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Section: 3

1) Decimation in frequency - Fast Fourier Transform (DIF - FFT)

By definition of DFT,

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n] W_N^{kn} + \sum_{n=\frac{N}{2}}^{N-1} x[n] W_N^{kn}$$

Replace 'n' by $(n+\frac{N}{2})$ w get

$$\therefore x[k] = \sum_{n=0}^{\frac{N}{2}-1} x[n] W_N^{kn} + \sum_{n+\frac{N}{2}=\frac{N}{2}}^{N-1} \cancel{x[n] W_N^{kn}}\, x\left[n+\frac{N}{2}\right] W_N^{k(n+\frac{N}{2})}$$

$$= \sum_{k=0}^{\frac{N}{2}-1} x[n] W_N^{kn} + \sum_{n=0}^{\frac{N}{2}-1} x\left[n+\frac{N}{2}\right] W_N^{kn+\frac{kN}{2}}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n] W_N^{kn} + \sum_{n=0}^{N/2-1} x\left[n+\frac{N}{2}\right] W_N^{kn} \cdot W_N^{kN/2}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n] W_N^{kn} + \varepsilon_N^{kN/2} \sum_{n=0}^{\frac{N}{2}-1} x\left[n+\frac{N}{2}\right] \cdot W_N^{kn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n] W_N^{kn} + \left(W_N^{N/2}\right)^k \sum_{n=0}^{\frac{N}{2}-1} x\left[n+\frac{N}{2}\right] W_N^{kn}$$

$$\left[W_N^{N/2}\right]^k = \left[e^{-\frac{j2\pi}{N}\cdot\frac{N}{2}}\right]^k = \left[e^{-j\pi}\right]^k = (-1)^k$$

$$\therefore x[k] = \sum_{n=0}^{N/2-1} x[n] W_N^{kn} + (-1)^k \sum_{n=0}^{\frac{N}{2}-1} x\left[n+\frac{N}{2}\right] W_N^{kn}$$

$$X[k] = \sum_{n=0}^{N/2-1} x[n] W_N^{kn} + (-1)^k \sum_{n=0}^{\frac{N}{2}-1} x\left[n+\frac{N}{2}\right] W_N^{kn}$$

$$X[k] \neq X[2k] + X[2k+1]$$

$$\therefore X[k] = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) W_N^{kn} + (-1)^k x\left(n+\frac{N}{2}\right) W_N^{kn}\right], \; k = 0, 1, \ldots (N$$

$$X[K] = X[2K] + X[2K+1], \; K = 0, 1, \ldots \left(\frac{N}{2}-1\right)$$

$$\therefore X[2K] = \sum_{n=0}^{\frac{N}{2}-1} \left(x(n) + (-1)^{2k} x\left(n+\frac{N}{2}\right)\right) W_N^{2kn}$$

$$\therefore X[2K+1] = \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + (-1)^{2K+1} x\left[n+\frac{N}{2}\right]\right) W_N^{2kn}$$

$$\therefore x[2K+1] = \sum_{n=0}^{\frac{N}{2}-1} \left[x[n] + (-1)^{(2K+1)} x\left[n+\frac{N}{2}\right]\right) W_N^{(2K+1)n}$$

$$(-1)^{2K} = 1$$

$$(-1)^{2K+1} = -1$$

$$W_N^{2kn} = e^{\frac{-j2\pi kn}{N} \cdot 2} = e^{\frac{-j\pi kn}{(N/2)}} = W_{N/2}^{kn}$$

$$W_N^{(2K+1)n} = W_N^{2kn} W_N^{n} = W_{N/2}^{kn} \cdot W_N^{n}$$

$$X[2K] = \sum_{n=0}^{N/2-1} \left[x(n) + (-1)^0 x\left(n+\frac{N}{2}\right)\right] W_{N/2}^{kn} \cdot W_N^{n}$$

where $K = 0, 1, \ldots, \left(\frac{N}{2}-1\right)$

$$\therefore X[2k+1] = \sum_{n=0}^{\frac{N}{2}-1} \left[\left(x(n) - x\left(n+\frac{N}{2}\right)\right) W_N^{n}\right] W_{N/2}^{kn}$$

let $g_1(n) = \left[x[n] + x\left(n+\frac{N}{2}\right)\right]$, $n = 0, 1, 2, 3$

$$g_2(n) = \left[x(n) - x\left(n+\frac{N}{2}\right)\right]$$

$$X[2K] = \sum_{n=0}^{N/2-1} g_1(n) W_{N/2}^{kn}$$

$$X[2k+1] = \sum_{n=0}^{\frac{N}{2}-1} g_2(n) W_{N/2}^{kn}$$

Calculation of N-point from $\frac{N}{2}$ point DFT

$$g_1(n) = \left[x(n) + x\left(n+\frac{N}{2}\right)\right], \; n = 0, 1, 2, \ldots \left(\frac{N}{2}-1\right)$$

for 8 point

$$g_1(n) = [x(n) + x(n+4)], \; n = 0, 1, 2, 3$$

for $n=0$, $g_1(0) = [x(0) + x(4)]$

$n=1$, $g_1(1) = [x(1) + x(5)]$

$n=2$, $g_2(2) = [x(2) + x(6)]$

$n=3$, $g_1(3) = [x(3) + x(7)]$

$$g_2(n) = \left[x(n) - x\left(n + \frac{N}{2}\right)\right] W_N^{\gamma}$$

$\Downarrow$

for $n = 0, 1, ---, \left(\frac{N}{2} - 1\right)$

for 8 point,

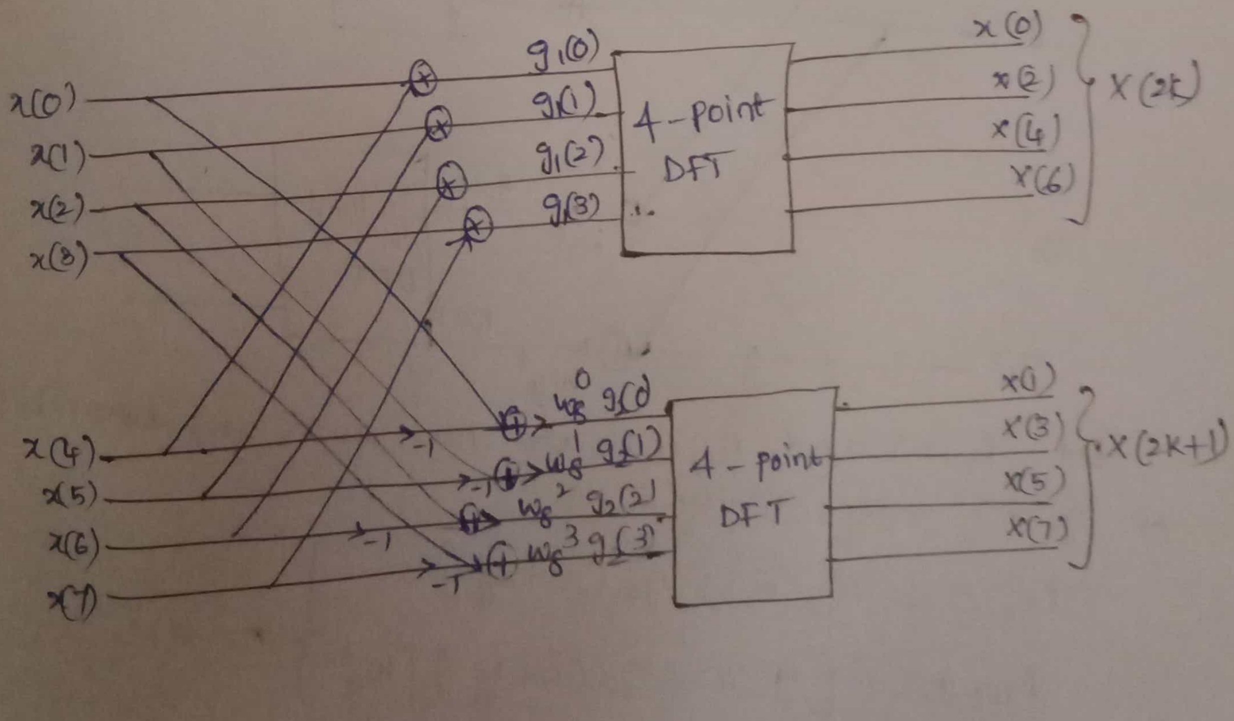
$\therefore g_2(n) = [x(n) - x(n+4)] W_8^n$, $n = 0, 1, 2, 3$

$\therefore g_2(0) = [x(0) - x(4)] W_8^0 = [x(0) - x(4)]$

$g_2(1) = [x(1) - x(5)] W_8^1$

$g_2(2) = [x(2) - x(6)] W_8^2$

$g_2(3) = [x(3) - x(7)] W_8^3$, $n = 0, 1, 2, 3$.



Let $P_{11}(n)$ and $P_{12}(n)$ is even and odd part of $g_1(n)$ respectively

$$P_{11}(n) = g_1(n) + g_1\left(n + \frac{N/2}{2}\right)$$

$$P_{12}(n) = \left[g_1(n) - g_2\left(n + \frac{N/2}{2}\right)\right] W_{N/2}^{n}$$

$$\therefore \left.\begin{aligned} P_{11}(n) &= g_1(n) + g_1\left(n + \frac{N}{4}\right) \\ P_{12}(n) &= \left[g_1(n) - g_1\left(n + \frac{N}{4}\right)\right] W_{N/2}^{n} \end{aligned}\right\} n = 0, 1$$

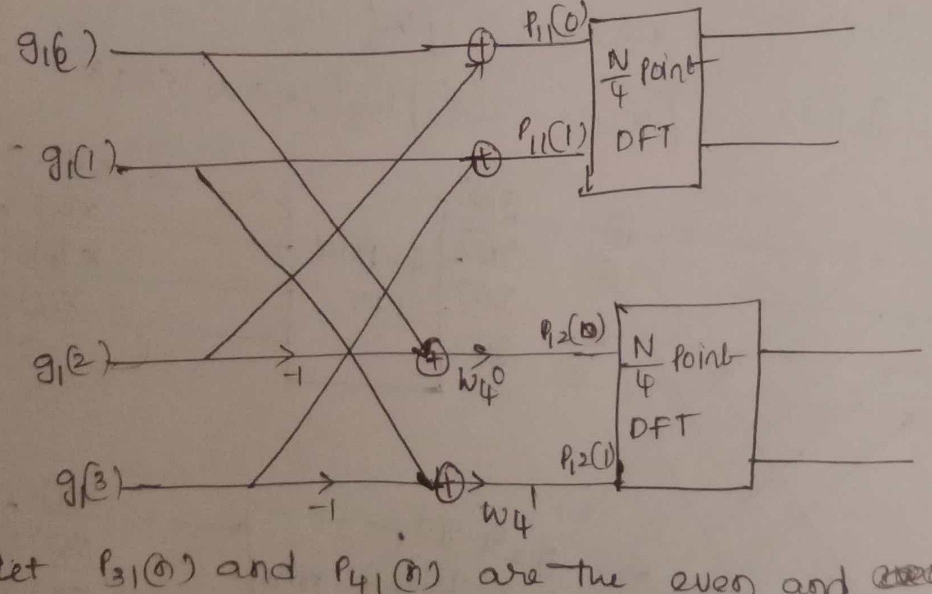
for 8-point

$$P_{11}(0) = g_{11}(0) + g_1(2)$$

$$P_{11}(1) = g_1(1) + g_1(3)$$

$$P_{12}(0) = [g_1(0) - g_1(2)] W_4^0$$

$$P_{12}(1) = [g_1(1) - g_1(3)] W_4^1$$



Let $P_{31}(n)$ and $P_{41}(n)$ are the even and odd part of $g_2(n)$ respectively.

$$\left.\begin{aligned} P_{31}(n) &= g_2(n) + g_2\left(n + \frac{N}{4}\right) \\ P_{41}(n) &= \left[g_2(n) - g_2\left(n + \frac{N}{4}\right)\right] W_4^n \end{aligned}\right\} n = 0, 1, -$$

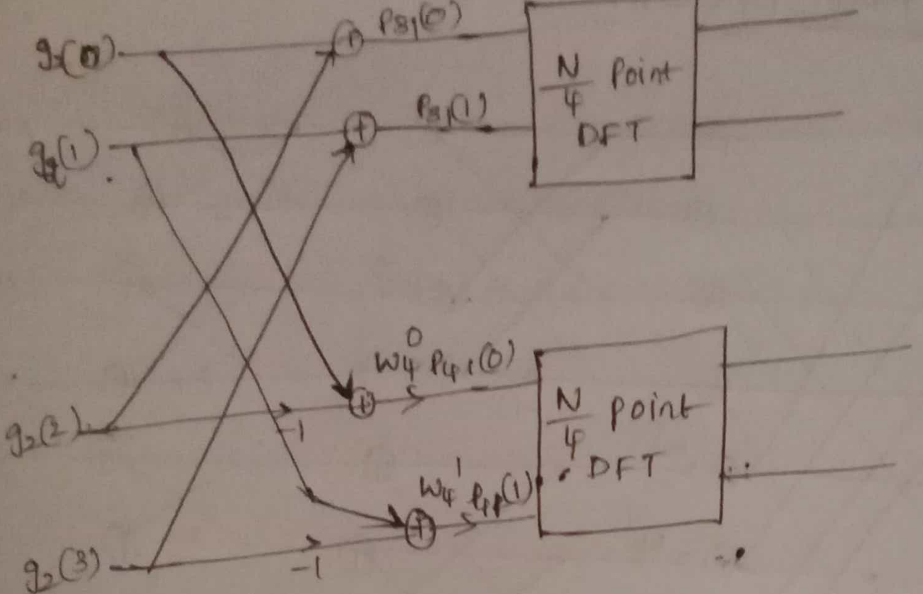
$$P_{31}(0) = g_2(0) + g_2(2)$$

$$P_{31}(1) = g_2(1) + g_2(3)$$

$$P_{41}(0) = [g_2(0) - g_2(1)] W_4^0$$

$$P_{41}(1) = \left[\cancel{g_2(n)} - g_2\left(n + \frac{N}{4}\right)\right] W_4^n$$

$$[g_2(1) - g_2(3)] W_4^1$$



$N=8 \Rightarrow \frac{N}{4} = 2$ points

$$P_{11}(k) = \sum_{n=0}^{1} P_{11}(n) W_N^{nk}$$

$K=0$

$$P_{11}(0) = \sum_{n=0}^{1} P_{11}(n) = P_{11}(0) + P_{11}(1)$$

$$P_{12}(k) = \sum_{n=0}^{1} P_{12}(n) W_N^{nk}$$

$K=0,$

$$P_{12}(0) = \sum_{n=0}^{1} P_{12}(n) W_N^0 = P_{12}(0) + P_{12}(1)$$

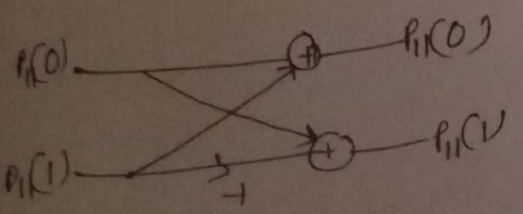
$K=1 \Rightarrow$

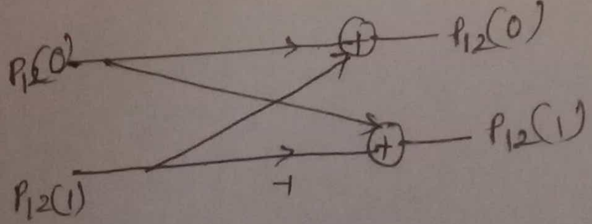
$$P_{11}(1) = \sum_{n=0}^{1} P_{11}(n) W_N^n = \sum_{n=0}^{1} P_{11}(n) W_2^{n(1)}$$

$$= P_{11}(0) + P_{11}(1) W_2^1 = P_{11}(0) - P_{11}(1)$$

$$P_{12}(1) = \sum_{n=0}^{1} P_{12}(n) W_2^n$$

$$= P_{11}(0) + P_{12}(1) W_2^1 = P_{11}(0) - P_{12}(1)$$





8 point DTF - FFT