efficient-frontier-assignment

September 13, 2024

[2]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import math

import warnings
warnings.filterwarnings('ignore')
import seaborn as sns
```

0.0.1 Efficient Frontier

Industry_Portfolios.xlsx contains monthly nominal (net) returns (expressed as percentages) for ten industry portfolios, over the ten-year period from Jan 2004 through Dec 2013.

[5]:
```
from google.colab import files
files.upload()
('Industry_Portfolios.xlsx')
```

```
<IPython.core.display.HTML object>
```

```
Saving Industry_Portfolios.xlsx to Industry_Portfolios.xlsx
```

[5]:
```
'Industry_Portfolios.xlsx'
```

[6]:
```
Data = pd.read_excel('Industry_Portfolios.xlsx',index_col=0)
```

Use these returns to estimate the vector of mean returns and the covariance matrix of returns for the ten industry portfolios:

Create a table showing the mean return and standard deviation of return for the ten industry portfolios.

[7]:
```
mean_std=pd.DataFrame({"mean_return":Data.mean(),"std_return":Data.std()})
mean_std
```

[7]:

	mean_return	std_return
NoDur	0.902833	3.345657
Durbl	0.733333	8.361852
Manuf	1.012833	5.310270

```
Enrgy      1.231167    6.081524
HiTec      0.766250    5.381191
Telcm      0.881417    4.448284
Shops      0.916333    4.093786
Hlth       0.783833    3.787172
Utils      0.907167    3.701763
Other      0.489083    5.582452
```

Plot the minimum-variance frontier (without the riskless asset) generated by the ten industry portfolios:

[8]:
```
V = Data.cov()
V_inv = pd.DataFrame(np.linalg.inv(V), columns=V.columns, index=V.index)
R = mean_std["mean_return"]
e = pd.Series([1]*len(R))
e.index = R.index
```

[9]:
```
alpha = R.dot(V_inv).dot(e)
zeta = R.dot(V_inv).dot(R)
delta = e.dot(V_inv).dot(e)
R_mv = alpha/delta
```

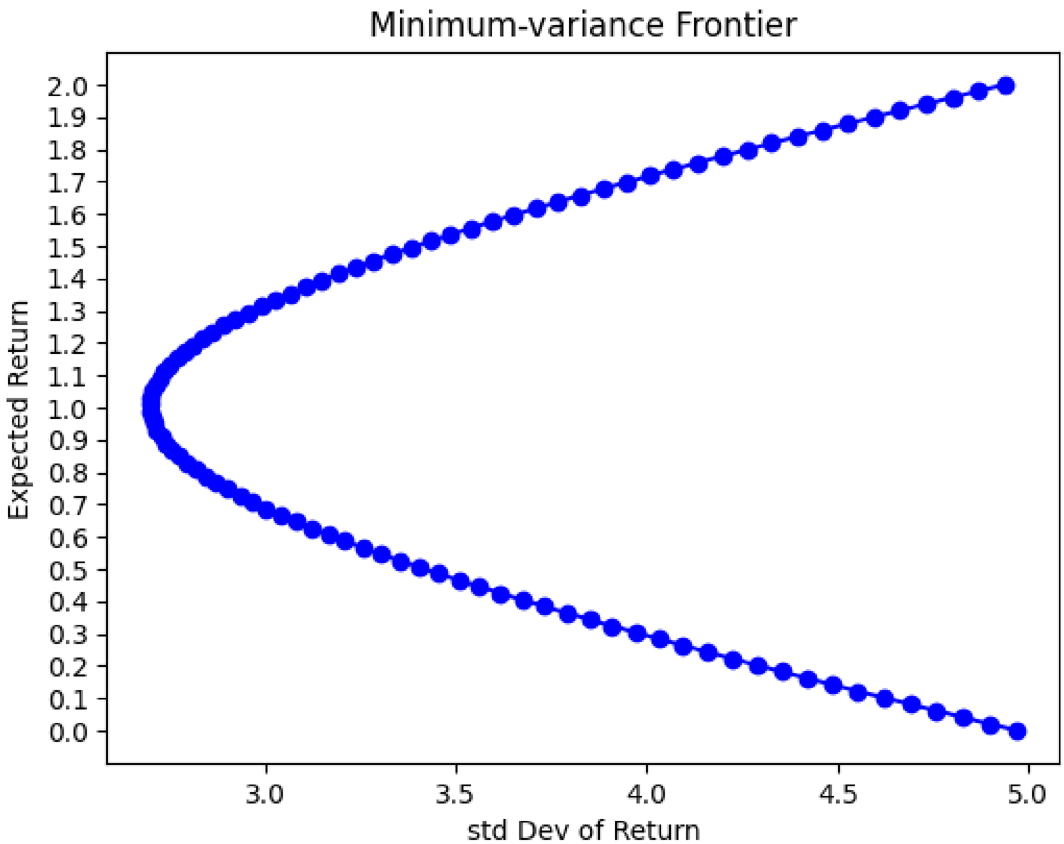
This graph must have expected (monthly) return on the vertical axis vs standard deviation of (monthly) return on the horizontal axis.

This graph must cover the range from 0% to 2% on the vertical axis, in increments of 0.1% (or less).

[10]:
```
R_p = np.linspace(0, 2, 100)

sigma_p = np.sqrt(1/delta + delta/(zeta*delta-alpha**2)*(R_p-R_mv)**2)
```

[11]:
```
plt.plot(sigma_p, R_p, color='blue', linestyle='-')
plt.scatter(sigma_p, R_p, color='blue', marker='o')
plt.yticks(np.arange(0, 2.1, 0.1))
plt.xlabel('std Dev of Return')
plt.ylabel('Expected Return')
plt.title('Minimum-variance Frontier')
plt.show()
```



Briefly explain (in words, without mathematical equations or formulas) the economic significance and relevance of the minimum-variance frontier to an investor.

Minimum-variance frontier represents outermost envelope of attainable portfolios   no attainable portfolio exists on the left of the minimum-variance frontier.

Risk-Return Tradeoff: The minimum-variance frontier helps investors understand the tradeoff between risk and return. It illustrates that, for a given level of expected return, there is an optimal portfolio with the lowest possible risk. Investors can use this information to make informed decisions about their risk tolerance and desired returns.

Portfolio Diversification: The concept of the minimum-variance frontier highlights the benefits of diversification. By investing in a mix of assets with low correlations, investors can achieve a lower overall portfolio risk than by holding individual assets. This diversification can improve risk-adjusted returns.

Efficient Frontier: The minimum-variance frontier is a key component of the efficient frontier, which shows all portfolios that provide the highest expected return for a given level of risk. Investors aim to construct portfolios that lie on or above the efficient frontier, as these portfolios offer the best risk-return tradeoff.

Portfolio Optimization: Investors can use the minimum-variance frontier to optimize their portfolios by selecting a mix of assets that aligns with their risk preferences and return expectations. This optimization helps investors achieve the best possible risk-return balance based on their specific goals and constraints.

Now suppose that the (net) risk-free rate is 0.13% per month:

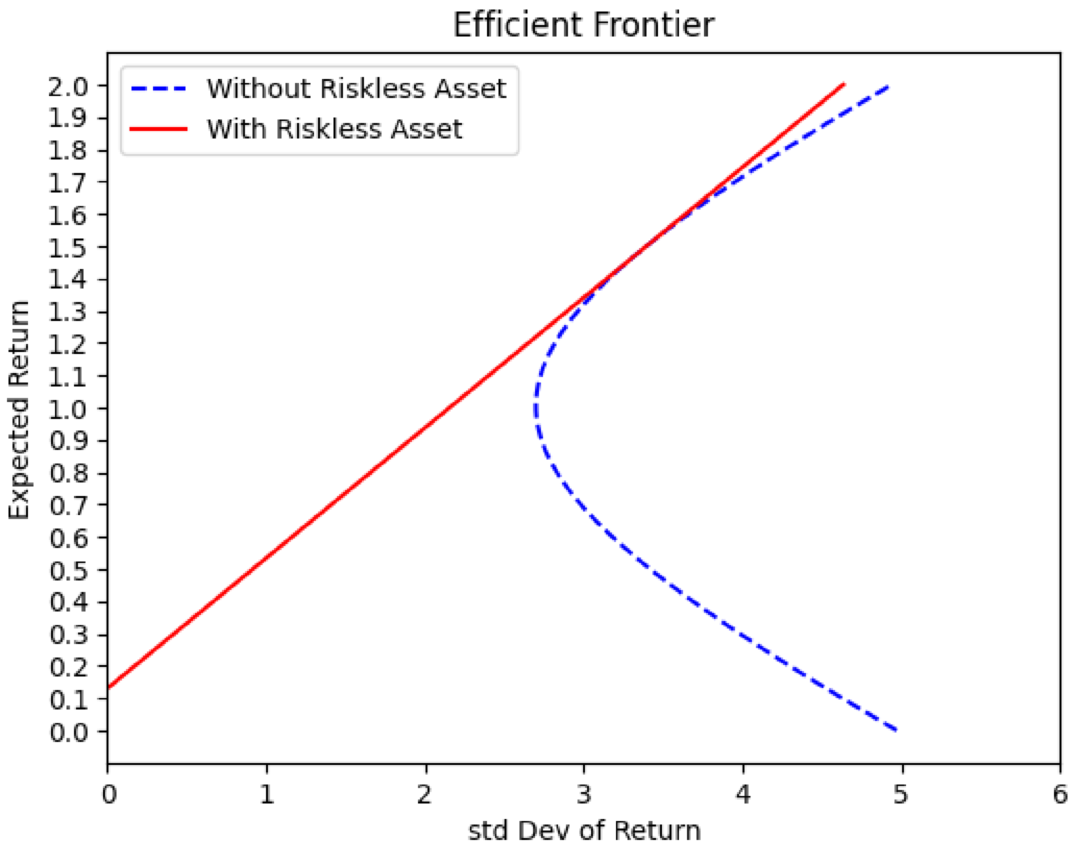
Plot the efficient frontier (with the riskless asset) on the same graph as the minimum-variance frontier generated by the ten industry portfolios.

[12]:
```
R_f = 0.13

R_p_riskless = np.linspace(0, 2, 100)
sigma_p_riskless = (R_p_riskless-R_f)/np.sqrt(zeta - 2*alpha*R_f +␣
 ↪delta*(R_f**2))
```

[13]:
```
plt.plot(sigma_p, R_p, color='blue', linestyle='--', label = "Without Riskless␣
 ↪Asset")
plt.plot(sigma_p_riskless, R_p_riskless, color='red', linestyle='-', label =␣
 ↪"With Riskless Asset")

plt.yticks(np.arange(0, 2.1, 0.1))
plt.xlim(0,6)
plt.xlabel('std Dev of Return')
plt.ylabel('Expected Return')
plt.title('Efficient Frontier')
plt.legend()
plt.show()
```



Briefly explain the economic significance and relevance of the efficient frontier to an investor.

Efficient frontier consists of portfolios with highest potential reward for given amount of risk risk-averse investor must invest in (optimal) portfolio on efficient frontier to maximise expected utility.

Risk-Return Tradeoff: The efficient frontier demonstrates the tradeoff between risk and return. It helps investors understand that there is an optimal portfolio mix that maximizes returns while minimizing risk. This knowledge is essential for making informed investment decisions aligned with their risk tolerance and financial goals.

Portfolio Diversification: The concept of the efficient frontier highlights the benefits of diversification. By combining assets with different risk and return profiles, investors can construct portfolios that lie on or above the efficient frontier, achieving superior risk-adjusted returns compared to holding individual assets.

Portfolio Optimization: Investors can use the efficient frontier to optimize their portfolios. By selecting a portfolio on the efficient frontier that matches their risk preferences and return expectations, they can achieve the best possible risk-return tradeoff, ensuring their investments align with their objectives.

Comparison and Evaluation: The efficient frontier allows investors to compare different portfolios and investment strategies. They can assess whether their current portfolio is efficient or if adjustments are needed to enhance their risk-adjusted returns.

Benchmark for Performance: The efficient frontier serves as a benchmark for measuring the performance of investment managers and investment products. Investors can evaluate how well a particular investment or fund performs in comparison to the efficient frontier, helping them make better investment choices.

The two frontiers will intersect at single point: the tangency portfolio:

Calculate the Sharpe ratio for the tangency portfolio, and also the tangency portfolio weights for the ten industry portfolios.

[14]:
```
sharpe_ratio = np.sqrt(zeta - 2*alpha*R_f + delta*R_f**2)
sharpe_ratio
```

[14]:
```
0.4035655993495088
```

[15]:
```
R_tg = (alpha*R_f - zeta)/(delta*R_f - alpha)
a = (zeta*V_inv.dot(e)-alpha*V_inv.dot(R))/(zeta*delta-alpha**2)
b = (delta*V_inv.dot(R) - alpha*V_inv.dot(e))/(zeta*delta-alpha**2)
w_star = a + b*R_tg
w_star
```

[15]:
```
NoDur    0.567972
Durbl   -0.214073
Manuf    0.714105
Enrgy    0.104087
HiTec   -0.363438
Telcm   -0.095463
Shops    0.991647
Hlth     0.075570
Utils    0.132643
Other   -0.913051
dtype: float64
```

Briefly explain the economic significance and relevance of the tangency portfolio to an investor.

The tangency portfolio is highly significant in modern portfolio theory because it represents the portfolio on the efficient frontier that offers the highest Sharpe ratio, meaning the best risk-adjusted return. Economically, it is relevant for investors in several key ways:

Optimal Risk-Return Tradeoff: The tangency portfolio provides the maximum possible return per unit of risk (as measured by standard deviation). This makes it the most efficient portfolio for an investor willing to take on some risk but aiming to maximize returns relative to that risk.

Capital Market Line (CML): The tangency portfolio is the point where the Capital Market Line (CML), representing combinations of a risk-free asset and a risky portfolio, touches the efficient frontier. Investors can achieve any desired return by mixing the tangency portfolio with the risk-free asset, tailoring their risk exposure to their preferences.

Foundation for Portfolio Construction: Since it has the best risk-adjusted return, all investors (under the assumptions of the theory) would hold a combination of the tangency portfolio and the risk-free asset. Investors with higher risk tolerance invest more in the tangency portfolio, while more risk-averse investors allocate more to the risk-free asset.

Diversification: The tangency portfolio is well-diversified, spreading risk across various assets in an optimal way. This minimizes the unsystematic (diversifiable) risk and leaves only the market risk (systematic risk), ensuring that no unnecessary risk is taken.