap + δ

= 112 + 14 +37.53 = 163.53 mm

Or

Free length = 165 mm (vi)

Pitch of the coil $= \dfrac{freelength}{(N_t - 1)} = \dfrac{165}{(8-1)}$

= 23.57 mm (vii)

10.(a) A safety valve of 70 mm dia is to blow off pressure at 1. MPa. it is placed on its seat by a closed coil helical spring of circule steel wirw. the mean dia of each coil is 150 mm and compression of the spring is 25 mm.Find the dia of the spring wire and active numer of turns required.if the allowale shear stress of wire material is 120 N/mm2. (Nov/Dec 2021)

Given data:

Valve diameter, $d_v = 70\ mm$

Valve pressure, $p_v = 1.1 MPa = 1.1\ N/mm^2$

Mean diameter of the coil, $D = 150\ mm$

Compression of the spring, $y = 25\ mm$

Modulus of rigidity, $G = 0.084 \times 10^6\ MPa = 0.084 \times 10^6\ N/mm^2$

Allowable shear stress, $\tau = 130\ MPa = 130\ N/mm^2$

To find:

(i) Diameter of the spring wire, d

(ii) Number of active turns, n

☺ ***Solution:***

(i) Diameter of the spring wire, d:

Area of safety valve, $A_v = \dfrac{\pi \times {d_v}^2}{4} = \dfrac{\pi \times 70^2}{4} = 3848.45\ mm^2$

Maximum load on the valve spring, P

$$P = \text{Valve pressure} \times \text{Valve area}$$

$$= 1.1 \times 3848.45 = 4233.296\ N$$

From equation (4.7),

Allowable shear stress, $\tau = K_s \dfrac{8PC}{\pi d^2} = K_s \dfrac{8PD}{\pi d^3}$ $\quad [\because Assume\ K_s = 1]$

$$130 = 1 \times \frac{8 \times 4233296 \times 150}{\pi d^3}$$

$$d^3 = 12438.46$$

$$d = 23.169\ mm$$

The nearest standard diameter of the spring wire,

$$d = \mathbf{23.6}\ mm$$

Spring index, $C = \dfrac{D}{d} = \dfrac{150}{23.6} = 6.36$

(ii) Number of active turns, n:

deflection of the spring,

$$y = \frac{8PC^3 n}{Gd}$$

$$25 = \frac{8 \times 4233296 \times (6.36)^3 \times n}{0.084 \times 10^6 \times 23.6}$$

$$n = \mathbf{6}$$

Assume end of the coil is squared and ground.

Total number of coils, $n_t = n + 2 = 6 + 2 = \mathbf{8}$

Result:

(i) Diameter of the spring wire, $d = 23.6\ mm$

(ii) Number of active turn, $n = 6$

(iii) Total number of coils, $n_t = 8$

10.(b)Design and draw a valve spring of a petrol engine for the following operating conditions: Spring load when the valve is open = 400 N Spring load when the valve is closed = 250 N Maximum inside diameter of spring = 25 mm Length of the spring when the valve is open = 40 mm Length of the spring when the valve is closed = 50 mm Maximum permissible shear stress = 400 MPa(A/M'2023)

Solution. Given: W1 = 400 N ; W2 = 250 N ; Di = 25 mm ; l 1 = 40 mm ; l 2 = 50 mm ; τ = 400 MPa = 400 N/mm2

1. Mean diameter of the spring coil

Let d = Diameter of the spring wire in mm, and

D = Mean diameter of the spring coil

= Inside dia. of spring + Dia. of spring wire = (25 + d) mm

Since the diameter of the spring wire is obtained for the maximum spring load (W1), therefore maximum twisting moment on the spring,

$$T = W_1 \times \frac{D}{2} = 400\left(\frac{25+d}{2}\right) = (5000 + 200\ d)\ \text{N-mm}$$

We know that maximum twisting moment (T),

$$(5000 + 200\ d) = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 400 \times d^3 = 78.55\ d^3$$

Solving this equation by hit and trial method, we find that $d = 4.2$ mm.

From Table 23.2, we find that standard size of wire is *SWG* 7 having $d = 4.47$ mm.

Now let us find the diameter of the spring wire by taking Wahl's stress factor (K) into consideration.

We know that spring index,

$$C = \frac{D}{d} = \frac{25 + 4.47}{4.47} = 6.6 \qquad \text{... } (\because D = 25 + d)$$

$\therefore$ Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 6.6 - 1}{4 \times 6.6 - 4} + \frac{0.615}{6.6} = 1.227$$

We know that the maximum shear stress (τ),

$$400 = K \times \frac{8\ W_1 . C}{\pi\ d^2} = 1.227 \times \frac{8 \times 400 \times 6.6}{\pi\ d^2} = \frac{8248}{d^2}$$

$$\therefore \qquad d^2 = 8248 / 400 = 20.62 \quad \text{or} \quad d = 4.54 \text{ mm}$$

Taking larger of the two values, we have

$$d = 4.54 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 6 having diameter (d) = 4.877 mm.

$\therefore$ Mean diameter of the spring coil

$$D = 25 + d = 25 + 4.877 = 29.877 \text{ mm } \textbf{Ans.}$$

and outer diameter of the spring coil,

$$D_o = D + d = 29.877 + 4.877 = 34.754 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the coil*

Let n = Number of active turns of the coil.

We are given that the compression of the spring caused by a load of ($W_1 - W_2$), *i.e.* 400 – 250 = 150 N is $l_2 - l_1$, *i.e.* 50 – 40 = 10 mm. In other words, the deflection (δ) of the spring is 10 mm for a load (W) of 150 N

We know that the deflection of the spring (δ),

$$10 = \frac{8\ W . D^3 . n}{G . d^4} = \frac{8 \times 150\ (29.877)^3\ n}{80 \times 10^3\ (4.877)^4} = 0.707\ n$$

... (Taking $G = 80 \times 10^3$ N/mm²)

$$\therefore \qquad n = 10 / 0.707 = 14.2 \text{ say } 15 \textbf{ Ans.}$$

Taking the ends of the springs as squared and ground, the total number of turns of the spring,

$$n' = 15 + 2 = 17 \textbf{ Ans.}$$

3. *Free length of the spring*

Since the deflection for 150 N of load is 10 mm, therefore the maximum deflection for the maximum load of 400 N is

$$\delta_{max} = \frac{10}{150} \times 400 = 26.67 \text{ mm}$$

∴ Free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\ \delta_{max}$$
$$= 17 \times 4.877 + 26.67 + 0.15 \times 26.67 = 113.58 \text{ mm } \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{113.58}{17 - 1} = 7.1 \text{ mm } \textbf{Ans.}$$

11. A concentric spring is used as a valve spring in a heavy duty diesel engine. It consists of two helical compression springs having the same free length and same solid length. The composite spring is subjected to maximum force of 6000N and the corresponding deflection is 50 mm. the maximum torsional shear stress induced in each spring is 800 N/mm^2. The spring index of each spring is 6. Assume the same material for two springs and the modulus of rigidity of spring material is 81370 N/mm^2. The diametral clearance between the coils is equal to the difference between their wire diameters. (16) (Apr/May 2010)
Calculate:

- **(i) The axial force transmitted by each spring**
- **(ii) Wire and mean coil diameter of each spring**
- **(iii) Number of active coils in each spring.**

Solution:

The diametral clearance between the coil is equal to the difference between their wire diameter.

$$\frac{d_1}{d_2} = \frac{C}{(C-2)} = \frac{6}{(6-2)} = 1.5$$

$$\frac{P_1}{P_2} = \frac{d_1^{\ 2}}{d_2^{\ 2}} = \left(\frac{d_1}{d_2}\right)^2 = (1.5)^2 = 2.25 \qquad \text{(a)}$$

$$P_1 + P_2 = P = 6000 \text{ N} \qquad \text{(b)}$$

Also,

Solving eqs. (a) and (b) simultaneously,

$$P_1 = 4153.85 \text{ N and P2} = 1486.15 \text{ N} \qquad \text{(i)}$$

$$\boldsymbol{K} = \frac{4C - 1}{4C + 1} + \frac{0.615}{C}$$

$$= \frac{4(6) - 1}{4(6) + 1} + \frac{0.615}{6} = 1.2525$$

Outer spring

From eq. (10.13),

$$\tau = K\left(\frac{8P_1C}{\pi {d_1}^2}\right)$$

Or

$$800 = (1.2525)\left(\frac{8(4153.85)(6)}{\pi {d_1}^2}\right)$$

d_1= 9.97 or 10 mm

D_1= Cd_1 = 6(10) = 60 mm

Inner spring

$$\tau = K\left(\frac{8P_2C}{\pi {d_2}^2}\right)$$

$$800 = (1.2525)\left(\frac{8(1846.15)(6)}{\pi {d_2}^2}\right)$$

d_2= 6.65 or 7 mm

D_2 = Cd_2 = 6(7) = 42 mm

We know that

$$\delta = \frac{8P_1{D_1}^3N_1}{G{d_1}^4}$$

$$50 = \frac{8(4153.85)(60)^3N_1}{81370(10)^4}$$

N_1 = 5.67 or 6 coils

It is assumed that the spring have square and ground ends. Therefore,

$(N_t)_1$ = N_1 + 2 = 6 + 2 = 8 coils

Since the springs have same solid length,

$(N_t)_1 d_1 = (N_t)_2 d_2$

$8(10) = (N_t)_2(7)$

$(N_t)_2$ = 11.43 or 12 coils

N_2 = 12-2 = 10 coils

12. A mechanism used in printing machinery consists of a tension spring assembled with a preload of 30 N. The wire diameter of spring is 2 mm with a spring index of 6. The spring has 18 active coils. The spring wire is hard drawn and oil tempered having following material

properties: Design shear stress = 680 MPa Modulus of rigidity = 80 kN/mm^2 Determine : 1. the initial torsional shear stress in the wire; 2. spring rate; and 3. the force to cause the body of the spring to its yield strength.

Solution. Given : *Wi* = 30 N ; *d* = 2 mm ; *C* = *D*/*d* = 6 ; *n* = 18 ; τ = 680 MPa = 680 N/mm2 ; *G* = 80 kN/mm^2 = 80 × 10^3 N/mm^2

1. *Initial torsional shear stress in the wire*

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 6-1}{4\times 6-4} + \frac{0.615}{6} = 1.2525$$

∴ Initial torsional shear stress in the wire,

$$\tau_i = K \times \frac{8\,W_i \times C}{\pi\,d^2} = 1.2525 \times \frac{8\times 30\times 6}{\pi\times 2^2} = 143.5 \text{ N/mm}^2$$

$$= 143.5 \text{ MPa} \textbf{ Ans.}$$

2. *Spring rate*

We know that spring rate (or stiffness of the spring),

$$= \frac{G.d}{8\,C^3.n} = \frac{80\times 10^3\times 2}{8\times 6^3\times 18} = 5.144 \text{ N/mm} \textbf{ Ans.}$$

3. *Force to cause the body of the spring to its yield strength*

Let W = Force to cause the body of the spring to its yield strength.

We know that design or maximum shear stress (τ),

$$680 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.2525 \times \frac{8W\times 6}{\pi\times 2^2} = 4.78\,W$$

∴ $W = 680 / 4.78 = 142.25$ N **Ans.**

13. Design a helical compression spring to support an axial load of 3000 N. The deflection under load is limited to 60mm. The spring index is 6.The spring is made of chrome vanadium steel and factor of safety is equal to 2. (Nov/Dec 2018)

Data

F = 3000N, y = 60mm, c = 6, FOS = 2

Solution

From DHB for chrome-vanadium steel refer standard table

τ_y = 690MPa=690N/mm^2 (0.69GPa)

G=79340MPa=79340N/mm^2 (79.34GPa)

$$\tau = \frac{\tau_y}{FOS} = \frac{690}{2} = 345\ N/mm^2$$

Diameter of wire

Shear stress $\tau = \frac{8FDk}{\pi d^3}$

Wahl's stress factor $k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4x6-1}{4x6-4} + \frac{0.615}{6} = 1.2525$

Spring index $c = \frac{D}{d}$

$6 = \frac{D}{d}$

$\therefore$ D = 6d

$$345 = \frac{8x3000x6dx1.2525}{\pi d^3}$$

$\therefore$ d = 12.89

Select standard diameter of wire from table

$\therefore$ d = 13 mm

Diameter of coil

$$c = \frac{D}{d}$$

$$6 = \frac{D}{13}$$

Mean diameter of coil= D = 78 mm

Outer diameter of coil $= D_o = D + d = 78 + 13 = 91$ mm

Inner diameter of coil $= D_i = D - d = 78 - 13 = 65$ mm

3. Number of coil or turns

Deflection $y = \frac{8FD^3 i}{Gd^4}$

$$60 = \frac{8x3000 78^3 \times i}{79840 x 13^4}$$

$$i = 11.93$$

$\therefore$ Number active turns $i = 12$

4. Free length

$$l \geq (i + n)\, d + y + a$$

Clearance a = 25% of maximum deflection $= \frac{25}{100} x 60 = 15 mm$

5. Pitch

$$p = \frac{l_o - 2d}{i} = \frac{257 - 2x13}{12} = 19.25\ mm$$

6. Stiffness or Rate of spring

$$F_o = \frac{F}{y} = \frac{3000}{60} = 50\ N/mm$$

7. Spring specification

Material Chrome vanadium steel
Wire diameter d =13 mm
Mean diameter D = 78 mm
Free length l_o = 257 mm
Total number of terms i' = 14
Style of end-square and ground
Pitch p = 19.25 mm
Rate of spring F_o = 50 N/mm

14. At the bottom of a mine shaft, a group of 10 identical close coiled helical springs are set in parallel to absorb the shock caused by the falling of the cage in case of a failure. The loaded cage weighs 75 kN, while the counter weight has a weight of 15 kN. If the loaded cage falls through a height of 50 metres from rest, find the maximum stress induced in each spring if it is

made of 50 mm diameter steel rod. The spring index is 6 and the number of active turns in each spring is 20. Modulus of rigidity, G = 80 kN/mm2. (Nov/Dec 2018)

Solution. Given : No. of springs = 10 ; W_1 = 75 kN = 75 000 N ; W_2 = 15 kN = 15 000 N ; h = 50 m = 50 000 mm ; d = 50 mm ; C = 6 ; n = 20 ; G = 80 kN/mm² = 80 × 10³ N/mm²

We know that net weight of the falling load,

$$P = W_1 - W_2 = 75\,000 - 15\,000 = 60\,000 \text{ N}$$

Let W = The equivalent static (or gradually applied) load on each spring which can produce the same effect as by the falling load P.

We know that compression produced in each spring,

$$\delta = \frac{8\,W.C^3.n}{G.d} = \frac{8\,W \times 6^3 \times 20}{80 \times 10^3 \times 50} = 0.008\,64\;W \text{ mm}$$

Since the work done by the falling load is equal to the energy stored in the helical springs which are 10 in number, therefore,

$$P\,(h + \delta) = \frac{1}{2}\,W \times \delta \times 10$$

$$60\,000\,(50\,000 + 0.008\,64\;W) = \frac{1}{2}\,W \times 0.008\,64\,W \times 10$$

$$3 \times 10^9 + 518.4\;W = 0.0432\;W^2$$

or $\quad W^2 - 12\,000\;W - 69.4 \times 10^9 = 0$

$$\therefore \qquad W = \frac{12\,000 \pm \sqrt{(12\,000)^2 + 4 \times 1 \times 69.4 \times 10^9}}{2} = \frac{12\,000 \pm 527\,000}{2}$$

$$= 269\,500 \text{ N} \qquad \text{... (Taking +ve sign)}$$

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.25$$

and maximum stress induced in each spring,

$$\tau = K \times \frac{8\,W.C}{\pi\,d^2} = 1.25 \times \frac{8 \times 269\,500 \times 6}{\pi\,(50)^2} = 2058.6 \text{ N/mm}^2$$

$$= 2058.6 \text{ MPa}$$ **Ans.**

Flywheel:

12. The intercepted areas between the output torque curve and the mean resistance line of a turning moment diagram for a multi cylinder engine, taken in order from one end are as follows: – 35, + 410, – 285, + 325, – 335, + 260, – 365, + 285, – 260 mm2.
The diagram has been drawn to a scale of 1 mm = 70 N-m and 1 mm = 4.5°. The engine speed is 900 r.p.m. and the fluctuation in speed is not to exceed 2% of the mean speed. Find the mass and cross-section of the flywheel rim having 650 mm mean diameter. The density of the material of the flywheel may be taken as 7200 kg / m3. The rim is rectangular with the width 2 times the thickness. Neglect effect of arms, etc. (8)

Given data:

N = 900 r.p.m. or ω = 2π × 900 / 60 = 94.26 rad/s;

ω1– ω2 = 2% ω orω1 − ω2

ω = CS = 2% = 0.02; D = 650 mm or R = 325 mm = 0.325 m; ρ = 7200 kg / m3

Solution:

1. **Mass of the flywheel rim**

Let m = Mass of the flywheel rim in kg.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram for amulti-cylinder engine is shown in Fig

Since the scale of turning moment is 1 mm = 70 N-m and scale of the crank angle is 1 mm = 4.5º = π / 40 rad, therefore 1 mm2 on the turning moment diagram.

$$= 70 \times \pi / 40 = 5.5 \text{ N-m}$$

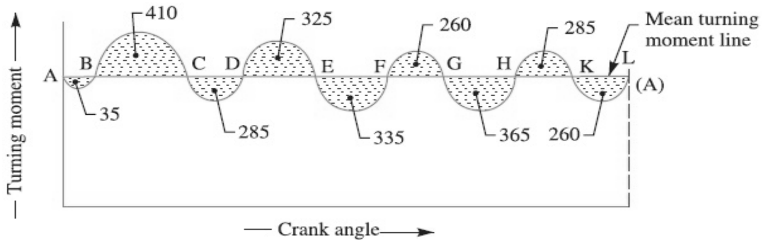


Fig 4.14

Let the total energy at A = E. Therefore from Fig. we find that

Energy at B = E – 35

Energy at C = E – 35 + 410 = E + 375

Energy at D = E + 375 – 285 = E + 90

Energy at E = E + 90 + 325 = E + 415

Energy at F = E + 415 – 335 = E + 80

Energy at G = E + 80 + 260 = E + 340

Energy at H = E + 340 – 365 = E – 25

Energy at K = E – 25 + 285 = E + 260

Energy at L = E + 260 – 260 = E = Energy at A

From above, we see that the energy is maximum at E and minimum at B. as shown in fig 4.14

∴ Maximum energy = E + 415

And minimum energy = E – 35

We know that maximum fluctuation of energy,

$$= (E + 415) – (E – 35) = 450 \text{ mm}^2$$

$$= 450 \times 5.5 = 2475 \text{ N-m}$$

We also know that maximum fluctuation of energy (ΔE),

$$2475 = m.R^2.\omega^2.CS = m\,(0.325)^2\,(94.26)^2\,0.02 = 18.77\,m$$

$$\therefore \quad m = 2475 / 18.77 = 132 \text{ kg } \textbf{Ans}.$$

2. **Cross-section of the flywheel rim**

Let, t = Thickness of the rim in meters, and

b = Width of the rim in meters = 2 t ... (Given)

∴ Area of cross-section of the rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim (m),

$$132 = A \times 2\,\pi\,R \times \rho = 2\,t^2 \times 2\,\pi \times 0.325 \times 7200 = 29\,409\,t^2$$

$$\therefore \quad t^2 = 132 / 29\,409 = 0.0044 \text{ or } t = 0.067 \text{ m} = 67 \text{ mm}$$

$$\text{and} \quad b = 2t = 2 \times 67 = 134 \text{ mm}$$

13.The areas of the turning moment diagram for one revolution of a multi-cylinderengine with reference to the mean turning moment, below and above the line, are– 32, + 408, – 267, + 333, – 310, + 226, – 374, + 260 and – 244 mm^2.The scale for abscissa and ordinate are: 1 mm = 2.4° and 1 mm = 650 N-m respectively. Themean speed is 300 r.p.m. with a percentage speed fluctuation of ± 1.5%. If the hoop stress in thematerial of the rim is not to exceed 5.6 MPa, determine the suitable diameter and cross-section forthe flywheel, assuming that the width is equal to 4 times the thickness. The density of the materialmay be taken as 7200 kg / m3. Neglect the effect of the boss and arms.(Nov/Dec-16)

Given:

: N = 300 r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s ; σ_t = 5.6 MPa $= 5.6 \times 10^6$ N/m^2 ; $\rho = 7200$ kg/m^3

To find: Diameter and cross-section for the flywheel

Solution:

Diameter of the flywheel

Let D = Diameter of the flywheel in metres.

We know that peripheral velocity of the flywheel,

$$v = \frac{\pi D N}{60} = \frac{\pi D \times 300}{60} = 15.71\,D \text{ m/s}$$

We also know that hoop stress (σ_t),

$$5.6 \times 10^6 = \rho \times v^2 = 7200\,(15.71\,D)^2 = 1.8 \times 10^6 D^2$$

$$\therefore \quad D^2 = 5.6 \times 10^6 / 1.8 \times 10^6 = 3.11 \quad \text{or} \quad D = 1.764 \text{ m } \textbf{Ans.}$$

Cross-section of the flywheel

Let t = Thickness of the flywheel rim in metres, and

b = Width of the flywheel rim in metres = $4\,t$ ...(Given)

∴ Cross-sectional area of the rim,

$$A = b \times t = 4\,t \times t = 4\,t^2 \text{ m}^2$$

Now let us find the maximum fluctuation of energy. The turning moment diagram for one revolution of a multi-cylinder engine is shown in Fig. 4.15

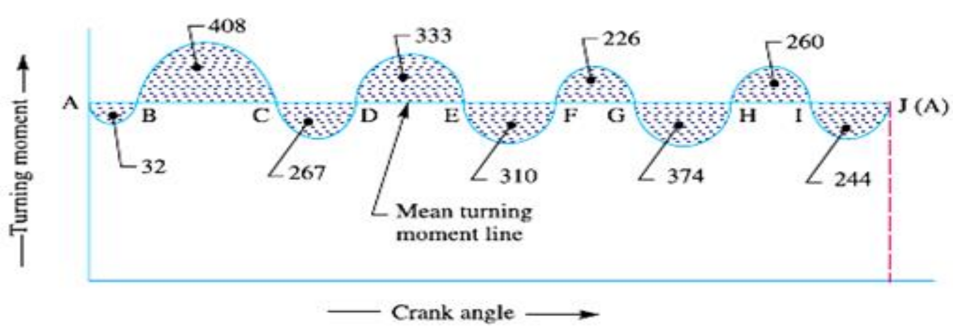


fig 4.15

Since the scale of crank angle is 1 mm = 2.4° = $2.4 \times \frac{\pi}{180}$ = 0.042 rad, and the scale of the turning moment is 1 mm = 650 N-m, therefore

1 mm^2 on the turning moment diagram

$= 650 \times 0.042 = 27.3$ N-m

Let the total energy at $A = E$. Therefore from Fig. 4.15, we find that

Energy at $B = E - 32$

Energy at $C = E - 32 + 408 = E + 376$

Energy at $D = E + 376 - 267 = E + 109$

Energy at $E = E + 109 + 333 = E + 442$

Energy at $F = E + 442 - 310 = E + 132$

$$\text{Energy at } G = E + 132 + 226 = E + 358$$
$$\text{Energy at } H = E + 358 - 374 = E - 16$$
$$\text{Energy at } I = E - 16 + 260 = E + 244$$
$$\text{Energy at } J = E + 244 - 244 = E = \text{Energy at } A$$

From above, we see that the energy is maximum at E and minimum at B.

$\therefore$ Maximum energy $= E + 442$

and minimum energy $= E - 32$

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$
$$= (E + 442) - (E - 32) = 474 \text{ mm}^2$$
$$= 474 \times 27.3 = 12\,940 \text{ N-m}$$

Since the fluctuation of speed is ± 1.5% of the mean speed, therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 3\% \text{ of mean speed} = 0.03\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.03$$

Let m = Mass of the flywheel rim.

We know that maximum fluctuation of energy (ΔE),

$$12\,940 = m.R^2.\omega^2.C_S = m\left(\frac{1.764}{2}\right)^2 (31.42)^2\ 0.03 = 23\ m$$

$$\therefore \quad m = 12\,940 / 23 = 563 \text{ kg } \textbf{Ans.}$$

We also know that mass of the flywheel rim (m),

$$563 = A \times \pi D \times \rho = 4\,t^2 \times \pi \times 1.764 \times 7200 = 159\,624\ t^2$$

$$\therefore \quad t^2 = 563 / 159\,624 = 0.00353$$

$$t = 0.0594 \text{ m} = 59.4 \text{ say } 60 \text{ mm } \textbf{Ans.}$$

$$b = 4\,t = 4 \times 60 = 240 \text{ mm } \textbf{Ans.}$$

14.The turning moment diagram of a multi-cylinder engine is drawn with a scale of (1 mm = 1°) on the abscissa and (1 mm = 250 N-m) on the ordinate. The intercepted areas between the torque developed by the engine and the mean. resisting torque of the machines, taken in order from one end are -350, +800, -600, + 900, -550, +450 and -650 mm2. The engine is running at a mean speed of 750 rpm and the coefficient of speed fluctuation is limited to 0.02. Arimmed flywheel made of grey cast iron FG 200 (density = 7100 kg/m ') is provided. The spokes, hub and shaft are assumed to contribute 10%of the required moment of inertia. The rim has rectangular cross-section and the ratio of width to thickness is 1.5. Determine the dimensions of the rim.(April/May 2018)

Given:

n = 750 rpm ,C_s = 0.02 , b/t = 1.5, K= 0.9 , ρ = 7100 kg/m^3

Step I: Energy output from flywheel (U_o)

The turning moment diagram is shown in Fig. 4.16 .It is assumed that the energy stored in the flywheel is U at point A. Therefore, .

Energy at B = U – 350

Energy at C = U - 350 + 800 = U + 450

Energy atD = U + 450 - 600 = U - 150

Energy at E = U - 150 + 900 = U + 750

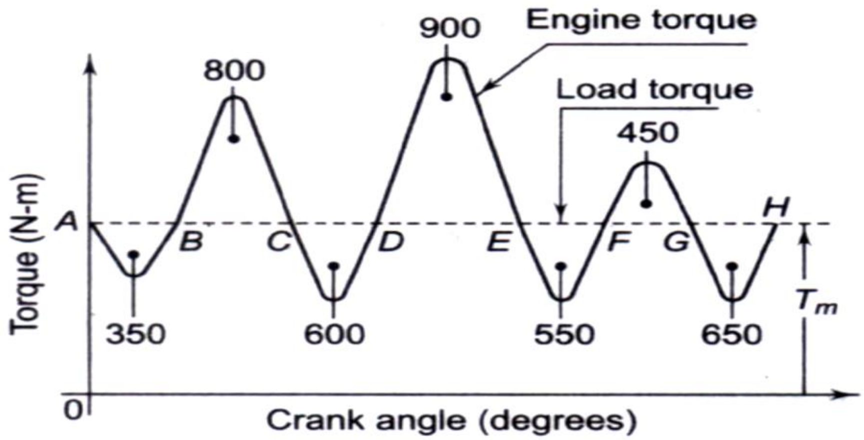


fig 4.16 Turning Moment Diagram

Energy atF= U+ 750 - 550 = U + 200

Energy at G = U+ 200 + 450 = U+ 650

Energy at H = U + 650 - 650 = U

The maximum and minimum energy occurs at points E and B. The angular velocity of the flywheel will be maximum at point E and minimum at point B.

$U_0 = U_E - U_B = (U + 750) - (U - 350)$

$= 1100\ mm^2$

The scale of the turning moment diagram is as follows:

X axis, $\quad 1\ mm = 1° = \left(\frac{\pi}{180}\right)$ radian

Y axis $\quad 1\ mm = 250$ N-m

Substituting scale factors in Eq. (a),

$$U_o = 1100(250)\left(\frac{\pi}{180}\right) \text{N-m} = 4799.66 \text{ N-m}$$

Step II: Dimensions of rim

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(750)}{60} = (25\pi)\ \text{rad/s}$$

$$I_r = \frac{U_o K}{\omega^2 C_s} = \frac{(4799.66)(0.9)}{(25\pi)^2(0.02)} = 35\ \text{kg-m}^2$$

the mean radius R of the rim is given by,

$$R < \frac{30}{\omega} \quad \text{or} \quad R < \frac{30}{(25\pi)} \quad \text{or} \quad R < 0.38\ \text{m}$$

Therefore, $R = 0.35$ m

The mass of the rim is given by

$$m_r = \frac{I_r}{R^2} = \frac{35}{(0.35)^2} = 285.71\ \text{kg}$$

The mass of the flywheel rim is given by,

$$m_r = 2\pi R\left(\frac{b}{1000}\right)\left(\frac{t}{1000}\right)\rho;$$

or $$285.71 = 2\pi(0.35)\left(\frac{1.5t}{1000}\right)\left(\frac{t}{1000}\right)(7100)$$

$$t = 110.45\ \text{mm}$$

$$b = 1.5(110.45) = 165.67\ \text{mm}$$

Therefore, $b = 170$ mm and $t = 110$ mm

15. A single cylinder double acting steam engine develops 150 kW at a mean speed of 80 r.p.m. The coefficient of fluctuation of energy is 0.1 and the fluctuation of speed is ± 2% of mean speed. If the mean diameter of the flywheel rim is 2 meters and the hub and spokes provide 5 percent of the rotational inertia of the wheel, find the mass of the flywheel and cross-sectional area of the rim. Assume the density of the flywheel material (which is cast iron) as 7200 kg / m^3. (16)

Given data:

$P = 150$ kW $= 150 \times 10^3$ W;

$N = 80$ r.p.m.; $CE = 0.1$; $\omega_1 - \omega_2 = \pm 2\%\ \omega$;

$D = 2$ m or $R = 1$ m; $\rho = 7200$ kg/m^3

Solution:

1. Mass of the flywheel rim

Let, m = Mass of the flywheel rim in kg.

We know that the mean angular speed,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 80}{60} = 8.4\ rad/s$$

Since the fluctuation of speed is ± 2% of mean speed (ω), therefore total fluctuation of speed,

$\omega 1 - \omega 2 = 4\ \%\ \omega = 0.04\ \omega$

and coefficient of fluctuation of speed

$$\omega = \frac{\omega_1 - \omega_2}{\omega} = 0.04$$

We know that the work done by the flywheel per cycle,

$$= \frac{P \times 60}{N} = \frac{150 \times 10^3 \times 60}{80} = 112500 N - m$$

We also know that coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Workdone}/\text{cycle}}$$

∴ Maximum fluctuation of energy,

$$\Delta E = CE \times \text{Work done / cycle}$$
$$= 0.1 \times 112\ 500 = 11\ 250 \text{ N-m}$$

Since 5% of the rotational inertia is provided by hub and spokes, therefore the maximumfluctuation of energy of the flywheel rim will be 95% of the flywheel.

∴ Maximum fluctuation of energy of the rim,

$$(\Delta E)_{rim} = 0.95 \times 11\ 250 = 10\ 687.5 \text{ N-m}$$

We know that maximum fluctuation of energy of the rim $(\Delta E)_{rim}$,

$$10\ 687.5 = m.R^2.\omega^2.C^s = m \times 12\ (8.4)2\ 0.04 = 2.82\ m$$

$$\therefore\ m = 10\ 687.5 / 2.82 = 3790 \text{ kg}$$

Cross-sectional area of the rim

Let,A= Cross-sectional area of the rim.

We know that the mass of the flywheel rim (m),

$$3790 = A \times 2\pi R \times \rho = A \times 2\pi \times 1 \times 7200 = 45\ 245\ A$$

$$\therefore\ A = 3790 / 45\ 245 = 0.084 \text{ m}^2$$

15.(a) A single cylinder internal combution engine working four stroe cycle develops 75 KW at 360 rpm.the fluctuation of energy can be assumed to be 0.9 times the energy developed per cycle.If the fluctuatin of speedis not to exceed 1% and the maximum centrfiugal stress in the fly wheel is to be 5.5 MPa,then estimate the mean diameter and cross setctional area of the rim.the material of the rim has density of 7200 kg/m3.(Nov/Dec 2021)

Given :-

Power = 75 kW
Speed N= 360 rpm,
Fluctuation o energy =E' =25000 x 0.9
maximum centrfiugal stress in the fly wheel = 5.5 MPa
rim density = 7200 kg/m3

Single cylinder engine 75 kW & 360rpm

Energy developed per cycle E = 75 x 10 ^{3} / (360 /2x60)

E = 25000 Nm

For 4 stroke engine 2 cycle = 1 power stroke

Fluctuation o energy =E' =25000 x 0.9

E' =I $\omega^2 C_s$

I=mR^2,

ω= 2πN /60

ω=37.7 rad/sec

C_s=1% =1/100

Substituting all the values,

25000 = m R^2x 37.7^2x 1/100

mR^2=1583 ----**eq-1**

mass of rimmed fly wheel

m =$\rho\pi dA$ -----**eq - 2**

A=cross section of fly wheel = t x w

Also maximum centrifugal stress,

$$\sigma = \rho v^2$$

$$5.5\ x\ 10^6 = 7200 \text{ x } v^2$$

$$v = 27.64\ m/s$$

Also V=$\frac{\pi dN}{60}$

$\frac{27.04\times 60}{\pi\times 360}$=d

$$d = 1.464\ m$$

Now ffrom Eq 1 & 2,

$$mR^2 = 1.583 \times 10^3$$

$$\rho\pi dAR^2 = 1.583 \times 10^3$$

$$7200 \times \pi \times 1.464 \times A \times 0.732^2 = 1.583 \times 10^3$$

A = 0.09 m²

16.A punching machine makes 25 working strokes per minute and is capable of punching 25 mm diameter holes in 18 mm thick steel plates having ultimate shear strength of 300 MPa. The punching operation takes place during 1/10 th of a revolution of the crank shaft. Estimate the power needed for the driving motor, assuming a mechanical efficiency of 95 per cent. Determine suitable dimensions for the rim cross-section of the flywheel, which is to revolve at 9 times the speed of the crank shaft. The permissible coefficient of fluctuation of speed is 0.1.The flywheel is to be made of cast iron having a working stress (tensile) of 6 MPa and density of 7250 kg / m³. The diameter of the flywheel must not exceed 1.4 m owing to space restrictions. The hub and the spokes may be assumed to provide 5% of the rotational inertia of the wheel. Check for the centrifugal stress induced in the rim. (Nov/Dec 2017)

Given data:

$n = 25$; $d_1 = 25$ mm; $t_1 = 18$ mm;
$\tau_u = 300$ MPa = 300 N/mm2;
$\eta_m = 95\% = 0.95$; CS = 0.1; $\sigma_t = 6$ MPa = 6 N/mm^2;
$\rho = 7250$ kg/m^3; D = 1.4 m or R = 0.7m

Solution:

1. **Power needed for the driving motor**

We know that the area of plate sheared,

$$A_S = \pi\, d_1 \times t_1 = \pi \times 25 \times 18 = 1414 \text{ mm}^2$$

∴ Maximum shearing force required for punching,

$$F_S = A_S \times \tau_u = 1414 \times 300 = 424\,200 \text{ N}$$

and energy required per stroke

= *Average shear force × Thickness of plate

$$= \frac{1}{2} \times F_S \times t_1 = \frac{1}{2} \times 424200 \times 18$$

$$= 3817.18 \times 10^3 N - mm$$

∴ Energy required per min

= Energy / stroke × No. of working strokes / min

= 3817.8 × 103 × 25 = 95.45 × 106 N-mm = 95 450 N-m

We know that the power needed for the driving motor,

$$= \frac{\text{Energy required per min}}{60 \times \eta_m} = \frac{95450}{60 \times 0.95} = 1675\text{W}$$

2. **Dimensions for the rim cross-section**

Considering the cross-section of the rim as rectangular and assuming the width of rim equal totwice the thickness of rim.

Let, t = Thickness of rim in metres,

b = Width of rim in metres = 2 t.

∴ Cross-sectional area of rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

Since the punching operation takes place (i.e. energy is consumed) during 1/10 th of a revolutionof the crank shaft, therefore during 9/10 th of the revolution of a crank shaft, the energy is stored in theflywheel.

∴ Maximum fluctuation of energy,

$$\Delta E = \frac{9}{10} \times {}^{\text{Energy}}/_{\text{stroke}} = \frac{9}{10} \times 38178. \times 10^3$$

$$= 3436 \times 103 \text{ N-mm} = 3436 \text{ N-m}$$

Let, m = Mass of the flywheel.

Since the hub and the spokes provide 5% of the rotational inertia of the wheel, therefore themaximum fluctuation of energy provided by the flywheel rim will be 95%.

∴ Maximum fluctuation of energy provided by the rim,

$$(\Delta E)_{rim} = 0.95 \times \Delta E = 0.95 \times 3436 = 3264 \text{ N-m}$$

Since the flywheel is to revolve at 9 times the speed of the crankshaft and there are 25 workingstrokes per minute, therefore mean speed of the flywheel,

$$N = 9 \times 25 = 225 \text{ r.p.m.}$$

and mean angular speed, $\omega = 2\,\pi \times 225 / 60 = 23.56$ rad/s

We know that maximum fluctuation of energy (Δ E),

$$3264 = m.R^2.\omega^2.C_S$$

$$= m\,(0.7)^2\,(23.56)^2\,0.1 = 27.2\,m$$

$$\therefore \quad m = 3264 / 27.2 = 120 \text{ kg}$$

We also know that mass of the flywheel (m),

$$120 = A \times \pi\,D \times \rho$$

$$= 2\,t^2 \times \pi \times 1.4 \times 7250 = 63\ 782\ t^2$$

$$\therefore \quad t^2 = 120 / 63\ 782 = 0.001\ 88 \text{ or } t = 0.044 \text{ m} = 44 \text{ mm}$$

$$\text{and} \quad b = 2\,t = 2 \times 44 = 88 \text{ mm}$$

3. **Check for centrifugal stress**

We know that peripheral velocity of the rim,

$$V = \frac{\pi D.N}{60} = \frac{\pi \times 1.4 \times 225}{60} = 16.5 \text{m/s}$$

∴ Centrifugal stress induced in the rim,

$$\sigma_t = \rho.v^2 = 7250\,(16.5)^2 = 1.97 \times 106 \text{ N/m}^2 = 1.97 \text{ MPa}$$

Since the centrifugal stress induced in the rim is less than the permissible value (i.e. 6 MPa),therefore it is safe.

17.A punching press pierces 35 holes per minute in a plate using 10 kN-m of energy per hole during each revolution. Each piercing takes 40 per cent of the time needed to make one revolution. The punch receives power through a gear reduction unit which in turn is fed by a motor driven belt pulley 800 mm diameter and turning at 210 r.p.m. Find the power of the electric motor if overall efficiency of the transmission unit is 80 per cent. Design a cast iron flywheel to be used with the punching machine for a coefficient of steadiness of 5, if the space considerations limit the maximum diameter to 1.3 m. Allowable shear stress in the shaft material = 50 MPaAllowable tensile stress for cast iron = 4 MPaDensity of cast iron = 7200 kg / m^3

Given data:

No. of holes = 35 per min;
Energy per hole = 10 kN-m = 10 000 N-m;
d = 800 mm = 0.8 m; N = 210 r.p.m.;
h = 80% = 0.8; $1/C_S$ = 5 or C_S = 1/5 = 0.2;
D_{max}= 1.3 m; τ = 50 MPa = 50 N/mm^2;
σt= 4 MPa = 4 N/mm^2; ρ = 7200 kg / m^3

Solution:

Power of the electric motor

We know that energy used for piercing holes per minute

= No. of holes pierced × Energy used per hole

= 35 × 10 000 = 350 000 N-m / min

∴ Power needed for the electric motor,

$$P = \frac{\text{Energy used per min}}{60 \times \eta}$$

$$= \frac{350000}{60 \times 0.8} = 7292 \text{ W} = 7.292\text{kW}$$

Design of cast iron flywheel

First of all, let us find the maximum fluctuation of energy.

Since the overall efficiency of the transmission unit is 80%, therefore total energy to be suppliedduring each revolution,

$$E_T = \frac{10000}{0.8} = 12500\text{N} - \text{m}$$

We know that velocity of the belt,

$$v = \pi\, d.N = \pi \times 0.8 \times 210 = 528 \text{ m/min}$$

∴ Net tension or pull acting on the belt

$$= \frac{P \times 60}{v} = \frac{7292 \times 60}{528} = 828.6\text{N}$$

Since each piercing takes 40 per cent of the time needed to make one revolution, therefore timerequired to punch a hole

= 0.4 / 35 = 0.0114 min

and the distance moved by the belt during punching a hole

= Velocity of the belt × Time required to punch a hole

= 528 × 0.0114 = 6.03 m

∴ Energy supplied by the belt during punching a hole, EB = Net tension × Distance travelled by belt

$= 828.6 \times 6.03 = 4996$ N-m

Thus energy to be supplied by the flywheel for punching during each revolution or maximumfluctuation of energy,

$$\Delta E = ET - EB = 12\ 500 - 4996 = 7504 \text{ N-m}$$

1. Mass of the flywheel

Let, m = Mass of the flywheel rim.

Since space considerations limit the maximum diameter of the flywheel as 1.3 m ; therefore letus take the mean diameter of the flywheel,

D = 1.2 m or R = 0.6 m

We know that angular velocity,

$$\omega = \frac{2\pi \times N}{60} = \frac{2\pi \times 210}{60} = 22 \text{ rad/s}$$

We also know that the maximum fluctuation of energy (Δ E),

$$7504 = m.R2.\omega 2.CS = m\ (0.6)2\ (22)2\ 0.2 = 34.85\ m$$

$$\therefore \quad m = 7504 / 34.85 = 215.3 \text{ kg}$$

2. Cross-sectional dimensions of the flywheel rim

Let, t = Thickness of the flywheel rim in metres, and

b = Width of the flywheel rim in metres = 2 t ... (Assume)

∴ Cross-sectional area of the rim,

$$A = b \times t = 2\ t \times t = 2\ t^2$$

We know that mass of the flywheel rim (m),

$$215.3 = A \times \pi D \times \rho = 2\ t^2 \times \pi \times 1.2 \times 7200 = 54.3 \times 103\ t^2$$

$$\therefore \quad t^2 = 215.3 / 54.3 \times 103 = 0.003\ 96$$

Or $t = 0.063$ say 0.065 m = 65 mm

and $b = 2\ t = 2 \times 65 = 130$ mm

3. Diameter and length of hub

Let, d = Diameter of the hub,

d1 = Diameter of the shaft, and

l = Length of the hub.

First of all, let us find the diameter of the shaft (d_1). We know that the mean torque transmittedby the shaft,

$$T_{max} = \frac{P \times 60}{2\pi.N} = \frac{7292 \times 60}{2\pi \times 210} = 331.5N$$

Assuming that the maximum torque transmitted by the shaft is twice the mean torque, thereforemaximum torque transmitted by the shaft,

$T_{max} = 2 \times T_{mean} = 2 \times 331.5 = 663$ N-m $= 663 \times 103$ N-mm

We know that maximum torque transmitted by the shaft (T_{max}),

$$663 \times 10^3 = \frac{\pi}{16} \times \tau \times (d_1)^3$$

$$663 \times 10^3 = \frac{\pi}{16} \times 50 \times (d_1)^3 = 9.82 (d_1)^3$$

$$(d_1)^3 = 663 \times 103 / 9.82 = 67.5 \times 103$$

or $d_1 = 40.7$ say 45 mm

The diameter of the hub (d) is made equal to twice the diameter of the shaft (d1) and length of hub (l) is equal to the width of the rim (b).

∴ d = 2 d_1 = 2 × 45 = 90 mm = 0.09 m and l = b = 130 mm

4. Cross-sectional dimensions of the elliptical cast iron arms

Let, a_1 = Major axis,

b_1 = Minor axis = 0.5 a_1 ... (Assume)

n = Number of arms = 6 ... (Assume)

We know that the maximum bending moment in the arm at the hub end, which is assumed ascantilever is given by

$$\mathrm{M} = \frac{\mathrm{T}}{\mathrm{R.n}} \times (R - r) = \frac{\mathrm{T}}{\mathrm{D.n}} \times (D - d)$$

$$= \frac{66}{1.2 \times 6} \times (1.2 - .009) = 102.2N - m = 102200\ N - mm$$

and section modulus for the cross-section of the arms,

$$\mathrm{z} = \frac{\pi}{32} \times b_1 \times (a_1)^3 = \frac{\pi}{32} \times 0.5a_1 \times (a_1)^3 = 0.05(a_1)^3$$

We know that bending stress (σ_t),

$$4 = \frac{\mathrm{M}}{\mathrm{Z}} = \frac{102200}{0.05(a_1)^3} = \frac{2044 \times 10^3}{0.05(a_1)^3}$$

∴ $(a_1)^3$ = 2044 × 103 / 4 = 511 × 103 or a_1 = 80 mm **Ans.**

and b_1 = 0.5 a_1 = 0.5 × 80 = 40 mm **Ans.**

5. Dimensions of key

The standard dimensions of rectangular sunk key for a shaft of diameter 45 mm are as follows:

Width of key, w = 16 mm

and thickness of key = 10 mm

The length of key (L) is obtained by considering the failure of key in shearing.

We know that maximum torque transmitted by the shaft (Tmax),

$$663 \times 10^3 = \mathrm{L} \times w \times \tau \times (\frac{d_1}{2})$$

$$663 \times 10^3 = \mathrm{L} \times 16 \times 50 \times \left(\frac{45}{2}\right) = 18 \times 10^3 L$$

∴ L = 663 × 103/18 × 103 = 36.8 say 38 mm

Let us now check the total stress in the rim which should not be greater than 4 MPa.

We know that the velocity of the rim,

$$\mathrm{V} = \frac{\pi\mathrm{D.N}}{60} = \frac{\pi \times 1.2 \times 210}{60} = 13.2\mathrm{m/s}$$

∴ Total stress in the rim,

$$\sigma = \rho.v^2 \left(0.75 + \frac{4.935R}{n^2.t}\right)$$

$$\sigma = 7200 \times (13.2)^2 \left(0.75 + \frac{4.935 \times 0.6}{6^2 \times 0.065}\right)$$

= 1.25 × 106 (0.75 + 1.26) = 2.5 × 106 N/m^2 = 2.5 MPa

Since it is less than 4 MPa, therefore the design is safe.

18. A punching machine carries out 6 holes per minute. Each hole of 40 mm diameter in 35 mm thick plate requires 8 N.m of energy/mm^2 of the sheared area. The punch has a stroke of 95 mm. Find the power of the motor required if the mean speed of the flywheel is 20 m/s. If total fluctuation of speed is not to exceed 3% of mean speed, determine the mass of the flywheel.(April/May 17)

Given: d = 40 mm, K = 0.03mm, Stroke = 95 mm, v = 20 m/s.

To find: (i) Mass of the flywheel.

Solution: As 6 hole are punched in one minute, time required to punch one hole is 10 s. Energy required/hole or energy supplied by the motor in 10 seconds

= area of hole x energy required / mm^2 = πdt x 8 = 35186 N.m

Energy supplied by the motor in 1 second = $\frac{35186}{10} = 3518.6$ N.m

Power of the motor, P = 3518.6 W or 3.5186 kW

The punch travels a distance of 190 mm (upstroke + down stroke) in 10 seconds. (6 holes are punched 1 minute)

Actual time required to punch a hole in 35 mm thick plate = $\frac{10}{190} \times 35 = 1.842\,\text{s}$

Energy supplied by the motor in 1.842 s = 3518.6 x 1.842 = 6481 N.m

Energy supplied by the flywheel

e = Energy required/hole – Energy supplied by the motor in 1.842 s

= 35186 – 6481 = 28705 N.m

or 2 x K x E = 28705

2 x 0.03 x E = 28705 or E = 48417

or $\frac{1}{2}mv^2 = 478417$ or $\frac{1}{2}m(20)^2 = 478417$ or $m = 2392$ kg

19. A punching machine carries out punching 10 holes per minute. Each hole of 36 mm diameter in 16 mm thick plate requires 7 N-m of energy/mm^2 of the sheared area. The punch has a stroke of 90 mm. determine the power of the motor required to operator the machine.If the total fluctuation of speed is not to exceed 2.5% of the mean speed; determine the mass of the flywheel. The mean speed of the flywheel is m/s. (April/May 17)

(Similar to the above problem)

20.A split type flywheel has outside diameter of the rim 1.80 m, inside diameter 1.35 m and the width 300 mm. the two halves of the wheel are connected by four bolts through the hub and near the rim joining the split arms and also by four shrink links on the rim. The speed is 250 r.p.m. and a turning moment of 15 kN-m is to be transmitted by the rim. Determine:

1. The diameter of the bolts at the hub and near the rim, σ_{tb} = 35 MPa.

2. The cross-sectional dimensions of the rectangular shrink links at the rim, σ_{tl} = 40 MPa;w = 1.25 h.

3. The cross-sectional dimensions of the elliptical arms at the hub and rim if the wheel has six arms, σ_{ta} = 15 MPa, minor axis being 0.5 times the major axis and the diameter of shaft being 150 mm. Assume density of the material of the flywheel as 7200 kg / m3.

(16)

Given data:s

D_0 = 1.8 m ;D_i= 1.35 m ;

b = 300 mm = 0.3 m ;

N = 250 r.p.m. ;

T = 15 kN-m = 15 000 N-m ;

σ_{tb}= 35 MPa = 35 N/mm^2 ;

σ_{tl} = 40 MPa = 40 N/mm^2 ; w = 1.25 h ; n = 6 ;

b_1 = 0.5 a_1 ;σ_{ta}= 15 MPa = 15 N / mm^2 ;

d_1 = 150 mm; ρ = 7200 kg / m^3.

Solution:

1. Diameter of the bolts at the hub and near the rim

Let, d_c= Core diameter of the bolts in mm.

We know that mean diameter of the rim,

$$D = \frac{D_O + D_i}{2} = \frac{1.8 + 1.35}{2} = 1.575m$$

and thickness of the rim,

$$t = \frac{D_O - D_i}{2} = \frac{1.8 - 1.35}{2} = 0.225m$$

Peripheral speed of the flywheel,

$$\mathrm{V} = \frac{\pi \mathrm{D.N}}{60} = \frac{\pi \times 1.575 \times 250}{60} = 20.6\mathrm{m/s}$$

We know that centrifugal stress (or tensile stress) at the rim,

$$\sigma_t = \rho \times v2 = 7200\,(20.6)^2 = 3.1 \times 106 \text{ N/m}^2 = 3.1 \text{ N/mm}^2$$

Cross-sectional area of the rim,

$$A = b \times t = 0.3 \times 0.225 = 0.0675 \text{ m}^2$$

∴ Maximum tensile force acting on the rim

$$= \sigma t \times A = 3.1 \times 106 \times 0.0675 = 209\,250 \text{ N}$$

We know that tensile strength of the four bolts,

$$= \frac{\pi}{4} \times (d_c)^2 \times \sigma_t \times No.of bolts$$

$$= \frac{\pi}{4} \times (d_c)^2 \times 35 \times 4 = 110(d_c)^2$$

Since the bolts are made as strong as the rim joint, therefore from equations **(i)** and **(ii)**, we have

$$(d_c)^2 = 209\,250 / 110 = 1903 \text{ or } dc = 43.6 \text{ mm}$$

The standard size of the bolt is M 56 with dc = 48.65 mm

2. Cross-sectional dimensions of rectangular shrink links at the rim

Let h = Depth of the link in mm, and

w = Width of the link in mm = 1.25 h ... (Given)

∴ Cross-sectional area of each link,

A_l= w × h = 1.25 h2 mm2

We know that the maximum tensile force on half the rim

= 2 × σtfor rim × Cross-sectional area of rim

= 2 × 3.1 × 106 × 0.0675 = 418 500 N **... (iii)**

and tensile strength of the four shrink links

= σ_{tl}× A_l× 4 = 40 × 1.25 h2 × 4 = 200 h_2**...(iv)**

From equations **(iii)** and **(iv)**, we have

h_2 = 418 500 / 200 = 2092.5 or h = 45.7 say 46 mm

And w = 1.25 h = 1.25 × 46 = 57.5 say 58 mm

3. Cross-sectional dimensions of the elliptical arms

Let, a_1 = Major axis,

b_1 = Minor axis = 0.5 a_1...(Given)

n = Number of arms = 6 ...(Given)

Since the diameter of shaft (d1) is 150 mm and the diameter of hub (d) is taken equal to twicethe diameter of shaft, therefore

d = 2 d_1 = 2 × 150 = 300 mm = 0.3 m

We know that maximum bending moment on arms at the hub end,

$$\mathrm{M} = \frac{\mathrm{T}}{\mathrm{R.n}} \times (R - r) = \frac{\mathrm{T}}{\mathrm{D.n}} \times (D - d)$$

$$= \frac{15000}{1.575 \times 6} \times (1.575 - 0.3) = 2024 N - m = 2024 \times 10^3 N - mm$$

Section modulus,

$$\mathrm{z} = \frac{\pi}{32} \times b_1 \times (a_1)^3 = \frac{\pi}{32} \times 0.5 a_1 \times (a_1)^3 = 0.05 (a_1)^3$$

We know that bending stress (σ_{ta}),

$$15 = \frac{\mathrm{M}}{\mathrm{Z}} = \frac{2024 \times 10^3}{0.05 (a_1)^3} = \frac{40.5 \times 10^3}{(a_1)^3}$$

∴ $(a_1)^3$ = 40.5 × 106 / 15 = 2.7 × 106 or a_1 = 139.3 say 140 mm

and **b_1 = 0.5 a_1 = 0.5 × 140 = 70 mm**

21.A multi-cylinder engine is to run at a constant load at a speed of 600 r.p.m. On drawing the crank effort diagram to a scale of 1 m = 250 N-m and 1 mm = 3º, the areas in sq mm above and below the mean torque line are as follows:

+ 160, – 172, + 168, – 191, + 197, – 162 sq mm. The speed is to be kept within ± 1% of the mean speed of the engine. Calculate the necessary moment of inertia of the flywheel. Determine suitable dimensions for cast iron flywheel with a rim whose breadth is twice its

radial thickness. The density of cast iron is 7250 kg / m3, and its working stress in tension is 6 MPa. Assume that the rim contributes 92% of the flywheel effect. (16)

Given data:
N = 600 r.p.m. or
ω = 2π × 600 / 60 = 62.84 rad /s;
ρ = 7250 kg / m^3;
σ_t= 6 MPa = 6 × 106 N/m^2
Solution:
1. Moment of inertia of the flywheel
Let,I= Moment of inertia of the flywheel.
First of all, let us find the maximum fluctuation of energy. The turning moment diagram isshown in Fig 4.17

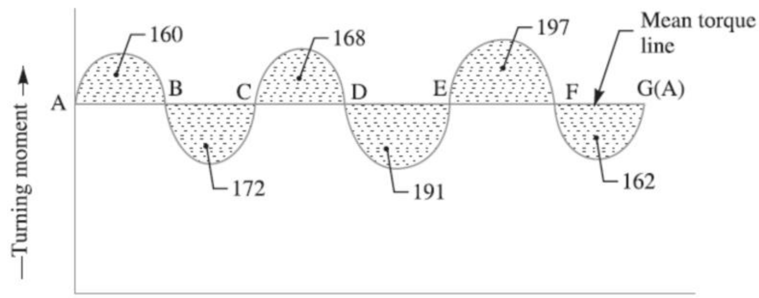


fig 4.17

` Since the scale for the turning moment is 1 mm = 250 N-m and the scale for the crank angle is1 mm = 3º = π/ 60rad, therefore
1 mm^2 on the turning moment diagram

$$= 250 \times \frac{\pi}{60} = 13.1N - m$$

Let, the total energy at A = E. Therefore from Fig, we find that
Energy at B = E + 160
Energy at C = E + 160 – 172 = E – 12
Energy at D = E – 12 + 168 = E + 156
Energy at E = E + 156 – 191 = E – 35
Energy at F = E – 35 + 197 = E + 162
Energy at G = E + 162 – 162 = E = Energy at A
From above, we find that the energy is maximum at F and minimum at E.
∴ Maximum energy = E + 162
and minimum energy = E – 35
We know that the maximum fluctuation of energy,

Δ E = Maximum energy – Minimum energy
= (E + 162) – (E – 35) = 197 mm2 = 197 × 13.1 = 2581 N-m

Since the fluctuation of speed is ± 1% of the mean speed (ω), therefore total fluctuation ofspeed,

ω1 – ω2 = 2% ω = 0.02 ω

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.02$$

We know that the maximum fluctuation of energy (ΔE),

$$2581 = I.\omega^2.C_S = I\ (62.84)2\ 0.02 = 79\ I$$

$$\therefore\ I = 2581 / 79 = 32.7 \text{ kg-m2 } \textbf{Ans.}$$

2. Dimensions of a flywheel rim

Let, t = Thickness of the flywheel rim in metres, and

b = Breadth of the flywheel rim in metres = 2 t ... (Given)

First of all let us find the peripheral velocity (v) and mean diameter (D) of the flywheel.

We know that tensile stress (σ_t),

$$6 \times 106 = \rho.v^2 = 7250 \times v^2$$

$$\therefore\ v^2 = 6 \times 106 / 7250 = 827.6 \text{ or } v = 28.76 \text{ m/s}$$

We also know that peripheral velocity (v),

$$28.76 = \frac{\pi D.N}{60} = \frac{\pi \times D \times 600}{60} = 31.42D$$

$$D = 28.76 / 31.42 = 0.915 \text{ m} = 915 \text{ mm}$$

Now let us find the mass of the flywheel rim. Since the rim contributes 92% of the flywheeleffect, therefore the energy of the flywheel rim (E_{rim}) will be 0.92 times the total energy of the flywheel(E). We know that maximum fluctuation of energy (ΔE),

$$2581 = E \times 2\ C_S = E \times 2 \times 0.02 = 0.04\ E$$

$$\therefore \quad E = 2581 / 0.04 = 64\,525 \text{ N-m}$$

and energy of the flywheel rim,

$$E_{rim} = 0.92\ E = 0.92 \times 64\,525 = 59\,363 \text{ N-m}$$

Let, m = Mass of the flywheel rim.

We know that energy of the flywheel rim (E_{rim}),

$$59363 = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times m \times (28.76)^2 = 413.6m$$

$$m = 59\,363 / 413.6 = 143.5 \text{ kg}$$

We also know that mass of the flywheel rim (m),

$$143.5 = b \times t \times \pi D \times \rho = 2\ t \times t \times \pi \times 0.915 \times 7250 = 41\,686\ t^2$$

$$t^2 = 143.5 / 41\,686 = 0.003\,44$$

or $\quad t = 0.0587$ say $0.06 \text{ m} = 60 \text{ mm}$

and $\quad b = 2\ t = 2 \times 60 = 120 \text{ mm}$

The mass of the flywheel rim may also be obtained by using the following relations. Since the rimcontributes 92% of the flywheel effect, therefore using

1. $I_{rim} = 0.92\ I_{flywheel}$ or $m.k^2 = 0.92 \times 32.7 = 30 \text{ kg–m}^2$

Since radius of gyration,

$$k = R = D / 2 = 0.915 / 2 = 0.4575 \text{ m, therefore}$$

$$m = \frac{30}{k^2} = \frac{30}{(0.4575)^2} = \frac{30}{0.209} = 143.5kg$$

2. $(\Delta E)_{rim} = 0.92\ (\Delta E)_{flywheel}$

$$m.v^2.C_S = 0.92\ (\Delta E)_{flywheel}$$

$$m(28.76)^2\ 0.02 = 0.92 \times 2581$$

$$16.55\ m = 2374.5 \text{ or } m = 2374.5 / 16.55 = 143.5 \text{ kg}$$

22. A cranked lever, as shown in 15.10, has the following dimensions:

Length of the handle = 300 mm

Length of the lever arm = 400 mm

Overhang of the journal = 100 mm

If the lever is operated by a single person exerting a maximum force of 400 N at a distance of1/3 rd length of the handle from its free end, find: 1. Diameter of the handle, 2. Cross-section of thelever arm, and 3. Diameter of the journal.The permissible bending stress for the lever material may be taken as 50 MPa and shear stressfor shaft material as 40 MPa. (16)

Given data:

l = 300 mm; L = 400 mm ;
x = 100 mm ;P= 400 N ;
σ_b= 50 Mpa= 50 N/mm^2 ;
τ = 40 MPa = 40 N/mm^2

Solution:

1. Diameter of the handle

Let, d = Diameter of the handle in mm.

Since the force applied acts at a distance of 1/3 rd length of the handle from its free end,thereforemaximum bending moment,

$$M = \left(1 - \frac{1}{3}\right) \times P \times l = \left(\frac{2}{3}\right) \times 400 \times 300N - mm$$

$= 80 \times 10^3$ N-mm ... **(i)**

Section modulus,

$$z = \frac{\pi}{32} \times d^3 = 0.0982d^3$$

∴ Resisting bending moment,

$M = \sigma_b \times Z = 50 \times 0.0982$ d3 = 4.91 d3 N-mm...**(ii)**

From equations **(i)** and **(ii),** we get

d^3 = 80 × 103 / 4.91 = 16.3 × 103 or d = 25.4 mm **Ans.**

2. Cross-section of the lever arm

Let, t = Thickness of the lever arm in mm, and

B = Width of the lever arm near the boss, in mm.

Since the lever arm is designed for 25% more bending moment, therefore maximum bendingmoment,

M = 1.25 P × L = 1.25 × 400 × 400 = 200 × 103 N-mm

Section modulus,

$$z = \frac{1}{6} \times t \times B^2 = \frac{1}{6} \times t \times (2t)^2 = 0.667t^3$$

We know that bending stress (σ_b),

$$50 = \frac{M}{Z} = \frac{200 \times 10^3}{0.667t^3} = \frac{300 \times 10^3}{t^3}$$

∴ t^3 = 300 × 103/50 = 6 × 103 or t = 18.2 say 20 mm

and B = 2 t = 2 × 20 = 40 mm

Let us now check the lever arm for induced bending and shear stresses.

Bending moment on the lever arm near the boss (assuming that the length of the arm extendsupto the centre of shaft) is given by

M = P × L = 400 × 400 = 160 × 103 N-mm

and section modulus,

$$z = \frac{1}{6} \times t \times B^2 = \frac{1}{6} \times 20 \times (40)^2 = 5333\ mm^3$$

∴ Induced bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{160 \times 10^3}{5333} = 30 N/mm^2$$

The induced bending stress is within safe limits.

We know that the twisting moment,

$$T = \frac{2}{3} \times P \times l = \frac{2}{3} \times 400 \times 300 = 80 \times 10^3 N - mm$$

We also know that the twisting moment (T),

$$80 \times 10^3 = \frac{2}{9} \times B \times t^2 \times \tau$$

$$= \frac{2}{9} \times 40 \times (20)^2 \times \tau = 3556\tau$$

∴ $\tau = 80 \times 10^3 / 3556 = 22.5$ N/mm^2 = 22.5 MPa

The induced shear stress is also within safe limits.

Let us now check the cross-section of lever arm for maximum principal or shear stress.

We know that maximum principal stress,

$$\sigma_{b(max)} = \frac{1}{2}\left[\sigma_{b+\sqrt{(\sigma_b)^2+4\tau^2}}\right]$$

$$= \frac{1}{2}\left[30_{+\sqrt{(30)^2+4(22.5)^2}}\right]$$

= ½ (30 + 54) = 42 N/mm^2 = 42 Mpa.

And maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_b)^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[\sqrt{(30)^2 + 4(22.5)^2}\right] = 27\frac{N}{mm^2} = 27 MPa$$

The maximum principal and shear stresses are also within safe limits.

3. *Diameter of the journal*

Let, *D* = Diameter of the journal.

Since the journal of the shaft is subjected to twisting moment and bending moment, therefore itsdiameter is obtained from equivalent twisting moment.

We know that equivalent twisting moment,

$$T = P\left[\sqrt{(\frac{2l}{3} + X)^2 + L^2}\right]$$

$$= 400\left[\sqrt{(\frac{2 \times 30}{3} + 100)^2 + 400^2}\right]$$

= 200 × 10^3 N-mm

We know that equivalent twisting moment (*Te*), we know that equivalent twisting moment (*Te*),

$$200 \times 10^3 = \frac{\pi}{16} \times D^3 \times \tau$$

$$= \frac{\pi}{16} \times D^3 \times 40 = 7.86D^3$$

D^3 = 200 × 103 / 7.86 = 25.4 × 103 or D = 29.4 say 30 mm

23. Design a plain carbon steel centre crankshaft for a single acting four stroke single cylinder engine for the following data*:*
Bore = 400 mm ; Stroke = 600 mm ; Engine speed = 200 r.p.m. ; Mean effective pressure = 0.5 N/mm²; Maximum combustion pressure = 2.5 N/mm²; Weight of flywheel used as a pulley = 50 kN; Total belt pull = 6.5 kN. When the crank has turned through 35° from the top dead centre, the pressure on the piston is n1N/mm² and the torque on the crank is maximum. The ratio of the connecting rod length to the crank radius is 5. Assume any other data required for the design. *(16)*

Given data:
D = 400 mm ;L= 600 mm
r = 300 mm ;pm= 0.5 N/mm² ; p = 2.5 N/mm² ;
W = 50 kN ;T1 + T2 = 6.5 kN ; θ = 35° ;
p′ = 1N/mm² ;l/ r = 5

Solution:
We shall design the crankshaft for the two positions of the crank, *i.e.* firstly when the crank is atthe dead centre ; and secondly when the crank is at an angle of maximum twisting moment.

1. Design of the crankshaft when the crank is at the dead centre
We know that the piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} \times (400)^2 \times 2.5$$

$$= 314200N = 314.2kN$$

Assume that the distance (*b*) between the bearings 1 and 2 is equal to twice the piston diameter (*D*).

∴ *b* = 2*D* = 2 × 400 = 800 mm

$$b_1 = b_2 = \frac{b}{2} = \frac{800}{2} = 400mm$$

We know that due to the piston gas load, there will be two horizontal reactions H1 and H2 atbearings 1 and 2 respectively, such that

$$H_1 = \frac{F_P \times b_1}{b} = \frac{314.2 \times 400}{800} = 157.1kN$$

$$H_2 = \frac{F_P \times b_2}{b} = \frac{314.2 \times 400}{800} = 157.1kN$$

Assume that the length of the main bearings to be equal, i.e. c1 = c2 = c / 2. We know that due to the weight of the flywheel acting downwards, there will be two vertical reactions V2 and V3 at bearings 2 and 3 respectively, such that

$$V_2 = \frac{W \times c_1}{c} = \frac{W \times c/2}{c} = \frac{W}{2} = \frac{50}{2} = 25kN$$

$$V_3 = \frac{W \times c_1}{c} = \frac{W \times c/2}{c} = \frac{W}{2} = \frac{50}{2} = 25kN$$

Due to the resultant belt tension ($T1$ + $T2$) acting horizontally, there will be two horizontalreactions$H2'$ and $H3'$ respectively, such that

$$H'_2 = \frac{(T_1 + \mathrm{T}_2)\,\mathrm{c}_1}{c} = \frac{(T_1 + \mathrm{T}_2) \times c/2}{c} = \frac{T_1 + \mathrm{T}_2}{2} = \frac{6.5}{2} = 3.25kN$$

$$H'_3 = \frac{(T_1 + \mathrm{T}_2)\,\mathrm{c}_1}{c} = \frac{(T_1 + \mathrm{T}_2) \times c/2}{c} = \frac{T_1 + \mathrm{T}_2}{2} = \frac{6.5}{2} = 3.25kN$$

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crankpin

Let, d_c= Diameter of the crankpin in mm;

l_c= Length of the crankpin in mm; and

σ_b= Allowable bending stress for the crankpin. It may be assumed as75 MPa or N/mm^2.

We know that the bending moment at the centre of the crankpin,

$M_C = H_1 \cdot b_2 = 157.1 \times 400 = 62\ 840$ kN-mm ... **(i)**

We also know that,

$$M_c = \frac{\pi}{32}(\mathrm{d}_c)^2 \sigma_b = \frac{\pi}{32}(\mathrm{d}_c)^2 75 = 7.364(\mathrm{d}_c)^2\, N - mm$$

$= 7.364 \times 10^{-3}\ (d_c)^3$kN-mm ...**(ii)**

Equating equations **(i)** and **(ii)**, we have

$(d_c)^3 = 62\ 840 / 7.364 \times 10–3 = 8.53 \times 10^6$

Or d_c= 204.35 say 205 mm **Ans.**

We know that length of the crankpin,

$$l_c = \frac{F_P}{d_c . p_b} = \frac{314.2 \times 10^3}{205 \times 10} = 153.33 say 155 mm$$

(b) Design of left hand crank web

We know that thickness of the crank web,

t = 0.65 dc + 6.35 mm

= 0.65 × 205 + 6.35 = 139.6 say 140 mm

and width of the crank web, w = 1.125 d_c + 12.7 mm

= 1.125 × 205 + 12.7 = 243.3 say 245 mm

We know that maximum bending moment on the crank web

$$M = H_1\left[b_2 - \frac{l_c}{2} - \frac{t}{2}\right]$$

$$=157.1\left[400-\frac{155}{2}-\frac{140}{2}\right]=39668 kN-mm$$

Section modulus, $Z=\frac{1}{6}\times w.t^2=\frac{1}{6}\times 245(140)^2=800\times 10^3 mm^3$

$\therefore$ Bending stress, $\sigma_b=\frac{M}{Z}=\frac{39668}{800\times 10^3}=49.6\times 10^{-3} kN/\text{mm}^2=49.6 N/\text{mm}^2$

We know that direct compressive stress on the crank web,

$$\sigma_b=\frac{H_1}{w.t}=\frac{157.1}{245\times 140}=4.58\times 10^{-3} kN/\text{mm}^2=4.58 N/\text{mm}^2$$

Total stress on the crank web

$= \sigma_b + \sigma_c = 49.6 + 4.58 = 54.18$ N/mm2 or MPa

Since the total stress on the crank web is less than the allowable bending stress of 75 MPa,therefore, the design of the left hand crank web is safe.

(c) Design of right hand crank web

From the balancing point of view, the dimensions of the right hand crank web (i.e. thickness andwidth) are made equal to the dimensions of the left hand crank web.

(d) Design of shaft under the flywheel

Let, ds = Diameter of the shaft in mm.

Since the lengths of the main bearings are equal, therefore

$$l_1=l_2=l_3=2\left(\frac{b}{2}-\frac{l_c}{2}-t\right)$$

$$=2\left(400-\frac{155}{2}-140\right)=365mm$$

Assuming width of the flywheel as 300 mm, we have

c = 365 + 300 = 665 mm

Allowing space for gearing and clearance, let us take c = 800 mm.

$$c_1=c_2=\frac{c}{2}=\frac{800}{2}=400mm$$

We know that bending moment due to the weight of flywheel,

$M_W = V_3 \cdot c_1 = 25 \times 400 = 10\,000$ kN-mm = 10 × 106 N-mm

and bending moment due to the belt pull,

$M_T = H_3' \cdot c_1 = 3.25 \times 400 = 1300$ kN-mm = 1.3 × 106 N-mm

$\therefore$ Resultant bending moment on the shaft,

$$M_S=\sqrt{(M_W)^2+(M_T)^2}=\sqrt{(10\times 10^6)^2+(1.3\times 10^6)^2}$$

$= 10.08 \times 10^6$ N-mm

We also know that bending moment on the shaft (MS),

$$10.08 \times 10^6 = \frac{\pi}{32} \times (d_s)^3 \times \sigma_b = \frac{\pi}{32} \times (d_s)^3 \times 42 = 4.12(d_s)^3$$

$(d_s)^3$ = 10.08 × 106 / 4.12 = 2.45 × 106 or *ds* = 134.7 say **135 mm**

2. Design of the crankshaft when the crank is at an angle of maximum twisting moment

We also know that piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p' = \frac{\pi}{4} \times (400)^2 1 = 125680N = 125.68kN$$

In order to find the thrust in the connecting rod (FQ), we should first find out the angle ofinclination of the connecting rod with the line of stroke (i.e. angle φ). We know that

$$\sin\phi = \frac{\sin\theta}{l/r} = \frac{\sin 35^o}{5} = 0.1147$$

∴ φ= sin–1 (0.1147) = 6.58°

We know that thrust in the connecting rod,

$$F_Q = \frac{F_P}{\cos\phi} = \frac{125.68}{\cos 6.58^o} = 126.5kN$$

Tangential force acting on the crankshaft,

$F_T = F_Q \sin(\theta + \varphi)$ = 126.5 sin (35° + 6.58°) = 84 kN

and radial force, $F_R = F_Q \cos(\theta + \varphi)$ = 126.5 cos (35° + 6.58°) = 94.6 kN

Due to the tangential force (FT), there will be two reactions at bearings 1 and 2, such that

$$H_{T1} = \frac{F_T \times b_1}{b} = \frac{84 \times 400}{800} = 42kN$$

$$H_{T2} = \frac{F_T \times b_1}{b} = \frac{84 \times 400}{800} = 42k\text{N}$$

Due to the radial force (FR), there will be two reactions at bearings 1 and 2, such that

$$H_{R1} = \frac{F_R \times b_1}{b} = \frac{94.6 \times 400}{800} = 47.3kN$$

$$H_{R2} = \frac{F_R \times b_1}{b} = \frac{94.6 \times 400}{800} = 47.3kN$$

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crankpin

Let, dc = Diameter of crankpin in mm.

We know that the bending moment at the Centre of the crankpin,

$M_C = H_{R1} \times b2$ = 47.3 × 400 = 18 920 kN-mm

and twisting moment on the crankpin,

$T_C = H_{T1} \times r$ = 42 × 300 = 12 600 kN-mm

∴ Equivalent twisting moment on the crankpin,

$$T_e = \sqrt{(M_c)^2 + (T_C)} = \sqrt{(18920)^2 + (12600)^2}$$

= 22 740 kN-mm = 22.74 × 106 N-mm

We know that equivalent twisting moment (*Te*),

$$22.74\times10^6 = \frac{\pi}{32}\times(\mathrm{d}_s)^3\times\tau = \frac{\pi}{32}\times(\mathrm{d}_s)^3\times35 = 6.837(\mathrm{d}_s)^3$$

... (Taking τ = 35 MPa or N/mm2)

∴ $(d_c)^3$ = 22.74 × 106 / 6.873 = 3.3 × 106 or d_c= 149 mm

Since this value of crankpin diameter (*i.e. dc* = 149 mm) is less than the already calculated valueofd_c= 205 mm, therefore, we shall take d_c= 205 mm. **Ans.**

(*b*) *Design of shaft under the flywheel*

Let, *ds* = Diameter of the shaft in mm.

The resulting bending moment on the shaft will be same as calculated earlier, *i.e.*

M_S = 10.08 × 106 N-mm

and twisting moment on the shaft,

$T_S = F_T \times r$ = 84 × 300 = 25 200 kN-mm = 25.2 × 106 N-mm

∴ Equivalent twisting moment on shaft,

$$T_e = \sqrt{(M_s)^2 + (T_s)}$$

$$= \sqrt{(10.08\times10^6)^2 + (25.2\times10^6)^2} = 27.14\times10^6\ N - mm$$

We know that equivalent twisting moment (*Te*),

$$27.14\times10^6 = \frac{\pi}{16}\times(\mathrm{d}_s)^3\times\tau = \frac{\pi}{16}\times(135)^3\times\tau = 483156\tau$$

∴ τ = 27.14 × 106 / 483 156 = 56.17 N/mm2

From above, we see that by taking the already calculated value of ds = 135 mm, the induced shear stress is more than the allowable shear stress of 31 to 42 MPa. Hence, the value of ds is calculated by taking τ = 35 MPa or N/mm2 in the above equation, i.e.

$$27.14\times10^6 = \frac{\pi}{16}\times(\mathrm{d}_s)^3\times35 = 6.837(\mathrm{d}_s)^3$$

∴ $(d_S)^3$ = 27.14 × 106 / 6.873 = 3.95 × 106 or ds = 158 say 160 mm

(c) Design of shaft at the juncture of right hand crank arm

Let, d_{s1} = Diameter of the shaft at the juncture of the right hand crank arm.

We know that the resultant force at the bearing 1,

$$R_1 = \sqrt{(H_{T1})^2 + (H_{R1})^2}$$

$$R_1 = \sqrt{(H_{T1})^2 + (H_{R1})^2} = \sqrt{(42)^2 + (47.3)^2} = 63.3kN$$

∴ Bending moment at the juncture of the right hand crank arm,

$$M_{S1} = R_1\left[b_2 + \frac{l_c}{2} + \frac{t}{2}\right] - F_Q\left[\frac{l_c}{2} + \frac{t}{2}\right]$$

$$= 63.33\left[400 + \frac{155}{2} + \frac{140}{2}\right] - 126.5\left[\frac{155}{2} + \frac{140}{2}\right]$$

= 34.7 × 103 – 18.7 × 103 = 16 × 103 kN-mm = 16 × 106 N-mm

and twisting moment at the juncture of the right hand crank arm,

$T_{S1} = F_T \times r$ = 84 × 300 = 25 200 kN-mm = 25.2 × 106 N-mm

∴ Equivalent twisting moment at the juncture of the right hand crank arm,

$$T_e = \sqrt{(M_{S1})^2 + (T_{S1})^2}$$

$$= \sqrt{(16\times10^6)^2 + (25.5\times10^6)^2} = 29.8\times10^6\ N - mm$$

We know that equivalent twisting moment (*Te*),

$$29.85 \times 10^6 = \frac{\pi}{16} \times (d_s)^3 \times 42 = 8.25(d_s)^3$$

... (Taking τ = 42 MPa or N/mm2)

∴ (ds_1)3 = 29.85 × 106 / 8.25 = 3.62 × 106 or ds_1 = 153.5 say 155 mm

CONNECTING ROD:

24.Explain why the standard I-Section is chosen for designing of connecting rod over other cross section without sacrificing the fundamentals and write the design equation for connecting rod based on crippling load.(April/May 2018)

A connecting rod is a machine member which is subjected to alternating direct compressive andtensile forces. Since the compressive forces are much higher than the tensile forces, therefore thecross-section of the connecting rod is designed as a strut and the Rankine's formula is used.

A connecting rod subjected to an axial load W may buckle with X-axis as neutral axis (i.e. in theplane of motion of the connecting rod) or Y-axis as neutral axis (i.e. in the plane perpendicular to theplane of motion). The connecting rod is considered like both ends hinged for buckling about X-axisand both ends fixed for buckling about Y-axis. A connecting rod should be equally strong in bucklingabout either axes.

A = Cross-sectional area of the connecting rod,

l = Length of the connecting rod,

σ_c = Compressive yield stress,

W_{cr} = Crippling or buckling load, by in fig 4.18

I_{xx} and I_{yy} = Moment of inertia of the section about X-axis and Y-axis respectively,

and

k_{xx} and k_{yy} = Radius of gyration of the section about X-axis and Y-axis respectively

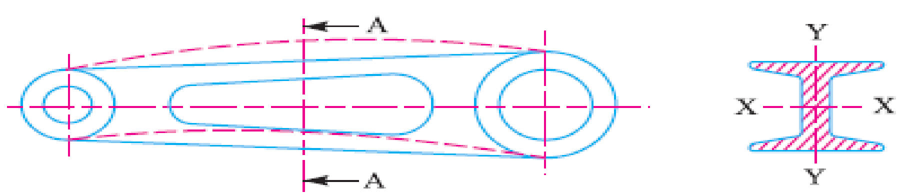


fig 4.18 Buckling of connecting rod.

According to Rankine's formula,

$$W_{cr} \text{ about } X\text{-axis} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k_{xx}}\right)^2} = \frac{\sigma_c \times A}{1 + a\left(\frac{l}{k_{xx}}\right)^2} \quad \text{... } (\because \text{ For both ends hinged, } L = l)$$

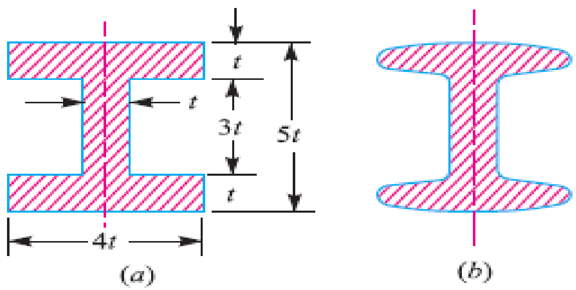
and
$$W_{cr} \text{ about } Y\text{-axis} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k_{yy}}\right)^2} = \frac{\sigma_c \times A}{1 + a\left(\frac{l}{2k_{yy}}\right)^2} \quad \text{... } \left(\because \text{ For both ends fixed, } L = \frac{l}{2}\right)$$

In order to have a connecting rod equally strong in buckling about both the axes, the buckling loads must be equal, *i.e.*

$$\frac{\sigma_c \times A}{1 + a\left(\frac{l}{k_{xx}}\right)^2} = \frac{\sigma_c \times A}{1 + a\left(\frac{l}{2\,k_{yy}}\right)^2} \quad \text{or} \quad \left(\frac{l}{k_{xx}}\right)^2 = \left(\frac{l}{2\,k_{yy}}\right)^2$$

$$\therefore \quad k^2_{xx} = 4\,k^2_{yy} \quad \text{or} \quad I_{xx} = 4\,I_{yy} \quad \text{... } (\because I = A \times k^2)$$

This shows that the connecting rod is fourtimes strong in buckling about Y-axis than aboutX-axis. If $I_{xx} > 4\,I_{yy}$, then buckling will occur aboutY-axis and if $I_{xx} < 4\,I_{yy}$, buckling will occur aboutX-axis. In actual practice, I_{xx} is kept slightly less than4 I_{yy}. It is usually taken between 3 and 3.5 and theconnecting rod is designed for buckling about X-axis.The design will alwyas be satisfactory for bucklingabout Y-axis.The most suitable section for the connectingrod is I-section with the proportions as shown in Fig.



4.19 I-section of connecting rod.

Area of the section

$$= 2\,(4\,t \times t) + 3\,t \times t = 11\,t^2$$

$\therefore$ Moment of inertia about X-axis,

$$I_{xx} = \frac{1}{12}\left[4\,t\,(5\,t)^3 - 3\,t\,(3\,t)^3\right] = \frac{419}{12}\,t^4$$

and moment of inertia about Y-axis,

$$I_{yy} = \left[2 \times \frac{1}{12}\,t \times (4\,t)^3 + \frac{1}{12}\,(3\,t)\,t^3\right] = \frac{131}{12}\,t^4$$

$$\therefore \quad \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

Since the value of $\frac{I_{xx}}{I_{yy}}$ lies between 3 and 3.5, therefore I-section chosen is quite satisfactory.

Notes : 1. The I-section of the connecting rod is used due to its lightness and to keep the inertia forces as low aspossible. It can also withstand high gas pressure.

2. Sometimes a connecting rod may have rectangular section. For slow speed engines, circular sections may be used.
3. Since connecting rod is manufactured by forging, therefore the sharp corners of I-section are rounded off as shown in Fig.4.19 for easy removal of the section from the dies.

25.Determine the dimensions of an I-section connecting rod for a petrol engine from the following data : Diameter of the piston = 110 mm, Mass of the reciprocating parts = 2 kg
Length of the connecting rod from centre to centre = 325 mm, Stroke length = 150 mm
R.P.M. = 1500 with possible over speed of 2500, Compression ratio = 4 : 1
Maximum explosion pressure = 2.5 N/mm^2 (16) (Nov/Dec – 2013)

Given :

D = 110 mm = 0.11 m ;m
R = 2 kg ;
l = 325 mm = 0.325 m ;
Stroke length = 150 mm = 0.15 m ;
N_{min}= 1500 r.p.m. ;N_{max}= 2500 r.p.m. ;
Compression ratio = 4 : 1 ;
p = 2.5 N/mm2

We know that the radius of crank,

$$r = Stroke\frac{length}{2} = \frac{150}{2} = 755mm = 0.075\ m$$

And ratio of the length of connection rod to the radius of crank,

$$n = \frac{l}{r} = \frac{325}{75} = 4.3$$

We know that the maximum force on the piston due to pressure,

$$F_L = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} \times 110^2 \times 2.5 = 23760\ N$$

And maximum angular speed,

$$\omega_{max} = \frac{2\pi \times N_{max}}{60} = \frac{2\pi \times 2500}{60} = 261.8\ m/s$$

We know that maximum inertia force of reciprocating parts,

$$F_1 = m_R(\omega_{max})^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right) - - - - - - - - - - - - - - - 1$$

The inertia force of reciprocating parts is maximum, when the crank is at inner dead centre, *i.e*. when θ = 0°.

$$F_1 = m_R(\omega_{max})^2 r\left(1 + \frac{1}{n}\right)$$

$$F_1 = 2(261.8)^2 0.075\left(1 + \frac{1}{4.3}\right) = 12672\ N$$

Since the connecting rod is designed by taking the force in the connecting rod (F_C) equal to the maximum force on the piston due to gas pressure (F_L), therefore Force in the connecting rod,
$F_C = F_L$ = 23 760 N
Consider the *I*-section of the connecting rod with the proportions as shown in Fig. 4.20

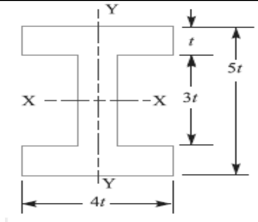


fig 4.20 $\frac{I_{xx}}{I_{yy}} = 3.2$

$$\frac{k^{2xx}}{k^{2yy}} = 3.2\ whichissatisfactory.$$

We have also discussed that the connecting rod is designed for buckling about *X*-axis (*i.e*. in a plane of motion of the connecting rod), assuming both ends hinged. Taking a factor of safety as 6, the buckling load,

$W_{cr}= F_C \times 6 = 23\,760 \times 6 = 142\,560$ N

and area of cross-section, $A = 2\,(4t \times t) + t \times 3t = 11\,t^2$ mm^2

Moment of inertia about *X*-axis,

$$I_{xx} = \left[\frac{4t(5\,t)^3}{12} - \frac{3t(3\,t)^3}{12}\right] = \frac{419t^4}{12}mm^4$$

Radius of gyration,

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{419t^4}{12} \times \frac{1}{11\,t^2}} = 1.78\,t$$

We know that equivalent length of the rod for both ends hinged,

$L = l = 325$ mm

Taking for mild steel, σ_c= 320 MPa = 320 N/mm2 and a = 1 / 7500, we have from Rankine's formula,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{K_{xx}}\right)^2}$$

$$142560 = \frac{320 \times 11\,t^2}{1 + \frac{1}{7500}\left(\frac{325}{1.78\,t}\right)^2}$$

$$40.5 = \frac{t^2}{1 + \frac{4.44}{t^2}} = \frac{t^4}{t^2 + 4.44}$$

$$t^4 - 40.5\,t^2 - 179.8 = 0$$

$$t^2 = \frac{40.5 \pm \sqrt{40.5^2 + 4 \times 179.8}}{2} = \frac{40.5 \pm 48.6}{2} = 44.55$$

t=6.67 say 6.8 mm

Therefore, dimensions of cross-section of the connecting rod are

Height = 5 t = 5 × 6.8 = 34 mm **Ans.**

Width = 4 t = 4 × 6.8 = 27.2 mm **Ans.**

Thickness of flange and web = t = 6.8 mm = 0.0068 m Ans.

Now let us find the bending stress due to inertia force on the connecting rod.

We know that the mass of the connecting rod per metre length,

m1 = Volume × density = Area × length × density

= $A \times l \times \rho = 11\, t^2 \times l \times \rho$ ..($\because A = 11\, t^2$)

= 11(0.0068)2 1 × 7800 = 3.97 kg ... (Taking ρ = 7800 kg / m^3)

Maximum bending moment

$$M_{max} = m\omega^2 r \times \frac{l}{9\sqrt{3}} = m_1\omega^2 r \times \frac{l^2}{9\sqrt{3}} m = m_1.l$$

$$= 3.97 \times 261.8^2 \times 0.075 \times \frac{0.325^2}{9\sqrt{3}} = 138.3\ N - m$$

And section modulus,

$$Z_{xx} = \frac{I_{xx}}{\frac{5t}{2}} = \frac{419t^4}{12} \times \frac{2}{5t}$$

$$= \frac{419}{30} t^3 = \frac{419}{30} \times 0.0068^3 = 4.4 \times 10^{-6} m^3$$

Maximum bending or whipping stress due to inertia bending forces,

$$\sigma_{b(max)} = \frac{M_{max}}{Z_{xx}} = \frac{138.3}{4.4 \times 10^{-6}} = 31.4 \times 10^6 N/m^2$$

Note :

The maximum compressive stress in the connecting rod will be,

$\sigma_{c(max)}$ = Direct compressive stress + Maximum bending stress

= (320/6)+ 31.4 = 84.7 MPa

26.Design a suitable connection rod for a petrol engine for the following details. Diameter of the piston=100mm, Weight of reciprocating pars per cylinder = 20N; connecting rod length = 300mm; compression ratio= 7:1 Maximum explosion pressure = 3 N/mm^2; stroke = 140 mm; speed of the engine = 2000 rpm. (Nov/ Dec–2012).

Given Data:

Diameter of the piston, d = 100 mm

Weight of reciprocating parts per cylinder R=20 N

Connecting rod length l=300 mm

Compression ratio = 7:1

Maximum explosion pressure = 3N/mm^2

Stroke = 140mm

Speed of the engine, N= 2000 rpm

To find:

Design of connecting rod

Solution:

The connecting rod used in I.C engine is mostly of I section with the proportions as shown in fig.

For the selected I- section from PSGDB 7.122

Area of section, a = 11 t^2

Moment of inertia of the section about Y-axis, I_{xx}= 419/12 t^4

Moment of inertia of the section about Y-axis, I_{yy}= 131/12 t^4

The ratio I_{xx}/ I_{yy} should be 3 to 3.5 for safety design.

For the selected I-section, I_{xx}/ I_{yy}= 3.2 which is satisfactory.

i) Dimensions of I-section:

Load due to maximum explosion pressure,

$$F_G = \frac{\pi}{4} \times d^2 \times p$$

$$= 23561.95\,N$$

We know that, radius of gyration about x- axis

$$k^2{}_{xx} = 3.18t^2$$

Rankine's formula for buckling load (F_{cr})

$$F_{cr} = \frac{\sigma_c . a}{1 + C(\frac{1}{K_{xx}})} \quad \text{---------------------1}$$

F_{cr} = factory of safety ×F_G

F_{cr} = 6*23561.95

= 141371.7 N

For cast iron, C= 1/1600

For mild steel, C = 1/7500

We assume that the connecting rod is made of mild steel,

Assume the stress value for mild steel, σ_c=330 N/mm

$$141371.7 = \frac{330 \times 11t^2}{1 + \frac{1}{7500}\left(\frac{300^2}{3.18t^2}\right)}$$

$$t^4 - 38.94t^2 - 3.92 \times 10^{-3} = 0$$

$$\therefore t = 4.68\ mm$$

$$thickness of\ I - section, t = 5mm$$

Height of the I-section, H=5t=25 mm

Width of the I-section B = 4t=20 mm

(ii) Design of pin for small end:

Length of small end pin = L_1

Diameter of small end pin = d_1

We know that,

L_1/d_1 = 1.5 to 2

Let, L_1/d_1=1.75

L1 = 1.75 d_1

Load due to steam pressure,

$$F_G = L_1.d_1.p_{b1}$$

$$assume, bearing pressure for small end,$$

$$p_{b1} = 13 N/mm^2$$

$$23561.96 = 1.75$$

$$23561.95 = 1.75 d_1 \times d_1 \times 13$$

$$d_1 = 32.18\ mm$$

$$say, diameter\ for\ small\ end\ pin,$$

$$d_1 = 33 mm$$

$$L_1 = 1.75 \times 33 = 57.5 mm$$

$$Say, length of\ small\ pin$$

$$L_1 = 58 mm$$

iii. Design of pin for big end:

$$\frac{L_2}{d_2} = 1.375$$

$$L_2 = 1.375 d_2$$

$$Assume, bearing\ pressure\ for\ big\ end,\ p_{b2} = 8 N/mm^2$$

$$Load\ due to\ steam\ pressure, F_G = L_2 * d_2 * p_{b2}$$

$$23561.95 = 1.375 d_2 \times d_2 \times 8$$

$$d_2 = 46.28 mm$$

$$\therefore d_2 = 47 mm$$

$$Say, length of\ big\ end\ L_2 = 65 mm$$

Diameter of bolts:

Angular velocity,

$$\omega = \frac{2\pi N}{60} = 209.439 rad/\sec$$

$$Radius of the crank, r = stroke/2 = 70 mm$$

$$Inertia\ force$$

$$F_i = \frac{R}{g} \times \omega^2 \times r \left[\cos\theta + \frac{\cos 2\theta}{l/r}\right] = 7720.66 N$$

Nominal diameter of bolt, $d_b = \dfrac{d_c}{0.84} = 5.95 mm$

$$diameter of\ bolt, d_b = 6 mm$$

v. **thickness of big end cap**

Distance between the bolt centres, X=(1.3 to 1.75)d_2

X=1.5 d2=1.5*47=70.5 mm

$$Z = \frac{65 \times t_c^{\,2}}{6} = 10.8 t_c^{\,2}$$

$$M_c = \sigma_b . Z$$

$$90717.755 = 120 * 10.8 t_c^{\,2}$$

$$\therefore t_c = 8.366 mm \, or \quad t_c = 8.5 mm$$

LEAF SPRING

27.Design a leaf spring for the following specifications for truck. Assume FOS =2.

Maximum load on springs=100 kN

No. of springs = 4,

Material of springs = Cr Va steel (σ_u = 1380 MPa and E=206 x 10^3MPa),

Span of spring = 1000 mm,

Width of central band = 150 mm,

Permissible deflection = 100mm,

Assume 2 full length leaves and 6 graduated leaves. (16) (Apr/May – 2015).

OR

Design a leaf spring for the following specifications :
Total load = 140 kN ; Number of springs supporting the load = 4 ;
Maximum number of leaves= 10; Span of the spring = 1000 mm ;
Permissible deflection = 80 mm.Take Young's modulus, E = 200 kN/mm^2 and allowable stress in spring material as 600 MPa(Nov/Dec-16)

Given data:

Maximum load on springs=100 kN

No. of springs = 4,

Material of springs = Cr Va steel (σ_u = 1380 MPa and E=206 x 10^3MPa),

Span of spring = 1000 mm,

Width of central band = 150 mm,

Permissible deflection = 100mm,

To find:

1. Design of the leaf spring

(i) Width of leaf spring, b

(ii) Thickness of the leaf, t

Solution:

Load on the spring,$2P = 140 \times 10^3$

$$2P = \frac{140 \times 10^3}{Number of spring}$$

$$2P = \frac{140 \times 10^3}{4}$$

P = 17500 N

Permissible stress,

$$\sigma = \frac{6PL}{nbt^2}$$

$$600 = \frac{6 \times 17500 \times 500}{10 \times bt^2}$$

$$bt^2 = 8750 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

Deflection of the spring,

$$y = \frac{6PL^3}{Enbt^3}$$

$$80 = \frac{6 \times 17500 \times 500}{200 \times 10^3 \times 10 \times bt^3}$$

$$bt^3 = 82031.25 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

$$bt^2.t = 82031.25$$

From equation (1) & (2)
8750t = 82031.25

$$t = \frac{82031.25}{8750} = 9.375mm$$

Say thickness of the leaves, t = 10mm

$$b = \frac{8750}{t^2} = \frac{8750}{10^2} = 87.5mm$$

Width of the leaves, b = 87.5mm
The nearest standard size, b = 90 mm

Result:

1.Width of the leaves, b = 87.5mm
2.The nearest standard size, b = 90 mm

28.Design a cantilever leaf spring to absorb 600 N-m energy without exceeding a deflection of 150mm and a stress of 800 N/mm^2. The length of the spring in 600mm. the material of the spring is steel.(16)

Given data:

Cantilever leaf spring to absorbed, E_1 = 600 N-m = 600 × 10^3 N-mm
Deflection of the spring, y = 150 mm
Bending stress of the spring,$\sigma_b = 800N/mm^2$
Length of the spring, L = 600 mm

To find:

Design of the cantilever leaf spring

(i) Width of the leaf spring, b

(ii) Thickness of the leaf spring, t

Solution:

Energy, $E_1 = \frac{1}{2} \times P \times y$

$$600 \times 10^3 = \frac{1}{2} \times P \times 150$$

Maximum load, P = 8000N

The maximum permissible stress in a leaf spring is

$$\sigma = \frac{6PL}{nbt^2}$$

$$800 = \frac{6 \times 8000 \times 600}{nbt^2}$$

$$nbt^2 = \frac{6 \times 8000 \times 600}{800}$$

$$nbt^2 = 36000 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Deflection of the spring,

$$y = \frac{12PL^3}{Enbt^2}$$

For steel spring, young's modulus,

E = 2×10^5 N/mm^2

$$150 = (6 \times 8000 \times 600^3)/(2 \times 10^5 \frac{\text{N}}{\text{mm}^2} bt^2\ )$$

$$nbt^2 = 345600$$

$$nbt^2.t = 345600 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Form equation (1) and (2)

36000 t = 345600

$$t = \frac{345600}{36000} = 9.6\ mm$$

Select standard size, t = 10 mm

$$nb = \frac{345600}{(9.6)^3} = 390.625$$

Select the width appropriate to, t = 10 mm

Standard size of the width is 80mm

Number of leaves,

$$n = \frac{390.625}{80} = 4.88\ say\ 5$$

Result:

Design of the cantilever leaf spring

(i) Width of the leaf spring, b = 80 mm

(ii) Thickness of the leaf spring, t = 10 mm (iii)Number of leaves, n = 5

CRANKSHAFT

29. Design a side or overhung crankshaft for a 250 mm × 300 mm gas engine. The weight of the flywheel is 30 kN and the explosion pressure is 2.1 N/mm2. The gas pressure at the maximum torque is 0.9 N/mm2, when the crank angle is 35° from I. D. C. The connecting rod is 4.5 times the crank radius. (16)

Given:

D = 250 mm ;
L = 300 mm or r = L / 2 = 300 / 2 = 150 mm ;
W = 30 kN = 30 × 103 N ;
p = 2.1 N/mm2, P′ = 0.9 N/mm2 ;l= 4.5 r or l / r = 4.5

Solution:

We shall design the crankshaft for the two positions of the crank, i.e. firstly when the crank is at the dead centre and secondly when the crank is at an angle of maximum twisting moment.

1. Design of crankshaft when the crank is at the dead centre
We know that piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} \times 250^2 \times 2.1 = 103 \times 10^3 N$$

Now the various parts of the crankshaft are designed as discussed below:
(a) Design of crankpin

Let d_c = Diameter of the crankpin in mm, and

l_c= Length of the crankpin = 0.8 d_c ...(Assume)

Considering the crankpin in bearing, we have

$F_P = d_c.l_c.p_b$

$103 \times 10^3 = dc \times 0.8\ dc \times 10 = 8\ (d_c)2$ ...(Taking p_b= 10 N/mm2)

$\therefore (d_c)2 = 103 \times 10^3 / 8 = 12\ 875$ or d_c= 113.4 say 115 mm

andl_c= 0.8 d_c= 0.8 × 115 = 92 mm

Let us now check the induced bending stress in the crankpin.

We know that bending moment at the crankpin,

$$M = \frac{3}{4} F_P \times l_c = \frac{3}{4} \times 103 \times 10^3 \times 92 = 7107 \times 10^3 N - mm$$

and section modulus of the crankpin,

$$Z = \frac{\pi}{32}(d_c)^3 == \frac{\pi}{32}(115)^3 A = 149 \times 10^3 mm^3$$

∴ Bending stress induced

$$= \frac{M}{Z} = \frac{7107 \times 10^3}{149 \times 10^3} = 47.7 \frac{N}{mm^2} or MPa$$

Since the induced bending stress is within the permissible limits of 60 MPa, therefore, design of crankpin is safe.

(*b*) *Design of bearings*

Let d_1 = Diameter of the bearing 1.

Let us take thickness of the crank web,

$t = 0.6\ d_c = 0.6 \times 115 = 69$ or 70 mm

and length of the bearing, $l_1 = 1.7\ d_c = 1.7 \times 115 = 195.5$ say 200 mm

We know that bending moment at the centre of the bearing 1,

$M = F_P\ (0.75 l_c + t + 0.5\ l_1) = 103 \times 10^3\ (0.75 \times 92 + 70 + 0.5 \times 200) = 24.6 \times 10^6 n$-mm

We also know that bending moment (M),

$$24.6 \times 10^6 = \frac{\pi}{32}(d_1)^3 \sigma_b = \frac{\pi}{32}(d_1)^3 60 = 5.9(d_1)^3$$

...(Taking σb= 60 MPa or N/mm^2)

∴ $(d_1)^3 = 24.6 \times 10^6 / 5.9 = 4.2 \times 106$ or $d_1 = 161.3$ mm say 162 mm **Ans.**

(*c*) *Design of crank web*

Let w = Width of the crank web in mm.

We know that bending moment on the crank web,

$M = F_P\ (0.75 l_c + 0.5\ t)$

$= 103 \times 10^3\ (0.75 \times 92 + 0.5 \times 70) = 10.7 \times 10^6$ N-mm

and section modulus,

$$Z = \frac{1}{6} \times w.t^2 = \frac{1}{6} \times w(70)^2 = 817\ w mm^3$$

∴ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{10.7 \times 10^6}{817\, w} = \frac{13 \times 10^3}{w} = N/mm^2$$

and direct compressive stress,

$$\sigma_b = \frac{F_P}{w.t} = \frac{103 \times 10^3}{w \times 70} = \frac{1.7 \times 10^3}{w} N/mm^2$$

We know that total stress on the crank web,

$$\sigma_T = \sigma_b + \sigma_d = \frac{13 \times 10^3}{w} + \frac{1.7 \times 10^3}{w} = \frac{14.47 \times 10^3}{w} \frac{N}{mm^2}$$

The total stress should not exceed the permissible limit of 60 MPa or N/mm2.

$$60 = \frac{14.47 \times 10^3}{w} or \frac{14.47 \times 10^3}{60} = 241\ say\ 245\ mm$$

(*d*) *Design of shaft under the flywheel.*

Let *ds* = Diameter of shaft under the flywheel.

First of all, let us find the horizontal and vertical reactions at bearings 1 and 2. Assume that the width of flywheel is 250 mm and *l*1 = *l*2 = 200 mm.

Allowing for certain clearance, the distance

$$b = 250 \frac{l_1}{2} + \frac{l_2}{2} + clearance$$

$$b = 250 \frac{200}{2} + \frac{200}{2} + 20 = 470\ mm$$

and*a*= 0.75 *lc*+ *t* + 0.5 *l*1

= 0.75 × 92 + 70 + 0.5 × 200 = 239 mm

We know that the horizontal reactions *H*1 and *H*2 at bearings 1 and 2, due to the piston gas load (*F*P) are

$$H_1 = \frac{F_P(a+b)}{b} = \frac{103 \times 10^3(239 + 470)}{470} = 155.4 \times 10^3 N$$

$$H_2 = \frac{F_P \times a}{b} = \frac{103 \times 10^3(239)}{470} = 52.4 \times 10^3 N$$

Assuming *b*1 = *b*2 = *b* / 2, the vertical reactions *V*1 and *V*2 at bearings 1 and 2 due to the weight of the flywheel are

$$V_1 = \frac{W.b_1}{b} = \frac{W \times \frac{b}{2}}{b} = \frac{W}{2} = \frac{30 \times 10^3}{2} = 15 \times 10^3 N$$

$$V_2 = \frac{W.b_2}{b} = \frac{W \times \frac{b}{2}}{b} = \frac{W}{2} = \frac{30 \times 10^3}{2} = 15 \times 10^3 N$$

Since there is no belt tension, therefore the horizontal reactions due to the belt tension are neglected.

We know that horizontal bending moment at the flywheel location due to piston gas load.

$$M_1 = F_P(a + b_2) - H_1.b_2$$

$$= 103 \times 10^3 \left(239 + \frac{470}{2}\right) - 155.4 \times 10^3 \times \frac{470}{2}$$

Since there is no belt pull, therefore, there will be no horizontal bending moment due to the belt pull, *i.e.* *M*2 = 0.

∴ Total horizontal bending moment,

Total horizontal bending moment,

*M*H = *M*1 + *M*2 = *M*1 = 12.3 × 106 N-mm

We know that vertical bending moment due to the flywheel weight,

$$M_V = \frac{W.b_1.b_2}{b} = \frac{W \times b \times b}{2 \times 2 \times b} = \frac{W \times b}{4} = \frac{30 \times 10^3 \times 470}{4} = 3.525 \times 10^6 N - mm$$

Resultant bending moment

$$M_R\sqrt{(M_H)^2 + (M_V)^2} = \sqrt{(12.3 \times 10^6)^2 + (3.525 \times 10^6)^2} = 12.8 \times 10^6 N - mm$$

We know that bending momet (M_R)

$$12.8 \times 10^6 = \frac{\pi}{32}(d_s)^3\sigma_b = \frac{\pi}{32}(d_s)^3 60 = 5.9\ (d_s)^3$$

$$(d_s)^3 = \frac{12.8 \times 10^6}{5.9} = 2.17 \times 10^6 or d_s = 129mm$$

Actually d_s should be more than d_1. There fore let us take

d_s =200 mm

2. *Design of crankshaft when the crank is at an angtle of maximum twisting moment*

We know that piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p' = \frac{\pi}{4} \times 250^2 \times 0.9 = 44200\ N$$

In order to find the thrust in the connecting rod (FQ), we should first find out the angle of inclination of the connecting rod with the line of storke (*i.e.* angle φ). We know that

$$\sin\emptyset = \frac{\sin\theta}{l/r} = \frac{\sin 35^o}{4.5} = 0.1275$$

$$\emptyset = \sin^{-1}(0.1275) = 7.32^o$$

We know that thrust in the connecting rod,

$$F_Q = \frac{F_P}{cos\emptyset} = \frac{44200}{\cos 7.32^o} = \frac{44200}{0.9918} = 44565\ N$$

Tangential force acting on the crankshaft,

$$F_T = F_Q(\theta + \emptyset) = 44565\sin(35^0 + 7.32^0) = 30 \times 10^3 N$$

And radial force,

$$F_R = F_Q(\theta + \emptyset) = 44565\cos(35^0 + 7.32^0) = 33 \times 10^3 N$$

$$H_{T1} = \frac{F_T(a+b)}{b} = \frac{30 \times 10^3(239 + 470)}{470} = 45 \times 10^3 N$$

$$H_{T2} = \frac{F_T \times a}{b} = \frac{30 \times 10^3(239)}{470} = 15.3 \times 10^3 N$$

Due to the radial force (FR), there will be two reactions at the bearings 1 and 2, such that

$$H_{R1} = \frac{F_R(a+b)}{b} = \frac{33 \times 10^3(239 + 470)}{470} = 49.8 \times 10^3 N$$

$$H_{R2} = \frac{F_R \times a}{b} = \frac{33 \times 10^3(239)}{470} = 16.8 \times 10^3 N$$

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crank web

We know that bending moment due to the tangential force,

$$M_{bT} = F_T\left(r - \frac{d_1}{2}\right) = 30 \times 10^3\left(150 - \frac{180}{2}\right) = 1.8 \times 10^6 N - mm$$

Bending stress due to tangintial force,

$$\sigma_{bT} = \frac{M_{bT}}{Z} = \frac{6M_{bT}}{t.w^2} = \frac{6 \times 1.8 \times 10^6}{70 \times 245^2} = 2.6\ N/mm^2 orMPa$$

Bending moment due to the radial force,

M_{bR} = FR (0.75 lc+ 0.5 t) = 33 × 10^3 (0.75 × 92 + 0.5 × 70) = 3.43 × 10^6 N-mm

∴ Bending stress due to the radial force,

$$\sigma_{bR} = \frac{M_{bR}}{Z} = \frac{6M_{bR}}{t.w^2} = \frac{6 \times 3.43 \times 10^6}{70 \times 245^2} = 17.1\ \ N/mm^2 orMPa$$

$$\ldots (\because Z = \frac{1}{6} \times t.w^2)$$

We know that direct compressive stress,

$$\sigma_d = \frac{F_R}{w.t} = \frac{33 \times 10^3}{70 \times 245} = 1.9\ \ N/mm^2 orMPa$$

Total compressive strees,

$$\sigma_C = \sigma_{bT} + \sigma_{bR} = 2.6 + 17.1 + 1.9 = 21.6\ MPa$$

We know that twisting moment due to the tangential force,

moment due to the tangential force,

T = F_T (0.75 l_c+ 0.5 t) = 30 × 10^3 (0.75 × 92 + 0.5 × 70) = 3.12 × 10^6 N-mm

Shear stress, $\tau = \frac{T}{Z_P} = \frac{4.5 \times 3.12 \times 10^6}{Z_P} = 11.7 \frac{N}{mm^2} orMPa$

We know that total or maximum stress,

$$\sigma_{max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2} = \frac{21.6}{2} + \frac{1}{2}\sqrt{(21.6)^2 + 4(11.7)^2}$$

$$= 10.8 + 15.9 = 26.7\ MPa$$

Since this stress is less than the permissible value of 60 MPa, therefore, the design is safe

(b) Design of shaft at the junction of crank

Let $ds1$ = Diameter of shaft at the junction of crank.

We know that bending moment at the junction of crank,

M = FQ (0.75lc + t) = 44 565 (0.75 × 92 + 70) = 6.2 × 106 N-mm

and twisting moment, T = FT × r = 30 × 103 × 150 = 4.5 × 106 N-mm

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(6.2 \times 10^6)^2 + (4.5 \times 10^6)^2} = 7.66\ \times 10^6 N - mm$$

We also know that equivalent twisting moment (T_e),

$$7.66\ \times 10^6 = \frac{\pi}{32}(d_{s1})^3\tau = \frac{\pi}{32}(180)^3\tau = 1.14 \times 10^6\tau$$

$$\tau = 7.66\ \times 10^6/1.14 \times 10^6 = 6.72\ N/mm^2 or MPa$$

Since the induced shear stress is less than the permissible limit of 30 to 40 MPa, therefore, the design is safe.

(*c*) *Design of shaft under the flywheel*

Let ds = Diameter of shaft under the flywheel.

We know that horizontal bending moment acting on the shaft due to piston gas load,

$$M_H = F_P(a + b) - \left[\sqrt{(H_{R1})^2 + (H_{T1})^2}\right] b_2$$

$$= 44200\left(239 + \frac{470}{2}\right) - \left[\sqrt{(49.8 \times 10^3)^2 + (45 \times 10^3)^2}\right]\frac{470}{2}$$

$$= 20.95 \times 10^6 - 15.77 \times 10^6 = 5.18 \times 10^6 N - mm$$

And bendin moment due to the flywheel wight

$$M_V = \frac{W.b_1.b_2}{b} = \frac{30 \times 10^3 \times 235 \times 235}{470} = 3.53 \times 10^6 N - mm$$

b_1=b_2=b/2=470/2 = 235 mm

Resultant bending moment

$$M_R\sqrt{(M_H)^2 + (M_V)^2} = \sqrt{(5.18 \times 10^6)^2 + (3.525 \times 10^6)^2} = 6.27 \times 10^6 N - mm$$

We know that twistin moment on the shaft,

$$T = F_T \times r = 30 \times 10^3 \times 150 = 145 \times 10^6 N - mm$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{M_R^2 + T^2} = \sqrt{(6.2 \times 10^6)^2 + (4.5 \times 10^6)^2} = 7.72 \times 10^6 N - mm$$

We also know that equivalent twisting moment (T_e),.

$$7.72 \times 10^6 = \frac{\pi}{16}(d_s)^3\tau = \frac{\pi}{16}(d_s)^3 30 = 5.9\, d_s{}^3$$

$$(d_s)^3 = \frac{7.72 \times 10^6}{5.9} = 1.31 \times 10^6 or d_s = 109\ mm$$

.

Actually, *ds* should be more than *d*1. Therefore let us take *ds* = 200 mm **Ans.**

30.A cast iron flywheel at 500 rpm is to furnish 100 kNcm energy during 0.1 revolutions. The total fluctuation of speed is 10%. Design the flywheel and find the power rating of the motor to drive th machine.(16)

Given :
Speed, N= 500 rpm
Energy, E=100 x 10^3Ncm = 10000 N-m

$$\frac{\mathrm{N_1 - N_2}}{\mathrm{N_1}} = 0.1 = \mathrm{C_s}(\mathrm{co-efficientoffluctuationofspeed})$$

K_s =0.1

To find:

1. Design the flywheel
2. Power rating of the motor

Solution:

Power rating of the motor:
We know that the energy developed per revolution

$$\mathrm{E} = \frac{\mathrm{P \times 60}}{\mathrm{N}}$$

Energy during one revolution is P = 83333.3 W
We know that the maximum fluctuation of energy

$$\Delta \mathrm{E} = 2 \times \mathrm{E} \times \mathrm{K_s} = 2 \times 10000 \times 0.1 = 20000\ \mathrm{N-m}$$

For cast iron flywheel the limited speed is 25 m/s

$$\mathrm{V} = \frac{\pi \mathrm{DN}}{60}$$
$$25 = \frac{\pi \times \mathrm{D} \times 500}{60}$$
$$\mathrm{D} = 0.955\ \mathrm{m}$$

Mass of flywheel

Energy stored by the flywheel per 0.1 revolution = E

$$E = \frac{1}{2} mv^2$$

$$\frac{1000}{0.1} = \frac{1}{2} \times m \times 25^2$$

m=32 kg

Mass of the flywheel is 32 kg

We know that the mean torque transmitted by the shaft,

$$(M_t)_{mean} = \frac{P \times 60}{N} = \frac{83 \times 10^3 \times 60}{2\pi \times 500} = 1585.1 \text{ N} - \text{m}$$

Assuming that the maximum torque transmitted $(M_t)_{max}$ by the shaft is twice the mean torque,

$$(M_t)_{max} = 2 \times (M_t)_{mean} = 3170.36 \text{ N} - \text{m}$$

We know that the maximum torque transmitted by the shaft

$$(M_t)_{max} = \frac{\pi}{16} \times \tau \times d_1^3$$

$$3170.36 \times 10^3 = \frac{\pi}{16} \times 40 \times d_1^3$$

$$\mathbf{d_1 = 73.90\ mm}$$

Round value d_1 = 75 mm

The diameter of the hub d is made equal to the twice diameter of the shaft d_1 and length of the hub l is equal to th width of the rim b.

3. Cross sectional dimensions ofthe flywheel rim:

h=thickness of flywheel rim in meters

b= width of the flywheel rim in meters

b=2h

Cross sectional area of rim,

A= bxh = 2h xh = $2h^2$

The mass of the flywheel rim(m)

$$m = A \times \pi D \times \rho = 2h^2 \times \pi \times 0.955 \times 7200$$

h = 0.272 m= 27.2 mm

$$b = 2h = 2 \times 0.0272 = 0.0544$$

b =54.4 mm

4. Diameter and length of hub:

d=diameter of the hub

d_1 = diameter of shaft and

l = length of the hub

d=2d_1 = 2x75 = 150 mm

l=b= 54.4 mm

Cross sectional dimensions of th elliptical arms:

a-minor axis

c-minor axis

Let us assume c=0.5 a

n= number of arms 6.

Let us assume n=6

$$\sigma_b = \text{Bendingstressforthematerialofthearm}$$

$$\sigma_b = 20\ \text{MPa} = 20\ \text{N/mm}^2$$

We know that the maximum bending moment in the arm at the hub (which is assumed as cantilever) is given by

$$M = \frac{M_t}{D \times n}(D - d) = \frac{1585.1}{0.955 \times 6}(0.955 - 0.15)$$

$$M = 236.52 \times 10^3 \text{N} - \text{mm}$$

And the section modulus for the cross section of the arm

$$Z = \frac{\pi}{32} \times ca^2 = \frac{\pi}{32} \times a^2 \times 0.5a = 0.05\ a^3$$

We know that

$$\sigma_b = \frac{M}{Z} \quad 20 = \frac{M}{Z} \quad 20 = \frac{236.52 \times 10^3}{0.05a^3}$$

Major axis a=61.84 mm

Minor axis c = 30.92 mm

5. Dimensions of key:

The standard dimensions of key for a shaft of 75 mm diameter are as follows.

Width of key, w=22mm

Thickness of key, t= 14mm

The length of key (L) is obtained by considering the failure of key in shearing

We know that the maximum torque transitted by the shaft (M_{tmax})

$$(M_t)_{max} = L_k \times w \times \tau \times \frac{d_1}{2}$$

$$3170.36 \times 10^3 = L_k \times 22 \times 40 \times 37.5$$

$$L_k = 96.37\ \text{mm}$$

Say length of the key, $L_k = 100\text{mm}$

Let us now check the total stress in the rim, which should not be greater than 20 MPa. We know that the total stress in the rim.

$$= \rho V^2 \left[0.75 + \frac{4.935R}{n^2 h}\right] = 7200 \times 25^2 \left[75 + \frac{4.935 \times 0.4775}{6^2 0.0272}\right]$$

$$= 14.2 \times 10^6 \mathrm{Pa} = 14.2\ \mathrm{MPa}$$

$$\text{Inducedstressislessthan } 20\ \text{MPa. Therefore, thedesignissafe}$$

31. Determine the dimensions of cross section of the connecting rod for a diesel engine with the following data. Cylinder bore = 100 mm, Length of connecting rod = 350 mm, Maximum gas pressure = 4 Mpa and factor of safety =2. (April/May 2019) Nov/Dec-20, Aprl/May-21

Solution

Given $D = 100$ mm $\quad p_{max.} = 4$ MPa $= 4$ N/mm^2

$L = 350$ mm $\quad (fs) = 6$

Step I *Force acting on the connecting rod*

$$P_c = \left(\frac{\pi D^2}{4}\right) p_{max.} = \left(\frac{\pi (100)^2}{4}\right)(4) = 31415.93 \text{ N}$$

Step II *Critical buckling load*

$$P_{cr} = P_c\,(fs) = 31\,415.93\,(6) = 188\,495.58 \text{ N}$$

Step III *Calculation of t*

Substituting,

$$A = 11t^2 \quad k_{xx} = 1.78t \quad a = \frac{1}{7500}$$

$$\sigma_c = 330 \text{ N/mm}^2$$

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\frac{L}{k_{xx}}\right)^2}$$

or $$188\,495.58 = \frac{(330)(11t^2)}{1 + \frac{1}{7500}\left(\frac{350}{1.78t}\right)^2}$$

$$\frac{188\,495.58}{(330)(11)} = \frac{t^2}{1 + \frac{5.16}{t^2}} \quad \text{or} \quad 51.93 = \frac{t^4}{t^2 + 5.16}$$

$t^4 - 51.93t^2 - 267.96 = 0$

The above expression is a quadratic equation in (t^2).

$$t^2 = \frac{51.93 \pm \sqrt{(51.93)^2 + 4(267.96)}}{2}$$

$$= \frac{51.93 \pm 61.39}{2}$$

$$t^2 = 56.66$$

$$t = 7.53 \text{ or } 8 \text{ mm}$$

Step IV *Dimensions of cross-section*

$B = 4t = 4(8) = 32$ mm

$H = 5t = 5(8) = 40$ mm

Thickness of web $= t = 8$ mm

Thickness of flanges $= t = 8$ mm

The width (B = 32 mm) is kept constant throughout the length of connecting rod.

Step V *Variation of height*

at the middle section, $H = 5t = 40$ mm

at the small end, $H_1 = 0.85\ H = 0.85\ (40) = 34$ mm

at the big end, $H_2 = 1.2\ H = 1.2\ (40) = 48$ mm

dimensions (B/H) of section at big end $= 32 \text{ mm} \times 48 \text{ mm}$

dimensions (B/H) of section at middle $= 32 \text{ mm} \times 40 \text{ mm}$

dimensions (B/H) of section at small end $= 32 \text{ mm} \times 34 \text{ mm}$

32. A semi-elliptic leaf spring used for automobile suspension consists of three extra full-length leaves and 15 graduated-length leaves, including the master leaf. The centre-to-centre distance between two eyes of the spring is 1 m. The maximum force that can act on the spring is 75 kN. For each leaf, the ratio of width to thickness is 9 : 1. The modulus of elasticity of the leaf material is 207 GPa. The leaves are pre-stressed in such a way that when the force is maximum, the stresses induced in all leaves are same and equal to 450 MPa. Determine (i) the width and thickness of the leave : (ii) the initial nip and (iii) the initial pre-load required to close the gap Cbetween extra full-length leaves and graduated length leaves.. Nov/Dec-20, Aprl/May-21

Solution

Given $2P = 75$ kN $\quad 2L = 1$ m $\quad b = 9t \quad n_f = 3$

$n_g = 15 \quad E = 207\,000$ N/mm^2 $\quad \sigma_b = 450$ N/mm^2

Step I *Width and thickness of the leaves*

$2P = 75$ kN $\quad$ or $\quad P = 37\,500$ N

$2L = 1$ m $\quad$ or $\quad L = 500$ mm

$b = 9t \quad\quad n = n_f + n_g = 3 + 15 = 18$

From Eq. (10.41),

$$\sigma_b = \frac{6PL}{nbt^2} \quad \text{or} \quad (450) = \frac{6(37\,500)(500)}{(15+3)(9t)t^2}$$

$$\therefore \quad t = 11.56 \text{ or } 12 \text{ mm}$$

$$b = 9\,t = 9\,(12) = 108 \text{ mm} \qquad \text{(i)}$$

Step II *Initial nip*

From Eq. (10.39),

$$C = \frac{2PL^3}{Enbt^3} = \frac{2(37\,500)(500)^3}{(207\,000)(18)(108)(12)^3}$$

$$= 13.48 \text{ mm} \qquad \text{(ii)}$$

Step III *Initial pre-load*

From Eq. (10.40),

$$P_i = \frac{2n_g n_f P}{n(3n_f + 2n_g)} = \frac{2(15)(3)(37\,500)}{18(3 \times 3 + 2 \times 15)}$$

$$= 4807.69 \text{ N} \qquad \text{(iii)}$$

32.a) A Locomotive semi elliptical spring has overall length of 1m and sustains a load of 80KN at this center. The spring has 3 full length leaves and 15 graduated leaves with a central band of 100mm width. All the leaves are to be stressed to 400N/mm2 when fully loaded. The ratio of the total spring depth to that of width is 2.let E=0.2*10^{6}N/mm^2(A/M'2023)

Determine:

i)the thickness and width of leaves

ii) the initial gap that should be provided between the full length and graduated leaves before the band load is applied

iii)the load exerted on the band after the spring is assembled.

FOLLOW THE SAME FOR ABOVE PROBLEM PROCEDURE

33.An Automative Single plate-clutch,with two pairs of friction surfaces,tramits surfaces,transmits a 300N-m torque at 1500 rpm. The inner and outer diameter of the friction disk are 170 and 270 respectively.the coeeficient of friction is 0.35.the normal force on the friction surfaces is exerted by nine helical-compression spring, so that the clutch is always engaged. The clutch is disengaed when the external force further compresses the spring. The spring index is 5 and the number of active coils are 6. The springs are made of patemted and cold-dawin steel wire is of grade 2.(G=81370N/mm2). The permissible shear stress for the spring wire is 30% of the ultimate tensile strength. Design the springs and specify the their dimensions.(N/D'2022)

Solution There are two pairs of contacting surfaces and the torque transmitted by each pair is (300/2) or 150 N-m. Assuming uniform-wear theory (Chapter 11), the total normal force P_t required to transmit the torque is given by Eq. (11.8), i.e.,

$$P_t = \frac{4(M_t)_f}{\mu(D+d)} = \frac{4(150 \times 10^3)}{0.35(270+170)} = 3896.1 \text{ N}$$

Since there are nine springs, the force exerted by each spring is

$$P = \frac{3896.1}{9} = 432.9 \text{ N}$$

$$K_s = \left(1 + \frac{0.5}{C}\right) = \left(1 + \frac{0.5}{5}\right) = 1.1$$

From Eq. (10.13),

$$\tau = K_s\left(\frac{8PC}{\pi d^2}\right) = (1.1)\left(\frac{8(432.9)(5)}{\pi d^2}\right)$$

or

$$\tau = \frac{6063.04}{d^2}\ \text{N/mm}^2 \tag{a}$$

The permissible shear stress τ_d is given by

$$\tau_d = 0.3S_{ut} \tag{b}$$

Equations (a) and (b) are solved by the trial and error method.

Trial 1

$$d = 3\ \text{mm}$$

$\therefore$

$$\tau = \frac{6063.4}{d^2} = \frac{6063.04}{(3)^2} = 673.67\ \text{N/mm}^2$$

From Table 10.2,

$$S_{ut} = 1570\ \text{N/mm}^2$$

$\therefore$

$$\tau_d = 0.3S_{ut} = 0.3(1570) = 471\ \text{N/mm}^2$$

Therefore,

$$\tau > \tau_d$$

Trial 2

$$d = 3.6\ \text{mm}$$

$$\tau = \frac{6063.04}{d^2} = \frac{6063.04}{(3.6)^2} = 467.83\ \text{N/mm}^2$$

From Table 10.2,

$$S_{ut} = 1510\ \text{N/mm}^2$$

$\therefore$

$$\tau_d = 0.3S_{ut} = 0.3(1510) = 453\ \text{N/mm}^2$$

Therefore,

$$\tau > \tau_d$$

Trial 3

$$d = 4\ \text{mm}$$

$$\tau = \frac{6063.04}{d^2} = \frac{6063.04}{(4)^2} = 378.94\ \text{N/mm}^2$$

From Table 10.2,

$$S_{ut} = 1480\ \text{N/mm}^2$$

$\therefore$

$$\tau_d = 0.3_{ut} = 0.3(1480) = 444\ \text{N/mm}^2$$

Therefore,

$$\tau < \tau_d$$

The design is satisfactory and the wire diameter should be 4 mm.

$$D = Cd = 5(4) = 20\ \text{mm}$$

It is assumed that the springs have square and ground ends.

$$N_t = N + 2 = 6 + 2 = 8$$

From Eq. (10.7),

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(432.9)(20)^3(6)}{(81370)(4)^4} = 7.98 \text{ mm}$$

This is the initial compression of the spring. The spring is further compressed during the disengagement of the clutch, which requires some margin for compression.

$$\begin{aligned} \text{free length} &= \text{solid length} + \delta + \text{margin} \\ &= 4(8) + 7.98 + \text{margin} \\ &\simeq 45 \text{ mm} \end{aligned}$$

33.A rimmed flywheel made of grey cast iron FG200 whose density is 7100 kg/m^3 is required to keep down fluctuations in speed from 200 to 220 rpm. The cyclic fluctuations in energy is 30,000 N-m, while the maximum torque during the cycle is 75,000 N-m. The outside diameter of the flywheel should not exceed 2 m. It can be assumed that there are six spokes and the rim contributes 90% of the required moment of inertia. The cross-section of the rim is rectangular and the ratio of width to thickness is 2. Determine the dimensions of the rim. Assuming suitable cross-section for spokes, calculate the stresses in the rim and spokes. Nov/Dec-20, Aprl/May-21

Solution

Given $n = 200$ to 220 rpm $\quad b/t = 2 \quad K = 0.9$

$\rho = 7100$ kg/m^3 $\quad U_o = 30\,000$ N-m

outer diameter < 2 m $\quad$ number of spokes $= 6$

Step I *Dimensions of rim*

The average speed of the flywheel is 210 rpm. Therefore,

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(210)}{60} = 21.99 \text{ rad/s}$$

$$C_s = \frac{\omega_{max.} - \omega_{min.}}{\omega} = \frac{n_{max.} - n_{min.}}{n}$$

$$= \frac{220 - 200}{210} = 0.095$$

From Eq. 21.14,

$$I_r = \frac{U_o K}{\omega^2 C_s} = \frac{(30\,000)(0.9)}{(21.99)^2 (0.095)} = 587.75\,\text{kg-m}^2$$

Since the outer diameter of flywheel is limited to 2 m, the mean radius of the rim (R) is assumed as 0.9 m.

The mass of the rim is given by Eq. (21.15)

$$m_r = \frac{I_r}{R^2} = \frac{587.75}{(0.9)^2} = 725.61\,\text{kg}$$

The mass of the flywheel rim is also given by,

$$m_r = 2\pi R \left(\frac{b}{1000}\right)\left(\frac{t}{1000}\right)\rho$$

or $$725.61 = 2\pi(0.9)\left(\frac{2t}{1000}\right)\left(\frac{t}{1000}\right)(7100)$$

$t = 95.06$ mm or 100 mm $b = 2(100) = 200$ mm

The cross-section of the rim is 200×100 mm.

Step II *Stresses in rim*

It is assumed that the spokes have elliptical cross-section with 200 mm as the major axis and 100 mm as the minor axis. The cross-sectional area (A_1) of the spokes is given by,

$$A_1 = \pi ab$$

where a and b are semi-major and semi-minor axes respectively.

$$A_1 = \pi(100)(50) = 15\,707.96 \text{ mm}^2$$

The cross-sectional area (A) of the rim is given by,

$$A = (200)(100) = 20\,000 \text{ mm}^2$$

$$\frac{A}{A_1} = \frac{20\,000}{15\,707.96} = 1.27$$

From Eq. 21.20,

$$C = \left[\frac{20\,280\,R^2}{t^2} + 0.957 + \frac{A}{A_1}\right]$$

$$= \left[\frac{20\,280\,(0.9)^2}{(100)^2} + 0.957 + 1.27\right] = 3.87$$

$$v = \omega R = 21.99(0.9) = 19.79 \text{ m/s}$$

The mass of the rim per millimetre of the circumference is given by,

$$m = bt\rho = \left(\frac{200}{1000}\right)\left(\frac{100}{1000}\right)(7100)(10^{-3})$$

$$= 0.142 \text{ kg/mm}$$

$$\left(\frac{1000mv^2}{bt}\right) = \frac{(1000)(0.142)(19.79)^2}{(200)(100)} = 2.78$$

There are six spokes and (2α) is the angle between two consecutive spokes.

$$(2\alpha) = 360/6 = 60° \quad \therefore \ \alpha = 30° = \left(\frac{\pi}{6}\right) \text{rad.}$$

From Eq. 21.18, the stresses in the rim are given by

At $\phi = 30°$,

$$\sigma_t = \frac{(1000)mv^2}{bt}\left[1 - \frac{\cos\phi}{3C\sin\alpha} \pm \frac{2(1000)R}{Ct}\left(\frac{1}{\alpha} - \frac{\cos\phi}{\sin\alpha}\right)\right]$$

$$= (2.78)\left[1 - \frac{\cos(30)}{3(3.87)\sin(30)} \pm \frac{2(1000)(0.9)}{(3.87)(100)}\left(\frac{6}{\pi} - \frac{\cos(30)}{\sin(30)}\right)\right]$$

$= 4.66$ N/mm^2 (using the positive sign)

At $\phi = 0°$,

$$\sigma_t = (2.78)\left[1 - \frac{\cos(0)}{3(3.87)\sin(30)} \pm \frac{2(1000)(0.9)}{(3.87)(100)}\left(\frac{6}{\pi} - \frac{\cos(0)}{\sin(30)}\right)\right]$$

$= 1.14$ N/mm^2 (using the negative sign)

Step III *Stresses in spoke*

From Eq. 21.17, the stress in the spokes is given by.

$$\sigma_t = \frac{2}{3}\left[\frac{(1000)mv^2}{CA_1}\right] = \frac{2}{3}\left[\frac{(1000)(0.142)(19.79)^2}{(3.87)(15\,707.96)}\right]$$

$= 0.61$ N/mm^2

34.The torque developed by an engine is given by the equation(N/D'2022)

$$T = 14\,250 + 2200\sin 2\theta - 1800\cos 2\theta$$

where T is the torque in N-m and θ is the crank angle displacement from inner dead centre position. The resisting torque of the machine is constant throughout the work cycle. The coefficient of speed fluctuations is 0.01. The engine speed is 150 r.p.m. A solid circular steel disk, 50 mm thick, is used as a flywheel. The mass density of steel is 7800 kg/m^3. Calculate the diameter of the flywheel disk.

Solution The fluctuating terms sin 2θ and cos 2θ have a zero mean. The mean torque is, therefore, given by

$$T_m = 14250 \text{ N-m}$$

When $T = T_m$,

$$2200\sin 2\theta - 1800\cos 2\theta = 0$$

$$\tan 2\theta = \frac{1800}{2200}$$

or $$2\theta = 39.29° \quad \text{or} \quad 180° + 39.29°$$

Therefore,

$$\theta = 19.645° \quad \text{or} \quad 109.645°$$

The turning moment diagram is shown in Fig. 4.21. The maximum and minimum angular velocities will occur at points A and B respectively.

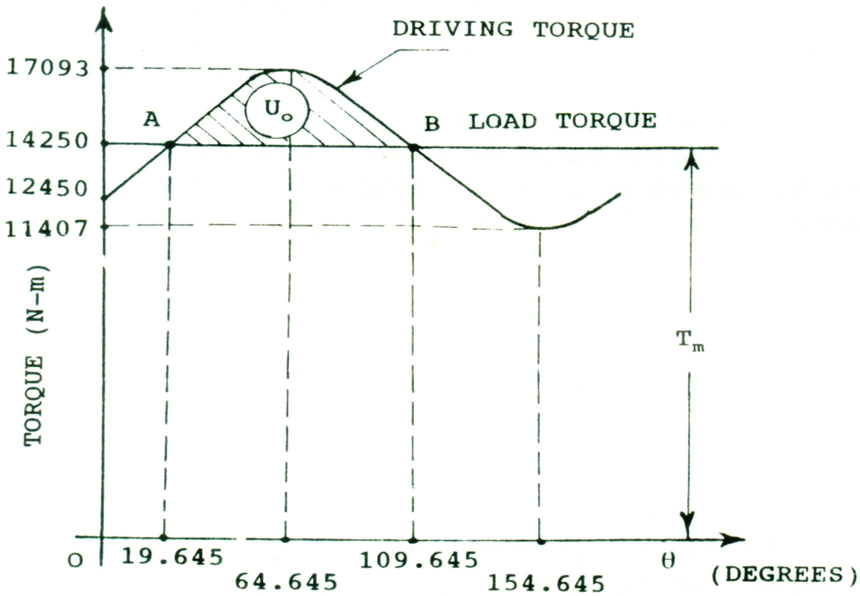


Fig. 4.21 Turning moment diagram

Therefore,

$$U_o = \int_A^B (T - T_m)\, d\theta$$

$$= \int_{19.645}^{109.645} (2200 \sin 2\theta - 1800 \cos 2\theta)\, d\theta$$

$$= \Big[-1100 \cos 2\theta - 900 \sin 2\theta\Big]_{19.645}^{109.645}$$

$$= -1100\,(-0.774 - 0.774) - 900\,(-0.633 - 0.633)$$

$$= 2842.2 \text{ N-m} \quad \text{(or J)}$$

From Eq. 21.4.

$$I = \frac{U_o}{\omega^2 C_s} = \frac{2842.2}{\left(\frac{2\pi \times 150}{60}\right)^2 (0.01)} = 1151.9 \text{ kg-m}^2$$

From Eq. 21.5,

$$R^4 = \frac{2I}{\pi\rho t} = \frac{2(1151.9)}{\pi(7800)\left(\frac{50}{1000}\right)}$$

or $$R = 1.171 \text{ m} = 1171 \text{ mm} \quad \text{or} \quad 1175 \text{ mm approx.}$$

DEPARTMENT OF MECHANICAL ENGINEERING

Subject Title: DESIGN OF MACHINE ELEMENTS
Subject Code: ME 3591
Year/ SEM : III / V

UNIT V – DESIGN OF BEARING AND MISCELLANEOUS ELEMENTS

SYLLABUS

Sliding contact and rolling contact bearings - Hydrodynamic journal bearings, Somerfield Number, Raimondi and Boyd graphs, -- Selection of Rolling Contact bearings.

SUMMARY

BEARING

A bearing is a machine element it permits a relative motion between the contact surfaces of the Members, while carrying the load

The bearings under this group are classified as:

(a) Sliding contact bearings (b) Rolling contact bearings.

- In sliding contact bearings the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as plain bearings.

- In rolling contact bearings the steel balls or rollers, are interposed between the moving and fixed elements. The balls offer rolling friction at two points for each ball or roller.

Depending upon the direction of load to be supported the bearings under this group Are Classified as:

(a) Radial bearings, and (b) Thrust bearings.

- In radial bearings, the load acts perpendicular to the direction of motion of the Moving element
- In thrust bearings, the load acts along the axis of rotation

SLIDING CONTACT BEARINGS

A bearing is a machine element which supports another moving machine element (known as journal). It permits a relative motion between the contact surfaces of the members, while carrying the load. A little consideration will show that due to the relative motion between the contact surfaces, a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are in direct contact, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid (known as lubricant) may be provided. The lubricant used to separate the journal and bearing is usually a mineral oil refined from petroleum, but vegetable oils, silicon oils, greases etc., may be used.

ROLLING CONTACT BEARINGS

In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings. We have already discussed that the ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction. It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction. Due to this low friction offered by rolling contact bearings, these are called antifriction bearings. Following are the two types of rolling contact bearings:Ball bearings; and Roller bearings.

PART – A (Two marks)

Introduction of bearings

1. What is bearing?

Bearing is a stationery machine element which supports a rotating shafts or axles and confines its motion.

2. Classify the types of bearings. (Nov/Dec 16)

i. Depending on the type of load coming upon the shaft:
a. Radial bearing
b. Thrust bearings.
ii. Depending upon the nature of contact:
a. Sliding contact
b. Rolling contact bearings or Antifriction bearings.

3. What are the required properties of bearing materials?

Bearing material should have the following properties.

i. High compressive strength
ii. Low coefficient of friction
iii. High thermal conductivity
iv. High resistance to corrosion
v. Sufficient fatigue strength
vi. It should be soft with a low modulus of elasticity
vii. Bearing materials should not get weld easily to the journal material.

4. What are the types of thrust ball bearings?

One directional flat race, one directional grooved race, two directional grooved race

5. What is the advantage of Teflon which is used for bearings?

It has low coefficient of friction; it can be used at higher temperature, and chemically inert.

6. State any points to be considered for selection of bearings.(or) List any six types of bearing materials.

Lead based babbit, tin based babbit, leaded bronze, copper lead alloy, gun metal, phosphor bronze.

7. What is known as self – acting bearing? (A/M'2023)

The pressure is created within the system due to rotation of the shaft, this type of bearing is known as self – acting bearing

8. What type of bearings can take axial load?(Nov/Dec 2017)

These two types of bearing can take both axial and radial load: Deep groove ball bearing and Taper roller bearing.

Sliding contact bearings

9. What is a journal bearing? What are the types of journal bearings depending onthe nature of contact?

A journal bearing is a sliding contact bearing which gives lateral support to the rotating shaft.

1. Full journal bearing
2. Partial bearing
3. Fitted bearing.

10. What are the types of journal bearing depending upon the nature of lubrication?

1. Thick film type
2. Thin film type
3. Hydrostatic bearings
4. Hydrodynamic bearing.

11. What is a Journal bearing? List any two applications. (May/June 2013)

A journal bearing is a sliding contact bearing which gives lateral support to the rotating shaft.

12. What is mean by square journal bearing? (NOV/DEC 2015)

When the length of the journal (l) is equal to the diameter ofthe journal (d), then the bearing is called square bearing.

13. What is Somerfield number? State it's important in the design of journal bearing? [Apr/May 2015]

The Somerfield number is a very useful non-dimensional number in the design of journal bearing. It is given by the expression.

$$S=\frac{Z^{F}n'}{p}\left(\frac{D}{C}\right)^{2}$$

Where,

Z' – absolute viscosity in N-s/m^2

n' – revolution per second

p – bearing pressure in N/m^2

14. In hydro dynamic bearing, what are factors which influence the formation of wedge fluid film? [Nov-Dec 2014]

1. Wedge clearance,
2. oil with significance viscosity

Significant relative velocity between mating elements

15. State the essential requirements to develop thin film for hydro dynamic action.(Nov/Dec 2021)

(Or)

List the basic assumption used in the theory of hydrodynamic lubrication (Nov/Dec 2011)

- The lubricant obeys Newton's law of viscous flow.
- The pressure is assumed to be constant throughout the film thickness.
- The lubricant is assumed to be incompressible.
- The viscosity is assumed to be constant throughout the film thickness.
- The flow is one dimensional.

15. (a) What tis Bearing Modulus? (Nov/Dec 2021)

Bearing Modulus (C) is C = (Zn/p) where

Z = oil viscosity

n = speed of rotation (rpm)

p = bearing pressure (N/MM2)

For any bearing, there is a value for indicated by C, for which the coefficient of friction is at a minimum.

The bearing should not be operated at this value of bearing modulus,

16. Classify the sliding contact bearings according to the thickness of layer of the lubricant between the bearing and journal. (May/ June 2012)

1. Thick film bearing 2. Thin film bearing 3. Zero film bearing 4. Hydrostatic bearing

17. List the advantages of hydrostatic bearings. (Nov/Dec 2017)

1. Very low friction (hydrodynamic means that there is a full film of oil between the bearing and race components)
2. Lower wear and longer life than standard bearings (no metal-metal contact within the wearing portions of the bearing)

3. Should run cooler since there is less friction and mainly viscous loss to the oil

Rolling contact bearings

18. Define anti friction bearing.

The contact between the bearing elements is rolling; this type has very small friction.

19. What is meant by life of anti-friction bearings? (Nov/Dec 2013) (A/M'2023)

For an individual rolling bearing, the number of revolutions which one of the bearing rings makes in relation to the other rings under the prevailing working conditions before the first evidence of fatigue develops in the material of one of the rings or rolling elements.

20.Why Rolling Contact Bearing Are Termed as Antifriction Bearing? (N/D'2022)

Due to large contact area friction between mating parts is high requiring greater lubrication. friction is much lesser than the sliding friction, hence these bearings are also known as antifriction bearing. due to low rolling friction these bearings are aptly called "antifriction" bearing.

21.A rolling contact bearing with number 6208 is chosen for assembly of gear box in relation with the bearing number 6208 provide the physical meaning of the digits from the right side end.(NOV/DEC'2022)

For 6208 number bearing the following physical meaning

Basic load rating cr=2910

Cor=17900

Limiting speeds

Open z=1000(1/min)

20. Give an example for anti-friction bearing. (NOV/DEC 2015)

Deep groove ball bearing, cylindrical roller, angular contact, taper roller, spherical roller, thrust ball bearing.

21. What is self aligning ball bearing? State its unique feature? [Apr/May 2015]

Self aligning ball bearings are used where a misalignment between the axes of shaft may be present. These bearing permit shaft deflections within 2^0 to 3^0.They are available in two types:
Externally self aligning bearings, and
Internally self aligning bearings.

22. What are various types of radial ball bearing? (May/ June 2012)

1. Single row deep groove ball bearing 2.Filling notch bearing 3. Angular contactbearing 4.Double row bearing 5. Self-aligning bearing

23. State the components of rolling contact bearing.

Outer race, inner race, rolling element, retaining cage.

24. Classify the roller bearings.

Depending on the type of rolling element:-ball bearing, roller bearingDependingon the load to be carried, radial, angular, and thrust bearings.

25. What is load rating?

The load carrying capacity of a rolling element bearing is called load rating.

26. Define dynamic load rating (Nov/Dec 2021) (or) Explain the term Dynamic load carrying capacities of rolling contact bearing.(Nov/Dec 2012)

Dynamic load rating is defined as the radial load in radial bearings that can be carried for a minimum life of one million revolutions.

27. List any four advantages of rolling contact bearings over sliding contact bearings.

Low starting torque can carry combined radial and axial torque. Required less axial space, maintain accurate alignment of shaft.

28. State the disadvantages of thrust ball bearing.

They are not suitable for high speeds; thrust loads try to shift the plane of rotation of balls.

29. What do you meant by life of an individual bearing? (May/ June 2013)

The life of individual bearing may be defined as the number of revolution which the bearing runs before the first evidence of fatigue develops in the material of one of the rings or any of the rolling elements.

30. What is a quill bearing?

Quill bearing are characterized by cylindrical rollers of very small diameter and relatively long. They are also called needle bearings.

31. What is meant by static load carrying capacity of a bearing?. [Nov-Dec 2014]Nov/Dec-20,Aprl/May-21

It is defined as that static radial load which is applied would cause the same total permanent deformation at the most heavily stressed ball and race contact which occurs under the actual condition of loading.

32. What is meant by hydrodynamic lubrication? (MAY/JUNE 2016)

The thick film bearings are those in which the working surfaces arecompletely separated from each other by the lubricant. Such types of bearings are also called as hydrodynamic lubricated bearings.

33. Define the term reliability of a bearing. (Nov/Dec 16)

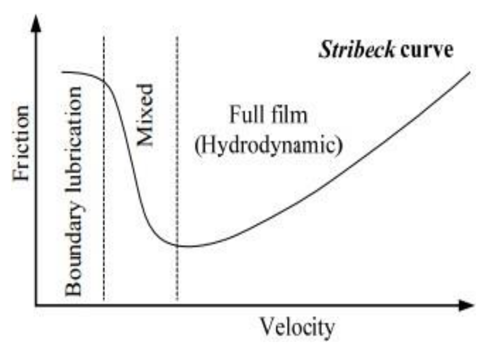
Reliability is define as the ability of an item to perform a required function under stated conditions for a stated period of time. Reliability is expressed quantitatively such as 0.9 or 90%.

34. What are antifriction bearings? (April/May 17)

Anti-friction bearing friction minimum to compare the sliding contact bearing. There are two types

- Roller bearing
- ball bearing

35. Plot the friction induced in various bearings based on shaft speed (April/May 17)



36. What are the essential conditions for wedge film formation in Hydro dynamic bearing? (April/May 2018)

- Viscosity and lubricant oil or working fluid.
- Relative velocity between contact members.
- Sufficient clearance between contact members.

37. Define load factor and explain its significance in related to bearing selection(April/May 2018)

To take into factor of dynamic loading due to factors like unbalance in masses etc.

Load factor>1

Design load = Load factor * actual load.

Life of the components increased due to safer design.

38. Differentiate sliding contact and rolling contact bearing (April/May 2019)

Sliding contact bearing	Rolling contact bearing
Friction is great.	Friction coefficient is lower.
Difficult to achieve sub	Easy to sub
Difficult to estimate life	Easy to estimate life
Difficult to estimate rigidity	Easy to estimate rigidity
Unsuitable for extremely low and high speed	Wide range of use from low to high speed

39. Define life of Bearing (April/May 2019)

Bearing life is usually expressed as the number of hours an individual bearing will operate before the first evidence of metal fatigue develops. The usual life rating for industrial applications is called "L-10" life. Simply put the L-10 life is the number of hours in service that 90% of bearings will survive.

40. List the advantage and disadvantage of hydrostatic bearing (Nov/Dec 2018)

Advantages	Disadvantages
Support very large loads. The load support is a function of the pressure drop across the bearing and the area of fluid pressure action. Load does not depend on film thickness or lubricant viscosity. Long life (infinite in theory) without wear of surfaces. Provide stiffness and damping coefficients of very large magnitude. Excellent for exact positioning and control.	Require ancillary equipment. Larger installation and maintenance costs. Need of fluid filtration equipment. Loss of performance with fluid contamination. High power consumption because of pumping losses. Potential to induce hydrodynamic instability in hybrid mode operation. Potential to show pneumatic hammer instability for highly compressible fluids, i.e. loss of damping at low and high frequencies of operation due to compliance and time lag of trapped fluid volumes.

41.Classify the bearings depending upon type of rolling element. Nov/Dec-20,Aprl/May-21

Rolling elements are classified into two types: balls and rollers. Rollers come in four types: cylindrical, needle, tapered, and spherical. ... Theoretically, rolling bearings are constructed to allow the rolling elements to rotate orbitally while also rotating on their own axes at the same time**.**

***PART- B*(16 marks)**

0. Discusses about lubrication of ball/roller bearing.(Nov/Dec2021)

- The selection of the lubrication method and the lubricant itself must be an essential part of the dimensioning, to ensure a failure-free operation.
- A statistical assessment of all roller bearing failures finds the highest failure rate of over 80% is attributed to the lubrication.
- The lubricant usually is contaminated, inappropriate or obsolete.
  - ✓ The main task of the lubrication is to reduce or prevent the metallic contact of the roll and gliding surface, and
  - ✓ Therefore generate a friction minimization and reduction wear in the roller bearing.
  - ✓ Besides the separation of the metallic surface, the lubrication has further functions:
  - ✓ Corrosion prevention. Heat dissipation (oil lubrications)
  - ✓ Contamination removal from the bearing inside (oil lubrication)
  - ✓ Sealing the bearing inside (grease collar and oil-air-lubrication)

- The actual lubricant for a rolling bearing is oil, which can be a mineral oil, fully synthetic or a blend of the two. Different types of additives are added to these oils to influence the corrosion resistance properties and/or build layers that protect the metal surface under extreme conditions.
- There are three methods of lubricating systems: manual greasing using a grease gun or manual pump, forced greasing with the aid of an automatic pump, and oil-bath lubrication.
- To achieve efficient lubrication, it is necessary to mount the grease nipple or the plumbing fixture ac- cording to the mounting orientation.
- There are three different types of lubrication: boundary, mixed and full film.
- Each type is different, but they all rely on a lubricant and the additives within the oils to protect against wear.

1. Suggest suitable material for the following parts stating the special properties which make it more suitable use in the manufacturing.1.Ball bearing, 2.helical spring, 3.keys.(Nov/Dec 2021)

(1)Ball bearings

- Common materials include carbon steel, stainless steel, chrome steel, brass, aluminium, tungsten carbide, platinum, gold, and titanium, plastic.
- Other less common materials include copper, monel, k-monel, lead, silver, glass, and niobium.
- The best bearing materials for the job
  - Chrome Steel – SAE 52100. ...
  - Extra clean 52100 chrome steel bearings. ...

- Stainless Steel. ...
- Carbon Alloy Steel. ...
- Ceramics. ...
- Plastics and Non-Metallic Materials

(2)Helical spring

- Steel alloys are the most commonly used spring materials.
- The most popular alloys include high-carbon (such as the music wire used for guitar strings), oil-tempered low-carbon, chrome silicon, chrome vanadium, and stainless steel
- Other metals that are sometimes used to make springs are beryllium copper alloy, phosphor bronze, and titanium.
- Rubber or urethane may be used for cylindrical, non-coil springs.
- Ceramic material has been developed for coiled springs in very high-temperature environments. One-directional glass fiber composite materials are being tested for possible use in springs

(3) Shaft key

- Shaft and hub keyways are often cut on key seating machines but can also be made using broaching, milling, shaping, slotting EDM. Retention elements such as Splines, flexible couplings, tapered joints etc are also used.
- Typically, shaft keys are made from either medium carbon steel or stainless steel.
- But they can be made from many different types of material such as aluminium alloy, bronze, copper, and brass to suit different application environments
- For example, brass or bronze keys for marine propeller shafts and stainless-steel grade for use in food servicing equipment.
- Generally, key steel is supplied as per BS46 and BS4235 and is an unalloyed medium carbon steel with reasonable tensile strength.
- Unalloyed medium carbon steels with carbon content ranging from 0.25% to 0.60% are used due to their ideal combinations of strength, toughness, and good machining characteristics.
- The most popular grade steel is AISI 1045 (equivalent C45, EN8, 080M40), which can be hardened by heating the material to approximately 820-850C (1508 -1562 F) to increase the Ultimate Tensile Strength.

- Keys manufactured using British standards should be manufactured from steel complying with BS 970 with a tensile strength of not less than 550 MN/m2.

<u>Sliding contact bearings</u>

1. Design a journal bearing for a centrifugal pump with the following data:
Diameter of the journal = 150 mm
Load on bearing = 40 kN
Speed of journal = 900 rpm (NOV/DEC 2007& MAY/JUNE 2012)

Given:

D = 150 mm, W = 40 kN, n = 900 rpm, Application = Centrifugal pump

Solution:

i. Diameter of journal is already given in the problem, D = 150 mm

ii. From PSG DDB Pg. No. 7.81, $\frac{L}{D}$ = 1.0 – 2.0

Bearing pressure allowable = 71014 kgf/cm^2,

$\left(\frac{Zn}{p}\right)_{\min}$ = 2844.5

Take $\frac{L}{D}$ = 1.5, $\therefore$ L = 1.5 D = 1.5 x 150 = 225 mm

iii. Bearing Pressure

P = $\frac{W}{L \times D} = \frac{40\times10^3}{225\times150}$ = 1.185 N/mm^2 = 1.185 x 10 kgf/cm^2 = 11.85 kgf/cm^2.
which is less than allowable, so L/D value is acceptable.

iv. From PSG DDB Pg. No. 7.32, **Diameter clearance C= 150 microns**
= 150 x 10^{-3} mm

Clearance ratio, $\frac{C}{D} = \frac{150\times10^{-3}}{150}$ = 1 x 10^{-3}

v. Selection of lubricating oil.

From PSG DDB Pg. No. 7.31,

$\frac{Zn}{P}$ = 2844.5

Z = $\frac{2844.5\times11.85}{900}$ = 37.45 = 40 centipoise.

From PSG DDB Pg. No. 7.41, for Z = 40 and temperature = 60^o (assume). The suitable lubricating oil is SAE40.

vi. Bearing Characteristics number.

$\frac{Zn}{P} = \frac{40\times900}{11.85}$ = 3037.97

It is higher than the minimum value given in PSG DDB Pg. No. 7.31.

vii. Calculation of μ.

From PSG DDB Pg. No. 7.34, μ = $\frac{33.25}{10^{10}}\left(\frac{Zn}{P}\right)\left(\frac{D}{C}\right)+K$

$\frac{Zn}{P}$ = 3037.97, $\frac{D}{C}$ = $\frac{1}{1\times10^{-3}}$,

K = 0.0025 (for L/D = 1.5, from PSG DDB Pg. No. 7.34)

$$\mu=\frac{33.25}{10^{10}}\times3037.97\times\frac{1}{1\times10^{-3}}+0.0025$$

$\mu=0.0126$

H_g and H_d

H_g = μ .w .v (Watts)

w in Newton,

v = $\frac{\pi Dn}{60}$ in m/min,

D in meters,

n in rpm

H_g = 0.0126 x 4000 x $\frac{\pi\times0.15\times900}{60}$ = **3562.56 W**

H_d = $\frac{(\Delta t+18)^2 L\times D}{k}$

L in meters

D in meters

K – constant, assume = 0.484 heat dissipation

Δt = temperature of bearing surface

Form ambient temperature

Δt = $\frac{1}{2}(t_o-t_a)$

t_o= oil temperature, t_a = ambient temperature

Δt = $\frac{1}{2}\left(60^o-28^o\right)=16^oC$

$$H_d=\frac{(16+18)^2\times0.225\times0.15}{0.484}$$

H_d = 80.61 W

Here H_g >H_d so artificial cooling is required to carry away the excess heat.

Diameter of the bearing D_b = D+C = 150 + 150 x 10^{-3} = **150.15 mm**

Material of Bearing

From PSG DDB Pg. No. 7.30, for pump application material is rubber or moulded plastic laminate.

Summary of Design

Material = Rubber or Moulded plastic laminate
Cooling = Artificial cooling required
Diameter of journal = 150 mm
Length of journal L = 225 mm
Diameter of bearing D_b = 150.15 mm
Diameter of clearance C = 150 microns
Lubricating oil suitable = SAE40
Operating temperature = 60^oC
Atmospheric temperature = 28^oC

2. Following data is given for a 360° hydro dynamic bearing:
Journal diameter = 100 mm, Radial clearance = 0.12 mm, Radial load=50 kN,Bearing length = 100 mm, Journal speed = 1440 rpm, Viscosity of lubricant = 16 centipoises.
Calculate: 1. Minimum film thickness, 2. Co-efficient of friction, 3. Power lost in friction. (MAY/JUNE 2009)

Given: D= 100 mm, Radial clearance = 0.12 mm, W = 50kN, L=100 mm, n = 1440 rpm, Z=16 centipoises = 16×10^{-3} Ns/m^2.

Solution:

i. Minimum film thickness

W.K.T. Radial clearance = Diameteral clearance /2 = C/2
0.12 = C/2,
C = 0.24 mm

From PSG DDB Pg. No., Sommerfield number s = $\frac{Z'n'}{p}\left(\frac{D}{C}\right)^2$

Z' = viscosity in Ns/m^2,
n' = speed of jouranal in rps
p = bearing pressure in N/m^2.
n' = 1440/60 rps

Bearing pressure p = $\frac{W}{L \times D} = \frac{50 \times 10^3}{(0.1)(0.1)}$

p = 5 x 10^6 N/mm^2.

$$s = \frac{16 \times 10^{-3} \times \left(\frac{1440}{60}\right)}{5 \times 10^6} \times \left(\frac{100}{0.24}\right)^2$$

s = 0.013

From PSG DDB Pg. No. 7.40, for β = 360°, s = 0.013 and corresponding to L/D =1,

The minimum film thickness variable = $\frac{2h_o}{C}$ **= 0.071**

$$h_o = \frac{0.071 \times C}{2} = 8.52 \times 10^{-3} \text{ mm} = 0.00852 \text{ mm}$$

ii. Co-efficient of Friction (μ):

From PSG DDB Pg. No. 7.40, for β = 360°, L/D = 1, s = 0.013

$$\mu \times \frac{D}{C} = 1, \quad \mu = 1 \times \frac{C}{D} = \frac{0.24}{100} = 2.4 \times 10^{-3}$$

iii. Power cost due to friction:

H_g = μ w v

$$= 2.4 \times 10^{-3} \times 50{,}000 \times \frac{\pi \times 100 \times 1440}{60}$$

H_g = 904.8 W

3. The load on the journal bearing is 150KN due to turbine of 300mm diameter running at 1800rpm determine the following
(1) Length of the bearing if the allowable bearing pressure is 1.6N/mm²
(2) Amount of heat to be removed by the lubricant per minute if the bearing temperature is60⁰c and viscosity of the oil at 60⁰c is 0.02kg/m-s and the bearing clearance is 0.25. (Nov/Dec 2011)

GIVEN:-

$W=150KN = 150X10^3N$

D= 300mm=0.3m

N= 1800rpm

P= 1.6N/mm^2

Z=0.02kg/m-s

C= 0.25mm

SOLUTION:-

1) Length of the bearing:-

Let, l = length of bearing(mm)

WKT, projected bearing area

A=lxd=lx300=300lmm^2

And alloeable bearing pressure (P),

$1.6 = \frac{W}{A} = \frac{150x10}{300l} = \frac{500}{l}$

l= 500/1.6 = 312.5mm ans

2) Amount of heat to be removed by the lubricant:-

Wkt, co efficient of friction for the bearing

$\pi = 33/10^8 (\frac{ZN}{P}) (\frac{D}{C}) + k$

$= 33/10^8 (\frac{0.02X1800)}{1.6}) (\frac{300}{0.25}) + 0.002$

= 0.009+0.002= 0.011

Rubbing velacity,

$V = \frac{\pi DN}{60} = (\frac{\pi \times 0.3 \times 1800}{60}) = 28.3 m/s$

∴ Amount of heat to removed by the lubricant.

$Q_8 = 0.11x150x10^3x28.3$

= 46.695 J/s or W

= 46.695kw

4. Design a journal bearing for a centrifugal pump from the following data:

Load on the journal =20000N
Speed of the journal=900rpm
Type of oil is =SAE10
For which absolute viscosity at 55°C =0.017 kg/ms
Ambient temperature of oil =15.5°C
Maximum bearing pressure for the pump=1.5N/mm²
(Nov/Dec013)(April/May 17)

Given:

W=20000N

N=900rpm

T_o= 55°C,t_a=15.5°C

Z=0.017 kg/ms, p=1.5 N/mm²

Solution:

i. To find length of the journal,(l):

Assume,dia and journal d=100mm

Take l/d =1.6

L =1.6d=1.6x100=160mm

ii. Checking of bearing pressure:

WKT,

p=w/ld = 20000/160x100 = 1.25<1.5N/mm^2 (given)

The value of l&d is safe

iii. Bearing characteristic number (ZN/P):

ZN/P = 0.017x900/1.25 = 12.24

WKT,

The minimum value of bearing modules 3K = ZN/P

Bearing module at the minimum point of friction:

K = 1/3(ZN/P) = 1/3x28 = 9.33 [ZN/P = 28 from table]

Since the calculated value13.24 is more than 9.33,

Therefore the bearing is operate under hydrodynamic condition.

iv. Clearance ratio: (c/d)

From the table c/d = 0.0013 (for centrifugal pump)

v. Co-efficient of friction(μ):

$$\mu = \frac{33}{108}\left(\frac{ZN}{P}\right)\frac{c}{d} + k$$

$$= \frac{33}{108} x\, 12.24\, x \frac{1}{0.0013} + 0.002$$

= 0.0051 Ans

vi. Heat generated (Q_g):

Q_g= μwv

$$= \mu w \left(\frac{\pi dN}{60}\right) W$$

$$= 0.0051x20000 \left(\frac{\pi x 0.1x900}{60}\right)$$

= 480.7 W

vii. Heat dissipated (Q_d):

$$Q_d = CA\ (t_b - t_a)$$
$$= cld\ (t_b - t_a)W$$

$$(t_b - t_a) = \frac{1}{2}(t_b - t_a) = \frac{1}{2}(55^o - 15.5^o) = 19.75^oC$$

$Q_d = 1232 \times 0.16 \times 0.1 \times 19.75$

$= 389.3$ W (l,d – in meters)

∴amount of artificial cooling required

$= Q_g - Q_d$

$= 4807.7 - 389.3 = 91.4$ W

5. A journal bearing is to be designed for a centrifugal pump for the following Data:-

Load on the journal = 12kN,
Diameter of the journal = 75mm
Speed N = 1440rpm
Atmospheric temperature of the oil =16^0
Operating temperature of the oil = 60^0
Absolute viscosity of oil at 60^0 = 0.23 kg/ms
Give the systematic design of the bearing (MAY-JUNE-2012).

Solution :- (Solve this problem as per the procedure of previous problem)

6. Design a journal bearing for a 49.9 mm diameter of journal. It is ground and hardened and is rotating at 1500 rpm in a bearing of diameter and length both 50mm. the inlet temperature of oil 65^0 C. determine max radial load that the journal can carry and power loss. (April/May 17)(Nov/Dec 2017)

Given:

D = 50 mm, n = 1500 rpm, oil temperature = 65^0 C

Solution:

i. Diameter of journal is already given in the problem, D = 50 mm

ii. From PSG DDB Pg. No. 7.81, $\frac{L}{D} = 1.0 - 2.0$

Bearing pressure allowable = 71014 kgf/cm^2,

$$\left(\frac{Zn}{p}\right)_{\min} = 2844.5$$

Take $\frac{L}{D}$ = 1, $\therefore$ L = 1D = 1 x 50 = 50 mm

iii. Bearing Pressure (to assume)

P = $\frac{W}{L \times D}$ = 1.185 N/mm^2 = 1.185 x 10 kgf/cm^2 = 11.85 kgf/cm^2.

which is less than allowable, so L/D value is acceptable.

iv. From PSG DDB Pg. No. 7.32, **Diameter clearance C= 150 microns**

= 150 x 10^{-3} mm

Clearance ratio, $\frac{C}{D} = \frac{150 \times 10^{-3}}{50}$ = 3 x 10^{-3}

v. Selection of lubricating oil.

From PSG DDB Pg. No. 7.31,

$\frac{Zn}{P}$ = 2844.5

Z = $\frac{2844.5 \times 11.85}{1500}$ = 25 centipoise.

From PSG DDB Pg. No. 7.41, for Z = 25 and temperature = 65^o (assume). The suitable lubricating oil is SAE40.

vi. Bearing Characteristics number.

$\frac{Zn}{P} = \frac{40 \times 1500}{11.85}$

= 5063.29

It is higher than the minimum value given in PSG DDB Pg. No. 7.31.

vii. Calculation of μ.

From PSG DDB Pg. No. 7.34, μ = $\frac{33.25}{10^{10}}\left(\frac{Zn}{P}\right)\left(\frac{D}{C}\right)+K$

$\frac{Zn}{P}$ = 5063.29, $\frac{D}{C} = \frac{1}{1 \times 10^{-3}}$,

K = 0.0025 (for L/D = 1.5, from PSG DDB Pg. No. 7.34)

$$\mu = \frac{33.25}{10^{10}} \times 5063.29 \times \frac{1}{1 \times 10^{-3}} + 0.0025$$

$$\mu = 0.0126$$

H_g and H_d

H_g = μ .w .v (Watts)

w in Newton,

v = $\frac{\pi Dn}{60}$ in m/min,

D in meters,

n in rpm

H_g = 0.0126 x 4000 x $\frac{\pi\times0.15\times900}{60}$ = **3562.56 W**

H_d = $\frac{(\Delta t+18)^2 L\times D}{k}$

L in meters
D in meters
K – constant, assume = 0.484 heat dissipation

Δt = temperature of bearing surface

Form ambient temperature

Δt = $\frac{1}{2}(t_o - t_a)$

t_o= oil temperature, t_a = ambient temperature

Δt = $\frac{1}{2}\left(60^o - 28^o\right) = 16^o C$

$$H_d = \frac{(16+18)^2\times0.225\times0.15}{0.484}$$

H_d = 80.61 W

Here H_g >H_d so artificial cooling is required to carry away the excess heat.

Diameter of the bearing D_b = D+C = 50 + 3 x 10^{-3} = **150.15 mm**

Material of Bearing

From PSG DDB Pg. No. 7.30, for pump application material is rubber or moulded plastic laminate.

6.(a)A Journal bearing is proposed for a steam engine. The load on the journal is 5KN. Diameter 50mm.length 75mm speed 1600 rpm, diametral clearance 0. 001mm.ambient temperature 15.5^0 C. Oil SAE 10 is used and the film temperature is 60^{0}C. Determine The Heat generated and heat dissipated. Take absolute viscosity of SAE 10 at 60^{0}C=0.014kg/m.s(A/M'2023)

Solution:- (Solve this problem as per the procedure of previous problem 4)

7.Design a journal bearing for 12MW, 1000 rpm steam turbine, which is supported by two bearings. Take the atmospheric temperatures as 16^0C and operating temperature of oil as 60^0C. Assume viscosity of oil as 23 centistokes

(No*Given data:*

Power to be transmitted, $P = 12\ MW = 12 \times 10^6\ W$

Speed, $n = 1000\ rpm$

Atmospheric temperature, $T_a = 16°C$

Operating temperature, $T = 60°C$

Viscosity of oil, $Z = 23\ centipoise = 23 \times 10^{-3}\ N\text{-}s/m^2$

Solution:

From PSGDB 7.31, the allowable bearing pressure for steam turbine is

$$p = 7 \text{ to } 20\ kgf/cm^2$$

From PSGDB 7.31,

Minimum value of $\frac{Zn}{p} = 1422.3$

$$\frac{23 \times 1000}{p} = 1422.3$$

$$p = 16.17\ kgf/cm^2 = 1.617\ N/mm^2$$

From PSGDB for steam turbine $\frac{L}{D}$ is 1.0 to 2.0. Let us assume $\frac{L}{D} = 1.5$.

$$\text{Power, } P = \frac{2\pi N (M_t)}{60}$$

$$M_t = 114591.56\ N\text{-}m$$

The suitable material for steam turbine bearing (from PSGDB 7.32) is **heavy babbit liner on steel or cast Iron**. It is having the tensile yield strength of $70kgf/cm^2 = 700N/mm^2$ and assuming the factor of safety 1.5. The permissible shear strength,

$$\tau = \frac{0.55\sigma_y}{FOS} = \frac{0.55 \times 700}{1.5} = 256.67\ N/mm^2$$

We know that $M_t = \frac{\pi}{16} \times \tau \times D^3$

$$114591.56 \times 10^3 = \frac{\pi}{16} \times 25667 \times D^3$$

$$D = 131.5\ mm \text{ say } \mathbf{D = 132\ mm}$$

We assumed $\frac{L}{D} = 1.5$

$$\therefore \quad \mathbf{L = 198\ mm}$$

Assume the clearance, $C = 150\ microns = 150 \times 10^{-3}\ mm$

Coefficient of friction,

$$\mu = \frac{33.25}{10^{10}}\left(\frac{Zn}{p}\right)\left(\frac{D}{C}\right) + k$$

where

C – Diametral clearance, it may be assumed as 150*microns* from PSGDB 7.32.

$$\therefore \frac{D}{C} = \frac{132}{150 \times 10^{-3}} = 880$$

$$k = 0.002 \text{ for } 0.75 < \frac{L}{D} < 2.8$$

$$\therefore \mu = \frac{33.25}{10^{10}}\left(\frac{23 \times 1000}{16.17}\right) \times 880 + 0.002$$

$$\mu = 0.00616$$

Linear velocity, $V = \frac{\pi D n}{60} = \frac{\pi \times 0.132 \times 1000}{60} = 6.91\ m/s$

$$W = p\,D\,L = 1.617 \times 133 \times 198 = 42261.9\ N$$

Heat generated, $H_g = \mu\ W\ V$

$$H_g = 0.00616 \times 42261.9 \times 6.91 = 1799.3\ W$$

Temperature rise of bearing surface from ambient temperature in °C

$$\Delta t = \frac{1}{2}(t_o - t_a) = \frac{1}{2}(60 - 16) = 22°C$$

Heat dissipated, $H_d = \frac{(\Delta t + 18)^2\ L\,D}{K}$

$$= \frac{(22+18)^2 \times 0.132 \times 0.198}{0.484} = 86.4\ W$$

Since the heat generated is more than the heat dissipated, artificial cooling arrangements must be provided to carry away the excess heat. This cooling arrangement can be done by providing cooling fans or by circulated water.

8. A 50 mm diameter journal bearing rotates at 1500 rpm, L/D = 1, radial clearance 0.05 mm, minimum film thickness=0.01mm.calculate the maximum radial load that the journal bearing can carry and still operate under hydro dynamic condition. For this load calculate power lost in friction and increase in the oil temperature. Assume Hg=Hd. absolute viscosity =20 x10 $^{-3}$ Pas, Sp.gravity of oil 0.8, Sp.heat of oil 2.1 kj/kg°c[Nov/Dec 2014]

From table, for h_o/c_r =0.01/0.05 = 0.2, Somerfield number =0.0446

$0.0446 = \frac{zN'}{P}(D/C)^2$

$= 0.02X1500/60\ P\ [\ (50/0.1)^2]$

$P = 2.8X10^6 Pa$

Maximum radial load=2.8 x10^6x 0.05 x0.05

=7000 radial

V=$\frac{\pi\times 50\times 1500}{60\times 100}$ 3.92 m/s

Corresponding to ho/c_r=0.2,

$\frac{\mu}{c}$=1.7 $\mu = 3.4x10 - 4$
Power lost = $\mu W r$
=93.3 J/sec = H_d

From table,
$\frac{Q}{rcrN'l}$ = 4.62> a =7218.75 mm^2

m = 7218.75 x0.88x 1000/10^9

H_d=H_g

93.3 = 2.1x7218.75/10^9x0.88x1000xΔt x 1000

Δt=6.993^0

9. Load on a hydrodynamic full journal bearing is 30 k N. The diameter and speed of the shaft are 150 mm and 1200 rpm respectively. diametral clearance 0.2 mm. Somerfield number is 0.631.L/D ratio 1:1.calculate temperature rise of oil, quantity of oil, heat generated and type of oil require. [Apr/May 2015]
Given: Load w=30000 N, D=150mm, n = 1200 rpm, C=0.2 mm, S=0.631 ,L/D=1

Solution:-

From DD: 7.36, table-I, $\frac{\rho \ c^F \Delta t0}{P} = 52.1$

WKT,P=$\frac{W}{L\times D}$=$\frac{30000/9.81}{15\times 15}$=13.59

$$\Delta t0 = \frac{52.1\times 13.59}{14.2} = 49.861 = 49^0 51'$$

∴ **Temperature raises of oil = 50^0**

From DD: 7.36, TABLE 1

$$\frac{4q}{DCn^{F}L} = 3.59$$

$$q = \frac{3.59 \times L \times D \times C \times n'}{4}$$

$$q = \frac{3.59 \times 15 \times 15 \times 0.02 \times 1200/60}{4}$$

Quantity of required oil q = 80.775 cm^3/sec

From DD: 7.36, TABLE 1

$\mu \frac{D}{C}$=12.8

$\mu = \frac{12.8 \times 0.02}{15}$=0.01706

Wkt, V = πDn

=$\pi \times 0.15 \times 1200$ =565.486 m/min

Heat generated H_g=μW v

= 0.01706 ×(30000/9.81)×565.486

= 29502.148 kgf m/min (or) 4917.02 w

From DD: 7.31, for L/D=1, assume centrifugal pump,

The value of $\frac{zn}{p}$ = 2884.5, from this, find absolute viscosity: z.

Z=32.5 centipoises.

From the graph DD: 7.38 for Z=32.5 and temperature 50^0 C the suitable oil SAE 20

10. A full journal bearing of 50 mm diameter and 100 mm long has a bearing pressure of 1.4 N/mm2. The speed of the journal is 900 r.p.m. and the ratio of journal diameter to the diametral clearance is 1000. The bearing is lubricated with oil whose absolute viscosity at the operating temperature of 75°C may be taken as 0.011 kg/m-s. The room temperature is 35°C. Find : 1. The amount of artificial cooling required, and 2. The mass of the lubricating oil required, if the difference between the outlet and inlet temperature of the oil is 10°C. Take specific heat of the oil as 1850 J / kg / °C

Solution. Given : d = 50 mm = 0.05 m ; l = 100 mm = 0.1 m; p = 1.4 N/mm^2 ; N = 900 r.p.m. ; d / c = 1000 ; Z = 0.011 kg / m-s ; t_0 = 75°C ; t_a = 35°C ; t = 10°C ; S = 1850 J/kg / °C

1. *Amount of artificial cooling required*

We know that the coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k = \frac{33}{10^8}\left(\frac{0.011 \times 900}{1.4}\right)(1000) + 0.002$$

$$= 0.002\,33 + 0.002 = 0.004\,33$$

Load on the bearing,

$$W = p \times d.l = 1.4 \times 50 \times 100 = 7000 \text{ N}$$

and rubbing velocity,

$$V = \frac{\pi d.N}{60} = \frac{\pi \times 0.05 \times 900}{60} = 2.36 \text{ m/s}$$

$\therefore$ Heat generated,

$$Q_g = \mu.W.V = 0.004\,33 \times 7000 \times 2.36 = 71.5 \text{ J/s}$$

Let t_b = Temperature of the bearing surface.

We know that

$$(t_b - t_a) = \frac{1}{2}(t_0 - t_a) = \frac{1}{2}(75 - 35) = 20°\text{C}$$

Since the value of heat dissipation coefficient (C) for unventilated bearing varies from 140 to 420 W/m^2/°C, therefore let us take

$$C = 280 \text{ W/m}^2/°\text{ C}$$

We know that heat dissipated,

$$Q_d = C.A\,(t_b - t_a) = C.l.d\,(t_b - t_a)$$
$$= 280 \times 0.05 \times 0.1 \times 20 = 28 \text{ W} = 28 \text{ J/s}$$

$\therefore$ Amount of artificial cooling required

$$= \text{Heat generated} - \text{Heat dissipated} = Q_g - Q_d$$
$$= 71.5 - 28 = 43.5 \text{ J/s or W } \textbf{Ans.}$$

2. *Mass of the lubricating oil required*

Let m = Mass of the lubricating oil required in kg / s.

We know that heat taken away by the oil,

$$Q_t = m.S.t = m \times 1850 \times 10 = 18\,500\, m \text{ J/s}$$

Since the heat generated at the bearing is taken away by the lubricating oil, therefore equating

$$Q_g = Q_t \text{ or } 71.5 = 18\,500\, m$$

$$\therefore \quad m = 71.5 / 18\,500 = 0.003\,86 \text{ kg / s} = 0.23 \text{ kg / min } \textbf{Ans.}$$

11. The following data is given for a full hydrodynamic bearing used for electric motor radial load = 1200N; journal speed = 1440rpm; journal diameter = 50mm static load on the bearing = 350 N. The values of surface roughness of the journal and the bearing are 2 and 1 micron respectively. The minimum oil film thickness should be five times the sum of surface roughness of the journal' and the bearings. Determine i) length of the bearing ii) radial clearance iii) minimum oil film thickness iv) viscosity of lubricant v) flow of lubricant select a suitable oil for this application assuming the operating temperature as 65°C.(April/May 2018)

Given

W = 1200 N

n = 1440 rpm

d = 50 mm static load = 350 N

h_o = 5(sum of surface roughness)

Step I: Length of the bearing

Starting condition

The starting load on the bearing is static load i.e. 350 N.

the start-up bearing pressure is usually taken as 2 N/ mm^2

$$p = \frac{W}{ld} (or) l = \frac{W}{pd} = \frac{350}{2 \times 50} = 3.5mm \quad \text{.......(a)}$$

Running condition

During running condition, the radial load on bearing is 1200 N. The of permissible bearing pressure in application electric motor is from 0.7 to 1.5 N/ mm^2. We will assume permissible bearing pressure as 1 N/ mm^{2}this range. Therefore, during running conditions,

$$p = \frac{W}{ld} (or) l = \frac{W}{pd} = \frac{1200}{1 \times 50} = 24mm \quad \text{..........(b)}$$

From (a) and (b), the minimum length of bearing is 24 mm.

$$\therefore \left(\frac{l}{d}\right) = \frac{24}{50} = 0.48$$

We will assume standard value for (l/d) ratio as 0.5.

$$\therefore \left(\frac{l}{d}\right) = 0.50$$

$$l = 0.5d = 0.5(50) = 25mm \text{.......} \quad (i)$$

Step II: Radial clearance

The standard value of radial clearance in case ofBabbitt bearing is given by,

c = (0.001) r = (0.001)(25) = 0.025 mm ......(ii)

Step III: Minimum oil film thickness

The minimum oil film thickness is given by,

h_o= 5(sum of values of surface roughness of journal and bearing)

h_o = 5(2 + 1) = 15 microns = 0.015mm .... (iii)

Step IV: Viscosity of lubricant

$$\left(\frac{l}{d}\right) = \frac{1}{2} \quad \text{and} \quad \left(\frac{h_o}{c}\right) = \frac{0.015}{0.025} = 0.6$$

For above mentioned values,

$$S = 0.779 \quad \text{and} \quad \frac{Q}{rcn_s l} = 4.29$$

Also, $\left(\frac{r}{c}\right) = \frac{25}{0.025} = 1000$

$$n_s = \frac{1440}{60} = 24 \text{ rev/s}$$

$$p = \frac{W}{ld} = \frac{1200}{(25)(50)} = 0.96 \text{ N/mm}^2$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu \, n_s}{p}$$

$$0.779 = (1000)^2 \frac{\mu \,(24)}{(0.96)}$$

$$\mu = (31.16)(10^{-9}) \text{ N-sec/mm}^2$$

or 31.16 c.P. (iv)

step V: Selection of lubricant

The values viscosity for SAE-30 and SAE-40 oils are 30 and38 c Prospectively at the operating temperature of65°C. We will select SAE-40 oil for this application,Which will satisfythe minimum viscosity of31.16 cP

Step VI: Flow of lubricant

$$\frac{Q}{rcn_s l} = 4.29;$$

$$Q = 4.29\, rcn_s l = 4.29\,(25)(0.025)\,(24)(25)$$

$$= 1608.75 \text{ mm}^3/\text{s} \qquad \text{(v)}$$

11.(a)A 100 mm long and 60 mm diameter journal earing supports a load of 2500 N at 600 rpm .If the room temperature is 20° C what should be the viscosity of the oil to limit the bearing surface temperature to 60° C ?.The diameteral clearance is 0.06 mm and energy dissipation coefficient based on projected area of earing is 210 W/m2/°C. (Nov/Dec 2021)

Given data:-

Length o journal (L) =100 mm

Dia o journal (d) =60 mm

Load P=2500 N

Speed N = 600 rpm

Room temp To= 20 oC

Surface temp Ts =60oC

Diameteral clearance Cd=0.06 mm

Energy dissipation coefficient (h) =210 W/m2/C

Mass low rate = 0.85 kg/min

Specific heat = 1950 KJ/Kg-K

Temp raise o the oil = 8oC

Solution:-

WKT,

$$\mu = \frac{33}{10^8}\{(ZN|P)(d|Cd)\}+k \text{ ----------1}$$

For

L/d=100/60=1.666,the value of k=0.002

Wkt, heat generation, Hg= μ v w

$h\,A(T_O\text{-}T_S)= \mu\, v\, w$ ------2

and,

v=πdN/60,

=π x0.06 x600 /60

=1.884 m/sec

And,

P= 2500/ (60 x100)

P=0.4I66 N/mm2

Considering equation-2,

$210 \times (0.1\times0.06)(60 - 20) = \mu \times 1.884 \times 2.5\times10^3$

μ =0.0107

Consider equation 1,

$$\mu = \frac{33}{10^8}\{(Zx600|0.4166)(60|0.06)\}+0.002$$

$$0.0107 = \frac{33}{10^8}\{(Zx600|0.4166)(60|0.06)\}+0.002$$

Solving this, we get,

$$Z=0.183 \text{ kg/ms}$$

Result:-

Viscosity o the oil =0.183 kg/ms

ROLLING CONTACT BEARINGS

12. Select a bearing for a 40 mm diameter shaft rotates at 400 rpm. Due to a bevel gear mounted in the shaft. The bearing will have to withstand a 5000 N radial load of the bearing thrust load. The life of the bearing expected to be at least 1000 hrs.

(A/M'2023)

Given: d= 40mm, n=400 rpm, F_r = 5000N, F_a = 3000N, L_h = 1000 hrs

Solution:

Select Series 62 and for d = 40 mm, From PSG DDB Pg. No. 4.13, bearing basic design no. SKF 6208. The values of C_o, C are

C_o = 1600 kgf = 1600 x 10 = 16000 N
C = 2280 kgf = 2280 x 10 = 22800 N

i. Equivalent diameter load (P):

P = (X.F_r + Y.F_a) x s

For X and Y values, from PSG DDB Pg. No. 4.4, $F_a$and e are given

F_a/C_o e

$$0.12\begin{cases}0.13 & 0.31 \\ 0.25 & 0.37\end{cases}\Bigg\}0.06$$

$$\frac{0.12}{10} = 0.012, \text{ and } \frac{0.06}{10} = 0.006$$

For $\frac{F_a}{C_o} = \frac{3000}{16000} = 0.1875 \cong 0.19$, For value of $\frac{F_a}{C_o}(0.19) = 0.13 + 5(0.012)$

Similarly, For 'e' value = $(5 \times 0.006 + 0.31) = 0.03 + 0.31 =$ 0.34

$$\therefore \frac{F_a}{C_o} = 0.19$$

$e = 0.34$

$\frac{F_a}{F_r} = \frac{3000}{5000} = 0.6$ >e,

So, X value = 0.56, 's' value from PSG DDB Pg. No. 4.2
Y value = 1.2

Therefore, P = [(0.56 x 5000) + (1.2 x 3000)] x 1.2

P = 7680 N

ii. Dynamic Load capacity(C)

From PSG DDB Pg. No. 4.6 (Ball bearing), For 400 rpm and 1000 hrs life

C/P = 2.88

$\frac{C}{7680} = 2.88$

C = 2.88 x 7680 = 22118.4 N

This dynamic load is less than the tabulated (allowable) value i.e. 22800 N. So the suitable bearing designation is **SKF 6208.**

13.A deep groove ball bearing No. 6308 selected for a particular application, carries a radial load of 2900 N and a thrust load of 1800 N ; both being steady. The inner race of the bearing rotates at 900 r.p.m. The bearing is required to have aminimum life of 9000 hours. Check whether the bearing selected can servethe purpose.(Nov/Dec 2017)

(Similar to above problem)

14. Select a suitable ball bearing to support the overhung countershaft. The shaft is 60 mm diameter and rotating at 1250 rpm. The bearings are to have 99% reliability corresponding to a life of 4000 hrs. The bearing is subjected to an equivalent radial load of 6000N.

Given:

d = 60 mm, n = 1250 rpm, Reliability = 99% = 0.99 = probability = p, L = 4000 hrs, F_r = 6000N.

Solution:

From PSG DDB Pg. No. 4.2 $\frac{L}{L'10} = \left[\frac{\ln(1/p)}{\ln(1/p_{10})}\right]^{1/b}$

Here, $\ln(1/p_{10})$ = 0.1053, L = 4000 hrs, b = 1.34, p = 0.99

Substitute all value,

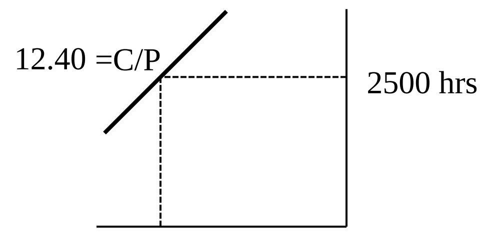
1250 rpm

$$\frac{4000}{L'10} = (0.09544)^{0.7463}$$

L'10 = 23.093 hrs

From PSG DDB Pg. No. 4.6, For life 23.093 hrs and 1250 rpm,

The $\frac{C}{P}$ value is 12.40



$\frac{C}{P}$ = 12.4

C = 12.4 x F_r

C = 12.4 x 6000

1250 rpm

C = 74400N

Select the bearing for C = 74400 N or C = 7440 kgf, and the diameter of the shaft is 60 mm. (From PSG DDB Pg. No. 4.15, series 64)

Result:

SKF 6412 is suitable bearing,

C_o = 7100 kgf,C = 8450 kgf.

15. A 70mm machine shaft is to supported at ends. If operates continuously for 8 hrs per day ,320 days per year for 8 years the load of speed cycle for one of the hearing are given below,

S.No	Fraction of cycle	Radial load in N	Thrust	Speed rpm	Factors		
					X	y	Z
1.	0.25	3500	1000	600	0.56	1.2	1.5
2.	0.25	3000	1000	800	0.56	1.2	1.5
3.	0.5	4000	2000	900	0.56	1.4	1.5

Select suitable bearing.

Solution:

i. Equivalent load (p)

P_1=(XFr+YFa)Z

=(0.56×3500+1.2×1000)1.5

=4740w

P_2=(0.56×3000+1.2×1000)1.5

=4320w

P_3=(0.56×4000+1.4×2000)1.5

=7560w

rompg 4.2:

$$\text{Cubic mean load } F_m = \left(\frac{p_1^3 n_1 t_1 + p_2^3 n_2 t_2 + p_3^3 n_3 t_3}{n_1 + n_2 + n_3} \right)^{1/3}$$

$$= \left(\frac{(4740)^3 x600x0.25 + (4320)^3 x800x0.25 + (7560)^3 x900x0.5}{600+800+900} \right)$$

Fm = 4618.16 N

W.k.t: equivalent load = cubic mean load (p=fm)

!!) Equivalent speed (N)

$N = n_1t_1+n_2t_2+n_3t_3$

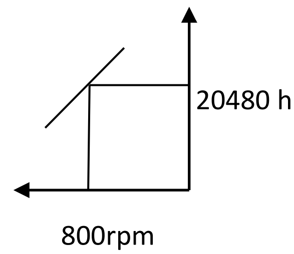
= 600x0.25+800x0.25+900x0.5

N = 800 rpm

!!!) Total life hrs = 8hrs/day , 320day s/yr For F yea

= 8x320x8

= 20,480 hrs



From pg no 4.6 , For lie 20.480 hrs&800 rpm

C/P= 9.83

C= 9.83x4618.16 = 45396 N

For, C= 45,396 N & d= 70mm

The suitable size of bearing in skf= 6214

16.a) A ball bearing operate on the following work cycle(A/M'2023)

A ball bearing operates on the following work cycle :

Element no	Radial load N	Speed, rpm	Element time, %
1	3000	720	30
2	7000	1440	50
3	5000	900	20

The dynamic load capacity of the bearing is 16.6 kN. Calculate

(i) The average speed of rotation (4)

(ii) The equivalent radial load (3)

(iii) The bearing life and (4)

(iv) The bearing life at 95% reliability (2)

Solve the problem using previous procedure

15. A single row deep groove ball bearing no: 6002 is subjected to an axial thrust load of 1000N and a radial load of 2200N. Find the expected life that 50% of the bearing will complete under this condition.

GIVEN;-

Deep groove ball bearing no: 6002

F_a= 1000N

F_a = 2200N

SOLUTION:-

From DDB:4.12 :- For bearing no 6002

C_o = 255kgf = 255*10N

C = 440kgf = 4400

F_a/c_o=1000/2550 = 0.392

From DDB 4.4 for Fa/co=0.392 the value of e=0.412

Since, f_a/c_o= 1000/2550 = 0.454>e

The radial load factor X=0.56

Thrust load factor Y= 1.83

Service factor from DDB 4.2 , S=1.1to 1.5 , say S=1.3

∴ equivalent load

(fromDDO: 4.2) $P=(XF_r+YF_a)S$

= (0.56*2200+1.83*1000)1.3

= 3980.6N

WKT,

$L= (c/p)^b = (4400/3980.6)^3$ =1.35 million revolution

Expected life at 50% reliability(L50) is obtained from

$$L50/L90=\left(\frac{\ln(1/R50)}{\ln(1/R90)}\right)^{1/b}$$

$$L50/1.35 = \left(\frac{\ln(1/0.5)}{\ln(1/0.9)}\right)^{1/b} = (0.693/0.105)^{0,85} = 4.058$$

L50= 4.958*1.35

L50= 6.69 million rev

16. A single row deep groove ball bearing is subjected to a radial force of 8kN and a thrust force of 3kN. the rotates at 1200rpm the expected life L_{10}^th of the bearing is 20000hr the minimum acceptable diameter of having for this application. (MAY/JUNE 2012) (April/May 17)

GIVEN:-

F_n = 8kn

F_a = 3kn

N = 1200rpm

$L_{10}h$ = 20,000

d = 75mm

SOLUTION :-

STEP1 - X and Y factor

X = 0.56 , Y= 1.5 , F_r = 8000N

F_a = 3000N

WKT,

$C = p(L_{10})^{1/3} = (8980)(1440)^{1/3}$

$= 101406.04N$

The shaft of 75mm diameter, bearing no.6315 (c= 112000) is suitable for the above data for this bearing,

C_o = 72000N

$\therefore , (\frac{Fa}{Fr}) = (\frac{3000}{8000}) = 0.375$

And $(\frac{Fa}{Co}) = (\frac{3000}{72000}) = 0.04167$

e = 0.24 (approximately) and $(\frac{Fa}{Fr})$>e

the value of Y is obtained by liner interpolation .

Y = 1.8 – $(\frac{(1.8-1.6)}{(0.07-0.04)} \text{x} (0.0416 - 0.04) = 1.79$

And X = 0.56

STEP2-

Dynamic load capacity

P =XFr+YF = 0.56(8000)+ 1.79 (3000)

= 9850N

C = p $(L_{10})^{1/3}$ = $9850(1440)^{1/3}$ = 111230.46N

STEP3 -

Selection of bearing

Bearing No.6315(c= 112000) is suitable for the above application.

RESULT:-

The suitable bearing is 6315

17. <u>Find the rated load of a deep groove ball bearing for the following load cycle.</u>

Sl.no	Radial load (N)	Axial load (N)	% of time
1	3000	1000	15
2	3500	1000	20
3	3500	100	30
4	500	2000	35

<u>Also find the 90% life of ball bearing if bearings used is 6207 with dynamic capacity 19620 N. [Nov/Dec 2014]</u>

P_{e1}=1.5(0.56 x 200 + 1.2x1000) = 4320 N

$\text{III}^{ly} P_{e2}$=474 N, P_{e3}=4200 N, P_{e4}=4020 N

N1=0.15 x 60= 90 cycle (assuming life =1 min)

N2=160 cycle
N3=270 cycle
N4=525 cycle
Ne=1045 cycle

1045 P_e^2= 90x $(4320)^3$ + 160$(4740)^3$ + 270$(4200)^3$+ 525$(4020)^3$

P_e=4217.70 N,

If C=19620 N,

Life expected =$10^6\left(\frac{19620}{4217.7}\right)$ =100.66 x10^6cycles

∴<u>Life in hours = 1605.42 hrs.</u>

18.(a) A ball bearing has to be selected for an application in which the radial load is 2000 N during 90% of the time and 8000 N during 10% of time. The shaft is to rotate at 150 rpm, determine the minimum value of the basic dynamic load rating for 5000 hours of the operation with more than 10 % failures.(Nov/Dec 2021)

Given

W_1=2000N n_1=0.9

W_2=8000N n_2=0.1

N=150 rpm

Step1:

Equivalence constant load (W) for ball bearings (k=3)

$$W = \frac{n_1(W_1)^3 + n_2(W_2)^3}{n_1 + n_2}$$

$$W = \frac{0.9(2000)^3 + 0.1(8000)^3}{0.1 + 0.9}$$

W = 3879.75 N

Step2:

Relation for life in revolutions (L) and life in working hours (L_H) is:

$$L = 60N.L_H revolutions$$

Given L_H=5000 hours

$$L = 60 * 150 * 5000$$
$$L = 45000000 = 45 * 10^6 revolutions$$

Step3:

Basic dynamic load Rating is

$$c = W\left(\frac{L}{10^6}\right)^{\frac{1}{k}}$$

K=3 for ball bearings.

$$c = 3879.75 * \left(\frac{45*10^6}{10^6}\right)^{\frac{1}{3}}$$

$$c = 3879.75 * 3.556$$
$$c = 13799.85N$$

c= 13.8 KN (ANS)

Result: - The minimum value of the basic dynamic load rating= **c= 13.8 KN**

18. Enumerate the detail steps involved in the selection of bearings from the manufactures catalogue. [Apr/May 2015]

1. Calculate the radial and axial forces acting on the bearing.

2. Calculate the shaft diameter.

3. Determine the radial load factor (X) and thrust load factor (Y) from the manufacturing's catalogue. The values of X and Y for ball and roller bearing are given in **DD: 4.4** the values depend upon two ratios (F_a/F_r) and (F_a/c_o), where C_O is the static load capacity. Select the series (60, 62, 63….) for the given diameter of the shaft and the valve of C_O **(Refer DD: 4.12-4.20)**

4. Calculate the equivalent dynamic load from the equation $P = (XF_r + YF_a)S$, **(Refer DD: 4.2)**

5. Decide the excepted life of the bearing. Convert life in hours into millions of revolutions.

6. Calculate the dynamic load capacity from the equation $L=(C/P)^b$

7.Check whether the selected bearing has the required dynamic load capacity. If yes, the selected bearing is suitable for this purpose. Otherwise, select another bearing from the next series and continue from step (3) .

20. Select a single row deep groove ball bearing for a radial load of 4000 N and an axial load of 5000 N, operating at a speed of 1600 r.p.m. for an average life of 5 years at 10 hours per day. Assume uniform and steady load.

Solution. Given : $W_R = 4000$ N ; $W_A = 5000$ N ; $N = 1600$ r.p.m.

Since the average life of the bearing is 5 years at 10 hours per day, therefore life of the bearing in hours,

$$L_H = 5 \times 300 \times 10 = 15\ 000 \text{ hours} \quad \text{... (Assuming 300 working days per year)}$$

and life of the bearing in revolutions,

$$L = 60\ N \times L_H = 60 \times 1600 \times 15\ 000 = 1440 \times 10^6 \text{ rev}$$

We know that the basic dynamic equivalent radial load,

$$W = X.V.W_R + Y.W_A \quad ...(i)$$

In order to determine the radial load factor (X) and axial load factor (Y), we require W_A / W_R and W_A / C_0. Since the value of basic static load capacity (C_0) is not known, therefore let us take $W_A / C_0 = 0.5$. Now from Table 27.4, we find that the values of X and Y corresponding to $W_A / C_0 = 0.5$ and $W_A/W_R = 5000 / 4000 = 1.25$ (which is greater than $e = 0.44$) are

$$X = 0.56 \quad \text{and} \quad Y = 1$$

Since the rotational factor (V) for most of the bearings is 1, therefore basic dynamic equivalent radial load,

$$W = 0.56 \times 1 \times 4000 + 1 \times 5000 = 7240 \text{ N}$$

From Table 27.5, we find that for uniform and steady load, the service factor (K_S) for ball bearings is 1. Therefore the bearing should be selected for $W = 7240$ N.

We know that basic dynamic load rating,

$$C = W\left(\frac{L}{10^6}\right)^{1/k} = 7240\left(\frac{1440 \times 10^6}{10^6}\right)^{1/3} = 81\ 760 \text{ N}$$

$$= 81.76 \text{ kN} \qquad \text{... } (\because\ k = 3, \text{ for ball bearings})$$

From Table 27.6, let us select the bearing No. 315 which has the following basic capacities,

$$C_0 = 72 \text{ kN} = 72\ 000 \text{ N} \quad \text{and} \quad C = 90 \text{ kN} = 90\ 000 \text{ N}$$

Now $\qquad W_A / C_0 = 5000 / 72\ 000 = 0.07$

$\therefore$ From Table 27.4, the values of X and Y are

$$X = 0.56 \quad \text{and} \quad Y = 1.6$$

Substituting these values in equation (*i*), we have dynamic equivalent load,

$$W = 0.56 \times 1 \times 4000 + 1.6 \times 5000 = 10\ 240 \text{ N}$$

$\therefore$ Basic dynamic load rating,

$$C = 10\ 240\left(\frac{1440 \times 10^6}{10^6}\right)^{1/3} = 115\ 635 \text{ N} = 115.635 \text{ kN}$$

the bearing number 319 having $C = 120$ kN, may be selected.

21. A ball bearing subjected to radial load of 10KN and a thrust load of 5 KN. The inner ring rotates at 1000 rpm. The average life is to be 5000 hours. What basic load rating must be used to select a bearing for this purpose? Take F_a/C_a=0.5 and assume service factor 1.5. This problem is to consider service factor Ks= 1.5 (April/May 2017)

(Similar to above problem)

22. A shaft rotating at constant speed is subjected to variable load. The bearings supporting the shaft are subjected to stationary equivalent radial load of 3 kN for 10 per cent of time, 2 kN for 20 per cent of time, 1 kN for 30 per cent of time and no load for remaining time of cycle. If the total life expected for the bearing is 20 × 106 revolutions at 95 per cent reliability, calculate dynamic load rating of the ball bearing.

Solution. Given : $W_1 = 3$ kN ; $n_1 = 0.1\ n$; $W_2 = 2$ kN ; $n_2 = 0.2\ n$; $W_3 = 1$ kN ; $n_3 = 0.3\ n$; $W_4 = 0$; $n_4 = (1 - 0.1 - 0.2 - 0.3)\ n = 0.4\ n$; $L_{95} = 20 \times 10^6$ rev

Let L_{90} = Life of the bearing corresponding to reliability of 90 per cent,

L_{95} = Life of the bearing corresponding to reliability of 95 per cent

$= 20 \times 10^6$ revolutions ... (Given)

We know that

$$\frac{L_{95}}{L_{90}} = \left[\frac{\log_e (1/R_{95})}{\log_e (1/R_{90})}\right]^{1/b} = \left[\frac{\log_e (1/0.95)}{\log_e (1/0.90)}\right]^{1/1.17} \quad \dots (\because b = 1.17)$$

$$= \left(\frac{0.0513}{0.1054}\right)^{0.8547} = 0.54$$

$\therefore$ $L_{90} = L_{95} / 0.54 = 20 \times 10^6 / 0.54 = 37 \times 10^6$ rev

We know that equivalent radial load,

$$W = \left[\frac{n_1 (W_1)^3 + n_2 (W_2)^3 + n_3 (W_3)^3 + n_4 (W_4)^3}{n_1 + n_2 + n_3 + n_4}\right]^{1/3}$$

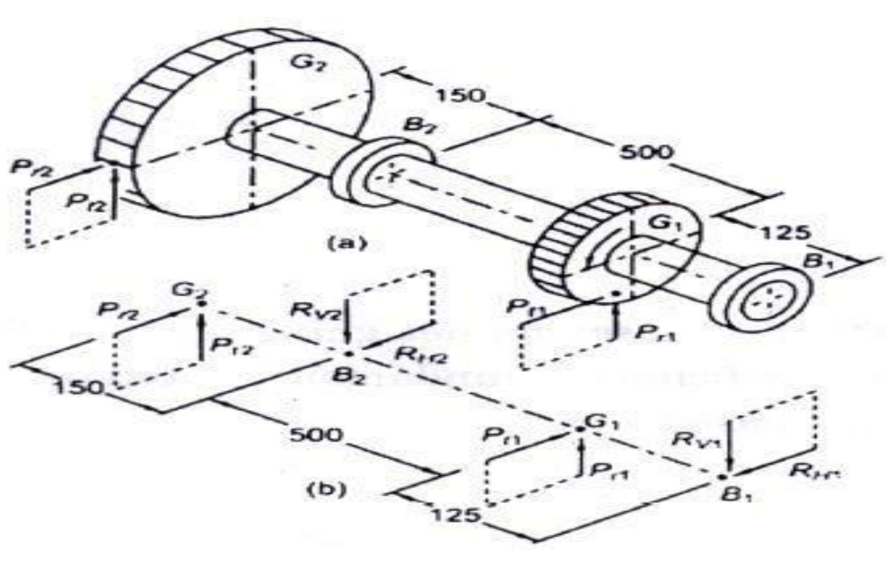
$$= \left[\frac{0.1n \times 3^3 + 0.2\ n \times 2^3 + 0.3n \times 1^3 + 0.4\ n \times 0^3}{0.1\ n + 0.2n + 0.3n + 0.4n}\right]^{1/3}$$

$$= (2.7 + 1.6 + 0.3 + 0)^{1/3} = 1.663 \text{ kN}$$

We also know that dynamic load rating,

$$C = W \left(\frac{L_{90}}{10^6}\right)^{1/k} = 1.663 \left(\frac{37 \times 10^6}{10^6}\right)^{1/3} = 5.54 \text{ kN Ans.}$$

23. A shaft transmitting 50 kW at 1255 rpm from the gear G_1 to the gear G_2 and mounted on two single-row deep groove ball bearings B_1 and B_2 as shown. The gear tooth forces are P_{t1} = 15915 N, P_{r1} = 5793 N, P_{t2} = 9549 Nand P_{r2} = 3476 N. The diameter of the shaft at bearings B_1 and B_2 is 75 mm. The load factor is 1.4 and the expected life for 90% of the bearings is 10000 h. Select suitable ball bearings. Refer figure.(April/May 2018)



Given:
kW=50 n=125rpm d=75mm L_{10h} = 10000 hr load factor = 1.4

I: Radial and axial forces
The forcesacting on the shaft are shown in Fig. considering forces in the vertical plane and takingmoments about bearing B_1'

$P_{r1}(125)+P_{t2}(125 + 500 + 150)-R_{v2}(125 + 500) = 0$
$5793(125) + 9549(775) - R_{v2}(625) = 0$
Therefore R_{V2} = 12999 N

Consideringequilibrium of vertical forces,
$P_{t2} + P_{r1} = R_{v2} + R_{v1}$
$9549 + 5793 = 12999 + R_{V1}$
R_{V1} = 2343 N

Considering forces in the horizontal plane and
taking moments about bearing B_1'
$P_{r1}(125) + P_{r2}(125 + 500 + 150) - R_{H2}(125 + 500) = 0$
or $15915(125) + 3476(775) - R_{H2}(625) = 0$
R_{H2} = 7493 N

Considering equilibrium of horizontal forces,
$P_{t1} + P_{r2} = R_{H1} + R_{H2}$
15915 + 3476 = RHi + 7493
R_{H1} = 11898 N

Theradial forces at the two bearings are given by

$$F_{r1} = \sqrt{(R_{V1})^2 + (R_{H1})^2} = \sqrt{(2343)^2 + (11898)^2}$$
$$= 12\,127 \text{ N}$$
$$F_{r2} = \sqrt{(R_{V2})^2 + (R_{H2})^2} = \sqrt{(12999)^2 + (7493)^2}$$
$$= 15\,004 \text{ N}$$

Since there is no axial thrust,
$F_{a1} = F_{a2} = 0$

StepII:Dynamic load capacities

$P_1 = F_{r1}$ = 12 127 N; $P_2 = F_{r2}$ = 15004 N

$$\mathbf{L_{10}} = \frac{60nL_{10h}}{10^6} = \frac{60(125)(10000)}{10^6} = 75 \text{ million rev.}$$

Considering load factor and using Eq, the dynamic load capacities are given by,$C_1 = P_1 (L_{10})^{1/3}$ (Load factor)
$= (12127)(75)^{1/3} (1.4) = 71598$ N
$C_2 = P_2 (L_{10})^{1/3}$ (Load factor)
$= (15004)(75)^{1/3} (1.4) = 88\,584$ N

Step III: Selection of bearings
From Table the available bearings at $B_1$and $B_2$are as follows:
B_1 andB_2 (d= 75 mm)
No.6015 (C = 39 700 N)
No.6215 (C = 66 300 N)
No.6315 (C= 112000 N)
No.6415 (C = 153000 N)
Therefore, Bearing No. 6315 is suitable at B_1 as well as B_2

24.A ball bearing subjected to a radial load of 5KN is expected to have a life of 8000 hours at 1450 r.p.m. with a reliability of 99%. Calculate the dynamic load capacity of the bearing so that it can be selected from the manufacturer's catalogue based on a reliability of 90%(Nov/Dec-16)
Given:

Radial load = 10 KN,

Thrust load = 5 KN,

Speed = 1000 rpm,

Average life = 5000 Hr**.**

To find:

Basic dynamic load rating

Solution:

Step I *Bearing life with 99% reliability*

$$L_{99} = \frac{60nL_{99h}}{10^6} = \frac{60(1450)(8000)}{10^6}$$
$$= 696 \text{ million rev.}$$

Step II *Bearing life with 90% reliability*
From Eq.

$$\left(\frac{L_{99}}{L_{10}}\right) = \left[\frac{\log_e\left(\frac{1}{R_{99}}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/1.17} = \left[\frac{\log_e\left(\frac{1}{0.99}\right)}{\log_e\left(\frac{1}{0.90}\right)}\right]^{1/1.17}$$
$$= 0.1342$$

Therefore,
$$L_{10} = \frac{L_{99}}{0.1342} = \frac{696}{0.1342} = 5186.29 \text{ million rev.}$$

Step III *Dynamic load carrying capacity of bearing*
$$C = P\,(L_{10})^{1/3} = 5000\,(5186.29)^{1/3} = 86\,547.7 \text{ N}$$

25. Following data is given for a 360^0 hydrodynamic bearing

Radial load = 3.2 KN

Journal speed = 1490 rpm

1/d ratio = 1

Unit bearing pressure = 1.3 Mpa

Radial clearance = 0.05 mm

Viscosity of oil = 25 centipoise.

Assume that the total heat generated in the bearing is carried by the total oil flow in the beraing, calculate the journal diameter, power lost in friction and the temperature rise. (April/May 2019) **Nov/Dec-20,Aprl/May-21(N/D'2022)**

Solution

Given $W = 3.2$ kN $\quad n = 1490$ rpm $\quad d = 50$ mm
$l = 50$ mm $\quad c = 0.05$ mm $\quad z = 25$ cP

Step I Performance parameters

$$p = \frac{W}{ld} = \frac{(3.2)(1000)}{(50)(50)} = 1.28 \text{ N/mm}^2$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p} = \left(\frac{25}{0.05}\right)^2 \left(\frac{25}{10^9}\right)\left(\frac{1490}{60}\right)\left(\frac{1}{1.28}\right)$$
$$= 0.121$$

$$\left(\frac{l}{d}\right) = \left(\frac{50}{50}\right) = 1$$

From Table 16.1,

$$\left(\frac{r}{c}\right) f = 3.22 \qquad \left(\frac{h_o}{c}\right) = 0.4 \qquad \frac{Q}{rcn_s l} = 4.33$$

Step II Coefficient of friction

$$f = 3.22\left(\frac{c}{r}\right) = 3.22\left(\frac{0.05}{25}\right) = 0.00644$$

Step III Power lost in friction

From Eq. (16.20),

$$(\text{kW})_f = \frac{2\pi n_s \, f W r}{10^6}$$
$$= \frac{2\pi\,(1490/60)\,(0.00644)\,(3.2)(1000)\,(25)}{10^6}$$
$$= 0.08$$

Step IV *Minimum oil film thickness*

$$h_o = 0.4c = 04(0.05) = 0.02 \text{ mm}$$

Step V *Flow requirement*

$$Q = 4.33\, rcn_s\, l = 4.33(25)(0.05)(1490/60)(50)$$
$$= 6720.5 \text{ mm}^3/\text{s}$$
$$Q = (6720.5)\,(10^{-3}) \text{ cc/s}$$
$$Q = (6720.5)\,(10^{-3})\,(10^{-3}) \text{ litres/s } (1000 \text{ cc} = 1 \text{litre})$$
$$Q = (6720.5)(10^{-6})\,(60) \text{ litre/min}$$
$$= 0.403 \text{ litre/min}$$

Step VI *Temperature rise*

From Eq. 16.23,

$$\Delta t = \frac{8.3p(\text{CFV})}{(\text{FV})}$$
$$= \frac{8.3(1.28)(3.22)}{(4.33)} = 7.9°C$$

26. A ball bearing is operating on a work cycle consisting of three parts a radial load of 3000 N at 1440 rpm for one quarter cycle, a radial load of 5000 N at 720 rpm for one half cycle and radial load of 2500 N at 1440 rpm for the remaining cycle. The expected life of the bearing is 10,000 hr. calculate the dynamic load carrying capacity of the bearing. (April/May 2019)

Solution

Given $L_{10h} = 10\,000$ h

Step I *Equivalent load for complete work cycle*

Considering the work cycle of one minute duration,

$$N_1 = \frac{1}{4}(1440) = 360 \text{ rev.}$$

$$N_2 = \frac{1}{2}(720) = 360 \text{ rev.}$$

$$N_3 = \frac{1}{4}(1440) = 360 \text{ rev.}$$

The average speed of rotation is given by,

$$n = N_1 + N_2 + N_3 = 1080 \text{ rpm}$$

From Eq. (15.13),

$$P_e = \sqrt[3]{\left[\frac{N_1 P_1^3 + N_2 P_2^3 + N_3 P_3^3}{N_1 + N_2 + N_3}\right]}$$

$$= \sqrt[3]{\left[\frac{360(3000)^3 + 360(5000)^3 + 360(2500)^3}{1080}\right]}$$

$$= 3823 \text{ N}$$

Step II *Dynamic load carrying capacity of bearing*

$$L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60(1080)(10\,000)}{10^6}$$

$$= 648 \text{ million rev.}$$

From Eq. (15.7),

$$C = P(L_{10})^{1/3} = 3823(648)^{1/3} = 33\,082 \text{ N}$$

27. A single row deep groove ball bearing is subjected to a radial force of 8 KN and a thrust force of 3 KN. The values of X and Y factors are 0.56 and 1.5 respectively. The shaft rotates at 1200 rpm. The diameter of the shaft is 75 mm and Bearing No. 6315 (C = 112000 N) is selected for this application. Estimate

(i) Life of the bearing with 90% reliability.

(ii) Reliability for 20000 hr. life. (April/May 2019) **Nov/Dec-20,Aprl/May-21(N/D'2022)**

Solution

Given $F_r = 8$ kN $F_a = 3$ kN $X = 0.56$ $Y = 1.5$

$n = 1200$ rpm $d = 75$ mm $C = 112\,000$ N

Step I *Bearing life with 90% reliability*

From Eq. (15.3),

$$P = XF_r + YF_a = 0.56\,(8000) + 1.5\,(3000) = 8980 \text{ N}$$

From Eq. (15.6),

$$L_{10} = \left(\frac{C}{P}\right)^3 = \left(\frac{112\,000}{8980}\right)^3 = 1940.10 \text{ million rev.}$$

$$L_{10h} = \frac{L_{10}(10^6)}{60n} = \frac{1940.10(10^6)}{60(1200)} = 26\,945.83 \text{ h} \quad \text{(i)}$$

Step II *Reliability for 20 000 hr life*

$$\left(\frac{L}{L_{10}}\right) = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

$$\left(\frac{L}{L_{10}}\right)^b = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]$$

Substituting the following values,

$L = 20\,000$ h $L_{10} = 26\,945.83$ h $R_{90} = 0.90$

$b = 1.17$ we get,

$$\left(\frac{20\,000}{26\,945.83}\right)^{1.17} = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{0.90}\right)}\right]$$

$$R = 0.9283 \text{ or } 92.83\%$$

28. A 100 mm diameter full journal bearing supports a radial load of 5000 N. the bearing is 100 mm long and operates at 400 rpm. Permissible min film thickness 25 micron. Diametral clearance 152 microns. Using Raimond & Boyd curves find (i) viscosity of suitable oil, (ii) μ, (iii) heat generation rate (iv) amount of oil pumped through bearing (v) amount of end leakage (vi) rise in temperature of oil. (Nov/Dec 2018)

Solution

1. The value of Sommerfeld number S can be determined from

$$\frac{2h_o}{C} = \frac{2 \times 0.025}{0.152} = 0.3289$$

Corresponding $S = 0.0875$ (for $L/D = 1$ and $\beta = 360°$)

$P = W/LD = 5000/(100 \times 100) = 0.5 \text{ N/mm}^2$
$= 0.5 \times 10^6 \text{ N/m}^2$

$$S = \frac{ZN'}{P}\left(\frac{D}{C}\right)^2$$

N' = revolutions/second
$= 400/60 = 6.67$ rps

$$0.0875 = \frac{Z \times 6.67}{0.5 \times 10^6}(100/0.152)^2$$

$Z = 1.515 \times 10^{-2} = 15.15 \times 10^{-3}$ kg/ms (required value)

2. From charts , $\mu D/C = 2.6$ (for $S = 0.0875$, $L/D = 1$ and $\beta = 360°$).

Friction coefficient, $\mu = 2.6\,(C/D) = \dfrac{2.6 \times 0.152}{100} = 0.00395$

3. Heat generated $= H_g = \mu WV$

$V = \pi DN'/1000$
$= \pi \times 100 \times (400/60)/1000$
$= 2.09$ m/s

$H_g = 0.00395 \times 5000 \times 2.09$
$= 41.28$ Nm/s $= 41.28$ watts

4. Flow variable is obtained from Table by interpolation.
$4q/DCN'L = 4.4572$ (for $S = 0.0875$ and $L/D = 1$)

$q = 4.4572 \times 0.1 \times 0.152 \times 10^{-3} \times 6.67 \times 0.1/4$
$= 0.01129 \times 10^{-3} \text{ m}^3/\text{s} = 11.29 \times 10^{-6} \text{ m}^3/\text{s}$

5. From Table , for $S = 0.0875$,

$q_s/q = 0.7511$ (by interpolation)

$q_s = 0.7511 \times 11.29 \times 10^{-6} = 8.48 \times 10^{-6}\ m^3/s$

6. From Table , for $S = 0.0875$,

$\rho C' \Delta t_o / P = 11.48$ (by interpolation)

$\rho C' = 14.2 \times 10^5\ N/m^2\ °C$

Temperature rise of oil, $\Delta t_o = 11.48 \times P/\rho C'$

$= (11.48 \times 0.5 \times 10^6)/(14.2 \times 10^5) = 4.04°C$

Assume inlet temperature of oil, $t_1 = 68°C$.

Outlet temperature of oil $= t_2 = t_1 + \Delta t_o = 68 + 4 = 72°C$

Oil film temperature (average) $= t_1 + (\Delta t_o/2) = 68 + (4/2) = 70°C$

From chart

We find SAE 20 has $Z = 14 \times 10^{-3}$ kg/ms at 70°C.

Oil inlet temperature has to be reduced to 66°C so that, oil film temperature becomes 68°C and SAE 20 has $Z = 15 \times 10^{-3}$ kg/ms (close to the required value). Selection of SAE 20 is now satisfactory.

29. A roller bearing is to be selected to withstand a radial load of 4000N and have an L_{10} life of 1200 hours at a speed of 600 rpm. (Nov/Dec 2018)

(i) What is the basic dynamic load rating of the bearing to be selected?

(ii) If the reliability requirement is 99%, what load rating would be used? Take b=1.17 and V=S=1.

a) $C = \left(\frac{L}{L_{10}}\right)^{1/k} \cdot P$

Since roller bearings do not take any axial load,

$P = VXF_r S$ $\quad$ $V = 1$ (assumed)

$X = 1$ (for cylindrical roller bearing)

$= 1 \times 1 \times 4000 \times 1$ $\quad$ $S = 1$ (assumed)

$= 4000$ N

$k = 10/3$ for roller bearings

$L = 1200 \times 60 \times 600 = 43.2$ mr (L_{10}-life or life with 10% failures)

$$C = \left(\frac{43.2}{1}\right)^{3/10} \cdot 4000 = 12379.6 \text{ N}$$

Note:- "L_{10}-life of 1200 hours" means that life of the bearing is 1200 hours at 600 rpm with load 4000 N for 90% survival or 10% failure of bearings. Therefore this "L_{10}-life" is the desired life. This should not be confused with the L_{10} we find in Eqn. 9.3, which is the 1mr during which the load C acts for 90% survival. Since dynamic capacity of the bearing is given in catalogue for 1 mr, with calculated C, bearing selection is made.

$$\text{(b)} \ \frac{L}{L'_{10}} = \left[\frac{\ln(1/p)}{\ln(1/p_{10})}\right]^{1/b} = \left[\frac{\ln(1/0.99)}{0.1053}\right]^{1/1.17} = 0.1343$$

$L =$ Expected life with a probability of survival, 0.99

$= L'_{10} \times 0.1343 = 43.2 \times 0.1343$

$= 5.8$ mr

Now, equivalent load corresponding to 5.8 mr is obtained.

$$\frac{L_1}{L_2} = \left[\frac{P_2}{P_1}\right]^k ; \quad \frac{43.2}{5.8} = \left[\frac{P_2}{4000}\right]^{10/3}$$

$P_2 = 7306$ N. Now, dynamic load rating is obtained.

$$C = \left(\frac{43.2}{1}\right)^{3/10} \cdot 7306 = 22611 \text{ N}$$