3: INTRODUCTION TO GRAPHING

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Begin

English

3.1: Graphing Equations by Hand

We begin with the definition of an *ordered pair*.

Ordered Pair

The construct (x, y), where x and y are any real numbers, is called an ordered pair of real numbers.

$(4, 3)$, $(-3, 4)$, $(-2, -3)$, and $(3, -1)$ are examples of ordered pairs.

Order Matters

Pay particular attention to the phrase “ordered pairs.” Order matters. Consequently, the ordered pair (x, y) is not the same as the ordered pair (y, x), because the numbers are presented in a different order.

The Cartesian Coordinate System

Pictured in Figure $3.1.1$ is a Cartesian Coordinate System. On a grid, we’ve created two real lines, one horizontal labeled x (we’ll refer to this one as the x-axis), and the other vertical labeled y (we’ll refer to this one as the y-axis).

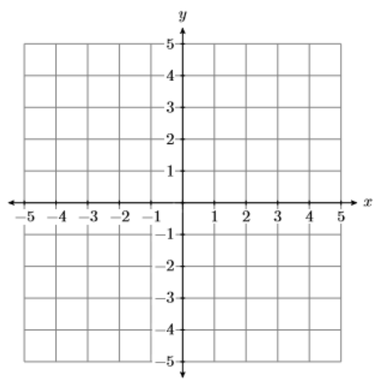


Figure $3.1.1$: The Cartesian Coordinate System.

Two Important Points:

Here are two important points to be made about the horizontal and vertical axes in Figure $3.1.1$.

1. As you move from left to right along the horizontal axis (the x-axis in Figure $3.1.1$), the numbers grow larger. The positive direction is to the right, the negative direction is to the left.
2. As you move from bottom to top along the vertical axis (the y-axis in Figure $3.1.1$), the numbers grow larger. The positive direction is upward, the negative direction is downward.

Additional Comments:

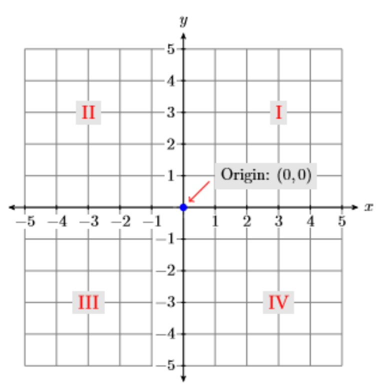


Figure $3.1.2$: Numbering the quadrants and indicating the coordinates of the origin.

Two additional comments are in order:

3. The point where the horizontal and vertical axes intersect in Figure $3.1.2$ is called the origin of the coordinate system. The origin has coordinates $(0, 0)$.
4. The horizontal and vertical axes divide the plane into four quadrants, numbered $\mathrm{I}, \mathrm{II}, \mathrm{MI}$, and IV (roman numerals for one, two, three, and four), as shown in Figure $3.1.2$. Note that the quadrants are numbered in a counter-clockwise order.

Note

Rene Descartes (1596-1650) was a French philosopher and mathematician who is well known for the famous phrase“cogito ergo sum” (I think, therefore I am), which appears in his Discours de la methode pour bien conduire sa raison, et chercher la verite dans les sciences (Discourse on the Method of Rightly Conducting the Reason, and Seeking Truth in the Sciences). In that same treatise, Descartes introduces his coordinate system, a method for representing points in the plane via pairs of real numbers. Indeed, the Cartesian plane of modern day is so named in honor of Rene Descartes, who some call the “Father of Modern Mathematics”

Plotting Ordered Pairs

Before we can plot any points or draw any graphs, we first need to set up a Cartesian Coordinate System on a sheet of graph paper? How do we do this? What is required?

how to setup cartesian coordinate system

Draw and label each axis.

If we are going to plot points (x,y), then, on a sheet of graph paper, perform each of the following initial tasks.

1. Use a ruler to draw the horizontal and vertical axes.
2. Label the horizontal axis as the x-axis and the vertical axis as the y-axis.

We don’t always label the horizontal axis as the x-axis and the vertical axis as the y-axis. For example, if we want to plot the velocity of an object as a function of time, then we would be plotting points (t, v). In that case, we would label the horizontal axis as the t-axis and the vertical axis as the v-axis.

Indicate the scale on each axis.

3. Label at least one vertical gridline with its numerical value.
4. Label at least one horizontal gridline with its numerical value.

The scales on the horizontal and vertical axes may differ. However, on each axis, the scale must remain consistent. That is, as you count to the right from the origin on the x-axis, if each gridline represents one unit, then as you count to the left from the origin on the x-axis, each gridline must also represent one unit. Similar comments are in order for the y-axis, where the scale must also be consistent, whether you are counting up or down.

The result of this first step is shown in Figure $3.1.3$.

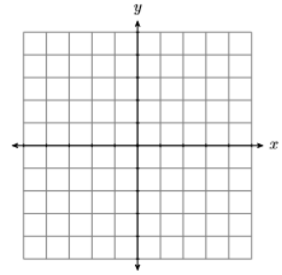


Figure 3.1.3: Draw and label each axis.

An example is shown in Figure 3.1.4. Note that the scale indicated on the x-axis indicates that each gridline counts as 1-unit as we count from left-to-right. The scale on the y-axis indicates that each gridlines counts as 2-units as we count from bottom-to-top.

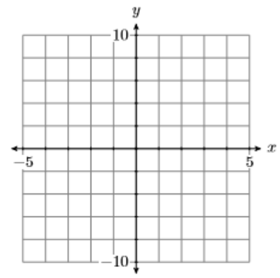


Figure 3.1.4: Indicate the scale on each axis.

Now that we know how to set up a Cartesian Coordinate System on a sheet of graph paper, here are two examples of how we plot points on our coordinate system.

To plot the ordered pair $(4, 3)$, start at the origin and move 4 units to the right along the horizontal axis, then 3 units upward in the direction of the vertical axis.

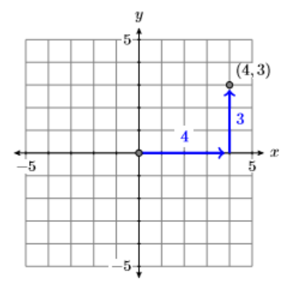


Figure 3.1.5: Plot of $(4, 3)$.

To plot the ordered pair $(-2, -3)$, start at the origin and move 2 units to the left along the horizontal axis, then 3 units downward in the direction of the vertical axis.

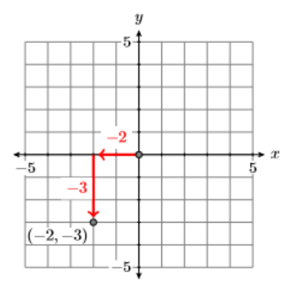


Figure 3.1.6: Plot of $(-2, -3)$

Continuing in this manner, each ordered pair (x, y) of real numbers is associated with a unique point in the Cartesian plane. Vice-versa, each point in the Cartesian point is associated with a unique ordered pair of real numbers. Because of this association, we begin to use the words “point” and “ordered pair” as equivalent expressions, sometimes referring to the “point” (x, y) and other times to the “ordered pair” (x, y).

Example $3.1.1$

Identify the coordinates of the point P in Figure $3.1.7$.

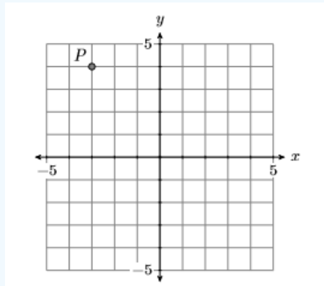


Figure $3.1.7$: Identify the coordinates of the point P.

Solution

In Figure $3.1.8$, start at the origin, move 3 units to the left and 4 units up to reach the point P. This indicates that the coordinates of the point P are $(-3, 4)$.

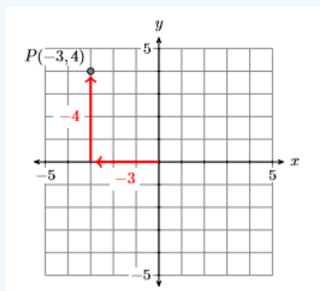
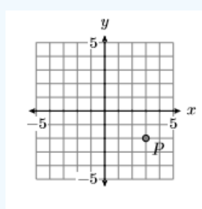


Figure $3.1.8$: Start at the origin, move 3 units left and 4 units up.

Exercise $3.1.1$

Identify the coordinates of the point P in the graph below.



Answer

$(3, -2)$

Equations in Two Variables

Note

The variables do not have to always be x and y. For example, the equation $v = 2 + 3.2t$ is an equation in two variables, v and t.

The equation $y = x + 1$ is an equation in two variables, in this case x and y. Consider the point $(x, y) = (2, 3)$. If we substitute 2 for x and 3 for y in the equation $y = x + 1$, we get the following result:

$$\begin{aligned} y &= x + 1 && \text{Original equation.} \\ 3 &= 2 + 1 && \text{Substitute: 2 for } x, 3 \text{ for } y \\ 3 &= 3 && \text{Simplify both sides.} \end{aligned}$$

Because the last line is a true statement, we say that $(2, 3)$ is a solution of the equation $y = x + 1$. Alternately, we say that $(2, 3)$ satisfies the equation $y = x + 1$. On the other hand, consider the point $(x, y) = (-3, 1)$. If we substitute -3 for x and 1 for y in the equation $y = x + 1$, we get the following result.

$$\begin{aligned} y &= x + 1 && \text{Original equation.} \\ 1 &= -3 + 1 && \text{Substitute: } -3 \text{ for } x, 1 \text{ for } y \\ 1 &= -2 && \text{Simplify both sides.} \end{aligned}$$

Because the last line is a false statement, the point $(-3, 1)$ is **not** a solution of the equation $y = x + 1$; that is, the point $(-3, 1)$ does **not** satisfy the equation $y = x + 1$.

Solutions of an equation in two variables

Given an equation in the variables x and y and a point $(x, y) = (a, b)$, if upon subsituting a for x and b for y a true statement results, then the point $(x, y) = (a, b)$ is said to be a solution of the given equation. Alternately, we say that the point $(x, y) = (a, b)$ satisfies the given equation.

Example 3.1.2

Which of the ordered pairs $(0, -3)$ and $(1, 1)$ satisfy the equation $y = 3x - 2$?

Solution

Substituting the ordered pairs $(0, -3)$ and $(1, 1)$ into the equation $y = 3x - 2$ lead to the following results:

Consider $(x, y) = (0, -3)$. Substitute 0 for x and -3 for y:

$$\begin{aligned} y &= 3x - 2 \\ -3 &= 3(0) - 2 \\ -3 &= -2 \end{aligned}$$

The resulting statement is false.

Consider $(x, y) = (1, 1)$. Substitute 1 for x and 1 for y:

$$\begin{aligned} y &= 3x - 2 \\ 1 &= 3(1) - 2 \\ 1 &= 1 \end{aligned}$$

The resulting statement is true.

Thus, the ordered pair $(0, -3)$ does not satisfy the equation $y = 3x - 2$, but the ordered pair $(1, 1)$ does satisfy the equation $y = 3x - 2$.

Exercise 3.1.2

Which of the ordered pairs $(-1, 3)$ and $(2, 1)$ satisfy the equation $y = 2x + 5$?

Answer

$(-1,3)$

Graphing Equations in Two Variables

Let’s first define what is meant by the graph of an equation in two variables.

The graph of an equation

The graph of an equation is the set of all points that satisfy the given equation.

Example 3.1.3

Sketch the graph of the equation $y=x+1$.

Solution

The definition requires that we plot all points in the Cartesian Coordinate System that satisfy the equation $y=x+1$. Let’s first create a table of points that satisfy the equation. Start by creating three columns with headers x, y, and (x,y), then select some values for x and put them in the first column.

Take the first value of x, namely $x=-3$, and substitute it into the equation $y=x+1$.

$$\begin{aligned}y&=x+1\\y&=-3+1\\y&=-2\end{aligned}$$

x	$y=x+1$	(x,y)
-3	-2	$(-3,-2)$
-2		
-1		
0		
1		
2		
3		

Thus, when $x=-3$, we have $y=-2$. Enter this value into the table.

Continue substituting each tabular value of x into the equation $y=x+1$ and use each result to complete the corresponding entries in the table.

$$\begin{aligned}y&=-3+1=-2\\y&=-2+1=-1\\y&=-1+1=0\\y&=0+1=1\\y&=1+1=2\\y&=2+1=3\\y&=3+1=4\end{aligned}$$

x	$y=x+1$	(x,y)
-3	-2	$(-3,-2)$
-2	-1	$(-2,-1)$
-1	0	$(-1,0)$
0	1	$(0,1)$
1	2	$(1,2)$
2	3	$(2,3)$
3	4	$(3,4)$

The last column of the table now contains seven points that satisfy the equation $y=x+1$. Plot these points on a Cartesian Coordinate System (see Figure \(\PageIndex{9}\)).

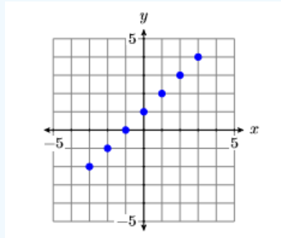


Figure 3.1.9: Seven points that satisfy the equation $y=x+1$.

In Figure $3.1.9$, we have plotted seven points that satisfy the given equation $y=x+1$. However, the definition requires that we plot all points that satisfy the equation. It appears that a pattern is developing in Figure $3.1.9$, but let's calculate and plot a few more points in order to be sure. Add the x-values -2.5, -1.5, -0.5, 0.5, 1.5, and 2.5 to the x-column of the table, then use the equation $y=x+1$ to evaluate y at each one of these x-values.

$$\begin{aligned}
&y=-2.5+1=-1.5\\
&y=-1.5+1=-0.5\\
&y=-0.5+1=0.5\\
&y=0.5+1=1.5\\
&y=1.5+1=2.5\\
&y=2.5+1=3.5
\end{aligned}$$

x	$y=x+1$	(x, y)
-2.5	-1.5	$(-2.5,-1.5)$
-1.5	-0.5	$(-1.5,-0.5)$
-0.5	0.5	$(-0.5,0.5)$
0.5	1.5	$(0.5,1.5)$
1.5	2.5	$(1.5,2.5)$
2.5	3.5	$(2.5,3.5)$

Add these additional points to the graph in Figure $3.1.9$ to produce the image shown in Figure $3.1.10$.

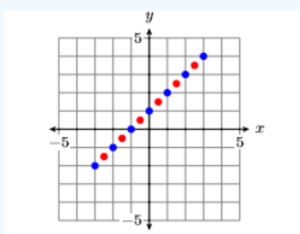


Figure 3.1.10: Adding additional points to the graph of $y=x+1$.

There are an infinite number of points that satisfy the equation $y=x+1$. In Figure $3.1.10$, we've plotted only 13 points that satisfy the equation. However, the collection of points plotted in Figure $3.1.10$ suggest that if we were to plot the remainder of the points that satisfy the equation $y=x+1$, we would get the graph of the line shown in Figure $3.1.11$.

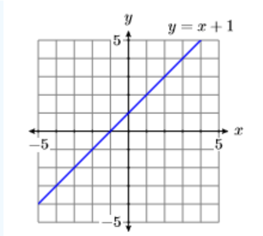
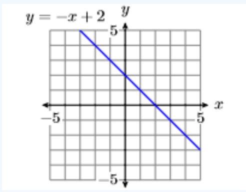


Figure 3.1.11: The graph of $y=x+1$ is a line.

Exercise 3.1.3

Sketch the graph of the equation $y=-x+2$.

Answer



Guidelines and Requirements

Example 3.1.3 suggests that we should use the following guidelines when sketching the graph of an equation.

Guidelines for drawing the graph of an equation

When asked to draw the graph of an equation, perform each of the following steps:

1. Set up and calculate a table of points that satisfy the given equation.
2. Set up a Cartesian Coordinate System on graph paper and plot the points in your table on the system. Label each axis (usually x and y) and indicate the scale on each axis.
3. If the number of points plotted are enough to envision what the shape of the final curve will be, then draw the remaining points that satisfy the equation as imagined. Use a ruler if you believe the graph is a line. If the graph appears to be a curve, freehand the graph without the use of a ruler.
4. If the number of plotted points do not provide enough evidence to envision the final shape of the graph, add more points to your table, plot them, and try again to envision the final shape of the graph. If you still cannot predict the eventual shape of the graph, keep adding points to your table and plotting them until you are convinced of the final shape of the graph.

Here are some additional requirements that must be followed when sketching the graph of an equation.

Graph paper, lines, curves, and rulers.

The following are requirements for this class:

5. All graphs are to be drawn on graph paper.
6. All lines are to be drawn with a ruler. This includes the horizontal and vertical axes.
7. If the graph of an equation is a curve instead of a line, then the graph should be drawn freehand, without the aid of a ruler.

Using the TABLE Feature of the Graphing Calculator

As the equations become more complicated, it can become quite tedious to create tables of points that satisfy the equation. Fortunately, the graphing calculator has a TABLE feature that enables you to easily construct tables of points that satisfy the given equation.

Example 3.1.4

Use the graphing calculator to help create a table of points that satisfy the equation $y=x^2-7$. Plot the points in your table. If you don't feel that there is enough evidence to envision what the final shape of the graph will be, use the calculator to add more points to your table and plot them. Continue this process until your are convinced of the final shape of the graph.

Solution

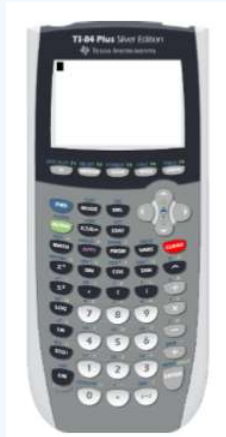


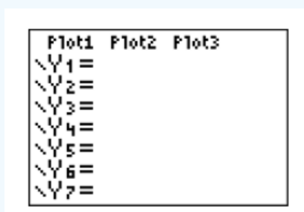
Figure 3.1.12: The graphing calculator.

The first step is to load the equation $y=x^2-7$ into the **Y=** menu of the graphing calculator. The topmost row of buttons on your calculator (see Figure 3.1.12) have the following appearance:

Y= WINDOW ZOOM TRACE GRAPH

Figure 3.1.13. Use the up-and-down arrow keys (see Figure 3.1.15) to move the cursor after **Y1**= in the **Y**= menu, then use the following keystrokes to enter the equation $y=x^2-7$. The result is shown in Figure 3.1.14.

X,T,θ,n ∧ 2 − 7 ENTER



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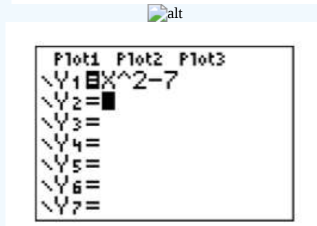


Figure 3.1.13: Opening the **Y**= menu. ***Figure*** **3.1.14**: Entering the equation $y=x^2-7$.

Figure 3.1.15: Arrow keys move the cursor.

The next step is to “set up” the table. First, note that the calculator has symbolism printed on its case above each of its buttons. Above the WINDOW button you’ll note the phrase TBLSET. Note that it is in the same color as the 2ND button. Thus, to open the setup window for the table, enter the following keystrokes.

alt

Figure 3.1.16) equal to the first x-value you want to see in your table. In this case, enter -4 after TblStart. SetTbl to the increment you want for your x-values. In this case, set Tbl equal to 1. Finally, set both the independent and dependent variables to “automatic.” In each case, use the arrow keys to highlight the word AUTO and press ENTER. The result is shown in Figure 3.1.16.

Next, note the word TABLE above the GRAPH button is in the same color as the 2ND key. To open the TABLE, enter the following keystrokes.

Figure 3.1.17. Note that you can use the up-and-down arrow keys to scroll through the table.

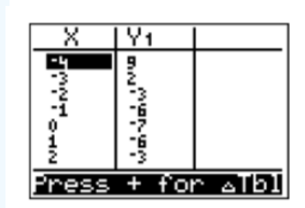


Figure 3.1.16: Opening the TBLSET menu. **Figure 3.1.17:** Table of points satisfying $y = x^2 - 7$.

Next, enter the results from your calculator’s table into a table on a sheet of graph paper, then plot the points in the table. The results are shown in Figure 3.1.18.

x	$y = x^2 - 7$	(x, y)
-4	9	$(-4, 9)$
-3	2	$(-3, 2)$
-2	-3	$(-2, -3)$
-1	-6	$(-1, -6)$
0	-7	$(0, -7)$
1	-6	$(1, -6)$
2	-3	$(2, -3)$
3	2	$(3, 2)$
4	9	$(4, 9)$

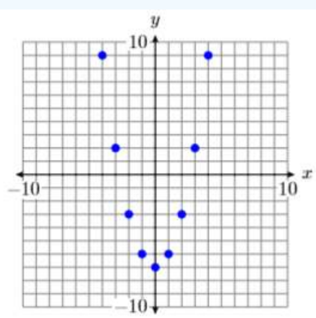
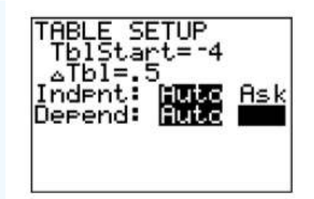


Figure 3.1.18: Plotting nine points that satisfy the equation $y = x^2 - 7$.

In Figure 3.1.18, the eventual shape of the graph of $y = x^2 - 7$ may be evident already, but let’s add a few more points to our table and plot them. Open the table “setup” window again by pressing 2ND WINDOW. Set TblStart to -4 again, then set the increment Tbl to 0.5. The result is shown in Figure 3.1.19.



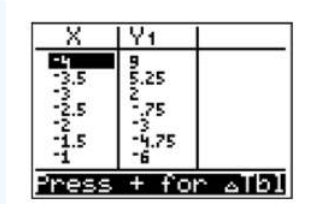


Figure 3.1.19: SetTbl equal to 0.5. **Figure 3.1.20:** More points satisfying $y=x^2-7$.

Add these new points to the table on your graph paper and plot them (see Figure $3.1.21$).

x	$y=x^2-7$	(x,y)
-3.5	5.25	$(-3.5, 5.25)$
-2.5	-0.75	$(-2.5, -0.75)$
-1.5	-4.75	$(-1.5, -4.75)$
-0.5	-6.75	$(-0.5, -6.75)$
0.5	-6.75	$(0.5, -6.75)$
1.5	-4.75	$(1.5, -4.75)$
2.5	-0.75	$(2.5, -0.75)$
3.5	5.25	$(3.5, 5.25)$

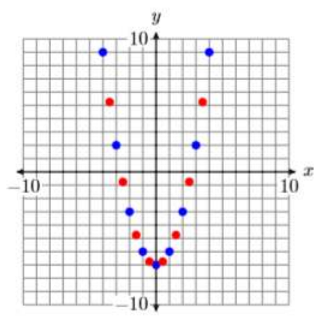


Figure 3.1.21: Adding additional points to the graph of $y=x^2-7$.

There are an infinite number of points that satisfy the equation $y=x^2-7$. In Figure $3.1.21$, we've plotted only 17 points that satisfy the equation $y=x^2-7$. However, the collection of points in Figure $3.1.21$ suggest that if we were to plot the remainder of the points that satisfy the equation $y=x^2-7$, the result would be the curve (called a parabola) shown in Figure $3.1.22$.

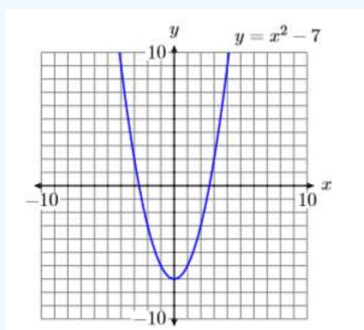


Figure 3.1.22: The graph of $y=x^2-7$ is a curve called a parabola.

3.2: The Graphing Calculator

It’s time to learn how to use a graphing calculator to sketch the graph of an equation. We will use the TI-84 graphing calculator in this section, but the skills we introduce will work equally well on the ancient TI-82 and the less ancient TI-83 graphing calculators.

Figure $3.2.1$: The Graphing Calculator.

In this introduction to the graphing calculator, you will need to use several keys on the upper half of the graphing calculator (see Figure $3.2.2$). The upand-down and left-and-right arrow keys are located in the upper right corner of Figure $3.2.2$. These are used for moving the cursor on the calculator view screen and various menus. Immediately below these arrow keys is the CLEAR button, used to clear the view screen and equations in the **Y**= menu.

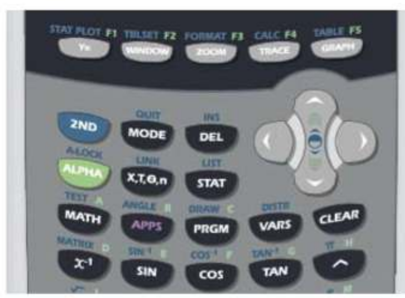


Figure $3.2.2$: Top half of calculator.

It is not uncommon that people share their graphing calculators with friends who make changes to the settings in the calculator. Let’s take a moment to make sure we have some common settings on our calculators.

Note

Above each button are one or more commands located on the calculator’s case. Press the 2ND key to access a command having the same color as the 2ND key. Press the ALPHA key to access a command having the same color as the ALPHA key.

Locate and push the MODE in the first row of Figure $3.2.2$. This will open the window shown in Figure $3.2.3$. Make sure that the mode settings on your calculator are identical to the ones shown in Figure $3.2.3$. If not, use the up-and-down arrow keys to move to the non-matching line item. This should place a blinking cursor over the first item on the line. Press the ENTER button on the lower-right corner of your calculator to make the selection permanent. Once you’ve completed your changes, press 2ND MODE again to quit the MODE menu.

Figure $3.2.3$: Settings in the MODE window.

Note

The QUIT command is located above the MODE button on the calculator case and is used to exit the current menu.

Next, note the buttons across the first row of the calculator, located immediately below the view screen.

Y= WINDOW ZOOM TRACE GRAPH

Figure $3.2.2$). Press the 2ND button, then the **Y**= button. This opens the stat plot menu shown in Figure $3.2.4$.

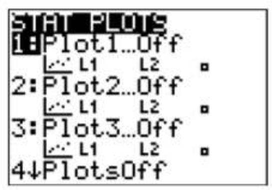


Figure $3.2.4$: The STAT PLOT menu.

We need all of the stat plots to be "off." If any of the three stat plots are "on," select **4:PlotsOff** (press the number 4 on your keyboard), then press the ENTER key on the lower right corner of your calculator. That's it! Your calculator should now be ready for the upcoming exercises.

Example $3.2.1$

Use the graphing calculator to sketch the graph of $y = x + 1$.

Solution

Recall that we drew the graph of $y = x + 1$ by hand in Example 3.1.1 of Section 3.1 (see Figure 3..1.11). In this example, we will use the graphing calculator to produce the same result.

Press the **Y**= button. The window shown in Figure $3.2.5$ appears. If any equations appear in the **Y**= menu of Figure $3.2.5$, use the up-and-down arrow keys and the CLEAR button (located below the up-and-down and left-and-right arrow keys) to delete them.

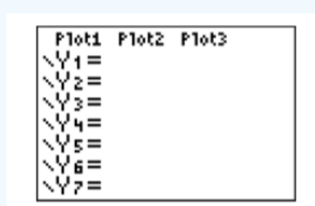


Figure $3.2.5$: Press the Y= button to open the Y= menu.

Next, move the cursor to **Y1**=, then enter the equation $y = x + 1$ in **Y1** with the following button keystrokes. The result is shown in Figure $3.2.6$.

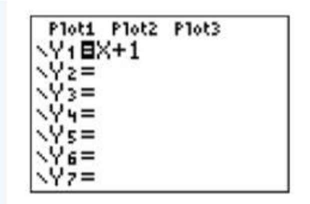


Figure 3.2.6: Enter $y = x + 1$ in **Y1**=.

Figure 3.2.7. From the ZOOM menu, select **6:ZStandard**.

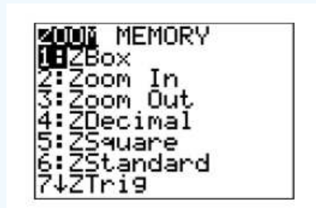


Figure 3.2.7: Select **6:ZStandard** from the **ZOOM** menu.

There are two ways to make this selection:

1. Use the down-arrow key to move downward in the **ZOOM** menu until **6:ZStandard** is highlighted, then press the **ENTER** key.
2. A quicker alternative is to simply press the number 6 on the calculator keyboard to select **6:ZStandard**.

The resulting graph of $y = x + 1$ is shown in Figure 3.2.8. Note that the result is identical to the graph of $y = x + 1$ drawn by hand in Example 3.1.3 of Section 3.1 (see Figure 3..1.11).

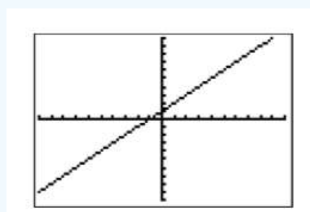
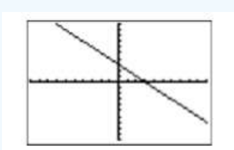


Figure 3.2.8: The graph of $y = x + 1$ is a line.

Exercise 3.2.1

Use the graphing calculator to sketch the graph of $y = -x + 3$.

Answer



Example 3.2.2

Use the graphing calculator to sketch the graph of $y = x^2 - 7$.

Solution

Recall that we drew the graph of $y = x^2 - 7$ by hand in Example 4 of Section 3.1 (see Figure 3.1.22). In this example, we will use the graphing calculator to produce the same result. Press the **Y**= button and enter the equation $y = x^2 - 7$ into **Y1** (see

Figure $3.2.9$) with the following keystrokes:

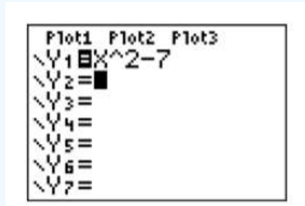


Figure $3.2.9$: Enter $y=x^2-7$ in **Y1**.

The caret (∧) symbol (see Figure $3.2.2$) is located in the last column of buttons on the calculator, just underneath the CLEAR button, and means “raised to.” The caret button is used for entering exponents. For example, x^2 is entered as $\mathrm{X}\wedge 2$, x^3 is entered as $\mathrm{X}\wedge 3$, and so on.

Press the **ZOOM** button, then select **6:ZStandard** from the **ZOOM** menu to produce the graph of $y=x^2-7$ shown in Figure $3.2.10$. Note that the result in Figure $3.2.10$ agrees with the graph of $y=x^2-7$ we drew by hand in Example 4 of Section 3.1 (see Figure 3.1.22).

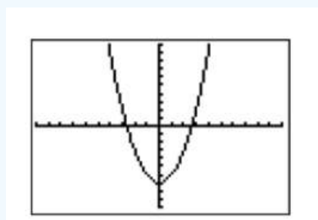
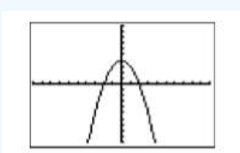


Figure $3.2.10$: The graph of $y=x^2-7$ is a parabola.

Exercise $3.2.2$

Use the graphing calculator to sketch the graph of $y=-x^2+4$.

Answer



Reproducing Calculator Results on Homework Paper

In this section we delineate recommendations and requirements when reproducing graphing calculator results on your homework paper.

Consider again the final result of Example $3.2.2$ shown in Figure $3.2.10$. To determine the scale at each end of each axis, press the WINDOW button on the topmost row of buttons on your calculator. The WINDOW settings for Figure $3.2.10$ are shown in Figure $3.2.11$. Figure $3.2.12$ presents a visual explanation of each of the entries Xmin, Xmax, Ymin and Ymax.

Figure 3.2.11: The **WINDOW** parameters.

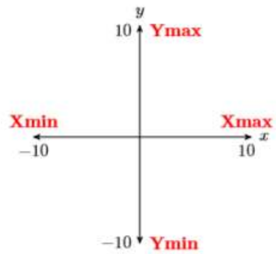


Figure 3.2.12: $\text{Xmin}, \text{Xmax}, \text{Ymin}$ and Ymax contain the scale at the end of each axis.

Xmin and Xmax indicate the scale at the left- and right-hand ends of the x-axis, respectively, while Ymin and Ymax indicate the scale at the bottomand top-ends of the y-axis. Finally, as we shall see shortly, Xscl and Yscl control the spacing between tick marks on the x- and y-axes, respectively.

When reproducing the graph in your calculator viewing window on your homework paper, follow the *Calculator Submission Guidelines*.

Calculator Submission Guidelines

1. All lines (including the horizontal and vertical axes) should be drawn with a ruler. All curves should be drawn freehand.
2. Set up a coordinate system on your homework paper that mimics closely the coordinate system in your calculator's view screen. Label your axes (usually with x and y).
3. Indicate the scale at each end of each axis. Use $\text{Xmin}, \text{Xmax}, \text{Ymin}$ and Ymax in the **WINDOW** menu for this purpose.
4. Copy the graph from your calculator's viewing window onto your coordinate system. Label the graph with its equation.

For example, to report the results of Figure $3.2.10$, draw the axes with a ruler, label the horizontal axis with x, the vertical axis with y, then place the values of $\text{Xmin}, \text{Xmax}, \text{Ymin}$ and Ymax at the appropriate end of each axis (see Figure $3.2.13$).

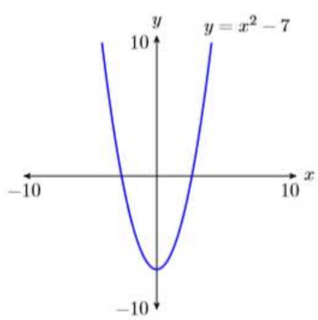


Figure 3.2.13: Reporting the graph of $y = x^2 - 7$ on your homework paper.

Because the graph is a curve, make a freehand copy that closely mimics the graph shown in Figure $3.2.10$, then label it with its equation, as shown in Figure $3.2.13$.

Adjusting the Viewing Window

Note that Figure $3.2.12$ gives us some sense of the meaning of the "standard viewing window" produced by selecting **6:ZStandard** from the ZOOM menu. Each time you select **6:ZStandard** from the ZOOM menu, Xmin is set to -10, Xmax to 10, and Xscl to

1 (distance between the tick marks on the x-axis). Similarly, Ymin is set to -10, Ymax to 10, and Yscl to 1 (distance between the tick marks on the y-axis). You can, however, override these settings as we shall see in the next example.

Example 3.2.3

Sketch the graphs of the following equations on the same viewing screen.

$$y=\frac{5}{4}x-3 \qquad \text{and} \qquad y=\frac{2}{3}x+4$$

Adjust the viewing window so that the point at which the graphs intersect (where the graphs cross one another) is visible in the viewing window, then use the TRACE button to approximate the coordinates of the point of intersection.

Solution

We must first decide on the proper syntax to use when entering the equations. In the case of the first equation, note that

$$y=\frac{5}{4}x-3$$

is pronounced "y equals five-fourths x minus three," and means "y equals five-fourths times x minus three." Enter this equation into **Y1** in the **Y**= menu as $5/4*\mathbf{X}-3$ (see Figure $3.2.14$), using the button keystrokes:

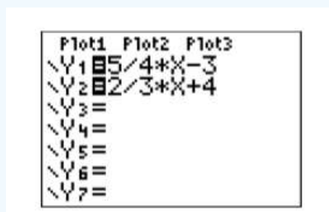


Figure $3.2.14$: Enter equations.

The division and times buttons are located in the rightmost column of the calculuator. Similarly, enter the second equation into **Y2** as $2/3*X+4$ using the button keystrokes:

Figure $3.2.15$.

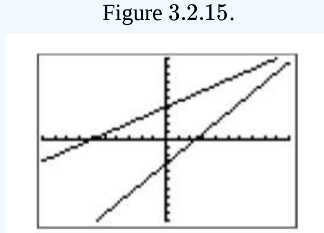


Figure $3.2.15$: The graphs.

When we examine the resulting graphs in Figure $3.2.15$, it appears that their point of intersection will occur off the screen above and to the right of the upper right-corner of the current viewing window. With this thought in mind, let's extend the x-axis to the right by increasing the value of Xmax to 20. Further, let's extend the y-axis upward by increasing the value of Ymin to 20. Press the WINDOW button on the top row of the calculator, then make the adjustments shown in Figure $3.2.16$.

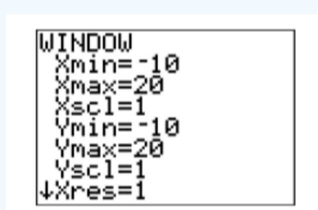


Figure $3.2.16$: Adjusting the viewing window.

If we select **6:ZStandard** from the ZOOM menu, our changes to the WINDOW parameters will be discarded and the viewing window will be returned to the "standard viewing window." If we want to keep our changes to the WINDOW parameters, the correct approach at this point is to push the GRAPH button on the top row of the calculator. The resulting graph is shown in Figure $3.2.17$. Note that the point of intersection of the two lines is now visible in the viewing window.

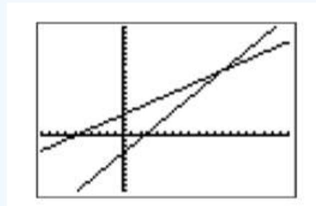


Figure $3.2.17$: The point of intersection is visible in the new viewing window.

The image in Figure $3.2.17$ is ready for recording onto your homework. However, we think we would have a better picture if we made a couple more changes:

1. It would be nicer if the point of intersection were more centered in the viewing window.
2. There are far too many tick marks.

With these thoughts in mind, make the changes to the WINDOW parameters shown in Figure $3.2.18$, then push the GRAPH button to produce the image in Figure $3.2.19$.

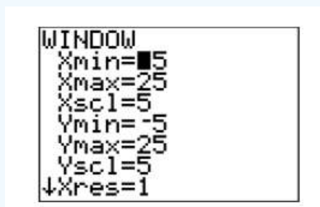


Figure $3.2.18$: A final adjustment of the viewing window.

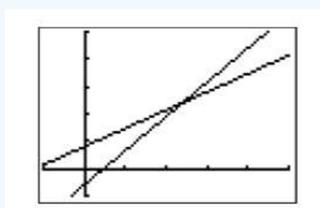


Figure $3.2.19$: A centered point of intersection and fewer tick marks.

Finally, push the TRACE button on the top row of buttons, then use the left- and right-arrow keys to move the cursor atop the point of intersection. An approximation of the coordinates of the point of intersection are reported at the bottom of the viewing window (see Figure $3.2.20$).

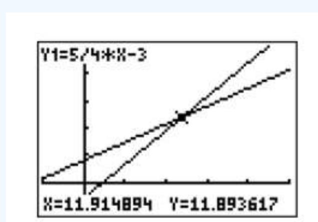


Figure $3.2.20$: Approximate coordinates of the point of intersection reported by the TRACE button.

Note

The TRACE button is only capable of providing a very rough approximation of the point of intersection. In Chapter 4, we'll introduce the intersection utility on the CALC menu, which will report a much more accurate result.

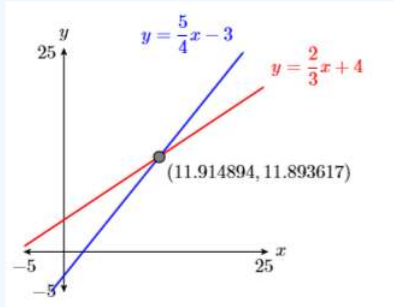


Figure 3.2.21: Reporting the answer on your homework.

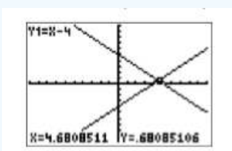
In reporting the answer on our homework paper, note that we followed the *Calculator Submission Guidelines*.

Exercise 3.2.3

Approximate the point of intersection of the graphs of $y = x - 4$ and $y = 5 - x$ using the TRACE button.

Answer

(4.68,0.68)



3.3: Rates and Slope

Let's open this section with an application of the concept of *rate*.

Independent versus dependent

It is traditional to place the independent variable on the horizontal axis and the dependent variable on the vertical axis.

Example 3.3.1

An object is dropped from rest, then begins to pick up speed at a constant rate of 10 meters per second every second ($10(\mathrm{m/s})/\mathrm{s}$ or $10\mathrm{m/s}^2$). Sketch the graph of the speed of the object versus time.

Solution

In this example, the speed of the object *depends* on the time. This makes the speed the *dependent variable* and time the *independent variable*.

Following this guideline, we place the time on the horizontal axis and the speed on the vertical axis. In Figure $3.3.1$, note that we've labeled each axis with the dependent and independent variables (v and t), and we've included the units ($\mathrm{m/s}$ and s) in our labels. Next, we need to scale each axis. In determining a scale for each axis, keep two thoughts in mind:

1. Pick a scale that makes it convenient to plot the given data.
2. Pick a scale that allows all of the given data to fit on the graph.

In this example, we want a scale that makes it convenient to show that the speed is increasing at a rate of 10 meters per second ($10\mathrm{m/s}$) every second ($\mathrm{m/s}$). One possible approach is to make each tick mark on the horizontal axis equal to $1\mathrm{s}$ and each tick mark on the vertical axis equal to $10\mathrm{m/s}$.

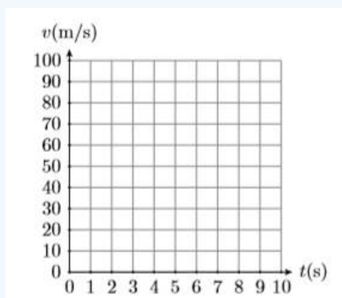


Figure $3.3.1$: Label and scale each axis. Include units with labels.

Next, at time $t = 0\mathrm{s}$, the speed is $v = 0\mathrm{m/s}$. This is the point $(t, v) = (0, 0)$ plotted in Figure $3.3.2$. Secondly, the rate at which the speed is increasing is $(10\mathrm{m/s})$ per second. This means that every time you move 1 second to the right, the speed increases by $(10\mathrm{m/s})$.

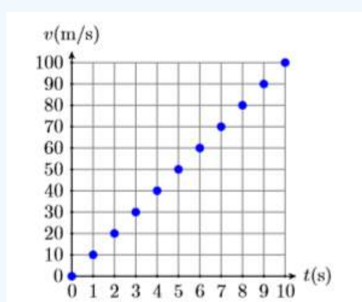


Figure $3.3.2$: Start at $(0, 0)$, then continuously move 1 right and 10 up.

In Figure $3.3.2$, start at $(0, 0)$, then move $1\mathrm{s}$ to the right and $(10\mathrm{m/s})$ up. This places you at the point $(1, 10)$, which says that after 1 second, the speed of the particle is $(10\mathrm{m/s})$. Continue in this manner, continuously moving $1\mathrm{s}$ to the right and $(10\mathrm{m/s})$

upward. This produces the sequence of points shown in Figure 3.3.2. Note that this constant rate of $10(\mathrm{m/s})/\mathrm{s}$ forces the graph of the speed versus time to be a line, as depicted in Figure 3.3.3.

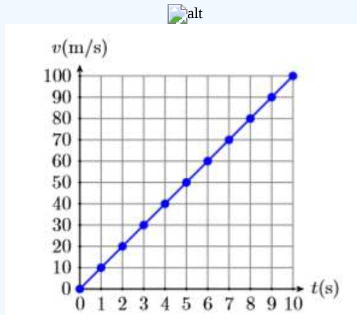
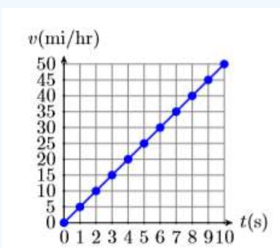


Figure 3.3.3: The constant rate forces the graph to be a line.

Exercise 3.3.1

Starting from rest, an automobile picks up speed at a constant rate of 5 miles per hour every second (5\((\mathrm{mi} / \mathrm{hr}) / \mathrm{s}\)). Sketch the graph of the speed of the object versus time.

Answer



Measuring the Change in a Variable

To calculate the change in some quantity, we take a difference. For example, suppose that the temperature in the morning is $40^\circ\mathrm{F}$, then in the afternoon the temperature measures $60^\circ\mathrm{F}$ (F stands for Fahrenheit temperature). Then the change in temperature is found by taking a difference.

$$\begin{aligned}\text{Change in temperature } &= \text{ Afternoon temperature - Morning temperature}\\ &= 60^\circ\mathrm{F} - 40^\circ\mathrm{F}\\ &= 20^\circ\mathrm{F}\end{aligned}$$

Therefore, there was a twenty degree increase in temperature from morning to afternoon.

Now, suppose that the evening temperature measures $50^\circ\mathrm{F}$. To calculate the change in temperature from the afternoon to the evening, we again subtract.

$$\begin{aligned}\text{Change in temperature } &= \text{ Evening temperature - Afternoon temperature}\\ &= 50^\circ\mathrm{F} - 60^\circ\mathrm{F}\\ &= -10^\circ\mathrm{F}\end{aligned}$$

There was a ten degree decrease in temperature from afternoon to evening.

Calculating the Change in a Quantity

To calculate the change in a quantity, subtract the earlier measurement from the later measurement.

Let T represent the temperature. Mathematicians like to use the symbolism ΔT to represent the change in temperature. For the change in temperature from morning to afternoon, we would write $\Delta T = 20^{\circ}\mathrm{F}$. For the afternoon to evening change, we would write $\Delta T = -10^{\circ}\mathrm{F}$.

Mathematicians and scientists make frequent use of the Greek alphabet, the first few letters of which are:

$$\begin{array}{ll} \alpha, \beta, \gamma, \delta, \ldots & \text{(Greek alphabet, lower case)} \\ A, B, \Gamma, \Delta, \ldots & \text{(Greek alphabet, upper case)} \\ a, b, c, d, \ldots & \text{(English alphabet)} \end{array}$$

Thus, the Greek letter ΔT, the upper case form of δ, correlates with the letter ‘d’ in the English alphabet. Why did mathematicians make this choice of letter to represent the change in a quantity? Because to find the change in a quantity, we take a difference, and the word “difference” starts with the letter ‘d.’ Thus, ΔT is also pronounced “the difference in T.”

Important Pronunciations

Two ways to pronounce the symbolism ΔT.

1. ΔT is pronounced “the change in T.”
2. ΔT is also pronounced “the difference in T.”

Slope as Rate

Here is the definition of the slope of a line.

Slope

The slope of a line is the rate at which the dependent variable is changing with respect to the independent variable. For example, if the dependent variable is y and the independent variable is x, then the slope of the line is:

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

Example $3.3.2$

In Example $3.3.1$, an object released from rest saw that its speed increased at a constant rate of 10 meters per second per second $(10(\mathrm{m/s})/\mathrm{s}$ or $10\mathrm{m/s}^2)$. This constant rate forced the graph of the speed versus time to be a line, shown in Figure $3.3.3$. Calculate the slope of this line.

Solution

Start by selecting two points $P(2,20)$ and $Q(8,80)$ on the line, as shown in Figure $3.3.4$. To find the slope of this line, the definition requires that we find the rate at which the dependent variable v changes with respect to the independent variable t. That is, the slope is the change in v divided by the change in t. In symbols:

$$\text{Slope} = \frac{\Delta v}{\Delta t}$$

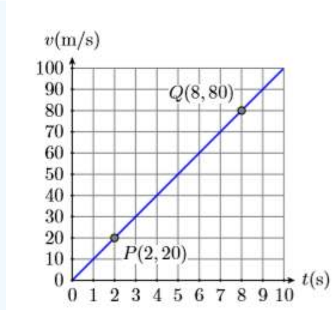


Figure 3.3.4: Pick two points to compute the slope.

Now, as we move from point $P(2,20)$ to point $Q(8,80)$, the speed changes from $20\mathrm{m/s}$ to $80\mathrm{m/s}$. Thus, the change in the speed is:

$$\begin{aligned}\Delta v &= 80\mathrm{m/s} - 20\mathrm{m/s} \\ &= 60\mathrm{m/s}\end{aligned}$$

Similarly, as we move from point P(2,20) to point Q(8,80), the time changes from 2 seconds to 8 seconds. Thus, the change in time is:

$$\begin{aligned}\Delta t &= 8\mathrm{s} - 2\mathrm{s} \\ &= 6\mathrm{s}\end{aligned}$$

Now that we have both the change in the dependent and independent variables, we can calculate the slope.

$$\begin{aligned}\text{Slope } &= \frac{\Delta v}{\Delta t} \\ &= \frac{60\mathrm{m/s}}{6\mathrm{s}} \\ &= 10\frac{\mathrm{m/s}}{\mathrm{s}}\end{aligned}$$

Therefore, the slope of the line is 10 meters per second per second ($10(\mathrm{m/s})/\mathrm{s}$ or $10\mathrm{m/s}^2$).

The slope of a line does not depend upon the points you select. Let's try the slope calculation again, using two different points and a more compact presentation of the required calculations. Pick points $P(3,30)$ and $Q(7,70)$ as shown in Figure $3.3.5$. Using these two new points, the slope is the rate at which the dependent variable v changes with respect to the independent variable t.

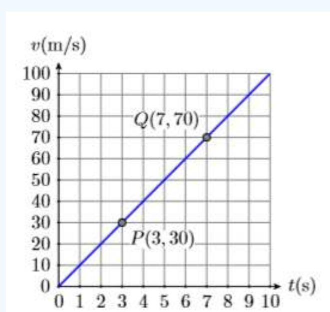


Figure $3.3.5$: The slope does not depend on the points we select on the line.

$$\begin{aligned}\text{Slope } &= \frac{\Delta v}{\Delta t} \\ &= \frac{70\mathrm{m/s} - 30\mathrm{m/s}}{7\mathrm{s} - 3\mathrm{s}} \\ &= \frac{40\mathrm{m/s}}{4\mathrm{s}} \\ &= 10\frac{\mathrm{m/s}}{\mathrm{s}}\end{aligned}$$

Again, the slope of the line is $10(\mathrm{m/s})/\mathrm{s}$.

Exercise 3.3.2

Starting from rest, an automobile picks up speed at a constant rate of 5 miles per hour every second (5(mi/hr)/s). The constant rate forces the graph of the speed of the object versus time to be a line. Calculate the slope of this line.

Answer

$5(\mathrm{mi/hr})/\mathrm{s}$

Example \(\PageIndex{2}\) points out the following fact.

Slope is independent of the selected points

It does not matter which two points you pick on the line to calculate its slope.

The next example demonstrates that the slope is also independent of the order of subtraction.

Example 3.3.3

Compute the slope of the line passing through the points $P(-1,-2)$ and $Q(3,3)$.

Solution

First, sketch the line passing through the points P(−1,−2) and Q(3,3) (see Figure \(\PageIndex{6}\)).

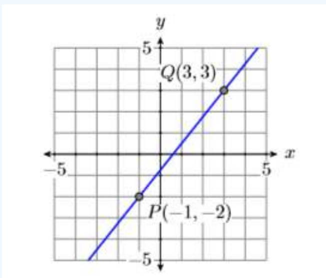


Figure 3.3.6: Computing the slope of the line passing through the points $P(-1,-2)$ and $Q(3,3)$.

To calculate the slope of the line through the points $P(-1,-2)$ and $Q(3,3)$, we must calculate the change in both the independent and dependent variables. We'll do this in two different ways.

Warning!

If you are not consistent in the direction you subtract, you will not get the correct answer for the slope.

For example:

$$\frac{3-(-2)}{-1-3}=-\frac{5}{4}$$

In this case, we subtracted the y-coordinate of point $P(-1,-2)$ from the y-coordinate of point $Q(3,3)$, but then we changed horses in midstream, subtracting the x-coordinate of point $Q(3,3)$ from the x-coordinate of point $P(-1,-2)$. Note that we get the negative of the correct answer.

Method 1

Subtract the coordinates of point $P(-1,-2)$ from the coordinates of point $Q(3,3)$.

$$\begin{aligned} \text{Slope } &= \frac{\Delta y}{\Delta x} \\ &= \frac{3-(-2)}{3-(-1)} \\ &= \frac{5}{4} \end{aligned}$$

Method 2

Subtract the coordinates of point $Q(3,3)$ from the coordinates of point $P(-1,-2)$.

$$\begin{aligned} \text{Slope } &= \frac{\Delta y}{\Delta x} \\ &= \frac{-2-3}{-1-3} \\ &= \frac{-5}{-4} \\ &= \frac{5}{4} \end{aligned}$$

Note that regardless of the direction of subtraction, the slope is $5/4$.

Exercise 3.3.3

Compute the slope of the line passing through the points P(−3,1) and Q(2,4).

Answer

$3/5$

Example $3.3.3$ demonstrates the following fact.

The direction of subtraction does not matter

When calculating the slope of a line through two points P and Q, it does not matter which way you subtract, provided you remain consistent in your choice of direction.

The Steepness of a Line

We need to examine whether our definition of slope matches certain expectations.

Slope and steepness of a line

The slope of a line is a number that tells us how steep the line is.

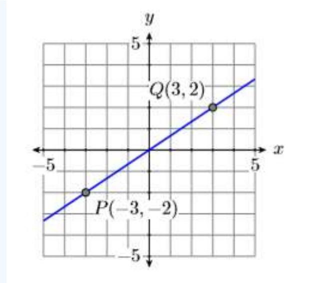
If slope is a number that measures the steepness of a line, then one would expect that a steeper line would have a larger slope.

Example 3.3.4

Graph two lines, the first passing through the points $P(-3,-2)$ and $Q(3,2)$ and the second through the points $R(-1,-3)$ and $S(1,3)$. Calculate the slope of each line and compare the results.

Solution

The graphs of the two lines through the given points are shown, the first in Figure $3.3.7$ and the second in Figure $3.3.8$. Note that the line in Figure $3.3.7$ is less steep than the line in Figure $3.3.8$.



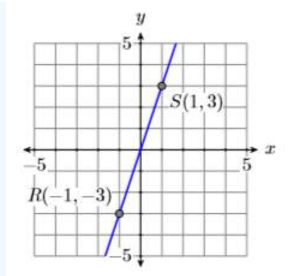


Figure 3.3.7: This line is less steep than the line on the right. **Figure 3.3.8:** This line is steeper than the line on the left.

Remember, the slope of the line is the rate at which the dependent variable is changing with respect to the independent variable. In both Figure 3.3.7 and Figure 3.3.8, the dependent variable is y and the independent variable is x.

Subtract the coordinates of point $P(-3,-2)$ from the coordinates of point $Q(3,2)$.

$$\begin{aligned}\text{Slope of first line } &= \frac{\Delta y}{\Delta x}\\ &= \frac{2-(-2)}{3-(-3)}\\ &= \frac{4}{6}\\ &= \frac{2}{3}\end{aligned}$$

Subtract the coordinates of the point $R(-1,-3)$ from the point $S(1,3)$.

$$\begin{aligned}\text{Slope of second line } &= \frac{\Delta y}{\Delta x}\\ &= \frac{3-(-3)}{1-(-1)}\\ &= \frac{6}{2}\\ &= 3\end{aligned}$$

Note that both lines go uphill and both have positive slopes. Also, note that the slope of the second line is greater than the slope of the first line. This is consistent with the fact that the second line is steeper than the first.

Exercise 3.3.4

Compute the slope of the line passing through the points $P(-2,-3)$ and $Q(2,5)$. Then compute the slope of the line passing through the points $R(-2,-1)$ and $S(5,3)$, and compare the two slopes. Which line is steeper?

Answer

The first line has slope 2, and the second line has slope $4/7$. The first line is steeper.

In Example $3.3.4$, both lines slanted uphill and both had positive slopes, the steeper of the two lines having the larger slope. Let's now look at two lines that slant downhill.

Example $3.3.5$

Graph two lines, the first passing through the points $P(-3, 1)$ and $Q(3, -1)$ and the second through the points $R(-2, 4)$ and $S(2, -4)$. Calculate the slope of each line and compare the results.

Solution

The graphs of the two lines through the given points are shown, the first in Figure $3.3.9$ and the second in Figure $3.3.10$. Note that the line in Figure $3.3.9$ goes downhill less quickly than the line in Figure $3.3.10$. Remember, the slope of the line is the rate at which the dependent variable is changing with respect to the independent variable. In both Figure $3.3.9$ and Figure $3.3.10$, the dependent variable is y and the independent variable is x.

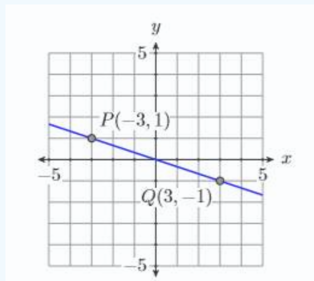


Figure $3.3.9$: This line goes downhill more slowly than the line on the right.

Figure $3.3.10$: This line goes downhill more quickly than the line on the left.

Subtract the coordinates of point $P(-3, 1)$ from the coordinates of point $Q(3, -1)$.

$$\begin{aligned} \text{Slope of first line } &= \frac{\Delta y}{\Delta x} \\ &= \frac{-1-1}{3-(-3)} \\ &= \frac{-2}{6} \\ &= -\frac{1}{3} \end{aligned}$$

Subtract the coordinates of point $R(-2, 4)$ from the coordinates of point $S(2, -4)$.

$$\begin{aligned} \text{Slope of second line } &= \frac{\Delta y}{\Delta x} \\ &= \frac{-4-4}{2-(-2)} \\ &= \frac{-8}{4} \\ &= -2 \end{aligned}$$

Note that both lines go downhill and both have negative slopes. Also, note that the magnitude (absolute value) of the slope of the second line is greater than the magnitude of the slope of the first line. This is consistent with the fact that the second line moves downhill more quickly than the first.

Exercise $3.3.5$

Compute the slope of the line passing through the points $P(-3, 3)$ and $Q(3, -5)$. Then compute the slope of the line passing through the points $R(-4, 1)$ and $S(4, -3)$, and compare the two slopes. Which line is steeper?

Answer

The first line has slope $-4/3$, and the second line has slope $-1/2$. The first line is steeper.

What about the slopes of vertical and horizontal lines?

Example 3.3.6

Calculate the slopes of the vertical and horizontal lines passing through the point $(2,3)$.

Solution

First draw a sketch of the vertical and horizontal lines passing through the point (2,3). Next, select a second point on each line as shown in Figures $3.3.11$ and $3.3.12$.

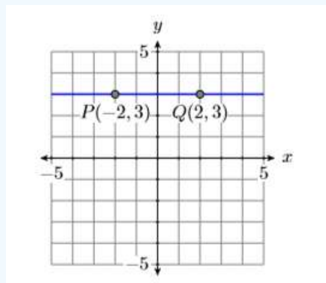


Figure $3.3.11$: A horizontal line through $(2,3)$.

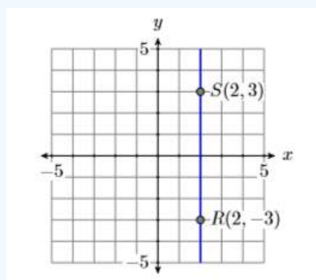


Figure $3.3.12$: A vertical line through $(2,3)$.

The slopes of the horizontal and vertical lines are calculated as follows.

Subtract the coordinates of the point $P(-2,3)$ from the coordinates of the point $Q(2,3)$.

$$\begin{aligned}\text{Slope of horizontal line } &= \frac{\Delta y}{\Delta x} \\ &= \frac{3-3}{2-(-2)} \\ &= \frac{0}{4} \\ &= 0\end{aligned}$$

Thus, the slope of the horizontal line is zero, which makes sense because a horizontal line neither goes uphill nor downhill.

Subtract the coordinates of the point $(2,-3)$ from the coordinates of the point $S(2,3)$.

$$\begin{aligned}\text{Slope of vertical line } &= \frac{\Delta y}{\Delta x} \\ &= \frac{3-(-3)}{2-2} \\ &= \frac{6}{0} \\ &= \text{ undefined}\end{aligned}$$

Division by zero is undefined. Hence, the slope of a vertical line is undefined. Again, this makes sense because as uphill lines get steeper and steeper, their slopes increase without bound.

Exercise 3.3.6

Calculate the slopes of the vertical and horizontal lines passing through the point $(-4,1)$.

Answer

The slope of the vertical line is undefined. The slope of the second line is 0.

The Geometry of the Slope of a Line

We begin our geometrical discussion of the slope of a line with an example, calculating the slope of a line passing through the points $P(2,3)$ and $Q(8,8)$. Before we begin we'll first calculate the change in y and the change in x by subtracting the coordinates of point $P(2,3)$ from the coordinates of point $Q(8,8)$.

$$\begin{aligned} \text{Slope} &= \frac{\Delta y}{\frac{\Delta x}{\Delta x}} \\ &= \frac{8-3}{8-2} \\ &= \frac{5}{6} \end{aligned}$$

Thus, the slope of the line through the points $P(2,3)$ and $Q(8,8)$ is $5/6$.

To use a geometric approach to finding the slope of the line, first draw the line through the points $P(2,3)$ and $Q(8,8)$ (see Figure $3.3.13$). Next, draw a right triangle with sides parallel to the horizontal and vertical axes, using the points $P(2,3)$ and $Q(8,8)$ as vertices. As you move from point P to point R in Figure $3.3.13$, note that the change in x is $\Delta x=6$ (count the tick marks[1]).

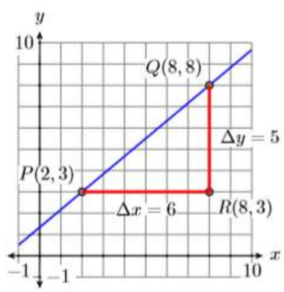


Figure $3.3.13$: Determining the slope of the line from the graph.

As you then move from point R to point Q, the change in y is $\Delta y=5$ (count the tick marks). Thus, the slope is $\Delta y/\Delta x=5/6$, precisely what we got in the previous computation.

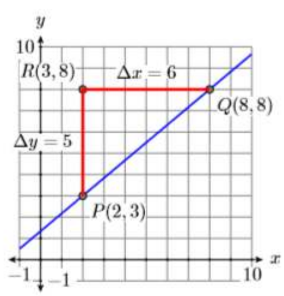


Figure $3.3.14$: Determining the slope of the line from the graph.

For contrast, in Figure $3.3.14$, we started at the point $P(2,3)$, then moved upward 5 units and right 6 units. However, the change in y is still $\Delta y = 5$ and the change in x is still $\Delta x = 6$ as we move from point $P(2,3)$ to point $Q(8,8)$. Hence, the slope is still $\Delta y/\Delta x = 5/6$.

Rise over run

In Figure $3.3.14$, we start at the point $P(2,3)$, then "rise" 5 units, then "run" 6 units to the right. For this reason, some like to think of the slope as "rise over run."

Consider a second example shown in Figure $3.3.15$. Note that the line slants downhill, so we expect the slope to be a negative number.

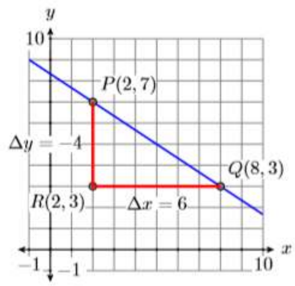


Figure 3.3.15: Determining the slope of the line from the graph.

In Figure $3.3.15$, we've drawn a right triangle with sides parallel to the horizontal and vertical axes, using the points $P(2,7)$ and $Q(8,3)$ as vertices. As you move from point P to point R in Figure $3.3.15$, the change in y is $\Delta y = -4$ (count the tick marks and note that your values of y are decreasing as you move from P to R). As you move from point R to point Q, the change is x is $\Delta x = 6$ (count the tick marks and note that your values of x are increasing as you move from R to Q). In this case, the "rise" is negative, while the "run" is positive.

Thus, the slope is $\Delta y/\Delta x = -4/6$, or $-2/3$. Note that the slope is negative, as anticipated.

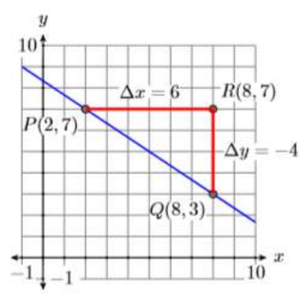


Figure 3.3.16: Determining the slope of the line from the graph.

In Figure $3.3.16$, we've drawn our triangle on the opposite side of the line. In this case, as you move from point P to point R in Figure $3.3.16$, the change in x is $\Delta x = 6$ (count the tick marks and note that your values of x are increasing as you move from P to R). As you move from point R to point Q, the change is y is $\Delta y = -4$ (count the tick marks and note that your values of y are decreasing as you move from R to Q). Thus, the slope is still $\Delta y/\Delta x = -4/6$, or $-2/3$.

We can verify our geometrical calculations of the slope by subtracting the coordinates of the point $P(2,7)$ from the point $Q(8,3)$.

$$\begin{aligned} \text{Slope } &= \frac{\Delta y}{\Delta x} \\ &= \frac{3-7}{8-2} \\ &= \frac{-4}{6} \\ &= -\frac{2}{3} \end{aligned}$$

This agrees with the calculations made in Figures $3.3.15$ and $3.3.16$.

Let’s look at a final example.

Example $3.3.7$

Sketch the line passing through the point $(-2,3)$ with slope $-2/3$.

Solution

The slope is $-2/3$, so the line must go downhill. In Figure $3.3.17$, we start at the point $P(-2,3)$, move right 3 units to the point $R(1,3)$, then move down 2 units to the point $Q(1,1)$. Draw the line through the points P and Q and you are done.

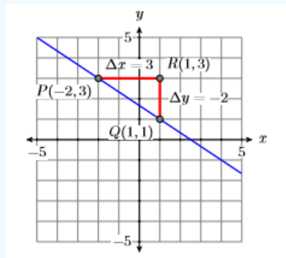


Figure $3.3.17$: Start at $P(-2,3)$, then move right 3 and down 2. The resulting line has slope $-2/3$.

In Figure $3.3.18$, we take a different approach that results in the same line. Start at the point $P(-2,3)$, move downward 4 units to the point $R(-2,-1)$, then right 6 units to the point $Q(4,-1)$. Draw a line through the points P and Q and you are done.

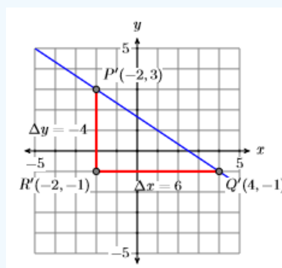


Figure $3.3.18$: Starting at $P(-2,3)$ and moving down 4 and right 6 also yields a slope of $-2/3$.

The triangle PQR in Figure $3.3.17$ is similar to the triangle PQR in Figure $3.3.18$, so their sides are proportional. Consequently, the slope of the line through points $P(-2,3)$ and $Q(4,-1)$,

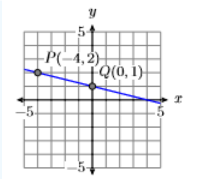
$$\begin{aligned} \text{Slope } &= \frac{\Delta y}{\Delta x} \\ &= \frac{-4}{6} \\ &= -\frac{2}{3} \end{aligned}$$

reduces to the slope of the line through the points P and Q in Figure $3.3.17$.

Exercise $3.3.7$

Sketch the line passing through the point $(-4,2)$ with slope $-1/4$.

Answer



A summary of facts about the slope of a line

We present a summary of facts learned in this section.

1. The slope of a line is the rate at which the dependent variable is changing with respect to the independent variable. If y is the dependent variable and x is the independent variable, then the slope is

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

where Δy is the change in y (difference in y) and Δx is the change in x (difference in x).
2. If a line has positive slope, then the line slants uphill as you "sweep your eyes from left to right." If two lines have positive slope, then the line with the larger slope rises more quickly.
3. If a line has negative slope, then the line slants downhill as you "sweep your eyes from left to right." If two lines have negative slope, then the line having the slope with the larger magnitude falls more quickly.
4. Horizontal lines have slope zero.
5. Vertical lines have undefined slope.

References

[1] When counting tick marks, make sure you know the amount each tick mark represents. For example, if each tick mark represents two units, and you count six tick marks when evaluating the change in x, then $\Delta x = 12$.

3.4: Slope-Intercept Form of a Line

We start with the definition of the y-intercept of a line.

The y-intercept

The point $(0, b)$ where the graph of a line crosses the y-axis is called the y-intercept of the line.

We will now generate the slope-intercept formula for a line having y-intercept $(0, b)$ and $\text{Slope} = m$ (see Figure $3.4.1$). Let (x, y) be an arbitrary point on the line.

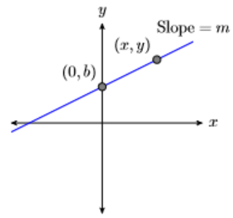


Figure $3.4.1$: Line with y-intercept at $(0, b)$ and $\text{Slope} = m$.

Start with the fact that the slope of the line is the rate at which the dependent variable is changing with respect to the independent variable.

$$\text{Slope} = \frac{\Delta y}{\Delta x} \qquad \text{Slope formula.}$$

Substitute m for the slope. To determine both the change in y and the change in x, subtract the coordinates of the point $(0, b)$ from the point (x, y).

$$\begin{aligned} m &= \frac{y-b}{x-0} && \text{Substitute } m \text{ for the Slope. } \Delta y = y - b \quad \text{and } \Delta x = x - 0. \\ m &= \frac{y-b}{x} && \text{Simplify.} \end{aligned}$$

Clear fractions from the equation by multiplying both sides by the common denominator.

$$\begin{aligned} mx &= \left[\frac{y-b}{x}\right] x && \text{Multiply both sides by } x \\ mx &= y - b && \text{Cancel.} \end{aligned}$$

Solve for y.

$$\begin{aligned} mx + b &= y - b + b && \text{Add } b \text{ to both sides.} \\ mx + b &= y && \text{Simplify.} \end{aligned}$$

Thus, the equation of the line is $y = mx + b$.

The Slope-Intercept form of a line

The equation of the line having y intercept $(0, b)$ and slope m is:

$$y = mx + b$$

Because this form of a line depends on knowing the slope m and the intercept $(0, b)$, this form is called the *slope-intercept* form of a line.

Example 3.4.1

Sketch the graph of the line having equation $y = \frac{3}{5}x + 1$.

Solution

Compare the equation $y = \frac{3}{5}x + 1$ with the slope-intercept form $y = mx + b$, and note that $m = 3/5$ and $b = 1$. This means that the slope is $3/5$ and the y-intercept is $(0, 1)$. Start by plotting the y-intercept $(0, 1)$, then move uward 3 units and right 5 units, arriving at the point $(5, 4)$. Draw the line through the points $(0, 1)$ and $(5, 4)$, then label it with its equation $y = \frac{3}{5}x + 1$.

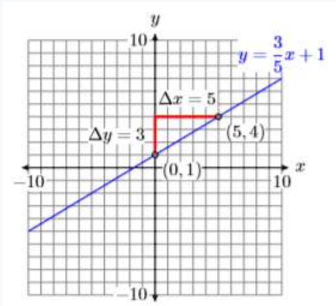


Figure 3.4.2: Hand-drawn graph of $y = \frac{3}{5}x + 1$.

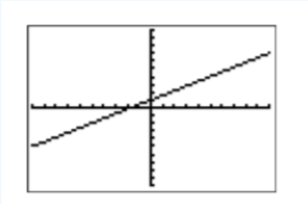


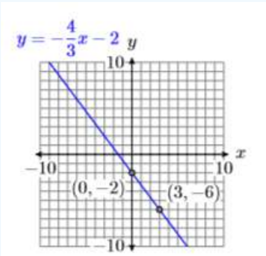
Figure 3.4.3: Select **5:ZSquare** from the ZOOM menu to draw the graph of $y = \frac{3}{5}x + 1$.

When we compare the calculator image in Figure $3.4.3$ with the hand-drawn graph in Figure $3.4.2$, we get a better match.

Exercise 3.4.1

Sketch the graph of the line having equation $y = -\frac{4}{3}x - 2$.

Answer



Example 3.4.2

Sketch the line with y-intercept $(0, 2)$ and slope $-7/3$. Label the line with the slope-intercept form of its equation.

Solution

Plot the y-intercept $(0, 2)$. Now use the slope $-7/3$. Start at $(0, 2)$, then move down seven units, followed by a three unit move to the right to the point $(3, -5)$. Draw the line through the points $(0, 2)$ and $(3, -5)$. (See Figure $3.4.4$).

Next, the y-intercept is $(0,2)$, so $b=2$. Further, the slope is $-7/3$, so $m=-7/3$. Substitute these numbers into the slope-intercept form of the line.

$$\begin{aligned} y &= mx+b && \text{Slope-intercept form.} \\ y &= -\frac{7}{3}x+2 && \text{Substitute: } -7/3 \text{ for } m, 2 \text{ for } b. \end{aligned}$$

Therefore, the slope-intercept form of the line is $y=-\frac{7}{3}x+2$. Label the line with this equation.

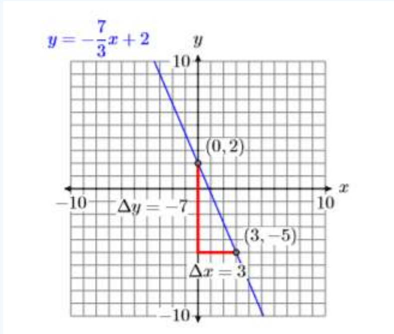


Figure 3.4.4: Hand-drawn graph of $y=-\frac{7}{3}x+2$

Check: To graph $y=-\frac{7}{3}x+2$, enter $-7/3*X+2$ in Y1 in the **Y=** menu. Select **6:ZStandard** from the ZOOM menu, followed by **5:ZSquare** from the ZOOM menu to produce the graph shown in Figure 3.4.6.

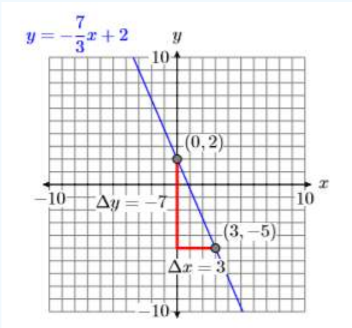


Figure 3.4.5: Hand-drawn graph of $y=-\frac{7}{3}x+2$

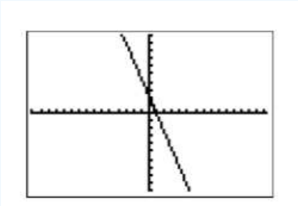


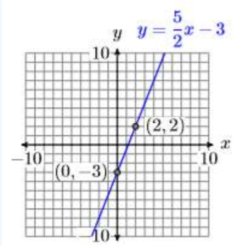
Figure 3.4.6: Select **6:ZStandard** from the ZOOM menu, followed by **5:ZSquare** from the ZOOM menu to produce the graph of the equation $y=-\frac{7}{3}x+2$

This provides a good match of the hand-drawn graph in Figure 3.4.5 and our graphing calculator result in Figure 3.4.6.

Exercise 3.4.2

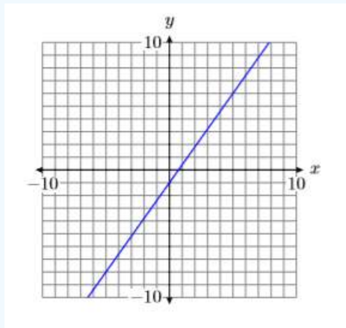
Sketch the line with y-intercept $(0,-3)$ and slope $5/2$. Label the line with the slope-intercept form of its equation.

Answer



Example 3.4.3

Use the graph of the line in the following figure to find the equation of the line.



Solution

Note that the y-intercept of the line is $(0, -1)$ (See Figure 3.4.7). Next, we try to locate a point on the line that passes directly through a lattice point, a point where a vertical and horizontal grid line intersect. In Figure 3.4.7, we chose the point $(5, 6)$. Now, starting at the y-intercept $(0, 1)$, we move up 7 units, then to the right 5 units. Hence, the slope is $m = \Delta y / \Delta x$, or $m = 7/5$.

Note

We could also subtract the coordinates of point $(0, -1)$ from the coordinates of point $(5, 6)$ to determine the slope.

$$\frac{6-(-1)}{5-0} = \frac{7}{5}$$

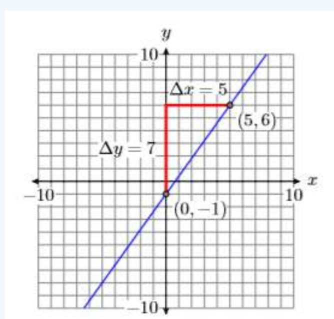


Figure 3.4.7: The line has y-intercept $(0, -1)$ and slope $7/5$.

Next, state the slope-intercept form, the substitute $7/5$ for m and -1 for b.

$$y = mx + b \quad \text{Slope-intercept form.}$$
$$y = \frac{7}{5}x + (-1) \quad \text{Substitute: } 7/5 \text{ for } m, -1 \text{ for } b$$

Thus, the equation of the line is $y=\frac{7}{5}x-1$.

Check: This is an excellent situation to run a check on your graphing calculator.

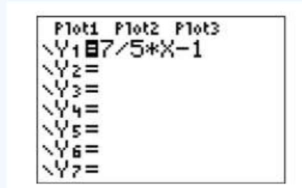


Figure $3.4.8$: Enter $y=\frac{7}{5}x-1$ in the **Y**= menu.

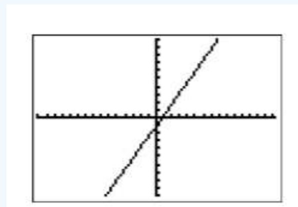
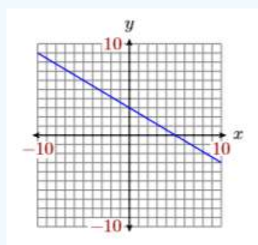


Figure $3.4.9$: Select **6:ZStandard** followed by **5:ZSquare** (both from the ZOOM menu) to produce this graph.

When we compare the result in Figure $3.4.9$ with the original hand-drawn graph (see Figure $3.4.7$), we're confident we have a good match.

Exercise $3.4.3$

Use the graph of the line in the figure below to find the equation of the line.



Answer

$$y=-\frac{3}{5}x+3$$

Applications

Let's look at a linear application.

Example $3.4.4$

Jason spots his brother Tim talking with friends at the library, located 300 feet away. He begins walking towards his brother at a constant rate of 2 feet per second (2 ft/s).

a. Express the distance d between Jason and his brother Tim in terms of the time t.
b. At what time after Jason begins walking towards Tim are the brothers 200 feet apart?

Solution

Because the distance between Jason and his brother is decreasing at a constant rate, the graph of the distance versus time is a line. Let's begin by making a rough sketch of the line. In Figure $3.4.10$, note that we've labeled what are normally the x- and y-axes with the time t and distance d, and we've included the units with our labels.

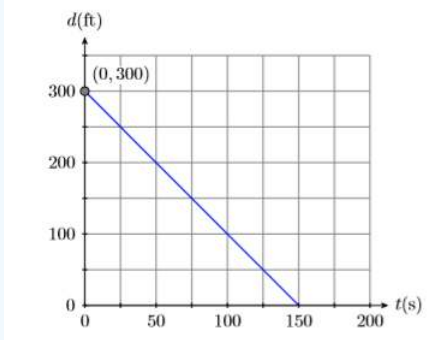


Figure $3.4.10$: A plot of the distance d separating the brothers versus time t.

Let $t = 0$ seconds be the time that Jason begins walking towards his brother Tim. At time $t = 0$, the initial distance between the brothers is 300 feet. This puts the d-intercept (normally the y-intercept) at the point $(0, 300)$ (see Figure $3.4.10$).

Because Jason is walking toward his brother, the distance between the brothers decreases at a constant rate of 2 feet per second. This means the line must slant downhill, making the slope negative, so $m = -2$ ft/s. We can construct an accurate plot of distance versus time by starting at the point $(0, 300)$, then descending $\Delta d = -300$, then moving to the right $\Delta t = 150$. This makes the slope $\Delta d/\Delta t = -300/150 = -2$ (See Figure $3.4.10$). Note that the slope is the rate at which the distance d between the brothers is changing with respect to time t.

Finally, the equation of the line is $y = mx + b$, where m is the slope of the line and b is the y-coordinate (in this case the d-coordinate) of the point where the graph crosses the vertical axis. Thus, substitute -2 for m, and 300 for b in the slope-intercept form of the line.

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form.} \\ y &= -2x + 300 && \text{Substitute: } -2 \text{ for } m, 300 \text{ for } b \end{aligned}$$

One problem remains. The equation $y = -2x + 300$ gives us y in terms of x.

a. The question required that we express the distance d in terms of the time t. So, to finish the solution, replace y with d and x with t (check the axes labels in Figure $3.4.10$) to obtain a solution:

$$d = -2t + 300$$

b. Now that our equation expresses the distance between the brothers in terms of time, let's answer part (b), "At what time after Jason begins walking towards Tim are the brothers 200 feet apart?" To find this time, substitute 200 for d in the equation $d = -2t + 300$, then solve for t.

$$\begin{aligned} d &= -2t + 300 && \text{Distance equation} \\ 200 &= -2t + 300 && \text{Substitute 200 for } d \end{aligned}$$

Solve this last equation for the time t.

$$\begin{aligned} 200 - 300 &= -2t + 300 - 300 && \text{Subtract 300 from both sides.} \\ -100 &= -2t && \text{Simplify both sides.} \\ \frac{-100}{-2} &= \frac{-2t}{-2} && \text{Divide both sides by } -2 \\ 50 &= t && \text{Simplify both sides.} \end{aligned}$$

Thus, it takes Jason 50 seconds to close the distance between the brothers to 200 feet.

Exercise $3.4.4$

A swimmer is 50 feet from the beach, and then begins swimming away from the beach at a constant rate of 1.5 feet per second (1.5 ft/s). Express the distance d between the swimmer and the beach in terms of the time t.

Answer

$d = 1.5t + 50$

3.5: Point-Slope Form of a Line

In the previous section we learned that if we are provided with the slope of a line and its y-intercept, then the equation of the line is $y = mx + b$, where m is the slope of the line and b is the y-coordinate of the line's y-intercept.

However, suppose that the y-intercept is unknown? Instead, suppose that we are given a point (x_0, y_0) on the line and we're also told that the slope of the line is m (see Figure $3.5.1$).

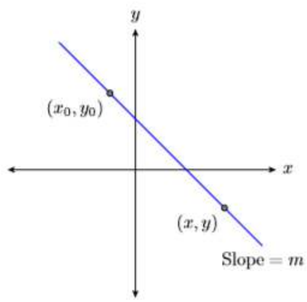


Figure $3.5.1$: A line through the point (x_0, y_0) with slope m.

Let (x, y) be an arbitrary point on the line, then use the points (x_0, y_0) and (x, y) to calculate the slope of the line.

$$\text{Slope } = \frac{\Delta y}{\Delta x} \quad \text{The slope formula.}$$

$$m = \frac{y - y_0}{x - x_0}$$

Substitute m for the slope. Use (x, y) and (x_0, y_0) to calculate the difference in y and the difference in x

Clear the fractions from the equation by multiplying both sides by the common denominator.

$$\begin{aligned} m(x - x_0) &= \left[\frac{y - y_0}{x - x_0}\right] x - x_0 && \text{Multiply both sides by } x - x_0 \\ m(x - x_0) &= y - y_0 && \text{Cancel.} \end{aligned}$$

Thus, the equation of the line is $y - y_0 = m(x - x - 0)$.

The Point-Slope form of a line

The equation of the line with slope m that passes through the point (x_0, y_0) is:

$$y - y_0 = m(x - x_0)$$

Example 3.5.1

Draw the line passing through the point $(-3, -1)$ that has slope $3/5$, then label it with its equation.

Solution

Plot the point $(-3, -1)$, then move 3 units up and 5 units to the right (see Figure $3.5.2$). To find the equation, substitute $(-3, -1)$ for (x_0, y_0) and $3/5$ for m in the point-slope form of the line.

$$\begin{aligned} y - y_0 &= m(x - x_0) && \text{Point-slope form.} \\ y - (-1) &= \frac{3}{5}(x - (-3)) && \text{Substitute: } 3/5 \text{ for } m, -3 \text{ for } x_0, \text{ and } -1 \text{ for } y_0 \end{aligned}$$

Simplifying, the equation of the line is $y + 1 = \frac{3}{5}(x + 3)$.

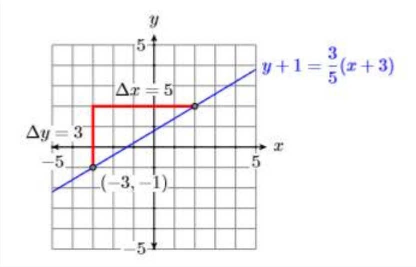
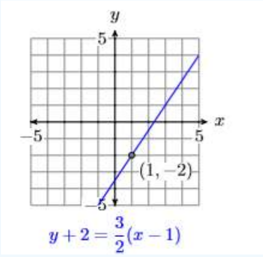


Figure 3.5.2: The line passing through $(-3,-1)$ with slope $3/5$.

Exercise 3.5.1

Draw the line passing through the point $(1,-2)$ that has slope $3/2$, then label it with its equation.

Answer



At this point, you may be asking the question "When should I use the slopintercept form and when should I use the point-slope form?" Here is a good tip.

Tip: Which form should I use

The form you should select depends upon the information given.

1. If you are given the y-intercept and the slope, use $y = mx + b$.
2. If you are given a point and the slope, use $y - y_0 = m(x - x_0)$.

Example 3.5.2

Find the equation of the line passing through the points $P(-1,2)$ and $Q(3,-4)$.

Solution

First, plot the points $P(-1,2)$ and $Q(3,-4)$ and draw a line through them (see Figure 3.5.3).

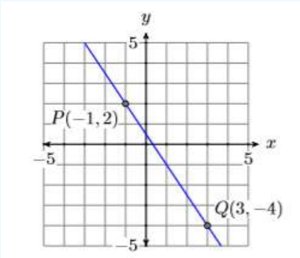


Figure 3.5.3: The line passing through $P(-1,2)$ and $Q(3,-4)$.

Next, let's calculate the slope of the line by subtracting the coordinates of the point $P(-1,2)$ from the coordinates of point $Q(3,-4)$.

$$\begin{aligned} \text{Slope } &= \frac{\Delta y}{\Delta x} \\ &= \frac{-4-2}{3-(-1)} \\ &= \frac{-6}{4} \\ &= -\frac{3}{2} \end{aligned}$$

Thus, the slope is $-3/2$.

Next, use the point-slope form $y - y_0 = m(x - x_0)$ to determine the equation of the line. It's clear that we should substitute $-3/2$ form. But which of the two points should we use? If we use the point $P(-1,2)$ for (x_0, y_0), we get the answer on the left, but if we use the point $Q(3,-4)$ for (x_0, y_0), we get the answer on the right.

$$y-2=-\frac{3}{2}(x-(-1)) \qquad \text{or} \qquad y-(-4)=-\frac{3}{2}(x-3)$$

At first glance, these answers do not look the same, but let's examine them a bit more closely, solving for y to put each in slope-intercept form. Let's start with the equation on the left.

$$\begin{aligned} y-2 &= -\frac{3}{2}(x-(-1)) && \text{Using } m=-3/2 \text{ and } (x_0,y_0)=(-1,2) \\ y-2 &= -\frac{3}{2}(x+1) && \text{Simplify.} \\ y-2 &= -\frac{3}{2}x-\frac{3}{2} && \text{Distribute } -3/2 \\ y-2+2 &= -\frac{3}{2}x-\frac{3}{2}+2 && \text{Add 2 to both sides.} \\ y &= -\frac{3}{2}x-\frac{3}{2}+\frac{4}{2} \\ & && \text{On the left, simplify. On the right, make equivalent fractions with a common denominator.} \\ y &= -\frac{3}{2}x+\frac{1}{2} && \text{Simplify.} \end{aligned}$$

Let's put the second equation in slope-intercept form.

> **Note**
>
> Note that the form $y=-\frac{3}{2}x+\frac{1}{2}$, when compared with the general slope-intercept form $y=mx+b$, indicates that the y-intercept is $(0,1/2)$. Examine Figure $3.5.3$. Does it appear that the y-intercept is $(0,1/2)$?

$$\begin{aligned} y-(-4) &= -\frac{3}{2}(x-3) && \text{Using } m=-3/2 \text{ and } (x_0,y_0)=(3,-4) \\ y+4 &= -\frac{3}{2}(x-3) && \text{Simplify.} \\ y+4 &= -\frac{3}{2}x+\frac{9}{2} && \text{Distribute } -3/2 \\ y+4-4 &= -\frac{3}{2}x+\frac{9}{2}-4 && \text{Subtract 4 from both sides.} \\ y &= -\frac{3}{2}x+\frac{9}{2}-\frac{8}{2} \\ & && \text{On the left, simplify. On the right, make equivalent fractions with a common denominator.} \\ y &= -\frac{3}{2}x+\frac{1}{2} \end{aligned}$$

Thus, both equations simplify to the same answer, $y=-\frac{3}{2}x+\frac{1}{2}$. This means that the equations $y-2=-\frac{3}{2}(x-(-1))$ and $y-(-4)=-\frac{3}{2}(x-3)$, though they look different, are the same.

Exercise 3.5.2

Find the equation of the line passing through the points $P(-2,1)$ and $Q(4,-1)$.

Answer

$$y=-\frac{1}{3}x+\frac{1}{3}$$

Example 3.5.2 gives rise to the following tip.

tip

When finding the equation of a line through two points P and Q, you may substitute either point P or Q for (x_0, y_0) in the point-slope form $y-y_0=m(x-x_0)$. The results look different, but they are both equations of the same line.

Parallel Lines

Recall that slope is a number that measures the steepness of the line. If two lines are parallel (never intersect), they have the same steepness.

Parallel lines

If two lines are parallel, they have the same slope.

Example 3.5.3

Sketch the line $y=\frac{3}{4}x-2$, then sketch the line passing through the point $(-1,1)$ that is parallel to the line $y=\frac{3}{4}x-2$. Find the equation of this parallel line.

Solution

Note that $y=\frac{3}{4}x-2$ is in slope-intercept form $y=mx+b$. Hence,its slope is $3/4$ and its y-intercept is $(0,-2)$. Plot the y-intercept $(0,-2)$, move up 3 units, right 4 units, then draw the line (see Figure $3.5.4$).

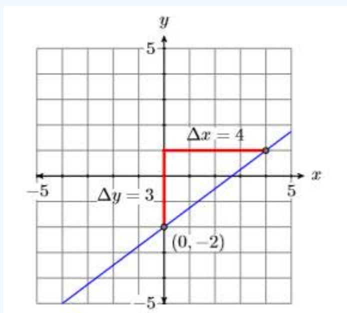


Figure 3.5.4: The line $y=\frac{3}{4}x-2$.

The second line must be parallel to the first, so it must have the same slope $3/4$. Plot the point $(-1,1)$, move up 3 units, right 4 units, then draw the line (see the red line in Figure $3.5.5$).

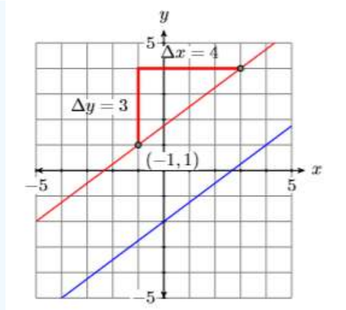


Figure 3.5.5: Adding a line parallel to $y = \frac{3}{4}x - 2$.

To find the equation of the parallel red line in Figure 3.5.5, use the point slope form, substitute $3/4$ for m, then $(-1, 1)$ for (x_0, y_0). That is, substitute -1 for x_0 and 1 for y_0.

$$\begin{aligned} y - y_0 &= m\,(x - x_0) && \text{Point-slope form.} \\ y - 1 &= \frac{3}{4}(x - (-1)) && \text{Substitute: } 3/4 \text{ for } m, -1 \text{ for } x_0 \text{ and } 1 \text{ for } y_0 \\ y - 1 &= \frac{3}{4}(x + 1) && \text{Simplify.} \end{aligned}$$

Check: In this example, we were not required to solve for y, so we can save ourselves some checking work by writing the equation

$$y - 1 = \frac{3}{4}(x + 1) \qquad \text{in the form} \qquad y = \frac{3}{4}(x + 1) + 1$$

by adding 1 to both sides of the first equation.

Next, enter each equation as shown in Figure 3.5.6, then change the WINDOW setting as shown in Figure 3.5.7.

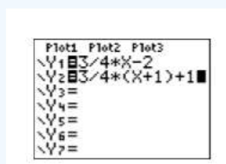


Figure 3.5.6: Enter equations of parallel lines.

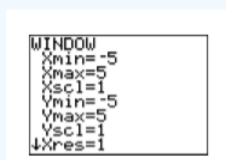


Figure 3.5.7: Adjust the **WINDOW** parameters as shown.

Next, press the GRAPH button, the select **5:ZSquare** to produce the image in Figure 3.5.9.

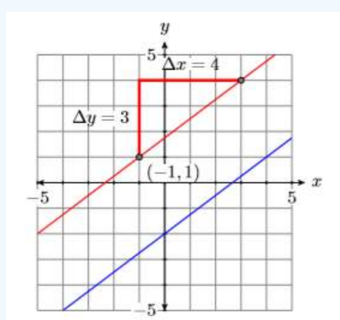


Figure 3.5.8: Hand-drawn parallel lines.

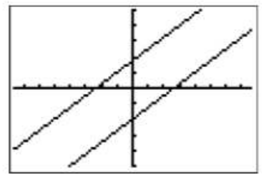


Figure $3.5.9$: Press the GRAPH button then select **5:ZSquare** to produce this image.

Note the close correlation of the calculator lines in Figure $3.5.9$ to the hand drawn lines in Figure $3.5.8$. This gives us confidence that we've captured the correct answer.

Exercise $3.5.3$

Find the equation of the line which passes through the point $(2, -3)$ and is parallel to the line $y = \frac{3}{4}x + 2$.

Answer

$$y = \frac{3}{2}x - 6$$

Perpendicular Lines

Two lines are perpendicular if they meet and form a right angle (90 degrees). For example, the lines $\mathcal{L}_1$ and $\mathcal{L}_2$ in Figure $3.5.10$ are perpendicular, but the lines $\mathcal{L}_1$ and $\mathcal{L}_2$ in Figure $3.5.11$ are not perpendicular.

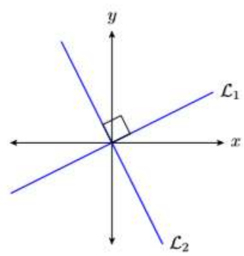


Figure $3.5.10$: The lines $\mathcal{L}_1$ and $\mathcal{L}_2$ are perpendicular. They meet and form a right angle (90 degrees).

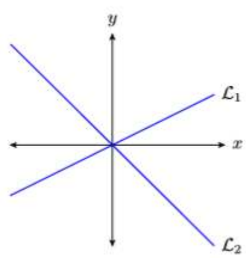


Figure $3.5.11$: The lines $\mathcal{L}_1$ and $\mathcal{L}_2$ are **not** perpendicular. They do **not** form a right angle (90 degrees).

Before continuing, we need to establish a relation between the slopes of two perpendicular lines. So, consider the perpendicular lines $\mathcal{L}_1$ and $\mathcal{L}_2$ in Figure $3.5.12$

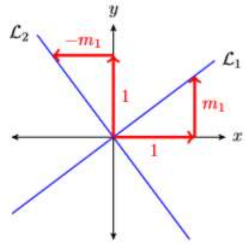


Figure 3.5.12: Perpendicular lines $\mathcal{L}_1$ and $\mathcal{L}_2$.

Note

1. If we were to rotate line $\mathcal{L}_1$ ninety degrees counter-clockwise, then $\mathcal{L}_1$ would align with the line $\mathcal{L}_2$, as would the right triangles revealing their slopes.
2. $\mathcal{L}_1$ has slope $\dfrac{\Delta y}{\Delta x} = \dfrac{m_1}{1} = m_1$.
3. $\mathcal{L}_2$ has slope $\dfrac{\Delta y}{\Delta x} = \dfrac{1}{-m_1} = -\dfrac{1}{m_1}$

Slopes of perpendicular lines

If $\mathcal{L}_1$ and $\mathcal{L}_2$ are perpendicular lines and $\mathcal{L}_1$ has slope m_1, the $\mathcal{L}_2$ has slope $-1/m_1$. That is, the slope of $\mathcal{L}_2$ is the negative reciprocal of the slope of $\mathcal{L}_1$.

Examples:

To find the slope of a perpendicular line, invert the slope of the first line and negate.

- If the slope of $\mathcal{L}_1$ is 2, then the slope of the perpendicular line $\mathcal{L}_2$ is $-1/2$.
- If the slope of $\mathcal{L}_1$ is $-3/4$, then the slope of the perpendicular line $\mathcal{L}_2$ is $4/3$.

Example $3.5.4$

Sketch the line $y = -\dfrac{2}{3}x - 3$, then sketch the line through $(2, 1)$ that is perpendicular to the line $y = -\dfrac{2}{3}x - 3$. Find the equation of this perpendicular line.

Solution

Note that $y = -\dfrac{2}{3}x - 3$ is in slope-intercept form $y = mx + b$. Hence, its slope is $-2/3$ and its y-intercept is $(0, -3)$. Plot the y-intercept $(0, -3)$, move right 3 units, down two units, then draw the line (see Figure $3.5.13$).

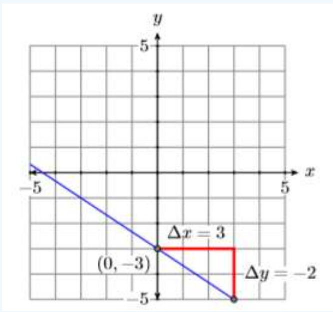


Figure $3.5.13$: The line $y = -\dfrac{2}{3}x - 3$.

Because the line $y = -\dfrac{2}{3}x - 3$ has slope $-2/3$, the slope of the line perpendicular to this line will be the negative reciprocal of $-2/3$, namely $3/2$. Thus, to draw the perpendicular line, start at the given point $(2, 1)$, move up 3 units, right 2 units, then draw the line (see Figure $3.5.14$).

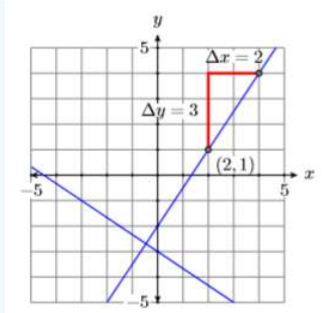


Figure 3.5.14: Adding a line perpendicular to $y = -\frac{2}{3}x - 3$.

To find the equation of the perpendicular line in Figure $3.5.14$, use the point slope form, substitute $3/2$ for m, then $(2, 1)$ for (x_0, y_0). That is, substitute 2 for x_0, then 1 for y_0.

$$\begin{aligned} y - y_0 &= m\,(x - x_0) && \color{red}{\text{Point-slope form.}} \\ y - 1 &= \frac{3}{2}(x - 2) && \color{red}{\text{Substitute: } 3/2 \text{ for } m,\ 2 \text{ for } x_0 \text{ and } 1 \text{ for } y_0} \end{aligned}$$

Check: In this example, we were not required to solve for y, so we can save ourselves some checking work by writing the equation

$$y - 1 = \frac{3}{2}(x - 2) \qquad \text{in the form} \qquad y = \frac{3}{2}(x - 2) + 1$$

by adding 1 to both sides of the first equation. Next, enter each equation as shown in Figure $3.5.15$, then select **6:ZStandard** to produce the image in Figure $3.5.16$.

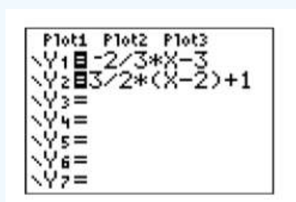


Figure 3.5.15: Enter equations of perpendicular lines.

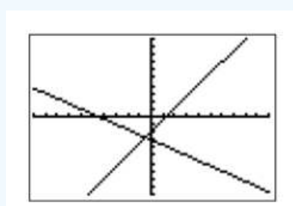


Figure 3.5.16: **6:ZStandard** produces two lines that do not look perpendicular.

Note that the lines in Figure $3.5.16$ do not appear to be perpendicular. Have we done something wrong? The answer is no! The fact that the calculator's viewing screen is wider than it is tall is distorting the angle at which the lines meet.

To make the calculator result match the result in Figure $3.5.14$, change the window settings as shown in Figure $3.5.17$, then select **5:ZSquare** from the ZOOM menu to produce the image in Figure $3.5.18$. Note the close correlation of the calculator lines in Figure $3.5.18$ to the hand-drawn lines in Figure $3.5.14$. This gives us confidence that we've captured the correct answer.

Figure 3.5.17: Change the **WINDOW** parameters as shown.

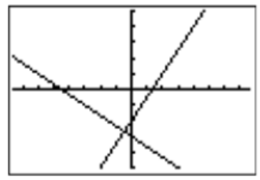


Figure 3.5.18: **5:ZSquare** produces two lines that do look perpendicular.

Exercise 3.5.4

Find the equation of the line that passes through the point $(-3, 1)$ and is perpendicular to the line $y = -\frac{1}{2}x + 1$.

Answer

$y = 2x + 7$

Applications

Let's look at a real-world application of lines.

Example 3.5.5

Water freezes at $32^{\circ}\mathrm{F}$ (Fahrenheit) and at $0^{\circ}\mathrm{C}$ (Celsius). Water boils at $212^{\circ}\mathrm{F}$ and at $100^{\circ}\mathrm{C}$. If the relationship is linear, find an equation that expresses the Celsius temperature in terms of the Fahrenheit temperature. Use the result to find the Celsius temperature when the Fahrenheit temperature is $113^{\circ}\mathrm{F}$.

Solution

In this example, the Celsius temperature depends on the Fahrenheit temperature. This makes the Celsius temperature the dependent variable and it gets placed on the vertical axis. This Fahrenheit temperature is the independent variable, so it gets placed on the horizontal axis (see Figure 3.5.19).

Next, water freezes at $32^{\circ}\mathrm{F}$ and $0^{\circ}\mathrm{C}$, giving us the point $(F, C) = (32, 0)$. Secondly, water boils at $212^{\circ}\mathrm{F}$ and $100^{\circ}\mathrm{C}$, giving us the point $(F, C) = (212, 100)$. Note how we've scaled the axes so that each of these points fit on the coordinate system. Finally, assuming a linear relationship between the Celsius and Fahrenheit temperatures, draw a line through these two points (see Figure 3.5.19).

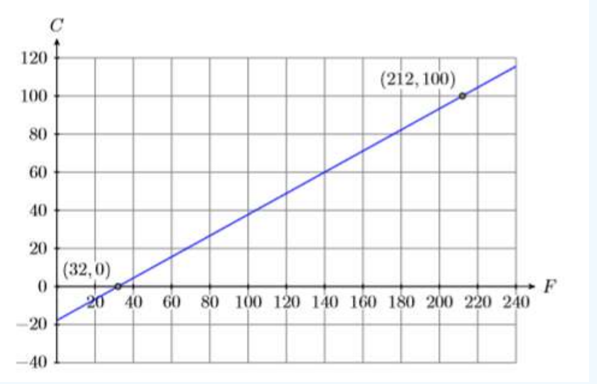


Figure 3.5.19: The linear relationship between Celsius and Fahrenheit temperature.

Calculate the slope of the line.

$m = \dfrac{\Delta C}{\Delta F}$ Slope formula.

$m = \dfrac{100-0}{212-32}$ Use the points $(32,0)$ and $(212,100)$ to compute the difference in C and F

$m = \dfrac{100}{180}$ Simplify.

$m = \dfrac{5}{9}$ Reduce.

You may either use $(32,0)$ or $(212,100)$ in the point-slope form. The point $(32,0)$ has smaller numbers, so it seems easier to substitute $(x_0, y_0) = (32,0)$ and $m = 5/9$ into the point-slope form $y - y_0 = m(x - x_0)$.

$$\begin{aligned} y - y_0 &= m\left(x - x_0\right) && \text{Point-slope form.} \\ y - 0 &= \frac{5}{9}(x - 32) && \text{Substitute: } 5/9 \text{ for } m, 32 \text{ for } x_0, \text{ and } 0 \text{ for } y_0 \\ y &= \frac{5}{9}(x - 32) && \text{Simplify.} \end{aligned}$$

However, our vertical and horizontal axes are labeled C and F (see Figure \(\PageIndex{19}\)) respectively, so we must replace y with C and x with F to obtain an equation expressing the Celsius temperature C in terms of the Fahrenheit temperature F.

$$C = \frac{5}{9}(F - 32) \tag{3.5.1}$$

Finally, to find the Celsius temperature when the Fahrenheit temperature is $113^{\circ}\mathrm{F}$, substitute 113 for F in Equation 3.5.1

$$
\begin{aligned}
C &= \frac{5}{9}(F-32) \quad \textcolor{red}{\text{Equation } (3.5.1)} \\
C &= \frac{5}{9}(113-32) \quad \textcolor{red}{\text{Substitute: } 113 \text{ for } F} \\
C &= \frac{5}{9}(81) \quad \textcolor{red}{\text{Subtract.}} \\
C &= 45 \quad \textcolor{red}{\text{Multiply.}}
\end{aligned}
$$

Therefore, if the Fahrenheit temperature is $113^\circ \mathrm{F}$, then the Celsius temperature is $45^\circ \mathrm{C}$.

Exercise $3.5.5$

Find an equation that expresses the Fahrenheit temperature in terms of the Celsius temperature. Use the result to find the Fahrenheit temperature when the Celsius temperature is $25^\circ \mathrm{C}$.

Answer

$77^\circ \mathrm{F}$

3.6: Standard Form of a Line

In this section we will investigate the standard form of a line. Let's begin with a simple example.

Example 3.6.1

Solve the equation $2x + 3y = 6$ for y and plot the result.

Solution

First we solve the equation $2x + 3y = 6$ for y. Begin by isolating all terms containing y on one side of the equation, moving or keeping all the remaining terms on the other side of the equation.

$$\begin{aligned} 2x+3y &= 6 && \text{Original equation.} \\ 2x+3y-2x &= 6-2x && \text{Subtract } 2x \text{ from both sides.} \\ 3y &= 6-2x && \text{Simplify.} \\ \frac{3y}{3} &= \frac{6-2x}{3} && \text{Divide both sides by 3} \end{aligned}$$

Note

Just as multiplication is distributive with respect to addition

$$a(b+c) = ab+ac$$

so too is division distributive with respect to addition.

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

When dividing a sum or a difference by a number, we use the distributive property and divide both terms by that number.

$$\begin{aligned} y &= \frac{6}{3} - \frac{2x}{3} && \text{On the left, simplify. On the right, divide both terms by 3} \\ y &= 2 - \frac{2x}{3} && \text{Simplify.} \end{aligned}$$

Finally, use the commutative property to switch the order of the terms on the right-hand side of the last result.

$$\begin{aligned} y &= 2 + \left(-\frac{2x}{3}\right) && \text{Add the opposite.} \\ y &= -\frac{2}{3}x + 2 && \text{Use the commutative property to switch the order.} \end{aligned}$$

Because the equation $2x + 3y = 6$ is equivalent to the equation $y = -\frac{2}{3}x + 2$, the graph of $2x + 3y = 6$ is a line, having slope $m = -2/3$ and y-intercept $(0, 2)$. Therefore, to draw the graph of $2x + 3y = 6$, plot they-intercept $(0, 2)$, move down 2 and 3 to the right, then draw the line (see Figure $3.6.1$).

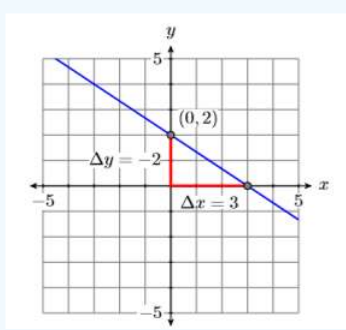
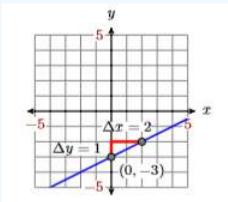


Figure 3.6.1: The graph of $2x + 3y = 6$, or equivalently, $y = -\frac{2}{3}x + 2$

Exercise 3.6.1

Add exercises text here.

Answer



In general, unless $B = 0$, we can always solve the equation $Ax + By = C$ for y:

$$\begin{aligned} Ax + By &= C && \text{Original equation.} \\ Ax + By - Ax &= C - Ax && \text{Subtract } Ax \text{ from both sides.} \\ By &= C - Ax && \text{Simplify.} \\ \frac{By}{B} &= \frac{C - Ax}{B} && \text{Divide both sides by } B \\ y &= \frac{C}{B} - \frac{Ax}{B} && \text{distribute the } B \\ y &= -\frac{A}{B}x + \frac{C}{B} && \text{Commutative property} \end{aligned}$$

Note that the last result is in slope-intercept form $y = mx + b$, whose graph is a line. We have established the following result.

FAct

The graph of the equation $Ax + By = C$, is a line.

Important points: A couple of important comments are in order.

1. The form $Ax + By = C$ requires that the coefficients A, B, and C are integers. So, for example, we would clear the fractions from the form

$$\frac{1}{2}x + \frac{2}{3}y = \frac{1}{4}$$

by multiplying both sides by the least common denominator.

$$\begin{aligned} 12\left(\frac{1}{2}x + \frac{2}{3}y\right) &= \left(\frac{1}{4}\right)12 \\ 6x + 8y &= 3 \end{aligned}$$

Note that the coefficients are now integers.

2. The form $Ax + By = C$ also requires that the first coefficient A is nonnegative; i.e., $A \geq 0$. Thus, if we have

$$-5x + 2y = 6$$

then we would multiply both sides by -1, arriving at:

$$\begin{aligned} -1(-5x + 2y) &= (6)(-1) \\ 5x - 2y &= -6 \end{aligned}$$

Note that $A = 5$ is now greater than or equal to zero.

3. If A, B, and C have a common divisor greater than 1, it is recommended that we divide both sides by the common divisor, thus “reducing” the coefficients. For example, if we have

$$3x + 12y = -24$$

then dividing both side by 3 “reduces” the size of the coefficients.

$$\begin{aligned}\frac{3x+12y}{3} &= \frac{-24}{3}\\ x+4y &= -8\end{aligned}$$

Standard form

The form $Ax+By=C$, where A, B, and C are integers, and $A \geq 0$, is called the standard form of a line.

Slope-Intercept to Standard Form

We’ve already transformed a couple of equations in standard form into slopeintercept form. Let’s reverse the process and place an equation in slope intercept form into standard form.

Example $3.6.2$

Given the graph of the line in Figure $3.6.2$, find the equation Given the graph of the line below, find the equation of the line in standard form.

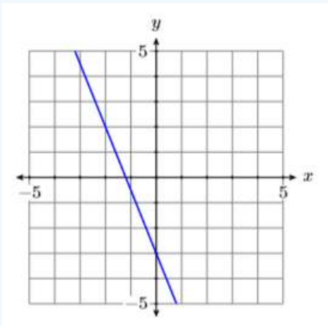


Figure $3.6.2$: Determine the equation of the line.

Solution

The line intercepts the y-axis at $(0,-3)$. From $(0,-3)$, move up 5 units, then left 2 units. Thus, the line has slope $\Delta y/\Delta x=-5/2$ (see Figure $3.6.3$). Substitute $-5/2$ form and -3 for b in the slope-intercept form of the line.

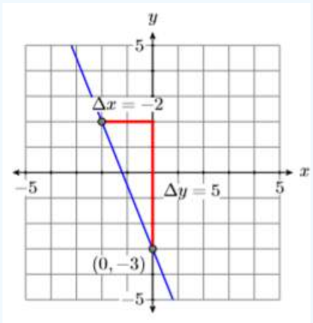


Figure $3.6.3$: The line has y-intercept $(0,-3)$ and slope $-5/2$.

$$\begin{aligned} y &= mx+b && \color{red}{\text{Slope-intercept form.}}\\ y &= -\frac{5}{2}x-3 && \color{red}{\text{Substitute: } -5/2 \text{ for } m, -3 \text{ for } b}\end{aligned}$$

To put this result in standard form $Ax+By=C$, first clear the fractions by multiplying both sides by the common denominator.

$$2y = 2\left[-\frac{5}{2}x - 3\right] \quad \text{Multiply both sides by 2}$$
$$2y = 2\left[-\frac{5}{2}x\right] - 2[3] \quad \text{Distribute the 2}$$
$$2y = -5x - 6 \quad \text{Multiply.}$$

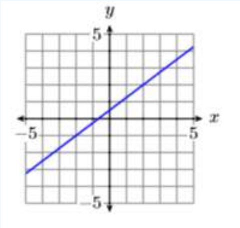
That clears the fractions. To put this last result in the form $Ax + By = C$, we need to move the term $-5x$ to the other side of the equation.

$$5x + 2y = -5x - 6 + 5x \quad \text{Add } 5x \text{ to both sides.}$$
$$5x + 2y = -6 \quad \text{Simplify.}$$

Thus, the standard form of the line is $5x + 2y = -6$. Note that all the coefficients are integers and the terms are arranged in the order $Ax + By = C$, with $A \geq 0$.

Exercise 3.6.2

Given the graph of the line below, find the equation of the line in standard form.



Answer

$3x - 4y = -2$

Point-Slope to Standard Form

Let's do an example where we have to put the point-slope form of a line in standard form.

Example 3.6.3

Sketch the line passing through the points $(-3, -4)$ and $(1, 2)$, then find the equation of the line in standard form.

Solution

Plot the points (−3,−4) and (1,2), then draw a line through them (see Figure 3.6.4).

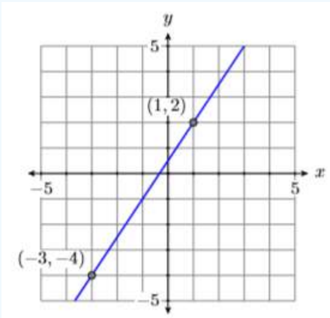


Figure 3.6.4: The line through $(-3, -4)$ and $(1, 2)$.

Use the points $(-3, -4)$ and $(1, 2)$ to calculate the slope.

$$\begin{aligned} \text{Slope } &= \frac{\Delta y}{\Delta x} \quad \color{red}{\text{Slope formula.}} \\ &= \frac{2-(-4)}{1-(-3)} \quad \color{red}{\text{Subtract coordinates of } (-3,-4)} \\ &= \frac{6}{4} \quad \text{Simplify.} \\ &= \frac{3}{2} \quad \text{Reduce.} \end{aligned}$$

Let's substitute $(x_0, y_0) = (1, 2)$ and $m = 3/2$ in the point-slope form of the line. (*Note: Substituting* $(x_0, y_0) = (-3, -4)$ *and* $m = 3/2$ *would yield the same answer.*)

$$\begin{aligned} y - y_0 &= m(x - x_0) \quad \color{red}{\text{Point-slope form.}} \\ y - 2 &= \frac{3}{2}(x-1) \quad \color{red}{\text{Substitute: } 3/2 \text{ for } m, 1 \text{ for } x_0} \end{aligned}$$

The question requests that our final answer be presented in standard form. First we clear the fractions.

$$\begin{aligned} y - 2 &= \frac{3}{2}x - \frac{3}{2} \quad \color{red}{\text{Distribute the } 3/2} \\ 2[y-2] &= 2\left[\frac{3}{2}x - \frac{3}{2}\right] \quad \color{red}{\text{Multiply both sides by 2}} \\ 2y - 2[2] &= 2\left[\frac{3}{2}x\right] - 2\left[\frac{3}{2}\right] \quad \color{red}{\text{Distribute the 2}} \\ 2y - 4 &= 3x - 3 \quad \color{red}{\text{Multiply.}} \end{aligned}$$

Now that we've cleared the fractions, we must order the terms in the form $Ax + By = C$. We need to move the term $3x$ to the other side of the equation.

$$\begin{aligned} 2y - 4 - 3x &= 3x - 3 - 3x \quad \color{red}{\text{Subtract } 3x \text{ from both sides.}} \\ -3x + 2y - 4 &= -3 \quad \color{red}{\text{Simplify, changing the order on the left-hand side.}} \end{aligned}$$

To put this in the form $Ax + By = C$, we need to move the term -4 to the other side of the equation.

$$\begin{aligned} -3x + 2y - 4 + 4 &= -3 + 4 \quad \color{red}{\text{Add 4 to both sides.}} \\ -3x + 2y &= 1 \quad \color{red}{\text{Simplify.}} \end{aligned}$$

It appears that $-3x + 2y = 1$ is in the form $Ax + By = C$. However, standard form requires that $A \geq 0$. We have $A = -3$. To fix this, we multiply both sides by -1.

$$\begin{aligned} -1[-3x + 2y] &= -1[1] \quad \color{red}{\text{Multiply both sides by } -1} \\ 3x - 2y &= -1 \quad \color{red}{\text{Distribute the } -1} \end{aligned}$$

Thus, the equation of the line in standard form is $3x - 2y = -1$.

Note

If we fail to reduce the slope to lowest terms, then the equation of the line would be:

$$y - 2 = \frac{6}{4}(x-1)$$

Multiplying both sides by 4 would give us the result

$$4y - 8 = 6x - 6$$

or equivalently:

$$-6x + 4y = 2$$

This doesn't look like the same answer, but if we divide both sides by -2, we do get the same result.

$$3x - 2y = -1$$

This shows the importance of requiring $A \geq 0$ and "reducing" the coefficients A, B, and C. It allows us to compare our answer with our colleagues or the answers presented in this textbook.

Exercise 3.6.3

Find the standard form of the equation of the line that passes through the points $(-2,4)$ and $(3,-3)$.

Answer

$7x+5y=6$

Intercepts

We've studied the y-intercept, the point where the graph crosses the y-axis, but equally important are the x-intercepts, the points where the graph crosses the x-axis.

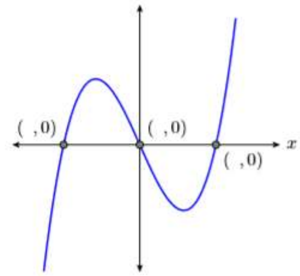


Figure 3.6.5: Each x-intercept has a y-coordinate equal to zero.

In Figure 3.6.5, the graph crosses the x-axis three times. Each of these crossing points is called an x-intercept. Note that each of these x-intercepts has a y-coordinate equal to zero. This leads to the following rule.

x Intercepts

To find the x-intercepts of the graph of an equation, substitute $y=0$ into the equation and solve for x.

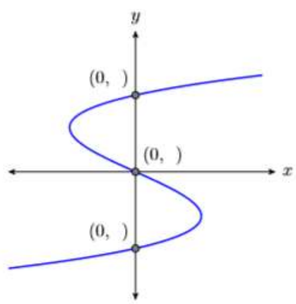


Figure 3.6.6: Each y-intercept has an x-coordinate equal to zero.

Similarly, the graph in Figure 3.6.6 crosses the y-axis three times. Each of these crossing points is called a y-intercept. Note that each of these y-intercepts has an x-coordinate equal to zero. This leads to the following rule.

y Intercepts

To find the y-intercepts of the graph of an equation, substitute $x=0$ into the equation and solve for y.

Let's put these rules for finding intercepts to work.

Example $3.6.4$

Find the x- and y-intercepts of the line having equation $2x-3y=6$. Plot the intercepts and draw the line.

Solution

We know that the graph of $2x-3y=6$ is a line. Furthermore, two points completely determine a line. This means that we need only plot the x- and y-intercepts, then draw a line through them.

To find the x-intercept of $2x-3y=6$, substitute 0 for y and solve for x.

$$\begin{aligned} 2x-3y &= 6 \\ 2x-3(0) &= 6 \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} \\ x &= 3 \end{aligned}$$

Thus, the x-intercept of the line is $(3,0)$.

To find the y-intercept of $2x-3y=6$, substitute 0 for x and solve for y.

$$\begin{aligned} 2x-3y &= 6 \\ 2(0)-3y &= 6 \\ -3y &= 6 \\ \frac{-3y}{-3} &= \frac{6}{-3} \\ y &= -2 \end{aligned}$$

Thus, the y-intercept of the line is $(0,-2)$.

Plot the x-intercept $(3,0)$ and the y-intercept $(0,-2)$ and draw a line through them (see Figure $3.6.7$).

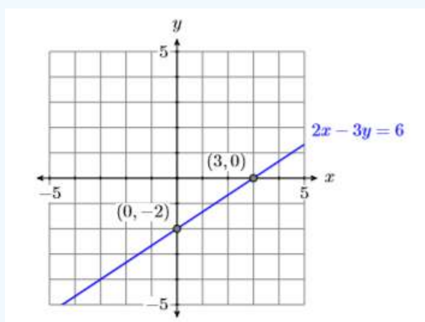


Figure $3.6.7$: The graph of $2x-3y=6$ has intercepts $(3,0)$ and $(0,-2)$.

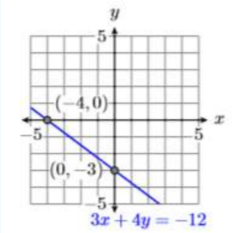
Exercise $3.6.4$

Find the x- and y-intercepts of the line having equation $3x+4y=-12$. Plot the intercepts and draw the line.

Answer

x-intercept: $(-4,0)$

y-intercept: $(0,-3)$



Example 3.6.5

Sketch the line $4x + 3y = 12$, then sketch the line through the point $(-2, -2)$ that is perpendicular to the line $4x + 3y = 12$. Find the equation of this perpendicular line.

Solution

Let's first find the x- and y-intercepts of the line $4x + 3y = 12$.

To find the x-intercept of the line $4x + 3y = 12$, substitute 0 for y and solve for x.

$$\begin{aligned} 4x + 3y &= 12 \\ 4x + 3(0) &= 12 \\ 4x &= 12 \\ \frac{4x}{4} &= \frac{12}{4} \\ x &= 3 \end{aligned}$$

Thus, the x-intercept of the line is $(3, 0)$.

To find the y-intercept of the line $4x + 3y = 12$, substitute 0 for x and solve for y.

$$\begin{aligned} 4x + 3y &= 12 \\ 4(0) + 3y &= 12 \\ 3y &= 12 \\ \frac{3y}{3} &= \frac{12}{3} \\ y &= 4 \end{aligned}$$

Thus, the y-intercept of the line is $(0, 4)$.

Plot the intercepts and draw a line through them. Note that it is clear from the graph that the slope of the line $3x + 4y = 12$ is $-4/3$ (see Figure $3.6.8$).

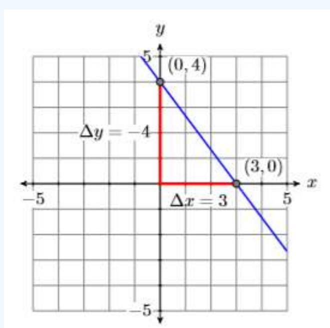


Figure $3.6.8$: The graph of $4x + 3y = 12$ has intercepts $(3, 0)$ and $(0, 4)$ and slope $-4/3$.

You could also solve for y to put $3x + 4y = 12$ in slope intercept form in order to determine the slope.

Because the slope of $3x + 4y = 12$ is $-4/3$, the slope of a line perpendicular to $3x + 4y = 12$ will be the negative reciprocal of $-4/3$, namely $3/4$. Our perpendicular line has to pass through the point $(-2, -2)$. Start at $(-2, -2)$, move 3 units upward, then 4 units to the right, then draw the line. It should appear to be perpendicular to the line $3x+4y=12$ (see Figure 3.6.9).

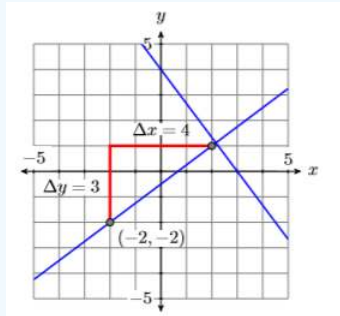


Figure 3.6.9: The slope of the perpendicular line is the negative reciprocal of $-4/3$, namely $3/4$.

Finally, use the point-slope form, $m=3/4$, and $(x_0, y_0)=(-2,-2)$ to determine the equation of the perpendicular line.

$$\begin{aligned} y-y_0 &= m\left(x-x_0\right) && \text{Point-slope form.} \\ y-(-2) &= \frac{3}{4}(x-(-2)) && \text{Substitute: } 3/4 \text{ for } m, -2 \text{ for } x_0 \text{ and } -2 \text{ for } y_0 \\ y+2 &= \frac{3}{4}(x+2) && \text{Simplify.} \end{aligned}$$

Let’s place our answer in standard form. Clear the fractions.

$$\begin{aligned} y+2 &= \frac{3}{4}x+\frac{6}{4} && \text{Distribute } 3/4 \\ 4[y+2] &= 4\left[\frac{3}{4}x+\frac{6}{4}\right] && \text{Multiply both sides by } 4 \\ 4y+4[2] &= 4\left[\frac{3}{4}x\right]+4\left[\frac{6}{4}\right] && \text{Distribute the } 4 \\ 4y+8 &= 3x+6 && \text{Multiply.} \end{aligned}$$

Rearrange the terms to put them in the order $Ax+By=C$.

$$\begin{aligned} 4y+8-3x &= 3x+6-3x && \text{Subtract } 3x \text{ from both sides.} \\ -3x+4y+8 &= 6 && \text{Simplify. Rearrange on the left.} \\ -3x+4y+8-8 &= 6-8 && \text{Subtract 8 from both sides.} \\ -3x+4y &= -2 && \text{Simplify.} \\ -1(-3x+4y) &= -1(-2) && \text{Multiply both sides by } -1 \\ 3x-4y &= 2 && \text{Distribute the } -1 \end{aligned}$$

Hence, the equation of the perpendicular line is $3x-4y=2$.

Exercise 3.6.5

Find the equation of the line that passes through the point $(3,2)$ and is perpendicular to the line $6x-5y=15$.

Answer

$5x+6y=27$

Horizontal and Vertical Lines

Here we keep an earlier promise to address what happens to the standard form $Ax+By=C$ when either $A=0$ or $B=0$. For example, the form $3x=6$, when compared with the standard form $Ax+By=C$, has $B=0$. Similarly, the form $2y=-12$, when compared with the standard form $Ax+By=C$, has $A=0$. Of course, $3x=6$ can be simplified to $x=2$ and $2y=-12$

can be simplified to $y=-6$. Thus, if either $A=0$ or $B=0$, the standard form Ax + By = C takes the form $x=a$ and $y=b$, respectively.

As we will see in the next example, the form $x=a$ produces a vertical line, while the form $y=b$ produces a horizontal line.

Example 3.6.6

Sketch the graphs of $x=3$ and $y=-3$.

Solution

To sketch the graph of $x=3$, recall that the graph of an equation is the set of all points that satisfy the equation. Hence, to draw the graph of $x=3$, we must plot all of the points that satisfy the equation $x=3$; that is, we must plot all of the points that have an x-coordinate equal to 3. The result is shown in Figure $3.6.10$.

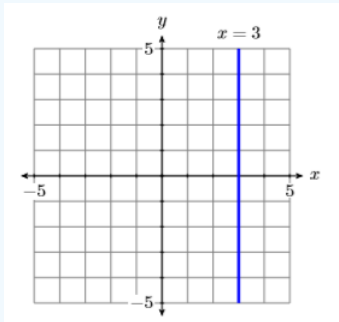


Figure $3.6.10$: The graph of $x=3$ is a vertical line.

Secondly, to sketch the graph of $y=-3$, we plot all points having a y-coordinate equal to -3. The result is shown in Figure $3.6.11$.

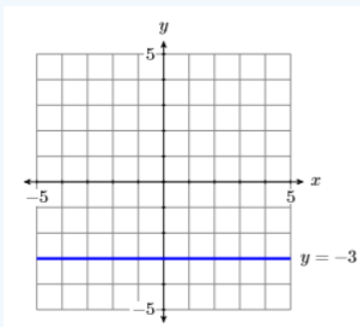


Figure $3.6.11$: The graph of $y=-3$ is a horizontal line.

Things to note:

A couple of comments are in order regarding the lines in Figures $3.6.10$ and $3.6.11$.

1. The graph of $x=3$ in Figure $3.6.10$, being a vertical line, has undefined slope. Therefore, we cannot use either of the formulae $y=mx+b$ or $y-y_0=m(x-x_0)$ to obtain the equation of the line. The only way we can obtain the equation is to note that the line is the set of all points (x,y) whose x-coordinate equals 3.
2. However, the graph of $y=-3$, being a horizontal line, has slope zero, so we can use the slope-intercept form to find the equation of the line. Note that the y-intercept of this graph is $(0,-3)$. If we substitute these numbers into $y=mx+b$, we get:

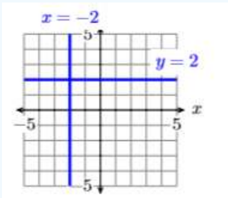
$$\begin{aligned} y &= mx+b && \text{Slope-intercept form.} \\ y &= 0x+(-3) && \text{Substitute: } 0 \text{ for } m,\ -3 \text{ for } b \\ y &= -3 && \text{Simplify.} \end{aligned}$$

However, it is far easier to just look at the line in Figures $3.6.11$ and note that it is the collection of all points (x,y) with $y=3$.

Exercise 3.6.6

Sketch the graphs of $x=-2$ and $y=2$.

Answer



3.E: Introduction to Graphing (Exercises)

3.1: Graphing Equations by Hand

In Exercises 1-6, set up a Cartesian Coordinate system on a sheet of graph paper, then plot the given points.

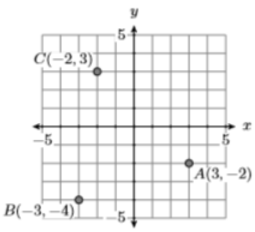
1) $A(2, -4)$, $B(-4, -3)$, and $C(-3, 2)$

Answer

2) $A(3, -4)$, $B(-3, -2)$, and $C(-4, 4)$

3) $A(3, -2)$, $B(-3, -4)$, and $C(-2, 3)$

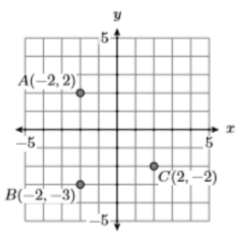
Answer



4) $A(3, -4)$, $B(-3, -4)$, and $C(-4, 2)$

5) $A(-2, 2)$, $B(-2, -3)$, and $C(2, -2)$

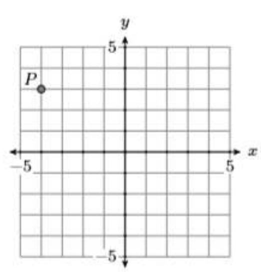
Answer



6) $A(-3, 2)$, $B(2, -4)$, and $C(-3, -4)$

In Exercises 7-10, identify the coordinates of point P in each of the given coordinate systems.

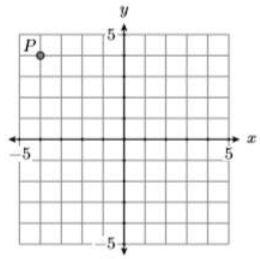
7)



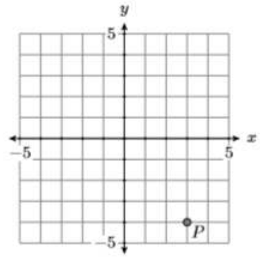
Answer

$(-4, 3)$

8)



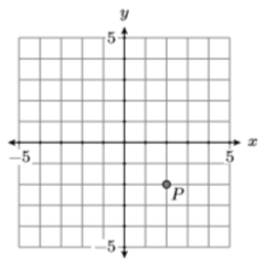
9)



Answer

$(3, -4)$

10)



11) Which of the points $(8, 36)$, $(5, 20)$, $(4, 13)$, and $(-2, -12)$ is a solution of the equation $y = 5x - 5$?

Answer

$(5, 20)$

12) Which of the points $(0, -9)$, $(6, -46)$, $(1, -14)$, and $(-6, 36)$ is a solution of the equation $y = -7x - 7$?

13) Which of the points $(-9, 49)$, $(1, 1)$, $(-6, 39)$, and $(2, -3)$ is a solution of the equation $y = -5x + 6$?

Answer

$(1, 1)$

14) Which of the points $(7, -15)$, $(-9, 30)$, $(-6, 19)$, and $(5, -11)$ is a solution of the equation $y = -3x + 3$?

15) Which of the points $(1, 12)$, $(7, 395)$, $(-7, 398)$, and $(0, 1)$ is a solution of the equation $y = 8x^2 + 3$?

Answer

$(7, 395)$

16) Which of the points $(-7, 154)$, $(7, 153)$, $(-2, 21)$, and $(2, 16)$ is a solution of the equation $y = 3x^2 + 6$?

17) Which of the points $(8, 400)$, $(5, 158)$, $(0, 3)$, and $(2, 29)$ is a solution of the equation $y = 6x^2 + 2x$?

Answer

$(8, 400)$

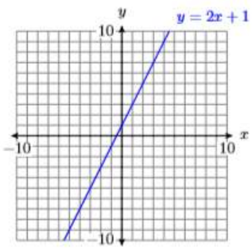
18) Which of the points $(-6, -114)$, $(6, -29)$, $(-7, -149)$, and $(4, -1)$ is a solution of the equation $y = -2x^2 + 7x$?

In Exercises 19-26, complete the table of points that satisfy the given equation. Set up a coordinate system on a sheet of graph paper, plot the points from the completed table, then draw the graph of the given equation.

19)

x	$y = 2x + 1$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		

Answer



20)

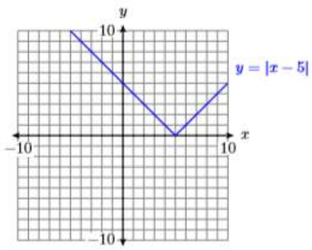
x	$y = x - 4$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		

21)

x	$y = \lvert x - 5 \rvert$	(x, y)
2		

x	$y=\|x-5\|$	(x,y)
3		
4		
5		
6		
7		
8		

Answer



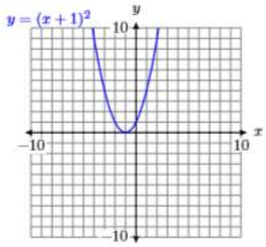
22)

x	$y=\|x+2\|$	(x,y)
-5		
-4		
-3		
-2		
-1		
0		
1		

23)

x	$y=(x+1)^2$	(x,y)
-4		
-3		
-2		
-1		
0		
1		
2		

Answer



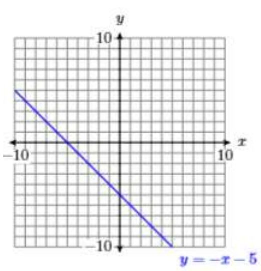
24)

x	$y=(x+5)^2$	(x,y)
-8		
-7		
-6		
-5		
-4		
-3		
-2		

25)

x	$y=-x-5$	(x,y)
-3		
-2		
-1		
0		
1		
2		
3		

Answer

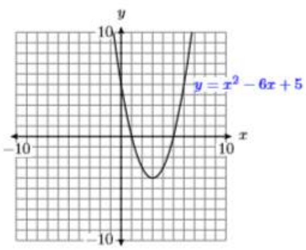


26)

x	$y = -2x + 3$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		

27) Use a graphing calculator to complete a table of points satisfying the equation $y = x^2 - 6x + 5$. Use integer values of x, starting at 0 and ending at 6. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = x^2 - 6x + 5$.

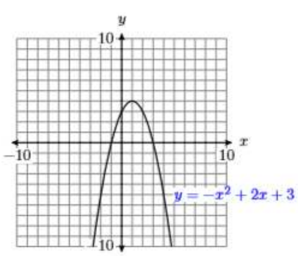
Answer



28) Use a graphing calculator to complete a table of points satisfying the equation $y = x^2 - 2x - 3$. Use integer values of x, starting at $-2$and ending at 4. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = x^2 - 2x - 3$.

29) Use a graphing calculator to complete a table of points satisfying the equation $y = -x^2 + 2x + 3$. Use integer values of x, starting at -2 and ending at 4. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = -x^2 + 2x + 3$.

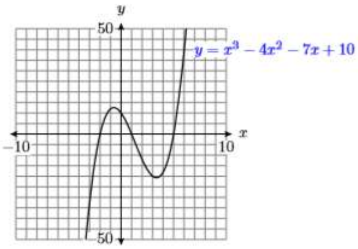
Answer



30) Use a graphing calculator to complete a table of points satisfying the equation $y = -x^2 - 2x + 8$. Use integer values of x, starting at -5 and ending at 3. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = -x^2 - 2x + 8$.

31) Use a graphing calculator to complete a table of points satisfying the equation $y = x^3 - 4x^2 - 7x + 10$. Use integer values of x, starting at -3 and ending at 6. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = x^3 - 4x^2 - 7x + 10$.

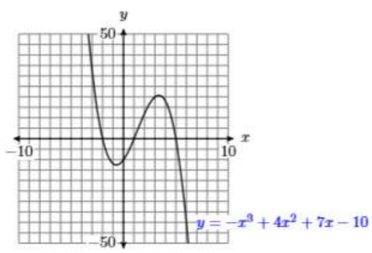
Answer



32) Use a graphing calculator to complete a table of points satisfying the equation $y = x^3 + 3x^2 - 13x - 15$. Use integer values of x, starting at -6 and ending at 4. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = x^3 + 3x^2 - 13x - 15$.

33) Use a graphing calculator to complete a table of points satisfying the equation $y = -x^3 + 4x^2 + 7x - 10$. Use integer values of x, starting at -3 and ending at 6. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = -x^3 + 4x^2 + 7x - 10$.

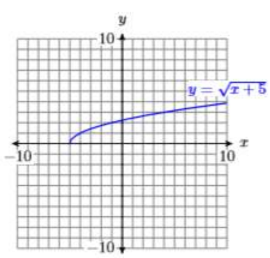
Answer



34) Use a graphing calculator to complete a table of points satisfying the equation $y = -x^3 + x^2 + 12x$. Use integer values of x, starting at -4 and ending at 5. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = -x^3 + x^2 + 12x$.

35) Use a graphing calculator to complete a table of points satisfying the equation $y = \sqrt{x+5}$. Use integer values of x, starting at -5 and ending at 10. Round your y-values to the nearest tenth. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = \sqrt{x+5}$.

Answer



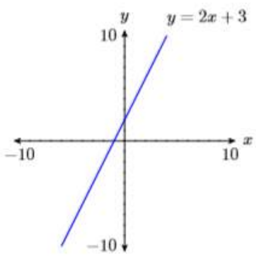
36) Use a graphing calculator to complete a table of points satisfying the equation $y = \sqrt{4-x}$. Use integer values of x, starting at -10 and ending at 4. Round your y-values to the nearest tenth. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = \sqrt{4-x}$.

3.2: The Graphing Calculator

In Exercises 1-16, enter the given equation in the Y= menu of your calculator, then select 6:ZStandard from the ZOOM menu to produce its graph. Follow the Calculator Submission Guidelines outlined in the subsection Reproducing Calculator Results on Homework Paper when submitting your result on your homework paper.

1) $y=2x+3$

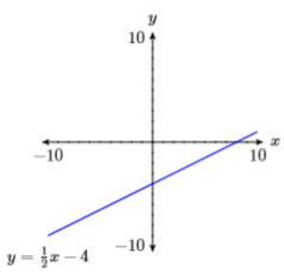
Answer



2) $y=3-2x$

3) $y=\frac{1}{2}x-4$

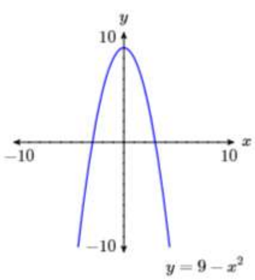
Answer



4) $y=-\frac{2}{3}x+2$

5) $y=9-x^2$

Answer



6) $y=x^2-4$

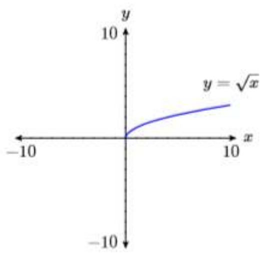
7) $y=\frac{1}{2}x^2-3$

Answer

8) $y = 4 - \frac{1}{3}x^2$

9) $y = \sqrt{x}$

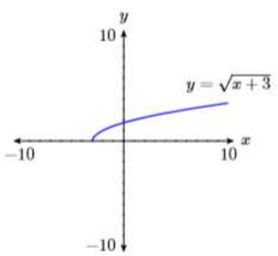
Answer



10) $y = \sqrt{-x}$

11) $y = \sqrt{x+3}$

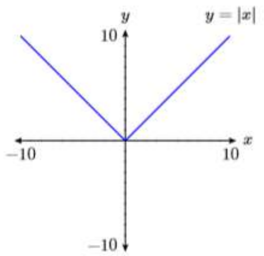
Answer



12) $y = \sqrt{5-x}$

13) $y = |x|$

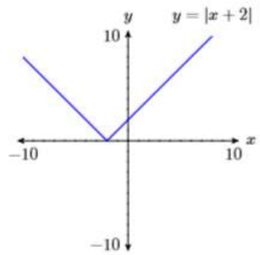
Answer



14) $y = -|x|$

15) $y = |x+2|$

Answer

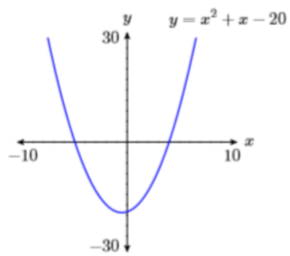


16) $y = -|x-3|$

In Exercises 17-22, the graph of each given equation is a parabola, having a “u-shape” similar to the graph in Figure 3.2.13 Some of the parabolas “open up” and some “open down.” Adjust Ymin and/or Ymax in the WINDOW menu so that the “turning point” of the parabola, called its vertex, is visible in the viewing screen of your graphing calculator. Follow the *Calculator Submission Guidelines* outlined in the subsection Reproducing Calculator Results on Homework Paper when submitting your result on your homework paper.

17) $y = x^2 + x - 20$

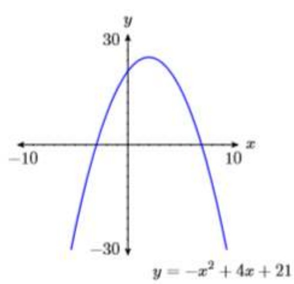
Answer



18) $y = -x^2 - 2x + 24$

19) $y = -x^2 + 4x + 21$

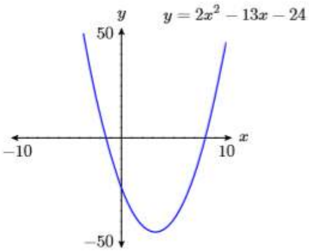
Answer



20) $y = x^2 + 6x - 16$

21) $y = 2x^2 - 13x - 24$

Answer

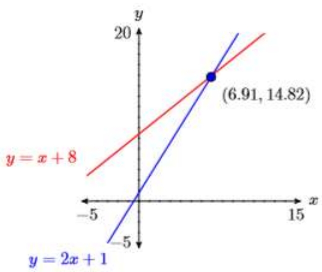


22) $y = -3x^2 - 19x + 14$

In Exercises 23-28, sketch the given lines, then adjust the WINDOW parameters so that the point of intersection of the two lines is visible in the viewing window. Use the TRACE key to estimate the point of intersection. Follow the *Calculator Submission Guidelines* outlined in the subsection Reproducing Calculator Results on Homework Paper when submitting your result on your homework paper. Be sure to also include your estimate of the point of intersection in your sketch.

23) $y = 2x + 1$ and $y = x + 8$

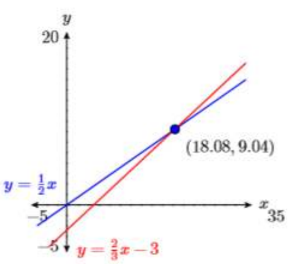
Answer



24) $y = 4 - 2x$ and $y = -6 - x$

25) $y = \frac{1}{2}x$ and $y = \frac{2}{3}x - 3$

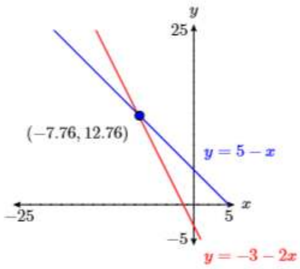
Answer



26) $y = 2x + 5$ and $y = x - 6$

27) $y = 5 - x$ and $y = -3 - 2x$

Answer



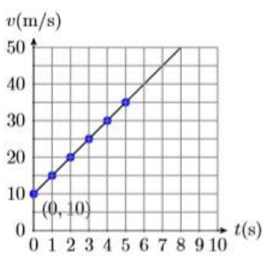
28) $y = 1 - \frac{1}{2}x$ and $y = -4 - \frac{4}{5}x$

3.3: Rates and Slope

1) An object's initial velocity at time $t = 0$ seconds is $v = 10$ meters per second. It then begins to pick up speed (accelerate) at a rate of 5 meters per second per second ($5\mathrm{m/s/s}$ or $5\mathrm{m/s}^2$).

a. Set up a Cartesian Coordinate System on a sheet of graph paper. Label and scale each axis. Include units with your labels.
b. Plot the point representing the initial velocity at time $t = 0$ seconds. Then plot a minimum of 5 additional points using the fact that the object is accelerating at a rate of 5 meters per second per second.
c. Sketch the line representing the object's velocity versus time.
d. Calculate the slope of the line.

Answer



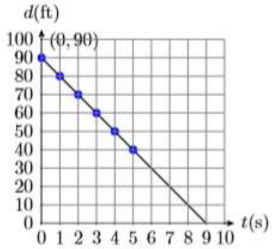
2) An object's initial velocity at time $t = 0$ seconds is $v = 40$ meters per second. It then begins to lose speed at a rate of 5 meters per second per second ($5\mathrm{m/s/s}$ or $5\mathrm{m/s}^2$).

a. Set up a Cartesian Coordinate System on a sheet of graph paper. Label and scale each axis. Include units with your labels.
b. Plot the point representing the initial velocity at time $t = 0$ seconds. Then plot a minimum of 5 additional points using the fact that the object is losing speed at a rate of 5 meters per second per second.
c. Sketch the line representing the object's velocity versus time.
d. Calculate the slope of the line.

3) David first sees his brother when the distance separating them is 90 feet. He begins to run toward his brother, decreasing the distance d between him and his brother at a constant rate of 10 feet per second (10 ft/s).

a. Set up a Cartesian Coordinate System on a sheet of graph paper. Label and scale each axis. Include units with your labels.
b. Plot the point representing David's initial distance from his brother at time t = 0 seconds. Then plot a minimum of 5 additional points using the fact that David's distance from his brother is decreasing at a constant rate of 10 feet per second ($10\mathrm{ft/s}$).
c. Sketch the line representing David's distance from his brother versus time.
d. Find the slope of the line.

Answer:



4) David initially stands 20 feet from his brother when he sees his girl friend Mary in the distance. He begins to run away from his brother and towards Mary, increasing the distance d between him and his brother at a constant rate of 10 feet per second ($10 \mathrm{ft}/\mathrm{s}$).

a. Set up a Cartesian Coordinate System on a sheet of graph paper. Label and scale each axis. Include units with your labels.
b. Plot the point representing David's initial distance from his brother at time $t = 0$ seconds. Then plot a minimum of 5 additional points using the fact that David's distance from his brother is increasing at a constant rate of 10 feet per second ($10 \mathrm{ft}/\mathrm{s}$).
c. Sketch the line representing David's distance from his brother versus time.
d. Find the slope of the line.

In Exercises 5-14, calculate the slope of the line passing through the points P and Q. Be sure to reduce your answer to lowest terms.

5) $P(9,0), Q(-9,15)$

Answer

$$-\frac{5}{6}$$

6) $P(19,-17), Q(-13,19)$

7) $P(0,11), Q(16,-11)$

Answer

$$-\frac{11}{8}$$

8) $P(-10,-8), Q(11,19)$

9) $P(11,1), Q(-1,-1)$

Answer

$$\frac{1}{6}$$

10) $P(16,-15), Q(-11,12)$

11) $P(-18,8), Q(3,-10)$

Answer

$$-\frac{6}{7}$$

12) $P(9,9), Q(-6,3)$

13) $P(-18,10), Q(-9,7)$

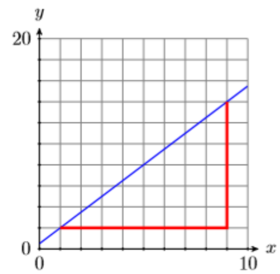
Answer

$$-\frac{1}{3}$$

14) $P(-7,20), Q(7,8)$

In Exercises 15-18, use the right triangle provided to help determine the slope of the line. Be sure to pay good attention to the scale provided on each axis when counting boxes to determine the change in y and the change in x.

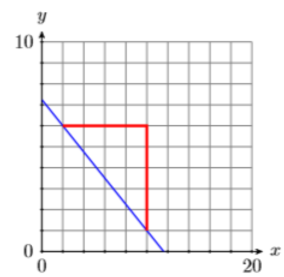
15)



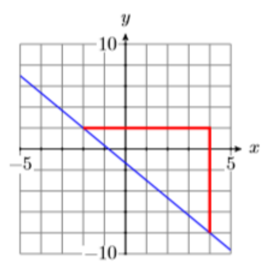
Answer

$$\frac{3}{2}$$

16)



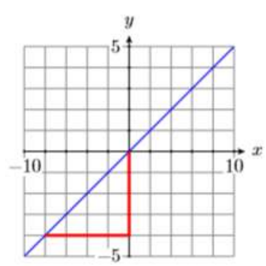
17)



Answer

$$-\frac{5}{3}$$

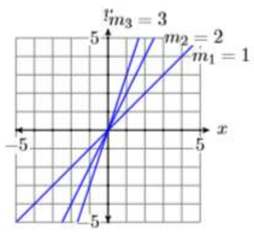
18)



19) On one coordinate system, sketch each of the lines that pass through the following pairs of points. Label each line with its slope, then explain the relationship between the slope found and the steepness of the line drawn.

a. $(0,0)$ and $(1,1)$
b. $(0,0)$ and $(1,2)$
c. $(0,0)$ and $(1,3)$

Answer



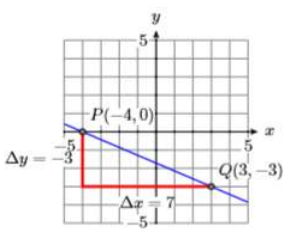
20) On one coordinate system, sketch each of the lines that pass through the following pairs of points. Label each line with its slope, then explain the relationship between the slope found and the steepness of the line drawn.

a. $(0,0)$ and $(1,-1)$
b. $(0,0)$ and $(1,-2)$
c. $(0,0)$ and $(1,-3)$

In Exercises 21-30, setup a coordinate system on graph paper, then sketch the line passing through the point P with the slope m.

21) $P(-4,0), m=-3/7$

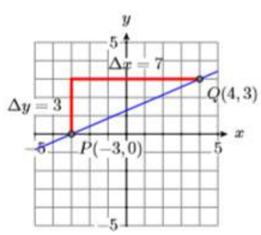
Answer



22) $P(-3,0), m=-3/7$

23) $P(-3,0), m=3/7$

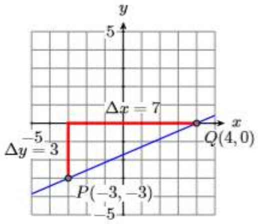
Answer



24) $P(-3,0), m=3/4$

25) $P(-3,-3), m=3/7$

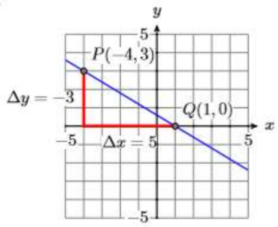
Answer



26) $P(-2,3), m=-3/5$

27) $P(-4,3), m=-3/5$

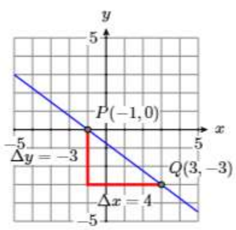
Answer



28) $P(-1,-3), m=3/4$

29) $P(-1,0), m=-3/4$

Answer



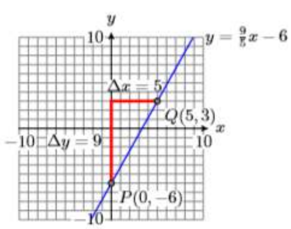
30) $P(-3,3), m=-3/4$

3.4: Slope-Intercept Form of a Line

In Exercises 1-6, setup a coordinate system on graph paper, then sketch the line having the given equation. Label the line with its equation.

1) $y=\dfrac{9}{5}x-6$

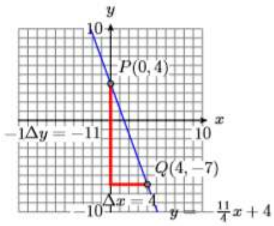
Answer



2) $y=\dfrac{8}{7}x-1$

3) $y=-\frac{11}{4}x+4$

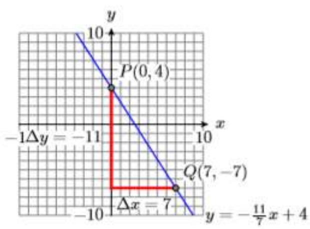
Answer



4) $y=\frac{5}{4}x$

5) $y=-\frac{11}{7}x+4$

Answer

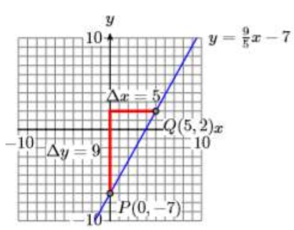


6) $y=-\frac{7}{5}x+7$

In Exercises 7-12, sketch the line with given y-intercept slope. Label the line with the slope-intercept form of its equation.

7) $(0,-7), 9/5$

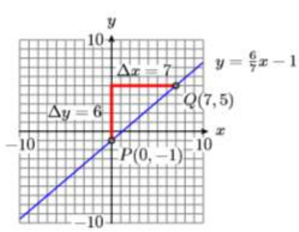
Answer



8) $(0,7), -4/5$

9) $(0,-1), 6/7$

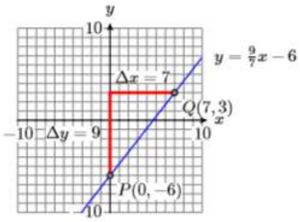
Answer



10) $(0,1), -7/5$

11) $(0,-6), 9/7$

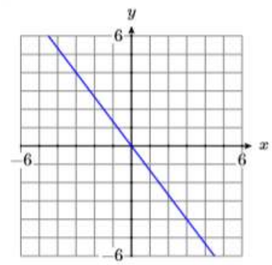
Answer



12) $(0,-5), 7/5$

In Exercises 13-20, determine the equation of the given line in slope-intercept form.

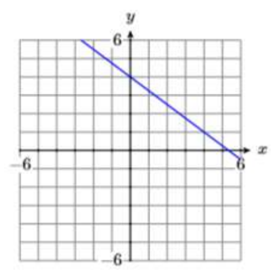
13)



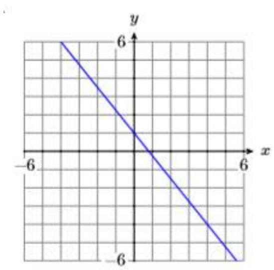
Answer

$$y=-\frac{4}{3}x$$

14)



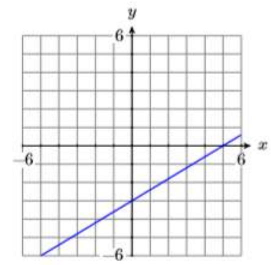
15)



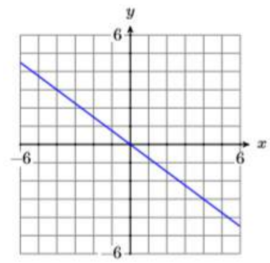
Answer

$$y=-\frac{5}{4}x+1$$

16)



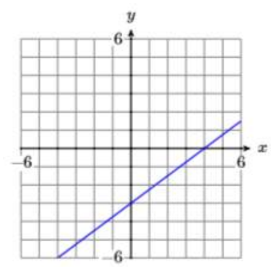
17)



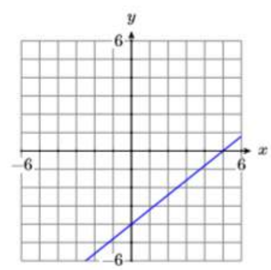
Answer

$$y=-\frac{3}{4}x$$

18)



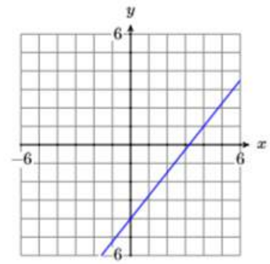
19)



Answer

$$y=\frac{4}{5}x-4$$

20)



21) Assume that the relationship between an object's velocity and its time is linear. At $t = 0$ seconds, the object's initial velocity is $20\mathrm{m/s}$. It then begins to speed up at a constant rate of 5 meters per second per second.

a. Set up a coordinate system, placing the time t on the horizontal axis and the velocity v on the vertical axis. Label and scale each axis. Include units in your labels.
b. Use the initial velocity and the rate at which the object's velocity is increasing to draw the line representing the object's velocity at time t. Use the slope-intercept form to determine the equation of the line.
c. Replace x and y in the equation found in part (b) with t and v, respectively. Use the result to determine the velocity of the object after 14 seconds.

Answer

c) $90\mathrm{m/s}$

22) Assume that the relationship between an object's velocity and its time is linear. At $t = 0$ seconds, the object's initial velocity is $80\mathrm{m/s}$. It then begins to lose speed at a constant rate of 4 meters per second per second.

a. Set up a coordinate system, placing the time t on the horizontal axis and the velocity v on the vertical axis. Label and scale each axis. Include units in your labels.
b. Use the initial velocity and the rate at which the object's velocity is increasing to draw the line representing the object's velocity at time t. Use the slope-intercept form to determine the equation of the line.
c. Replace x and y in the equation found in part (b) with t and v, respectively. Use the result to determine the velocity of the object after 13 seconds.

23) A water tank initially (at time $t = 0$ min) contains 100 gallons of water. A pipe is opened and water pours into the tank at a constant rate of 25 gallons per minute. Assume that the relation between the volume V of water in the tank and time t is linear.

a. Set up a coordinate system, placing the time t on the horizontal axis and the volume of water V on the vertical axis. Label and scale each axis. Include units in your labels.
b. Use the initial volume of water in the tank and the rate at which the volume of water is increasing to draw the line representing the volume V of water in the tank at time t. Use the slope-intercept form to determine the equation of the line.
c. Replace x and y in the equation found in part (b) with t and V, respectively. Use the result to predict how much time must pass until the volume of water in the tank reaches 400 gallons.

Answer

c) 12 min

24) A water tank initially (at time $t = 0$ min) contains 800 gallons of water. A spigot is opened at the bottom of the tank and water pours out at a constant rate of 40 gallons per minute. Assume that the relation between the volume V of water in the tank and time t is linear.

a. Set up a coordinate system, placing the time t on the horizontal axis and the volume of water V on the vertical axis. Label and scale each axis. Include units in your labels.
b. Use the initial volume of water in the tank and the rate at which the volume of water is decreasing to draw the line representing the volume V of water in the tank at time t. Use the slope-intercept form to determine the equation of the line.

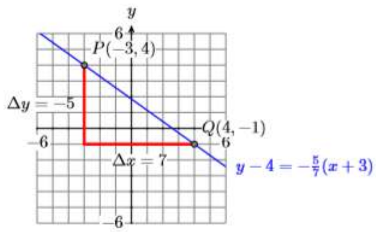
c. Replace x and y in the equation found in part (b) with t and V, respectively. Use the result to predict how much time must pass until the water tank is empty.

3.5: Point-Slope Form of a Line

In Exercises 1-6, set up a coordinate system on a sheet of graph paper, then sketch the line through the given point with the given slope. Label the line with its equation in point-slope form.

1) $m=-5/7, P(-3,4)$

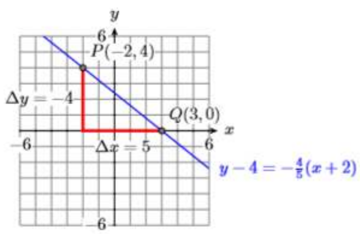
Answer



2) $m=3/4, P(-2,-4)$

3) $m=-4/5, P(-2,4)$

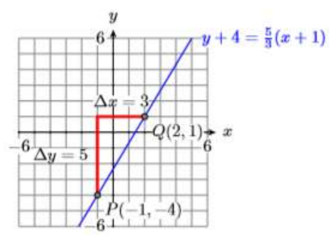
Answer



4) $m=5/6, P(-3,-1)$

5) $m=5/3, P(-1,-4)$

Answer

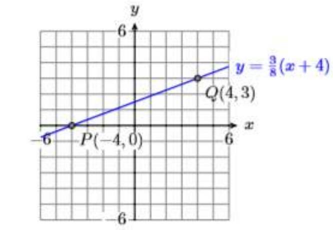


6) $m=-3/8, P(-4,0)$

In Exercises 7-12, set up a coordinate system on a sheet of graph paper, then sketch the line through the given points. Label the line with the point-slope form of its equation.

7) $P(-4,0)$ and $Q(4,3)$

Answer

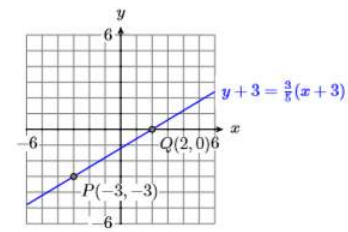


Alternate answer: $y-3=\frac{3}{8}(x-4)$

8) $P(-2,4)$ and $Q(2,-1)$

9) $P(-3,-3)$ and $Q(2,0)$

Answer

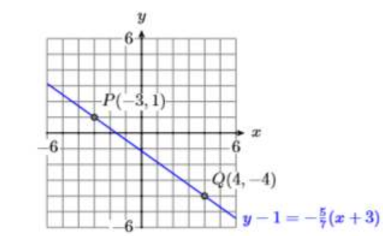


Alternate answer: $y=\frac{3}{5}(x-2)$

10) $P(-3,4)$ and $Q(2,0)$

11) $P(-3,1)$ and $Q(4,-4)$

Answer



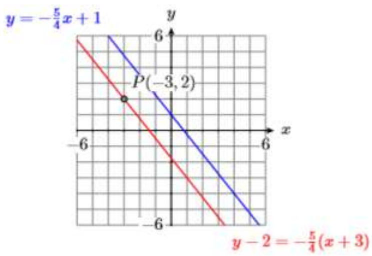
Alternate answer: $y+4=-\frac{5}{7}(x-4)$

12) $P(-1,0)$ and $Q(4,3)$

In Exercises 13-18, on a sheet of graph paper, sketch the given line. Plot the point P and draw a line through P that is parallel to the first line. Label this second line with its equation in point-slope form.

13) $y=-\dfrac{5}{4}x+1, P(-3,2)$

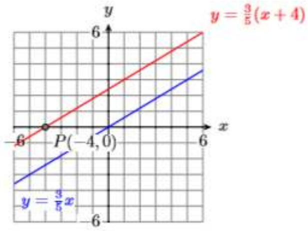
Answer



14) $y=-\dfrac{3}{5}x, P(-4,0)$

15) $y=\frac{3}{5}x, P(-4,0)$

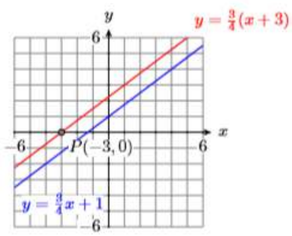
Answer



16) $y=\frac{5}{3}x-1, P(-2,-2)$

17) $y=\frac{3}{4}x+1, P(-3,0)$

Answer

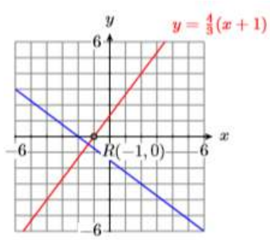


18) $y=\frac{5}{4}x-4, P(-2,-2)$

In Exercises 19-24, on a sheet of graph paper, sketch the line passing through the points P and Q. Plot the point R and draw a line through R that is perpendicular to the line passing through the points P and Q. Label this second line with its equation in point-slope form.

19) $P(-2,0), Q(2,-3)$,and $R(-1,0)$

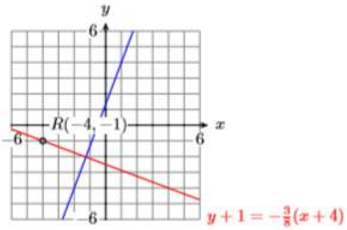
Answer



20) $P(-1,3), Q(2,-2)$,and $R(-1,0)$

21) $P(-2,-4), Q(1,4)$,and $R(-4,-1)$

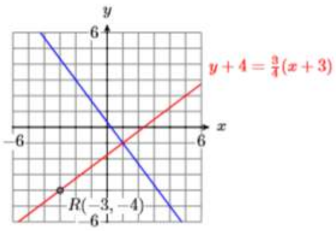
Answer



22) $P(-4,-4), Q(-1,3)$, and $R(-4,2)$

23) $P(-2,3), Q(1,-1)$, and $R(-3,-4)$

Answer



24) $P(-4,4), Q(1,-4)$, and $R(-4,-3)$

25) Assume that the relationship between an object's velocity and its time is linear. At 3 seconds, the object's velocity is $50 \mathrm{ft}/\mathrm{s}$. At 14 seconds, the object's velocity is $30 \mathrm{ft}/\mathrm{s}$.

a. Set up a coordinate system, placing the time t on the horizontal axis and the velocity v on the vertical axis. Label and scale each axis. Include units in your labels.
b. Plot the points determined by the given data and draw a line through them. Use the point-slope form of the line to determine the equation of the line.
c. Replace x and y in the equation found in part (b) with t and v, respectively, then solve the resulting equation for v.
d. Use the result of part (c) to determine the velocity of the object after 6 seconds.

Answer

$44.5 \mathrm{ft}/\mathrm{s}$

26) Water freezes at approximately $32^{\circ}\mathrm{F}$ and $273^{\circ}\mathrm{K}$, where F represents the temperature measured on the Fahrenheit scale and K represents the temperature measured on the Kelvin scale. Water boils at approximately $212^{\circ}\mathrm{F}$ and $373^{\circ}\mathrm{K}$. Assume that the relation between the Fahrenheit and Kelvin temperatures is linear.

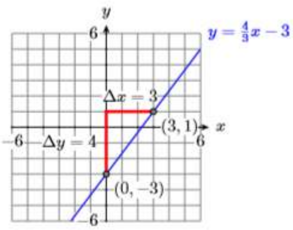
a. Set up a coordinate system, placing the Kelvin temperature K on the horizontal axis and the Fahrenheit temperature F on the vertical axis. Label and scale each axis. Include units in your labels.
b. Plot the points determined by the given data and draw a line through them. Use the point-slope form of the line to determine the equation of the line.
c. Replace x and y in the equation found in part (b) with K and F, respectively, then solve the resulting equation for F.
d. Use the result of part (c) to determine the Fahrenheit temperature of the object if the Kelvin temperature is $212^{\circ}\mathrm{K}$.

3.6: Standard Form of a Line

In Exercises 1-6, place the given standard form into slope-intercept form and sketch its graph. Label the graph with its slope-intercept form.

1) $4x-3y=9$

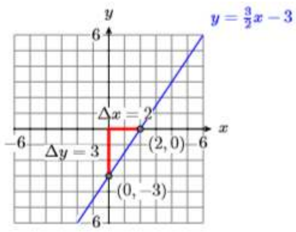
Answer



2) $2x - 3y = 3$

3) $3x - 2y = 6$

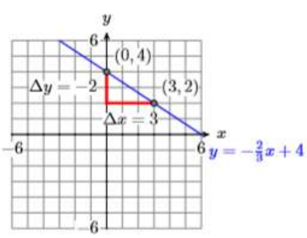
Answer



4) $5x - 3y = 3$

5) $2x + 3y = 12$

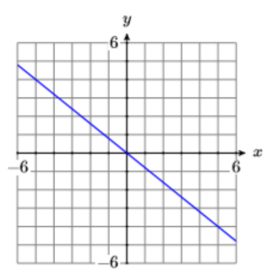
Answer



6) $3x + 4y = 8$

In Exercises 7-10, determine an equation of the given line in standard form.

7)

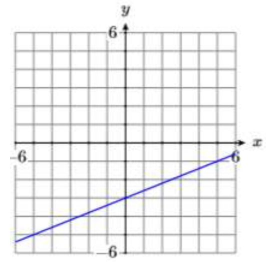


Answer

$4x + 5y = 0$

8)

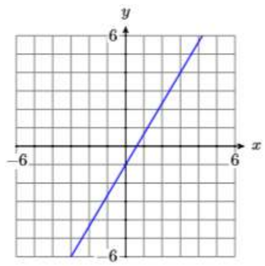
9)



Answer

$2x-5y=15$

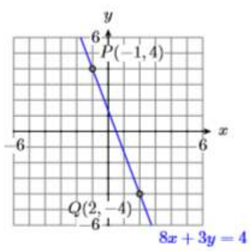
10)



In Exercises 11-16, sketch the line passing through the points P and Q. Label the line with its equation in standard form.

11) $P(-1,4)$ and $Q(2,-4)$

Answer



12) $P(-1,4)$ and $Q(3,1)$

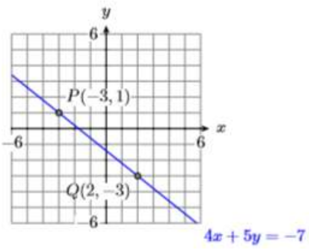
13) $P(-1,-1)$ and $Q(3,4)$

Answer

14) $P(2,-1)$ and $Q(4,4)$

15) $P(-3,1)$ and $Q(2,-3)$

Answer

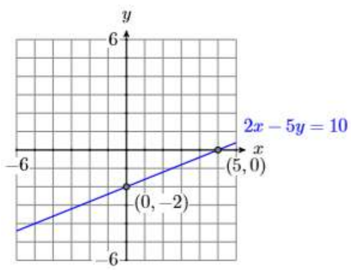


16) $P(-4,3)$ and $Q(0,0)$

In Exercises 17-22, plot the x- and y-intercepts of the line having the given equation, then draw the line through the intercepts and label it with its equation.

17) $2x-5y=10$

Answer



18) $2x+3y=-6$

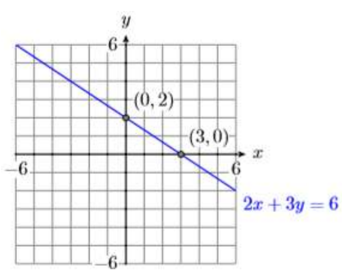
19) $3x-2y=6$

Answer

20) $3x-4y=12$

21) $2x+3y=6$

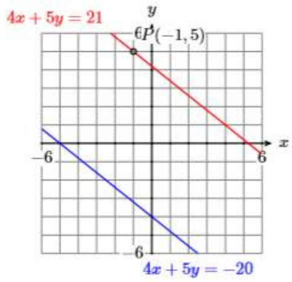
Answer



22) $2x-3y=-6$

23) Find an equation of the line (in standard form) that passes through the point $P(-1,5)$ that is parallel to the line $4x+5y=-20$.

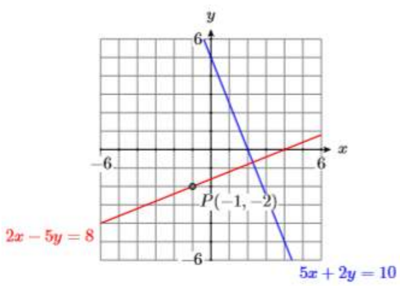
Answer



24) Find an equation of the line (in standard form) that passes through the point $P(-3,2)$ that is parallel to the line $3x+5y=-15$.

25) Find an equation of the line (in standard form) that passes through the point $P(-1,-2)$ that is perpendicular to the line $5x+2y=10$.

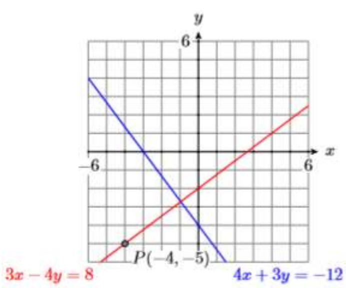
Answer



26) Find an equation of the line (in standard form) that passes through the point $P(-1,-2)$ that is perpendicular to the line $2x+5y=-10$.

27) Find an equation of the line (in standard form) that passes through the point $P(-4,-5)$ that is perpendicular to the line $4x+3y=-12$.

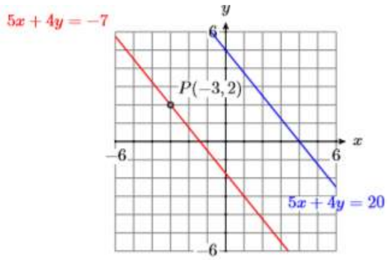
Answer



28) Find an equation of the line (in standard form) that passes through the point $P(-2,-3)$ that is perpendicular to the line $3x+2y=6$.

29) Find an equation of the line (in standard form) that passes through the point $P(-3,2)$ that is parallel to the line $5x+4y=20$.

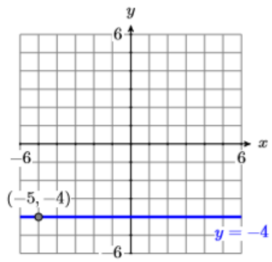
Answer



30) Find an equation of the line (in standard form) that passes through the point $P(-1, 3)$ that is parallel to the line $3x + 5y = -15$.

31) Sketch the equation of the horizontal line passing through the point $P(-5, -4)$ and label it with its equation.

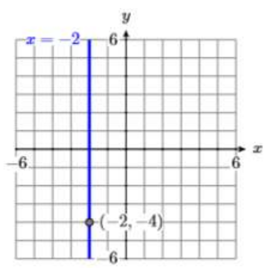
Answer



32) Sketch the equation of the vertical line passing through the point $P(-4, 4)$ and label it with its equation.

33) Sketch the equation of the vertical line passing through the point $P(-2, -4)$ and label it with its equation.

Answer



34) Sketch the equation of the vertical line passing through the point $P(1, 3)$ and label it with its equation.