3.6: Standard Form of a Line

In this section we will investigate the standard form of a line. Let's begin with a simple example.

Example 3.6.1

Solve the equation $2x+3y=6$ for y and plot the result.

Solution

First we solve the equation $2x+3y=6$ for y. Begin by isolating all terms containing y on one side of the equation, moving or keeping all the remaining terms on the other side of the equation.

$$\begin{aligned} 2x+3y &= 6 && \text{Original equation.} \\ 2x+3y-2x &= 6-2x && \text{Subtract } 2x \text{ from both sides.} \\ 3y &= 6-2x && \text{Simplify.} \\ \frac{3y}{3} &= \frac{6-2x}{3} && \text{Divide both sides by 3} \end{aligned}$$

Note

Just as multiplication is distributive with respect to addition

$$a(b+c)=ab+ac$$

so too is division distributive with respect to addition.

$$\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$$

When dividing a sum or a difference by a number, we use the distributive property and divide both terms by that number.

$$\begin{aligned} y &= \frac{6}{3}-\frac{2x}{3} && \text{On the left, simplify. On the right, divide both terms by 3} \\ y &= 2-\frac{2x}{3} && \text{Simplify.} \end{aligned}$$

Finally, use the commutative property to switch the order of the terms on the right-hand side of the last result.

$$\begin{aligned} y &= 2+\left(-\frac{2x}{3}\right) && \text{Add the opposite.} \\ y &= -\frac{2}{3}x+2 && \text{Use the commutative property to switch the order.} \end{aligned}$$

Because the equation $2x+3y=6$ is equivalent to the equation $y=-\frac{2}{3}x+2$, the graph of $2x+3y=6$ is a line, having slope $m=-2/3$ and y-intercept $(0,2)$. Therefore, to draw the graph of $2x+3y=6$, plot they-intercept $(0,2)$, move down 2 and 3 to the right, then draw the line (see Figure 3.6.1).

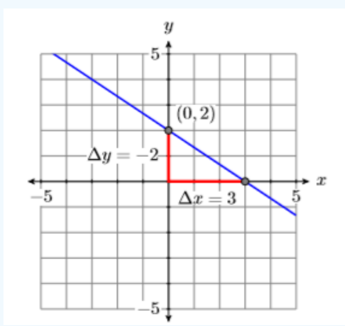

Figure 3.6.1: The graph of $2x+3y=6$, or equivalently, $y=-\frac{2}{3}x+2$

Exercise 3.6.1

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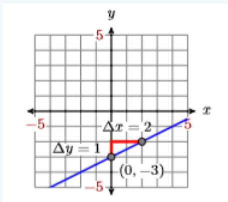
Answer



In general, unless $B = 0$, we can always solve the equation $Ax + By = C$ for y:

$$\begin{aligned}
Ax + By &= C && \color{red}{\text{Original equation.}} \\
Ax + By - Ax &= C - Ax && \color{red}{\text{Subtract } Ax \text{ from both sides.}} \\
By &= C - Ax && \color{red}{\text{Simplify.}} \\
\frac{By}{B} &= \frac{C - Ax}{B} && \color{red}{\text{Divide both sides by } B} \\
y &= \frac{C}{B} - \frac{Ax}{B} && \color{red}{\text{distribute the } B} \\
y &= -\frac{A}{B}x + \frac{C}{B} && \color{red}{\text{Commutative property}}
\end{aligned}$$

Note that the last result is in slope-intercept form $y = mx + b$, whose graph is a line. We have established the following result.

FAct

The graph of the equation $Ax + By = C$, is a line.

Important points: A couple of important comments are in order.

1. The form $Ax + By = C$ requires that the coefficients A, B, and C are integers. So, for example, we would clear the fractions from the form

$$\frac{1}{2}x + \frac{2}{3}y = \frac{1}{4}$$

by multiplying both sides by the least common denominator.

$$\begin{aligned}
12\left(\frac{1}{2}x + \frac{2}{3}y\right) &= \left(\frac{1}{4}\right)12 \\
6x + 8y &= 3
\end{aligned}$$

Note that the coefficients are now integers.

2. The form $Ax + By = C$ also requires that the first coefficient A is nonnegative; i.e., $A \geq 0$. Thus, if we have

$$-5x + 2y = 6$$

then we would multiply both sides by -1, arriving at:

$$\begin{aligned}
-1(-5x + 2y) &= (6)(-1) \\
5x - 2y &= -6
\end{aligned}$$

Note that $A = 5$ is now greater than or equal to zero.

3. If A, B, and C have a common divisor greater than 1, it is recommended that we divide both sides by the common divisor, thus “reducing” the coefficients. For example, if we have

$$3x + 12y = -24$$

then dividing both side by 3 “reduces” the size of the coefficients.

$$\begin{aligned}\frac{3x+12y}{3} &= \frac{-24}{3} \\ x+4y &= -8\end{aligned}$$

Standard form

The form $Ax+By=C$, where A, B, and C are integers, and $A \geq 0$, is called the standard form of a line.

Slope-Intercept to Standard Form

We’ve already transformed a couple of equations in standard form into slopeintercept form. Let’s reverse the process and place an equation in slope intercept form into standard form.

Example $3.6.2$

Given the graph of the line in Figure $3.6.2$, find the equation Given the graph of the line below, find the equation of the line in standard form.

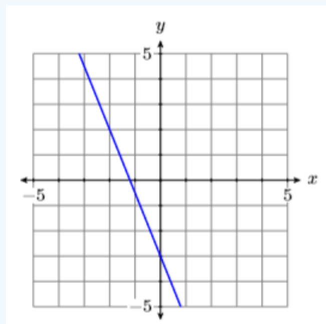


Figure $3.6.2$: Determine the equation of the line.

Solution

The line intercepts the y-axis at $(0,-3)$. From $(0,-3)$, move up 5 units, then left 2 units. Thus, the line has slope $\Delta y/\Delta x=-5/2$ (see Figure $3.6.3$). Substitute $-5/2$ form and -3 for b in the slope-intercept form of the line.

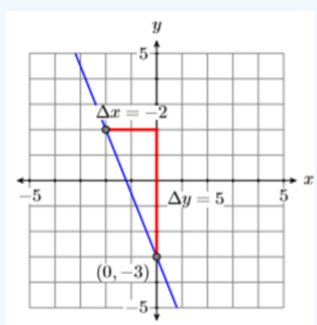


Figure $3.6.3$: The line has y-intercept $(0,-3)$ and slope $-5/2$.

$$\begin{aligned} y &= mx+b && \text{Slope-intercept form.} \\ y &= -\frac{5}{2}x-3 && \text{Substitute: } -5/2 \text{ for } m, -3 \text{ for } b \end{aligned}$$

To put this result in standard form $Ax+By=C$, first clear the fractions by multiplying both sides by the common denominator.

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$$2y = 2\left[-\frac{5}{2}x - 3\right] \quad \text{Multiply both sides by 2}$$

$$2y = 2\left[-\frac{5}{2}x\right] - 2[3] \quad \text{Distribute the 2}$$

$$2y = -5x - 6 \quad \text{Multiply.}$$

That clears the fractions. To put this last result in the form $Ax + By = C$, we need to move the term $-5x$ to the other side of the equation.

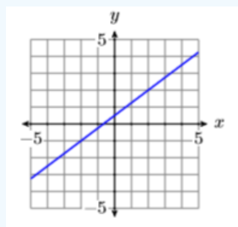
$$5x + 2y = -5x - 6 + 5x \quad \text{Add } 5x \text{ to both sides.}$$

$$5x + 2y = -6 \quad \text{Simplify.}$$

Thus, the standard form of the line is $5x + 2y = -6$. Note that all the coefficients are integers and the terms are arranged in the order $Ax + By = C$, with $A \geq 0$.

Exercise 3.6.2

Given the graph of the line below, find the equation of the line in standard form.



Answer

$3x - 4y = -2$

Point-Slope to Standard Form

Let's do an example where we have to put the point-slope form of a line in standard form.

Example 3.6.3

Sketch the line passing through the points $(-3, -4)$ and $(1, 2)$, then find the equation of the line in standard form.

Solution

Plot the points (−3,−4) and (1,2), then draw a line through them (see Figure 3.6.4).

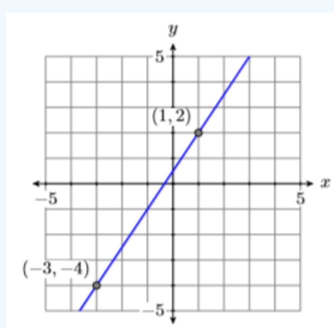


Figure 3.6.4: The line through $(-3, -4)$ and $(1, 2)$.

Use the points $(-3, -4)$ and $(1, 2)$ to calculate the slope.

$$\begin{aligned}\text{Slope} &= \frac{\Delta y}{\Delta x} \quad \text{Slope formula.}\\ &= \frac{2-(-4)}{1-(-3)} \quad \text{Subtract coordinates of } (-3,-4)\\ &= \frac{6}{4} \quad \text{Simplify.}\\ &= \frac{3}{2} \quad \text{Reduce.}\end{aligned}$$

Let's substitute $(x_0, y_0) = (1, 2)$ and $m = 3/2$ in the point-slope form of the line. (*Note: Substituting* $(x_0, y_0) = (-3, -4)$ *and* $m = 3/2$ *would yield the same answer.*)

$$\begin{aligned} y - y_0 &= m(x - x_0) \quad \text{Point-slope form.}\\ y - 2 &= \frac{3}{2}(x-1) \quad \text{Substitute: } 3/2 \text{ for } m, 1 \text{ for } x_0\end{aligned}$$

The question requests that our final answer be presented in standard form. First we clear the fractions.

$$\begin{aligned} y - 2 &= \frac{3}{2}x - \frac{3}{2} \quad \text{Distribute the } 3/2\\ 2[y-2] &= 2\left[\frac{3}{2}x - \frac{3}{2}\right] \quad \text{Multiply both sides by } 2\\ 2y - 2[2] &= 2\left[\frac{3}{2}x\right] - 2\left[\frac{3}{2}\right] \quad \text{Distribute the } 2\\ 2y - 4 &= 3x - 3 \quad \text{Multiply.}\end{aligned}$$

Now that we've cleared the fractions, we must order the terms in the form $Ax + By = C$. We need to move the term $3x$ to the other side of the equation.

$$\begin{aligned} 2y - 4 - 3x &= 3x - 3 - 3x \quad \text{Subtract } 3x \text{ from both sides.}\\ -3x + 2y - 4 &= -3 \quad \text{Simplify, changing the order on the left-hand side.}\end{aligned}$$

To put this in the form $Ax + By = C$, we need to move the term -4 to the other side of the equation.

$$\begin{aligned} -3x + 2y - 4 + 4 &= -3 + 4 \quad \text{Add 4 to both sides.}\\ -3x + 2y &= 1 \quad \text{Simplify.}\end{aligned}$$

It appears that $-3x + 2y = 1$ is in the form $Ax + By = C$. However, standard form requires that $A \geq 0$. We have $A = -3$. To fix this, we multiply both sides by -1.

$$\begin{aligned} -1[-3x + 2y] &= -1[1] \quad \text{Multiply both sides by } -1\\ 3x - 2y &= -1 \quad \text{Distribute the } -1\end{aligned}$$

Thus, the equation of the line in standard form is $3x - 2y = -1$.

Note

If we fail to reduce the slope to lowest terms, then the equation of the line would be:

$$y - 2 = \frac{6}{4}(x - 1)$$

Multiplying both sides by 4 would give us the result

$$4y - 8 = 6x - 6$$

or equivalently:

$$-6x + 4y = 2$$

This doesn't look like the same answer, but if we divide both sides by -2, we do get the same result.

$$3x - 2y = -1$$

This shows the importance of requiring $A \geq 0$ and “reducing” the coefficients A, B, and C. It allows us to compare our answer with our colleagues or the answers presented in this textbook.

Exercise 3.6.3

Find the standard form of the equation of the line that passes through the points $(-2, 4)$ and $(3, -3)$.

Answer

$7x + 5y = 6$

Intercepts

We’ve studied the y-intercept, the point where the graph crosses the y-axis, but equally important are the x-intercepts, the points where the graph crosses the x-axis.

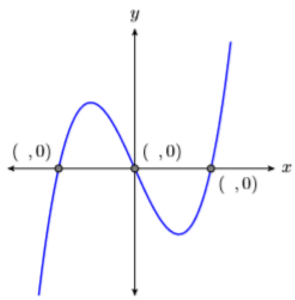


Figure 3.6.5: Each x-intercept has a y-coordinate equal to zero.

In Figure 3.6.5, the graph crosses the x-axis three times. Each of these crossing points is called an x-intercept. Note that each of these x-intercepts has a y-coordinate equal to zero. This leads to the following rule.

x Intercepts

To find the x-intercepts of the graph of an equation, substitute $y = 0$ into the equation and solve for x.

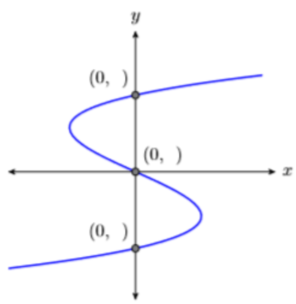


Figure 3.6.6: Each y-intercept has an x-coordinate equal to zero.

Similarly, the graph in Figure 3.6.6 crosses the y-axis three times. Each of these crossing points is called a y-intercept. Note that each of these y-intercepts has an x-coordinate equal to zero. This leads to the following rule.

y Intercepts

To find the y-intercepts of the graph of an equation, substitute $x = 0$ into the equation and solve for y.

Let’s put these rules for finding intercepts to work.

Example $3.6.4$

Find the x- and y-intercepts of the line having equation $2x-3y=6$. Plot the intercepts and draw the line.

Solution

We know that the graph of $2x-3y=6$ is a line. Furthermore, two points completely determine a line. This means that we need only plot the x- and y-intercepts, then draw a line through them.

To find the x-intercept of $2x-3y=6$, substitute 0 for y and solve for x.

$$\begin{aligned} 2x-3y &= 6 \\ 2x-3(0) &= 6 \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} \\ x &= 3 \end{aligned}$$

Thus, the x-intercept of the line is $(3,0)$.

To find the y-intercept of $2x-3y=6$, substitute 0 for x and solve for y.

$$\begin{aligned} 2x-3y &= 6 \\ 2(0)-3y &= 6 \\ -3y &= 6 \\ \frac{-3y}{-3} &= \frac{6}{-3} \\ y &= -2 \end{aligned}$$

Thus, the y-intercept of the line is $(0,-2)$.

Plot the x-intercept $(3,0)$ and the y-intercept $(0,-2)$ and draw a line through them (see Figure $3.6.7$).

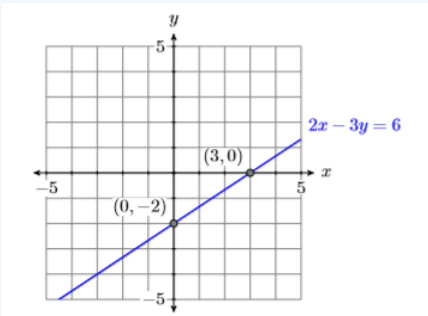


Figure $3.6.7$: The graph of $2x-3y=6$ has intercepts $(3,0)$ and $(0,-2)$.

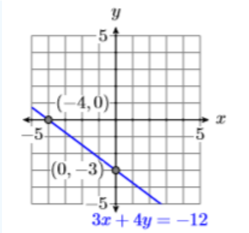
Exercise $3.6.4$

Find the x- and y-intercepts of the line having equation $3x+4y=-12$. Plot the intercepts and draw the line.

Answer

x-intercept: $(-4,0)$

y-intercept: $(0,-3)$



Example 3.6.5

Sketch the line $4x+3y=12$, then sketch the line through the point $(-2,-2)$ that is perpendicular to the line $4x+3y=12$. Find the equation of this perpendicular line.

Solution

Let's first find the x- and y-intercepts of the line $4x+3y=12$.

To find the x-intercept of the line $4x+3y=12$, substitute 0 for y and solve for x.

$$\begin{aligned} 4x+3y &= 12 \\ 4x+3(0) &= 12 \\ 4x &= 12 \\ \frac{4x}{4} &= \frac{12}{4} \\ x &= 3 \end{aligned}$$

Thus, the x-intercept of the line is $(3,0)$.

To find the y-intercept of the line $4x+3y=12$, substitute 0 for x and solve for y.

$$\begin{aligned} 4x+3y &= 12 \\ 4(0)+3y &= 12 \\ 3y &= 12 \\ \frac{3y}{3} &= \frac{12}{3} \\ y &= 4 \end{aligned}$$

Thus, the y-intercept of the line is $(0,4)$.

Plot the intercepts and draw a line through them. Note that it is clear from the graph that the slope of the line $3x+4y=12$ is $-4/3$ (see Figure 3.6.8).

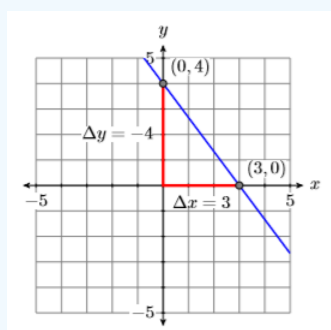


Figure 3.6.8: The graph of $4x+3y=12$ has intercepts $(3,0)$ and $(0,4)$ and slope $-4/3$.

You could also solve for y to put $3x+4y=12$ in slope intercept form in order to determine the slope.

Because the slope of $3x+4y=12$ is $-4/3$, the slope of a line perpendicular to $3x+4y=12$ will be the negative reciprocal of $-4/3$, namely $3/4$. Our perpendicular line has to pass through the point $(-2,-2)$. Start at $(-2,-2)$, move 3 units upward,

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then 4 units to the right, then draw the line. It should appear to be perpendicular to the line $3x+4y=12$ (see Figure 3.6.9).

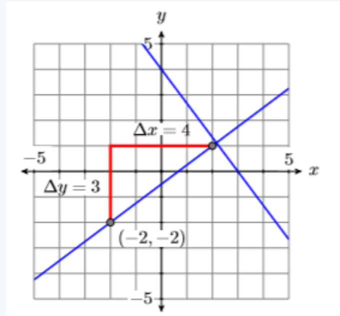


Figure 3.6.9: The slope of the perpendicular line is the negative reciprocal of $-4/3$, namely $3/4$.

Finally, use the point-slope form, $m=3/4$, and $(x_0,y_0)=(-2,-2)$ to determine the equation of the perpendicular line.

$$\begin{aligned} y-y_0 &= m\left(x-x_0\right) && \text{Point-slope form.} \\ y-(-2) &= \frac{3}{4}(x-(-2)) && \text{Substitute: } 3/4 \text{ for } m, -2 \text{ for } x_0 \text{ and } -2 \text{ for } y_0 \\ y+2 &= \frac{3}{4}(x+2) && \text{Simplify.} \end{aligned}$$

Let's place our answer in standard form. Clear the fractions.

$$\begin{aligned} y+2 &= \frac{3}{4}x+\frac{6}{4} && \text{Distribute } 3/4 \\ 4[y+2] &= 4\left[\frac{3}{4}x+\frac{6}{4}\right] && \text{Multiply both sides by } 4 \\ 4y+4[2] &= 4\left[\frac{3}{4}x\right]+4\left[\frac{6}{4}\right] && \text{Distribute the } 4 \\ 4y+8 &= 3x+6 && \text{Multiply.} \end{aligned}$$

Rearrange the terms to put them in the order $Ax+By=C$.

$$\begin{aligned} 4y+8-3x &= 3x+6-3x && \text{Subtract } 3x \text{ from both sides.} \\ -3x+4y+8 &= 6 && \text{Simplify. Rearrange on the left.} \\ -3x+4y+8-8 &= 6-8 && \text{Subtract 8 from both sides.} \\ -3x+4y &= -2 && \text{Simplify.} \\ -1(-3x+4y) &= -1(-2) && \text{Multiply both sides by } -1 \\ 3x-4y &= 2 && \text{Distribute the } -1 \end{aligned}$$

Hence, the equation of the perpendicular line is $3x-4y=2$.

Exercise 3.6.5

Find the equation of the line that passes through the point $(3,2)$ and is perpendicular to the line $6x-5y=15$.

Answer

$5x+6y=27$

Horizontal and Vertical Lines

Here we keep an earlier promise to address what happens to the standard form $Ax+By=C$ when either $A=0$ or $B=0$. For example, the form $3x=6$, when compared with the standard form $Ax+By=C$, has $B=0$. Similarly, the form $2y=-12$, when compared with the standard form $Ax+By=C$, has $A=0$. Of course, $3x=6$ can be simplified to $x=2$ and $2y=-12$

can be simplified to $y=-6$. Thus, if either $A=0$ or $B=0$, the standard form Ax + By = C takes the form $x=a$ and $y=b$, respectively.

As we will see in the next example, the form $x=a$ produces a vertical line, while the form $y=b$ produces a horizontal line.

Example 3.6.6

Sketch the graphs of $x=3$ and $y=-3$.

Solution

To sketch the graph of $x=3$, recall that the graph of an equation is the set of all points that satisfy the equation. Hence, to draw the graph of $x=3$, we must plot all of the points that satisfy the equation $x=3$; that is, we must plot all of the points that have an x-coordinate equal to 3. The result is shown in Figure 3.6.10.

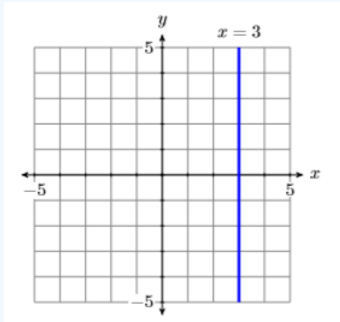


Figure 3.6.10: The graph of $x=3$ is a vertical line.

Secondly, to sketch the graph of $y=-3$, we plot all points having a y-coordinate equal to -3. The result is shown in Figure 3.6.11.

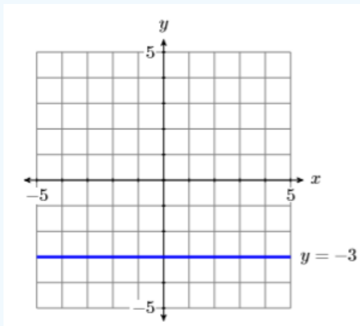


Figure 3.6.11: The graph of $y=-3$ is a horizontal line.

Things to note:

A couple of comments are in order regarding the lines in Figures 3.6.10 and 3.6.11.

1. The graph of $x=3$ in Figure 3.6.10, being a vertical line, has undefined slope. Therefore, we cannot use either of the formulae $y=mx+b$ or $y-y_0=m(x-x_0)$ to obtain the equation of the line. The only way we can obtain the equation is to note that the line is the set of all points (x,y) whose x-coordinate equals 3.
2. However, the graph of $y=-3$, being a horizontal line, has slope zero, so we can use the slope-intercept form to find the equation of the line. Note that the y-intercept of this graph is $(0,-3)$. If we substitute these numbers into $y=mx+b$, we get:

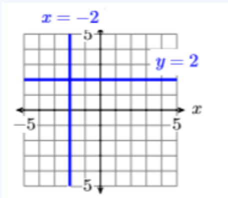
$$\begin{aligned} y &= mx+b && \text{Slope-intercept form.} \\ y &= 0x+(-3) && \text{Substitute: 0 for } m, -3 \text{ for } b \\ y &= -3 && \text{Simplify.} \end{aligned}$$

However, it is far easier to just look at the line in Figures 3.6.11 and note that it is the collection of all points (x,y) with $y=3$.

Exercise 3.6.6

Sketch the graphs of $x=-2$ and $y=2$.

Answer



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