• **Introduction :—**

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R.C.C.
&
PRESTRESSED CONCRETE
OR (Design of RC structures).

इन्सान कभी हारता नहीं है, या तो वो जीतता है या फिर सीखता है।

BE RUTHLESS TO THE THINGS THAT DON'T MATTER

R.C.C. Basics

- Plane cement concrete (P.C.C.) → 2400 kg/m³ OR 24 kN/m³ (unit wt.)
- Reinforced cement concrete (R.C.C.) → 2500 kg/m³ OR 25 kN/m³

Take ⇓

{taken from IS 875 (Part 1) : (1987) Page No (7) & (8).
OR
IS 456 : 2000.}

[Unless more accurate calculations are warranted]

(with sand & gravel or crushed natural stone aggregate)

15/09/2024 Saturday

R.C.C.

#01

Limit State Method (LSM) — IS-456:2000

- **Limit state of collapse**
  - This method deals with SF, BM, TM, AF and their combinations.
  - Load combinations :-
    - $1.5DL + 1.5IL$ (Dead load) (Imposed Load) → 500kN
    - $1.2(DL + IL + (WL/EQ))$ → 750kN (wind Load) (Earth quake load) } take max. value.
    - $1.5\,DL + 1.5\,WL/EQ$ → 630kN
    - $0.9\,DL + 1.5\,WL/EQ$ → 600kN

    (B.M. & S.F. ke liye bhi lage sakte hai.)
  - This method deals with the 'safety' of the structure.

- **Limit state of serviceability**
  - This method deals with deflection, cracking, corrosion, abrasion.
  - Load combinations :-
    - $1.0DL + 1.0IL$
    - $1.0\,DL + 1.0WL/EQ$
    - $DL + 0.8(IL + WL/EQ)$

    we'll design for worst case
  - This method deals with the durability of the structure.

Definitions :-

(1) **Characteristic strength** :- The strength below which not more than 5% results are expected to fall.

eg: (M20)

100 sample

> 95 samples — strength > 20 N/mm²

< 5 sample — strength < 20 N/mm²

fck → characteristic compressive strength of concrete.

fy → characteristic strength or yield tensile strength of concrete. (characteristic strength of steel)

(2) Design strength :-

$$\text{Design strength } (f_d) = \frac{\text{characteristic strength}}{FOS}$$

(to be on safe side)

⇒ (FOS > 1) → Always so Design strength ↓

for concrete; FOS = 1.5

for steel; FOS = 1.15

Que:- Why F.O.S. is more for concrete then steel?

Ans:- Because concrete made on site & on site bez of uncontrolable factors like dust on ground surface, variable temperature (can vary) water can be of not of good quality - it may have solid or may be wastewater, due to storage of cement in open area clumps in cement can be there. तो ऐसी किसी condition में concrete की quality decrease ना हो जाए इसीलिए concrete की FOS हम steel से ज्यादा लेकर चलते हैं। because steel तो factories में बनता है।

3. Characteristic Load :-

$\longrightarrow$ 1000 kN
95 % ✓ (5% chance of failure)

The Load which has 95% Probability of not exceeding in the entire service life of the structure.

4. Design Load :→

$$\boxed{\text{Design Load} = \text{FOS} \times \text{characteristic Load}}$$

Think :→ why were multiplying here by FOS whereas in case of design strength we had divided.
(क्योंकि strength में हम कम लेकर बात करते हैं और Load में हम ज्यादा Load लेकर चलते हैं।)

जैसे → 100 kN के लिए Design करना है।
तो $100 \times 1.5 = 150$ kN
⇒ (50 kN margin लेकर चलो) safe side

(IS 456 : 2000) $\boxed{\text{F.O.S.} = 1.5}$

※ PROBABILITY of Failure of a structure

- Structure may fail due to :-

(i) Strength of material is less than characteristic strength. ⇒ 5% ⇒ 0.05
(ii) The applied load exceeds the characteristic Load ⇒ 5% ⇒ 0.05
(iii) When condition (i) and (ii) occur together ⇒ 0.05 × 0.05 ⇒ $\boxed{0.0025}$

Reasons of failure of a structure :—

(i) & (ii) are individual events.

so $P(iii) = P(i) \times P(ii)$

$= 0.05 \times 0.05 = 0.0025$

Total probability of failure of the structure/building $\Rightarrow (0.05)\times(0.95) + (0.05)\times(0.95) + (0.0025)$ (0.05×0.05)

$\Rightarrow \boxed{0.0975}$

$\simeq 0.1$

$\simeq 10^{-1}$

so total probability of failure of the structure

$\Rightarrow$ $\boxed{0.0975 \text{ OR } 0.1 \text{ OR } 10^{-1}}$

$\Rightarrow$ $\boxed{9.75\%}$

Take care :- (can be Round off)

direct GATE 2015 2 Marks

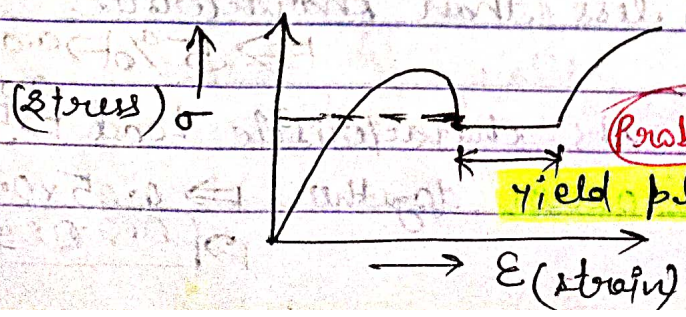
- $\boxed{\text{Types of Steel}}$ (सरिया)

(i) Mild Steel (MS) :— (a) Fe 250.

(b) $f_y = 250 \, N/mm^2$ (yield st.).

(c) Highly ductile.

(d) But yield plateu is present which is not desired. (but costly defn.)



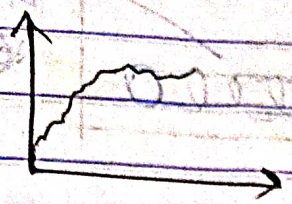
(बहुत बेकार चीज, load नहीं लगा रहे पर भी मिल रहा है)

(MS will be use in earthquake prone buildings.)

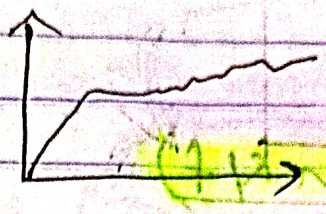
(ज्यादा wind & earthquake वाली जगह काम करेगा।) (5)

Que:- Tell which material is highly ductile

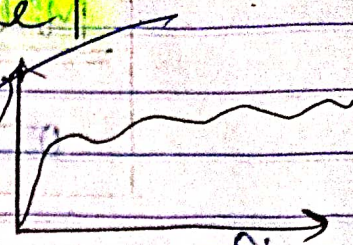
(i)



(ii)



(iii)



(खूब ज्यादा खींच सकते हैं।)

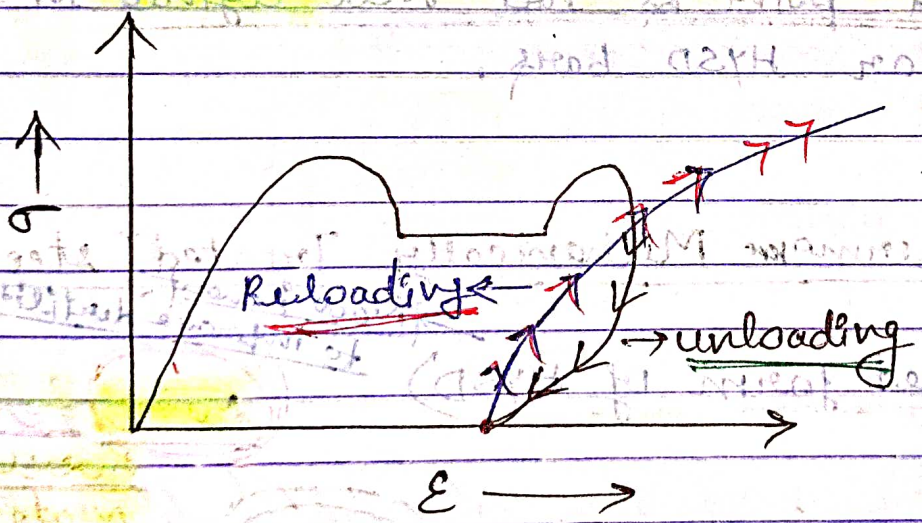
⇒ well defined yield point → (+point) (earthquake आने पर suitable)

⇒ Mild steel is having less yield (250 N/mm²) strength comparative to other steel.

ii Cold twisted deformed bars :- (CTD)

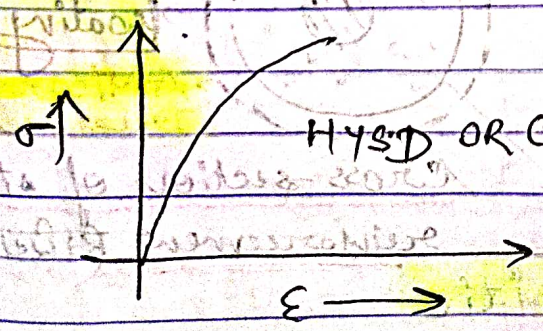
or

High yield strength deformed bars - (HYSD)



HYSD forms from Mild steel only. (process showing by graph)

(unloading → Reloading)



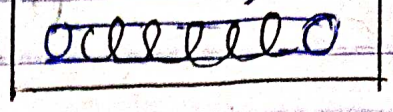
Advantages :-

⇒ Fe 415, Fe 500

⇒ No yield plateau

⇒ Higher yield strength

⇒ Higher Bond strength

Mild Steel	HYSD.
	

(Plain R/f)

(Grooves पर हुए होते हैं)

Disadvantages:- (i) Length of stress-strain curve of HYSD has reduced compare to MS.

& •) Don't know the location of yield point.

(ii) Less ductility. (in CDT or HYSD)

⇓

So; Fe 500 is not permitted in the design of Earthquake resistant building.

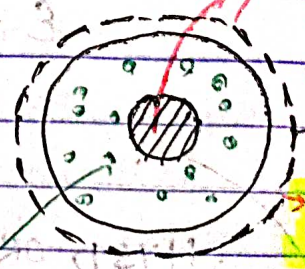
Yield point is not well defined in the case of CTD or HYSD bars.

iii TMT [Thermo Mechanically Treated] steel:-

(Now i may to improve ductility of HYSD)

(upgraded form of HYSD)

- HYSD/CTD.
- Fe 415, Fe 500
- High yield strength
- High Bond strength.
- No yield Plateau
- Ductility is good.

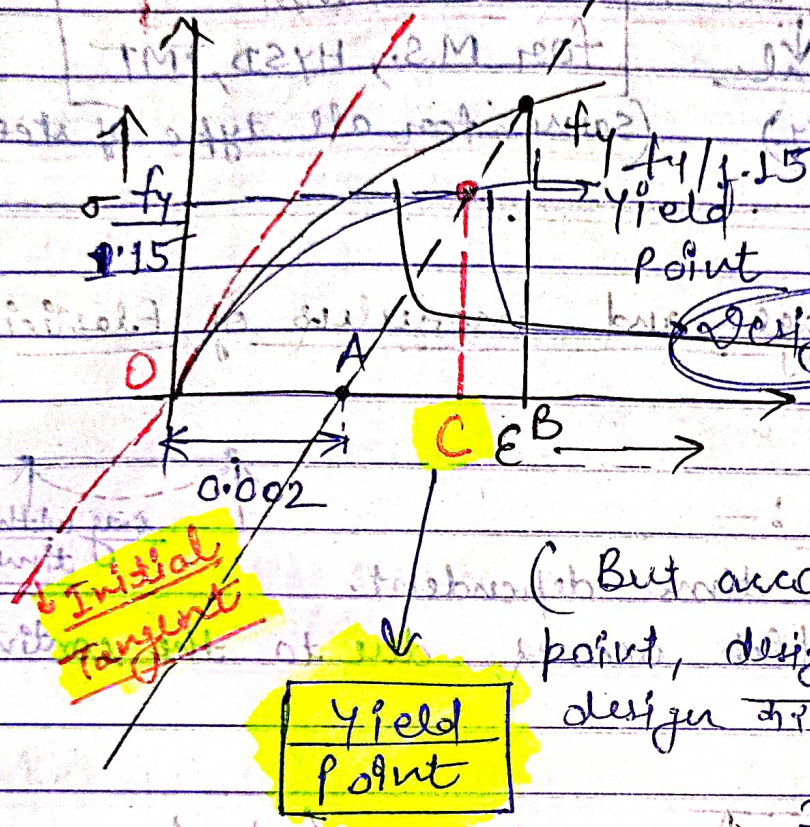


Cross-section of steel reinforcement (सरिया)

⇒ **Ductility** :- M.S. > Fe415 > Fe500

TMT → No yield point known.

Determination of Yield Point in HYSD/CTD Bars :-



(Almost सभी Metal 0.2% strain पर Yielding start देते हैं।) 0.002

(But according to IS code yield point, design curve के According design करना है।)

Yield Point

$$\text{Design strength} = \frac{f_y}{FOS}$$

$$\boxed{FOS \text{ of steel} = 1.15}$$

So yield point will be $\boxed{\varepsilon_C}$

$$\varepsilon = \varepsilon_B = \frac{f_y}{E_s}$$

$$\varepsilon = \varepsilon_C = \frac{f_y}{1.15 E_s}$$

E_s → modulus of elasticity of steel

min. strain

strain at A = 0.002

strain at B = $\left(0.002 + \frac{f_y}{E_s}\right)$

strain at C = $(0.002) + \left(\frac{f_y}{1.15 E_s}\right)$

$$\boxed{\varepsilon_{steel} = 0.002 + \frac{f_y(0.87)}{E_s}}$$

Date 06/09/2020 Sunday

(इतना तो होना ही चाहिए।) $\frac{1}{F.O.S_s} \Rightarrow \left(\frac{1}{1.15} = 0.87\right)$

So; Minimum strain of steel:-

$$\boxed{\varepsilon_{steel} = 0.0024 + \frac{0.87 f_y}{E_s}}$$

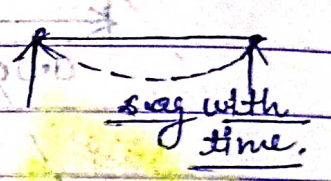
(Min. strain)

(Modulus of Elasticity of steel) (Material Property) $E_s \Rightarrow \boxed{2 \times 10^5 \ N/mm^2 \text{ OR } 200 \ kN/mm^2}$ for M.S., HYSD, TMT (same for all type of steel)

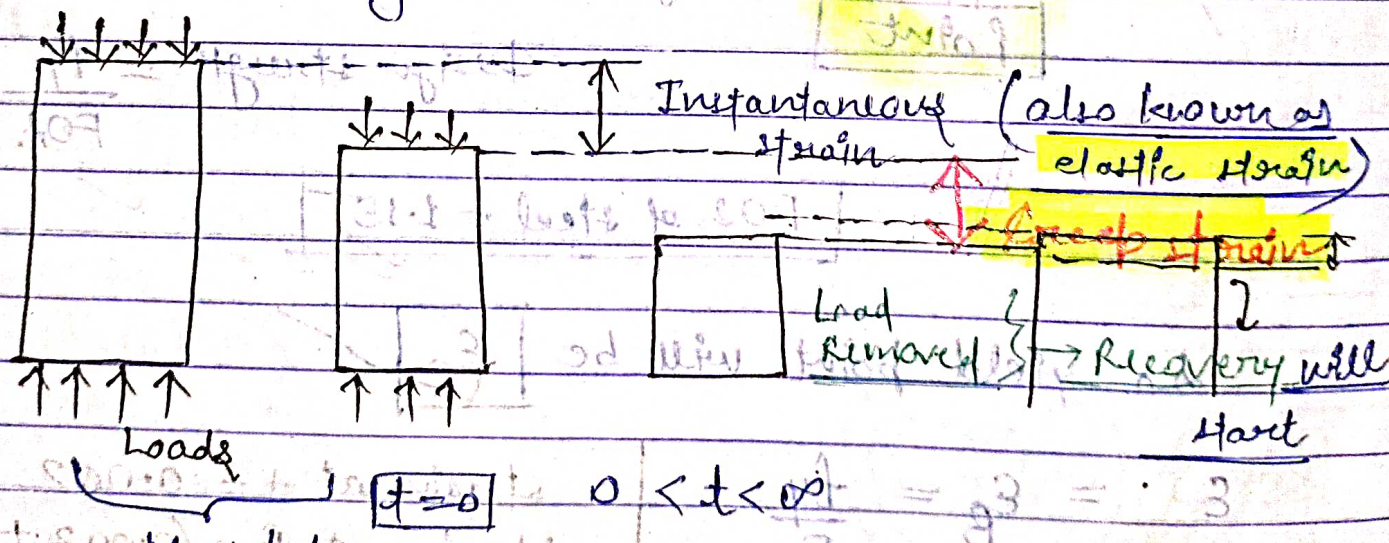
#02

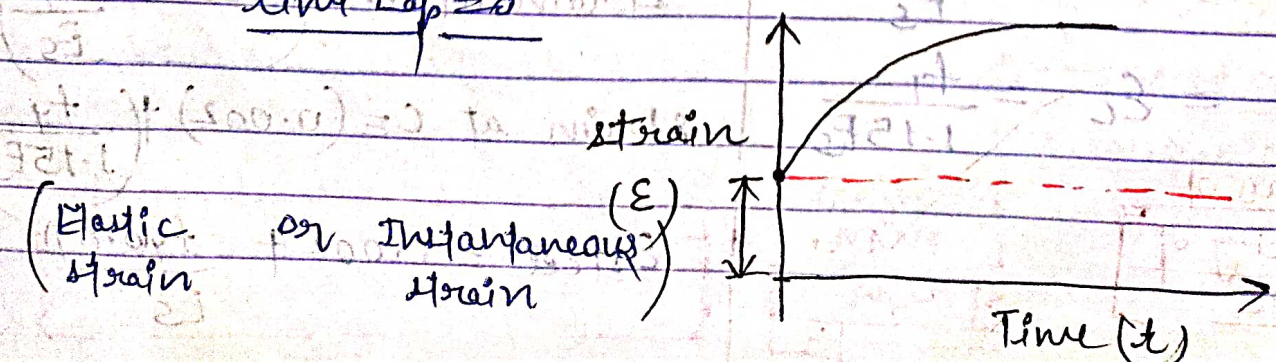
Lecture 2:- Creep, Shrinkage and modulus of Elasticity of concrete.

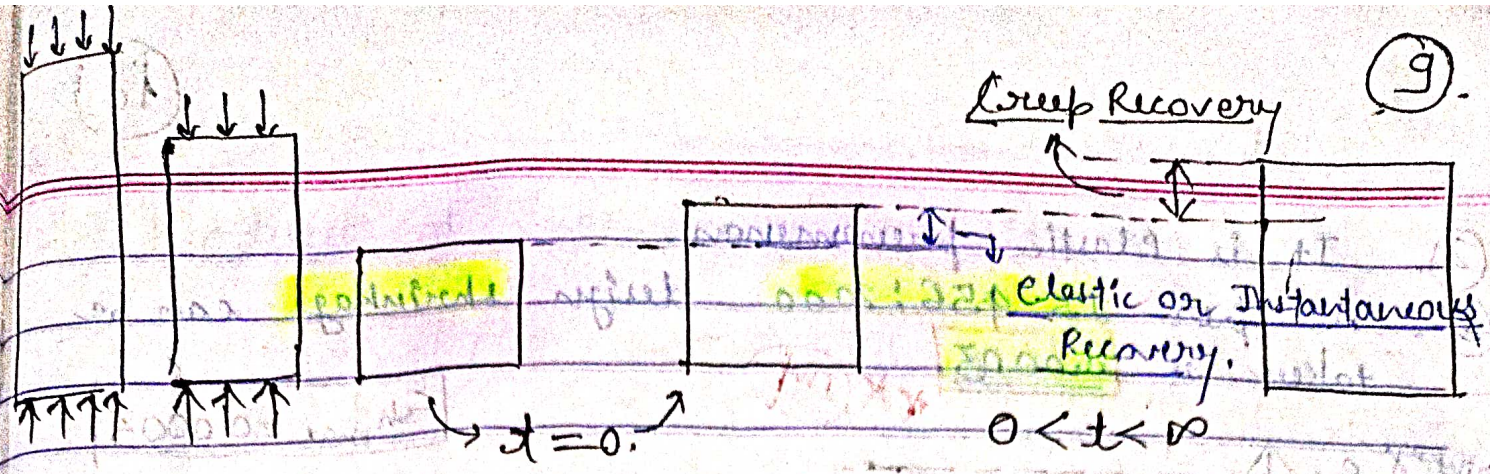
Creep and Shrinkage:-

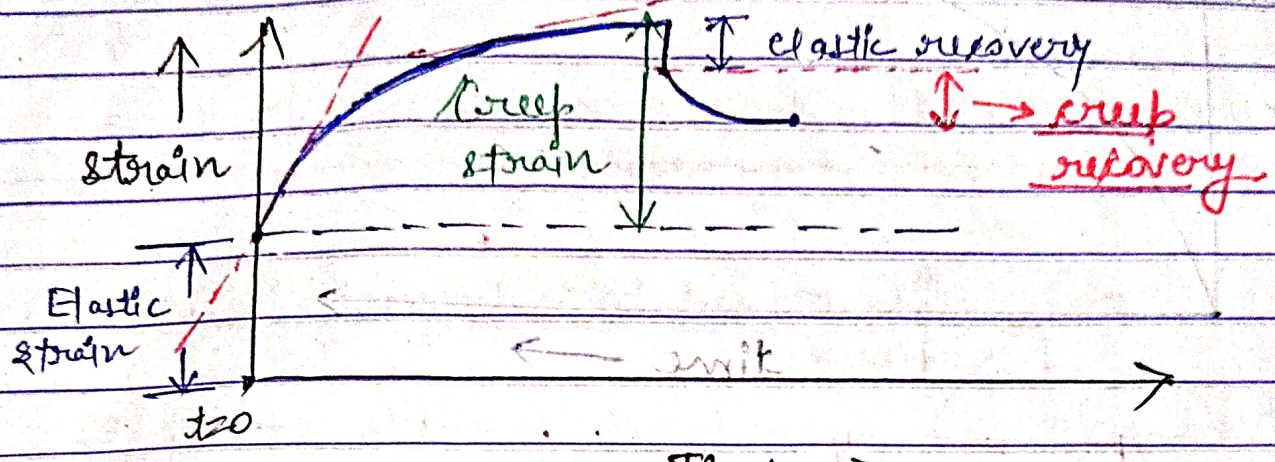


Creep :→ It is a time dependent phenomenon which occurs due to the continuous dead loading.









Creep coefficient (ϕ) =	Creep strain / Elastic strain

~~Shrinkage~~ :→ As Per IS 456 : 2000 :→

Time	7 days	28 days	1 year
ϕ	2.2	1.6	1.1

ϕ is decreasing (↓) with the time (because Rate of increment of creep is decreasing)
[see curve (slope ↓)]

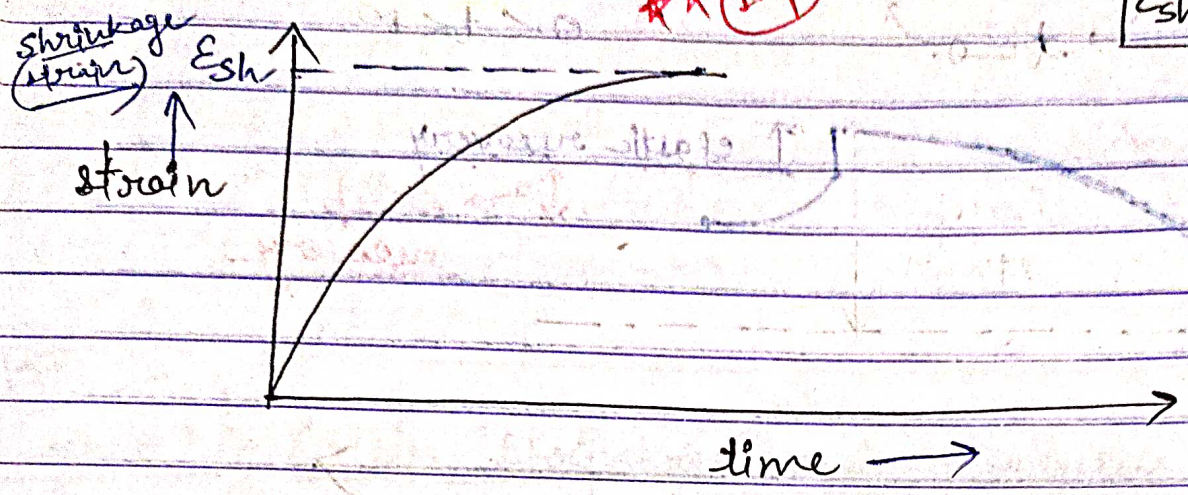
Shrinkage :→ (Time dependent plastic phenomenon)

① Reduction in the volume of concrete due to loss of water is called shrinkage of concrete.

(2). It is Plastic phenomenon.

(3). As per IS 456:2000 design shrinkage can be taken as 0.0003. ★★ (Imp)

$\varepsilon_{sh_{max}} = 0.0003$



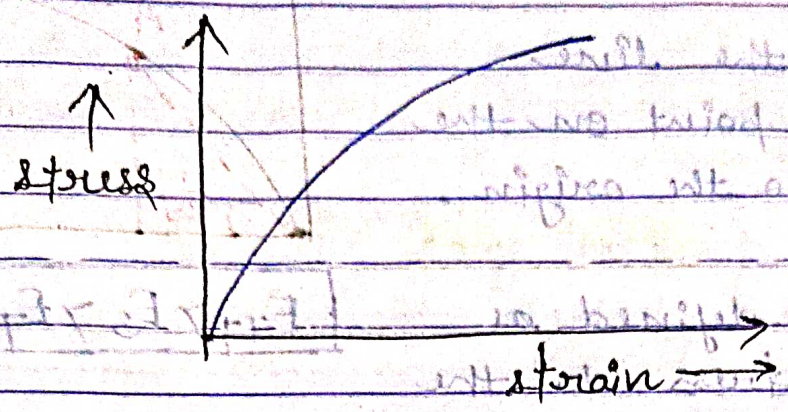
Reason of water loss can be → Ambient Temperature
→ Hydration of cement.
→ Chemical hydration

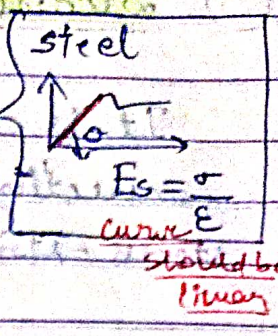
◉ Factors Affecting creep & shrinkage :—

- If w/c Ratio ↑ → creep ↑ → shrinkage ↑
  strain
  → (Cement Paste ✓ ↓)
- Cement Content ↑ → creep ↑ → shrinkage ↑
- Ambient Temperature ↑ → creep ↑ → shrinkage ↑
- Age of Concrete ↑ → creep ↓ → shrinkage ↓
- Relative Humidity ↑ → creep ↓ → shrinkage ↓

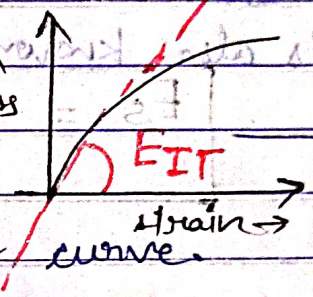
Imp Topic
** GATE

Modulus of Elasticity of Concrete (Brittle Material) :→





① Initial tangent Modulus → (E_{IT})

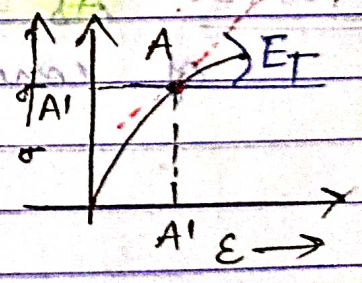


It is the slope of the tangent from the origin of the curve.

E_{IT} → Highest

It is also known as 'dynamic modulus of elasticity' of concrete. (क्योंकि starting में, motion or disturbance करने पर vibration or disturbance हल्के से आ जाते हैं।)

② Tangent Modulus [E_T] :→



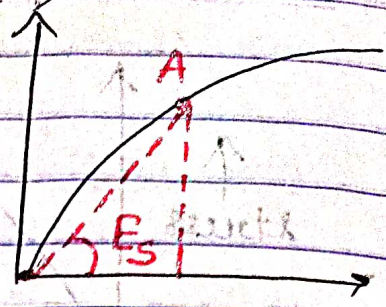
It is the slope of the tangent drawn at any point on the stress-strain curve.

$$E_T = \frac{\text{Instantaneous stress}}{\text{Instantaneous strain}}$$

Tangent modulus can also be define as the ratio of instantaneous stress to the instantaneous strain.

3. Secant Modulus of Elasticity :- (E_s)

It is the slope of the line connecting the any point on the stress-strain curve to the origin.



- It can also be defined as the ratio of total stress to the total strain.

$$\boxed{E_{IT} > E_s > E_T}$$

It is also known as Static modulus of Elasticity.

$$\boxed{E_s = \frac{\text{total stress}}{\text{total strain}}}$$

Imp

NOTE :-

$$\boxed{E_{IT} > E_s > E_T}$$

E_{IT} — dynamic, E_s — static

NOTE:- As per IS 456 : 2000 The Modulus of Elasticity of concrete can be given as,

$$\boxed{E_c = 5000\sqrt{f_{ck}}} \ N/mm^2$$ **

$E_c = 5000\sqrt{f_{ck}}$ is known as Short term Static Modulus of Elasticity.

(bcz we're not considering creep behaviour)

(Independent of Time)

so $\boxed{E_c \text{ is static or secant modulus of Elasticity.}}$

Long term static modulus of Elasticity:-

As per code short term static modulus of Elasticity can be given as $5000\sqrt{f_{ck}}$ = E_c

In the long term; static modulus of elasticity, we consider the effect of creep.

(Imp)

$$E_{cr} = \frac{E_c}{1+\phi}$$

E_{cr} → creep

OR

ϕ → creep coefficient

★★ (GATE Imp)

$$E_{cr} = \frac{5000\sqrt{f_{ck}}}{1+\phi}$$

(f_{ck} in N/mm² OR MPa)

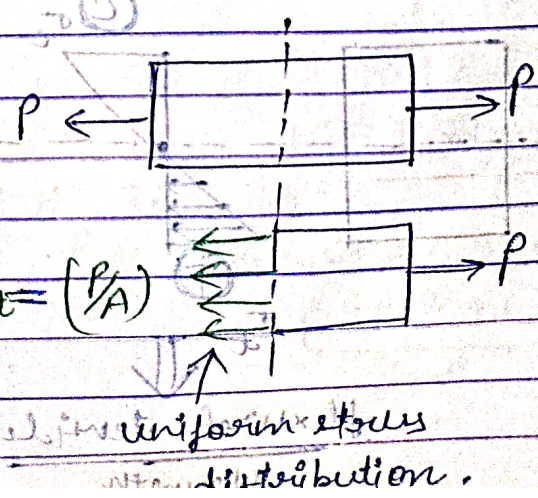
time	7 days	28 days	1 year
ϕ	2.2	1.6	1.1

Date 07/09/2020 Monday

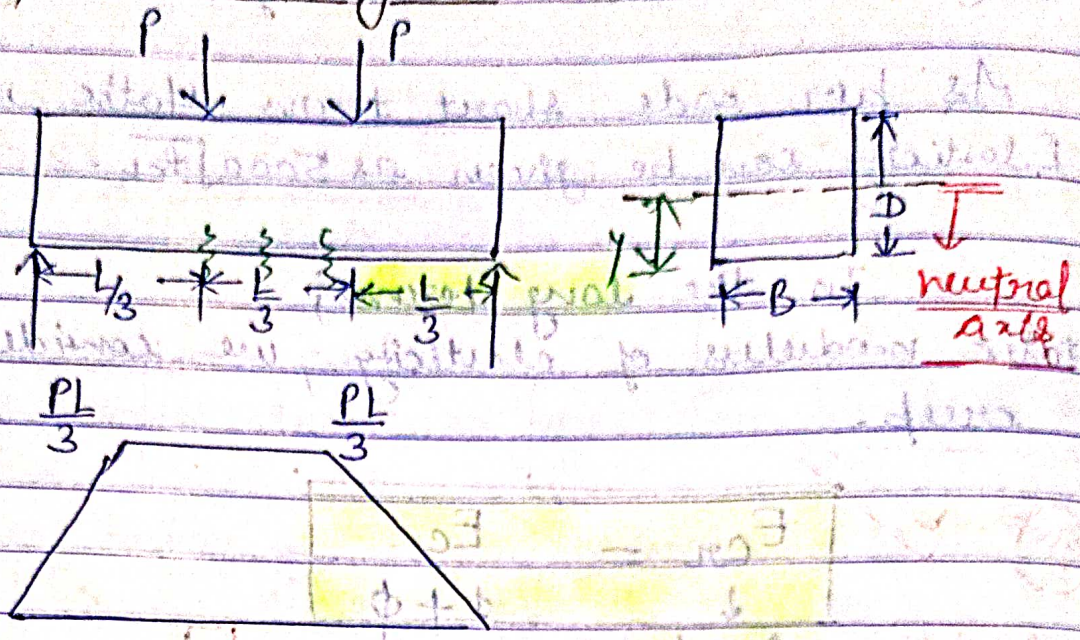
#03

Tensile strength of concrete

(i) Direct Tensile strength (σ_{dt})



2. Flexural Tensile strength :→



$$\boxed{B.M_{max} \Rightarrow \frac{PL}{3}}$$

$$\boxed{I = \frac{BD^3}{12}}$$

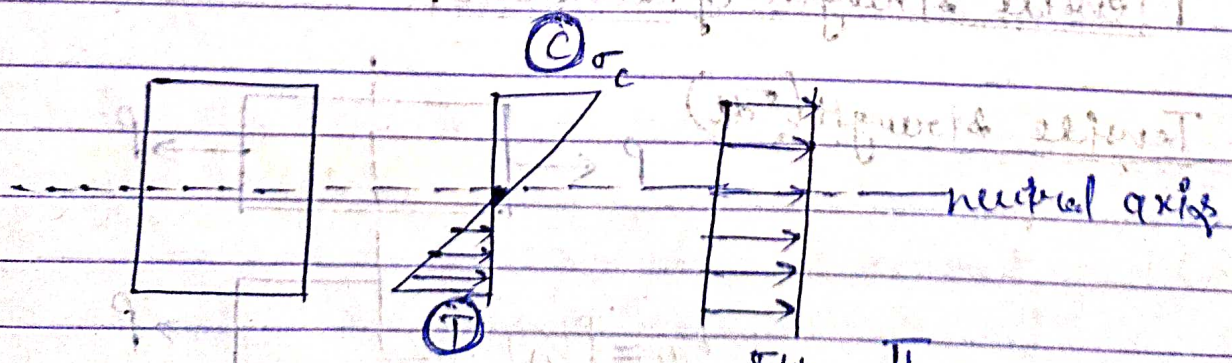
⊨ **Bending equation :—**

$$\frac{M}{I} = \frac{\sigma}{y}$$

So, $\frac{B.M_{max}}{I} \cdot y = \sigma$

$$\boxed{\sigma_t = \frac{(B.M_{max})}{I}\, y}$$

$\boxed{y = D/2}$ (distance from neutral axis)



σ_t ⇓ flexural tensile strength

σ_{dt} ⇓ direct tensile strength

As per IS 456 : 2000,
The flexural tensile strength of concrete.

★★★ $f_{cr} = 0.7\sqrt{f_{ck}}$ (N/mm²)

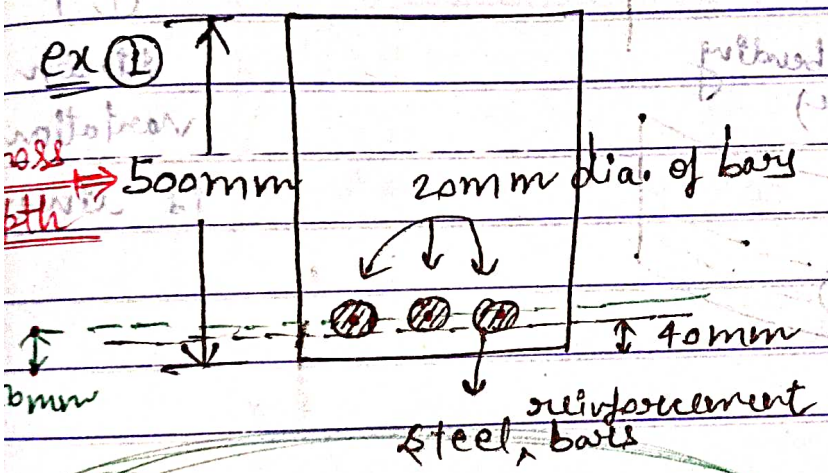
f_{cr} ⇒ flexural tensile strength or modulus of rupture (Imp)

flexure → means bending

Nominal clear cover :→

It is the minimum clear cover to the outer most steel R/f.

ex (1)



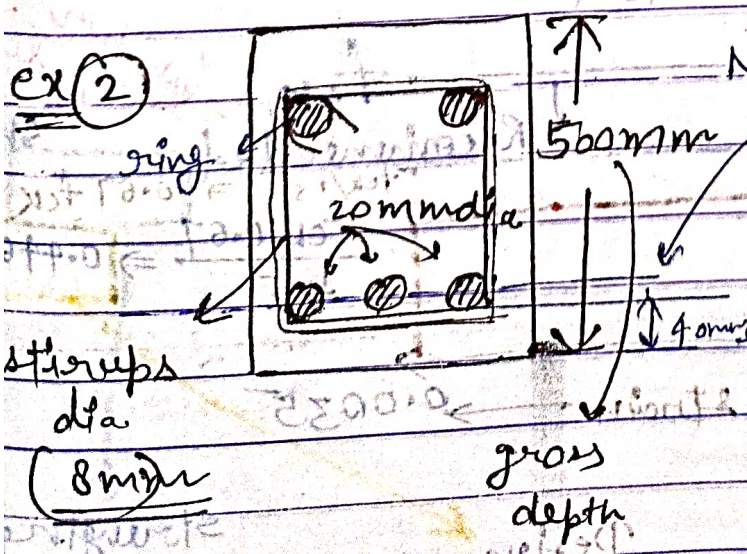
effective cover ⇒ (from ~~centre~~ c.g. of bar)
⇒ 50 mm

effective depth (d) ⇒
D − effective cover

Nominal clear cover = 40mm (here)

⇒ d = 500 − 50
⇒ d = 450mm

ex (2)



Nominal clear cover ⇒ (40mm)

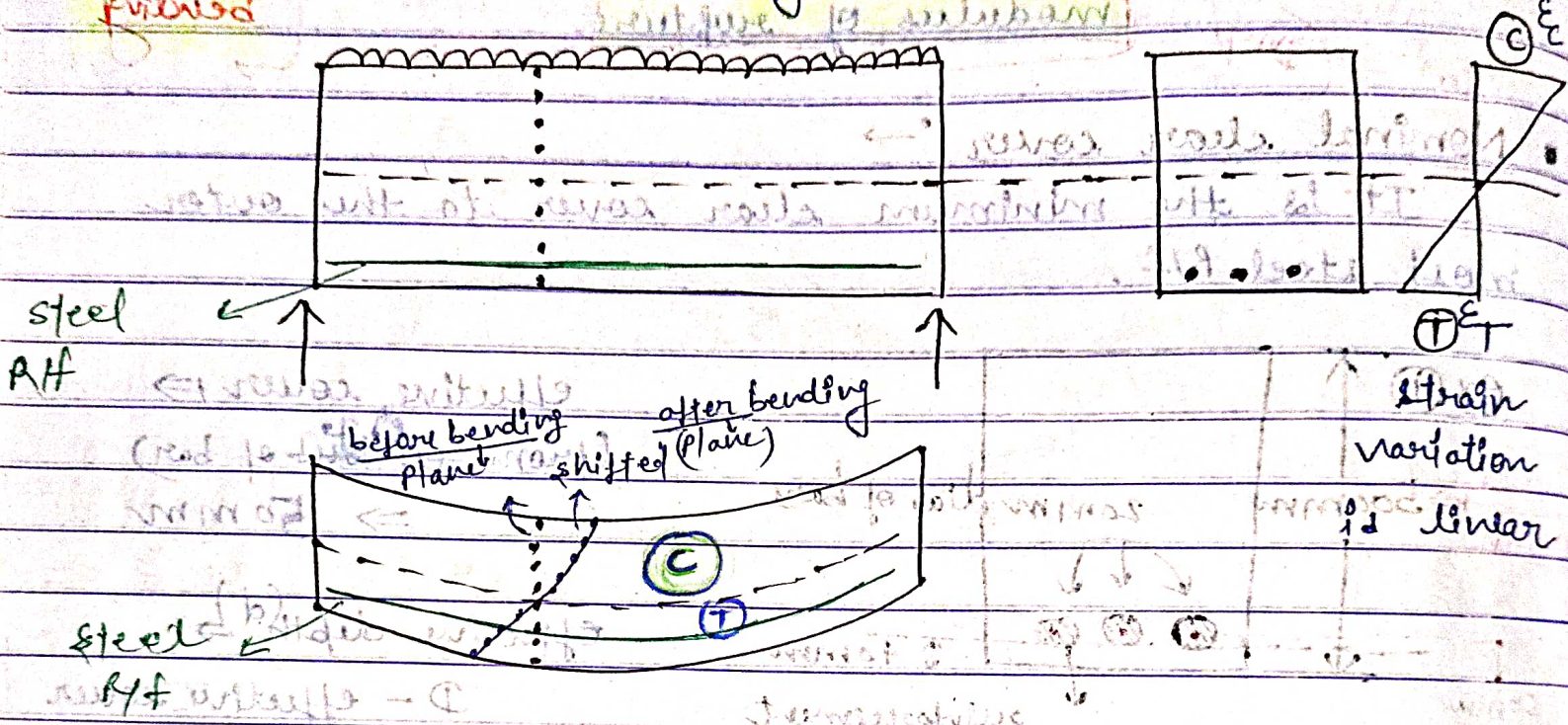
effective cover ⇒ 40 + 8 + 10
⇒ (58 mm)

effective depth ⇒
500mm − 58 mm
⇒ (442 mm)

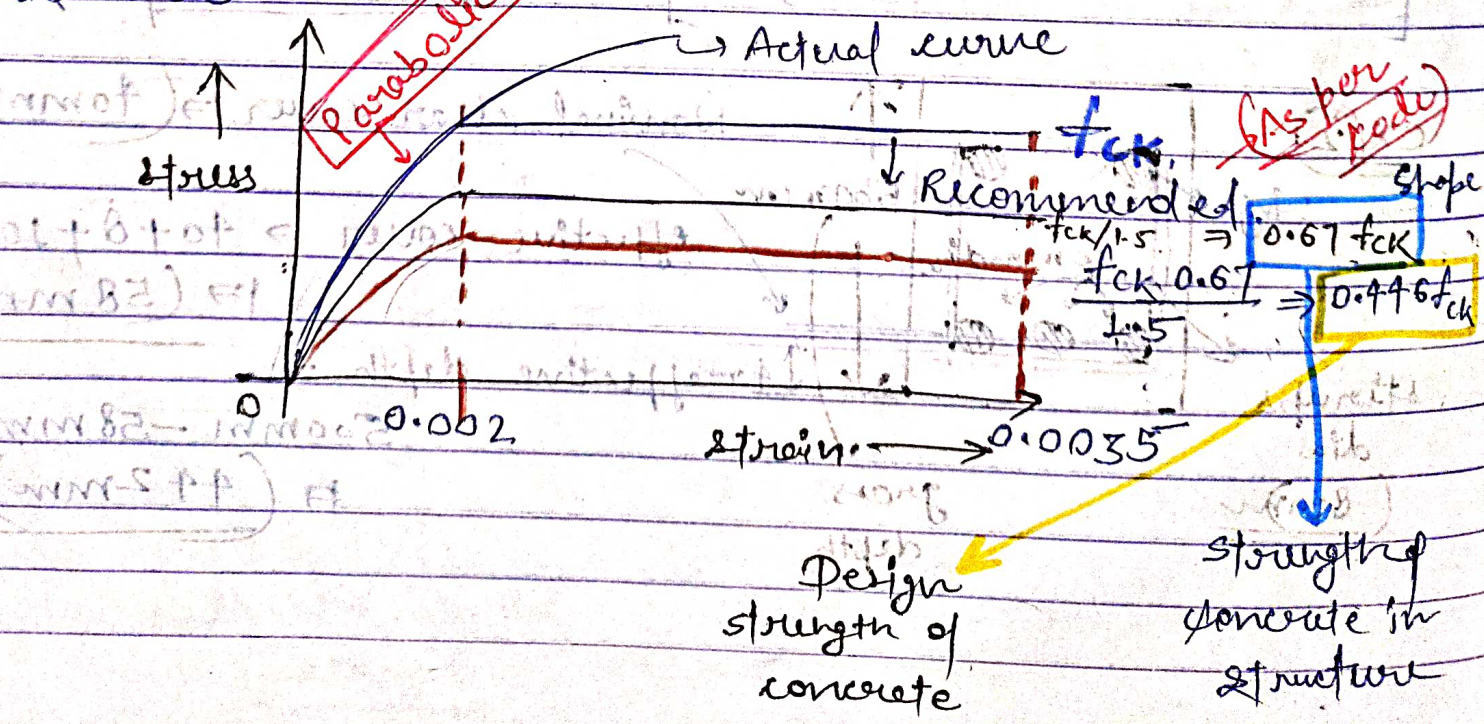
Singly Reinforced Beam :-

Limit state of collapse : Flexure ⇒

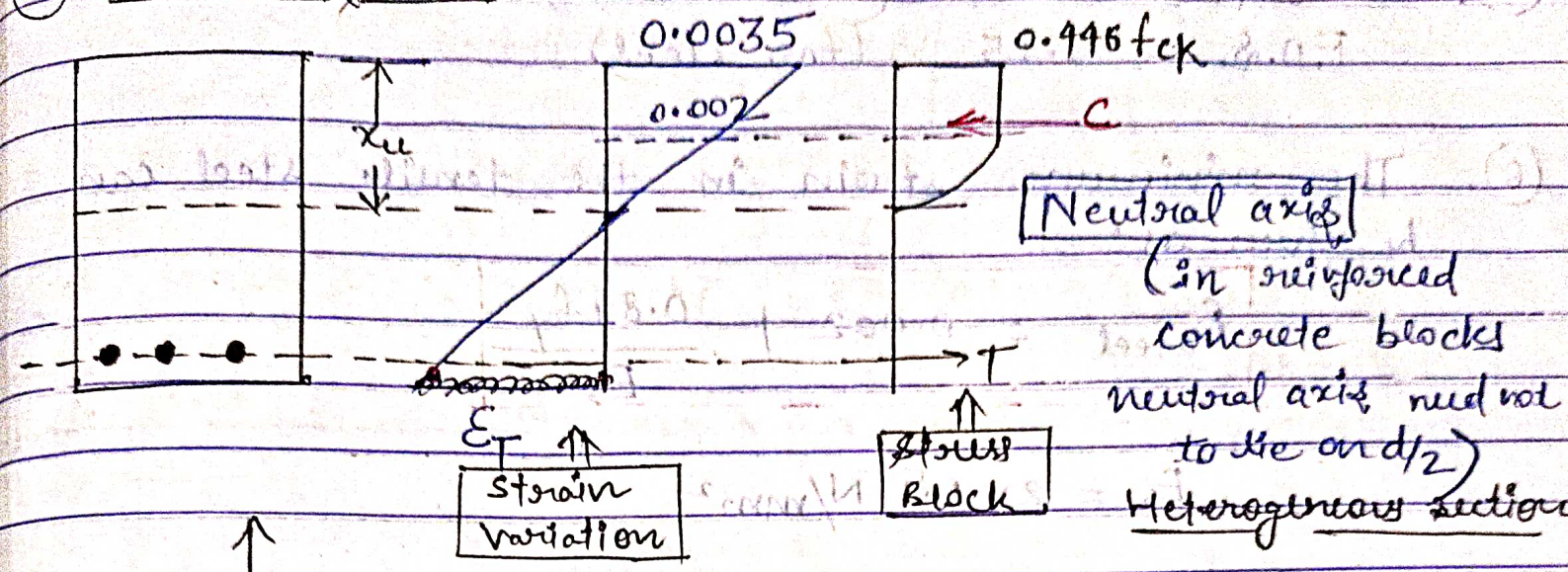
• Assumption :- ① The plane section remains plane before and after the bending.

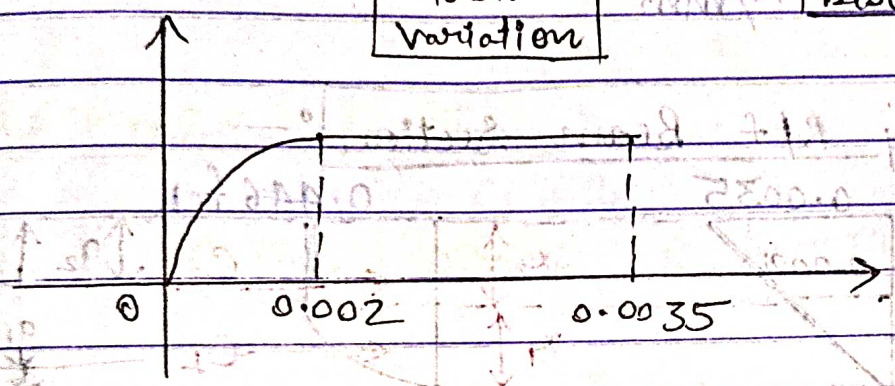


② The max compressive strain in concrete in flexure is 0.0035.

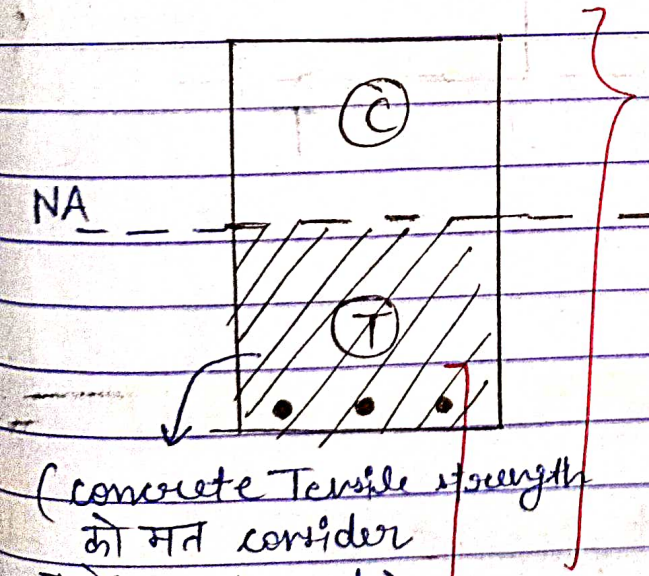


3 Stress Block :—





4. All the tensile stresses shall be taken by steel only.

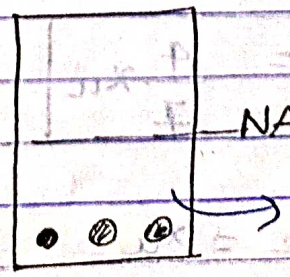


(concrete Tensile strength को मत consider करो acc. to code)

(Ineffective concrete below N.A.)

That means the area of concrete below the neutral axis is ignored.

This is based on cracked section theory.



concrete Below the NA is assumed to be cracked.

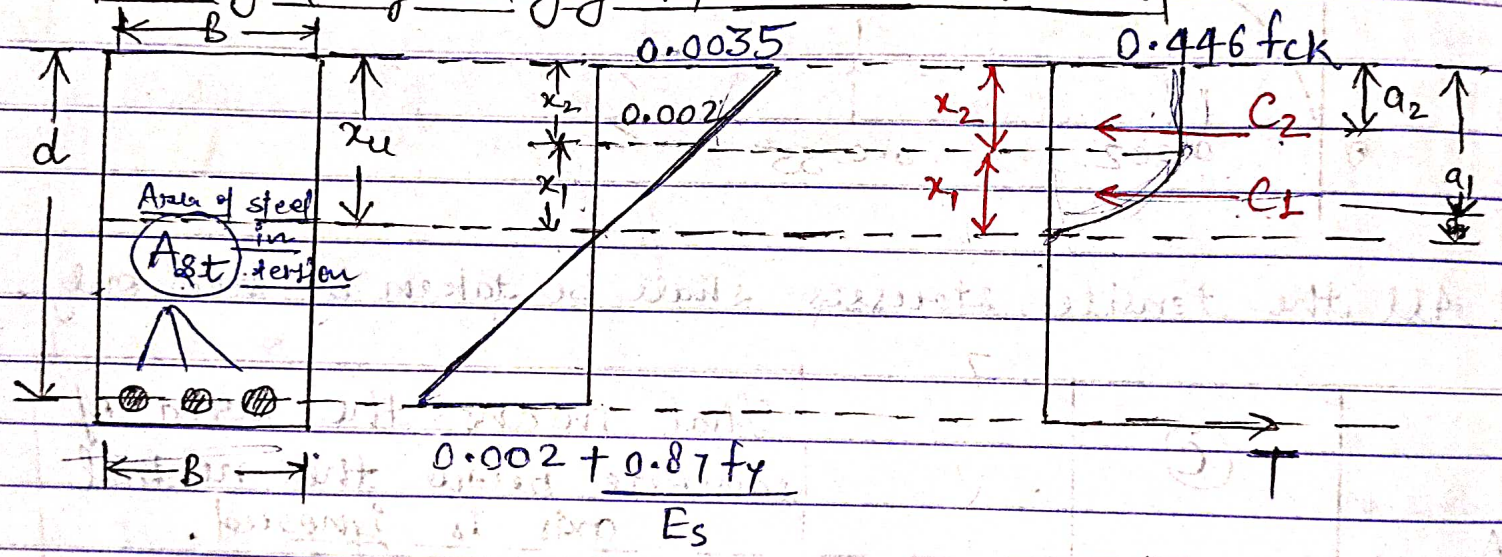
5. F.O.S. = 1.5 (for concrete)
F.O.S. = 1.15 (for steel)

6. The minimum strain in the tensile steel can be given as :-

$$\boxed{\varepsilon_{steel} = 0.002 + \frac{0.87 f_y}{E_s}}$$

$$E_s = 2 \times 10^5 \ N/mm^2$$

Analysis of singly R/f Beam section :-



$$\frac{0.0035}{x_u} = \frac{0.002}{x_1}$$

$$\boxed{x_1 = \frac{4}{7} x_u}$$

$$x_1 + x_2 = x_u$$

$$\boxed{x_2 = \frac{3}{7} x_u}$$

force = stress × Area

compressive force

$$C_1 = (0.446 f_{ck})\left[\frac{2}{3} \times x_1 \times B\right]$$ (Area of Parabola)

$$C_1 = 0.446 f_{ck} \times \frac{2}{3} \times \frac{4}{7} x_u \times B$$

$$C_2 = (0.446 f_{ck})[x_2 \times B]$$ (Area of Rectangle)

$$C_2 = 0.446 f_{ck} \times \frac{3}{7} x_u \times B$$

Total compressive force :-

$$C = C_1 + C_2$$

$$C = 0.36 f_{ck} b x_u$$ → compressive force with partial safety factor

$$a_2 = \frac{x_2}{2} = \frac{3}{7} \times \frac{1}{2} x_u$$

$$a_1 = x_2 + \frac{3}{8} x_1$$

$$a_1 = \frac{3}{7} x_u + \frac{3}{8} \times \frac{4}{7} x_u$$

GATE 1991

compressive force without partial safety factor;

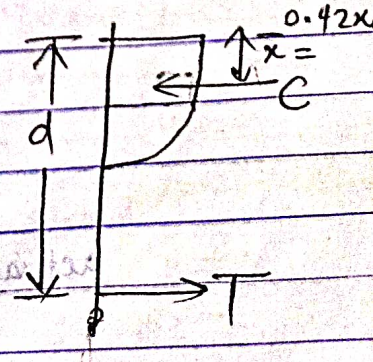
$$C = 0.36 \times 1.5 f_{ck} b x_u$$

$$C = 0.54 f_{ck} b x_u$$

Location of total compressive force "C"

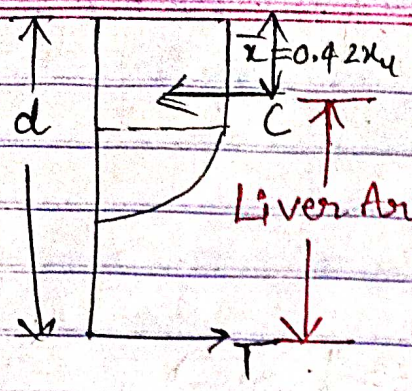
$$\bar{x} = \frac{C_1 a_1 + C_2 a_2}{C_1 + C_2}$$

$$\bar{x} = 0.42 x_u$$ from top fibre



(C & T applying along the length)

$\frac{f_y}{1.15} = 0.87 f_y$



Estimation of tensile force (T)

$\Rightarrow \sigma \times \text{Area}$

$T = 0.87 f_y \times A_{st}$

★★★ $\boxed{T = 0.87 f_y A_{st}}$ → (At the centre (c.g.) of steel reinforcement)

(C & T trying to rotate, creating a moment)

★ $\boxed{\text{Lever Arm} = d - 0.42x_u}$

Moment of Resistance $(MOR)_C = C \cdot LA$

depth of neutral axis

★★★ $\boxed{(MOR)_C = 0.36 f_{ck} b x_u (d - 0.42x_u)}$

grade of concrete

(मोमेंट) equal same

$(MOR)_T = T \cdot LA$

★★★ $\boxed{(MOR)_T = 0.87 f_y A_{st} (d - 0.42x_u)}$

How to calculate depth of neutral axis (x_u)

Put $C = T$

$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$

$\Rightarrow \boxed{x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}}$

actual depth of neutral axis ★★

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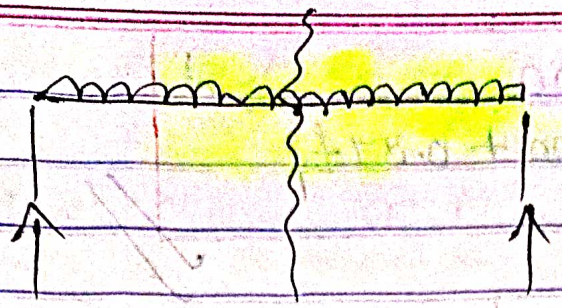
Road



(तोड़ने वाली ताकत) → $B.M. = \frac{wl^2}{8}$ kN.m

(बचाने वाली ताकत) → $MOR = 0.36 f_u b x_u (d - 0.42 x_u)$ kN.m

So we require, ⇒ $\boxed{M.O.R. > B.M.}$ Safe

$\boxed{\text{8-september 2020}}$:- $\boxed{\#04}$

•) Singly reinforced beam :-

$C = 0.36 f_{ck} b x_u$

$T = 0.87 f_y A_{st}$

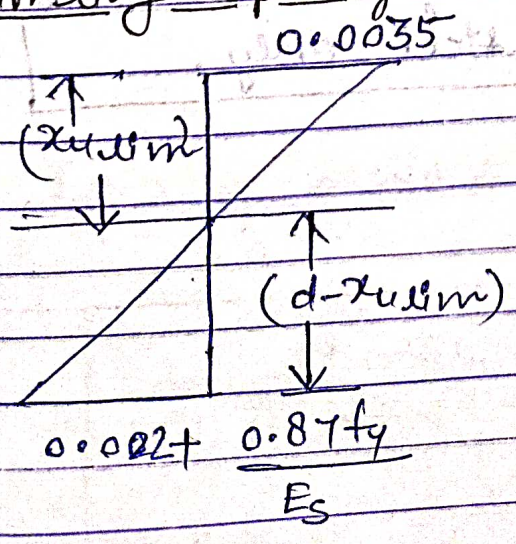
$(MOR)_C = 0.36 f_{ck} b x_u (d - 0.42 x_u)$ → Lever Arm

$(MOR)_T = 0.87 f_y A_{st} (d - 0.42 x_u)$

⇒ $$\boxed{x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}}$$

•) limiting depth of neutral axis :- (OR Max. depth)

$\boxed{x_{u,lim} \text{ OR } x_{u,max}}$



$$\frac{0.0035}{x_{u,lim}} = \frac{\left(0.002 + \frac{0.87 f_y}{E_s}\right)}{d - x_{u,lim}}$$

OR ⇒ $$\frac{d - x_{u,lim}}{x_{u,lim}} = \frac{0.002 + \frac{0.87 f_y}{2 \times 10^5}}{0.0035}$$

$\Rightarrow$ $$x_{u\,lim} \text{ OR } x_{u\,max} = \frac{700}{1100 + 0.87 f_y} \cdot d$$

$$x_{u\,lim} = K \cdot d$$

K → neutral axis constant.

$$K = \frac{700}{1100 + 0.87 f_y}$$

- for Fe250; $$K = \frac{700}{1100 + 0.87(250)} = 0.53$$

$$x_{u\,lim} \text{ OR } x_{u\,max} = 0.53d$$

- for Fe415; $$x_{u\,lim} = 0.48d$$

- for Fe500; $$x_{u\,lim} = 0.46d$$

The maximum/limiting depth of neutral axis depends upon the grade of steel only.

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

x_u ↓ actual depth of neutral axis

we've studied; MOR = C.LA
= T.LA

Now $M_{u,lim}$ OR $M_{u,max}$ ⇒ ?

⇒ $(MOR)_c = 0.36 f_{ck} b x_u (d - 0.42 x_u)$

$M_{u,lim}$ OR $M_{u,max} = 0.36 f_{ck} b x_{u,lim} (d - 0.42 x_{u,lim})$

• for fe 250; $M_{u,lim} = 0.36 f_{ck} b (0.53d)[d - 0.42(0.53d)]$

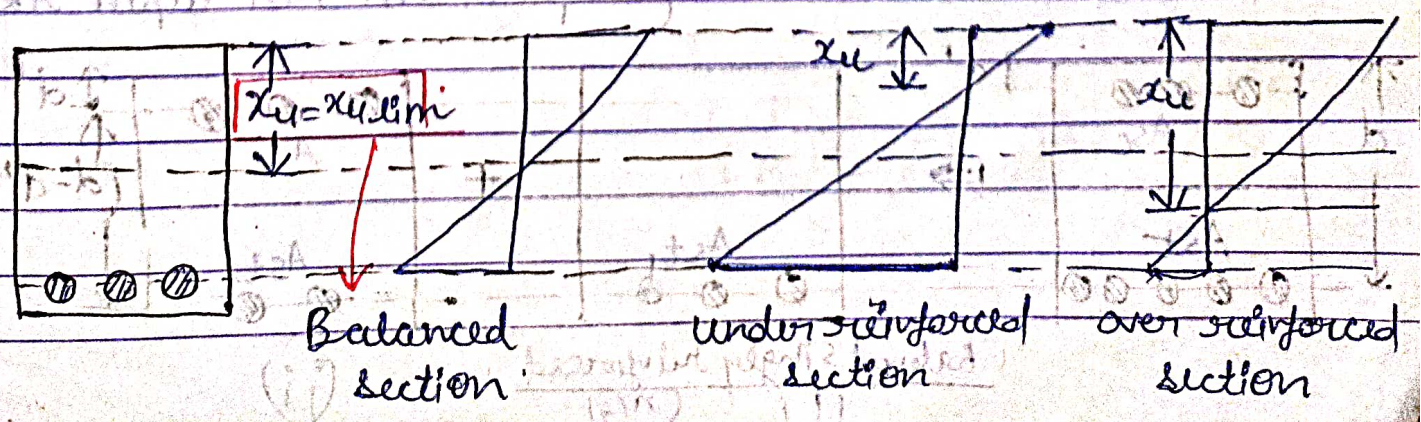
remember

⇒ ★★★ $\boxed{M_{u,lim} = 0.148 f_{ck} b d^2}$

Grade of steel	$x_{u,lim} = Kd$	$M_{u,lim}$
Fe250	0.53d	$0.148 f_{ck} b d^2$
Fe415	0.48d	$0.138 f_{ck} b d^2$
Fe500	0.46d	$0.133 f_{ck} b d^2$

} Calculate similarly as for Fe 250.

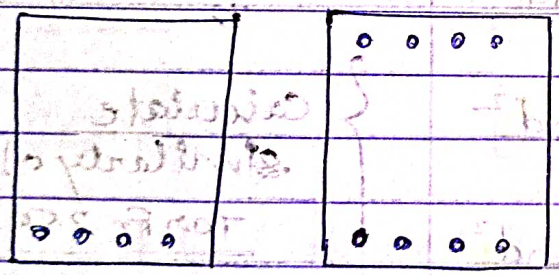
Types of sections :-



$\sigma_{c\,perm.} = 0.446 f_{ck}$
$\sigma_{s\,perm.} = 0.87 f_y$

Balanced section	Under reinforced section	Over Reinforced section
$\sigma_c \geq \sigma_{c\,perm}$	$\sigma_s \geq \sigma_{s\,perm}$	$\sigma_c \geq \sigma_{c\,perm}$
$\sigma_s \geq \sigma_{s\,perm}$	$\sigma_c < \sigma_{c\,perm}$	$\sigma_s < \sigma_{s\,perm}$
$\left\{ x_u = x_{u\,lim},\ MOR = M_{u\,lim} \right\}$	$x_u < x_{u\,lim}$	$x_u > x_{u\,lim}$
	Ductile failure	Brittle failure
	$\boxed{A_{st} < (A_{st})_{Bal.}}$	$\boxed{A_{st} > (A_{st})_{Bal.}}$
	we get Alarm	No alarm. (Strictly prohibited)

•) **Analysis of Doubly Reinforced Beam :—**

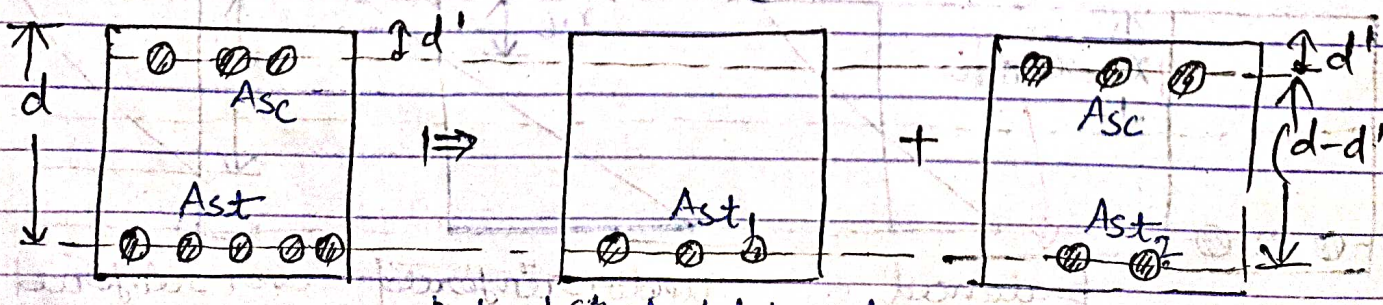


singly reinforced — doubly reinforced

Reason for design maybe :—

(i) $M_u > M_{u\,lim}$ $\propto d^2$
↓ 100 kN-m ↓ 50 kN-m

(ii) : depth restricted.
(aap ekdm km depth rkhna chahte ho)



Balanced singly reinforced (i) — (ii)

if — $M_u = 200\,kN.m$

Balanced singly

$M_{OR} \rightarrow M_{u\,lim}$

$x_u \longrightarrow x_{u\,lim}$

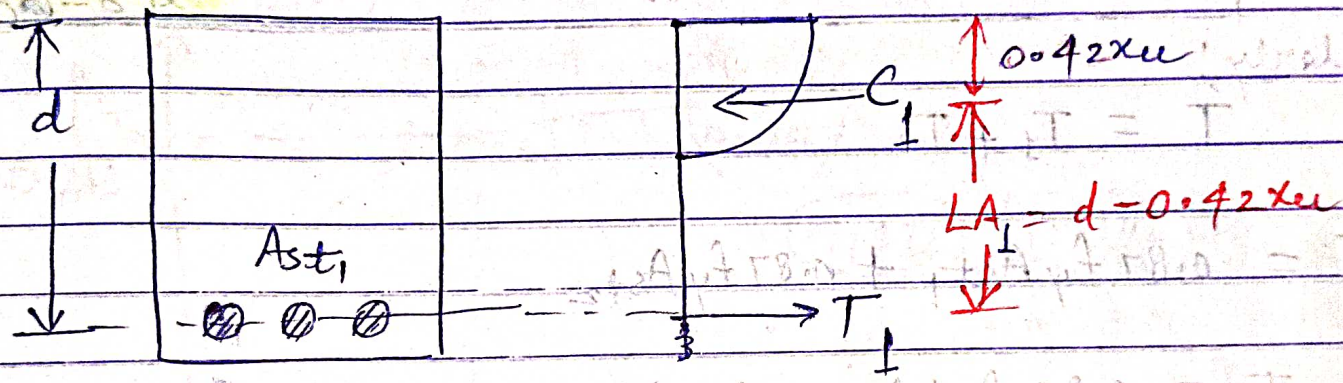
eg (50-kN-m)

Capacity $\rightarrow (M_u - M_{u\,lim})$

eg (150kN-m)

fig(i) | fig(ii)

fig(i) Analysis :



$\Rightarrow$ $C_1 = 0.36 f_{ck}\, b\, x_u$

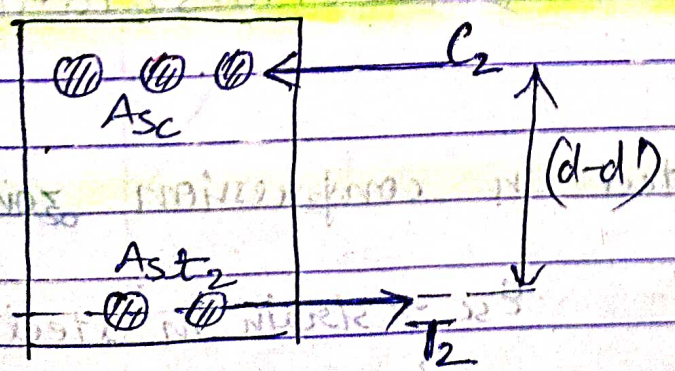
$T_1 = 0.87 f_y Ast_1$

$LA_1 = (d - 0.42x_u)$

fig(ii) Analysis $\rightarrow$

let (compressive stress)

$C_2 = f_{sc} Asc - 0.446 f_{ck} Asc$



$T_2 = 0.87 f_y Ast_2$

$LA_2 = (d - d')$

$$\boxed{M_u - M_{u,lim} = f_{sc} A_{sc} (d-d')}$$

Total compressive force

$$C = C_1 + C_2$$

$$C = 0.36 f_{ck} b x_u + A_{sc} (f_{sc} - 0.446 f_{ck})$$

$$(M.O.R)_C \Rightarrow C_1 LA_1 + C_2 LA_2$$

$$\boxed{\underset{bd^2}{(MOR)_C} \Rightarrow \underbrace{0.36 f_{ck} b x_u (d - 0.42 x_u)}_{bd^2} + \underbrace{A_{sc}(f_{sc} - 0.446 f_{ck})}_{b'd^2} \times (d - d')}$$

similarly;

$$\Rightarrow T = T_1 + T_2$$

$$\Rightarrow T = 0.87 f_y A_{st_1} + 0.87 f_y A_{st_2}$$

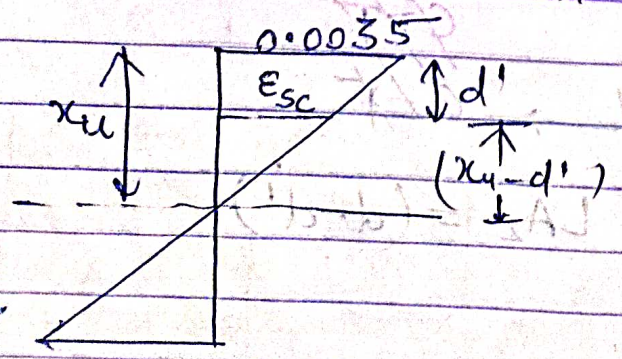
$$\Rightarrow T = 0.87 f_y (A_{st_1} + A_{st_2})$$

$$\boxed{T = 0.87 f_y A_{st}}$$

$$(MOR)_T = T_1 LA_1 + T_2 LA_2$$

$$\boxed{(MOR)_T = 0.87 f_y A_{st_1} (d - 0.42 x_u) + 0.87 f_y A_{st_2} (d - d')}$$

$\Rightarrow$ $f_{sc} \rightarrow$ stress in steel in compression zone.



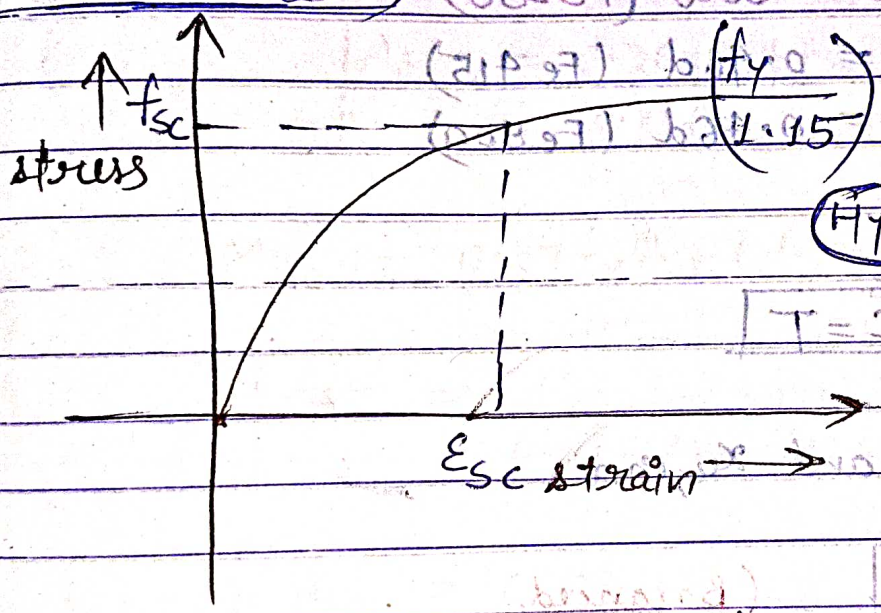
$\varepsilon_{sc} \rightarrow$ strain in steel in compression zone.

See: IS 456 : 2000 Page No. 96

$$\frac{0.0035}{x_u} = \frac{\varepsilon_{sc}}{x_u - d'}$$

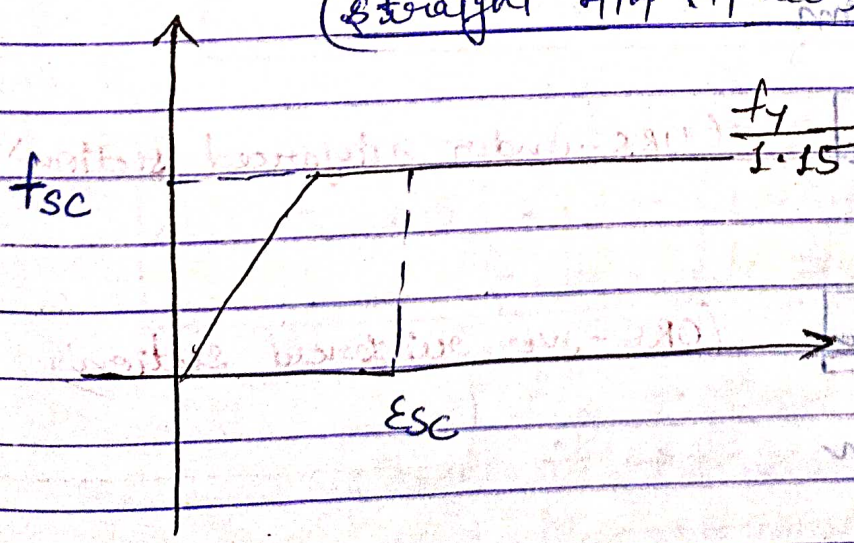
$$\Rightarrow \varepsilon_u = 0.0035\left(\frac{x_u - d'}{x_u}\right)$$

★★★ $\Rightarrow$ (strain in steel in compression zone) $\rightarrow$ $\boxed{\varepsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_u}\right)}$



HYSD / TMT / CTD

(straight मान लो acc to code)



$f_{sc} \rightarrow$ fixed

$\Rightarrow \boxed{f_{sc} = 0.87 f_y}$

Design of Over reinforced section is strictly prohibited by IS code.

- Estimation of Moment of Resistance in singly R/f & Doubly R/f sections :—

Step-1 Calculate $x_{u,lim}$

$$x_{u,lim} = 0.53d \ (Fe\,250)$$
$$= 0.48d \ (Fe\,415)$$
$$= 0.46d \ (Fe\,500)$$

Step-2 Calculate x_u

Put $\boxed{C = T}$

Step-3 Compare x_u and $x_{u,lim}$:

if $\boxed{x_u = x_{u,lim}}$ (Balanced)

⇒ use $x_{u,lim}$

if $\boxed{x_u < x_{u,lim}}$ (URS - under reinforced section)

⇒ use x_u.

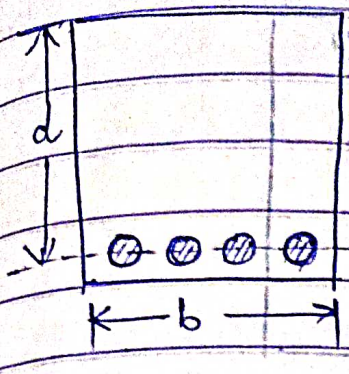
if $\boxed{x_u > x_{u,lim}}$ (ORS - over reinforced section)

⇒ use $x_{u,lim}$

Step-4 Calculate MoR

In singly / doubly R/f Beam.

Design of Singly R/F Beam :-



known → $B, d, f_{ck}, f_y, x_u, M_u$

Put $C = T$

$\Rightarrow 0.36 f_{ck} b x_u = 0.87 f_y A_{st}$

$\Rightarrow \boxed{A_{st} = \frac{0.36 f_{ck} b x_u}{0.87 f_y}}$ → put $x_{u\,lim}$

design for balanced section;

$$\boxed{A_{st} = \frac{0.36 f_{ck} b x_{u\,lim}}{0.87 f_y}} \quad (1)$$

$\because$ $MOR = 0.87 f_y A_{st} (d - 0.42 x_u)$ ⇐ (T (lever Arm))

Factored B.M.

Take care here

$\Rightarrow$ $$\boxed{A_{st} = \frac{M_u}{0.87 f_y (d - 0.42 x_u)}}$$ → put $x_{u\,lim}$

$\Rightarrow$ $$\boxed{A_{st} = \frac{M_u}{0.87 f_y (d - 0.42 x_{u\,lim})}} \quad (2)$$

Imp ★★

$$\boxed{A_{st} = 0.5 \frac{f_{ck}}{f_y} \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}\right] bd} \quad (3)$$

(No need of x_u here)

$f_{ck} \rightarrow N/mm^2$ OR MPa

$f_y \rightarrow N/mm^2$ OR MPa

$M_u \rightarrow N\text{-}mm$ $\quad \boxed{1\,kN\text{-}m = 10^6\,N\text{-}mm}$

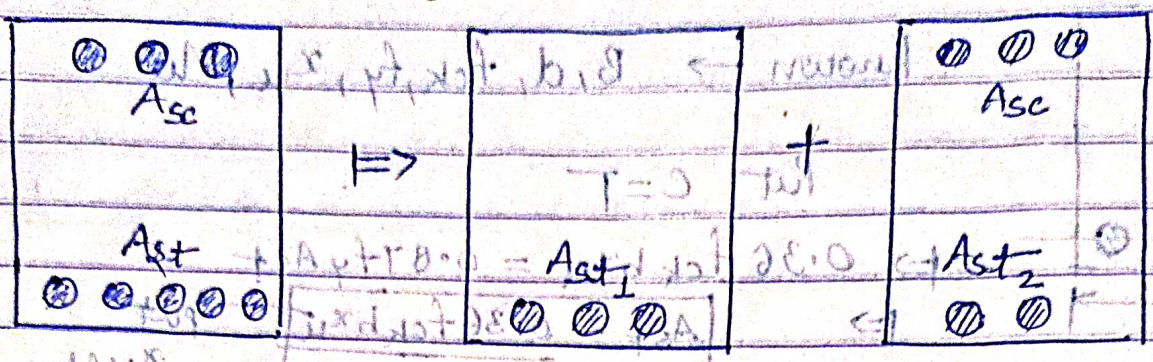
$b \rightarrow mm$

$d \rightarrow mm$

$x_{u\,lim} \rightarrow mm$

⊗ ⊓ → why not
↳ corners पर stress concn बढ़ने लगता है। (20M)
(sharp point नहीं होने चाहिए)

Design of doubly R/f Beam :-



fig(1) fig(2)

⇒ Balanced singly R/f section
⇒ $M_{u,lim}$
⇒ $LA \Rightarrow d - 0.42x_u$

⇒ Mu−Mulim
⇒ LA = d−d'

(1)

⇒ $$\boxed{A_{st_{(1)}} = \frac{M_{u,lim}}{0.87 f_y (d - 0.42 x_{u,lim})}}$$

MOR OR M = T × LA

⇒ $M_u - M_{u,lim} = 0.87 f_y A_{st_2} (d - d')$

⇒ $$\boxed{A_{st_{(2)}} = \frac{(M_u - M_{u,lim})}{0.87 f_y (d - d')}}$$

$$\boxed{A_{st_2} = \frac{A_{sc} f_{sc}}{0.87 f_y}}$$

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so; $$\boxed{A_{st} = A_{st_1} + A_{st_2}}$$

$(MOR)_c = C_2 LA$

$(MOR)_c = A_{sc}(f_{sc} - 0.446 f_{ck})(d - d')$

⇒ $M_u - M_{u,lim} = A_{sc}(f_{sc} - 0.446 f_{ck})(d - d')$

⇒ $$\boxed{A_{sc} = \frac{M_u - M_{u,lim}}{(f_{sc} - 0.446 f_{ck})(d - d')}}$$

To calculate A_{sc} :-

⇒ Put $C_2 = T_2$

⇒ $A_{sc}(f_{sc} - 0.446 f_{ck}) = 0.87 f_y A_{st_2}$

⇒ $$\boxed{A_{sc} = \frac{0.87 f_y A_{st_2}}{f_{sc} - 0.446 f_{ck}}}$$ (2)

Maximum & Minimum Area of steel R/f :-

Case (I) :- Minimum area of steel R/f in tention zone.

⇒ $$\boxed{\frac{A_{st\,min}}{bd} \geq \frac{0.85}{f_y}}$$ why min?

source — code

Case (II) :- Maximum area of steel R/f in tention zone.

⇒ $A_{st,max}$ = 4% of total cross-sectional area

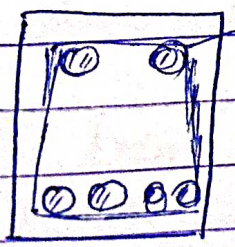
⇒ $$\boxed{A_{st,max} = 0.04\,BD}$$ why max?

→ Gross depth

Case (III) Maximum area of compression R/f.

⇒ $A_{sc,max}$ = 4% of total cross-sectional area

⇒ $$\boxed{A_{sc} = 0.04\,BD}$$



12mm (even though singly R/f we provide at least 2 bars of 12mm at top to support the stirrups)

IS 456 : 2000 Page No (47) (48).

$$\boxed{\frac{A_s}{bd} = \frac{0.85}{f_y}}$$

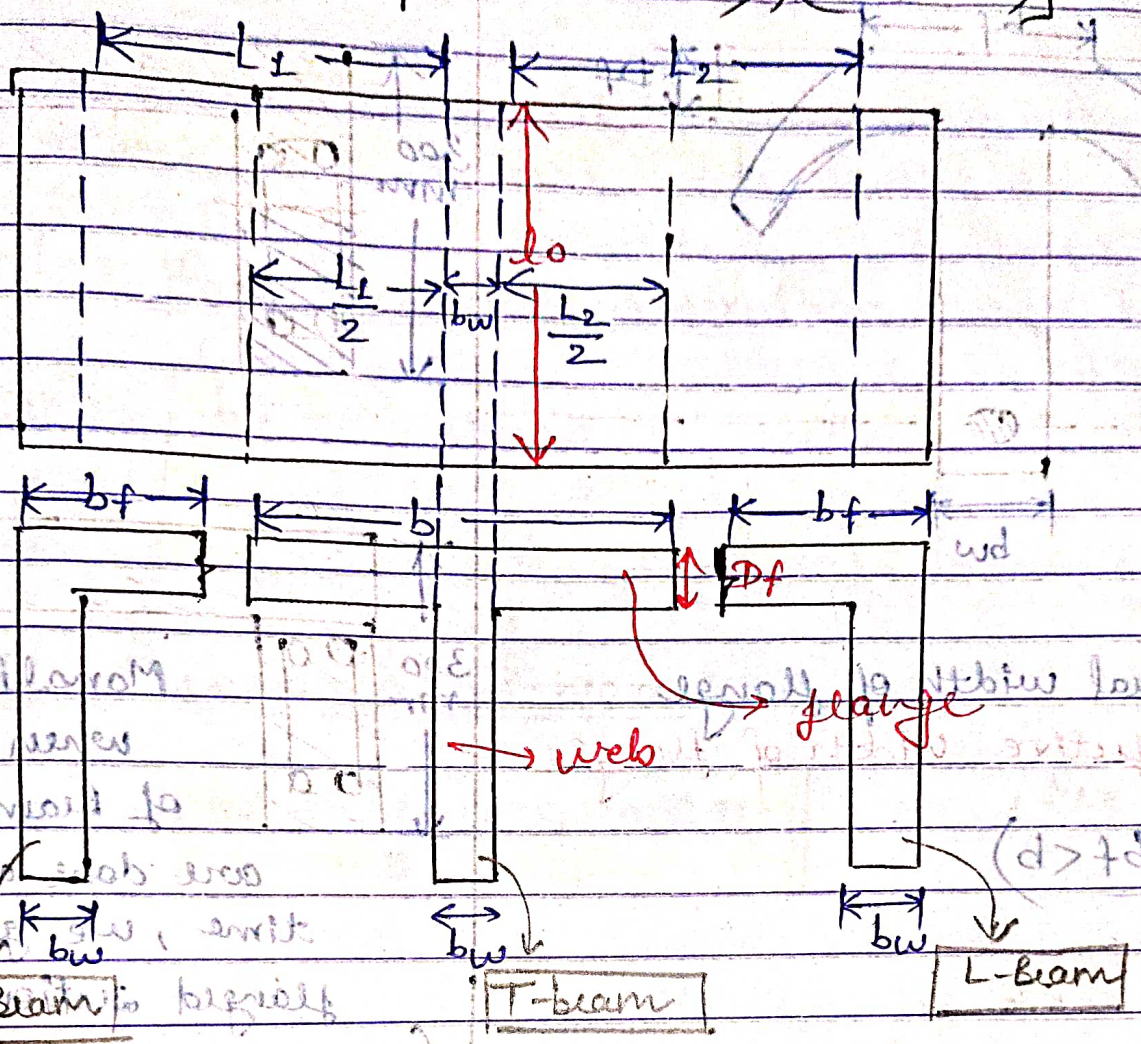
- **Why min :—**

minimum area of steel reinforcement is provided to have minimum ductility in the member. It is also provided to avoid the problem of sudden collapse/ failure.

- **Why max :—**

Is code [IS 456 : 2000] Page No (47) has given the maximum limit of area of steel r/f to avoid problem of compaction of concrete due to congestion of Reinforcement.

Flanged Beams :- [(T-beam), (L-Beam)]

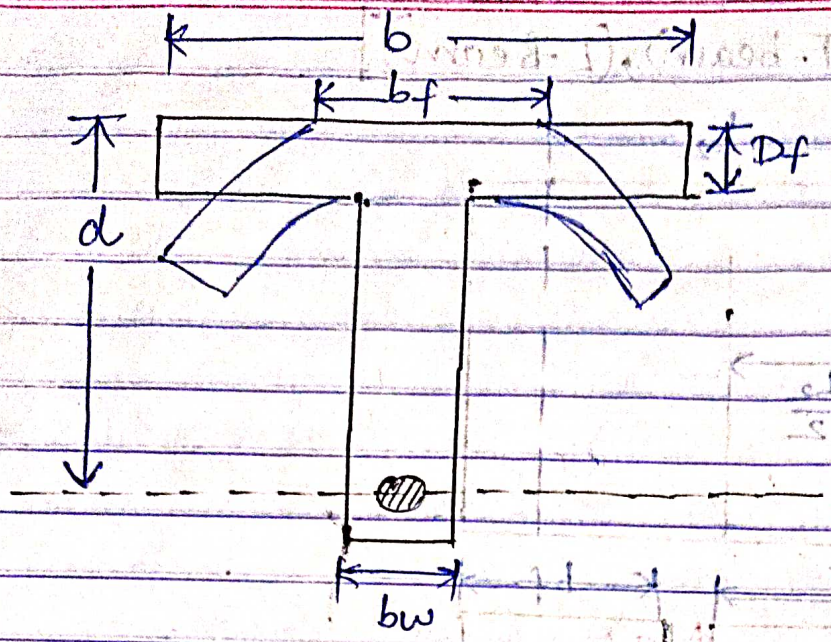


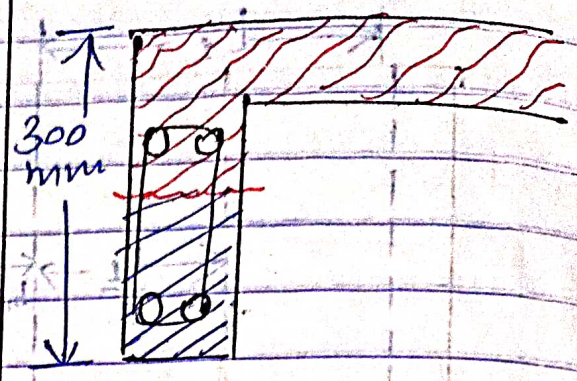
- In general, the floor slabs are casted monolithically (concreting of both beam & slab done together) with beam. & hence the behaviour of such beams is different from simple rectangular beams.

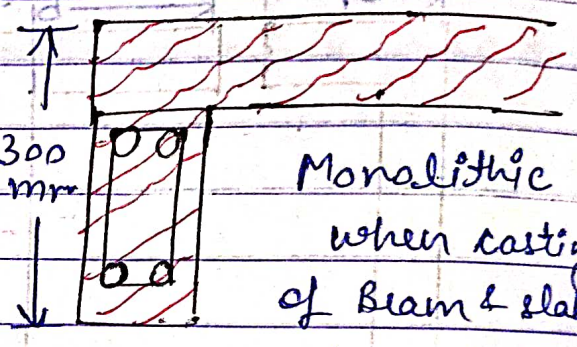
- In such type of beams, the slab acts in conjunction with beam.

→ The slab portion is called the 'flange' of 'T' or 'L' beam.

→ The beam portion which is below the slab (flange) is called the web of 'T' or 'L' Beam.







Monolithic when casting of Beam & slab are done at a time, we get flanged section.

(Moment carrying capacity increases in Monolithic construction)

b → Actual width of flange

bf → effective width of flange.

$(b_f < b)$

As per IS 456 : 2000

Case (I) For Isolated T-beam

$$\Rightarrow \quad b_f = \frac{l_0}{\frac{l_0}{b_w} + 4} + b_w$$

Case-II For Continuous T-beam :-

$$\boxed{b_f = \frac{l_o}{6} + b_w + 6D_f}$$

Case-III For Isolated L-Beam :-

$$\boxed{b_f = \frac{0.5 l_o}{\left(\frac{l_o}{b}\right) + 4} + b_w}$$

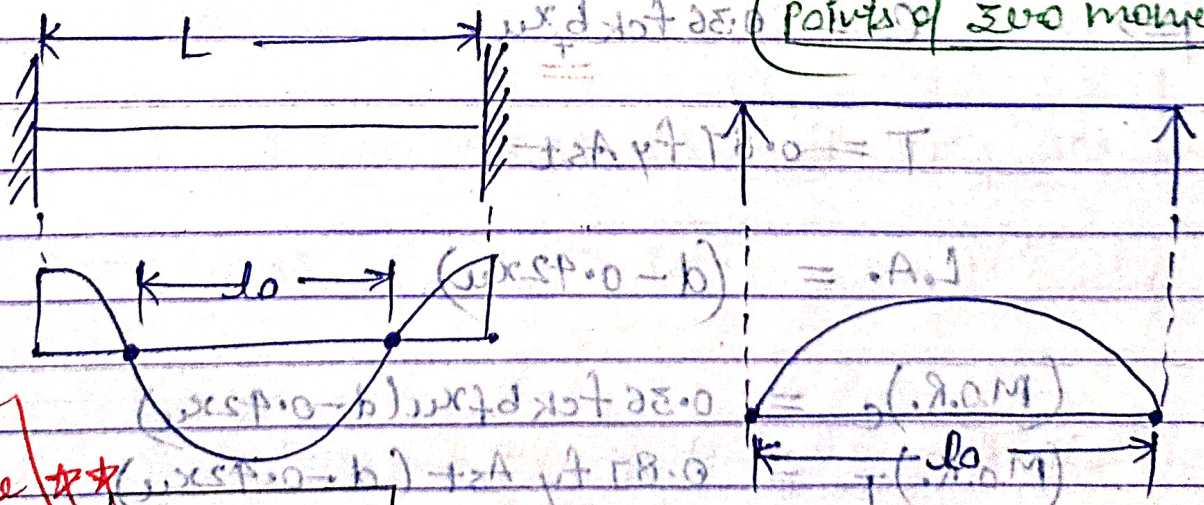
Case-IV Continuous T-beam :-

$$\boxed{b_f = \frac{l_o}{12} + b_w + 3D_f}$$

$\left(b_f = \frac{l_o}{12} + b_w + 3D_f\right)$

$l_o \rightarrow$ it is the span of the beam between two points of contraflexure.

(points of zero moment)



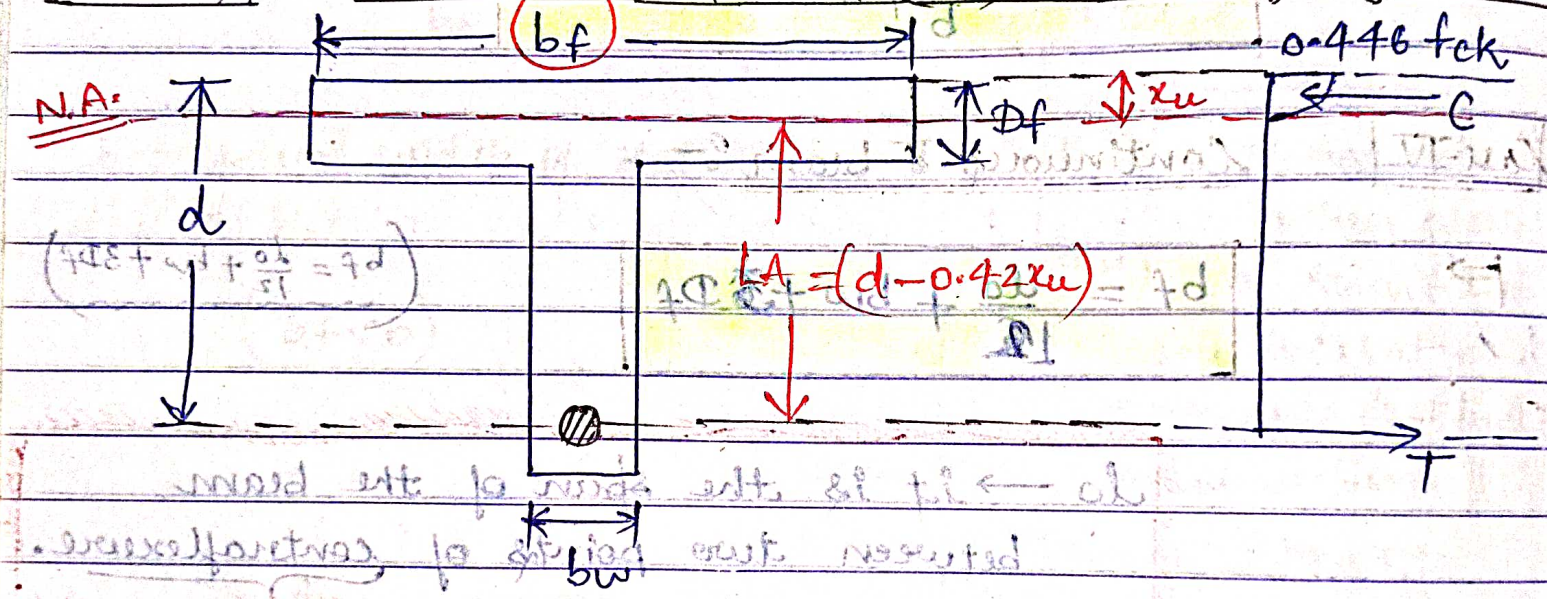
Acc. to IS code ** $\boxed{l_o = 0.7L}$

(no need to analyse the beam with moment distribution method)

Analysis of Flanged Beams Section :-

In a flanged beam, the neutral axis may either in the flange or web of the section which depends upon the dimensions of effective flange width (b_f), web width (b_w) & thickness of flange (D_f) of the flanged section.

Case (I) When Neutral Axis (N.A.) is in flange portion ($x_u < D_f$)



$\Rightarrow$ $C = 0.36\, f_{ck}\, b_f\, x_u$

$T = 0.87\, f_y\, A_{st}$

$L.A. = (d - 0.42x_u)$

$(M.O.R.)_C = 0.36\, f_{ck}\, b_f\, x_u\, (d - 0.42x_u)$

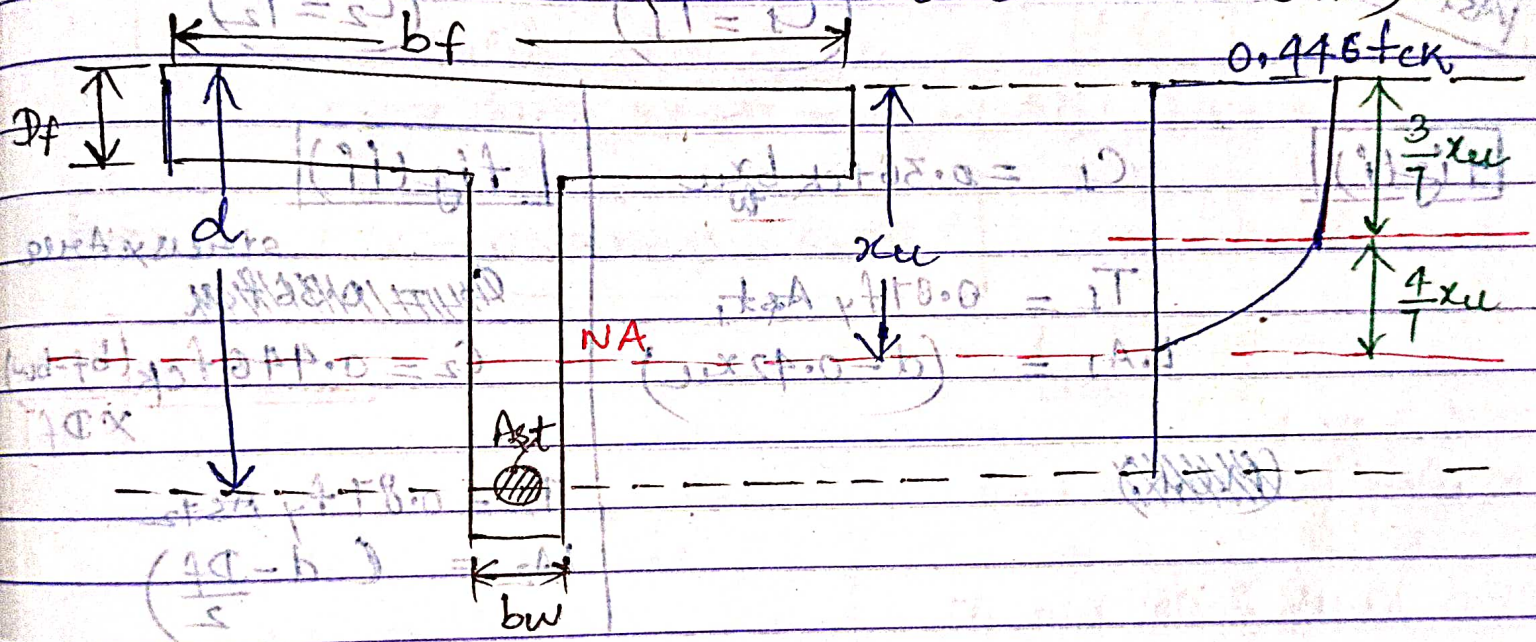
$(M.O.R.)_T = 0.87\, f_y\, A_{st}\, (d - 0.42x_u)$

To determine x_u; put $C = T$

$\Rightarrow$ $$\boxed{x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}}$$

$\Rightarrow$ compare it with $x_{u\,lim}$ & check the section (balanced / underreinf / over reinf) then put in MOR equation.

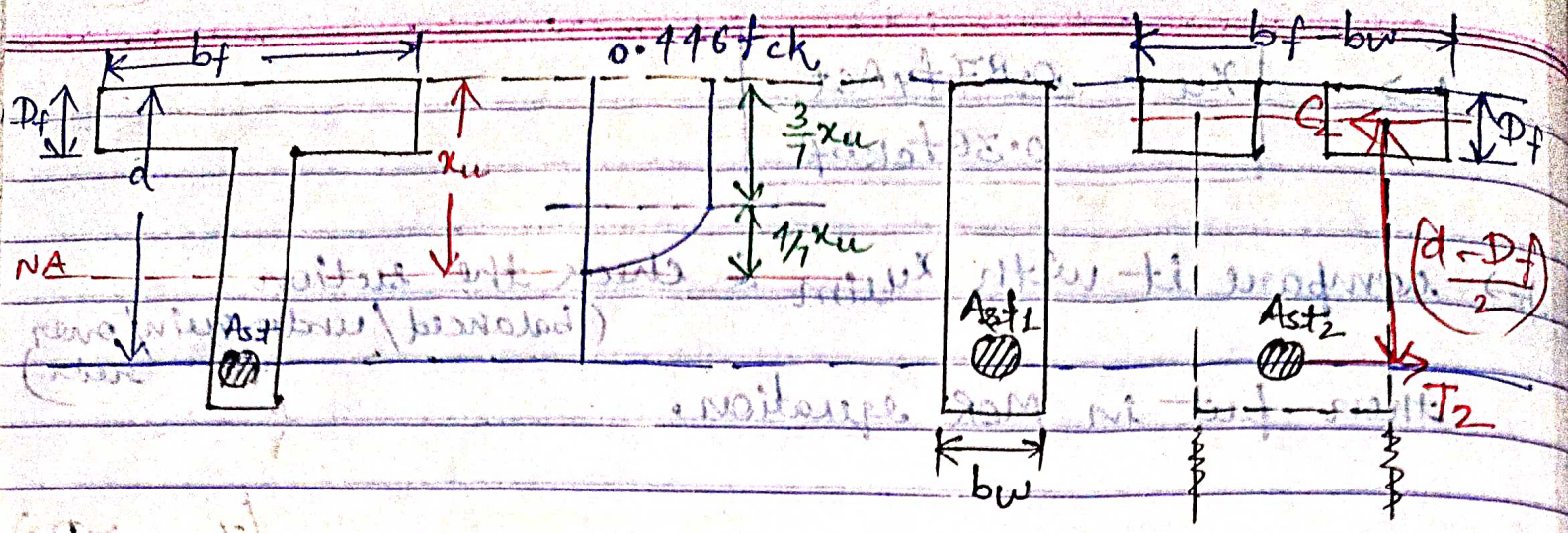
Case (II) When Neutral axis (N.A.) in the web and the section is balanced (i.e. $x_u = x_{u\,lim}$) $(x_{u\,lim} > D_f)$



1) Subcase [II (a)] When flange is uniformly stressed. (linear)

That means $\boxed{\frac{3}{7} x_u > D_f}$

(Rectangular portion of block covering full flange here)



$\boxed{A_{st} = A_{st1} + A_{st2}}$

fig(i) $(C_1 = T_1)$ fig(ii) $(C_2 = T_2)$

fig(i)

$C_1 = 0.36 f_{ck}\, b_w x_u$

$T_1 = 0.87 f_y A_{st1}$

$L.A_1 = (d - 0.42 x_u)$

fig(ii)

stress × Area

$C_2 = 0.446 f_{ck} [(b_f - b_w) \times D_f]$

$T_2 = 0.87 f_y A_{st2}$

$L.A_2 = \left(d - \frac{D_f}{2}\right)$

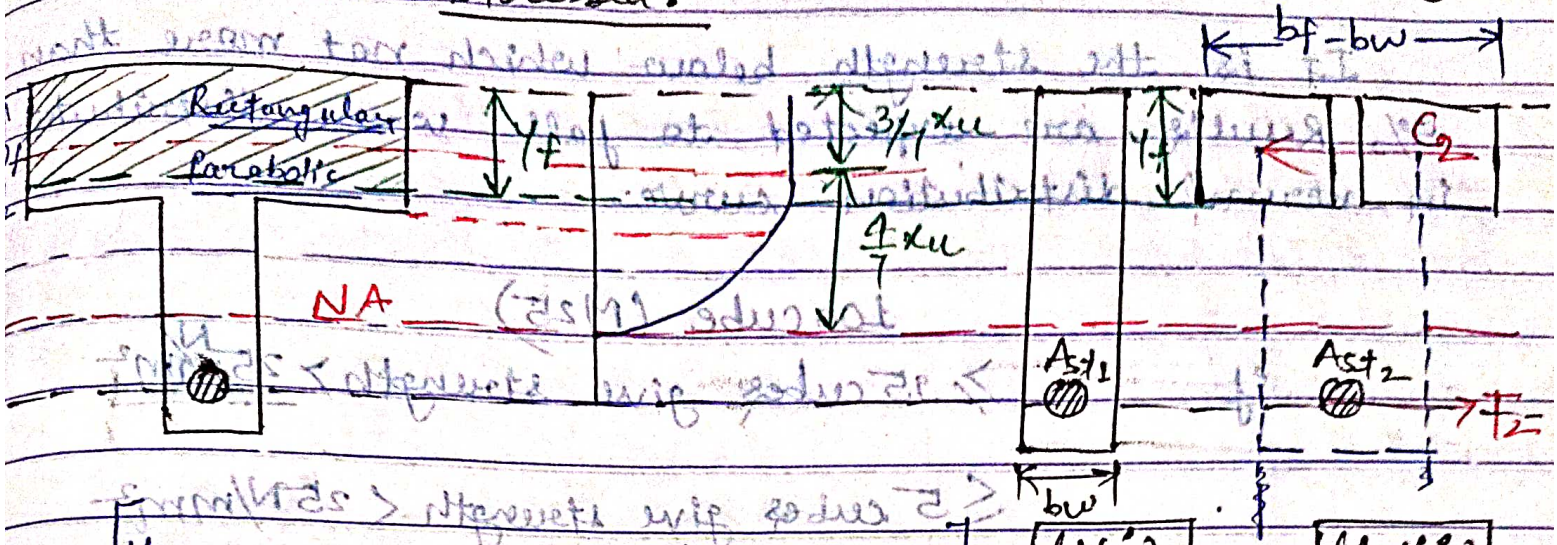
$\Rightarrow \boxed{(MOR)_C = C_1 L.A_1 + C_2 L.A_2}$

$\boxed{(MOR)_T = T_1 L.A_1 + T_2 \times L.A_2}$

To calculate x_u put $C = T$

$\Rightarrow \boxed{C_1 + C_2 = T_1 + T_2}$

Subcase [II(b)] When flanged is not uniformly stressed.



fig(i) fig(ii)

$y_f \rightarrow$ equivalent depth of flange

(मान लिया y_f तक straight है।)

IS456:2000

$$\boxed{y_f = 0.15x_u + 0.65D_f}$$

fig(i)

$C_1 = 0.36 f_{ck} b_w x_u$

$T_1 = 0.87 f_y A_{st}$

$L.A_1 = d - 0.42x_u$

fig(ii)

stress × Area

$C_2 = 0.446 f_{ck} (b_f - b_w) \times y_f$

$T_2 = 0.87 f_y A_{st_2}$

$L.A_2 = \left(d - \frac{y_f}{2}\right)$

[जहाँ-जहाँ D_f था, वहाँ-वहाँ y_f रखना है।]

$(M.O.R.)_C = C_1 L.A_1 + C_2 L.A_2$

$(M.O.R.)_T = T_1 L.A_1 + T_2 L.A_2$

To calculate x_u put $C = T$

$$\boxed{C_1 + C_2 = T_1 + T_2}$$

07 | 11-September 2020

Characteristic strength of Concrete :-

It is the strength below which not more than 5% Results are expected to fall when distributed in normal distribution curve.

100 cube (M25)

if $\geq$ 95 cubes give strength $> 25 N/mm^2$

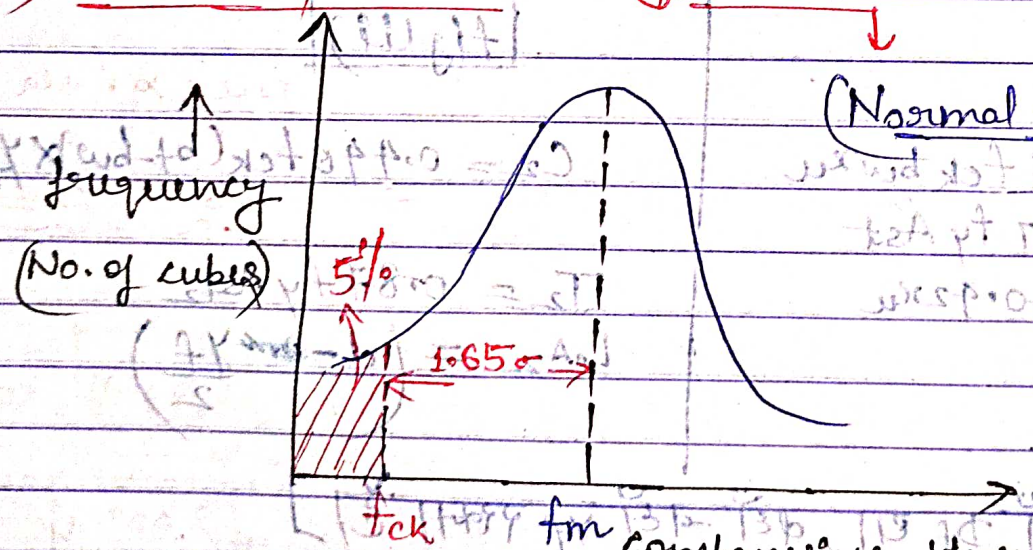
$\leq$ 5 cubes give strength $< 25 N/mm^2$

In that case $f_{ck} = 25 N/mm^2$

f_{ck} → characteristic compressive strength.

⇒ Probability – Curve

(Normal-distribution)



fm ⇒ target mean strength (in N/mm^2 or MPa)
(mean value)

⇒ $\boxed{f_m = f_{ck} + 1.65\sigma}$

$f_m = f_{ck} + 1.65\sigma$

where $\sigma \Rightarrow$ standard deviation.

Acc. to IS 456 : 2000 (Page No. 23 Table 8)

grades	σ (N/mm²)
M10, M15	3.5
M20, M25	4.0
M30, upto M55	5.0

(where there is deviation from this table values shall be increased by 1 N/mm²)

$f_m \rightarrow$ It is the characteristic compressive strength below which not more than 5% results are expected to fall.

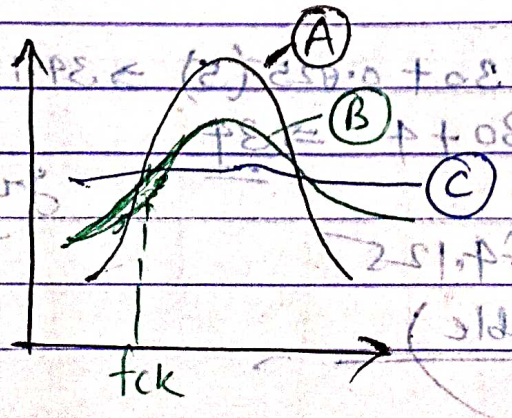
$$\boxed{f_m = f_{ck} + 1.65\sigma}$$

Probability factor

Qu: Calculate f_m for M30.

Sol^n: $\Rightarrow f_m = 30 + 1.65 \times 5$

$= 30 + 8.25 \Rightarrow 38.25\,MPa$



good quality control → A
Avg quality control → B
Poor quality control → C

Acceptance Criteria :—

IS code 456:2000 from table 8 (Page No. 23)

1st criteria

IS code 456:2000
Table 11
Page No. 30

$f_{avg} \geq f_{ck} + 0.825\sigma$ or $f_{ck} + 4$ } whichever is greater

($f_{ck} \geq 20 N/mm^2$) (M20, M30, M35)

$f_{avg} \geq f_{ck} + 0.825\sigma$ or $f_{ck} + 3$ } whichever is greater

($f_{ck} < 20 N/mm^2$) (M15)

$f_{avg} \rightarrow$ It is the average compressive strength of 4 non-overlapping consecutive test samples. (OR more than 4)

eg; for M30;

29.3 ✓
31.2 ✓
31.2 ✗
33.6 ✓
28.7 ✓
28.7 ✗
30.5 ✓

$$f_{avg} \Rightarrow \frac{(29.3 + 31.2 + 33.6 + 28.7 + 30.5)}{5} \Rightarrow 30.66$$

$f_{ck} + 0.825\sigma \Rightarrow 30 + 0.825(5) \Rightarrow 34.125$

$f_{ck} + 4 \Rightarrow 30 + 4 \Rightarrow 34$

greater 34.125

$f_{avg} < 34.125$

(Not acceptable)

2nd criteria :-

$$\boxed{f_{c\,individual} \geq f_{ck} - 3} \quad (\text{when } f_{ck} < 20\,N/mm^2)$$

$$\boxed{f_{c\,individual} \geq f_{ck} - 4} \quad (\text{when } f_{ck} \geq 20\,N/mm^2)$$

⇒ As per New Rule ; $\boxed{f_{c\,individual} \geq f_{ck} - 3}$ (for all grades)

eg, for M25 ;

- ✓ 22.5 > 22
- ✓ 24.7 > 22
- ✓ 26.5 > 22
- ✓ 27.2 > 22
- ✓ 25.7 > 22

25 − 3 → 22

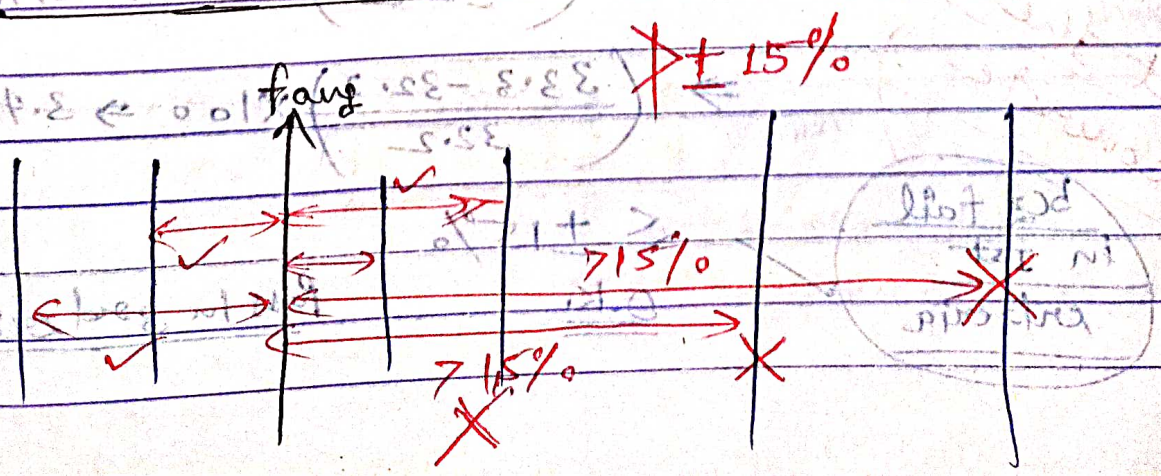
3rd criteria :-

(a). Minimum three cubes shall be tested.

(b). % variation in compressive strength ~~shall not be~~ from its average compressive strength shall not exceed by ± 15%.

$$\boxed{\%\ \text{variation} = \frac{(f_{c\,individual} - f_{avg})}{f_{avg}} \times 100}$$

eg;



(for Accepting the concrete all three criterias should be fulfil.)

Qu^n Three samples are tested for M30 grade of concrete. Their compressive strengths are 31.1, 32.2, 33.3 (N/mm^2). Check for acceptance :—

Sol^n :→ Check for Acceptance :—

$$f_{avg} = \left(\frac{31.1 + 32.2 + 33.3}{3}\right) = 32.2\ N/mm^2$$

1st criteria :— $f_{ck} + 4 \Rightarrow 34$

$\Rightarrow f_{ck} + 0.825\sigma \Rightarrow 34.125\ N/mm^2$

$\Rightarrow f_{avg} < 34.125$

∴ fail in 1st criteria.

2nd criteria :— $f_{ck} - 3 \Rightarrow 30 - 3 \Rightarrow 27\ N/mm^2$

$f_{individual}$ — $31.1 > 27$ ✓

$32.2 > 27$ ✓

$33.3 > 27$ ✓

Pass in 2nd criteria.

3rd criteria :— % variation $= \left(\frac{31.1 - 32.2}{32.2}\right) \times 100$

3 cubes ✓

$= -3.42\%$ O.K.

$\Rightarrow \left(\frac{32.2 - 32.2}{32.2}\right) \times 100 \Rightarrow 0\%$ O.K.

$\Rightarrow \left(\frac{33.3 - 32.2}{32.2}\right) \times 100 \Rightarrow 3.42\%$ O.K

$\leq \pm 15\%$

O.K.

Pass in 3rd criteria

Reject the concrete

bcz fail in Ist criteria ✓

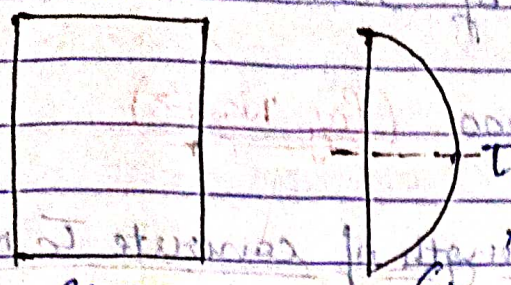
08 | 12-September 2020

(Rectangular rings → shear stirrups)

Limit state of collapse : shear

IS 456 : 2000
Page No (72)

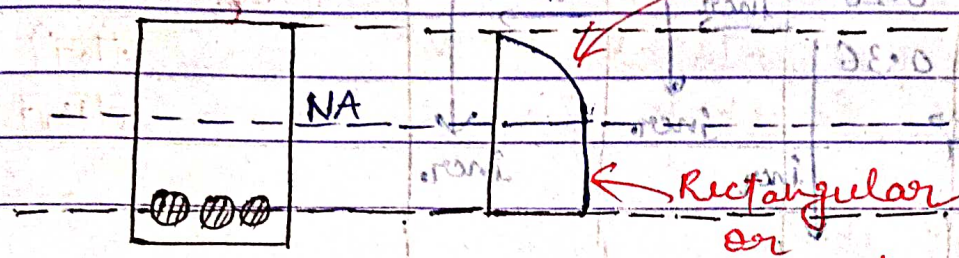
SOM



$$\tau_{max} = \frac{VA\bar{y}}{IB}$$

Plain Cement Concrete (PCC)

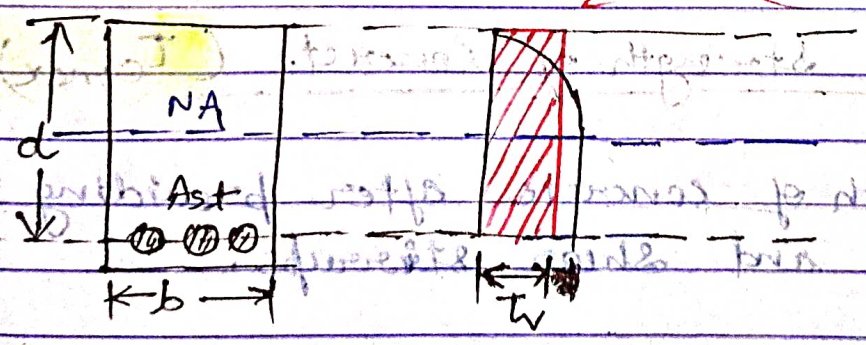
(Shear stress distribut^n)



non-uniform figure (difficult to analyse)

RCC section (Reinforced Cement Concrete)

(Acc to code) → (~~ज्ञ~~ uniform मान लो।)



① Nominal shear stress (τ_v) :-

{ V_u → shear force (max) }

(तोड़ने वाली ताकत)

$$\tau_v = \frac{V_u}{bd} \quad N/mm^2$$

{ b → width of beam
d → effective depth of beam }

(it is bcz of concrete & longitudinal steel)

2. Shear strength of Concrete (τ_c)

τ_c depends upon the % of tensile steel R/f and grade of concrete.

⇒ use table 19 from IS 456 : 2000 (Page No. 73)

Table 19 Design shear strength of concrete τ_c, N/mm²

% A_{st}/bd	M15	M20	M25	M30
≤0.15	0.28	0.28	incr	incr
0.25	0.35	0.36	incr	incr
	incr	incr	incr	incr
	increase			

(τ_c → बचाने वाली ताकत)

if f_{ck} ↑ ⇒ τ_c ↑

if % A_{st} ↑ ⇒ τ_c ↑

3. Maximum Shear Strength of Concrete (τ_{cmax}) :-

⇒ It is the strength of concrete after providing the longitudinal steel and shear stirrups.

⇒ As per code it depends upon the grade of concrete only.

Table 20 → (No need to learn)

⇒ $\boxed{\tau_{cmax} = 0.62\sqrt{f_{ck}}}$

Design of shear stirrups :-

Case (I) :- when $\tau_v < \tau_c$

✓ Minimum shear R/f shall be designed.

Imp

$$\boxed{\frac{A_{sv\,min}}{b.s_v} \geq \frac{0.4}{0.87 f_y}}$$

where; $s_v \rightarrow$ c/c (spacing of stirrups.

$f_y \rightarrow \not> 415\ N/mm^2$

~~Fe 500~~

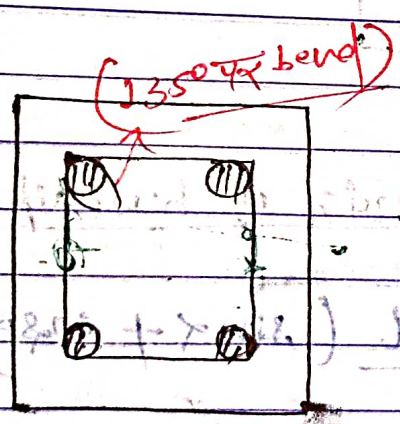
(code recommended)

$A_{sv} \rightarrow n.\frac{\pi}{4}\phi^2$ (cross-sectional area of stirrups)

$\phi \rightarrow$ diameter

$n \rightarrow$ no. of legs. - vertical

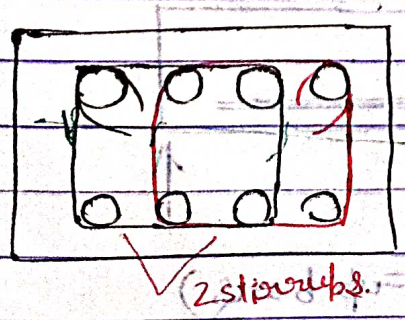
eg;



$\rightarrow$ 2 legs

$$\boxed{A_{sv} = 2 \times \frac{\pi}{4}\phi^2}$$

2-legged stirrups



$$\boxed{A_{sv} = 4 \times \frac{\pi}{4}\phi^2}$$

4-legged stirrups

Case (II) :- when $\tau_v > \tau_c$; (but $\tau_v < \tau_{c\,max}$)

→ Design the shear stirrup for design shear force of $V_{us} = (\tau_v - \tau_c) b d$

(suppose; $\tau_v \Rightarrow 0.7$

$\tau_c \Rightarrow 0.5$) $(\tau_v - \tau_c)$

$0.2\ N/mm^2$

stirrup design

$$*** \quad V_{us} = (\tau_v - \tau_c) b d$$

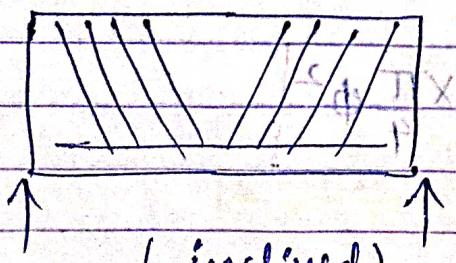
[II(a)] Vertical stirrups :-

$$\Rightarrow \quad V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

[II(b)] Inclined stirrups :- / Series of bent up bars :-

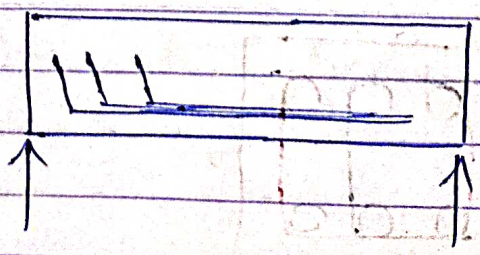
$$\Rightarrow \quad V_{us} = \frac{0.87 f_y A_{sv} d}{s_v} (\sin\alpha + \cos\alpha)$$

eg;



(inclined)

OR

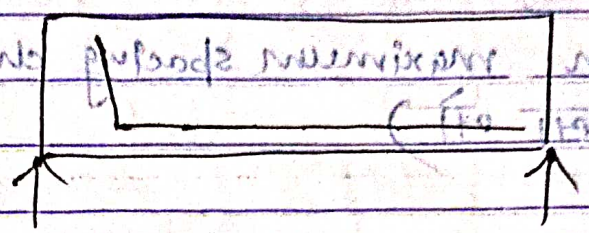


(bent-up bars)

[II.(C)] Bent up bars :- (single location पर)

⇒ $\boxed{V_{us} = 0.87 f_y A_{sv} \sin\alpha}$

eg



Case (III) When $\boxed{\tau_v > \tau_{cmax}}$

⇒ Redesign the section.

(cross-sectional dimensions increase करने पड़ेंगे क्योंकि $\tau_v = V_u/bd$)

Maximum Spacing Criteria

(i) $0.75d$ [for vertical stirrups] ✓

(ii) d [for inclined stirrups]

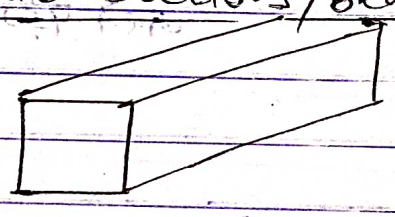
(iii) 300mm (in any case) ✓ 300

(भले ही case-II आए फिर भी case I से min. shear R/f जरूर check करेंगे।
& spacing (s_v) minimum को Prefer करेंगे

Then maximum spacing check करो और सबसे min वाला लो)

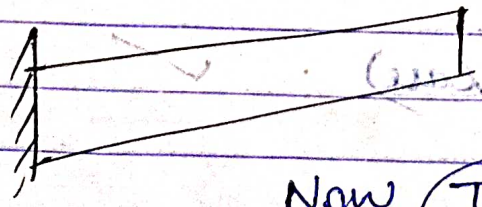
Note:- In all the cases check for minimum shear R/f.

•) for prismatic sections/beam :-



constant b and d

So we'll use $\boxed{\tau_v = \frac{V_u}{bd}}$



(Non-prismatic beam/section)

Now $\tau_v \neq \frac{V_u}{bd}$

Nominal shear stress in haunches / varying depth sections :-

(width is constant)

(nominal shear stress)

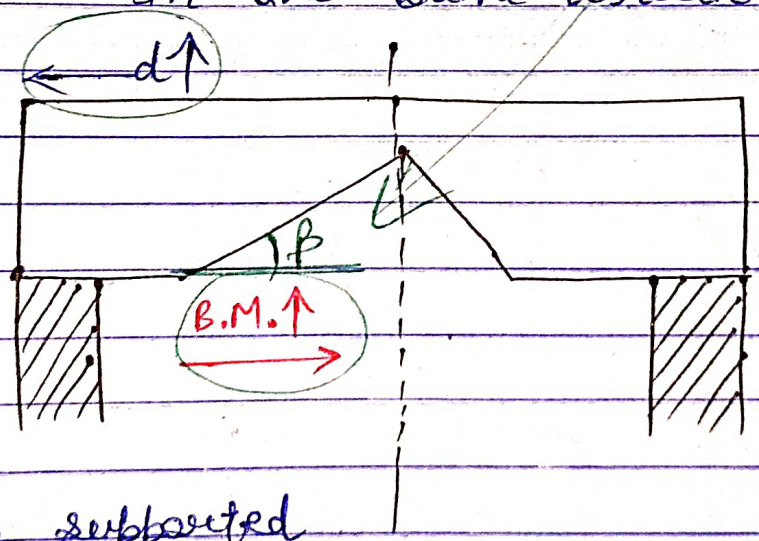
$$\tau_v = \frac{V_u \pm \frac{M_u}{d}\tan\beta}{b.d}$$

(at any particular section)

+ive → when depth and B.M. both increases in two opposite directions.

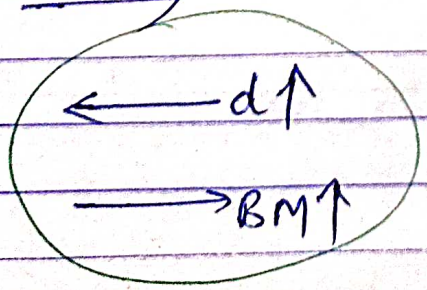
-ive → when depth and B.M. both increases in the same direction.

eg (1)



Haunch

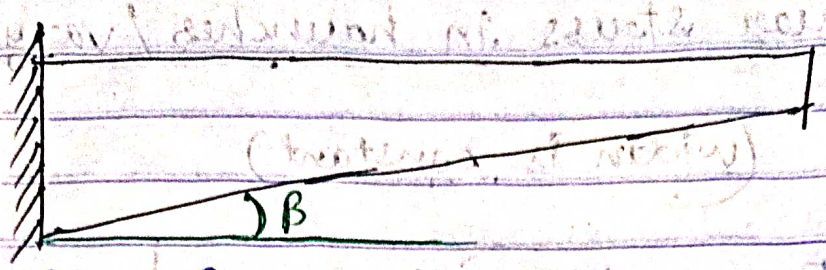
(simply supported beam)



use +ive sign.

$$\tau_v = \frac{V_u + \frac{M_u}{d}\tan\beta}{bd}$$

eg ②.



Cantilever Beam

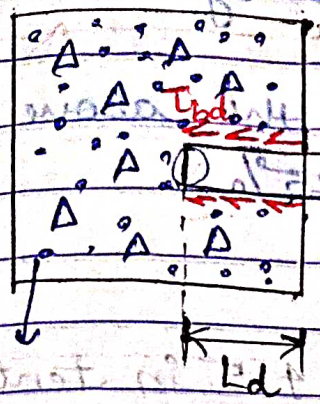
$\leftarrow d \uparrow$

$\leftarrow BM \uparrow$

use -ive sign.

so $\boxed{\tau_v = \frac{V_u - \frac{M_u}{d}\tan\beta}{bd}}$

Development Length :-



$$P = 0.87 f_y \left[\frac{\pi \phi^2}{4}\right]$$

steel bar

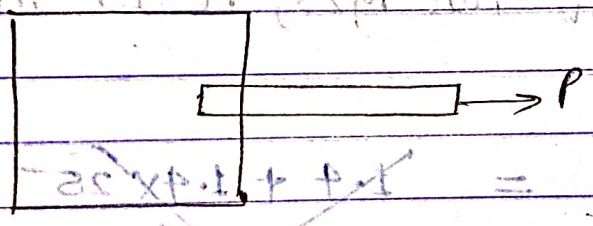
diameter of steel bar

$$\left[\tau_{bd\,perm} = 0.87 f_y \left[\frac{\pi \phi^2}{4}\right] \Big/ (\pi \phi L_d)\right]$$

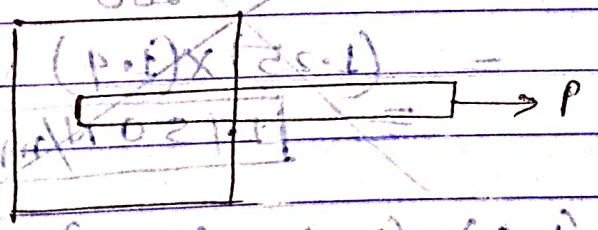
$$\Rightarrow \boxed{L_d = \frac{0.87 f_y \phi}{4(\tau_{bd\,perm})}}$$

bond strength

eg;



✓ fail early (here surface area in contact of concrete is very less.)



$\tau_{bd\,perm}$ has been given by IS 456 : 2000

For Plain R/f in tention zone :- P.N. 81 (for Tensile Mild steel Fe250)

fck	M20	M25	M30	M35	M40 & above
$\tau_{bd\,perm}$ (N/mm²)	1.2	1.4	1.5	1.7	1.9

V. Imp.

Note: In case of HYSD/CTD/TMT the above value of $\tau_{bd\,perm}$ shall be increased by 60%.

For compression reinforcement the above values of $\tau_{bd\,perm}$ shall be increased by 25%.

Que Determine $\tau_{bd\,perm}$ for M20, Fe415 in tension zone.

Soln:- So $\tau_{bd\,perm} = 1.2 + \frac{1.2 \times 60}{100}$

$= (1.6 \times 1.2)$

$= \boxed{1.92\ N/mm^2}$

Que Determine $\tau_{bd\,perm}$ for M25, Fe415 in compression R/f.

Soln:- $\tau_{bd\,perm} = 1.4 + \frac{1.4 \times 25}{100}$

$= (1.25) \times (1.4)$

$= \boxed{1.750\ N/mm^2}$

$= (1.4) \times (1.6) \times (1.25)$

$= \boxed{2.8\ N/mm^2}$

Qu:- $L_d = ?$ Fe 250

$\phi = 20mm$ compression R/f.

$f_{ck} = 20 N/mm^2$

Soln:-

$$L_d = \frac{(0.87)(250)(20)}{4 \times [(1.2) \times (1.25)]}$$

$$= \boxed{725 mm}$$

Qu The development length of a deformed reinforcement bar can be expressed as $\left(\frac{1}{K}\right)\left(\frac{\phi \sigma_s}{\tau_{bd}}\right)$. From the IS 456:2000, the value of K can be calculated as:-

[GATE 2015 : IIT Kanpur]

$\sigma_s = 0.87 f_y$

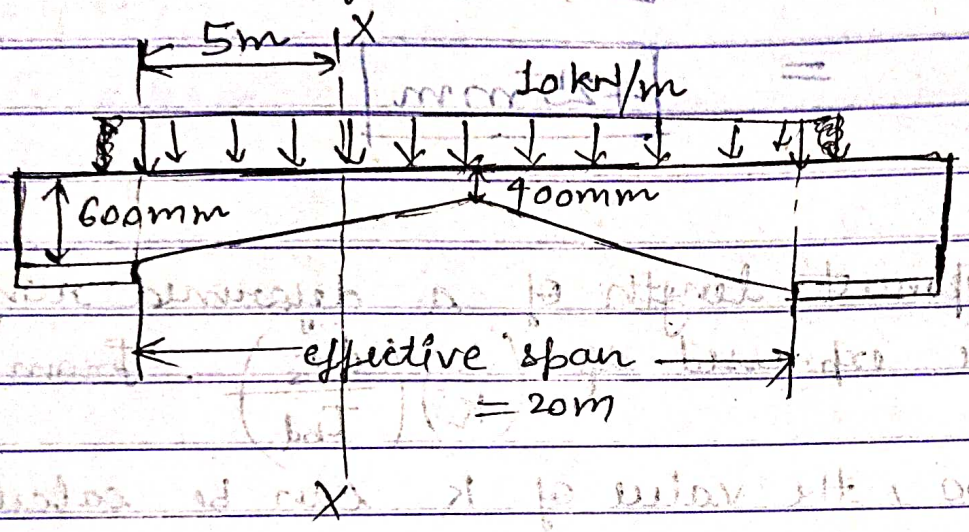
Soln:- for deformed bars

$$L_d = \frac{(0.87 f_y)(\phi)}{4 \tau_{bd} \times 1.6}$$

$$= \frac{1}{(6.4)} \frac{(\sigma_s \phi)}{\tau_{bd}}$$

$\Rightarrow$ $\boxed{K = 6.4}$ Ans

Qu A haunched (varying depth) reinforced concrete beam is simply supported at both ends, as shown in the figure. The beam is subjected to uniformly distributed factored load of intensity 10 kN/m. The design shear force (expressed in kN) at the section X-X of the beam is ____ (GATE 2016, IISC Banglore)



Soln: →

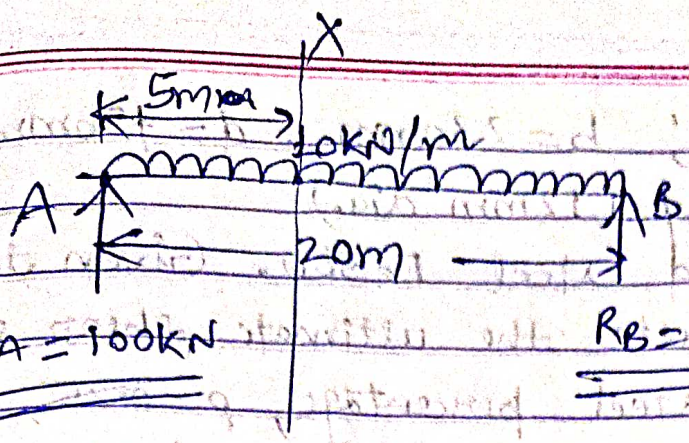
$$T_v = V_u \pm \frac{M_u}{d} \tan\beta$$

$M_u \uparrow$
$d \uparrow$
(+)

$$V_d = V_u + \frac{M_u}{d} \tan\beta$$

design shear force in varying depth section

$$= \boxed{V_{u(x-x)} + \frac{M_{u(x-x)}}{d_{(x-x)}} \tan\beta}$$



$R_A = 100kN$ $\quad$ $R_B = 100kN$

$V_{u(x-x)} = 100 - 5 \times 10$

$= \boxed{50\ kN}$

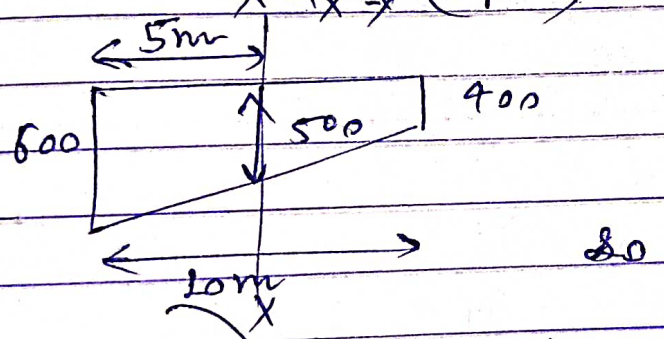
X

$M_{u\,x-x} = 100(5) - (5) \times (10) \times \left(\frac{5}{2}\right)$

$= 500 - 125$

$= \boxed{375\ kN.m}$

X d_{x-x} (depth) $\Rightarrow$ $\boxed{500mm}$ $\Rightarrow$ 0.5m



$V_d = 50 + \frac{375}{(500 \times 10^{-3})}(0.02)$

$= 50 + 15$

$= \boxed{65\ kN}$

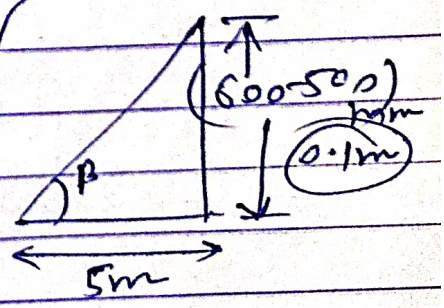
OR

$\left(\frac{600-400}{10}\right) \times 5$

100mm

$600 - 100$

500mm



$\tan\beta \Rightarrow \frac{0.1}{5}$ $\Rightarrow \boxed{0.02}$

OR $\left(\frac{600-400}{10}\right) \times 10^{-3} = 0.02$

Lab P.n 13 Q4

Qu A rectangular beam; $b = 230mm$, $d = 450mm$ ... $n = 4$ bars of 12ϕ (12mm dia.) concrete M20 and steel Fe500. Given that for M20 grade of concrete the ultimate shear strength $\tau_{uc} = 0.36\ N/mm^2$ for steel percentage, $p = 0.25$, and $\tau_{uc} = 0.48\ N/mm^2$ for $p = 0.50$. For a factored shear force of 45 kN, the diameter (in mm) of Fe500 steel two legged stirrups to be used at spacing of 375mm, should be

(A) 8 (B) 10

(C) 12 (D) 16.

[GATE 2014 – IIT Kharagpur]

#L10 :-

Limit state of collapse : Torsion

Equivalent shear Force :-

Imp

$$V_{ueq} = V_u + \frac{1.6\,T_u}{b}$$

V_u → bcz of self & other load

$\frac{1.6\,T_u}{b}$ → Additional shear force bcz of Torque

here; T_u ⇒ factored Torque (kN-m)
V_u ⇒ factored shear force (kN)
b ⇒ width of beam (m)

Equivalent Nominal shear stress :-

$$\tau_{v_{eq}} = \frac{V_{ueq}}{bd} \quad (N/mm^2)$$

⇒ τ_c → shear strength of concrete

(i) if $\tau_{v_{eq}} < \tau_c$

⟹ Design longitudinal tensile R/f for factored Bending moment M_u.

(ii) if $\tau_{v_{eq}} > \tau_c$

⟹ Design longitudinal tensile reinforcement for factored equivalent Moment (M_{ueq})

$$M_{ueq} = M_u + M_T$$

$$\boxed{M_{ueq} = M_u + \frac{T_u\left(1 + \frac{D}{b}\right)}{1.7}}$$

All the formulas of LSM of collapse of flexure & shear are valid here also only V_u shall be replaced by V_{ueq} & M_u shall be replaced by M_{ueq}.

Step (I)

$$M_{u\,lim} = Q f_{ck} b d^2$$

$Q \rightarrow$
- 0.148 → (Fe 250)
- 0.138 → (Fe 415)
- 0.133 → (Fe 500)

Step-II

Calculate M_{ueq}

$$M_{ueq} = M_u + \frac{T_u\left(1 + \frac{D}{b}\right)}{1.7}$$

Step-III Compare M_{ueq} & $M_{u\,lim}$

if $M_{ueq} < M_{u\,lim}$

⟹ Design single reinforced beam.

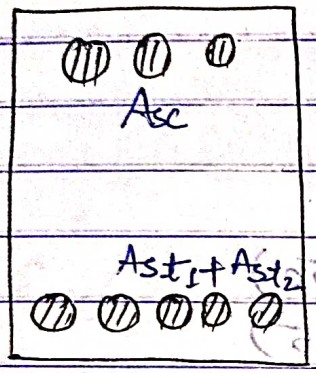
Put M_{ueq}

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_{ueq}}{f_{ck} b d^2}}\right] b d$$

if $M_{ueq} > M_{ulim}$

(if depth is restricted)

⟹ Design doubly R/f beam.



$$A_{st1} = \frac{M_{ulim}}{0.87 f_y (d - 0.42 x_u)}$$

$$A_{st2} = \frac{(M_u - M_{ulim})}{0.87 f_y (d - d')}$$

Put M_{ueq}

$$A_{st} = A_{st1} + A_{st2}$$

$$A_{sc} = \frac{M_{ueq} - M_{ulim}}{(f_{sc} - 0.446 f_{ck})(d - d')}$$

Special case

if $M_T > M_u$

⟹ Provide additional R/f in the top of the beam for moment M_{e2}.

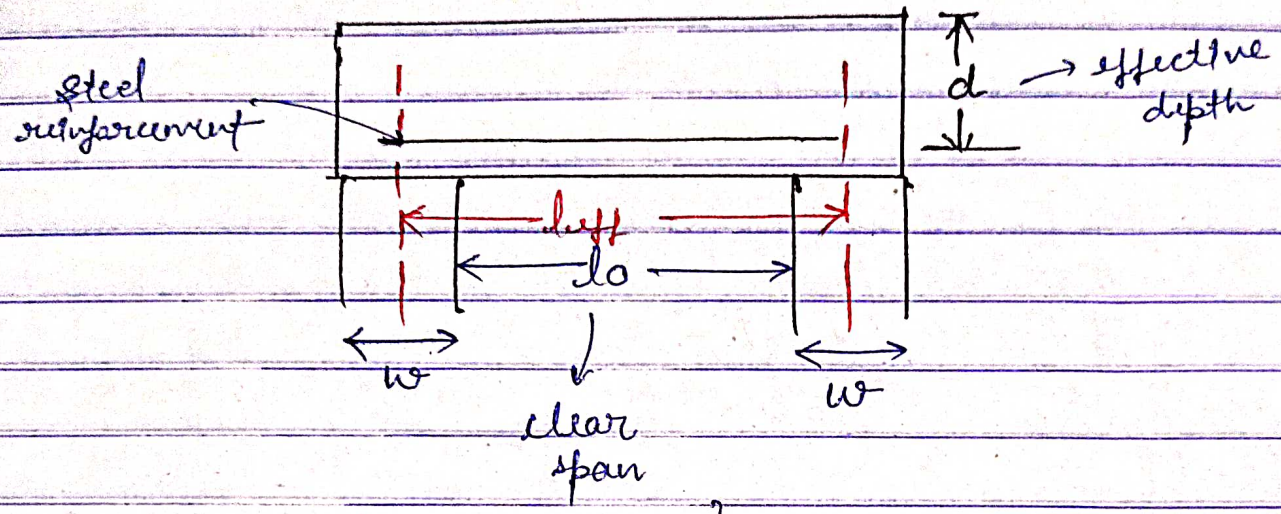
$$M_{e2} = M_T - M_u$$

$$\boxed{A_{st_3} = \frac{M_{e_2}}{0.87 f_y (d - d')}}$$

Slab

Effective span :-

Case (I) :- Simply supported Beam/Slab :-



$$l_{eff} = l_o + \frac{w}{2} + \frac{w}{2}$$

or

$$l_o + d$$

whichever is less

Case-II :- Continuous Beam/Slab :-

condition (1) $w \leq \frac{l_o}{12}$ or 600mm

$w \rightarrow$ width of support

$$l_{eff} = l_o + \frac{w}{2} + \frac{w}{2}$$

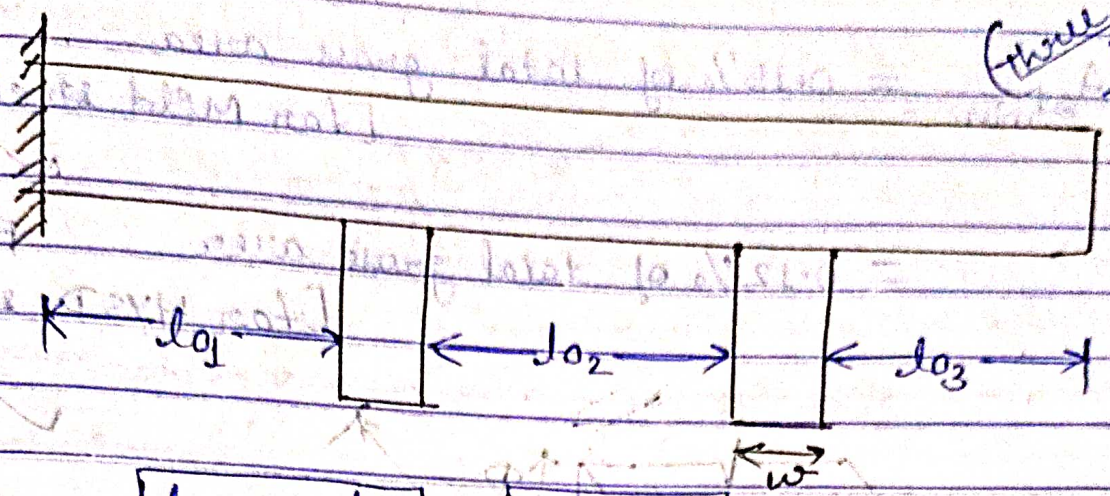
OR

$$l_o + d$$

whichever is less

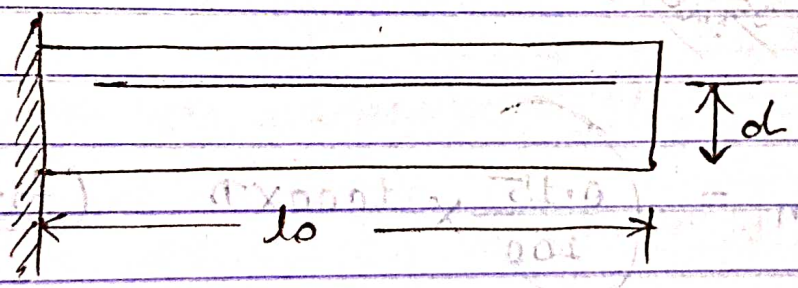
Condition (II):- $w > l_0/12$ or 600mm

(three continuous slabs)



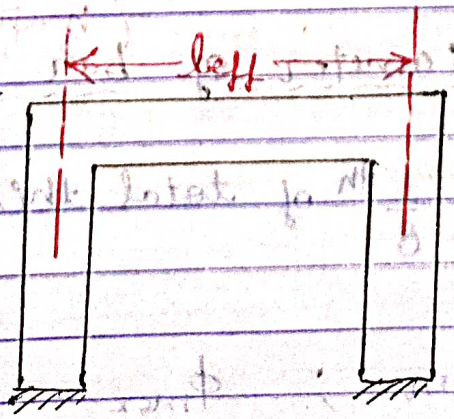
$\boxed{l_{eff} = l_{o_1}}$ $\boxed{l_{eff} = l_{o_2}}$ $l_{eff} = l_{o_3} + \frac{w}{2}$ OR $l_{o_3} + \frac{d}{2}$ } whichever is less

case (III) Cantilever Beam/slab



$$\boxed{l_{eff} = l_o + \frac{d}{2}}$$

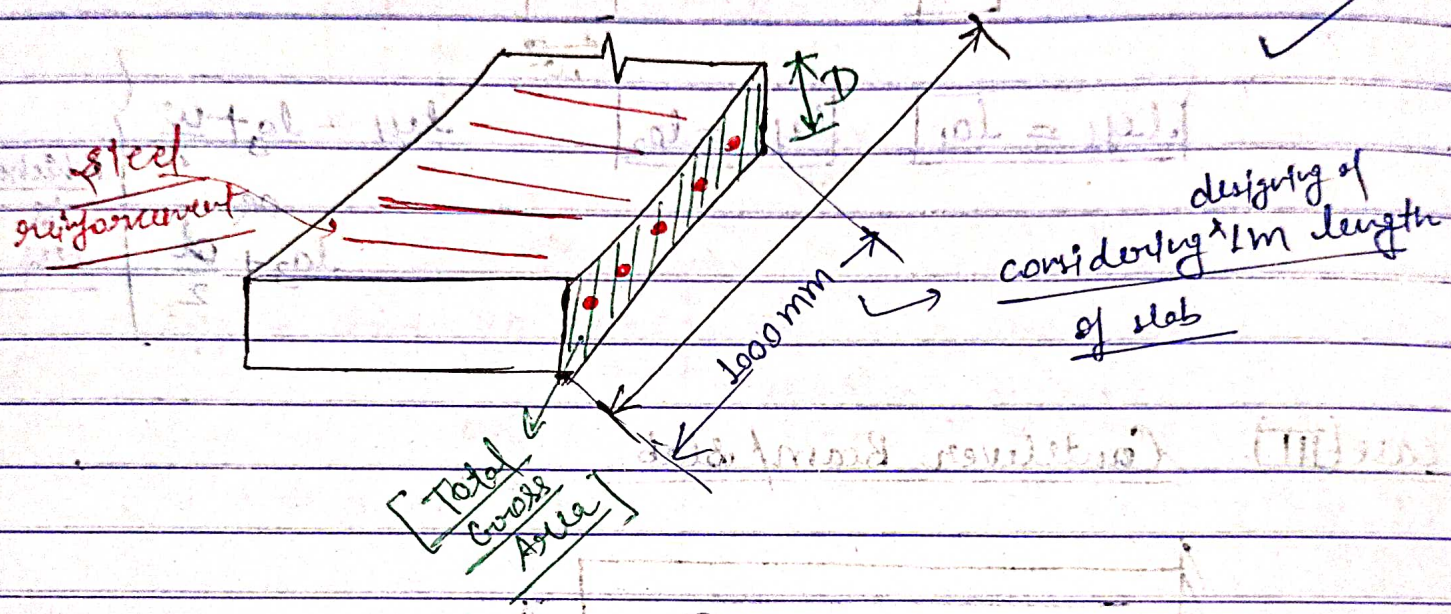
case (IV) Frames



Minimum area of steel R/f in slab :-

$A_{st_{min}}$ = 0.15% of total gross area [for Mild steel] — Fe 250

= 0.12% of total gross area [for HYSD steel] — Fe 415



$$A_{st_{min}} = \frac{0.15}{100} \times 1000 \times D \quad \text{(for MS)}$$

$$A_{st_{min}} = \frac{0.12}{100} \times 1000 \times D \quad \text{(for HYSD)}$$

Maximum diameter of bar (main R/f) in slab

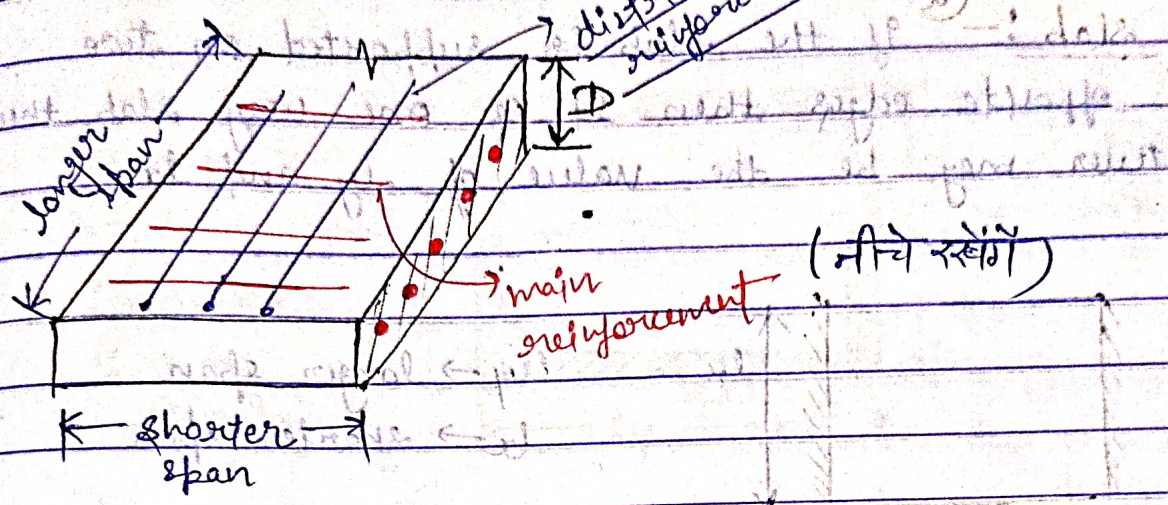
(for main R/f) $\phi_{max} = \frac{1}{8}$ th of total thickness of slab

Ex:- $D = 75mm$; $\phi_{max} = \frac{1}{8}(75) \Rightarrow 9.4\,mm$ → max

≃ ~~10mm~~

$\phi_{use} \Rightarrow 8mm$

Maximum spacing Criteria :-



⇒ For main reinforcement (R/f) :-

(i) 3d
(ii) 300mm
} whichever is less

⇒ For distribution reinforcement (R/f) :-

(i) 5d
(ii) 450mm
} whichever is less.

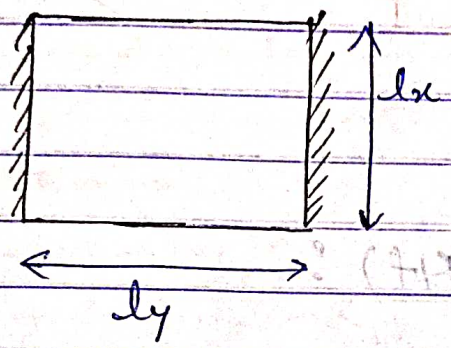
Why distribution reinforcement (R/f) :-

(•) to take temperature & shrinkage stresses.
(•) it reduces deflection upto an appreciable extent.

Types of slabs

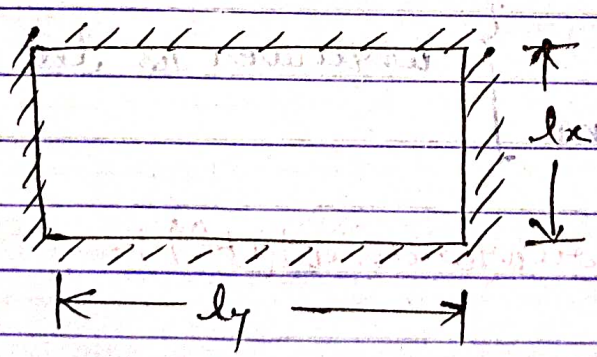
(i) One way slab : (ii) Two way slab

(i) One way slab :- (a) If the slab is supported on two opposite edges then it is one way slab then whatever may be the value of l_y and l_x.



$l_y \rightarrow$ longer span
$l_x \rightarrow$ shorter span

(b) When the given slab is ~~reqe~~ supported on all the four edges and $\frac{l_y}{l_x} > 2.0$ then it is one way slab.

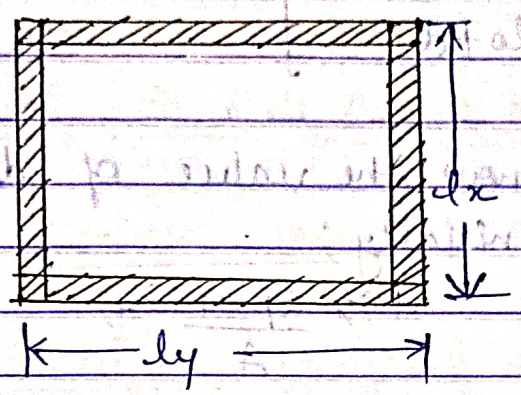


If the slab is supported on two supports than no need to check ratio of l_y/l_x. It is one way slab.

If slab is supported on all four sides then check the ratio if $\frac{l_y}{l_x} > 2$ then one way slab.

(ii) Two way slab :- If the slab is supported on all four edges and $\frac{l_y}{l_x} \leq 2.0$ then it is called two way slab.

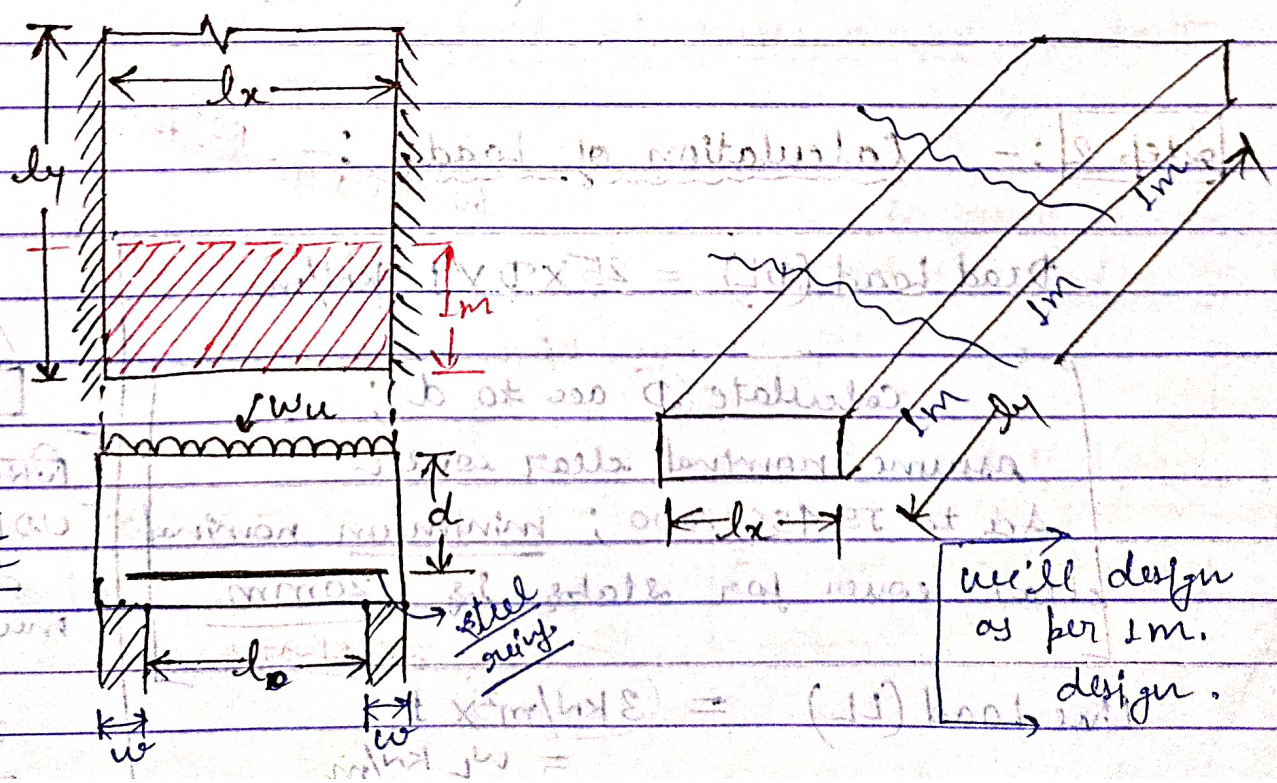
$l_y \rightarrow$ longer span
$l_x \rightarrow$ shorter span
irrespective of x & y direction.



$\frac{l_y}{l_x} < 2.0$

Note :- Two way slab can be simply supported or continuous.

Analysis and design steps of one way slab :-



Step (I) :- Estimate effective span

Simply supported

$$l_{eff} = l_o + \frac{w}{2} + \frac{w}{2}$$
(l_x)

OR

$$l_o + d$$

whichever is less

but we don't know the value of d ; [effective depth of slab]

so take initially ;

$$d \geq \frac{span\ (l_x)}{A}$$

[assume, it can be modified further]

A is basic span to (effective) depth ratio.

Acc to (IS 456 : 2000 provisions)

A = 7 (cantilever)

A = 20 (simply supported)

A = 26 (continuous)

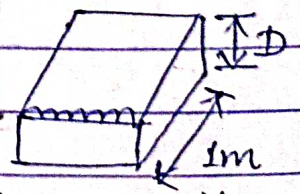
IS 456 : 2000 P.No. 37 Clause 23.2.1

Step 2 :- Calculation of Loads :-

Dead Load (DL) = 25 × D × 1 kN/m (unit wt of concrete) RCC

Calculate (D) acc to d ; (Gross depth)
assume nominal clear cover
Acc to IS 456 : 2000 ; minimum nominal clear cover for slabs is 20mm.

Live Load (LL) = 3 kN/m² × 1 (IS 875 Part-2)
= w_L kN/m.



[जिस dir^n में UDL है उसकी dimension को multiply मत करो बस, बाकी का unit wt. से * कर दो।]

unit wt of concrete

Floor finishing $= 24 \times t_f \times 1 = W_f$ KN/m

Total Load $\Rightarrow W_T = W_d + W_L + W_f$

factored load UDL $\boxed{W_u = 1.5 W_T}$ (in L.S.M.)

Step 3 :→ Calculate Bending Moment :-

$M_u = \frac{W_u l_{eff}^2}{8}$ (simply support)

$M_u = \frac{M_u l_{eff}^2}{2}$ (cantilever slab) $M_u \rightarrow$ max B.M.

$M_u = ???$ (continuous slab) either do by 'stad-pro' or by moment distribution method or by direct values from IS 456 : 2000 in terms of coefficients.

Step 4 :- Estimate required effective depth against above B.M.

$M_{u,lim} = Q f_{ck} b d^2$ (for balanced section) we had discussed earlier

$\Rightarrow$ $\boxed{d_{req.} = \sqrt{\frac{M_u}{Q f_{ck} b}}}$

$b \rightarrow$ width of slab 1m or $\boxed{1000mm}$ for what we're analysing

$Q = 0.148$ [Fe 250]
0.138 [Fe 415]
0.133 [Fe 500]

if ; $\boxed{d_{req.} > d_{assumed}}$ — change & repeat the steps

$\therefore$ $d_{assumed}$ (or provided) $> d_{required}$ It is safe & O.K.

$D = d +$ effective cover

$$\boxed{D = d + \text{clear cover} + \frac{\phi}{2}}$$

Step 5 :- Design of main steel R/f :-

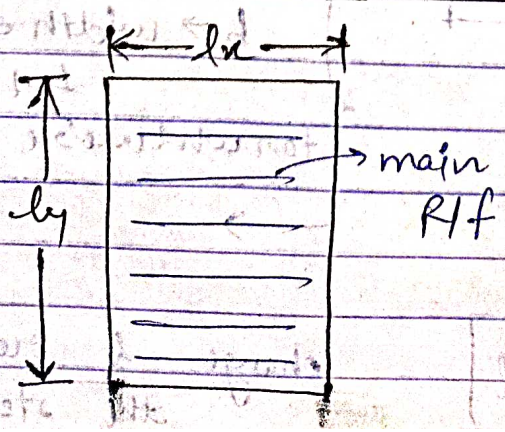
$$\boxed{A_{st\,(main)} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck}\, b d^2}}\right] b d} \quad (mm^2)$$

[along the shorter span]

from the above formula the area of tension steel R/f is being calculated for 1m step of the slab. Therefore put $b = 1000$ mm in the above formula.

OR

$$\boxed{A_{st\,(Main)} = \frac{M_u}{0.87 f_y (d - 0.42 x_{u\,lim})}} \quad (mm^2)$$



spacing (?)

Step 6 Spacing of main steel R/f :-

$$\text{Spacing} = \frac{\frac{\pi}{4}\phi^2}{A_{st}} \times 1000$$

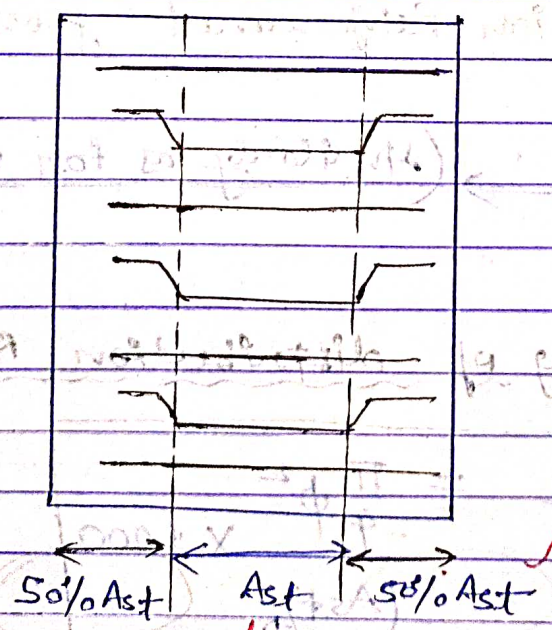
(per meter mai nikal rhe h isliye b = 1000mm)

Step 7 :- Check maximum spacing ratio critiya :-

$$\text{Spacing} \leq \left.\begin{array}{l} 3d \\ \text{or} \\ 300mm \end{array}\right\} \text{whichever is less}$$

Step 6th 280mm
Step 7th 300mm
3d → 420mm

Step 8 :- Provide main R/f along the shorter span :-



(Bcz supports pr zyada R/f ka requirement nahi hota hai.)

(Bcz deflection is max. here)

Step 9 :- Distribution R/f along the longer span

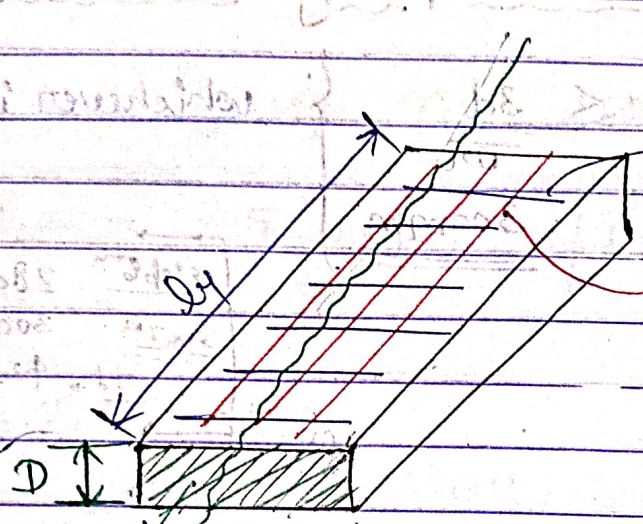
$$(A_{st})_D = \frac{0.15}{100} \times BD \Rightarrow \boxed{(A_{st})_D = \frac{0.15}{100} \times 1000 \times D}$$

Distribution

(Fe 250)

$$(A_{st})_D = \frac{0.12}{100} \times BD \Rightarrow \boxed{(A_{st})_D = \frac{0.12}{100} \times 1000 \times D}$$

(HYSD/CTD/TMT)



(Distribution Reinforcement gross Area)

1m

OR

1000 mm → (similarly as for main R/f)

Step-10 :- Spacing of distribution R/f

$$\boxed{\text{Spacing} = \frac{\frac{\pi}{4}\phi^2}{(A_{st})_D} \times 1000}$$

Now cross check with maximum spacing criteria

Note:- Diameter of distribution R/f ≤ dia of main R/f.

cross-check $\phi_{max} = \frac{1}{8} \times D$

(for main R/f)

Check max. spacing criteria for distribution R/f.

$\not> 5d$
$\not> 450mm$ } whichever is less.

(तीनों में से जो min होगा wo provide करेंगे)

Difference between working stress & limit stress method of design :-

WSM (Working stress Method)	LSM (Limit stress Method)
1. This insure the safety of structure under service load only.	This ensures the safety at ultimate loads and serviceability at working loads.
2. Permissible (or allowable) stresses are kept well below this ultimate strength of their material.	the material can be stressed beyond elastic limit.
3. It assumes linear-elastic behaviour of the material.	The structure will be designed on the basis of the most critical limit state and checked for other limit states.
4. A single factor of safety is applied for all type of loads such as dead load, live load.	A different factor of safety is applied ~~from~~ for different type of loads.

5. ~~A~~ Que :- As compared to the WSM of design, the LSM of design permits that the concrete can admit.

Ans- A higher stress level.